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Monday

Unit VI - Matrices

Linear dependence :-

The Set of vector $x_1, x_2, \dots x_n$ is said to be linearly dependent if there exists Scalar $k_1, k_2, \dots k_n$ not all zero.

Such that $k_1x_1 + k_2x_2 + k_3x_3 + \dots k_nx_n = 0$

where 0 is null vector.

If $k_1, k_2, \dots k_n = 0$ then vectors are called linearly independent.

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Q.1 Find whether the set of vectors are linearly dependent or otherwise if linearly dependent. Find the relation between them.

$X_1 = [1,1,1,3] \quad X_2 = [1,2,3,4] \quad X_3 = [2,3,4,7]$

Solution:- Given Vectors :- X_1, X_2, X_3

Let k_1, k_2, k_3 be three Scalar

Such that $k_1X_1 + k_2X_2 + k_3X_3 = 0$ — ①

$k_1[1,1,1,3] + k_2[1,2,3,4] + k_3[2,3,4,7] = 0$

$k_1 + k_2 + 2k_3 = 0$

$k_1 + 2k_2 + 3k_3 = 0$

$k_1 + 3k_2 + 4k_3 = 0$

$3k_1 + 4k_2 + 7k_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 4 & 0 \\ 3 & 4 & 7 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 3 & 4 & 0 \\ 3 & 4 & 7 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 3 & 4 & 7 & 0 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & 7 & 0 \end{array}\right]$$

$R_4 \to R_4 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_4 \to R_4 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$K_1 + K_2 + 2K_3 = 0$$
$$K_2 + K_3 = 0$$

Here we have to find values of 3 variables and we have only two eq^n. therefore $3-2=1$ we have to asign 1 arbitary value to one of the variables.

Let $K_2 = z$ $\quad K_1 + z - 2z = 0$

$K_3 = -z$

$K_1 = z$

Thus there exists K_1, K_2, K_3 not all zero.
Therefore given vectors are linearly dependent
The relation between them

$$zx_1 + zx_2 - zx_3 = 0$$
$$\Rightarrow x_1 + x_2 - x_3 = 0$$

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Q.2. Find whether the Set of Vectors are linearly dependent or otherwise if linearly dependent. Find the relation

$X_1 = (1, 2, 4)$ $X_2 = (2, -1, 3)$ $X_3 = (0, 1, 2)$

$X_4 = (-3, 7, 2)$

Sol^n → Given vectors : X_1, X_2, X_3, X_4

Let K_1, K_2, K_3, K_4 be Scalar

Such that $K_1X_1 + K_2X_2 + K_3X_3 + K_4X_4 = 0$ —(1)

$K_1(1,2,4) + K_2(2,-1,3) + K_3(0,1,2) + K_4(-3,7,2) = 0$

$K_1 + 2K_2 + 0K_3 - 3K_4 = 0$

$2K_1 - K_2 + K_3 + 7K_4 = 0$

$4K_1 + 3K_2 + 2K_3 + 2K_4 = 0$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 2 & -1 & 1 & 7 & 0 \\ 4 & 3 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - 4R_1$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & -5 & 1 & 13 & 0 \\ 0 & -5 & 2 & 14 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

endent
on
1,2)

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & -5 & 1 & 13 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

— (1)

,2) = 0

$$K_1 + 2K_2 + 0K_3 - 3K_4 = 0$$
$$-5K_2 + K_3 + 13K_4 = 0$$
$$K_3 + K_4 = 0$$

Here we have to find value of 4 Variables & we have only 3 equations therefore $4-3=1$
i.e. we have to asign one arbitary value to one of the Variables

let $\boxed{K_4 = z}$

$\boxed{K_3 = -z}$

$$-5K_2 + (-z) + 13(z) = 0$$
$$-5K_2 - z + 13z = 0$$
$$-5K_2 + 12z = 0$$
$$-5K_2 = -12z$$
$$\boxed{K_2 = \frac{12}{5}z}$$

Put K_2 in eq^n (1) $\Rightarrow K_1 + 2\left(\frac{12}{5}z\right) - 3(z) = 0$

$$K_1 + \left[\frac{24}{5} - 3\right]z = 0$$

$$\boxed{K_1 = -\frac{9}{5}z}$$

K_1, K_2, K_3, K_4 not all zero

$\therefore$ Given vectors are linearly dependent

The relation between this vectors

$$-\frac{9}{5} z X_1 + \frac{12}{5} z X_2 - z X_3 + z X_4 = 0$$

Multiply by 5 & divide by z

$$-9X_1 + 12X_2 - 5X_3 + 5X_4 = 0$$

Q.3 : Investigate whether the given vector are linearly dependance or otherwise, if linearly dependence then find the relation between them.

