B.E. (Computer Engineering / Information Technology) Third Semester (C.B.S.)

Applied Mathematics – III

P. Pages : 3

NIR/KW/18/3327/3332

Time : Three Hours

* 1 8 6 1 *

Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \overline{f}(s)$ then prove that **7**

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f}(s)\,ds$$

Hence Find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$

b) Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ by using convolution theorem. **7**

OR

2. a) Express **7**

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$$

in terms of unit step function and hence find its Laplace transform.

b) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given $y(0) = 1$ by Laplace transform method. **7**

3. Find Fourier transform of **7**

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$ and hence

find $\int_0^\infty \frac{\sin x}{x}\,dx$.

OR

4. Using Fourier integral, show that **7**

$$\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin\lambda x\, d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

5. a) Find Z-transform of $\sin n\theta$ and $\cos n\theta$. **7**

b) If $Z\{f(n)\} = F(z)$ then prove that **7**

$$Z\left\{\frac{f(n)}{n+k}\right\} = z^k \int_z^\infty \frac{F(z)}{z^{k+1}} dz$$

OR

6. a) Find the inverse Z-transform of **7**

$$\frac{z^2+z}{(z-1)(z^2+1)}$$

b) Solve the difference equation **7**

$y_{n+2} + 4y_{n+1} + 3y_n = 2^n, y_0 = 0, y_1 = 1$
using Z-transform.

7. a) Show that the vectors $x_1 = [2, -1, 3, 2]$ $x_2 = [1, 3, 4, 2]$, $x_3 = [3, -5, 2, 2]$. are linearly dependent. Also express one of these as the linear combination of the others. **6**

b) Find eigen value, eigen vector and modal matrix for **6**

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

c) Verify Cayley-Hamilton theorem for the matrix **6**

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

and hence find A^{-1}.

OR

8. a) Using Sylvester's theorem, verify **6**

$$\log_e e^A = A \text{ where } A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

b) Solve by matrix method **6**

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0, \text{ given } y(0) = 5,\ y'(0) = 8$$

c) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^3 - 12x, x_2 + 4x, x_3 - 8x_2x_3$ to canonical form by orthogonal transformation. **6**

9. a) Three machines A, B, C produce respectively 60%, 30% and 10% of the total no. of items in a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%, An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **7**

b) The distribution function of random variable X is $F(x)=\begin{cases}1-e^{-2x}, & x\geq 0\\ 0 & , x<0\end{cases}$ **7**

Find

i) Density function f(x) ii) $P(X>2)$ iii) $P(-3<X<4)$.

OR

10. a) A random variable X denotes the number of heads in three tosses of a fair coin. Find the probability function f(x) and the distribution function F(x). **7**

b) Let X and Y be continuous random variables having joint density function **7**

$$f(x, y)=\begin{cases}C\left(x^2+y^2\right), & 0\leq x\leq 1,\ 0\leq y\leq 1\\ 0 & , \quad \text{otherwise}\end{cases}$$

Determine

i) Constant C

ii) $P\left(X<\frac{1}{2},\ Y>\frac{1}{2}\right)$

iii) Marginal density function of X and Y.

11. a) Let X be random variable with density function **7**

$$f(x)=\begin{cases}2e^{-2x} & , \quad x\geq 0\\ 0 & , \quad \text{otherwise}\end{cases}$$

Find (i) E(X) (ii) $E\left(X^2+5\right)$ (iii) var(X) (iv) S.D. of X.

b) Find the moment generating function of random variable **6**

$$X=\begin{cases}1, & \text{Prob. } \frac{1}{2}\\ -1, & \text{Prob. } \frac{1}{2}\end{cases}$$

Hence find first four moments about origin.

OR

12. a) The joint density function of two random variables X and Y is given by **7**

$$f(x, y)=\begin{cases}\frac{xy}{96}, & 0<x<4, 1<y<5\\ 0 & , \text{otherwise}\end{cases}$$

Find: (i) E(X) (ii) E(Y) (iii) E(XY) (iv) E(2x+3y).

b) Suppose that the customers are arriving at a ticket counter according to a Poisson process with mean rate of 2 per minutes. Then in an arrival of 5 minutes, find the probability that the number of customers arriving is **6**

(i) Exactly 5 (ii) Less than 4 (iii) greater than 3.

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