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Unit - VI

Special Probability Distribution And Random Process

Binomial Distribution

Consider A random experiment with only two possible outcomes success or failure let 'p' denotes Probability of success and $q = 1-p$ denotes probability of failure. Suppose that the experiment is independently repeated 'n' times and x represents the number of successes in n trials.

Hence x assumes the values

$0, 1, 2, \ldots n$

Then probability of exactly x successes in n trials is given by the probability function

$$f(x) = P(X=x) = {}^nC_x p^x q^{n-x} \quad x = 1, 2, 3, \ldots n$$

This probability distribution is called Binomial or Bernoulli distribution

Note :- $(p+q)^n = {}^nC_0 p^0 q^n + {}^nC_1 p^1 q^{n-1} + {}^nC_2 p^2 q^{n-2} \ldots\ldots {}^nC_n p^n q^{n-n}$

The discrete random variable x having above probability distribution $f(x)$ is said to be binomially distributed

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If the random variable x is binomially distributed then

① Mean $\mu = np$

② Variance $\sigma^2 = npq$

③ Standard deviation $\sigma = \sqrt{npq}$

④ Moment generating function

$$M_x(t) = (q + pe^t)^n$$

⑤ Characteristic function

$$\phi(\omega) = (q + pe^{i\omega})^n$$

Q.1) Out of 800 family with 5 children each How many would you expect to have

(i) 3 boys (ii) 5 girls

(iii) either 2 or 3 boys

Assume equal probability for boys and girls

→

Total No of families

$N = 800$

Let p and q be the probabilities of success and failure. Let n be the of trials. Probability of boys and girls are same

$p = q = 1/2$

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$n = 5$

i) $P(x=3) = {}^nC_x p^x q^{n-x}$

$= {}^5C_3 (1/2)^3 (1/2)^{5-3}$

$\boxed{P(x=3) = 5/16}$

$\therefore$ No of families having exactly 3 boys is $5/16 \times 800 = 250$

(ii) $P(x=5) = {}^5C_5 (1/2)^5 (1/2)^0$

$P(x=5) = 1/32$

$\therefore$ No of families having 5 girls are $\frac{1}{32} \times 800 = 25$

(iii) $P(x=2, x=3) = P(x=2) + P(x=3)$

$= {}^xC_n p^x q^{n-x} + {}^xC_n p^x q^{n-x}$

$= {}^2C_5 p^2 q^3 + {}^3C_5 p^3 q^2$

$= 5/8$

$\therefore$ No of families having 2 or 3 boys are $5/8 \times 800 = 500$

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Q.2) An insurance sells man sells policies to 5 men all of identical edge and in good Health the probability of that a man of an particular edge will be alive 30 years is 2/3. Find the probability that in 30 years

i) all 5 men 2) at least 3 men

(iii) at the most 1 man

(iv) at least 1 men will be alive

→ Here The $p = 2/3$

$q = 1 - p = 2/3 - 1$

$q = 1/3$

we have no of trials $n = 5$

Let p be the probability of success

$\boxed{p = 2/3}$

$q = 1 - 2/3$

$\boxed{q = 1/3}$

(i) $P(x=5) = \cancel{x C_n}$

$= nC_x p^x q^{n-x}$

$= 5C_5 p^5 q^0$

$= 5C_5 \times (2/3)^5$

$$\boxed{P(x=5) = \frac{32}{243}}$$

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(2) $P(\text{at least } 3) = P(x \geq 3)$

$= P(x=3) + P(x=4) + P(x=5)$

$= {}^nC_x p^x q^{n-x} + {}^nC_x p^x q^{n-x} + {}^nC_x p^x q^{n-x}$

$= {}^5C_3 p^3 q^2 + {}^5C_4 p^4 q^1 + {}^5C_5 p^5$

$= \frac{40}{243} + \frac{10}{243} + \frac{32}{243}$

$= \frac{82}{243}$

$= \frac{64}{81}$

(3) $P(\text{at most 1 man})$.

$= P(x \leq 1) = P(x=0) + P(x=1)$

$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4$

$= {}^5C_0 p^0 \times (\frac{2}{3})^5 + {}^5C_1 (\frac{1}{3})^1 (\frac{2}{3})^4$

$= \frac{11}{243}$

(4) $P(\text{At least one man will come}) = P(x \geq 1)$

$= P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= 1 - P(x=0)$

$= 1 - {}^5C_0 (\frac{1}{3})^0 (\frac{2}{3})^5$

$= 1 -$

$= \frac{242}{243}$

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Q.3) Find the probability of getting total of 7
① At least once
② At the most twice
In the 5 tosses of a pair of fair dice

→ In a given event

$n(S) = 36$

$$p = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$q = 1 - p$

$q = 1 - 1/6$

$\boxed{q = 5/6}$

$\boxed{n = 5}$

① P(getting total 7 at least once)

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - {}^nC_x \, p^x q^{n-x}$

$= 1 - 5C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5$

$= \frac{4651}{7776}$

$\boxed{P(X \geq 1) = 0.5981}$

② P (at the most twice) $= P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= 5C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 + 5C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4 + 5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

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$P(X \le 2) = \frac{625}{648}$

$\boxed{P(X \le 2) = 0.9645}$

(W-15)

The probability of an item produced by certain machine will be defective is 0.05. If the produced items are sent to the market in packet of 20 find the no of packet containing atleast, exactly and at the most two defective items in the consignment of 1000 packets.

