Unit-2

* Method used to simplify Boolean function

(1) Algebric method
(2) Karnaugh-map technique (K-map)
(3) Quine-McCluskay method
(4) Variable Entered mapping (VEM) tech.

* Two variable k-map

B \ A	0	1
0	AB 00	AB 10
1	AB 01	AB 11

A \ B	0	1
0	AB 00	AB 01
1	AB 10	AB 11

B \ A	0	1
0	0	2
1	1	3

A \ B	0	1
0	0	1
1	2	3

* Three variable k-map.

C \ AB	00	01	11	10
0	ABC 000	010	110	100
1	001	011	111	101

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

* 4- variable K- map

CD \ AB	00	01	11	10
00	ABCD 0000	0100	1100	1000
01	0001	0101	1101	1001
11	0011	0111	1111	1011
10	0010	0110	1110	1010

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

* Standard Representation of logic function

i) Sum of product (SOP)

e.g. $Y = AB + \bar{A}C + BC$

ii) Product of sum (POS)

e.g. $Y = (A+B)(\bar{A}+C)(B+C)$

SOP	POS
1) minterm	1) maxterm
2) denotated by 1	2) denoted by 0.
3) $Y = \Sigma ( )$	3) $Y = \Pi ( )$
4) $0 \rightarrow \bar{A}$, $1 \rightarrow A$	4) $0 \rightarrow A$, $1 \rightarrow \bar{A}$

A variable in complimentated and of uncomplimentated is known as literal.
ex. A, $\bar{A}$, B

If each term is SOP and POS form contains all the literals then these are known as canomical SOP and POS respectively

Each Indivial term in canomical SOP form is called as minterm and canomical POS form as maxterm.

★ Convert following function in canomical form.

Que: $Y = AB + A\bar{C} + BC \cdots$

$(\because A + \bar{A} = 1)$

Sol ⇒ $\therefore Y = AB(C + \bar{C}) + A\bar{C}(B + \bar{B}) + BC(A + \bar{A})$

$= ABC + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC + \bar{A}BC$

$$\boxed{Y = ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC}$$

canomical form of expression.

$Y = \underset{111}{ABC} + \underset{110}{AB\bar{C}} + \underset{100}{A\bar{B}\bar{C}} + \underset{011}{\bar{A}BC}$

$7 \quad 6 \quad 4 \quad 3$

$$Y = \Sigma m(3,4,6,7)$$ ← standard representation

Que: $Y = XY + YZ$

$\Rightarrow \therefore f = xy(z+\bar{z}) + yz(x+\bar{x})$

$= xyz + xy\bar{z} + xyz + \bar{x}yz$

$$f = xyz + xy\bar{z} + \bar{x}yz$$

Que: $f = AB + B\bar{D} + AD$

Soln $\therefore f = AB(D+\bar{D}) + B\bar{D}(A+\bar{A}) + AD(B+\bar{B})$

$= ABD + AB\bar{D} + AB\bar{D} + \bar{A}B\bar{D} + ABD + A\bar{B}D$

$$f = ABD + AB\bar{D} + \bar{A}B\bar{D} + A\bar{B}D$$

Que $f = (A+B)(A+C)(B+\bar{C})$

Soln $\Rightarrow f = (A+B)(A+C)(B+\bar{C})$

$= (A+B+C\bar{C})(A+B\bar{B}+C)(A\bar{A}+B+\bar{C})$

$= (A+B+C)(A+B+\bar{C})(A+B+C)(A+\bar{B}+C)$
$(A+B+\bar{C})(\bar{A}+B+\bar{C})$.

$$f = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+\bar{C})$$

(Canomical form of representation)

$$f = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+\bar{C})$$

$$0\ 0\ 0 \quad 0\ 0\ 1 \quad 0\ 1\ 0 \quad 1\ 0\ 1$$

$$0 \quad 1 \quad 2 \quad 5$$

$$f = \prod M(0,1,2,5)$$ Stardard form of representation.

