



Computer Graphics 05101301

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CHAPTER-3

2D and 3D Transformation





Transformation

- Transformation means changing some graphics into something else by applying rules.
- It is used to reposition the graphics on the screen and change their size or orientation.
- The basic types of transformations are …
 - 1) Translation,
 - 2) Rotation,
 - 3) Scaling,
 - 4) Shearing,
 - 5) Reflection

When a transformation takes place on a 2D plane, it is called 2D transformation

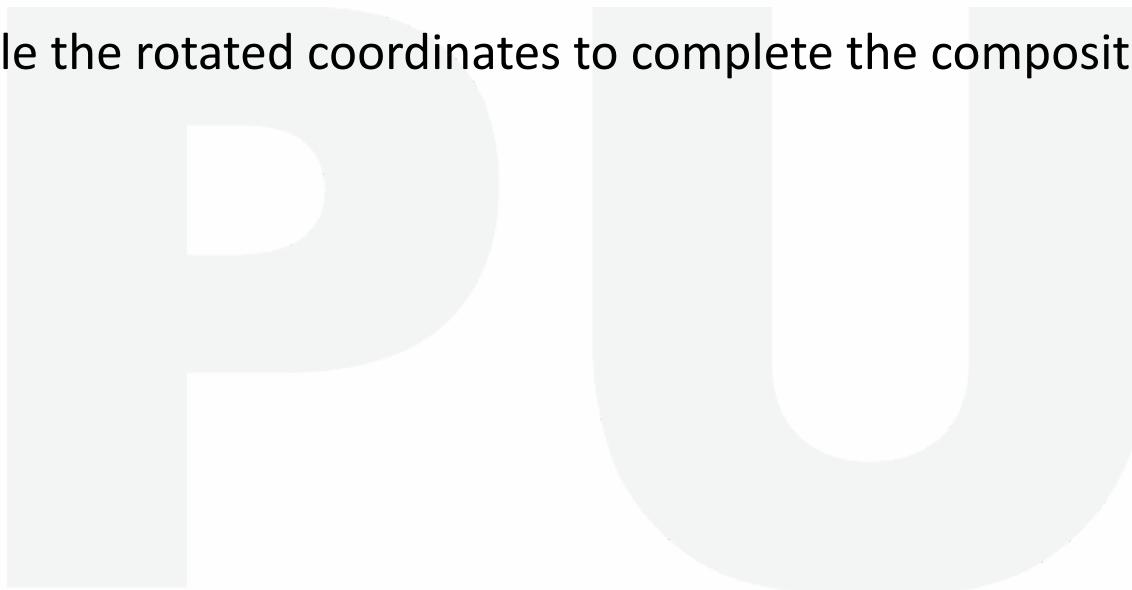




Transformation

To perform any type of transformation we need to follow a sequential process –

1. Translate the coordinates,
2. Rotate the translated coordinates, and then
3. Scale the rotated coordinates to complete the composite transformation.





Homogenous Coordinate System

A 3×3 transformation matrix can be used instead of 2×2 transformation matrix. To convert a 2×2 matrix to 3×3 matrix, we have to add an extra dummy coordinate W.

Any point can be represented by using 3 numbers instead of 2 numbers, which is called **Homogenous Coordinate** system.

All the transformation equations can be represented in matrix multiplication. Any Cartesian point $P(X, Y)$ can be converted to homogenous coordinates by $P' (X_h, Y_h, h)$.



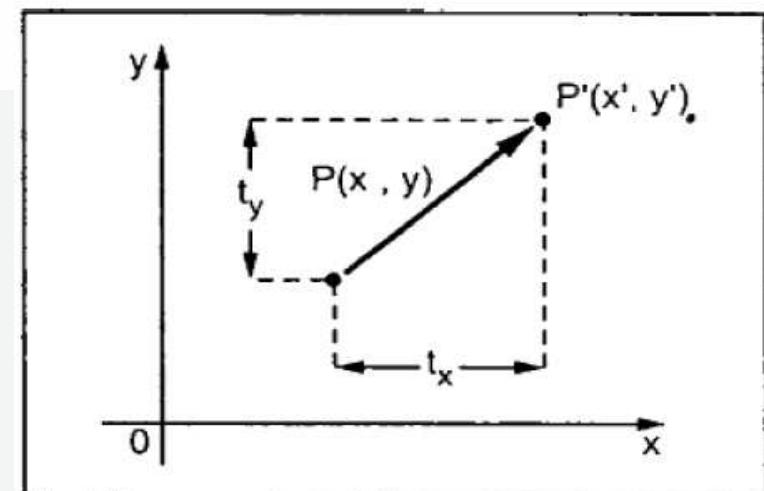


Translation

Translation refers to the shifting of a point to some other place, **whose distance with regard to the present point is known.**

A translation moves an object to a different position on the screen.

You can translate a point in 2D by adding translation coordinate (t_x, t_y) to the original coordinate (X, Y) to get the new coordinate (X', Y').





Translation

From the above figure, we can write

$$\mathbf{X}' = \mathbf{X} + \mathbf{t}_x$$

$$\mathbf{Y}' = \mathbf{Y} + \mathbf{t}_y$$

Where (t_x, t_y) is called the **translation vector or shift vector**.