→

let the probability of an item produced by certain machine is defective is

$p = 0.05$

$q = 1-p = 1-0.05$

$q = 0.95$

$n = 20$

$N = 1000$

(i) $P(\text{at least } 2) = P(x \ge 2)$

$= 1 - P(x \le 1)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [nC_x p^x q^{n-x} + nC_x p^x q^{n-x}]$

$= 1 - [20C_0 (0.05)^0 (0.95)^{20} + 20C_1 (0.05)^1 \times (0.95)^{19}]$

$= 1 - [0.3584 + 0.3773]$

$P(x \ge 2) = 0.2643$

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from: 1000 items

$\therefore$ Probability of at least 2 defective item

$= \cancel{0.2642 \times 1000}$

$= \cancel{264.2}$ 264.2

ii) $P(\text{exactly } 2) = P(x=2)$

$= {}^nC_x p^x q^{n-x}$

$= {}^{20}C_2 (0.05)^2 (0.95)^{18}$

$\boxed{P(x=2) = 0.1886}$

$\therefore$ Probability of exactly 2 defective items from 1000 is 0.1886×1000
$= 188.6$

iii) $P(\text{at the most } 2) = P(x \leq 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= 0.3584 + 0.3773 + {}^{20}C_2 (0.05)^2 (0.95)^{18}$

$= 0.3584 + 0.3773 + 0.1886$

$P(x \leq 2) = \cancel{0.9448}$ 0.9243

$\therefore$ Probability of at the most 2 defective items of 1000 items $= 0.9243 \times 1000$

$= \cancel{944.8}$

$= 924.3$

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(5-16)

If 3% of electric bulb manufactured by company are defective. Find the probability that in a sample of 100 bulbs

i) Exactly 2

ii) Between 1 and 3

iii) At most 2 bulb will be defective.

→ Let p be the probability of defective items to be produced by the manufacturing company.

$$p = \frac{3}{100} = 0.03$$

$$q = 1 - 0.03$$

$$\therefore \boxed{q = 0.97}$$

Here, $\boxed{n = 100}$

(i) P(Exactly two bulbs are defective) $= P(x=2)$

$$P(x=2) = {}^nC_x \, p^x \, q^{n-x}$$

$$= {}^{100}C_2 \, (0.03)^2 \, (0.97)^{98}$$

$$P(x=2) = 0.2251$$

ii) P(defective bulbs between 1 and 3) $= P(1 \le x \le 3)$

$$= P(x=1) + P(x=2) + P(x=3)$$

$$= {}^{100}C_1 \, (0.03)^1 \, (0.97)^{99} + 0.2251 + {}^{100}C_3 \, (0.03)^3 \, (0.97)^{97}$$

$$= 0.1470 + 0.2251 + 0.2274$$

$$= 0.5995$$

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iii) P (at most 2) = P ($x \le 2$)

$= P(x=0) + P(x=1) + P(x=2)$

$= 0.0478 + 0.1470 + 0.2254$

$= 0.4196$

The probability of any ship of a company being destroyed on a certain voyage is 0.02. The company owns 6 ships for the voyage. What is the probability of i) losing one ship
ii) losing at the most 2 ships
iii) losing none?

→ The probability of any ship being destroyed p

$\boxed{p = 0.02}$

$\therefore q = 1 - p = 1 - 0.02$

$\boxed{q = 0.98}$

Here $n = 6$

(i) P (losing one ship) = $P(x=1)$

$= {}^nC_x p^x q^{n-x}$

$= {}^6C_1 (0.02)^1 (0.98)^5$

$\boxed{P(x=1) = 0.1084}$

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ii) P(losing at most 2 ships) = $P(x \le 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= 0.8858 + 0.1084 + 0.005$

$\boxed{P(x \le 2) = 0.9992}$

iii) P(losing none) = $P(x=0)$

$= {}^nC_x p^x q^{n-x}$

$= 6C_0 (0.02)^0 (0.98)^6$

$\boxed{P(x=0) = 0.8858}$

In sampling a large no of parts manufactured by a machine. The mean no of defectives in a sample of 20 is 2 out of 1000 such samples how many would be expected to contain at least 3 defective parts.

→ Mean, $\mu = np$

Here $n = 20$

$\therefore p = \frac{2}{20} = 0.1$

$\therefore \boxed{p = 0.1}$

$\therefore q = 1 - p$

$\boxed{q = 0.9}$

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4. P(at least 3 defective sample)

$= P(X \geq 3)$

$= 1 - P(X \leq 3)$

$\therefore P(X \geq 3) = 1 - [P(x=0) + P(x=1) + P(x=2)]$

$P(X \geq 3) = 1 - [{}^nC_x p^x q^{n-x} + {}^nC_x p^x q^{n-x} + {}^nC_x p^x q^{n-x})$

$= 1 - [{}^{20}C_0 (0.1)^0 (0.9)^{20} + {}^{20}C_1 (0.1)^1 (0.9)^{19} + {}^{20}C_2 (0.1)^2 (0.9)^{18}.$

$= 1 - [0.1215 + 0.2701 + 0.2851]$

A discrete random variable x is binomially distributed with mean 6 and variance 2 find probability that $5 \leq x \leq 9$

