

final

August 29, 2024

```
[2]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

1 Question 1

```
[4]: df = pd.read_csv("SeoulBikeData.csv")

# Remove non functioning Days
df = df[df['Functioning Day'] == 'Yes']

# select winter seasons
df = df[df["Seasons"] == "Winter"]
```

```
[5]: # separate into snowfall and no snowfall data
df_snowfall = df[df["Snowfall (cm)"] > 0]
df_no_snowfall = df[df["Snowfall (cm)"] == 0]

print("Number of points with snowfall: ", len(df_snowfall))
print("Number of points with no snowfall: ", len(df_no_snowfall))
```

Number of points with snowfall: 392
Number of points with no snowfall: 1768

```
[6]: # Average bike rentals during both seasons
mean_snowfall = df_snowfall["Rented Bike Count"].mean()
mean_noSnowfall = df_no_snowfall["Rented Bike Count"].mean()

print("Average bike rentals during snowfall: ", mean_snowfall)
print("Average bike rentals during no snowfall: ", mean_noSnowfall)
```

Average bike rentals during snowfall: 157.30357142857142
Average bike rentals during no snowfall: 240.670814479638

```
[8]: sigmaSnowfall = 0

sigmaSnowfall = (df_snowfall["Rented Bike Count"] - mean_snowfall).pow(2).sum()
sigmaSnowfall = sigmaSnowfall / len(df_snowfall)

sigmaNoSnowfall = 0

sigmaNoSnowfall = (df_no_snowfall["Rented Bike Count"] - mean_noSnowfall).
    ↪pow(2).sum()
sigmaNoSnowfall = sigmaNoSnowfall / len(df_no_snowfall)

print("Standard deviation of bike rentals during snowfall: ", np.
    ↪sqrt(sigmaSnowfall))
print("Standard deviation of bike rentals during no snowfall: ", np.
    ↪sqrt(sigmaNoSnowfall))
```

Standard deviation of bike rentals during snowfall: 109.10574437962767
 Standard deviation of bike rentals during no snowfall: 153.98643284765893

```
[9]: print("Variance of bike rentals during snowfall: ", sigmaSnowfall)
print("Variance of bike rentals during no snowfall: ", sigmaNoSnowfall)
```

Variance of bike rentals during snowfall: 11904.063456632653
 Variance of bike rentals during no snowfall: 23711.821501146576

```
[10]: # Calculate Test Statistic T
T = (mean_snowfall - mean_noSnowfall) / np.sqrt(sigmaSnowfall/len(df_snowfall)
    ↪+ sigmaNoSnowfall/len(df_no_snowfall))
print("Test Statistic T: ", T)
```

Test Statistic T: -12.599742371278047

```
[11]: # Do Z test
from scipy.stats import norm

alpha = 0.05
z = norm.ppf(1-alpha/2)

print("Z value: ", z)
```

Z value: 1.959963984540054

```
[13]: if(T >= -z):
    print("Null Hypothesis is accepted")
else:
    print("Failed to accept Null Hypothesis")
```

Failed to accept Null Hypothesis

```
[15]: # Calculate p value
p = norm.cdf(T)
print("P value: ", p)
```

P value: 1.0591764873897083e-36

```
[ ]:
```

2 Question II

```
[16]: # Read again the data
df = pd.read_csv("SeoulBikeData.csv")

# Remove the non functioning days
df = df[df['Functioning Day'] == 'Yes']

# Filter out the data with non zero rainfall
df = df[df['Rainfall(mm)'] != 0]

print("number of rows with non zero rainfall: ", df.shape[0])
```

number of rows with non zero rainfall: 516

```
[17]: import matplotlib.pyplot as plt
import seaborn as sns

# Set the aesthetic style of the plots
sns.set(style="whitegrid")

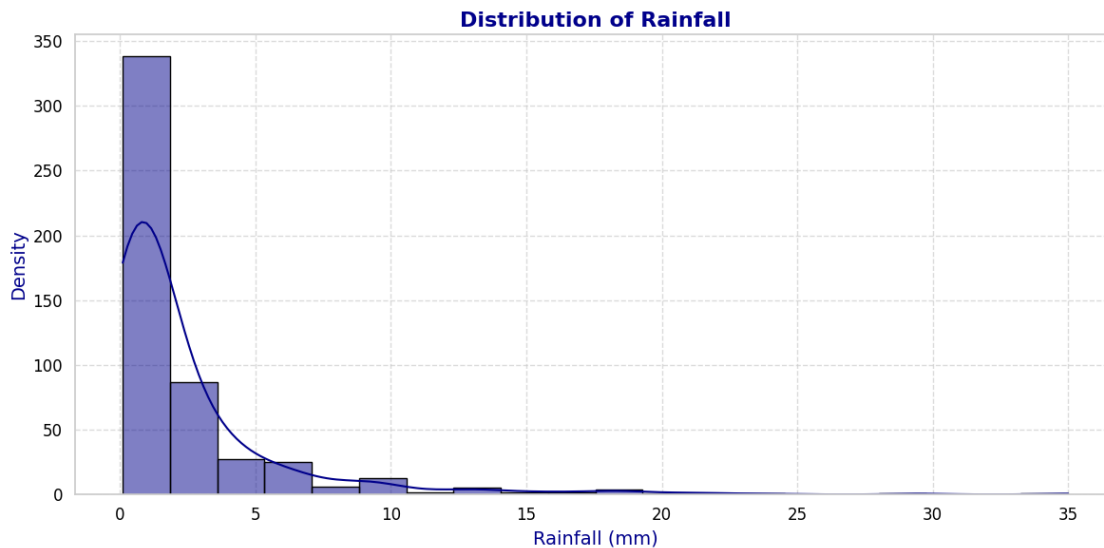
# Plot
plt.figure(figsize=(12, 6))
sns.histplot(df['Rainfall(mm)'], bins=20, color='darkblue', edgecolor='black',
             kde=True, linewidth=1)