$X_1 = [1, 2, -1, 3]$, $X_2 = [1, 2, 3, 4]$,

$X_3 = [2, 3, 4, 7]$

→ Given vectors :- X_1, X_2, X_3

Let K_1, K_2, K_3 be the three vectors scalar such that $K_1X_1 + K_2X_2 + K_3X_3 = 0$ —— ①

$$K_1[1,2,-1,3] + K_2[1,2,3,4] + K_3[2,3,4,7] = 0$$

$$1K_1 + K_2 + 2K_3 = 0$$

$$2K_1 + 2K_2 + 3K_3 = 0$$

$$-K_1 + 3K_2 + 4K_3 = 0$$

$$3K_1 + 4K_2 + 7K_3 = 0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & 2 & 3 & 0 \\ -1 & 3 & 4 & 0 \\ 3 & 4 & 7 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$ & $R_3 \rightarrow R_3 + R_1$ & $R_4 \rightarrow R_4 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

Interchange $R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_4 \rightarrow 4R_4 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$R_4 \rightarrow R_4 - 2R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$K_1 + K_2 + 2K_3 = 0$$
$$4K_2 + 6K_3 = 0$$
$$-K_3 = 0$$

$\therefore K_1 = 0 \quad K_2 = 0 \quad K_3 = 0$

K_1, K_2, K_3 are all zero

Therefore given vectors are linearly independence

Therefore there is no relation between them.

Matrix

- Characteristic Equation Eigen Values, Eigen Vectors
  - let A be a Square matrix of order n then $A - \lambda I$, where λ is a Scalar & I is Identity matrix of order n is called characteristic matrix
  - The Determinant of characteristic matrix is equate with zero.

    i.e $|A - \lambda I| = 0$ is called characteristic equation,
  - if we Solve this equation we get n, number of eigen Values.
  - $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ This values are also called latent roots or characteristic Values, Corresponding to this Value we get Eigen vectors

- **Diagonalization of matrix or Reduction to a diagonal (canonical) form :-**

If A is a square matrix of order 'n' having 'n' linearly independent vectors then we can find a non-singular matrix B such that $B^{-1}AB$ will be the diagonal form of given matrix.

If A is a square matrix of order 3 then there exists 3 Eigens values

$\lambda_1, \lambda_2, \lambda_3$ $\quad \lambda_1 \; \lambda_2 \; \lambda_3$

Then the diagonalization of given matrix

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

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Q.1 Find the modal matrix B corresponding to matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ and verify that $B^{-1}AB$ is diagonal form.

Sol^n →

Given matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

Characteristic matrix $= [A - \lambda I] = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}$

Characteristic equation $= |A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(2-\lambda) - 6 = 0$$

$$2 - 3\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = -1, 4$$

Eigen Values $\lambda_1 = -1 \quad \lambda_2 = 4$

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ eigen vector corresponding to eigen value λ,

Such that $[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$(1-\lambda)x_1 + 2x_2 = 0 \quad \text{---} ①$$
$$3x_1 + (2-\lambda)x_2 = 0 \quad \text{---} ②$$

For $\lambda = -1$

$eq^n ① \div 2 \qquad eq^n ② \div 3$

$$\left.\begin{array}{l} ① \Rightarrow 2x_1 + 2x_2 = 0 \\ ② \Rightarrow 3x_1 + 3x_2 = 0 \end{array}\right\} \longrightarrow x_1 + x_2 = 0$$

We have to find values of 2 variable and we have only 1 eq^n.
Therefore asign arbitary values to one of the variable.