* Simplification logic function using K-map

1. Identify once which cannot be combine with any other once and encircle them this are essential prime implicants

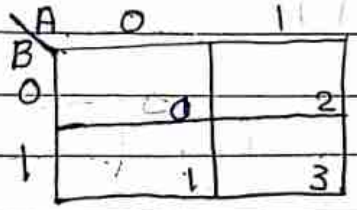
2. Identify once that can combing in grop of two an only one way encircle such group once

3. Identify once that can be combined with three other once to make a group of four adjustant ones in only one way encircle such group of once

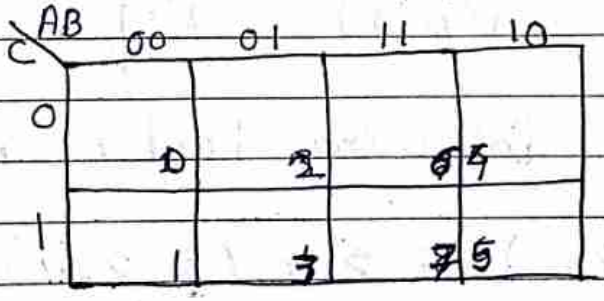
4. Identify the one that can be combined with seven other one to make a group eight adjustant one., in only one way encircle such group of one.

5. After identifying essential two, four, eight one, if there still remain some one which have not been encircle then these are to be combined with each other or other already encircle once.

*(i) 2- variable K-map



3- variable K-map.



* 4- Variable K- map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Que: Minimise the two variable logic function with K-map.

$f = \Sigma m(0,1)$

Sol ⇒

B \ A	0	1
0	1 (0)	2
1	1 (1)	3

$$f = \Sigma m(0,1) = \bar{A}$$

Que: Simply following boolan function using K- map

$f(A,B,C) = \Sigma m(0,2,7)$

C \ AB	00	01	11	10
0	1 (0)	1 (2)	6	4
1	1	3	1 (7)	5

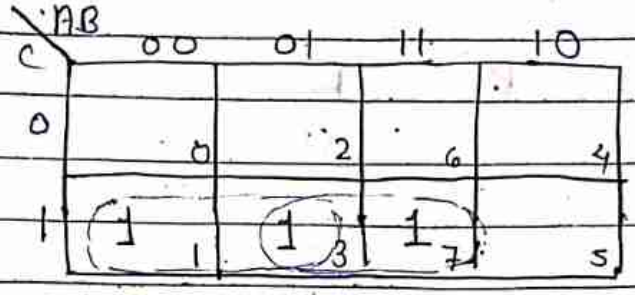
$$f(A,B,C) = \bar{A}\bar{C} + ABC$$

Que: Simply following logic equation using K-map

$f(A,B,C) + \bar{A}BC + \bar{A}\bar{B}C$
(1,1,1) (0,1,1) (0,0,1)

Sol:

$f = \Sigma m\,(1,3,7)$



$$f = \bar{A}\bar{C} + BC$$

Que: Minimize the four variable logic function using K-map $f(A,B,C,D) = \Sigma m(0,1,2,3,5,7,8,9,11,14)$

Ans: We required four variable K-map.

CD \ AB	00	01	11	10
00	1 (0)	4	12	1 (8)
01	1 (1)	1 (5)	13	1 (9)
11	1 (3)	1 (7)	15	1 (11)
10	1 (2)	6	1 (14)	10

$$f = \bar{A}D + \bar{B}D + \bar{A}\bar{B} + \bar{B}\bar{C} + ABC\bar{D}$$

Que: Determine minimize expression in SOP form for the truth table given below:

Inputs			output
B_0	B_1	B_0	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$f(A,B,C) = \Sigma M\ (0,1,4,5)$

C \ AB	00	01	11	10
0	1 (0)	(2)	(6)	1 (4)
1	1 (1)	(3)	(7)	1 (5)

$$\boxed{f(A,B,C) = \overline{B}}$$

Que minimize the following function k-map.

$$f = \underset{0\,0\,0}{\bar{A}\bar{B}\bar{C}} + \underset{0\,0\,1}{\bar{A}\bar{B}C} + \underset{1\,0\,0}{A\bar{B}\bar{C}} + \underset{4\;0\;1}{A\bar{B}C}$$

