Friday

Unit-II Z-TRANSFORM

discrete

- Z-Transform :-

Defination :- Let F(n) or Fn be a Sequence then z-Transform of F(n) is given by

$$z[F(n)] = \sum_{n=0}^{\infty} F(n) z^{-n}, n>0$$

Where z on LHS is an Operator and z on RHS is Complex number.

Ex:1 Find z-Transform of unity and hence find z transform of Constant Sequence k.

Sol^n → By defination

$$z[F(n)] = \sum_{n=0}^{\infty} F(n) z^{-n}$$

$$z[1] = \sum_{n=0}^{\infty} 1 z^{-n}$$

$$= \sum \frac{1}{z^n}$$

$z^0=1$

$$= \frac{1}{z^0} + \frac{1}{z^1} + \frac{1}{z^2} + \text{------}$$

$$= \left(1-\frac{1}{z}\right)^{-1}$$

$$= \frac{1}{1-\frac{1}{z}} = \frac{1}{\frac{z-1}{z}}$$

$$\boxed{z[1] = \frac{z}{z-1}}$$

Monday

for Constant Sequence

$$z[K] = K\,z[1]$$
$$= K\frac{z}{z-1}$$

$$\boxed{z[K] = \frac{zK}{z-1}}$$

- Properties of z-transform :-

① Linearity Property :

$$z[aF(n) + bg(n) - ch(n)] = a\,z[F(n)] + b\,z[g(n)] - c\,z[h(n)]$$
$$= aF(n) + bG(n) - cH(n)$$

② Change of scale Property :

If $z[f(n)] = F(z)$ then $z[a^n f(n)] = F(z/a)$

Soln : $z[f(n)] = F(z)$

By definition

$$z[f(n)] = F(z) = \sum_{n=0}^{\infty} f(n)\,z^{-n}$$

$$z[a^n f(n)] = \sum_{n=0}^{\infty} a^n f(n)\,z^{-n}$$
$$= \sum_{n=0}^{\infty} f(n)\left(\frac{z^{-n}}{a^{-n}}\right)$$
$$= \sum_{n=0}^{\infty} f(n)\left(\frac{z}{a}\right)^{-n}$$
$$= F(z/a)$$

Ex. Find $z[a^n]$ by using Change Scale Property

Sol^n →

By Change Scale Property

$z[F(n)] = F(z)$

$z[a^n F(n)] = F(z/a)$

$$z[a^n F(n)] = F\left(\frac{z}{a}\right)$$

$$z[a^n . 1] = \frac{z/a}{z/a - 1}$$

$$= \frac{z/a}{\frac{z-a}{a}}$$

$$z[a^n . 1] = \frac{z}{z-a}$$

Property ③ : Multiplicity by n :

If $z[F(n)] = F(z)$ then

$$z[nF(n)] = -z \frac{d}{dz} F(z)$$

Sol^n :- $z[F(n)] = F(z) = \sum_{n=0}^{\infty} F(n) z^{-n}$

$$F(z) = \sum_{n=0}^{\infty} F(n) z^{-n}$$

$$\frac{d}{dz} F(z) = \frac{d}{dz} \sum_{n=0}^{\infty} F(n) z^{-n}$$

$$\frac{d}{dz} F(z) = \sum_{n=0}^{\infty} F(n)(-n)\, z^{-n-1}$$

multiply Both Side by $-z$

$$-z \frac{d}{dz} F(z) = -z \sum_{n=0}^{\infty} F(n)(-n)\, z^{-n-1}$$

$$= \sum_{n=0}^{\infty} (n F(n))\, z^{-n-1+1}$$

$$= \sum_{n=0}^{\infty} (n F(n))\, z^{-n}$$

$$= z[n F(n)]$$

Ex: Find z-Transform of n and use it to find $z[n^2]$.