Let $x_1 = 1 \Rightarrow x_2 = -1$

$\therefore$ Therefore the eigen vector corresponding to $\lambda = 1$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

For $\lambda = 4$

$$\left.\begin{array}{l} ① \Rightarrow -3x_1 + 2x_2 = 0 \\ ② \Rightarrow 3x_1 \mp 2x_2 = 0 \end{array}\right\} \xrightarrow{eq^n ① \times -1} 3x_1 - 2x_2 = 0$$

Let $x_1 = 2 \Rightarrow x_2 = 3$

$\therefore$ Therefore the eigen vector corresponding to $\lambda = 4$ is $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Therefore Modal matrix $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$B^{-1} = \begin{bmatrix} 3/5 & -2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

$$B^{-1}AB = \frac{1}{5}\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$$

Q.2 Reduce the matrix $A = \begin{bmatrix} 0 & 1 \\ 12 & -4 \end{bmatrix}$ to diagonal form $B^{-1}AB$

→ Given matrix : $A = \begin{bmatrix} 0 & 1 \\ 12 & -4 \end{bmatrix}$

Characteristic matrix : $[A - \lambda I] = \begin{bmatrix} -\lambda & 1 \\ 12 & -4-\lambda \end{bmatrix}$

Characteristic equation : $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 \\ 12 & -4-\lambda \end{vmatrix} = 0$$

$$-\lambda(-4-\lambda) - 12 = 0$$

$$4\lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 + 4\lambda - 12 = 0$$

$$\lambda = 2, -6$$

Eigen Values $\lambda_1 = 2 \quad \lambda_2 = -6$

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$\Rightarrow$ Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be the eigen vector corresponding to eigen value λ such that

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} -\lambda & 1 \\ 12 & -4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-\lambda x_1 + x_2 = 0 \quad \text{--- (1)}$$

$$12 x_1 + (-4-\lambda) x_2 = 0 \quad \text{--- (2)}$$

$\Rightarrow$ For $\lambda = 2$ Eq^n (2) $\div -6$

$$\left.\begin{aligned} (1) &\Rightarrow -2x_1 + x_2 = 0 \\ (2) &\Rightarrow 12x_1 - 6x_2 = 0 \end{aligned}\right\} \Rightarrow -2x_1 + x_2 = 0$$

Let $x_1 = 1 \Rightarrow x_2 = 2$

$\Rightarrow$ For $\lambda = -6$ Eq^n (2) $\div 6$

$$\left.\begin{aligned} (1) &\Rightarrow 6x_1 + x_2 = 0 \\ (2) &\Rightarrow 12x_1 + 2x_2 = 0 \end{aligned}\right\} \Rightarrow 6x_1 + x_2 = 0$$

Let $x_1 = 1 \Rightarrow x_2 = -6$

Therefore Modal Matrix $B = \begin{bmatrix} 1 & 1 \\ 2 & -6 \end{bmatrix}$

$$B^{-1} = \begin{bmatrix} 3/4 & 1/8 \\ 1/4 & -1/8 \end{bmatrix}$$

$$B^{-1}AB = \begin{bmatrix} 3/4 & 1/8 \\ 1/4 & -1/8 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 12 & -4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & -6 \end{bmatrix}$$

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Q.3 Find a matrix B which reduces given matrix to a diagonal form by transformation $B^{-1}AB$. Hence find diagonal form of A.

Given matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Sol^n →

Given matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Characteristic matrix $[A-\lambda I] = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$

Characteristic equation $|A-\lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

S_1 = Sum of diagonal element

S_2 = Sum of minors of diagonal element

$5 + (-10) + 0$

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$$\lambda^3 - (8+7+3)\lambda^2 + \left[\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}\right]\lambda - 0 = 0$$

$\lambda^3 - 18\lambda^2 + (5 + 20 + 20)\lambda = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\lambda = 0, 3, 15$

$\therefore$ eigen Values $\lambda_1 = 0 \quad \lambda_2 = 3 \quad \lambda_3 = 15$

let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be eigen vectors corresponding to eigen value λ

Such that

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$(8-\lambda)x_1 - 6x_2 + 2x_3 = 0$ — ①

$-6x_1 + (7-\lambda)x_2 - 4x_3 = 0$ — ②

$2x_1 - 4x_2 + (3-\lambda)x_3 = 0$ — ③

For $\lambda = 0$

① $\Rightarrow 8x_1 - 6x_2 + 2x_3 = 0$

② $\Rightarrow -6x_1 + 7x_2 - 4x_3 = 0$

③ $\Rightarrow 2x_1 - 4x_2 + 3x_3 = 0$

By Cramers Rule,

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$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 8 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}}$$

$$\frac{x_1}{10} = \frac{-x_2}{-20} = \frac{x_3}{20}$$

Divide by 10

$\therefore x_1 = 1, \quad x_2 = 2, \quad x_3 = 2$

$$\cancel{x_1} = \quad X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

For $\lambda = 3$

$(A-\lambda I)B$ $\quad 5x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 + 4x_2 - 4x_3 = 0$