$\rightarrow f = \Sigma m(0, 1, 4, 5)$

$$\boxed{f = \bar{B}}$$

Que minimize the following function using k-map

$\rightarrow f(A, B, C) = \Pi M(2, 3, 6, 7)$

C \ AB	00	01	11	10
0	0	0 2	0 6	4
1	1	0 3	0 7	5

$$\boxed{f = \bar{B}}$$

$\rightarrow f(A, B, C) = \Pi M(2, 3, 4, 5)$

C \ AB	00	01	11	10
0	0	0 2	6	0 4
1	1	0 3	7	0 5

$$\boxed{f(A, B, C) = B\bar{B} \cdot (A + \bar{B})(\bar{A} + B)}$$

que: Minimize the following logic function in POS form

⇒

$f(x,y,z,w) = \prod M(4,6,10,12,13,15)$

zw \ xy	00	01	11	10
00	0	0 (4)	0 (12)	8
01	1	5	0 (13)	9
11	3	7	0 (15)	11
10	2	0 (6)	14	0 (10)

$f(x,y,z,w) = (x+\bar{y}+w)(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{w})(\bar{x}+y+\bar{z}+w)$

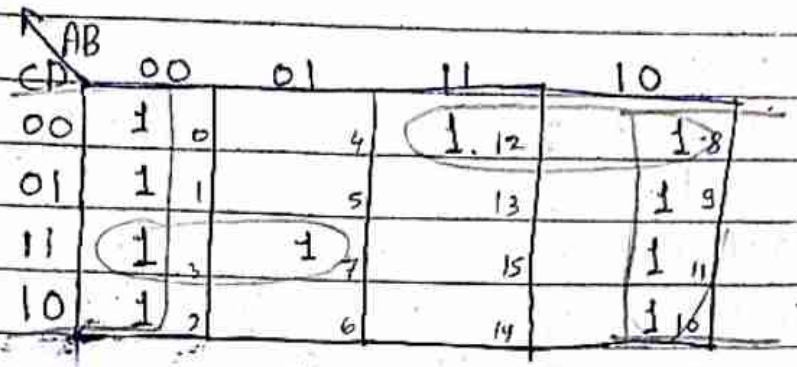
que: ⇒ $f(A,B,C,D) = \sum m(0,1,2,3,5,7,8,9,11,14)$ (SOP)

$= \prod M(4,6,10,12,13,15)$ (POS)

$f(A,B,C,D) = (A+\bar{B}+D)(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{D})(\bar{A}+B+\bar{C}+D)$

Que: Minimize the four variable logic function

$$f = \sum m(0,1,2,3,7,8,9,10,11,12)$$



$$f = \cancel{\bar{A}\bar{B}} + \bar{A}CD + A\bar{C}\bar{D} + \bar{B}$$

$$\boxed{f = \bar{B} + \bar{A}CD + A\bar{C}\bar{D}}$$

Que: Minimize the four variable logic function.

$$f(A,B,C,D) = AB\bar{C}D + \bar{A}BCD + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + A\bar{C} + A\bar{B}C + \bar{B}$$

$AB\bar{C}D = 13$
1101

$\bar{A}BCD = 7$
0111

$\bar{A}\bar{B}\bar{C}X$
0000 = 0
0001 = 1

$\bar{A}\bar{B}X\bar{D}$
0000 = 0
0010 = 2

$AX\bar{C}X =$
1000 = 8
1001 = 9
1100 = 12
1101 = 13

A	$\bar{B}$	C	X
1	0	1	0 = 10
1	0	1	1 = 11.

X	$\bar{B}$	X	X	
0	0	0	0	= 0.
0	0	0	1	= 1
0	0	1	0	= 2
0	0	1	1	= 3
1	0	0	0	= 8
1	0	0	1	= 9
1	0	1	0	= 10
1	0	1	1	= 11

0, 1, 2, 3, 4, 5, 6, 7,
8, 9, 10, 11, 12, 13, 14, 15

$f(A,B,C,D) = \sum m(0,1,2,3,7,8,9,10,11,12,13)$

CD \ AB	00	01	11	10
00	1 (0)	(4)	1 (12)	1 (8)
01	1 (1)	(5)	1 (13)	1 (9)
11	1 (3)	1 (7)	(15)	1 (11)
10	1 (2)	(6)	(14)	1 (10)

$f(A,B,C,D) = \bar{B} + A\bar{C} + \bar{A}CD$

* Don't Care Condition

There are certain combination of inputs which never occurs due to which output corresponds to these combination doesn't matter this condition is known as Don't care condition, it's also known Don't happen condition.