The above equations can also be represented using the column vectors.

$$\mathbf{P} = [\mathbf{X}][\mathbf{Y}]$$

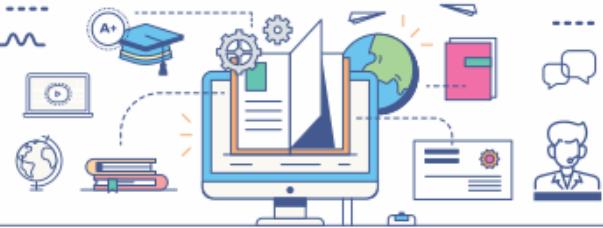
$$\mathbf{P}' = [\mathbf{X}'][\mathbf{Y}']$$

$$\mathbf{T} = [t_x][t_y]$$

Which can be written as

$$\mathbf{P}' = \mathbf{P} + \mathbf{T}$$



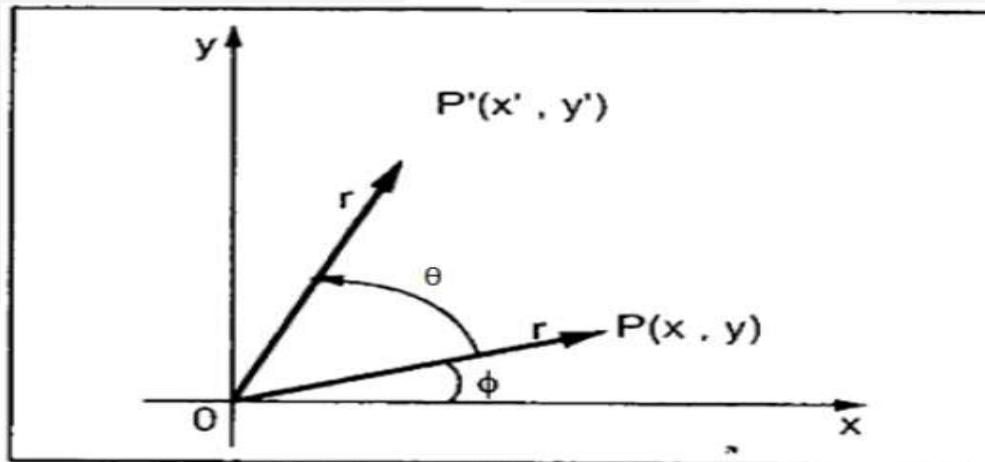


Rotation

Rotation means to rotate a point about an axis.

In rotation, an object is rotated at a particular angle θ (theta) from its origin.

From the following figure, we can see that the point $P(x, y)$ is located at angle ϕ from the horizontal X coordinate with distance r from the origin





Rotation

Let us suppose you want to rotate it at the angle θ . After rotating it to a new location, you will get a new point $P' (X', Y')$.

Using standard trigonometric the original coordinate of point $P(X, Y)$ can be represented as –

$$X = r \cos \phi \dots\dots (1)$$

$$Y = r \sin \phi \dots\dots (2)$$

Same way we can represent the point $P' (X', Y')$ as –

$$x' = r \cos(\phi + \theta) = r \cos \phi \cos \theta - r \sin \phi \sin \theta \dots\dots (3)$$

$$y' = r \sin(\phi + \theta) = r \cos \phi \sin \theta + r \sin \phi \cos \theta \dots\dots (4)$$





Rotation

Substituting equation (1) & (2) in (3) & (4) respectively, we will get

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

Representing the above equation in matrix form,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \text{OR}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{OR} \quad P' = P \cdot R$$

Where R is the rotation matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$





Rotation

The rotation angle can be positive and negative.

For positive rotation angle, we can use the above rotation matrix. However, for negative angle rotation, the matrix will change as shown below –

$$R = [\cos(-\theta) \quad -\sin(-\theta) \quad \sin(-\theta) \quad \cos(-\theta)]$$

$$R = [\cos(-\theta) \quad \sin(-\theta) \quad -\sin(-\theta) \quad \cos(-\theta)]$$

$$= [\cos\theta \quad \sin\theta \quad -\sin\theta \quad \cos\theta]$$

$$(\because \cos(-\theta) = \cos\theta \text{ and } \sin(-\theta) = -\sin\theta)$$





Rotation

Example

Given a triangle with corner coordinates $(0, 0)$, $(1, 0)$ and $(1, 1)$. Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

We rotate a polygon by rotating each vertex of it with the same rotation angle.

Given-

- Old corner coordinates of the triangle = A $(0, 0)$, B $(1, 0)$, C $(1, 1)$
- Rotation angle = $\theta = 90^\circ$





Rotation

For Coordinates A(0, 0)

Let the new coordinates of corner A after rotation = (x', y') .

Applying the rotation equations, we have-

$$X' = x * \cos\theta - Y * \sin\theta$$

$$= 0 * \cos 90^\circ - 0 * \sin 90^\circ$$

$$= 0$$

$$Y' = x * \sin\theta + y * \cos\theta$$

$$= 0 * \sin 90^\circ + 0 * \cos 90^\circ$$

$$= 0$$

Thus, New coordinates of corner A after rotation = $(0, 0)$.





Rotation

For Coordinates B(1, 0)

Let the new coordinates of corner B after rotation = (x',y').

$$X' = x * \cos\theta - y * \sin\theta = 1 \times \cos 90^\circ - 0 \times \sin 90^\circ = 0$$

$$Y' = x * \sin\theta + y * \cos\theta = 1 \times \sin 90^\circ + 0 \times \cos 90^\circ = 1 + 0 = 1$$

Thus, New coordinates of corner B after rotation = (0, 1).

For Coordinates C(1, 1)

Let the new coordinates of corner C after rotation = (X_{new}, Y_{new}).

$$X' = x * \cos\theta - y * \sin\theta = 1 \times \cos 90^\circ - 1 \times \sin 90^\circ = 0 - 1 = -1$$

$$Y' = x * \sin\theta + y * \cos\theta = 1 \times \sin 90^\circ + 1 \times \cos 90^\circ = 1 + 0 = 1$$

Thus, New coordinates of corner C after rotation = (-1, 1).

Thus, New coordinates of Triangle after Rotation = A(0,0), B(0,1), C(-1,1)





Scaling

- To change the size of an object, scaling transformation is used.
- **Scaling** is the concept of increasing (or decreasing) the size of a picture in any direction. (When it is done in both directions, the increase or decrease in both directions need not be same)
- Scaling can be achieved by **multiplying the original coordinates of the object with the scaling factor** to get the desired result.

$X' = X \cdot S_x$ and $Y' = Y \cdot S_y$ Where, original coordinates are (X, Y)

Scaling factors are (S_x, S_y) , and the

Produced coordinates are (X', Y') .