$2x_1 - 4x_2 + 0x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$\frac{x_1}{16} = \frac{-x_2}{-8} = \frac{x_3}{-16}$$

Divide by 8

$x_1 = 2 \qquad x_2 = 1 \qquad x_3 = -2$

$$\therefore X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

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For $\lambda = 15$

$\cancel{-7\lambda_2}$

① $\Rightarrow \quad -7x_1 - 6x_2 + 2x_3 = 0$

② $\Rightarrow \quad -6x_1 - 8x_2 - 4x_3 = 0$

③ $\Rightarrow \quad 2x_1 - 4x_2 - 12x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$$

$$\frac{x_1}{40} = \frac{-x_2}{40} = \frac{x_3}{20}$$

$\therefore x_1 = 2, \quad x_2 = -2, \quad x_3 = 1$

$$X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{Modal Matrix } B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \end{bmatrix}$$

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$$B^{-1}AB = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

Q4 Find the modal matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Sol^n ⇒ Given matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Characteristic matrix $[A - \lambda I] = \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$

Characteristic equation $|A - \lambda I| = 0$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix}$$

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$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

$$\lambda^3 + \lambda^2 + [-12 - 3 - 6]\lambda - 45 = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\lambda = 5, -3, -3$$

eigen values $\lambda_1 = 5 \quad \lambda_2 = -3 \quad \lambda_3 = -3$

let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(-2-\lambda)x_1 + 2x_2 - 3x_3 = 0$ — (1)

$2x_1 + (1-\lambda)x_2 - 6x_3 = 0$ — (2)

$-x_1 - 2x_2 - \lambda x_3 = 0$ — (3)

For $\lambda = 5$

(1) ⇒ $-7x_1 + 2x_2 - 3x_3 = 0$

(2) ⇒ $2x_1 - 4x_2 - 6x_3 = 0$

(3) ⇒ $-x_1 - 2x_2 - 5x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -4 & -6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & 2 \\ 2 & -4 \end{vmatrix}}$$

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$$\frac{x_1}{-24} = \frac{-x_2}{48} = \frac{x_3}{24}$$

Divide by -24

$\therefore \ x_1 = 1 \quad x_2 = 2 \quad x_3 = -1$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

For $\lambda = -3$

$$\left.\begin{aligned} ① &\Rightarrow x_1 + 2x_2 - 3x_3 = 0 \\ ② &\Rightarrow 2x_1 + 4x_2 - 6x_3 = 0 \\ ③ &\Rightarrow -x_1 - 2x_2 - \lambda x_3 = 0 \end{aligned}\right\} x_1 + 2x_2 - 3x_3 = 0$$

Here we have to find values of 3 variable and we have only one eqn

$3 - 1 = 2$

Therefore we have to asign two arbitary values for two of the variables.

Ans should not come in fraction

Let $x_1 = 0 \quad x_2 = 3 \Rightarrow x_3 = 2$

$$X_2 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

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For $\lambda = -3$

$x_1 + 2x_2 - 3x_3 = 0$

Let $x_1 = 1 \quad x_2 = 1 \Rightarrow x_3 = 1$

$$X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Modal Matrix $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

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Monday

(ii) $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ Q.1

$A^5[A^3 - 5A^2 + 7A - 3I] + [A^4 - 5A^3 + 8A^2 - 2A + I]$ →

$A^5[0] + [A^4 - 5A^3 + 8A^2 - 2A + I]$

$$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

- **Sylvester's theorem**

If A is a Square matrix of order n with 'n' number of eigen values $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ all are distinct and $P(A)$ is a polynomial then $P(A)$ can be expressed as

$$P(A) = \sum_{r=1}^{n} P(\lambda_r) Z(\lambda_r)$$

where $Z(\lambda_r) = \dfrac{\text{adj}\,|\lambda I - A|}{\phi'(\lambda)}$

$$\phi(\lambda) = \cancel{|A - \lambda I|}\ |\lambda I - A|$$

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Q.1 Find A^{50} by using Sylvester's theorem for
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