In is denoted by 'd' in standard form of function.
In K-map 'don't care' condition ('x')

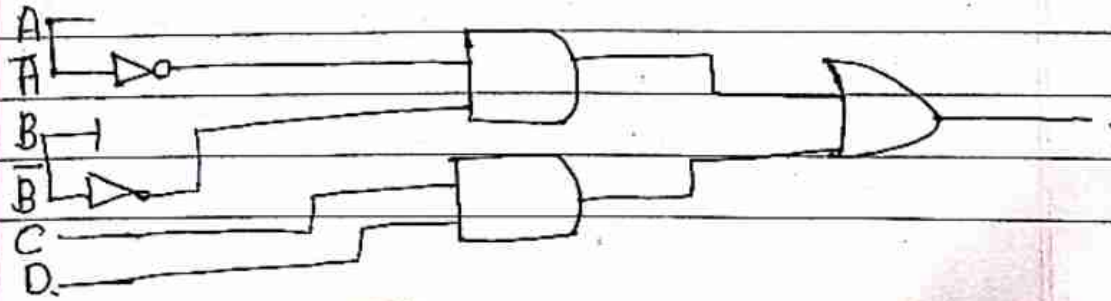
Que: minimize the following function using K-map.

$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$
and relise the function using gets

Sol ⇒

CD \ AB	00	01	11	10
00	X (0)	(4)	(12)	(8)
01	1 (1)	X (5)	(13)	(9)
11	1 (3)	1 (7)	1 (15)	1 (11)
10	X (2)	(6)	(14)	(10)

$f(A,B,C,D) = CD + \bar{A}\bar{B}$

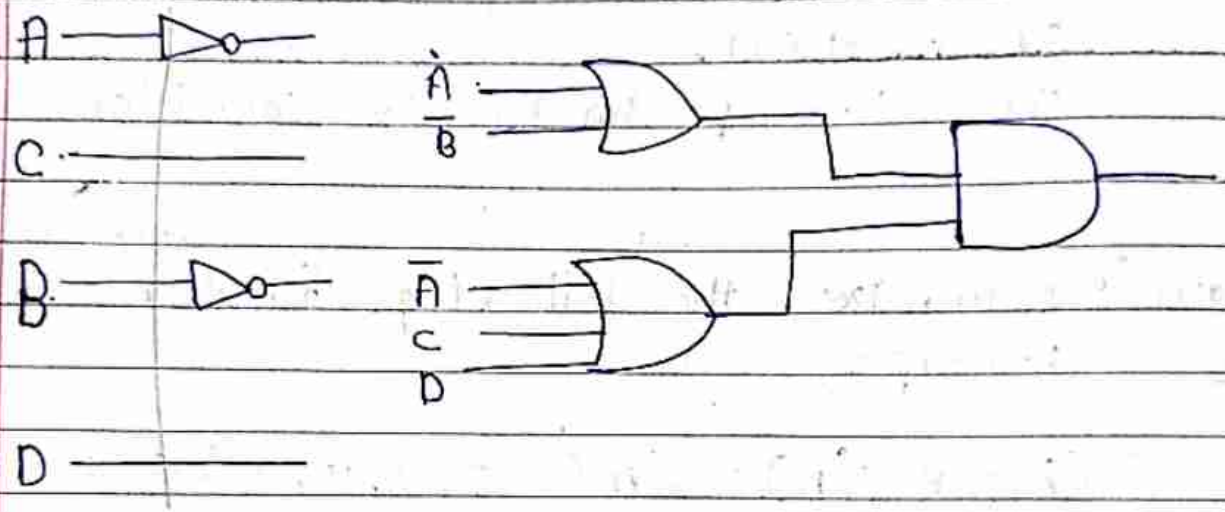


Que: $f(A,B,C,D) = \Pi M(4,5,6,7,8,12) \cdot d(1,2,3,9,11,14)$

Soln ⇒

CD \ AB	00	01	11	10
00	0	0 4	0 12	0 8
01	X 1	0 3	13	X 9
11	X 3	0 7	15	X 11
10	X 2	0 6	X 14	10

$f(A,B,C,D) = (A+\overline{B})(\overline{A}+C+D)$.