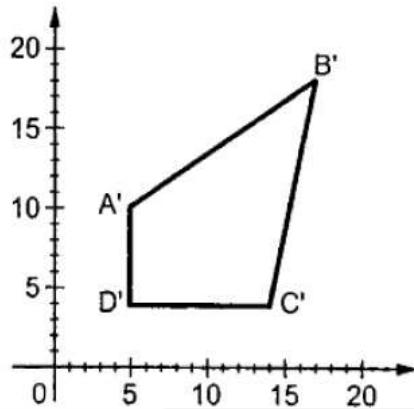
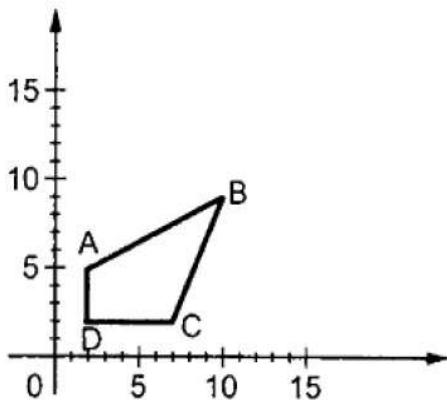


Scaling

The above equations is represented in matrix form

$$\text{OR } P' = P \cdot S$$

Where S is the scaling matrix. The scaling process is shown in the following figure.



If S value is less than 1, then size of object reduces

If S greater than 1, then size of object increase





Scaling: Example

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

Scaling factor along X axis = $S_x=2$

Scaling factor along Y axis = $S_y=3$

For Coordinates A(0, 3):x=0 and y=3

Let the new coordinates of corner A after scaling = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the scaling equations, we have-

$$X' = X * S_x = 0 \times 2 = 0$$

$$Y' = Y * S_y = 3 \times 3 = 9$$

Thus, New coordinates of corner A after scaling = A'(0, 9).





Scaling: Example

For Coordinates B(3, 3) : B'(6,9)

Let the new coordinates of corner B after scaling = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the scaling equations, we have-

$$X_{\text{new}} = X_{\text{old}} \times S_x = 3 \times 2 = 6$$

$$Y_{\text{new}} = Y_{\text{old}} \times S_y = 3 \times 3 = 9$$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0): C'(6,0)

$$X_{\text{new}} = X_{\text{old}} \times S_x = 3 \times 2 = 6$$

$$Y_{\text{new}} = Y_{\text{old}} \times S_y = 0 \times 3 = 0$$

Thus, New coordinates of corner C after scaling = (6, 0).





Scaling: Example

For Coordinates D(0, 0) :

D'(0,0)

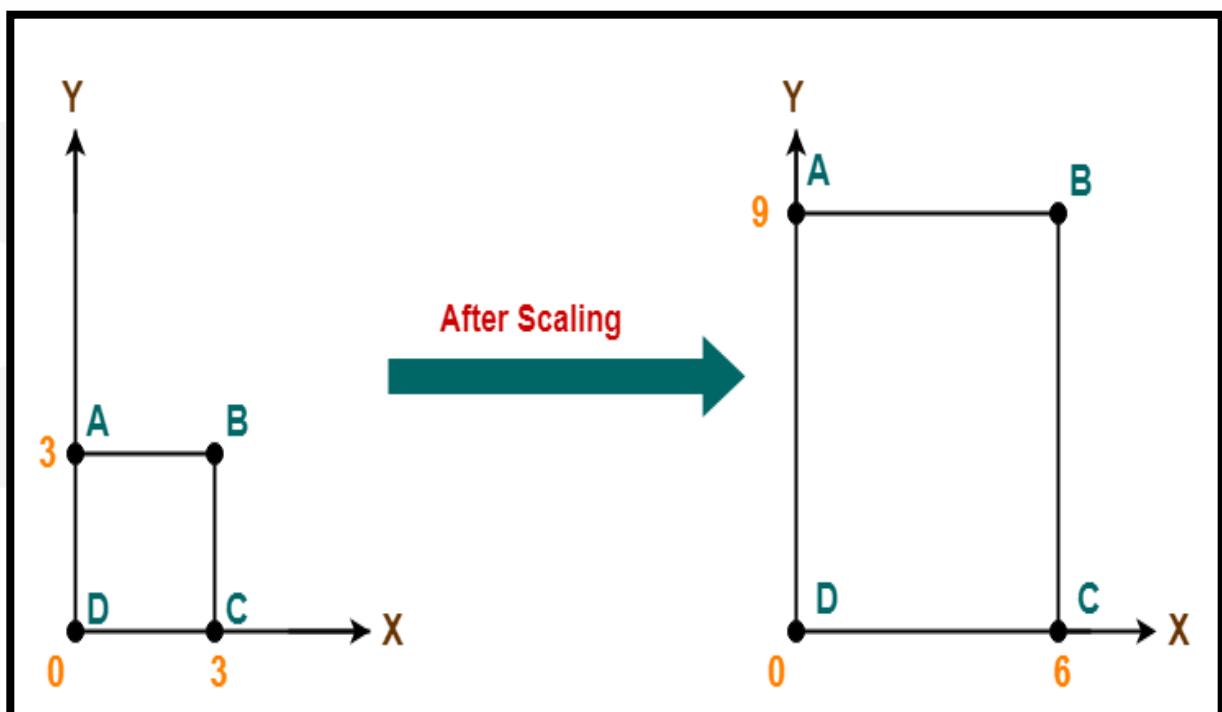
Let the new coordinates of corner D after scaling = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the scaling equations, we have-

$$X_{\text{new}} = X_{\text{old}} \times S_x = 0 \times 2 = 0$$

$$Y_{\text{new}} = Y_{\text{old}} \times S_y = 0 \times 3 = 0$$

Thus, New coordinates of corner D after scaling = (0, 0).





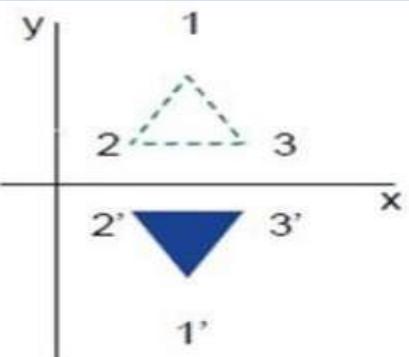
Reflection

- Reflection is the mirror image of original object. In other words, we can say that it is a rotation operation with 180° . In reflection transformation, the size of the object does not change.
- The following figures show reflections with respect to X and Y axes, and about the origin respectively.