→ let $p(A) = A^{50}$ — ①

By Sylvester theorem

$$p(A) = p(\lambda_1) z(\lambda_1) + p(\lambda_2) z(\lambda_2)$$

$$\phi(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-3 \end{vmatrix}.$$

$$\phi(\lambda) = \lambda^2 - 4\lambda + 3$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = 1, 3$$

$$\lambda_1 = 1, \quad \lambda_2 = 3$$

$$p(\lambda_1) = p(1) = 1^{50}$$

$$p(\lambda_2) = p(3) = 3^{50}$$

$$z(\lambda_2) = \frac{\operatorname{adj} |\lambda I - A|}{\phi'(\lambda)}$$

$$= \frac{\operatorname{adj} \begin{bmatrix} \lambda-1 & 0 \\ 0 & \lambda-3 \end{bmatrix}}{2\lambda - 4}$$

$$= \frac{\begin{bmatrix} \lambda-3 & 0 \\ 0 & \lambda-1 \end{bmatrix}^T}{2\lambda - 4}$$

$$z(\lambda_2) = \begin{bmatrix} \lambda-3 & 0 \\ 0 & \lambda-1 \end{bmatrix} \quad \text{— ③}$$

$$z(\lambda_1) = z(1) = \frac{\begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}}{-2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$z(\lambda_2) = z(3) = \frac{\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}}{2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore (2) \Rightarrow A^{50} = 1^{50}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 3^{50}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{50} = \begin{bmatrix} 1^{50} & 0 \\ 0 & 3^{50} \end{bmatrix}$$

Q.2 Using Sylvester's theorem find A^{-1}

$$A = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\rightarrow$ let $p(A) = A^{-1}$ — (1)

By Sylvester's theorem

$$P(A) = p(\lambda_1)z(\lambda_1) + p(\lambda_2)z(\lambda_2) + p(\lambda_3)z(\lambda_3) \quad \text{—(2)}$$

$$\phi(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-2 & 0 \\ 0 & 0 & \lambda-3 \end{vmatrix}$$

$$\phi(\lambda) = \lambda^3 - S_1\lambda^2 + S_2\lambda - |A|$$

$$\phi(\lambda) = \lambda^3 - 6\lambda^2 + 11\lambda - 6$$

$$\lambda_1 = 1 \quad , \quad \lambda_2 = 2 \quad , \quad \lambda_3 = 3$$

$$P(\lambda_1) = P(1) = 1^{-1} = 1$$

$$P(\lambda_2) = P(2) = 2^{-1} = \frac{1}{2}$$

$$P(\lambda_3) = P(3) = 3^{-1} = \frac{1}{3}$$

$$Z(\lambda_r) = \frac{\text{adj}\,[\lambda I - A]}{\phi'(\lambda)}$$

$$= \frac{\text{adj}\begin{bmatrix} \lambda-1 & 2 & 3 \\ 0 & \lambda-2 & 0 \\ 0 & 0 & \lambda-3 \end{bmatrix}}{3\lambda^2 - 12\lambda + 11}$$

$$= \frac{\begin{bmatrix} \lambda^2-5\lambda+6 & 0 & 0 \\ -2\lambda+6 & \lambda^2-4\lambda+3 & 0 \\ -3\lambda+6 & 0 & \lambda-3\lambda+2 \end{bmatrix}^T}{3\lambda^2 - 12\lambda + 11}$$

$Z(\lambda_1)$ ②

$$= \frac{\begin{bmatrix} \lambda^2-5\lambda+6 & -2\lambda+6 & -3\lambda+6 \\ 0 & \lambda^2-4\lambda+3 & 0 \\ 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}}{3\lambda^2 - 12\lambda + 11}$$

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$$z(\lambda_1) = z(1) = \frac{\begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}{2}$$

$$z(\lambda_2) = z(2) = \frac{\begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}{-1}$$

$$z(\lambda_3) = z(3) = \frac{\begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}}{2}$$

Q.3

$\Rightarrow$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 & 6 & 9 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & 6 & 6 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Q.3 If $M = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ find the value of $M^2 - 3M + I$ and verify the result by Sylvester theorem.