Que: Minimize the following logic function and realise using nand / nor gate.

univer

i) $f(A,B,C,D) = \Sigma m(1,3,5,8,9,11,15) + d(2,13)$ (sop)

ii) $f(X,Y,Z,W) = \Pi M(1,2,3,8,9,10,11,14) \cdot d(7,15)$ (pos)

Sol

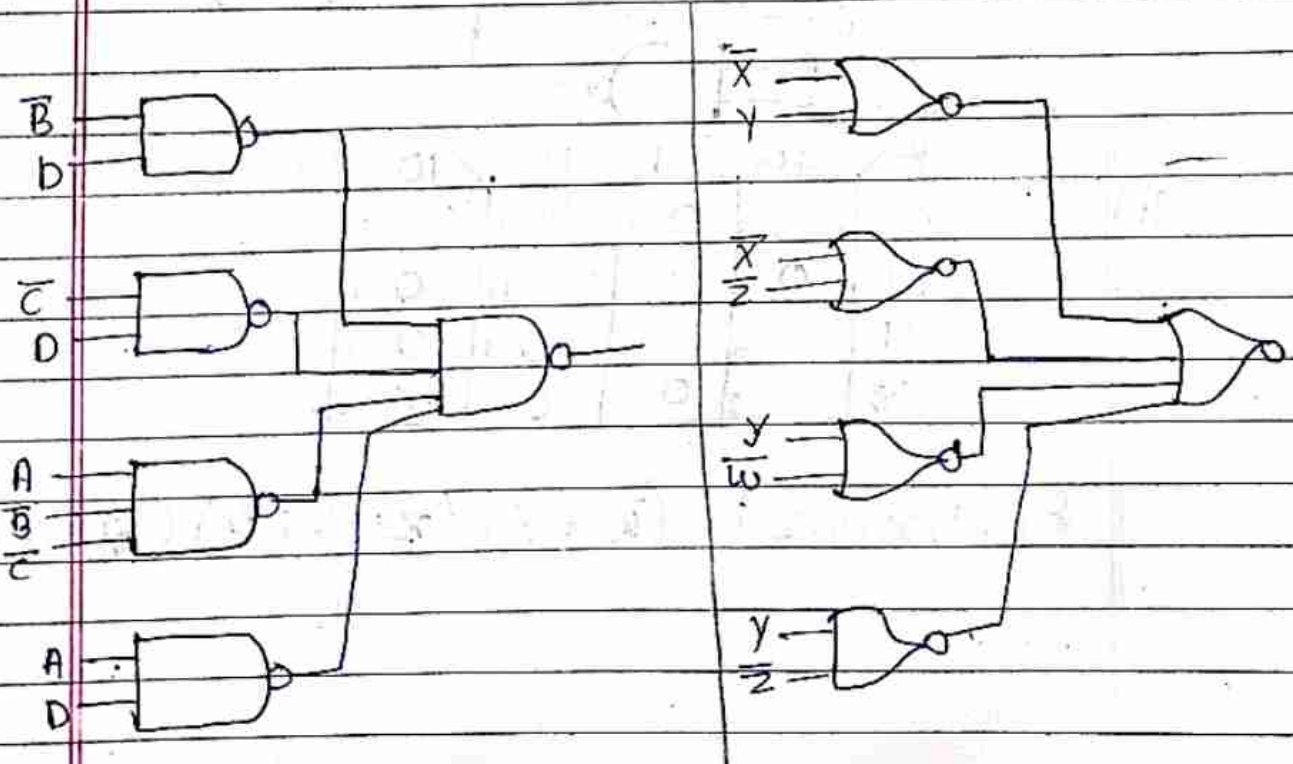
CD \ AB	00	01	11	10
00	0	4	12	1 (8)
01	1 (1)	1 (5)	X (13)	1 (9)
11	1 (3)	7	1 (15)	1 (11)
10	X (2)	6	14	10