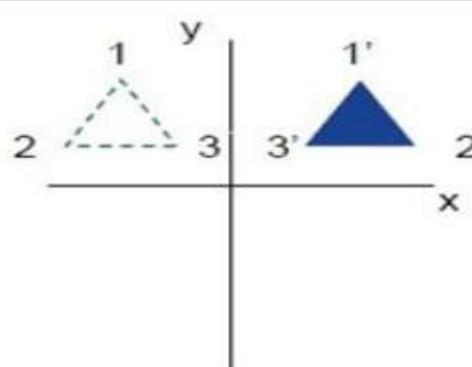




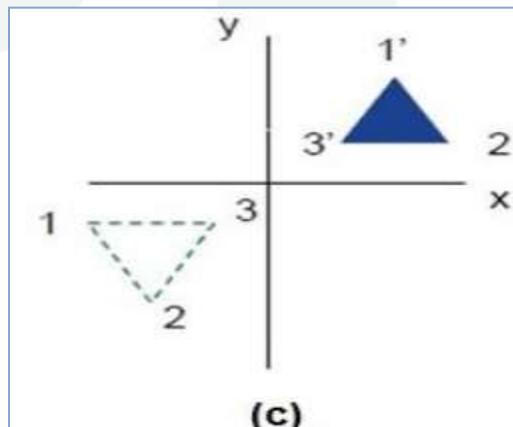
Reflection



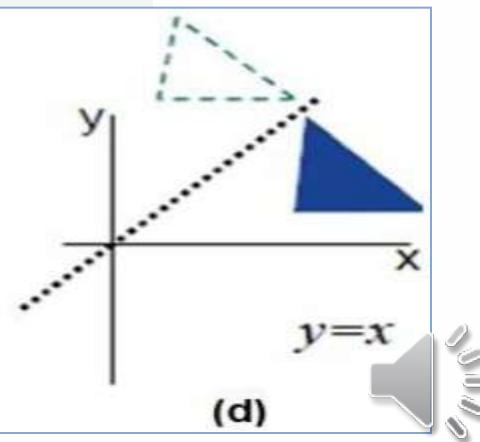
(a)



(b)



(c)



(d)



Reflection

Reflection On X-Axis:

This reflection is achieved by using the following reflection equations- (X' and Y' are the new coordinates)

$$X' = X$$

$$Y' = -Y$$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)





Reflection

Reflection On Y-Axis:

This reflection is achieved by using the following reflection equations- (X' and Y' are the new coordinates)

$$X' = -X$$

$$Y' = Y$$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Reflection Matrix
(Reflection Along Y Axis)





Reflection: Example

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Given-

Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)

Reflection has to be taken on the X axis





Reflection: Example

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

$$X_{\text{new}} = X_{\text{old}} = 3$$

$$Y_{\text{new}} = -Y_{\text{old}} = -4$$

Thus, New coordinates of corner A after reflection = $(3, -4)$.





Reflection: Example

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

$$X_{\text{new}} = X_{\text{old}} = 6$$

$$Y_{\text{new}} = -Y_{\text{old}} = -4$$

Thus, New coordinates of corner B after reflection = $(6, -4)$.





Reflection: Example

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

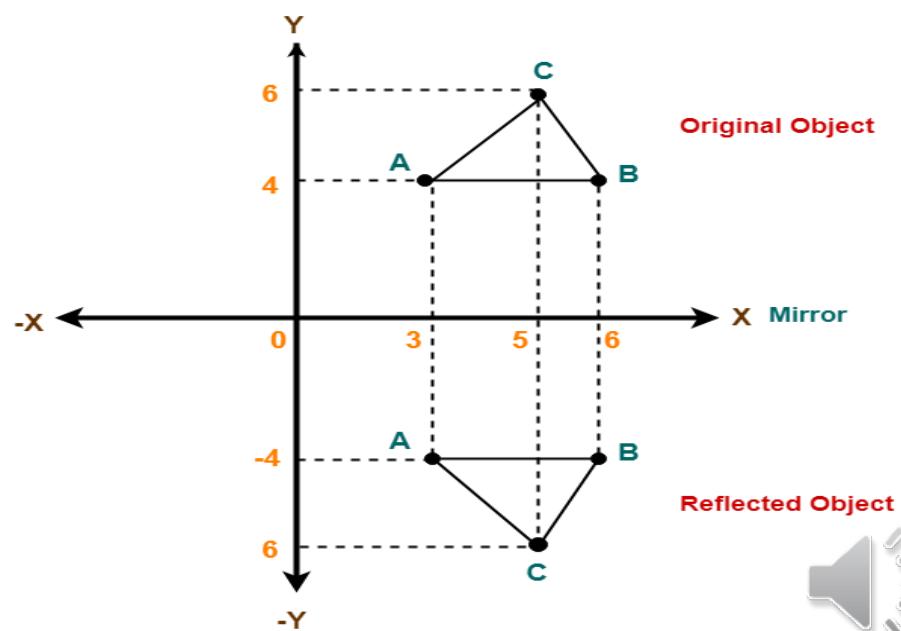
Applying the reflection equations, we have-

$$X_{\text{new}} = X_{\text{old}} = 5$$

$$Y_{\text{new}} = -Y_{\text{old}} = -6$$

Thus, New coordinates of corner C after reflection = $(5, -6)$.

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).





Shear

A transformation that slants the shape of an object is called the shear transformation.

Types of shear transformations :

X-Shear and Y-Shear.

One shifts X coordinates values and other shifts Y coordinate values. However; in both the cases only one coordinate changes its coordinates and other preserves its values. Shearing is also termed as **Skewing**.