$zF(n) = F(z)$

Sol^n → $\cancel{z}$ By property $z[n F(n)] = -z \frac{d}{dz} F(z)$

$z(n) = -z \frac{d}{dz}$

$$z[n \cdot 1] = -z \frac{d}{dz} \frac{z}{z-1}$$

$z(1) = \frac{z}{z-1}$

$$= -z \left[\frac{(z-1) \cdot 1 - z \cdot 1}{(z-1)^2} \right]$$

$$= -z \left[\frac{-1}{(z-1)^2} \right]$$

$$\boxed{z[n] = \frac{z}{(z-1)^2}}$$

$z(n) = -z \frac{d}{dz}$

Page No. 70
Date: 9/1/2019

Wednesday $z[n] = \frac{z}{(z-1)^2}$

for $z[n^2] = z[n \cdot n] = -z \frac{d}{dz} \frac{z}{(z-1)^2}$

$$= -z \left[\frac{(z-1)^2 \cdot 1 - 2z(z-1)}{(z-1)^4} \right]$$

$$= -z \left[\frac{(z-1) - 2z}{(z-1)^3} \right]$$

$$= -z \left[\frac{-z-1}{(z-1)^3} \right]$$

$$= \frac{z(z+1)}{(z-1)^3}$$

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Ex:-1 Prove that $z[n^p] = -z \frac{d}{dz} z[n^{p-1}]$, where p is any positive integer & hence deduce that $z[n] = \frac{z}{(z-1)^2}$, $z[n^2] = \frac{z(z+1)}{(z-1)^3}$

Solⁿ → By definition

$$z[n^{p-1}] = \sum_{n=0}^{\infty} n^{p-1} z^{-n}$$

$$z[f(n)] = \sum_{n=0}^{\infty} f(n) z^{-n}.$$

Taking $\frac{d}{dz}$ on both side

$$\frac{d}{dz} z[n^{p-1}] = \frac{d}{dz} \sum_{n=0}^{\infty} n^{p-1} z^{-n}$$

$$\frac{d}{dz} z[n^{p-1}] = \sum_{n=0}^{\infty} n^{p-1}(-n) z^{-n-1}$$

Multiply both Side by $(-z)$

$$-z\frac{d}{dz} z[n^{p-1}] = -z\sum_{n=0}^{\infty} n^{p-1}(-n) z^{-n-1}$$

$$= \sum n^{p-1} n z z^{-n-1}$$

$$= \sum_{n=0}^{\infty} n^{p} z^{-n}$$

$$= z[n^{p}]$$

$$\Rightarrow \boxed{z[n^{p}] = -z\frac{d}{dz} z[n^{p-1}]}$$

Hence proved.

Put $p=1$

$$z[n] = -z\frac{d}{dz} z[n^{1-1}]$$

$$= -z\frac{d}{dz} z[1] \qquad \because [n^{0}=1]$$

$$= -z\frac{d}{dz}\left[\frac{z}{z-1}\right]$$

$$= -z\left[\frac{(z-1)\cdot 1 - z\cdot 1}{(z-1)^2}\right]$$

$$\boxed{z[n] = \frac{z}{(z-1)^2}}$$

Put $p = 2$

$$Z[n^2] = -z \frac{d}{dz} Z[n^{2-1}]$$

$$= -z \frac{d}{dz} Z[n]$$

$$= -z \frac{d}{dz} \frac{z}{(z-1)^2}$$

$$= -z \left[\frac{(z-1)^2 \cdot 1 - 2z(z-1)}{(z-1)^4} \right]$$

$$= -z \left[\frac{(z-1) - 2z}{(z-1)^3} \right]$$

$$= -z \left[\frac{-z-1}{(z-1)^3} \right]$$

$$\boxed{Z[n^2] = \frac{z(z+1)}{(z-1)^3}}$$

- Property IV :- shifting Property

If $z[f(n)] = F(z)$ then

$$z[f(n+k)] = z^k\left[F(z) - \sum_{i=0}^{k-1} f(i) z^{-i}\right]$$

$k > 0$

→ Proof :-

$$z[f(n)] = F(z) = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$z[f(n+k)] = \sum_{n=0}^{\infty} f(n+k) z^{-n}$$

$$= \sum_{n=0}^{\infty} f(n+k) z^{-(n+k)} \cdot z^k$$

for balance.

$$z[f(n+k)] = z^k \sum_{n=0}^{\infty} f(n+k) z^{-(n+k)} \quad \text{--- (1)}$$

put $n+k = m$ on RHS :-

If $n = 0 \Rightarrow m = k$
& $n \to \infty \Rightarrow m \to \infty$

$$z[f(n+k)] = z^k \sum_{m=k}^{\infty} f(m) z^{-m} \quad \text{--- (2)}$$

$$\sum_{m=0}^{\infty} f(m) z^{-m} = \sum_{m=0}^{k-1} f(m) z^{-m} + \sum_{m=k}^{\infty} f(m) z^{-m} \quad \text{--- (3)}$$