$f(A,B,C,D) = \bar{B}D + \bar{A}\bar{C}D + A\bar{B}\bar{C} + AD$

$f(X,Y,Z,W)$

ZW \ XY	00	01	11	10
00	0	4	12	0 8
01	0 1	5	13	0 9
11	0 3	X 7	X 15	0 11
10	0 2	6	0 14	0 10

$f(X,Y,Z,W) = (\bar{X}+Y)(\bar{X}+\bar{Z})(Y+\bar{W})(Y+\bar{Z})$



Que: Simplify the given function using K-map and implement using NAND / NOR gate.

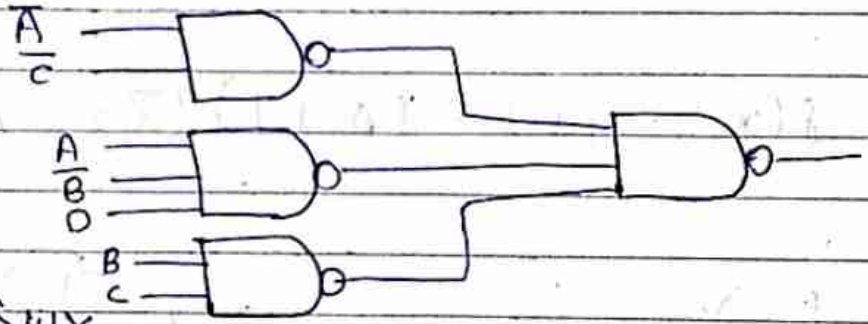
Winter-19 7-marks i) $f(A,B,C,D) = \sum m(0,1,5,6,7,9,11,15) + d(10,14)$

ii) $f(w,x,y,z) = \prod M(1,4,6,9,10,11,14,15)$ IMP

⟹ i)

CD \ AB	00	01	11	10
00	1 (0)	1 (4)	(12)	(8)
01	1 (1)	1 (5)	(13)	1 (9)
11	(3)	1 (7)	1 (15)	1 (11)
10	(2)	1 (6)	X (14)	X (10)

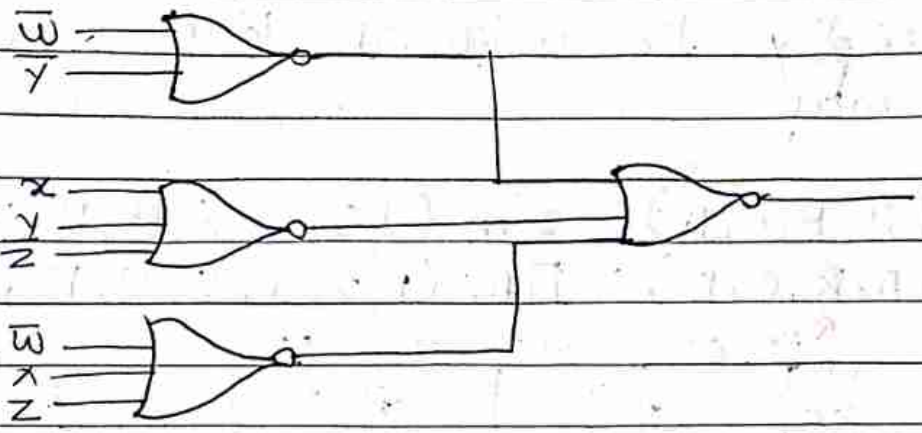
$f(A,B,C,D) = \bar{A}\bar{C} + \bar{A}\bar{B} + BC + A\bar{B}D$



ii)

yz \ wx	00	01	11	10
00	(0)	0 (4)	(12)	(8)
01	0 (1)	(5)	(13)	0 (9)
11	(3)	(7)	0 (15)	0 (11)
10	(2)	0 (6)	0 (14)	0 (10)