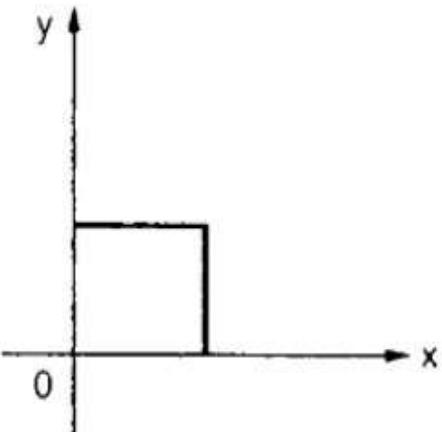




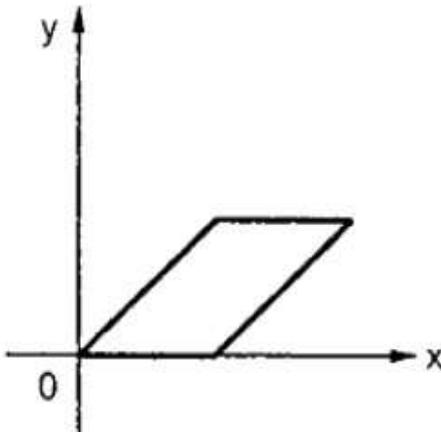
X-Shear

The X-Shear preserves the Y coordinate and changes are made to X coordinates, which causes the vertical lines to tilt right or left as shown in below figure.

The transformation matrix for X-Shear can be represented as –



(a) Original object



(b) Object after x shear

Transformation Matrix:

$$XS = \begin{bmatrix} 1 & 0 & 0 \\ Sx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_1 = x + Sx.y$

$y_1 = y$



X-Shear

Shearing in X Axis-

$$X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}}$$

$$Y_{\text{new}} = Y_{\text{old}}$$

The transformation matrix for X-Shear can be represented as –

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Shearing Matrix

(In X axis)





Y-Shear

The Y-Shear preserves the X coordinates and changes the Y coordinates which causes the horizontal lines to transform into lines which slopes up or down as shown in the following figure.

The Y-Shear can be represented in matrix from as:

Transformation Matrix:

$$XY = \begin{bmatrix} 1 & S_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x_1 = x$$

$$y_1 = y + S_x$$

y

0

y

0

(a) Original object

0

(b) Object after y shear





Y-Shear

Shearing in Y axis is achieved by using the following shearing equations-

$$X_{\text{new}} = X_{\text{old}}$$

$$Y_{\text{new}} = Y_{\text{old}} + Sh_y \times X_{\text{old}}$$

The Y-Shear can be represented in matrix form as –

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Shearing Matrix

(In Y axis)





Shear - Example

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Given-

Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)

Shearing parameter towards X direction (Sh_x) = 2

Shearing parameter towards Y direction (Sh_y) = 2





Shear - Example

Shearing in X Axis- For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

$$X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}} = 1 + 2 \times 1 = 3$$

$$Y_{\text{new}} = Y_{\text{old}} = 1$$

Thus, New coordinates of corner A after shearing = (3, 1).





Shear - Example

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

$$X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}} = 0 + 2 \times 0 = 0$$

$$Y_{\text{new}} = Y_{\text{old}} = 0$$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Thus, New coordinates of corner C after shearing = (1, 0).

New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).





Shear - Example

Shearing in Y Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

$$X_{\text{new}} = X_{\text{old}} = 1$$

$$Y_{\text{new}} = Y_{\text{old}} + Sh_y \times X_{\text{old}} = 1 + 2 \times 1 = 3$$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0), New coordinates of corner B after shearing = (0, 0).

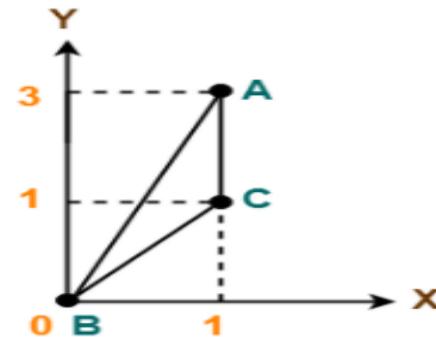
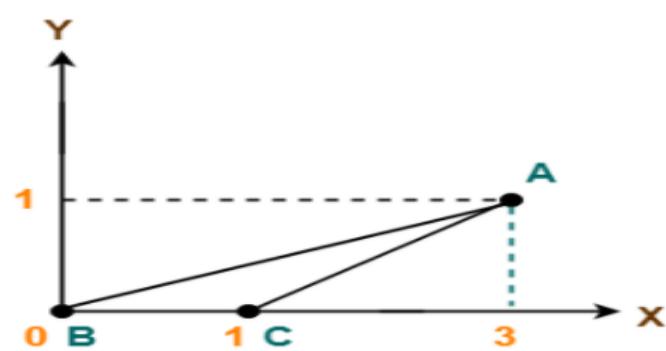
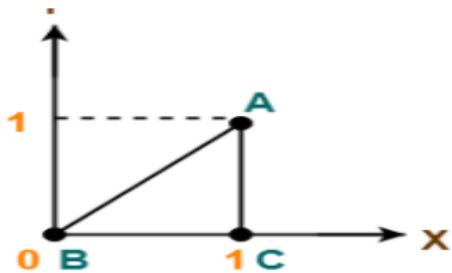
For Coordinates C(1, 0), New coordinates of corner B after shearing = (1, 2).

New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).





Shear - Example





Composite Transformation :

- A number of transformations or sequence of transformations can be combined into single one called as composition. The resulting matrix is called as composite matrix. The process of combining is called as concatenation.
- Suppose we want to perform rotation about an arbitrary point, then we can perform it by the sequence of three transformations
 - ✓ Translation
 - ✓ Rotation
 - ✓ Reverse Translation
- The ordering sequence of these numbers of transformations must not be changed. If a matrix is represented in column form, then the composite transformation is performed by multiplying matrix in order from right to left side. The output obtained from the previous matrix is multiplied with the new coming matrix.