# Title and labels
plt.title('Distribution of Rainfall', fontsize=16, fontweight='bold',
         color='darkblue')
plt.xlabel('Rainfall (mm)', fontsize=14, color='darkblue')
plt.ylabel('Density', fontsize=14, color='darkblue')

# Customize tick parameters
plt.xticks(fontsize=12, color='black')
plt.yticks(fontsize=12, color='black')

# Add gridlines
plt.grid(True, linestyle='--', alpha=0.7)
```

```
# Enhance the layout and show plot
plt.tight_layout()
plt.show()
```



```
[18]: import matplotlib.pyplot as plt
import seaborn as sns

# Set the aesthetic style of the plots
sns.set(style="whitegrid")

# Plot
plt.figure(figsize=(12, 6))
sns.boxplot(x='Rainfall(mm)', data=df, palette='Set2', linewidth=2)

# Title and labels
plt.title('Boxplot of Rainfall', fontsize=16, fontweight='bold', color='darkblue')
plt.xlabel('Rainfall (mm)', fontsize=14, color='darkblue')

# Customize tick parameters
plt.xticks(fontsize=12, color='black')
plt.yticks(fontsize=12, color='black')

# Add gridlines
plt.grid(True, linestyle='--', alpha=0.7)

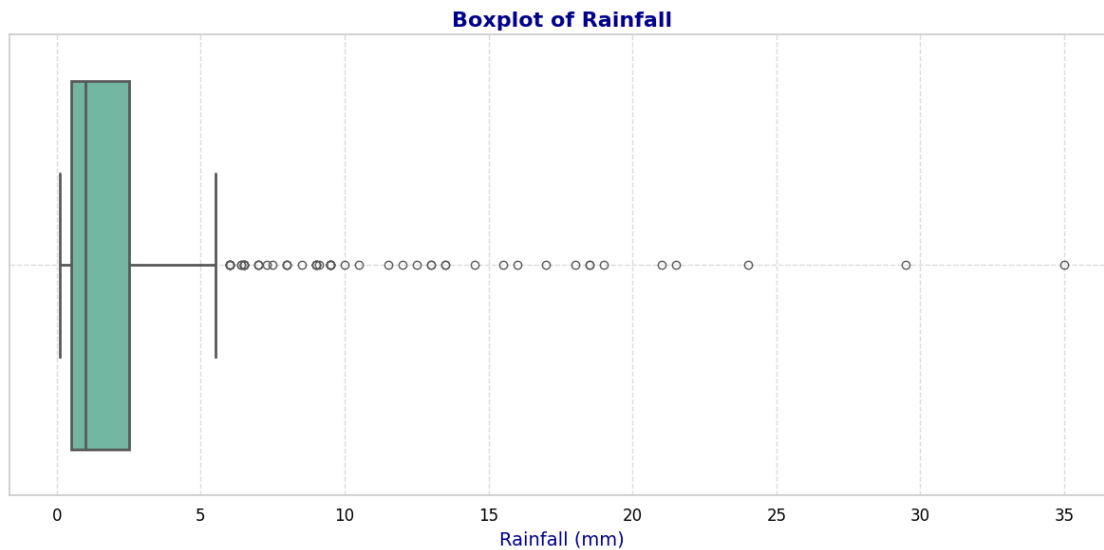
# Enhance the layout and show plot
```

```
plt.tight_layout()
plt.show()
```

/tmp/ipykernel_368280/1555155085.py:9: FutureWarning:

Passing `palette` without assigning `hue` is deprecated and will be removed in v0.14.0. Assign the `y` variable to `hue` and set `legend=False` for the same effect.

```
sns.boxplot(x='Rainfall(mm)', data=df, palette='Set2', linewidth=2)
```



```
[19]: # Separate based on Quartiles
```

```
dfQuartile1 = df[df['Rainfall(mm)'] < 0.5]
dfQuartile2 = df[(df['Rainfall(mm)'] >= 0.5) & (df['Rainfall(mm)'] < 1.00)]
dfQuartile3 = df[(df['Rainfall(mm)'] >= 1.00) & (df['Rainfall(mm)'] < 2.50)]
dfQuartile4 = df[df['Rainfall(mm)'] >= 2.50]
```

```
[20]: print("Number of rows in Quartile 1: ", dfQuartile1.shape[0])
print("Number of rows in Quartile 2: ", dfQuartile2.shape[0])
print("Number of rows in Quartile 3: ", dfQuartile3.shape[0])
print("Number of rows in Quartile 4: ", dfQuartile4.shape[0])
```