Multiply both Side by z^k

$$z^k \sum_{m=0}^{\infty} f(m) z^{-m} = \sum_{m=0}^{k-1} f(m) z^{-m} + z^k \sum_{m=k}^{\infty} f(m) z^{-m}$$

$z[f(n)] = F(z) = \sum_{n=0}^{\infty} f(n) z^{-n}$

$$z^k F(z) = z^k \sum_{m=0}^{k-1} f(m) z^{-m} + z[f(n+k)]$$

— from eqⁿ (2).

$$z[f(n+k)] = z^k F(z) - z^k \sum_{m=0}^{k-1} f(m) z^{-m}$$

$$= z^k \left[F(z) - \sum_{m=0}^{k-1} f(m) z^{-m}\right]$$

$$= z^k \left[F(z) - \left(f(0) + \frac{f(1)}{z} + \frac{f(2)}{z^2} + \cdots\right)\right]$$

$$= z^k \left[F(z) - \sum_{i=0}^{k-1} f(i) z^{-i}\right]$$

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Ex: Using defination find $z\left[\frac{1}{n!}\right]$ and then by using Shifting property find $z\left[\frac{1}{(n+1)!}\right]$

→ $$z\left[\frac{1}{n!}\right] = \sum_{n=0}^{\infty} \frac{1}{n!} z^{-n}$$

$$= \sum \frac{1}{n! z^n}$$

$$= \frac{1}{0! z^0} + \frac{1}{1! z^1} + \frac{1}{2! z^2} + \cdots$$

$0! = 1$

$$Z[f(n)] = F(z)$$

$$= 1 + \frac{1}{1!}\left(\frac{1}{z}\right) + \frac{1}{2!}\left(\frac{1}{z}\right)^2 + \frac{1}{3!}\left(\frac{1}{z}\right)^3 + \cdots$$

$$= e^{1/z}$$

$$\boxed{Z\left[\frac{1}{n!}\right] = e^{1/z}}$$

By shifting property

$$Z[f(n+k)] = z^k\left[F(z) - \sum_{i=0}^{k-1} f(i) z^{-i}\right]$$

$$Z[f(n+1)] = z^1\left[F(z) - \sum_{i=0}^{1-1} f(i) z^{-i}\right]$$

$$Z[f(n+1)] = z\left[F(z) - f(0)\right]$$

$$Z\left[\frac{1}{(n+1)!}\right] = z\left[e^{1/z} - \frac{1}{0!}\right]$$

$0! = 1$

$$= z\left[e^{1/z} - 1\right]$$

Numerator

Monday

Q. Find inverse z transform of $\frac{z-4}{(z-1)(z-2)^2}$

$$\frac{z-4}{(z-1)(z-2)^2} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

$$(z-4) = A(z-2) + B(z-1)(z-2) + C(z-1)$$

$z=2$

$-2 = C \Rightarrow \boxed{C=-2}$

$z=1$

$-3 = A \Rightarrow \boxed{A=-3}$

$z=0$

$-4 = 4A + 2B - C \Rightarrow -4 = -12 + 2B + 2$

$2B = 6 \Rightarrow \boxed{B=3}$

$$= z^{-1}\left[\frac{z-4}{(z-1)(z-2)^2}\right] = -3z^{-1}\left[z^{-1}\left[\frac{z}{z-1}\right]\right]$$

$$+3z^{-1}\left[z^{-1}\left[\frac{z}{z-2}\right]\right] - 2z^{-1}\left[z^{-1}\frac{z}{(z-2)^2}\right]$$

$$= -3\,1^{n-1}u(n-1) + 3\,2^{n-1}u(n-1) - 2(n-1)\,2^{n-2}u(n-1)$$

$z[2^n] = \frac{z}{z-2}$

Wednesday

• Solution of Difference eq^n By z-transform:

→ To Solve Difference eq^n by z-Transform method

<u>step 1:</u> first take z-transform on both side of eq^n

<u>step 2:</u> By using property & initial values or condition solve z transform.

<u>step 3:</u> Take inverse z-transform & find sol^n.

