B.E. (Computer Engineering / Information Technology) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NJR/KS/18/4382/4387**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $L[f(t)] = F(s)$, then show that $L[t\,f(t)] = -\frac{d}{ds}F(s)$ **7**

Hence find $L\left[t\,e^{3t}\sin 2t\right]$.

b) Find $L^{-1}\left[\frac{1}{(s+1)\left(s^2+1\right)}\right]$ using convolution theorem. **7**

OR

2. a) Express f(t) in terms of unit step function and hence find its Laplace transform **7**

$$f(t) = \begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$$

b) Solve the D.E. by L-T. method $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 5\sin t$ given $y(0)=0$ & $y'(0)=0$. **7**

3. Find Fourier transform of $e^{-|x|}$ and hence show that **7**

$$\int_0^\infty \frac{x\sin mx}{1+x^2}dx = \frac{\pi}{2}e^{-m},\ m>0.$$

OR

4. Express $f(x) = \begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$ as Fourier integral hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$. **7**

5. a) Prove that $z\left\{n^p\right\} = -z\frac{d}{dz}\left(n^{p-1}\right)$, where p is any positive integer and hence deduce that **7**

$z\{n\} = \frac{z}{(z-1)^2}$ and $z\left\{n^2\right\} = \frac{z(z+1)}{(z-1)^3}$.

b) Using convolution theorem show that $\frac{1}{n!} * \frac{1}{n!} = \frac{2^n}{n!}$. 7

OR

6. a) Find Z-Transform of $\sin n\theta$ and hence find $z\left[a^n \sin n\theta\right]$. 7

b) Solve $x_{n+2} + 3x_{n+1} + 2x_n = u_n$ given that $x_0 = 1$ and $x_n = 0$ for n<0. 7

7. a) Find modal matrix for the matrix 6

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

b) Using Sylvester's theorem prove that $\log_e e^A = A$ where $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. 6

c) Investigate the linear dependence of the vectors 6
$X_1 = (1,2,4),\ X_2 = (2,-1,3),\ X_3 = (0,1,2),\ X_4 = (-3,7,2)$
and if possible find relation between them.

OR

8. a) Verify Cayley Hamilton theorem for the given matrix 6

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$ and hence find A^{-1}.

b) Solve $\frac{dx_1}{dt} = x_1 + x_2$ & $\frac{dx_2}{dt} = x_2$ 6
Given $x_1(0) = 1$, $x_2(0) = 1$ by matrix method.

c) Determine largest eigen value and the corresponding eigen vector of the matrix 6

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

9. a) Three machines, A, B and C produce respectively 50%, 30% and 20% of the total number of items of a factory. The percentage of defective output of these machines are 3%, 4% and 5% respectively. One item is selected at random and is found to be defective find the probability that the item was produced by machine A. 7

b) A random variable X denotes the number of heads in three tosses of a fair coin. Find the probability function f(x) and the distribution function F(x). 7

OR

10. a) A random variable X has density function **7**

$$f(x)=\begin{cases} cx^2, & 1\le x\le 2 \\ cx, & 2<x<3 \\ 0, & \text{otherwise} \end{cases}$$

Find

i) the constant C

ii) $P[X>2]$

iii) $P[\frac{1}{2}<X<\frac{3}{2}]$

iv) distribution function.

b) The joint probability function of two discrete random variables X and Y is given by **7**

$$f(x,y)=\begin{cases} c(2x+y), & 0\le x\le 3 \\ & 0\le y\le 2 \\ 0\quad, & \text{otherwise} \end{cases}$$

Find

i) Constant C

ii) Marginal probability functions

iii) $P[1<X<2,\ Y>2]$

iv) $P[X<2]$

11. a) Let X be a random variable with density function **7**

$$f(x)=\begin{cases} 2e^{-2x}, & x\ge 0 \\ 0\ , & \text{otherwise} \end{cases}$$

Find

i) $E(X)$

ii) $E(X^2+5)$

iii) $V(X)$

iv) S.D. of X

b) Find the moment generating function of random variable **6**

$$X=\begin{cases} \frac{1}{2} \text{ prob. } -\frac{1}{2} \\ -\frac{1}{2} \text{ prob. } \frac{1}{2} \end{cases}$$

Hence find four moments about origin.

12. a) Let X and Y be joint density function **7**

$$f(x,y)=\begin{cases} x+y, & 0\le x\le 1,\ 0\le y\le 1 \\ 0\ , & \text{otherwise} \end{cases}$$

Find

i) $E(X+Y)$

ii) the conditional expectation of X given Y and Y given X.

iii) conditional variance of Y given X.

b) Defind **6**

i) Stochastic matrix.

ii) Bernoulli's process.

iii) Poisson process.

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