B.E. (Computer Engineering / Information Technology) Third Semester (CBS)

Applied Mathematics-III

P. Pages : 4 AHK/KW/19/2091/2096

Time : Three Hours * 1 6 1 1 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\}=\bar{f}(s)$, then show that **7**

$$L\left\{\int_0^t f(u)\,du\right\}=\frac{\bar{f}(s)}{s}.$$ Hence find the value of $L\left\{\int_0^t \sin u\,du\right\}$.

b) Find $L^{-1}\left\{\dfrac{1}{s\left(s^2+9\right)}\right\}$ by convolution theorem. **7**

OR

2. a) Express $f(t)=\begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$, in terms of unit step function and hence find its Laplace transform. **7**

b) Solve : **7**

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t,\ y(0)=0,\ y'(0)=1.$$

3. Find Fourier transform of $f(x)=\begin{cases} 1, & \text{for}\ |x|<1 \\ 0, & \text{for}\ |x|>1 \end{cases}$ **7**

hence find $\int_0^\infty \frac{\sin x}{x}\,dx.$

OR

4. Solve the integral equation $\int_0^\infty f(x)\cos\lambda x\,dx=e^{-\lambda},\lambda>0.$ **7**

5. a) If $z\{f(n)\} = F(z)$ then prove that 7

$$z\left\{\frac{f(n)}{n+k}\right\} = z^k \int_z^\infty \frac{F(z)}{Z^{k+1}} dz.$$

b) Find z-transform of $\sin n\theta$ and $\cos n\theta$. 7

OR

6. a) Find the inverse z-transform of $\left\{\frac{z^2}{(z-1)(z-3)}\right\}$ using convolution theorem. 7

b) Solve the difference equation. 7

$$y_{n+2} + y_n = 2 \;, \quad y_0 = 0, \; y_1 = 0$$

7. a) Using the concept of matrix, show that the vectors 6
$X_1 = [2,3,1,-1]$, $X_2 = [2,3,1,-2]$, $X_3 = [4,6,2,-3]$.
are linearly dependent. Find the relation bet^n them.

b) Find eigen value, eigen vector and model matrix for 6

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

c) Use Sylvester's theorem to verify $\log_e e^A = A$, where 6

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}.$$

OR

8. a) Verify Cayley - Hamilton theorem & hence find A^{-1}, where 6

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}.$$

b) Solve $\frac{d^2y}{dx^2} + 4y = 0$ given that $y(0) = 8$, $y'(0) = 0$ by matrix method. 6

c) Find largest eigen value & corresponding eigen vector for matrix 6

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

9. a) Three Machines A, B, C produce respectively 60%, 30% and 10% of the total number of items in a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and found defective. Find the probability that item was produced by machine B. **7**

b) Can the function **7**

$$f(x) = \begin{cases} C(1-x^2) & , \quad 0 \le x \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

be a distribution function? Explain.

OR

10. a) Let X and Y be continuous random variables having joint density function **7**

$$f(x, y) = \begin{cases} C(x^2 + y^2) & , \quad 0 \le x \le 1, 0 \le y \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Determine

i) Constant.

ii) $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$.

iii) Marginal density functions of x & y.

b) Let X and Y be two random variables with mass function **7**

$$f(x, y) = \begin{cases} \frac{x+2y}{27} & , \quad x = 0, 1, 2, \; y = 0, 1, 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find

i) Find marginal probability function of x & y.

ii) Conditional probability function y given x and x given y.

11. a) Find the moment generating function of random variable **7**

$$X = \begin{cases} 1/2 & \text{prob } 1/2 \\ -1/2 & \text{prob } 1/2 \end{cases}$$

Hence find four moments about origin.

b) A density function of random variable X is **6**

$$f(x) = \begin{cases} 2e^{-2x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Find

i) E (X) ii) Var (X)

iii) σ_X iv) $E\left[(X-1)^2\right]$.

OR

12. a) The joint density function of two random variables x and y is given by **7**

$$f(x,y)=\begin{cases}\dfrac{xy}{96}, & 0<x<4,\ 1<y<5\\ 0, & \text{otherwise}\end{cases}$$

Find

i) E (x)

ii) E (y)

iii) E (xy)

iv) E (2x + 3y)

b) State the postulates of Poisson process and prove that a Poisson process follow a Poisson distribution. **6**

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