```
Number of rows in Quartile 1: 89
Number of rows in Quartile 2: 121
Number of rows in Quartile 3: 160
Number of rows in Quartile 4: 146
```

```

[21]: import matplotlib.pyplot as plt
import seaborn as sns

# Set the aesthetic style of the plots
sns.set(style="whitegrid")

# Plot
plt.figure(figsize=(14, 7))

# Quartile 1
sns.histplot(dfQuartile1['Hour'], bins=24, color='darkblue', edgecolor='black',
             ↪kde=False, linewidth=2, label='Quartile 1', alpha=0.7)

# Quartile 2
sns.histplot(dfQuartile2['Hour'], bins=24, color='red', edgecolor='black',
             ↪kde=False, linewidth=2, label='Quartile 2', alpha=0.7)

# Quartile 3
sns.histplot(dfQuartile3['Hour'], bins=24, color='green', edgecolor='black',
             ↪kde=False, linewidth=2, label='Quartile 3', alpha=0.7)

# Quartile 4
sns.histplot(dfQuartile4['Hour'], bins=24, color='orange', edgecolor='black',
             ↪kde=False, linewidth=2, label='Quartile 4', alpha=0.7)

# Title and labels
plt.title('Hourly Bike Rental in All Quartiles', fontsize=18,
         ↪fontweight='bold', color='darkblue')
plt.xlabel('Hour of the Day', fontsize=15, color='darkblue')
plt.ylabel('Count', fontsize=15, color='darkblue')

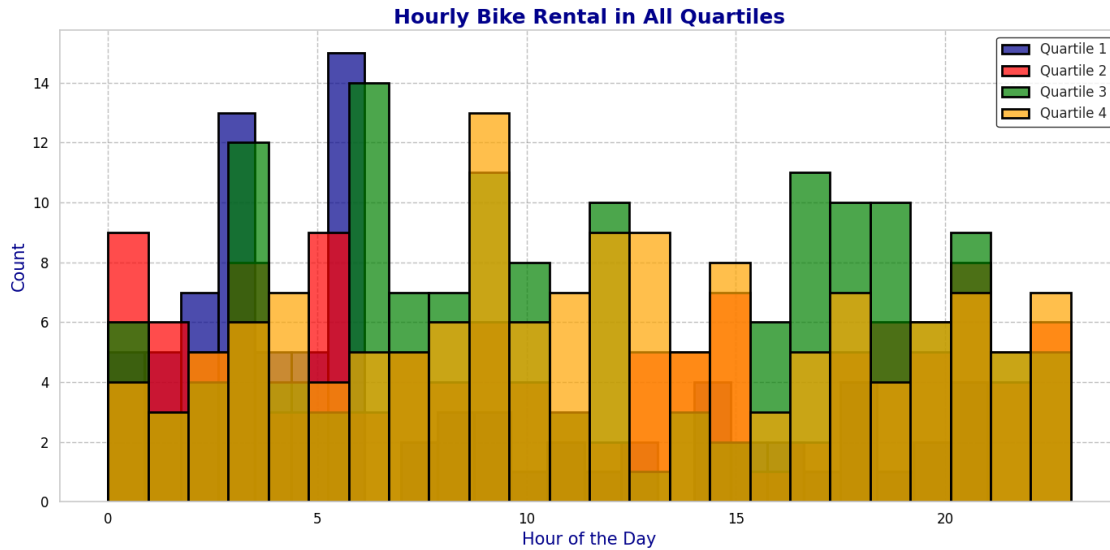
# Customize tick parameters
plt.xticks(fontsize=12, color='black')
plt.yticks(fontsize=12, color='black')