→ Mean $\mu = np$ — ①

variance $\sigma^2 = npq$ — ②

Random variable x is binomially distributed

$npq = 2 \Rightarrow 6q = 2$

$q = \frac{2}{6} = 1/3$

$\boxed{q = 0.3333}$

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$q = 1 - p$

$p = 1 - 0.3333$

$\boxed{p = 0.6667}$

$\therefore \boxed{p = \frac{2}{3}}$

$npq = 6$

$\therefore n = \frac{6}{\frac{2}{3} \cdot \frac{1}{3}}$

$\boxed{n = 9}$

$\therefore P(5 \le x \le 7) = P(x=5) + P(x=6) + P(x=7)$

$= {}^9C_5 (2/3)^5 (1/3)^4 + {}^9C_6 (2/3)^6 (1/3)^3$
$+ {}^9C_7 (2/3)^7 (1/3)^2$

$= 0.2048 + 0.2731 + 0.2341$

$= 0.712$

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If 20% of bolts produced by machine are defective, determine the probability that out of 4 bolts choose at random
i) 1 ii) 0 3) less than 2 bolts will be defective

→

20% of bolts produced by machine are defective

$\therefore \quad p = \frac{20}{100} =$

$\boxed{p = 0.2}$

$\therefore \quad q = 1 - p$

$\therefore \quad q = 1 - 0.2$

$\boxed{q = 0.8}$

Here $n = 4$.

i) P(1 bolt is defective) = $P(X=1)$

$P(X=1) = {}^nC_x \, p^x q^{n-x}$

$= {}^4C_1 (0.2)^1 (0.8)^3$

$P(X=1) = 0.4096$

ii) P(getting 0 bolts defective) = $P(X=0)$

$= {}^4C_0 (0.2)^0 (0.8)^4$

$\boxed{P(X=0) = 0.4096}$

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iii) P(defective bolts less than 2) = $P(X<2)$

$= P(X=0) + P(X=1)$

$= 0.4096 + 0.4096$

$= 0.8192$

Find variance, standard deviation of the no of coins which comes upon a single toss of a die.

→ No of head points are

$n = 1$

$p = 1/6$

$q = 1 - p = 1 - \frac{1}{6} = 5/6$

variance $= npq = 1 \times \frac{1}{6} \times 5/6$

$\boxed{\sigma^2 = \frac{5}{36}} = 0.13888$

standard deviation = $\sqrt{\text{variance}}$

$= \sqrt{5/36}$

$\boxed{\sigma = \frac{\sqrt{5}}{6}} = 0.37267$

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Poisson Distribution

If the discrete random variable X has the probability function

$$f(x) = P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$x = 0, 1, 2, 3 \ldots$$

where λ is positive constant then X is said to be poisson distributed and the probability distribution $f(x)$ is called poisson distribution.

1. mean $= \mu = np = \lambda$
2. variance $\sigma^2 = \lambda$
3. standard deviation $\sigma = \sqrt{\lambda}$
4. Moment generating function

$$M_X(t) = e^{\lambda(e^t - 1)}$$

5. characteristic function

$$\boxed{\phi(\omega) = e^{\lambda(e^{i\omega} - 1)}}$$

6. coefficient of skewness

$$\boxed{\alpha_3 = \frac{1}{\sqrt{\lambda}}}$$

7. coeff of kurtosis =

$$\boxed{\alpha_4 = 3 + (1/\lambda)}$$

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Q.1) If 3% of the electric bulb manufactured by a company are defective find the probability that in a sample of 100 bulbs

(i) Exactly 2

(ii) More than 5

(iii) between 1 and 3 (inclusive 1 and 3)

(4) At the most 2

(5) At least 2 bulbs.

→ let $P = 3\%$

$$\boxed{P = 0.03}$$

mean $\lambda = np = 100 \times 0.03$

$$\therefore \boxed{\lambda = 3}$$

i) $P(X=2) = \dfrac{e^{-\lambda}\lambda^{x}}{x!} = \dfrac{e^{-3}\,3^{2}}{2!}$

$$\boxed{P(X=2) = 0.2240}$$

ii) $P(x>5) = 1 - P(X \le 5)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$

$= 1 - \left[\dfrac{e^{-\lambda}\lambda^{x}}{x!} + \dfrac{e^{-\lambda}\lambda^{x}}{x!} + \dfrac{e^{-\lambda}\lambda^{x}}{x!} + \dfrac{e^{-\lambda}\lambda^{x}}{x!} + \dfrac{e^{-\lambda}\lambda^{x}}{x!} + \dfrac{e^{-\lambda}\lambda^{x}}{x!}\right]$

$= 1 - [0.0497 + 0.1493 + 0.2240 + 0.2240 + 0.1680 + 0.1008]$

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$$\boxed{P(X \geq 5) = 0.0842}$$

iii) $P(1 \leq x \leq 3) = P(x=1) + P(x=2) + P(x=3)$

$$= \frac{e^{-3} \times (3)^1}{1!} + \frac{e^{-3} \times (3)^2}{2!} + \frac{e^{-3} (3)^3}{3!}$$

$$= 0.1493 + 0.2240 + 0.2240$$

$$\boxed{P(1 \leq x \leq 3) = 0.5973}$$

(4) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$$= 0.4231$$

(5) $P(x \geq 2) = 1 - P(x < 2)$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 0.8008$$

Q.2) Between 2pm and 4pm the average no of phone calls per minute comming to the switch board of the company is 2.35 find the probability that during one perticular minute there will be at the most two phone calls.

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→ Average no of phone calls are given
i.e. mean $= \lambda$

$$\therefore \boxed{\lambda = 2.35}$$

P(at the most 2 phone called per minute)

$$= P(X \le 2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \left[\frac{e^{-\lambda}\lambda^{x}}{x!}\right] \text{ for } x = 0,1,2.$$

$$= \frac{e^{-2.35}(2.35)^0}{0!} + \frac{e^{-2.35}(2.35)^1}{1} + \frac{e^{-2.35}(2.35)^2}{2!}$$

$$= 0.5282.$$

If the continuous random variable X has the density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$$

where μ and σ are mean and standard deviation respectively. Then X is said to be normally distributed and probability function f(x) is called normal distribution.