<u>Ex 1:</u> Solve the difference eq^n $u_{n+2} + 4u_{n+1} + 3u_n = 2^n$, given $u_0 = 0$, $u_1 = 1$

→ Given difference eq^n $u_{n+2} + 4u_{n+1} + 3u_n = 2^n$

$\cancel{z[u_{n+2} + 4u_{n+1}}$

$$z[u_{n+2}] + 4z[u_{n+1}] + 3z[u_n] = z[2^n]$$

rough
$$z[F(n+k)] = z^k\left[F(z) - \sum_{i=0}^{k-1} F(i)\, z^{-i}\right]$$
$$\Rightarrow z^k\left[U(z) - u_0 - \frac{u_1}{z}\right]$$

$$z^2\left[U(z) - u_0 - \frac{u_1}{z}\right] + 4\left[z\left[U(z) - u_0\right]\right] + 3U(z) = \frac{z}{z-2}$$

$u_0 = 0$, $u_1 = 1$

$$z^2\left[U(z) - \frac{1}{z}\right] + 4zU(z) + 3U(z) = \frac{z}{z-2}$$

$$U(z)\left[z^2 + 4z + 3\right] - z = \frac{z}{z-2}$$

$$\cancel{U(z) = \frac{z}{z-2}}$$

$$U(z)\left(z^2+4z+3\right) = \frac{z}{z-2} + z$$

$$= \frac{z + z^2 - 2z}{z-2}$$

$$U(z) = \frac{z^2 - z}{(z-2)(z^2+4z+3)}$$

$$= \frac{z(z-1)}{(z-2)(z+3)(z+1)} \quad \text{--- (1)}$$

$$\frac{z-1}{(z-2)(z+3)(z+1)} \Rightarrow \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{z+1}$$

$$z-1 = A(z+3)(z+1) + B(z-2)(z+1) + C(z-2)(z+3)$$

$z=-3$	$z=-1$	$z=2$
$-4 = 10B$	$-2 = -6C$	$1 = 15A$
$\boxed{B = -\frac{2}{5}}$	$\boxed{C = \frac{1}{3}}$	$\boxed{A = \frac{1}{15}}$

$$U(z) = \frac{1}{15}\frac{z}{z-2} - \frac{2}{5}\frac{z}{z+3} + \frac{1}{3}\frac{z}{z+1}.$$

Take inverse z Transform on both side.

$$z^{-1}[U(z)] = \frac{1}{15} z^{-1}\left[\frac{z}{z-2}\right] - \frac{2}{5} z^{-1}\left[\frac{z}{z+3}\right] + \frac{1}{3} z^{-1}\left[\frac{z}{z+1}\right]$$

$$U_n = \frac{1}{15} 2^n - \frac{2}{5}(-3)^n + \frac{1}{3}(-1)^n$$

Ex:2 Use z-Transform to solve the difference eq^n $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = 0 = y_1$

→

Given difference eq^n

$$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$$

taking z on both Side

$$\Rightarrow z[y_{n+2}] + 6z[y_{n+1}] + 9z[y_n] = z[2^n]$$

$$\Rightarrow z^2\left[y(z) - y_0 - \frac{y_1}{z}\right] + 6z[y(z) - y_0] + 9y(z) = \frac{z}{z-2}$$

$y_0 = y_1 = 0$

$$\Rightarrow z^2 y(z) + 6z y(z) + 9y(z) = \frac{z}{z-2}$$

$$\Rightarrow y(z)[z^2 + 6z + 9] = \frac{z}{z-2}$$

$$\frac{}{(z+3)^2} \quad n(-3)^{n-1}$$

$$\Rightarrow Y(z) = \frac{z}{(z-2)(z^2+6z+9)}$$

$$\Rightarrow Y(z) = \frac{z}{(z-2)(z+3)^2} \quad \text{—} (1)$$

$$\frac{1}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2)$$

$z=-3$

$1 = -5C \Rightarrow \boxed{C = -1/5}$

$z=2$

$1 = A(25)$

$\boxed{A = \frac{1}{25}}$

$z=0$

$1 = \frac{1}{25}(9) + B(-2)(3) - \frac{1}{5}(-2)$

$\Rightarrow 1 - \frac{9}{25} - \frac{2}{5} = -6B$

$\Rightarrow \boxed{B = \frac{-1}{25}}$

$$(1) \Rightarrow Y(z) = \frac{1}{25}\left(\frac{z}{z-2}\right) - \frac{1}{25}\frac{z}{z+3} - \frac{1}{5}\frac{z}{(z+3)^2}$$

$$\Rightarrow z^{-1}[Y(z)] = \frac{1}{25} z^{-1}\left[\frac{z}{z-2}\right] - \frac{1}{25} z^{-1}\left[\frac{z}{z+3}\right] - \frac{1}{5} z^{-1}\left[\frac{z}{(z+3)^2}\right]$$

$$\Rightarrow y_n = \frac{1}{25} 2^n - \frac{1}{25}(-3)^n - \frac{1}{5} n(3)^{n-1}$$