# Add gridlines
plt.grid(True, linestyle='--', color='gray', alpha=0.5)

# Add legend
plt.legend(fontsize=12, loc='upper right', frameon=True, facecolor='white',
         ↪edgecolor='black')

# Enhance the layout and show plot
plt.tight_layout()
plt.show()

```



```
[22]: import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd

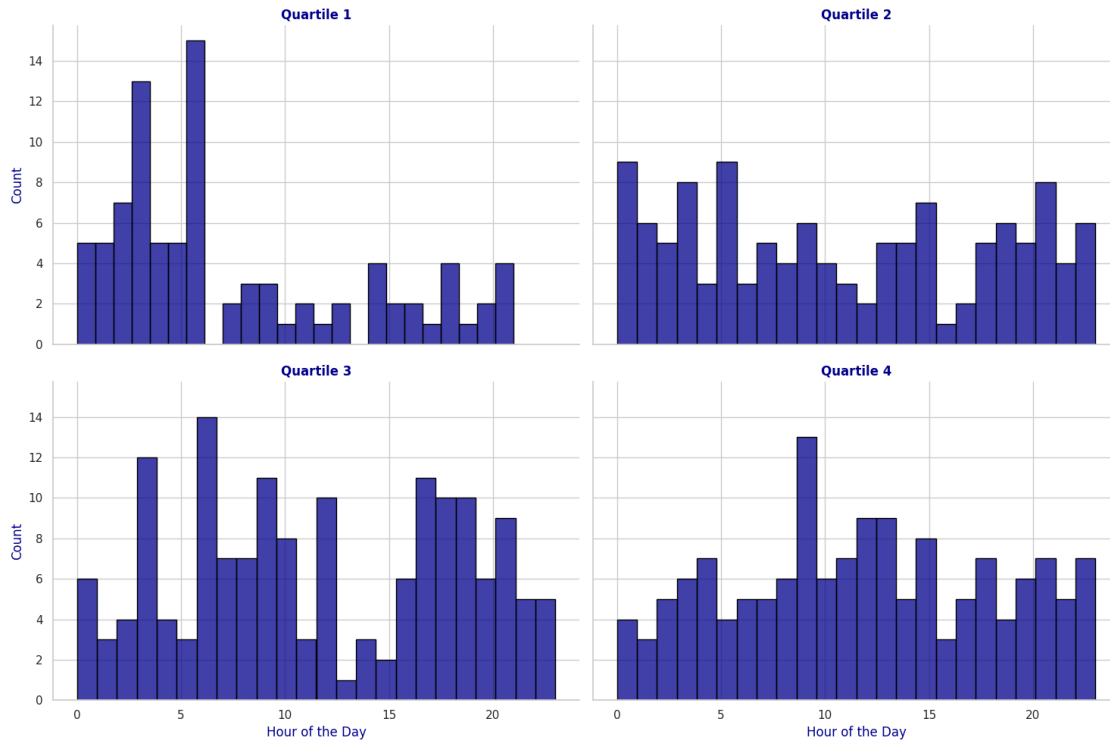
# Combine quartiles into a single DataFrame for facet plotting
df_combined = pd.concat([
    dfQuartile1.assign(Quartile='Quartile 1'),
    dfQuartile2.assign(Quartile='Quartile 2'),
    dfQuartile3.assign(Quartile='Quartile 3'),
    dfQuartile4.assign(Quartile='Quartile 4')
])

# Plot
plt.figure(figsize=(14, 7))
g = sns.FacetGrid(df_combined, col='Quartile', col_wrap=2, height=5, aspect=1.5)
g.map(sns.histplot, 'Hour', bins=24, color='darkblue', edgecolor='black',
      kde=False)

# Titles and labels
g.set_titles(col_template="{col_name}", fontsize=14, fontweight='bold',
            color='darkblue')
g.set_axis_labels('Hour of the Day', 'Count', fontsize=12, color='darkblue')

# Adjust layout and show plot
plt.tight_layout()
plt.show()
```

<Figure size 1400x700 with 0 Axes>



Note: I have used chatGPT to make these plots more beautiful

```
[23]: mean1 = dfQuartile1['Rented Bike Count'].mean()
mean2 = dfQuartile2['Rented Bike Count'].mean()
mean3 = dfQuartile3['Rented Bike Count'].mean()
mean4 = dfQuartile4['Rented Bike Count'].mean()

print("Mean of Rented Bike Count in Quartile 1: ", mean1)
print("Mean of Rented Bike Count in Quartile 2: ", mean2)
print("Mean of Rented Bike Count in Quartile 3: ", mean3)
print("Mean of Rented Bike Count in Quartile 4: ", mean4)
```

```
Mean of Rented Bike Count in Quartile 1: 252.0561797752809
Mean of Rented Bike Count in Quartile 2: 252.27272727272728
Mean of Rented Bike Count in Quartile 3: 127.1875
Mean of Rented Bike Count in Quartile 4: 89.02054794520548
```

```
[24]: SSW = 0
SSW += sum((dfQuartile1['Rented Bike Count'] - mean1) ** 2)
SSW += sum((dfQuartile2['Rented Bike Count'] - mean2) ** 2)
SSW += sum((dfQuartile3['Rented Bike Count'] - mean3) ** 2)
SSW += sum((dfQuartile4['Rented Bike Count'] - mean4) ** 2)

print("SSW: ", SSW)
```


SSW: 33201816.03245729

```
[25]: SSB = 0

SSB += len(dfQuartile1) * (mean1 - df['Rented Bike Count'].mean()) ** 2
SSB += len(dfQuartile2) * (mean2 - df['Rented Bike Count'].mean()) ** 2
SSB += len(dfQuartile3) * (mean3 - df['Rented Bike Count'].mean()) ** 2
SSB += len(dfQuartile4) * (mean4 - df['Rented Bike Count'].mean()) ** 2

print("SSB: ", SSB)
```

SSB: 2665086.6865349603

```
[26]: F = (SSB / 3) / (SSW / (len(dfQuartile1) + len(dfQuartile2) + len(dfQuartile3) +
    ↪ len(dfQuartile4) - 4))

print("F: ", F)
```

F: 13.6992946627977

```
[27]: # Import the f-distribution
from scipy.stats import f

# Define the number of groups and the degrees of freedom
num_groups = 4
dfn = num_groups - 1
dfd = len(dfQuartile1) + len(dfQuartile2) + len(dfQuartile3) + len(dfQuartile4) +
    ↪ num_groups

# Calculate the critical value
critical_value = f.ppf(0.95, dfn, dfd)

print("Critical Value: ", critical_value)
```

Critical Value: 2.6223153068438396

```
[28]: # Compare the F-value with the critical value
if F > critical_value:
    print("Reject the null hypothesis: There is a significant difference_
    ↪ between the group means.")
else:
    print("Fail to reject the null hypothesis: There is no significant_
    ↪ difference between the group means.")
```

Reject the null hypothesis: There is a significant difference between the group means.