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Normal curve is the graph of probability density function form is bell shaped and symetrical about the mean line $x = \mu$.

and X and P an asymptote to the curve. The mean mode and median of the distribution coincide.

If Z is the standard variable corresponding to X then $\boxed{z = \frac{x-\mu}{\sigma}}$

The mean of Z is zero and standard deviation is 1.

Properties

1. Mean of X is μ
2. variance σ^2
3. standard deviation σ
4. Moment generating function

$$M_x(t) = e^{\mu t + (\sigma^2 t^2/2)}$$

5. characteristic function

$$\phi(\omega) = e^{i\mu\omega - (\sigma^2\omega^2/2)}$$

6. coefficient of skewness. $\boxed{\alpha_3 = 0}$
7. coeff of kurtosis $\boxed{\alpha_4 = 3}$

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Poisson Numerical

Q.3) It is known that on an average 0.3% of the results obtained on a computer are not suitable. Assuming that poisson probability law is followed, find the probability that from a total of 1000 results

i) Exactly 2 are not suitable

ii) At the most 3 results are not suitable

→ Let P be the probability of getting not suitable result.

0.3% of the results are not suitable

$\therefore \quad P = 0.3\%$

$P = \frac{0.3}{100} = 0.003$

Here $n = 1000$

$\therefore$ mean r $\lambda = np$

$\lambda = 0.003 \times 1000$

$\boxed{\lambda = 3}$

i) $P(x=2) = \frac{e^{-\lambda} \lambda^x}{x!}$

$= \frac{e^{-3} (3)^2}{2!}$

$\boxed{P(x=2) = 0.2240}$

ii) $P(X \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= \sum_{x=0}^{3} \frac{e^{-\lambda} \lambda^x}{x!} = \sum_{x=0}^{3} \frac{e^{-3} 3^x}{x!}$

$\boxed{P(X \leq 3) = 0.6472}$

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Q.4) A car hire firm has two cars, which it hires out day by day. The no of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.

→ Given, mean $= \lambda = 1.5$

i) $P(\text{neither car used}) = P(X=0)$

$$= \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \frac{e^{-1.5}(1.5)^{0}}{0}$$

$$\boxed{P(X=0) = 0.2231}$$

ii) Probability of demand is refused

$$\text{or} \quad P(X \geq 2) = 1 - P(X \leq 2)$$

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= 1 - \sum_{x=0}^{2} \frac{e^{-1.5}(1.5)^{x}}{x!}$$

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$\therefore \quad P(x>2) = 1 - 0.8088$

$$\boxed{P(x>2) = 0.19115}$$

Q.5) Out of 1000 balls 50 are red and the rest white. If 80 balls are selected at random what is the probability of selection of (i) 3 red balls
(ii) Not more than 3 red ball in the sample

→ Let p be the probability of getting red balls.

$$p = \frac{50}{1000}$$

$$\boxed{p = 0.05}$$

Here $n = 80$

$\therefore \quad \lambda = np = 80 \times 0.05$

$$\boxed{\lambda = 4}$$

i) P (getting 3 red balls) = $P(X=3)$

$$= \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \frac{e^{-4}(4)^3}{3!}$$

$$\boxed{P(X=3) = 0.19536}$$

ii) P (getting not more than 3 red balls)

$= P(X \leq 3)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$= \sum_{x=0}^{3} \frac{e^{-\lambda}\lambda^{x}}{x!} = \sum_{x=0}^{3} \frac{e^{-4}(4)^{x}}{x!}$$

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$\therefore \boxed{P(x \le 3) = 0.4334}$

*

Q.6) If the no of telephones calles coming into a telephone exchange between 8 am and 9 am and between 9 am and 10 am are independent. and follows poisson distributions with parameters 3 and 5 respectively. what is the probability that more than 4 calls comes between 8 am and 10 am

→ Two parameters for two different slots are given.

$$\lambda_1 = 3, \quad \lambda_2 = 5$$

$$\lambda = \lambda_1 + \lambda_2 = 8$$

$$\boxed{\lambda = 8}$$

$\therefore$ Probability of more than 4 calls comes between 8am and 10am.

$$P(x > 4) = 1 - P(x \le 4)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)]$$

$$= 1 - \sum_{x=0}^{4} \left[\frac{e^{-\lambda} \lambda^{x}}{x!} \right]$$

$$= 1 - \left[\frac{e^{-8} 8^{0}}{0!} + \frac{e^{-8} 8^{1}}{1!} + \frac{e^{-8} 8^{2}}{2!} + \frac{e^{-8} 8^{3}}{3!} + \frac{e^{-8} 8^{4}}{4!} \right]$$

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$\therefore P(x>4) = 1 - 0.0996$

$\boxed{P(x>4) = 0.9003}$

7) Suppose that x have a poisson distribution if $P(x=2) = 2/3\, P(x=1)$ find

i) Mean

ii) variance

iii) $P(x=0)$ iv) $P(x=3)$

v) $E[x^2]$

→ Given, x has poisons distribution and

$P(x=2) = 2/3\, P(x=1)$

$\frac{e^{-\lambda}\lambda^{x}}{2!} = 2/3 \left[\frac{e^{-\lambda}\lambda^{1}}{1!}\right]$

from given.