$\Rightarrow$ $M^2 - 3M + I = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 3 \\ 9 & 8 \end{bmatrix} \quad \text{--- (1)}$$

$p(M) = M^2 - 3M + I$ --- (2)

By Sylvester theorem,

$p(M) = p(\lambda_1)\, z(\lambda_1) + p(\lambda_2)\, z(\lambda_2)$ --- (3)

$\phi(\lambda) = |\lambda I - M| = \begin{vmatrix} \lambda - 2 & -1 \\ 9 & \lambda - 4 \end{vmatrix}$

$= \lambda^2 - 6\lambda + 8 - 3$

$\phi(\lambda) = \lambda^2 - 6\lambda + 5$

$\lambda = 1, 5$

$\lambda_1 = 1 \qquad \lambda_2 = 5$

$p(\lambda_1) = p(1) = -1$

$p(\lambda_2) = p(5) = 11$

$$z(\lambda r) = \frac{\operatorname{adj}[\lambda I - A]}{\phi'(\lambda)}$$

$$= \frac{\operatorname{adj}\begin{bmatrix} \lambda-2 & -1 \\ -3 & \lambda-4 \end{bmatrix}}{2\lambda - 6}$$

$$\lambda = 1 \quad z(\lambda r) = \frac{\begin{bmatrix} \lambda-4 & 3 \\ 1 & \lambda-2 \end{bmatrix}}{2\lambda - 6}$$

$$z(\lambda_1) = z(1) = \frac{\begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}^T}{-4}$$

$$z(\lambda_2) = z(5) = \frac{\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}^T}{4} = \frac{\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}}{4}$$

$$M^2 - 3M + I = \begin{bmatrix} 2 & 3 \\ 9 & 8 \end{bmatrix} \text{ verified}$$

Q.4

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Q.4 Use Sylvester's theorem to show that $3\tan A = (\tan 3)A$, where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$

→ let $p(A) = 3\tan A$ — (1)

By Sylvester theorem

$$p(A) = p(\lambda_1)z(\lambda_1) + p(\lambda_2)z(\lambda_2) \quad \text{— (2)}$$

$$\phi(\lambda) = |\lambda I - A|$$

$$= \begin{vmatrix} \lambda+1 & -4 \\ -2 & \lambda-1 \end{vmatrix} = \lambda^2 - 1 - 8$$

$$\phi(\lambda) = \lambda^2 - 9 \quad \text{— (3)}$$

$$= \lambda = \pm 3$$

$$\lambda_1 = 3, \quad \lambda_2 = -3$$

$$p(\lambda_1) = p(3) = 3\tan 3$$

$$p(\lambda_2) = p(-3) = 3\tan(-3)$$

$$z(\lambda_r) = \frac{\text{adj}[\lambda I - A]}{\phi'(\lambda)}$$

$$= \frac{\text{adj}\begin{bmatrix} \lambda+1 & -4 \\ -2 & \lambda-1 \end{bmatrix}}{2\lambda}$$

$$= \frac{\begin{bmatrix} \lambda-1 & 2 \\ 4 & \lambda+1 \end{bmatrix}}{2\lambda}$$

$$= \frac{\begin{bmatrix} \lambda-1 & 4 \\ 2 & \lambda+1 \end{bmatrix}}{2\lambda} \quad \text{— (4)}$$

$$z(\lambda_1) = z(3) = \frac{\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}}{6}$$

$$z(\lambda_2) = z(-3) = \frac{\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}}{-6}$$

$$① \Rightarrow 3\tan A = 3\tan 3 \frac{\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}}{6} + \frac{3\tan(-3)}{1} \frac{\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}}{-6}$$

$$\frac{-1}{6}\left\{3\tan 3\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} + 3\tan 3\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}\right\}$$

$$= \frac{3\tan 3}{6}\begin{bmatrix} -2 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \tan 3\begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= (\tan 3)A$$

Q.5 If $M = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ find e^M — ①

$\Rightarrow$ let $p(M) = e^M$

By Sylvester theorem

$$p(M) = p(\lambda_1)Z(\lambda_1) + p(\lambda_2)Z(\lambda_2) \quad — ②$$

$$\phi(\lambda) = |\lambda I - A| = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-2 \end{vmatrix}$$

$$\phi(\lambda) = (\lambda-1)(\lambda-2) - 0$$

$$= \lambda^2 - 2\lambda - 1\lambda + 2 = 0$$

$$= \lambda^2 - 3\lambda + 2$$

$\lambda_1 = 2 \qquad \lambda_2 = 1$

$p(\lambda_1) = p(2) = e^2$

$p(\lambda_2) = p(1) = e^1$

$$z(\lambda r) = \frac{\operatorname{adj}\begin{bmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 2 \end{bmatrix}}{\phi_1(\lambda)}$$

$$= \frac{\begin{bmatrix} \lambda - 2 & 0 \\ 0 & \lambda - 1 \end{bmatrix}^T}{2\lambda - 3}$$

$$= \frac{\begin{bmatrix} \lambda - 2 & 0 \\ 0 & \lambda - 1 \end{bmatrix}}{2\lambda - 3}$$

$$z(\lambda_1) = z(1) = \frac{\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}}{-1}$$

$$z(\lambda_2) = z(0) = \frac{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}{1}$$