Example

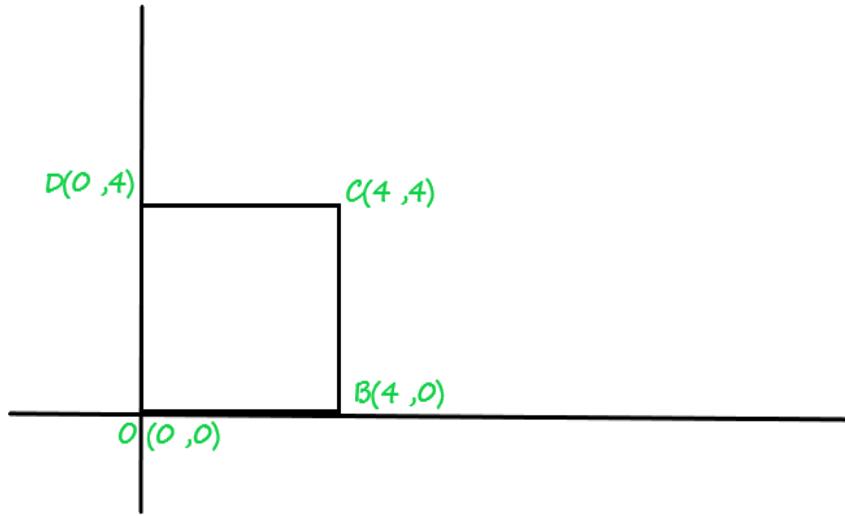
- Consider we have a 2-D object on which we first apply transformation T_1 (**2-D matrix condition**) and then we apply transformation T_2 (**2-D matrix condition**) over the 2-D object and the object get transformed, the very equivalent effect over the 2-D object we can obtain by multiplying T_1 & T_2 (**2-D matrix conditions**) with each other and then applying the T_{12} (**resultant of $T_1 \times T_2$**) with the coordinates of the 2-D image to get the transformed final image.





Example

- Consider we have a square $O(0, 0)$, $B(4, 0)$, $C(4, 4)$, $D(0, 4)$ on which we first apply **T1**(scaling transformation) given scaling factor is $S_x=S_y=0.5$ and then we apply **T2**(rotation transformation in clockwise direction) it by 90^* (angle), in last we perform **T3**(reflection transformation about origin).
- The square O, B, C, D looks like :





Example

- First, we perform scaling transformation over a 2-D object :

Representation of scaling condition :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sx \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

For coordinate O(0, 0) :

$$O \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate B(4, 0) :

$$B \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$





Example

For coordinate C(4, 4) :

$$C \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

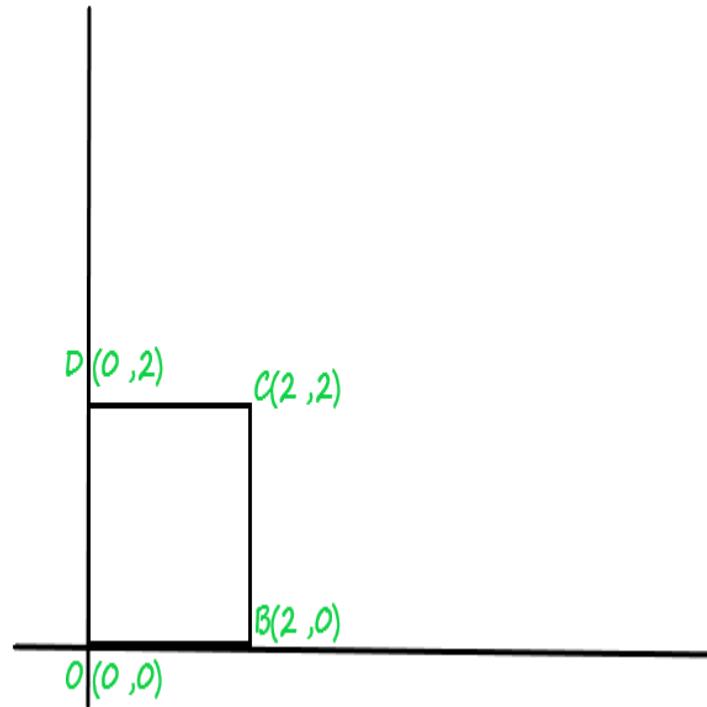
$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

For coordinate D(0, 4) :

$$D \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

2-D object after scaling :





Example

*Now, we'll perform rotation transformation in clockwise-direction on Fig.2 by 90^θ :

The condition of rotation transformation of 2-D object about origin is :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\cos 90 = 0$$

$$\sin 90 = 1$$





Example

For coordinate O(0, 0) :

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate B(2, 0) :

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

For coordinate C(2, 2) :

$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

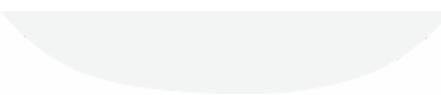
$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

For coordinate D(0, 2) :

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

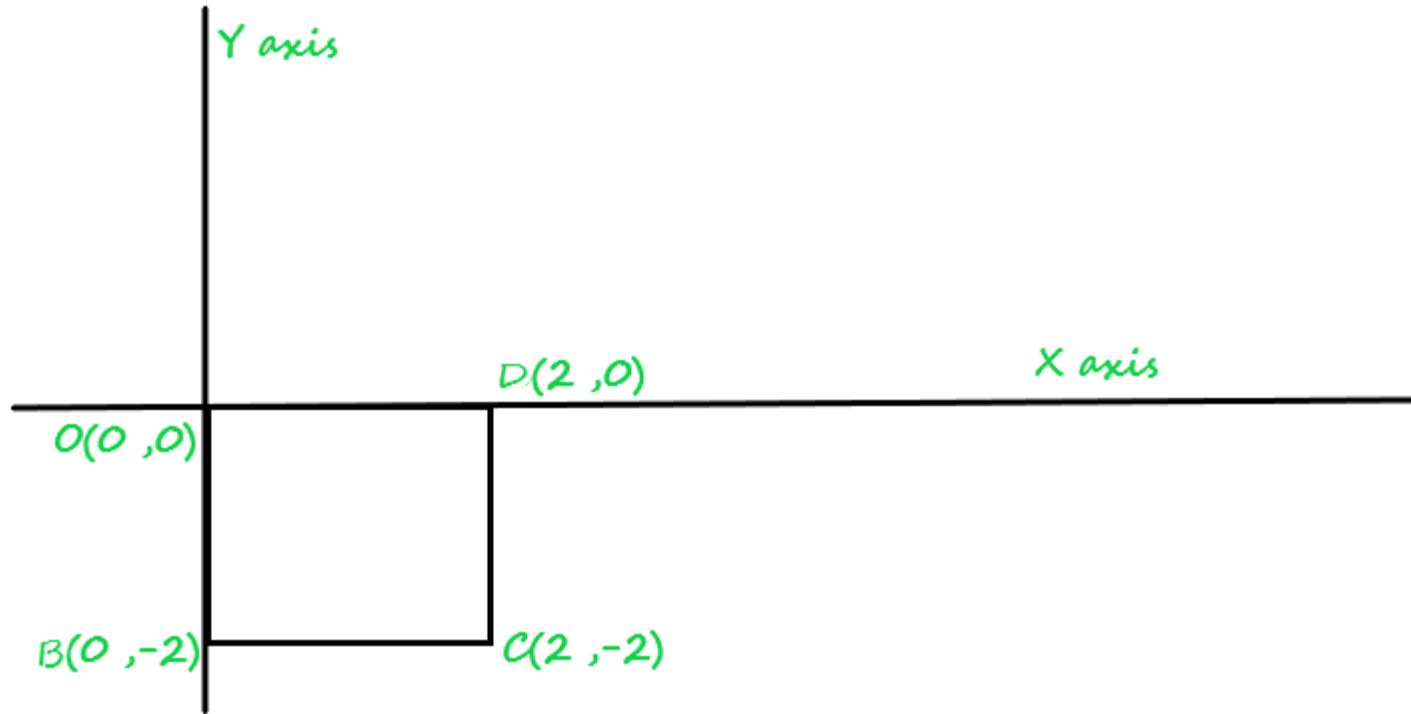
$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

