1 Definitions used

In the paper New common ancestor problems in trees and directed acyclic graphs, we have the following results.

Lemma 1 *Let G be a DAG and T_G its corresponding LSA-tree. Further, let v,w $\in$ V be two arbitrary nodes in G. Then* $LSCA_G(v,w) = LCA_{T_G}(v,w)$

Lemma 2 *For any node* $v \neq \perp$, $LSA_G(v) = LSCA_G(parents_G(v))$

Definition 1 $LSCA_G(w1, ..., wk) = LSCA_G(w1, LSCA_G(w2, ..., wk))$.

Definition 2 $LCA_{T_G}(u_1, u_2, ...u_n) = LCA_{T_G}(u_1, LCA_{T_G}(u_2, ..., u_n))$.

2 Labelling scheme

Since the LSA-tree T_G is well defined for a DAG G, we maintain on the nodes of G an ordered set of nodes L.

The label for a node u in the DAG is the ordered set of nodes L_u on the path from the root of T_G to u. Since T_G is a tree, the path from the root to u in T_G is unique and so is the label L_u.

Theorem 1 $LCA_{T_G}(v,w)$ *is the node of the maximum depth in* $L_u \cap L_v$

Proof. Let $l = LCA_{T_G}(v,w)$. We need to show that l is the deepest node in $L_v \cap L_w$.

Let's assume that there is a node l' that lies on the path from l to both v and w in T_G, for otherwise l' would not lie on all paths from $\perp$ to v or w in G. This assumption implies that the depth of l' is greater than the depth of l. If l' lies on these paths then the following should be true $l' \in L_v$ and $l' \in L_w$. This inturn implies that $l' \in L_v \cap L_w$. But l' cannot be the deepest member of $\in L_v \cap L_w$ otherwise l would not be the $LCA_{T_G}(v,w)$ which means that our assumption contradicts the definition $l = LCA_{T_G}(v,w)$. $\square$

We can rewrite lemma 2 using definotion 1 as follows:

$$LSA_G(v) = LSCA_G(p_1, LSCA_G(p_2, ..., p_k)) \text{ where } p_1, p_2, ...p_k \text{ are the parents of } v \tag{1}$$

Definition 2 can be rewritten using theorem 1 as follows:

$$LCA_{T_G}(u_1, u_2, ...u_n) = L_{u_1} \cap (L_{u_2} \cap ... \cap L_{u_n}) = L_{u_1} \cap L_{u_2} \cap ... \cap L_{u_n} \tag{2}$$

Using equations 1 and 2, we can say that the $LSA_G(v)$ is the deepest node in the set given by the relation:

$$\{ \bigcap_{u \in parents(v)} L_u \}$$

We store this set as the label of a node.

3 Finding the LSCA of nodes using their labels

According to lemma 1, $LSCA_G(v,w) = LCA_{T_G}(v,w)$. From equation 2 we know that the LCA_{T_G} can be found by a set intersection of their labels. We can use this relation to say that $LSCA_G(v,w)$ is the deepest node in the set $\{L_v \cap L_w\}$.

We maintain the labels as ordered sets. The order is defined by their depth in T_G. So, to save on computing depths explicitly, we simply pick the last element of the labels as the LSCA