B.E. (Computer Engineering / Information Technology) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 4

Time : Three Hours

NRT/KS/19/3327/3332

Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $L[f(t)] = \bar{f}(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} \bar{f}(s)\,ds$, hence find. $L\left\{\frac{\sin^2 t}{t}\right\}$. **7**

b) Find $L^{-1}\left[\frac{1}{(S+1)(S^2+1)}\right]$ by using convolution theorem. **7**

OR

2. a) Express **7**

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$$

In terms of unit step function and hence find its Laplace transform.

b) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$, $y(0) = 0$, $y'(0) = 1$, by L. T. method. **7**

3. Find the Fourier Sine Transform of **7**

$$f(x) = \frac{e^{-ax}}{x}$$

OR

4. Solve the integral equation. **7**

$$\int_0^{\infty} f(x)\sin tx\,dx = \begin{cases} 1 & 0 \le t < 1 \\ 2 & 1 \le t < 2 \\ 3 & t > 2 \end{cases}$$

5. a) If $Z[f(n)] = F(z)$, then prove that. 7

$$z[f(n+k)] = z^k \left[F(z) - \sum_{i=0}^{k-1} f(i) z^{-i} \right].$$

b) Find $Z^{-1}\left\{\frac{z^2}{(z-1)(z-3)}\right\}$ using convolution theorem. 7

OR

6. a) Find $Z[\sinh n\theta]$ and $Z[\cosh n\theta]$. 7

b) Solve: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = 0 = y_1$. 7

7. a) Using the concept of matrix show that the vectors are linearly dependent. 6

$x_1 = [1, 0, 2, 1]$, $x_2 = [3, 1, 2, 1]$

$x_3 = [4, 6, 2, -4]$, $x_4 = [-6, 0, -3, -4]$

b) Find the modal matrix for the matrix. 6

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

c) Use Sylvester's theorem to prove that $\sin^2 A + \cos^2 A = I$, where $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. 6

OR

8. a) Find the matrix represented by 6

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

Where

$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ by Cayley-Hamilton theorem.

b) Solve by matrix method: 6

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 0; \; y(0) = 2, \; y'(0) = 2.$$

c) Determine the largest eigen value and corresponding eigen vector at the matrix. 6

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

9. a) Three machines A, B, C produce respectively 60%, 30% and 10% of the total no. of items in a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found be defective. Find the probability that the item was produced by machine C. **7**

b) Let x be the random variable giving the number of heads in three tosses of a fair coin. Find: **7**

i) Probability function f (x),

ii) Distribution function F(x),

iii) Also draw the graphs of f (x) and F (x).

OR

10. a) A random variable x has density function. **7**

$$f(x) = \begin{cases} cx^2 & 1 \le x \le 2 \\ cx & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Find:

i) C,

ii) $P(x < 2)$,

iii) Distribution fun. F (x)

b) Find the conditional density function of **7**

i) X given Y,

ii) Y given X for the distribution function

$$f(x,y) = \begin{cases} \frac{3}{2}\left(x^2 + y^2\right) & 0 \le x \le 1 \\ & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

11. a) A density function of random variable **7**

$$x \text{ is } f(x) = \begin{cases} 2e^{-2x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Find

i) E(x)

ii) Var (x)

iii) σ_X

iv) $E\left[(x-1)^2\right]$.

b) Find the moment generating function and first four moments about origin of random variable. **6**

$$x = \begin{cases} \frac{1}{2}, & \text{Prob.}\ \frac{1}{2} \\ -\frac{1}{2}; & \text{Prob.}\ \frac{1}{2} \end{cases}$$

OR

12. a) The joint density function of two random variables X and Y is. 7

$$f(x, y) = \begin{cases} x + y, & 0 \le x \le 1; \quad 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find:

i) Conditional expectation of X given Y

ii) Conditional variance of X given Y.

b) State the Postulates of Poisson Process and prove that a Poisson Process follows a Poisson distribution. 6

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