$\lambda = 2/3 \cdot 2$

$\boxed{\lambda = 4/3}$

$\therefore$ i) Mean $= \lambda = 4/3 = E[x]$

ii) variance $= \lambda = 4/3$

iii) $P(x=0) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-4/3} \cdot (4/3)^0}{0!}$

$\boxed{P(x=0) = 0.2635}$

iv) $P(x=3) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-4/3}(4/3)^3}{3!}$

$\boxed{P(x=3) = 0.1041}$

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v) Variance of $x = E[x^2] - [E(x)]^2$

$$4/3 = E(x^2) - [4/3]^2$$

$$4/3 + \frac{16}{9} = E[x^2]$$

$$\boxed{E[x^2] = \frac{28}{9}}$$

Normal Distribution

Q.1) If the diameters of ball bearing are normally distributed with mean 15.60 mm and standard deviation 0.06 mm. Determine the percentage of ball bearing with diameters a) between 15.50 and 15.70 mm [Mumbai]

b) Greater than 15.70 mm.

c) less than 15.40 mm.

d) Equal to 15.60 mm.

Assume the measurements to be recorded to the nearest 0.01 mm

→ Let X denotes the diameter of ball bearings which is normally distributed with mean $\mu = 15.60$ and standard deviation $\sigma = 0.06$

Let Z be the standardised random variable given by

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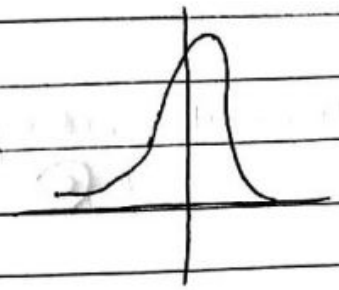
$$Z = \frac{x-\mu}{\sigma} = \frac{x-15.60}{0.06}$$

(a) (15.49 in std units)

$$= \frac{15.49-15.60}{0.06}$$



$$= -1.833$$

(15.71 in standard unit) $= \frac{15.71-15.60}{0.06}$

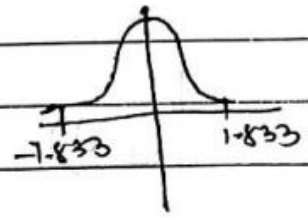
$$= 1.833$$

Area between (15.50 and 15.70)

= (Area between -1.833 and 1.833)

= 2 [Area between 0 to 1.833]

= 2 [0.4664]



= 0.9328

= 93.28%

∴ Percentage of ball bearing with diameter between 15.50 mm and 15.70 mm is 93.28%

b) Greater than 15.70mm

15.71 in standard units

$$= \frac{15.71-15.60}{0.06} = \frac{x-\mu}{\sigma}$$

$$= 1.833$$

(greater than 15.70 is greater than 1.833

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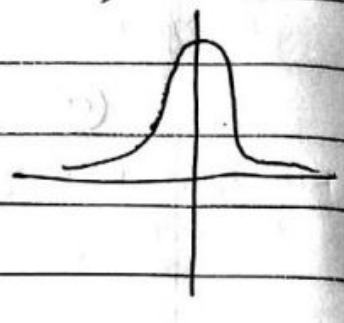
$= 0.5 -$ (Area between 1.833)

$= 0.5 - 0.4664$

$= 0.0336$

$= 0.0336 \times 100 = 3.36\%$



(c) (less than 15.30)

(15.39 in standard unit)

$= \frac{15.39 - 15.60}{0.06}$

$= -3.5$

$\therefore$ (less than 15.39 i.e less than -3.5)

$= 0.5 -$ (Area between 0 to -3.5)

$= 0.5 - 0.4998$

$= 0.0002$

d) (Equal to 15.60)

i.e (area between 15.59 and 15.61)

Standard value of 15.59

$= \frac{15.59 - 15.60}{0.06}$

$= \frac{0.01}{0.06}$

$= -0.1667$

Standard value for 15.61.

$= \frac{15.61 - 15.60}{0.06}$

$= 0.1667.$

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σ equal to (15.60) = (Area between 15.59 to 15.61)

= 2 × (Area between 0 to 15.59)

= the area between −0.1667 to 0.1667)

= 2 × [Area between 0 to 0.1667)

= 2 × 0.0636

= 0.1272

(S-18) If the diameter of the ball bearings are normally distributed with mean 0.614 inches and standard deviation 0.0025 inches - Determine the percentage of ball bearings with diameters

(i) between 0.61 and 0.618 inches (inclusive)

(ii) Greater than 0.617 inches

(iii) equal to 0.615 inches.

→ Given, mean $\mu = 0.614$

Standard deviation $\sigma = 0.0025$

$$z = \frac{x-\mu}{\sigma} = \frac{x-0.614}{0.0025}$$

(i) Between 0.61 and 0.618 inclusive

(substract minimum distance 0.005 from left limit and add 0.005 in right limit)

$\therefore P[0.605 < x < 0.623)$

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(0.605 in standard unit) $= \dfrac{0.605 - 0.614}{0.0025}$

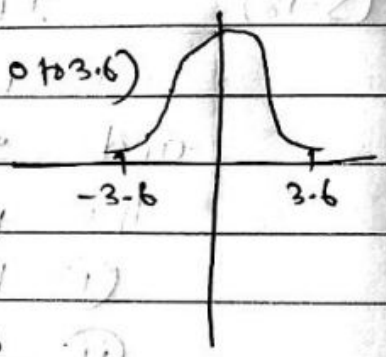
$= -3.6$

(0.623 in standard unit) $= \dfrac{0.623 - 0.614}{0.0025}$

$= +3.6$

$\therefore P(0.605 < x < 0.623) = P(-3.6 < x < 3.6)$

$P(-3.6 < x \leq 3.6) = 2 \times$ (Area between 0 to 3.6)



$= 2 \times (0.4998)$

$= 0.9996$

$= 0.9996 \times 100 = 99.96\%$

(2) Greater than 0.617

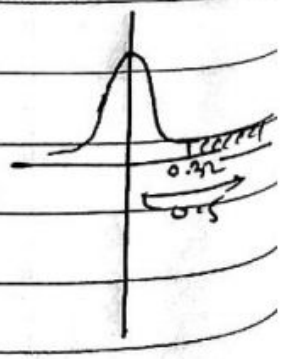
$\therefore P(x > 0.622) =$

(0.622 in standard unit)

$z = \dfrac{0.622 - 0.614}{0.0025}$.