Ex: 3 $\quad y_{n+2} + y_n = 2 \qquad y_0 = y_1 = 0$

$\rightarrow$ Given differential eq^n is

$$y_{n+2} + y_n = 2$$

taking z on both side

$$Z[y_{n+2}] + Z[y_n] = Z[2]$$

$$z^2\left[y(z) - y_0 - \frac{y_1}{z}\right] + y(z) = 2 \cdot Z[1]$$

$$= \frac{2z}{z-1}$$

$y_0 = y_1 = 0 \rightarrow$

$$[z^2+1]\, y(z) = \frac{2z}{z-1}$$

$$y(z) = \frac{2z}{(z^2+1)(z-1)} \quad \text{--- (1)}$$

$$\frac{1}{(z^2+1)(z-1)} = \frac{A}{z-1} + \frac{Bz+C}{z^2+1}$$

$$1 = A(z^2+1) + (Bz+C)(z-1)$$

Put $z = 1$

$1 = 2A \quad \boxed{A = \frac{1}{2}}$

Put $z = 0$

$1 = A - C \Rightarrow C = \frac{1}{2} - 1 \Rightarrow -\frac{1}{2}$

Put $z = -1$

$1 = 2A + (-B + C)(-2)$

$1 = 2\left(\frac{1}{2}\right) + \left(-B - \frac{1}{2}\right)(-2)$

$1 = 1 + 2B + 1$

$2B = -1 \qquad \boxed{B = -\frac{1}{2}}$

① ⇒ $y(z) = 2 \times \frac{1}{2}\left[\frac{z}{z-1}\right] + 2z\left[\frac{-\frac{1}{2}z - \frac{1}{2}}{z^2+1}\right]$

$$y(z) = \frac{z}{z+1} + \frac{-z^2}{z^2+1} - \frac{z}{z^2+1}$$

$$\cancel{z^{-1}[y(z)] = z^{-1}\left[\frac{z}{z+1}\right] - z^{-1}\left[\frac{z^2+z}{z^2+1}\right]}$$

$$z^{-1}[y(z)] = z^{-1}\left[\frac{z}{z+1}\right] - z^{-1}\left[\frac{z^2}{z^2+1}\right] - z^{-1}\left[\frac{z}{z^2+1}\right]$$

$$y_n = 1^n - \cos\frac{n\pi}{2} - \sin\frac{n\pi}{2}$$

Field is known

Thursday

$$z^{-1}[z^{-k}F(z)] = f(n-k)u(n-k)$$

Page No. 88

Date: 17/1/19

1] $y_{n+1} + 2y_n = \delta_n$, given $y_0 = 0$

$\rightarrow$

$$Z[y_{n+1}] + 2Z[y_n] = Z[\delta_n]$$

$$z[y(z) - y_0] + 2y(z) = 1$$

$$y(z)[z+2] = 1$$

$$y(z) = \frac{1}{z+2}$$

$$z^{-1}[y(z)] = z^{-1}\left[\frac{1}{z+2}\right]$$

$$y_n = z^{-1}\left[z^{-1}\left(\frac{z}{z+2}\right)\right]$$

$$= (-2)^{n-1}u(n-1)$$

2] $x_{n+2} + 3x_{n+1} + 2x_n = u_n$, $x_0 = 1$, $x_n = 0$ for $n < 0$.