```
[29]: # Calculate the p-value
p_value = 1 - f.cdf(F, dfn, dfd)

print("P-Value: ", p_value)
```

P-Value: 1.3142755661554872e-08

```
[30]: # Verify the results using the ANOVA test
from scipy.stats import f_oneway

# Perform the ANOVA test
f_statistic, p_value = f_oneway(dfQuartile1['Rented Bike Count'],
    dfQuartile2['Rented Bike Count'], dfQuartile3['Rented Bike Count'],
    dfQuartile4['Rented Bike Count'])

print("F-Statistic: ", f_statistic)
print("P-Value: ", p_value)
```

F-Statistic: 13.699294662797698

P-Value: 1.3142755691851183e-08

```
[ ]:
```

3 Question III

```
[31]: # Read the data
df = pd.read_csv("SeoulBikeData.csv")

# Remove the non functioning days
df = df[df['Functioning Day'] == 'Yes']

# Separate Spring and Winter data
df_spring = df[df['Seasons'] == 'Spring']
df_summer = df[df['Seasons'] == 'Summer']

print("Number of rows in Spring data: ", df_spring.shape[0])
print("Number of rows in Winter data: ", df_summer.shape[0])
```

Number of rows in Spring data: 2160

Number of rows in Winter data: 2208

```
[32]: import matplotlib.pyplot as plt

# Group by 'Hour' and calculate the mean 'Rented Bike Count' for spring
avg_hourly_rentals_spring = df_spring.groupby('Hour')['Rented Bike Count'].
    mean()
```

```

# Plot
plt.figure(figsize=(12, 6))
bars = plt.bar(avg_hourly_rentals_spring.index, avg_hourly_rentals_spring,
               ↪color='skyblue', edgecolor='black')

# Add value labels on top of the bars
for bar in bars:
    yval = bar.get_height()
    plt.text(bar.get_x() + bar.get_width()/2, yval, int(yval), va='bottom',
             ↪ha='center', fontsize=10, color='black')

# Title and labels
plt.title('Average Hourly Bike Rental Count in Spring', fontsize=16,
         ↪fontweight='bold', color='navy')
plt.xlabel('Hour of the Day', fontsize=14)
plt.ylabel('Average Rental Count', fontsize=14)

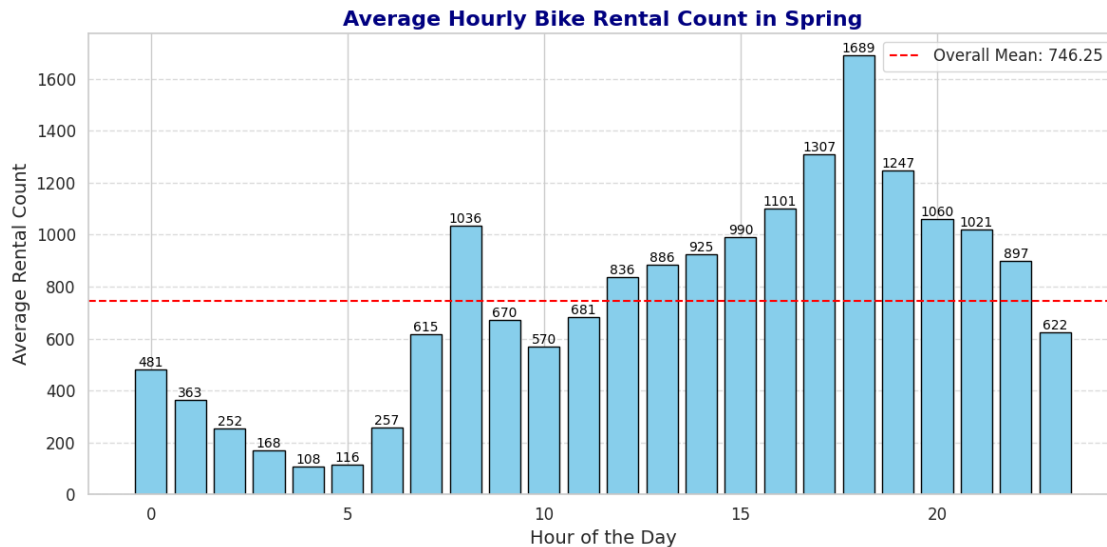
# Customize tick parameters
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

# Add gridlines for the y-axis
plt.grid(axis='y', linestyle='--', alpha=0.7)

# Add a horizontal line for the mean
mean_value = avg_hourly_rentals_spring.mean()
plt.axhline(mean_value, color='red', linestyle='--', linewidth=1.5,
           ↪label=f'Overall Mean: {mean_value:.2f}')
plt.legend(fontsize=12, loc='upper right')

# Enhance the layout and show plot
plt.tight_layout()
plt.show()

```



```
[33]: import matplotlib.pyplot as plt

# Group by 'Hour' and calculate the mean 'Rented Bike Count' for summer
avg_hourly_rentals_summer = df_summer.groupby('Hour')['Rented Bike Count'].
    ↪mean()

# Plot
plt.figure(figsize=(12, 6))
bars = plt.bar(avg_hourly_rentals_summer.index, avg_hourly_rentals_summer,
    ↪color='lightcoral', edgecolor='black')