$z = 3.2$

$P(x > 3.2) = 0.5 -$ [Area between 0 to 3.2)



$= 0.5 - 0.4995$

$= 0.0007$

$= 0.0007 \times 100 = 0.07\%$

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(iii) Equal to 0.615 inches.

$\therefore P(0.61 < x < 0.62)$

(0.61 in standard unit) $= z = \frac{0.61 - 0.614}{0.0025}$

$= -1.6$

(0.62 in standard unit) $= z = \frac{0.62 - 0.614}{0.0025}$

$= 2.4$

$\therefore P(0.61 < x < 0.62) = P(-1.6 < x < 2.4)$

$=$ Area between 0 to -1.6 & 0 to 2.4

$= 0.4452 + 0.4918$

$= 0.937$

$= 0.937 \times 100$

$= 93.7\%$

Relation Between Binomial and normal distribution.

If the number of trials 'n' in the Binomial distribution is large and the probabilities of success and failure i.e p and q are not too close to zero then binomial distribution can be closly approximated by normal distribution with mean

$\mu = np$

and standard deviation $\sigma = \sqrt{npq}$.

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If x is random variable then standard normal variable

$$z = \frac{x - np}{\sqrt{npq}}.$$

Q.1) A machine produces bolts which are 10% defective find the probability that in a random sample of 400 bolts produced by the machine

1. between 30 and 50
2. At the most 30
3. 55 or more

bolts will be defective

→ $p = 10\% = 0.1$

$n = 400$

$\mu = np$

$\mu = 0.1 \times 400$

$\boxed{\mu = 40}$

$q = 1 - p$

$q = 1 - 0.1$

$q = 0.9$

$\therefore \sigma = \sqrt{npq}$

$\sigma = \sqrt{400 \times 0.1 \times 0.9}$

$\boxed{\sigma = 6}$

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$$z = \frac{x-\mu}{\sigma} = \frac{x-40}{6}$$

① between 30 and 50

$P(29.5 < x < 50.5)$.

(29.5 in standard unit) $= \frac{29.5-40}{6}$

$= -1.75$

(50.5 in standard unit) $= \frac{50.5-40}{6}$

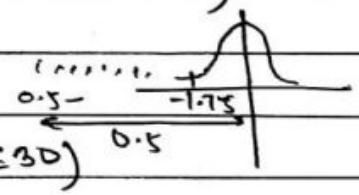
$= 1.75$

$\therefore P(29.5 < x < 50.5) = P(-1.75 < x < 1.75)$

$= 2 \times$ (Area between 0 to 1.75)

$= 2 \times (0.4599)$

$= 0.9198$



② At the most 30 $= P(x \leq 30)$

$P(x \leq 29.5) = P(x \leq -1.75)$

$= 0.5 -$ Area between (0 to 0.1.75)

$= 0.5 - 0.4599$.

$= 0.0401$

③ 55 or more

$P(x \geq 55) = P(x \geq 55.5)$

(55.5 in standard unit)

$= \frac{55.5-40}{6}$

$= 2.583$

$P(x \geq 55.5) = P(x \geq 2.583) = 0.5 -$ [Area between 0 to 2.583]

$= 0.5 - 0.4951 = 0.0049$

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Q.2) Find the probability of getting between 2 Heads to 4 Heads in 10 tosses of fair coin using ① Binomial distribution
② The normal approximation to the binomial distribution.

→ let p be the probability of getting head in toss of fair coin.

$$p = 1/2$$

$$q = 1/2$$

$$n = 10$$

① Binomial distribution

$$P(2 \le x \le 4) = P(x=2) + P(x=3) + P(x=4)$$

$$= \left[{}^{n}C_{x}\, p^{x} q^{n-x} \right]_{x=2,3,4}$$

$$= \sum_{x=2}^{4} \left[{}^{10}C_{x} \left(\tfrac{1}{2}\right)^{x} \left(\tfrac{1}{2}\right)^{10-x} \right]$$

$$\boxed{P(2 \le x \le 4) = 0.3662}$$

② $\mu = np = 10 \times 1/2$

$$\boxed{\mu = 5}$$

$$\sigma = \sqrt{npq} = \sqrt{5 \times 1/2}$$

$$\boxed{\sigma = 1.5811}$$

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$$z = \frac{x-\mu}{\sigma} = \frac{x-5}{1.5811}$$

$\therefore P(2 \le x \le 4) = P(1.9 \le x \le 4.1)$

(1.9 in standard unit) $= \frac{1.9-5}{1.5811}$

$= -1.9606$

(4.1 in standard unit) $= \frac{4.1-5}{1.5811}$

$= -0.5692$

$\therefore P(-1.9606 \le x \le -0.5692)$

= (Area between 0 to -1.9606)

− (Area between 0 to -0.5692)