$f(w,x,y,z) = (\bar{w} + \bar{y})(x + y + \bar{z})(\bar{w} + x + z)$



Que: Simplify the given function using K-map and relising using NOR gate

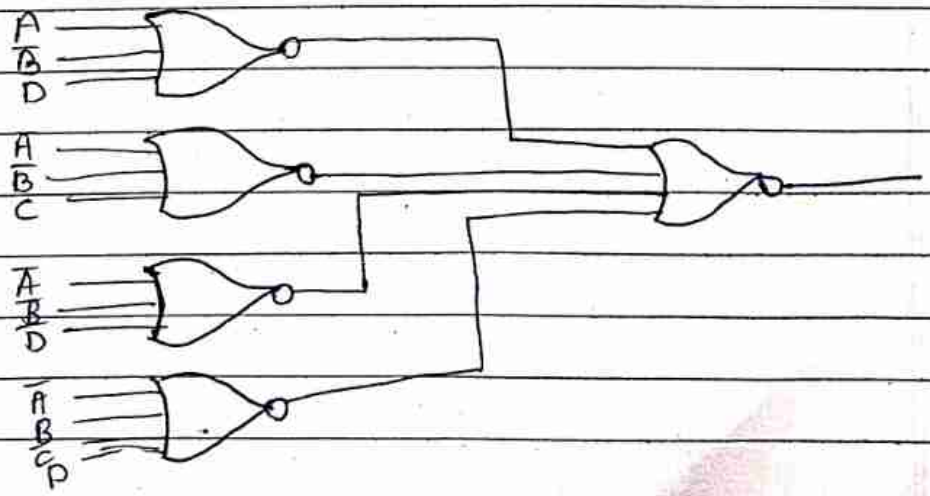
7-marks →

i) $f(A,B,C,D) = \pi M(4,6,10,12,13,15)$

⇒

CD \ AB	00	01	11	10
00	0	0 4	0 12	8
01	1	5	0 13	9
11	3	7	0 15	11
10	2	0 6	14	0 10

$f(A,B,C,D) = (A+\overline{B}+D)(\overline{A}+\overline{B}+C)(\overline{A}+\overline{B}+\overline{D})$
$(\overline{A}+B+\overline{C}+D)$



JIMIN

que: Simplify the following logic function using k-map

i) $f(A,B,C,D) = \Sigma m(1,3,5,8,9,11,15) + d(2,13)$

ii) $f(A,B,C,D) = \Pi M(1,2,3,8,9,10,11,14) \cdot d(7,15)$

CD \ AB	00	01	11	10
00	0	4	12	1 (8) (2)
01	1 (1)	1 5	X 13	1 (9)
11	1 3	7	1 15	1 11
10	X 2	6	14	10

$f(A,B,C,D) = AD + \bar{B}D + \bar{C}D + AB\bar{C}$

ii)

CD \ AB	00	01	11	10
00	0	4	12	0 8
01	0 1	3	13	0 9
11	0 3	X 7	X 15	0 11
10	0 2	6	0 14	0 10

$f(A,B,C,D) = (\bar{A}+B)(B+\bar{C})(B+\bar{D})(\bar{A}+\bar{C})$

J-HOPE

* 5- Variable K-map :

winter-A 8-marks

Que: minimise the following expresion using K-map and relise using NAND Gates.

$$f(A,B,C,D,E) = \Sigma m\,(0,5,6,8,9,10,11,16,20,24,25,26,27,29,31)$$

A = 0

DE \ ABC	00	01	11	10
00	1 (0)	4	12	1 (8)
01	1	1 (5)	13	1 (9)
11	3	7	15	1 (11)
10	2	1 (6)	14	1 (10)

A = 1

DE \ BC	00	01	11	10
00	1 (16)	1 (20)	28	1 (24)
01	17	21	1 (29)	1 (25)
11	19	23	1 (31)	1 (27)
10	18	22	30	1 (26)

$$f = B\bar{C} + ABE + A\bar{B}\bar{D}\bar{E} + \bar{A}\bar{C}\bar{D}\bar{E} + \bar{A}\bar{B}CD\bar{E} + \bar{A}\bar{B}CDE$$