# Add value labels on top of the bars
for bar in bars:
    yval = bar.get_height()
    plt.text(bar.get_x() + bar.get_width()/2, yval, int(yval), va='bottom',
    ↪ha='center', fontsize=10, color='black')

# Title and labels
plt.title('Average Hourly Bike Rental Count in Summer', fontsize=16,
    ↪fontweight='bold', color='darkred')
plt.xlabel('Hour of the Day', fontsize=14)
plt.ylabel('Average Rental Count', fontsize=14)

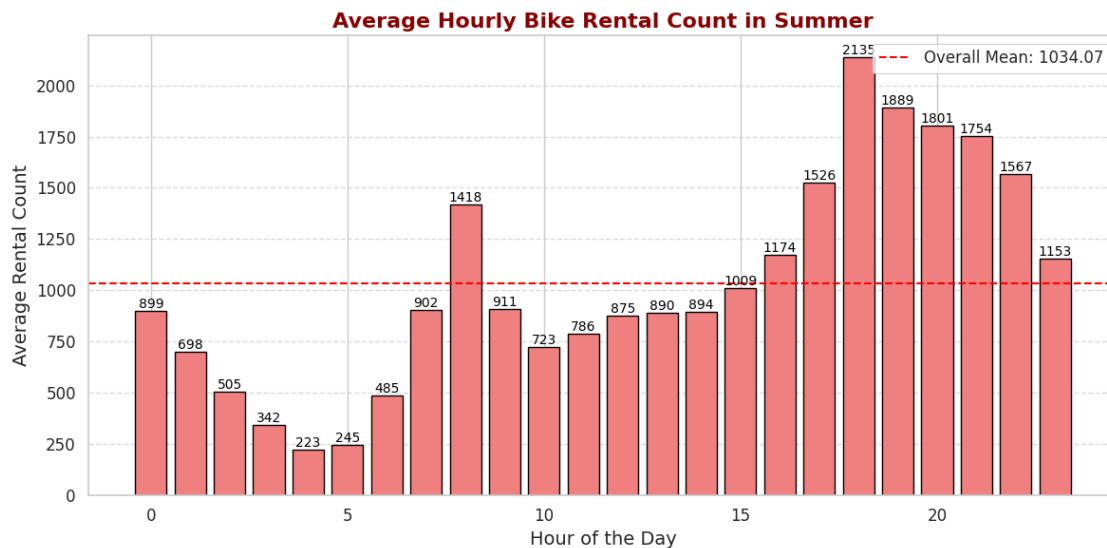
# Customize tick parameters
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

# Add gridlines for the y-axis
```

```
plt.grid(axis='y', linestyle='--', alpha=0.7)

# Add a horizontal line for the mean
mean_value = avg_hourly_rentals_summer.mean()
plt.axhline(mean_value, color='red', linestyle='--', linewidth=1.5,
            label=f'Overall Mean: {mean_value:.2f}')
plt.legend(fontsize=12, loc='upper right')

# Enhance the layout and show plot
plt.tight_layout()
plt.show()
```



```
[34]: # Make a dataframe with 2 columns: Spring and summer and rows as average hourly
      ↪ bike rental count
```

```
df_spring_avg = df_spring.groupby('Hour')['Rented Bike Count'].mean()
df_summer_avg = df_summer.groupby('Hour')['Rented Bike Count'].mean()
```

```
[35]: # Merge the two dataframes
```

```
df_merged = pd.merge(df_spring_avg, df_summer_avg, on='Hour')
df_merged.columns = ['Spring', 'Summer']
```

```
[36]: # Add Row sum and Column sum
```

```
df_merged.loc['Column Sum'] = df_merged.sum()
df_merged['Row Sum'] = df_merged.sum(axis=1)
```

```
[37]: # Calculate the Expected Values for the Summer Season using formula (Row Total
      ↪ * Column Total) / Grand Total
```

```
df_merged["Expected Spring"] = 3
```

```
df_merged["Expected Summer"] = 3
```

```
[42]: try:
        colSum1 = df_merged.loc['Column Sum'].iloc[0]
        print(f'colSum1: {colSum1}')

        colSum2 = df_merged.loc['Column Sum'].iloc[1]
        print(f'colSum2: {colSum2}')

        total = df_merged.loc['Column Sum'].iloc[2]
        print(f'total: {total}')
    except KeyError as e:
        print(f'KeyError: {e} - Check if the index exists in the DataFrame')
    except IndexError as e:
        print(f'IndexError: {e} - Check if the position indices are correct')
```

```
colSum1: 17910.1
colSum2: 24817.760869565216
total: 42727.860869565215
```

```
[39]:
```

```
17910.1
24817.760869565216
42727.860869565215

/tmp/ipykernel_368280/4156945129.py:1: FutureWarning: Series.__getitem__
treating keys as positions is deprecated. In a future version, integer keys will
always be treated as labels (consistent with DataFrame behavior). To access a
value by position, use `ser.iloc[pos]`
    colSum1 = df_merged.loc['Column Sum'][0]
/tmp/ipykernel_368280/4156945129.py:4: FutureWarning: Series.__getitem__
treating keys as positions is deprecated. In a future version, integer keys will
always be treated as labels (consistent with DataFrame behavior). To access a
value by position, use `ser.iloc[pos]`
    colSum2 = df_merged.loc['Column Sum'][1]
/tmp/ipykernel_368280/4156945129.py:7: FutureWarning: Series.__getitem__
treating keys as positions is deprecated. In a future version, integer keys will
always be treated as labels (consistent with DataFrame behavior). To access a
value by position, use `ser.iloc[pos]`
    total = df_merged.loc['Column Sum'][2]
```