$= 0.4750 - 0.2123$

$= 0.2627.$

Uniform distribution

A continuous random variable x is said to be uniformly distributed in $a \le x \le b$ if its density function is $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$

Properties

1. Mean $\mu = \frac{a+b}{2}$
2. Variance $\sigma^2 = \frac{(b-a)^2}{12}$
3. Standard deviation $\sigma = \frac{b-a}{2\sqrt{3}}$
4. Moment generating function

$$M_x(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$$

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Q.1) let x be uniformly distributed in $-2 \le x \le 2$ find

(i) $P(x<1)$

(ii) $P(|x-1| \ge 1/2)$

→ soln :-

x is uniformly distributed

∴ its density function is $f(x)$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{4} & -2 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) $$P(x<1) = \int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{1} f(x)\,dx$$

$$= \int_{-\infty}^{-2} 0 + \int_{-2}^{1} \frac{1}{4}\,dx$$

$$= \left[\frac{1}{4}x\right]_{-2}^{1}$$

$$= \frac{1}{4} + \frac{2}{4}$$

$$\boxed{P(x<1) = 3/4}$$

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(ii) $P(|x-1| \geq 1/2) = P(1/2 \leq x \leq 3/2)$

$$= \int_{1/2}^{3/2} \frac{1}{4}\, dx$$

$$= \left[\frac{1}{4}x\right]_{1/2}^{3/2}$$

$$= 1/4 \left[3/2 - 1/2\right]$$

$$\boxed{P(|x-1| \geq 1/2) = 1/4}$$

Q.2) A continuous random variable x is uniformly distributed having probability density function

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

Find (i) mean (ii) variance

(iii) moment generating function

→ $$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^2}{2} - \frac{a^2}{2}\right]$$

$$= \frac{b^2-a^2}{2(b-a)} = \frac{b+a}{2} = \frac{a+b}{2}$$

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(2) Variance $= E(X-\mu)^2 = E\left[X - \frac{a+b}{2}\right)^2$

$$= E\left[x^2 - 2x\frac{(a+b)}{2} + (a+b)^2\right]$$

$$= E(x^2) - (a+b)E(x) + E\left[\frac{a+b}{2}\right)^2 \quad \text{—(1)}$$

$$E(x^2) = \int_a^b x^2 \frac{1}{b-a}\,dx$$

$$= \left[\frac{x^3}{3}\right]_a^b \frac{1}{b-a}$$

$$= \left[\frac{b^3}{3} - \frac{a^3}{3}\right] \frac{1}{b-a}$$

$$= \frac{b^3 - a^3}{3(b-a)} = \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}}$$

$$= \frac{b^2+ab+a^2}{3}$$

put in eqn (1)

$$= \frac{a^2+ab+b^2}{3} - \frac{(a+b)(a+b)}{2} + \frac{(a+b)^2}{4}$$

$$= \frac{a^2+ab+b^2}{3} - \frac{1}{4}(a+b)^2$$

$$= \frac{4a^2}{12} + \frac{4ab}{12} + \frac{4b^2}{12} - \frac{1}{12}3a^2 - \frac{1}{12}6ab - \frac{3b^2}{12}$$

$$= \frac{a^2}{12} - \frac{2ab}{12} + \frac{b^2}{12}$$

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$= \frac{1}{12}[a-b)^2 = \text{variance}$

(iii) Moment generating function

$$M_X t = E(e^{tx})$$

$$= \int_a^b e^{tx} f(x)\,dx$$

$$= \int_a^b e^{tx} \frac{1}{b-a}\,dx$$

$$= \left[\frac{e^{tx}}{t}\right]_a^b \frac{1}{b-a}$$

$$= \left[\frac{e^{bt} - e^{at}}{t}\right]\left[\frac{1}{b-a}\right]$$

$=$

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Exponential Distribution

A continuous random variable x is said to be exponentially distributed if its density function is given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Properties

① $\mu = 1/\alpha$

② $\sigma^2 = \frac{1}{\alpha^2}$

③ standard deviation $\sigma = 1/\alpha$

④ $M_x(t) = \frac{\alpha}{\alpha - 1}$

(W-16) Define exponential distribution and find its mean, variance moment generating function

→ Definition :-

A C.R.V X is said to be exponentially distributed if its density function is given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

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$$\text{mean} = \mu = E(x)$$

$$= \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{0}^{\infty} x \alpha e^{-\alpha x} dx$$

$$= \alpha \int_{0}^{\infty} x e^{-\alpha x} dx$$

$$= \alpha \left[x \frac{e^{-\alpha x}}{-\alpha} - \int \frac{e^{-\alpha x}}{-\alpha} dx \right]$$

$$= \alpha \left[x \frac{e^{-\alpha x}}{-\alpha} - \frac{e^{-\alpha x}}{\alpha^2} \right]_0^{\infty}$$

$$= \alpha \left[0 + 1/\alpha^2 \, e^0 \right]$$

$$= 1/\alpha$$

$$\boxed{\mu = 1/\alpha}$$

(2) $\text{Variance} = E[(x-\mu)^2] =$

$$= E[x^2] - 2/\alpha \, E(x) + E(1/\alpha^2)$$

now,

$$E[x^2] = \int_{0}^{\infty} x^2 f(x)\, dx$$

$$= \alpha \int_{0}^{\infty} x^2 e^{-\alpha x} dx$$

$$= \alpha \left[x^2 \frac{e^{-\alpha x}}{-\alpha} - \int 2x \frac{e^{-\alpha x}}{-\alpha} \right]$$