$\rightarrow$

$$z[x_{n+2}] + 3Z[x_{n+1}] + 2Z[x_n] = Z[u_n]$$

$$z^2\left[X(z) - x_0 - \frac{x_1}{z}\right] + 3z[X(z)] + 2X(z)$$

$$= z^2\left[X(z) - x_0 - \frac{x_1}{z}\right] + 3z\left[X(z) - x_0\right] + 2X(z) = \frac{z}{z-1}$$

$$z^2\left[X(z) - 1 - \frac{x_1}{z}\right] + 3z\left[X(z) - 1\right] + 2X(z) = \frac{z}{z-1}$$

$$= X(z)\left[z^2+3z+2\right] - z^2 - zx_1 - 3z = \frac{z}{z-1} \quad -(1)$$

Put $n=-1$ in given difference eq^n.

$$x_1 + 3x_0 + 2x_{-1} = u_{-1}$$

$$x_1 + 3 + 0 = 0$$

$$\boxed{x_1 = -3}$$

$$(1) \Rightarrow X(z)\left[z^2+3z+2\right] - z^2 + \cancel{3z} - \cancel{3z} = \frac{z}{z-1}$$

$$X(z)(z^2+3z+2) = \frac{z}{z-1} + \frac{z^2}{1}$$

$$X(z) = \frac{z + z^3 - z^2}{(z-1)(z^2+3z+2)}$$

$$= \frac{z(1-z+z^2)}{(z-1)(z+1)(z+2)} \quad -(2)$$

Page No.
Date :

(3)

$$\frac{(1-z+z^2)}{(z-1)(z+1)(z+2)} = \frac{A}{(z+1)} + \frac{B}{(z-1)} + \frac{C}{z+2}$$

$$(1-z+z^2) = A(z-1)(z+2) + B(z+1)(z+2) + C(z+1)(z-1)$$

$z=1$

$1 = 6B \quad \boxed{B = \frac{1}{6}}$

$z=-2$

$7 = 3C \Rightarrow \boxed{C = \frac{7}{3}}$

$z=-1$

$3 = -2A \Rightarrow \boxed{A = -3/2}$

$$X(z) = \frac{-3}{2}\left[\frac{z}{z+1}\right] + \frac{1}{6}\left[\frac{z}{z-1}\right] + \frac{7}{3}\left[\frac{z}{z+2}\right]$$

$$z^{-1}[X(z)] = -\frac{3}{2} z^{-1}\left[\frac{z}{z+1}\right] + \frac{1}{6} z^{-1}\left[\frac{z}{z-1}\right] + \frac{7}{3} z^{-1}\left[\frac{z}{z+2}\right]$$

$$x_n = \frac{-3}{2}(-1)^n + \frac{1}{6}(1)^n + \frac{7}{3}(-2)^n$$

(3) $x_{n+2} - 3x_{n+1} + 2x_n = 4^n, \; x_0 = 0, \; x_1 = 1$

$$x_{n+2} - 3x_{n+1} + 2x_n = 4^n$$

$$Z[x_{n+2}] - 3Z[x_{n+1}] + 2Z[x_n] = Z[$$

$$z^2\left[x(z) - x_0 - \frac{x_1}{z}\right] - 3z\left[x(z) - x_0\right] + 2x(z) = \frac{z}{z-4}$$

$x_0 = 0 \qquad x_1 = 1$

$$\Rightarrow z^2\left[x(z) - \frac{1}{z}\right] - 3z\left[x(z)\right] + 2x(z) = \frac{z}{z-4}$$

$$\Rightarrow z^2\left[x(z) - 1/z\right] - 3z\,x(z) + 2x(z) = \frac{z}{z-4}$$

$$\Rightarrow x(z)\left[z^2 - 3z + 2\right] - z = \frac{z}{z-4}$$

$z^{-1}\left[\frac{z}{z+2}\right]$

$$\Rightarrow x(z)\left[z^2 - 3z + 2\right] = \frac{z}{z-4} + z$$

$$= \frac{z + z^2 - 4z}{z-4}$$

$$x(z) = \frac{z^2}{(z-4)(z^2 - 3z + 2)}$$

Q. Ex: $\sin(3n+5)$ and $\cos(3n+5)$

→ we have

$$e^{i(3n+5)} = e^{3in} \cdot e^{5i}$$

$$= e^{5i}\left[e^{i(3n)}\right]$$

$$Z\left[e^{i(3n+5)}\right] = e^{5i} Z\left[e^{i(3n)}\right]$$

$$= e^{5i} \sum_{n=0}^{\infty} e^{i3n} z^{-n}$$

$$= e^{5i}\left[1 + \frac{e^{3i}}{z} + \frac{e^{6i}}{z^2} + \cdots\right\}$$

$$= e^{5i}\left[1 + \frac{e^{3i}}{z} + \left(\frac{e^{3i}}{z}\right)^2 + \left(\frac{e^{3i}}{z}\right)^3 + \cdots\right.$$