```
[45]: # Ensure 'Expected Spring' and 'Expected Summer' columns exist
df_merged['Expected Spring'] = 0.0
df_merged['Expected Summer'] = 0.0

# Calculate expected values
for i in range(24):
```

```

expectedValue1 = df_merged.iloc[i]['Row Sum'] * colSum1 / total
expectedValue2 = df_merged.iloc[i]['Row Sum'] * colSum2 / total

# Use .at[] for scalar assignments to avoid warnings
df_merged.at[i, 'Expected Spring'] = expectedValue1
df_merged.at[i, 'Expected Summer'] = expectedValue2

```

```

[46]: import seaborn as sns
import matplotlib.pyplot as plt

# Set the aesthetic style of the plots to a dark background
sns.set(style="darkgrid")

# Increase the figure size
plt.figure(figsize=(12, 8))

# Plot the heatmap with a different color palette and enhanced styling
heatmap = sns.heatmap(
    df_merged.iloc[:, :3],
    annot=True,
    fmt=".2f",
    cmap='cividis', # Changed to a vibrant color palette
    linewidths=0.3, # Subtle grid lines
    linecolor='black', # Grid line color
    cbar_kws={'shrink': 0.8, 'aspect': 10}, # Customized color bar
    annot_kws={"size": 11, "weight": "bold", "color": "white"}, # Annotation
    ↪text style
)

# Add a title with custom styling
plt.title('Heatmap of Observed and Expected Values for Spring and Summer ↪
    ↪Seasons',
          fontsize=18, weight='bold', color='white', pad=20)

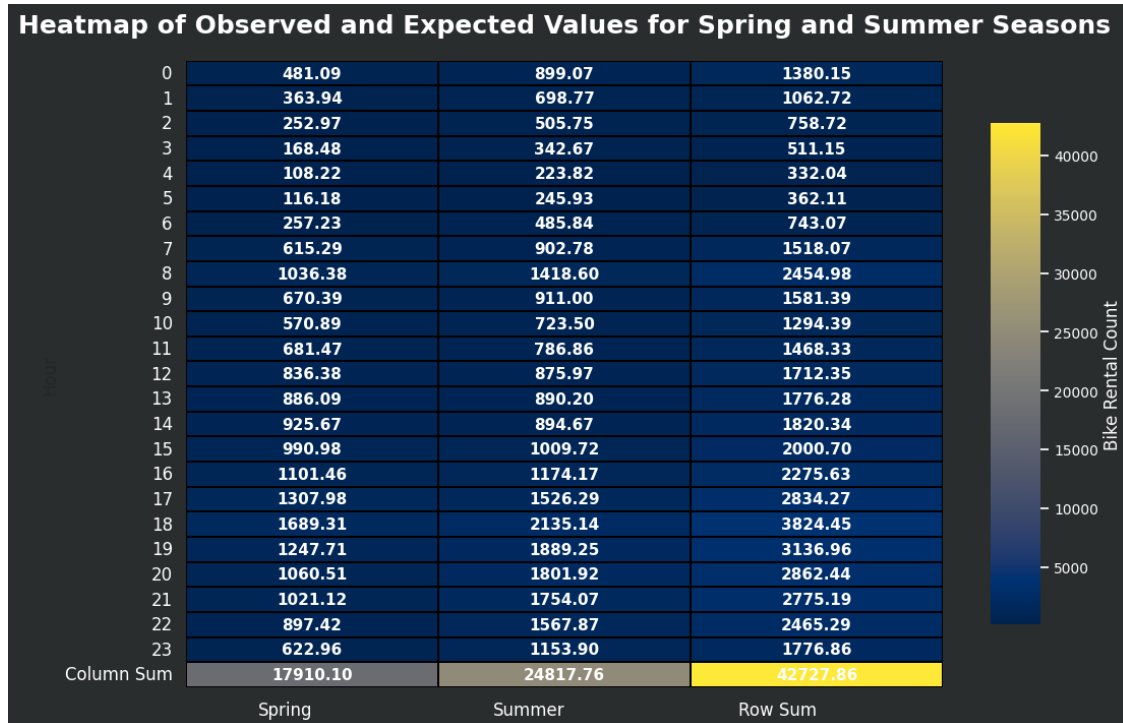
# Rotate the x and y labels for better readability
plt.xticks(ha='right', fontsize=12, color='white')
plt.yticks(rotation=0, fontsize=12, color='white')

# Customize axes and background
heatmap.figure.set_facecolor('#2a2d2e') # Set background color
heatmap.set_facecolor('#2a2d2e') # Set the face color of the plot

# Add a color bar with a label
colorbar = heatmap.collections[0].colorbar
colorbar.set_label('Bike Rental Count', fontsize=12, color='white')
colorbar.ax.tick_params(labelsize=10, colors='white')

```