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$$= \alpha \left[\frac{x^2 e^{-\alpha x}}{-\alpha} - 2 \int \frac{x e^{-\alpha x}}{-\alpha} dx \right]$$

$$= \alpha \left[\frac{x^2 e^{-\alpha x}}{-\alpha} - 2 \left[\right. \right.$$

$$= \alpha \left[2/\alpha \cdot \int x e^{-\alpha x} dx \right]$$

$$= \alpha \left[2/\alpha \times 1/\alpha \right] = \alpha \left[2/\alpha^2 \right]$$

$$\boxed{\text{variance} = 2/\alpha^2}$$

$$\therefore \text{variance} = 2/\alpha - 2/\alpha \cdot 1/\alpha + 1/\alpha^2$$

$$= 2/\alpha^2 - 2/\alpha^2 + 1/\alpha^2$$

$$= \cancel{2/\alpha - 3/\alpha^2}$$

$$= \cancel{2/\alpha - 1/\alpha^2}$$

$$= 1/\alpha^2$$

(3) Moment generating function.

$$M_x(t) = E\left[e^{tx}\right]$$

$$= \int_0^\infty e^{tx} f(x)\, dx$$

$$= \alpha \int_0^\infty e^{tx} e^{-\alpha x} dx$$

$$= \alpha \int_0^\infty e^{x(t-\alpha)} dx$$

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$$= \left[\alpha \frac{e^{x(t-\alpha)}}{t-\alpha} \right]_0^\infty$$

$$= \alpha \left[0 - \frac{1}{t-\alpha} \right]$$

$$\boxed{M_x(t) = \frac{\alpha}{\alpha - t}}$$

Geometric Distribution

Let the random variable x denotes the number of Bernoulli trials of a random experiment required to obtain the first success

$\therefore$ x assumes the values

$x = 1, 2, 3, 4 \ldots$

If $x = r$ then first $(r-1)$ trials results in failure and r trial results in success.

The probability function for x is given by

$$f(x) = P(x=r) = pq^{r-1}$$

where p = probability of success in single trial, and $q = 1-p$ is probability of failure.

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Q.1) If the probability that an applicant for drivers license will pass the road test on any given trial is 0.8 what is the probability that he will finally pass the test

① on the 4th trial

② within three trials.

→ Here $p = 0.8$

$q = 1 - p$

$q = 0.2$.

① $P(\text{on the 4th trial}) = P(X=4)$

$= pq^{r-1}$

$= 0.8\,(0.2)^{4-1}$

$= 0.0064$

② $P(\text{within three trials}) = P(X \le 3)$

$= P(X=1) + P(X=2) + P(X=3)$

$= pq^{1-1} + pq^{2-1} + pq^{3-1}$

$= 0.8 \times (0.2)^0 + 0.8 \times (0.2)^1 + 0.8 \times (0.2)^2$

$= 0.992$

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(1.2) Find the probability that in successive tosses of a fair die 3 comes up for the first time on the 5th toss.

→ Let p be the probability of 3 coming up

$$p = 1/6$$

$$q = 1 - p$$

$$\boxed{q = 5/6}$$

$\therefore$ P(at 5th toss) = $P(x=5)$

$$= pq^{x-1}$$

$$= 1/6 \, (5/6)^{5-1}$$

$$\boxed{P(x=5) = 0.08037}$$

#

A continuous random variable x is said to be weibull distributed with parameters $\alpha, \beta > 0$ if its density function is

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} \cdot e^{-\alpha x^{\beta}} & , x > 0 \\ 0 & x \le 0 \end{cases}$$

If $\beta = 1$ - weibull distribution reduces to exponential distribution.

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Q.1) Suppose that the life time of a certain kind of battery in hours is a random variable having weibull distribution with $\alpha = 0.1$, $\beta = 0.5$

find ① The mean life time of these batteries.

② The probability that such a battery will last more than 300 hours.

③ The probability that such a battery will not last 100 Hours.

→ ① Mean $= \mu = \alpha^{-1/\beta} \Gamma(1/\beta + 1)$

$$\mu = \frac{1}{(0.1)^{(0.5)}} \Gamma\left(\frac{1}{0.5} + 1\right)$$

$$\mu = 100 \, \Gamma(2+1)$$

$$\mu = 100 \times 2!$$

$$\boxed{\mu = 200 \text{ hour}}$$

② $P(x > 300) = P(300 < x < \infty)$

$$= \int_{300}^{\infty} f(x)\,dx = \int_{300}^{\infty} \alpha\beta x^{\beta-1} \cdot e^{-\alpha x^{\beta}}\,dx$$

$$= \alpha\beta \int (0.1)(0.5)\, x^{0.5}\, e^{-0.1 x^{0.5}}\,dx$$

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$$= \int \frac{d}{dx}\left[-e^{-0.1x^{0.5}}\right] dx$$

$$= \left[-e^{-0.1x^{0.5}}\right]_{300}^{\infty}$$

$$= 0 - e^{-0.1\times 300^{0.5}}$$

$$\boxed{P(X>300) = 0.176\,gm}$$

(iii) $P(x<100) = P(\cancel{100<x<\infty})$

$P = (-\infty < x < 100)$

$$= \int_{0}^{100} (0.1)(0.5)\, x^{0.5}\, e^{-0.1x^{0.5}}\, dx$$

$$= \int_{-\infty}^{100} \frac{d}{dx}\left[-e^{-0.1x^{0.5}}\right] dx$$

$$= \left[-e^{-0.1x^{0.5}}\right]_{0}^{100}$$

$$= 0.3678 - e^{-0.1\times 0^{0.5}}$$

$$= -0.3678 + 1$$

$$= +0.6321$$

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