2-D object after rotating about origin by 90° angle :





Example





Example

*Now, we'll perform third last operation on Fig.3, by reflecting it about origin :

The condition of reflecting an object about origin is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

For coordinate O(0, 0) :

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$





Example

For coordinate $B'(0, 0)$:

$$B, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$B, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

For coordinate $C'(0, 0)$:

$$C, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$C, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

For coordinate $D'(0, 0)$:

$$D, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$D, \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

The final 2-D object after reflecting about origin, we get :

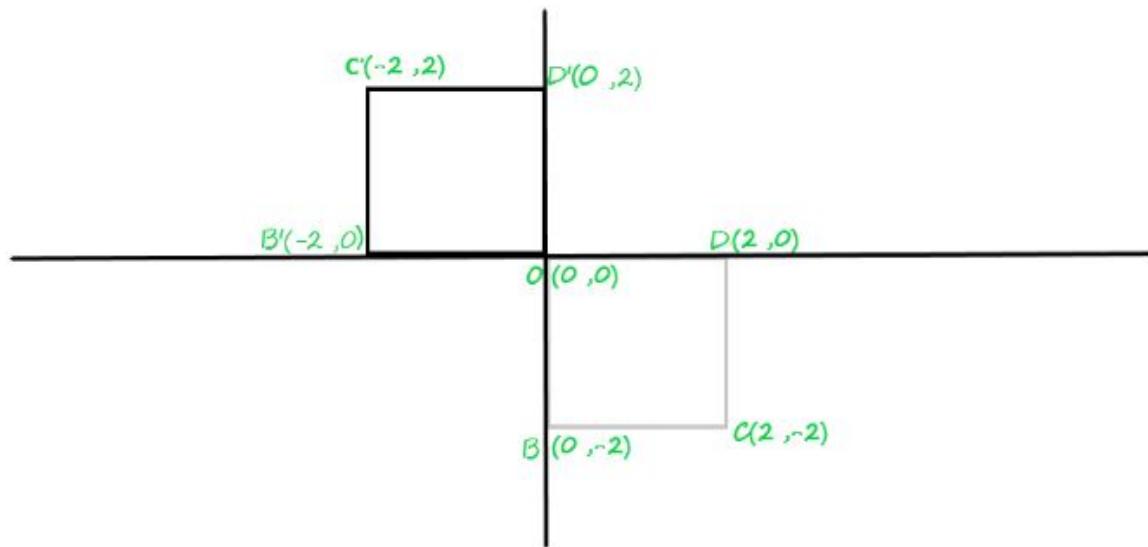


Fig.4



Example

Note : The above finale result of **Fig.4**, that we get after applying all transformation one after one in a serial manner. We could also get the same result by combining all the transformation 2-D matrix conditions together and multiplying each other and get a resultant of multiplication(R). Then, applying that 2D-resultant matrix(R) at each coordinate of the given square(above). So, you will get the same result as you have in **Fig.4**.

Solution using Composite transformation :

*First we multiplied 2-D matrix conditions of **Scaling transformation** with **Rotation transformation** :





Example

$$[R_1] = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} R_1 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}$$

*Now, we multiplied Resultant 2-D matrix(R_1) with the third last given Reflecting condition of transformation(R_2) to get Resultant(R) :

$$[R] = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} R \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}$$

Now, we'll applied the Resultant(R) of 2d-matrix at each coordinate of the given object (square) to get the final transformed or modified object.





Example

First transformed coordinate O'(0, 0) is :

$$O' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Second, transformed coordinate B'(4, 0) is :

$$B' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix} * \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$B' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Third transformed coordinate C'(4, 4) is :

$$C' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix} * \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$C' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Fourth transformed coordinate D'(0, 4) is :