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Q. (2) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$, Show that

(i) $\sin^2 A + \cos^2 A = I$

(ii) $\sec^2 A - \tan^2 A = I$

→

$$e^M = e\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + e^2\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix}$$

⇒ (i) $p(A) = \sin^2 A + \cos^2 A$ — (1)

$p(A) = p(\lambda_1)\, z(\lambda_1) + p(\lambda_2)\, z(\lambda_2)$

$\phi(\lambda) = (\lambda I - A) = \begin{bmatrix} \lambda - 2 & -4 \\ -3 & \lambda - 1 \end{bmatrix}$

$\phi(\lambda) = \lambda^2 - 3\lambda + 2 - 12$

$\phi(\lambda) = \lambda^2 - 3\lambda - 10$

$\lambda = -2, 5$

$p(\lambda_1) = p(-2) = \sin^2(-2) + \cos^2(-2)$

$p(\lambda_2) = p(5) = \sin^2(5) + \cos^2(5)$

$$z(\lambda_r) = \frac{\operatorname{adj}\begin{bmatrix} \lambda - 2 & -4 \\ -3 & \lambda - 1 \end{bmatrix}}{2\lambda - 3}$$

$$= \begin{bmatrix} \lambda - 1 & 3 \\ 4 & \lambda - 2 \end{bmatrix}$$

$$z(\lambda_r) = \frac{\begin{bmatrix} \lambda - 1 & 4 \\ 3 & \lambda - 2 \end{bmatrix}}{2\lambda - 3}$$

$$z(2) = \frac{\begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix}}{-7}$$

$$z(5) = \frac{\begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}}{7}$$

$\Rightarrow$ (2) $\sin^2 A + \cos^2 A = (\sin^2(-2)) + \cos^2(-2)) \dfrac{\begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix}}{-7}$

$$+ \sin^2 5 + \cos^2 5 \frac{\begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}}{7}$$

$$= \frac{1}{7}\left\{\begin{bmatrix} 3 & -4 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}\right\}$$

$$= \frac{1}{7}\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\sin^2 A + \cos^2 A = 1$$

(ii) $p(A) = \sec^2 A + \tan^2 A$ — (1)

$p(A) = p(\lambda_1) z(\lambda_1) + p(\lambda_2) z(\lambda_2)$

$\phi(\lambda) = [\lambda I - A] = \begin{bmatrix} \lambda - 2 & -4 \\ -3 & \lambda - 1 \end{bmatrix}$

$$\phi(\lambda) = \lambda^2 - 3\lambda + 2 - 12$$
$$\phi(\lambda) = \lambda^2 - 3\lambda - 10$$
$$\lambda = 2, 5$$
$$p(\lambda_1) = p(-2) = \sec^2(-2) - \tan^2(-2)$$
$$p(\lambda_2) = p(5) = \sec^2(5) - \tan^2(5) = 1$$

$$Z(\lambda_r) = \frac{\operatorname{adj}\begin{bmatrix} \lambda-2 & 4 \\ -3 & \lambda-1 \end{bmatrix}}{2\lambda - 3}$$

$$= \frac{\begin{bmatrix} \lambda-1 & 3 \\ 4 & \lambda-2 \end{bmatrix}^T}{2\lambda-3}$$

$$Z(\lambda_r) = \frac{\begin{bmatrix} \lambda-1 & 4 \\ 3 & \lambda-2 \end{bmatrix}}{2\lambda - 3} \Rightarrow Z(-2) = \frac{\begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix}}{-7}$$

$$\Rightarrow Z(5) = \frac{\begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}}{7}$$

(2) $\sec^2 A - \tan^2 A = -\frac{1}{7}\begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix} + \frac{1}{7}\begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}$

$$= \frac{1}{7}\left[\begin{bmatrix} 3 & -4 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}\right]$$

$$= \frac{1}{7}\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$