$$= e^{5i}\left[\left(1 - \frac{e^{3i}}{z}\right)^{-1}\right]$$

$$= e^{5i}\left[\frac{1}{1 - e^{3i}/z}\right]$$

$$= e^{5i} \frac{z}{z - e^{3i}}$$

$$= e^{5i}\left\{\frac{z}{z - e^{3i}} \times \frac{z - e^{-3i}}{z - e^{-3i}}\right]$$

$$= e^{5i}\left\{\frac{z^2 - z e^{-3i}}{z^2 - z(e^{-3i} + e^{3i}) + e^0}\right\}$$

$e^{5i} \to \cos 5 + i\sin 5$

$$= e^{5i}\left\{\frac{z^2 - z e^{-3i}}{z^2 - z(e^{-3i}+e^{3i}) + e^0}\right\}$$

$$= e^{5i}\left\{\frac{z^2 - z(\cos 3 - i\sin 3)}{z^2 - 2z\cos 3 + 1}\right\}$$

$$= \frac{(\cos 5 + i\sin 5)(z^2 - z\cos 3 + iz\sin 3)}{z^2 - 2z\cos 3 + 1}$$

$$= \frac{z^2\cos 5 - z\cos 3\cos 5 + iz\cos 5\sin 3 + iz^2\sin z - iz\sin 5\cos 3 - z\sin 3\sin 5)}{z^2 - 2z\cos 3 + 1}$$

$$= \frac{z^2\cos 5 - z[\cos 3\cos 5 + \sin 3\sin 5) + iz[\cos 5\sin 3 - \sin 5\cos 3] + iz^2\sin z}{z^2 - 2z\cos 3 + 1}$$

$$= \frac{z^2\cos 5 - z\cos(3-5) + iz\sin(5+3) + iz^2\sin z}{z^2 - 2z\cos 3 + 1}$$

$$= z\left[e^{i(3n+5)}\right] = \frac{z^2\cos 5 - z\cos 2 + iz\sin 2 + i z^2\sin z}{z^2 - 2z\cos 3 + 1}$$

$$z\left[\cos(3n+5) + i\sin(3n+5)\right] = \frac{z^2\cos 5 - z\cos 2 + iz\sin 2 + iz^2\sin z}{z^2 - 2z\cos 3 + 1}$$

$$\therefore\ z\left[\cos(3n+5)\right] = \frac{z^2\cos 5 - z\cos 2}{z^2 - 2z\cos 3 + 1}$$

$$z\left[\sin(3n+5)\right] = \frac{-z\sin 2 + z^2\sin 5}{z^2 - 2z\cos 3 + 1}$$

Find z transform of $\frac{(n+1)(n+2)}{2!}a^n$

$$\rightarrow Z\left[\frac{(n+1)(n+2)}{2!}a^n\right] = \sum_{n=0}^{\infty}\frac{(n+1)(n+2)}{2!}a^n z^{-n}$$

Put $n=0$

$$= \frac{1\cdot\cancel{2}}{\cancel{2}!}\frac{a^0}{z^0} + \frac{\cancel{2}\cdot 3}{\cancel{2}!}\frac{a^1}{z^1} + \frac{3\cdot\overset{2}{\cancel{4}}}{\cancel{2}!}\frac{a^2}{z^2} + \frac{\overset{2}{\cancel{4}}\cdot 5}{\cancel{2}!}\frac{a^3}{z^3} + \cdots$$

$$= \cancel{1 + \frac{3a}{z} + 3\cdot 2\left(\frac{a}{z}\right)^2}$$

$$= 1 + \frac{3a}{z} + 3\cdot 2\left(\frac{a}{z}\right)^2 + 2\cdot 5\left(\frac{a}{z}\right)^3 + \cdots$$

$$= 1 + \frac{3a}{z} + \frac{3\cdot 4(a/z)^2}{2!} + \frac{3\cdot 4\cdot 5}{3!}\left(\frac{a}{z}\right)^3 + \cdots$$

$$= \left(1-\frac{a}{z}\right)^{-3}$$