$$D' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

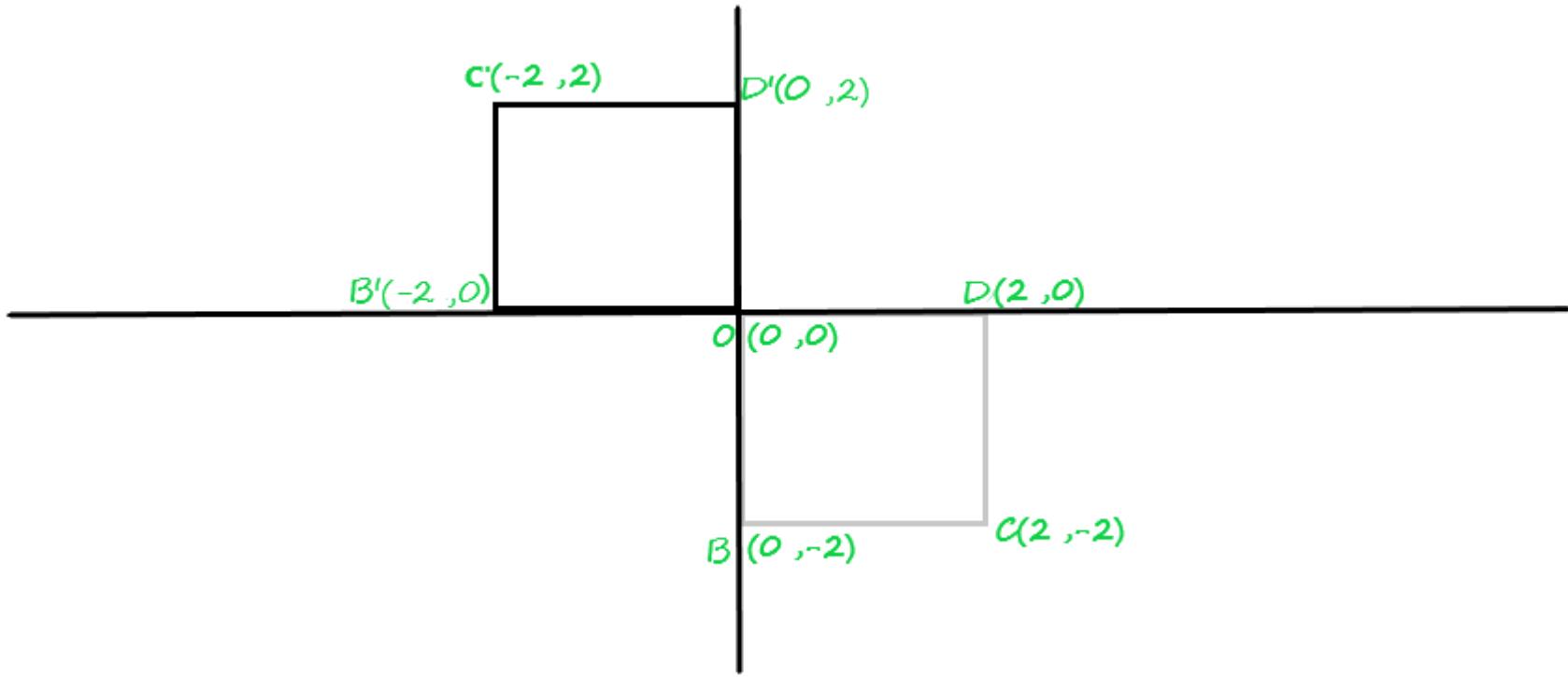
$$D' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

The final result of the transformed object that you get would be same as above :



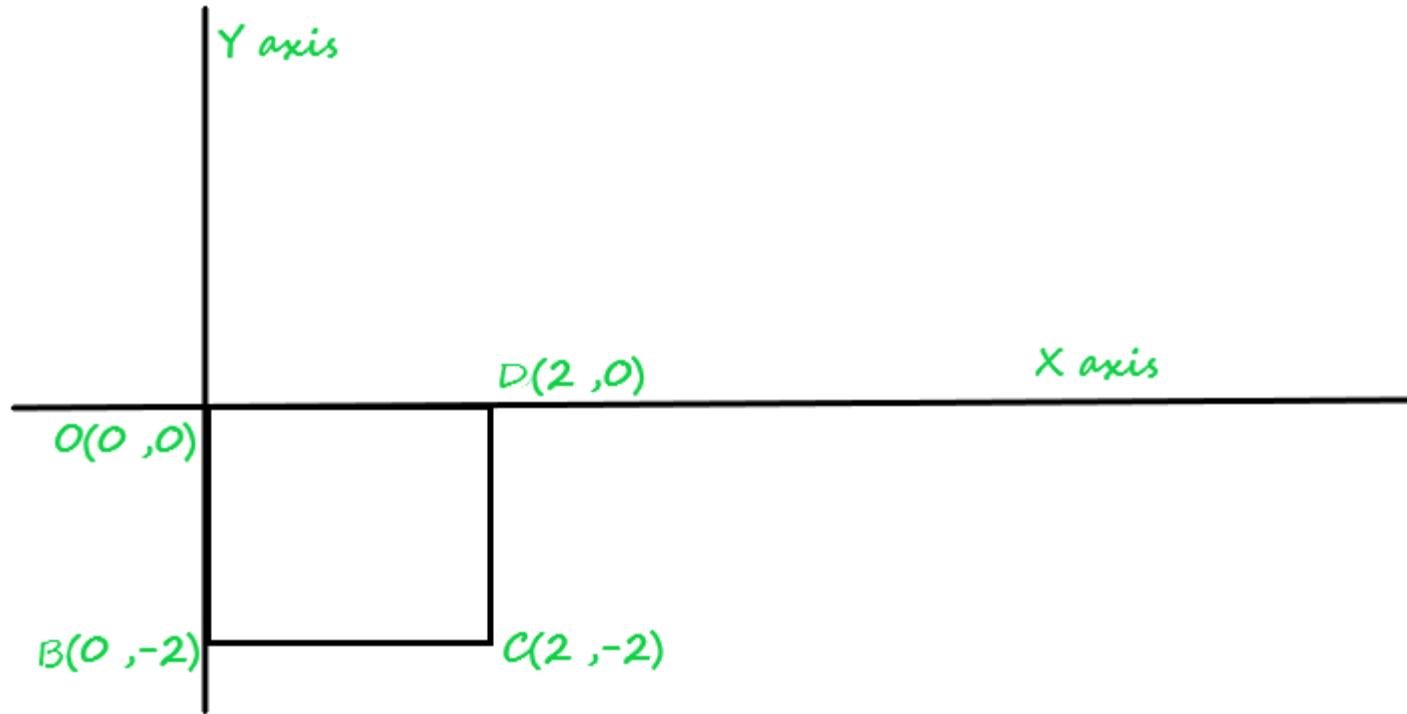


Example





Example



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