## B.E. (Computer Engineering / Information Technology) Third Semester (CBS)

## Applied Mathematics-III

P. Pages: 4

Time: Three Hours



AHK/KW/19/2091/2096

Max. Marks: 80

Notes: 1. All questions carry marks as indicated.

- 2. Solve Question 1 OR Questions No. 2.
- 3. Solve Question 3 OR Questions No. 4.
- 4. Solve Question 5 OR Questions No. 6.
- 5. Solve Question 7 OR Questions No. 8.
- 6. Solve Question 9 OR Questions No. 10.
- 7. Solve Question 11 OR Questions No. 12.
- 8. Assume suitable data whenever necessary.
- Use of non programmable calculator is permitted.

1. a) If 
$$L\{f(t)\}=\overline{f}(s)$$
, then show that

 $L\left\{\int_{0}^{t} f(u) du\right\} = \frac{\overline{f}(s)}{s}. \text{ Hence find the value of } L\left\{\int_{0}^{t} \sin u du\right\}$ 

b) Find 
$$L^{-1}\left\{\frac{1}{s(s^2+9)}\right\}$$
 by convolution theorem.

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OR

2. a) Express 
$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$
, in terms of unit step function and hence find its Laplace

transform

b) Solve:

3.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t, \ y(0) = 0, \ y'(0) = 1.$$

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Find Fourier transform of  $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ hence find  $\int_{0}^{\infty} \frac{\sin x}{x} dx$ .

OR

4. Solve the integral equation 
$$\int_{0}^{\infty} f(x) \cos \lambda x \, dx = e^{-\lambda}, \lambda > 0.$$

5. a) If  $z\{f(n)\}=F(z)$  then prove that

$$z\left\{\frac{f(n)}{n+k}\right\} = z^{k} \int_{z}^{\infty} \frac{F(z)}{z^{k+1}} dz.$$

b) Find z-transform of  $sinn \theta$  and  $cos n\theta$ .

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OR

- 6. a) Find the inverse z-transform of  $\left\{\frac{z^2}{(z-1)(z-3)}\right\}$  using convolution theorem.
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b) Solve the difference equation.

$$y_{n+2} + y_n = 2$$
,  $y_0 = 0$   
 $y_1 = 0$ 

7. a) Using the concept of matrix, show that the vectors  $X_1 = [2,3,1,-1], X_2 = [2,3,1,-2], X_3 = [4,6,2,-3].$  are linearly dependent. Find the relation bet<sup>n</sup> them.

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b) Find eigen value, eigen vector and model matrix for

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

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Use Sylvester's theorem to verify 
$$\log_e e^A = A$$
, where  $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ .

OR

8. a) Verify Cayley - Hamilton theorem & hence find  $A^{-1}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}.$$

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b) Solve 
$$\frac{d^2y}{dx^2} + 4y = 0$$
 given that  $y(0) = 8$ ,  $y'(0) = 0$  by matrix method.

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c) Find largest eigen value & corresponding eigen vector for matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

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- Three Machines A, B, C produce respectively 60%, 30% and 10% of the total number of items in a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and found defective. Find the probability that item was produced by machine B.

b) Can the function

$$f(x) = \begin{cases} C(1-x^2) & , & 0 \le x \le 1 \\ 0 & , & \text{otherwise} \end{cases}$$

be a distribution function? Explain.

Let X and Y be continuous random variables having joint density function 10. a)

$$(x,y) = \begin{cases} C(x^2 + y^2), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Determine

- i) Constant.
- $P(X < \frac{1}{2}, Y > \frac{1}{2}).$ ii)
- iii) Marginal density functions of x & y.
- Let X and Y be two random variables with mass function b)

Let X and Y be two random variables with mass function 
$$f\left(x,y\right) = \begin{cases} \frac{x+2y}{27}, & x=0,1,2, \ y=0,1,2\\ 0, & \text{otherwise} \end{cases}$$

- Find marginal probability function of x & y.
- Conditional probability function y given x and x given y.
- Find the moment generating function of random variable  $X = \begin{cases} 1/2 & \text{prob } 1/2 \\ -1/2 & \text{prob } 1/2 \end{cases}$

Hence find four moments about origin.

b) A density function of random variable X is

$$f(x) = \begin{cases} 2e^{-2x} & x \ge 0\\ 0 & \text{otherwise} \end{cases}$$

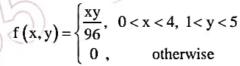
Find

ii) Var (X)

 $E[(X-1)^2].$ 

OR

12. a) The joint density function of two random variables x and y is given by



Find

i) E (x)

ii) E (y)

iii) E(xy)

iv) E(2x + 3y)

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b) State the postulates of Poisson process and prove that a Poisson process follow a Poisson distribution.