```
# Show the plot
plt.show()
```



```
[47]: import seaborn as sns
import matplotlib.pyplot as plt

# Set the aesthetic style of the plots
sns.set(style="darkgrid")

# Increase the figure size
plt.figure(figsize=(12, 8))

# Plot the heatmap for the next two columns with enhanced aesthetics
heatmap = sns.heatmap(
    df_merged.iloc[:-1, 3:5], # Select the next two columns
    annot=True,
    fmt=".2f",
    cmap='Spectral', # Use a vibrant color palette
    linewidths=0.3, # Add subtle grid lines
    linecolor='black', # Grid line color
    cbar_kws={'shrink': 0.8, 'aspect': 10}, # Customize color bar
    annot_kws={"size": 11, "weight": "bold", "color": "black"}, # Annotation
    text style
)
```



```

# Add a title with custom styling
plt.title('Heatmap of Observed and Expected Values for Spring and Summer',
          fontsize=18, weight='bold', color='white', pad=20)

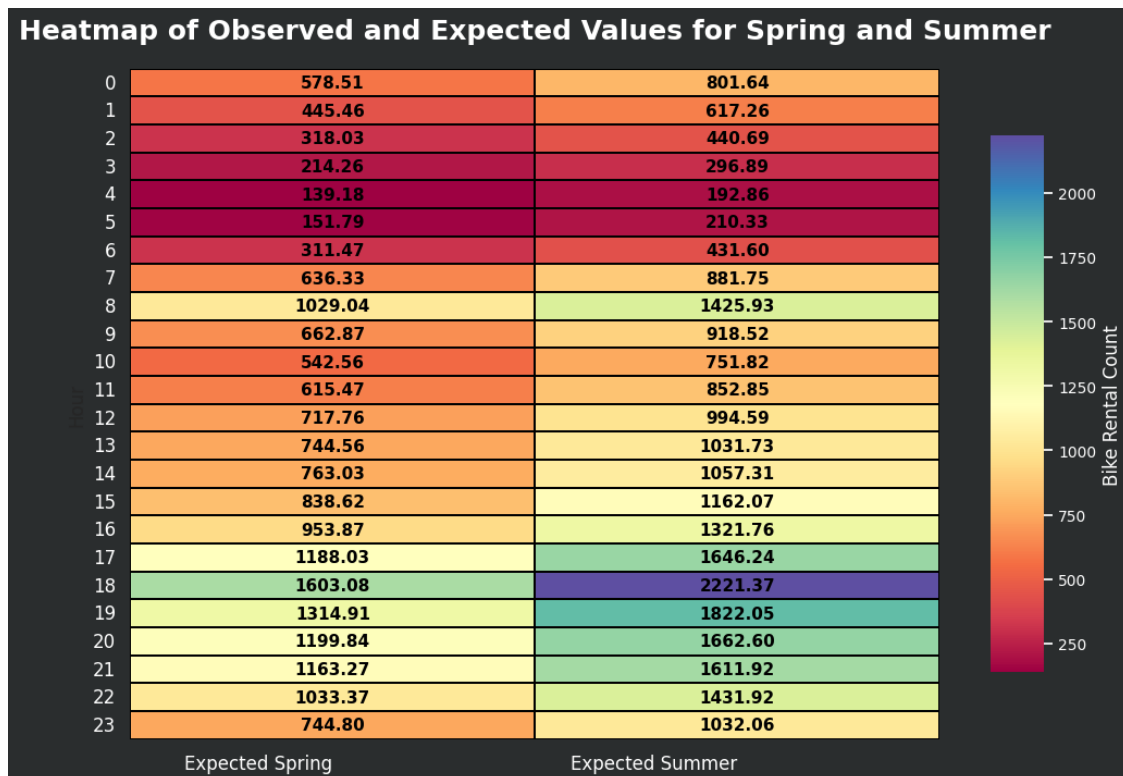
# Rotate the x and y labels for better readability
plt.xticks(ha='right', fontsize=12, color='white')
plt.yticks(rotation=0, fontsize=12, color='white')

# Customize axes and background
heatmap.figure.set_facecolor('#2a2d2e') # Set background color
heatmap.set_facecolor('#2a2d2e') # Set the face color of the plot

# Add a color bar with a label
colorbar = heatmap.collections[0].colorbar
colorbar.set_label('Bike Rental Count', fontsize=12, color='white')
colorbar.ax.tick_params(labelsize=10, colors='white')

# Show the plot
plt.show()

```



```
[48]: # Compute the Test Statistic for Chi-Square Test where Observed is the summer_
      ↪column except for the last row and Expected is the Expected Summer column
T = 0
for i in range(0, 24):
    E = df_merged.iloc[i]["Expected Summer"]
    O = df_merged.iloc[i]["Summer"]

    T += ((O - E) ** 2) / E

    E = df_merged.iloc[i]["Expected Spring"]
    O = df_merged.iloc[i]["Spring"]

    T += ((O - E) ** 2) / E

T
```

[48]: 536.676606781657

```
[49]: DOF = 24
      alpha = 0.05

      from scipy.stats import chi2

      critical_value = chi2.ppf(1 - alpha, DOF)
      critical_value
```

[49]: 36.41502850180731

```
[50]: # Compare the Test Statistic with the Critical Value
      if T > critical_value:
          print("Reject the Null Hypothesis")
      else:
          print("Fail to Reject the Null Hypothesis")
```

Reject the Null Hypothesis

```
[51]: import scipy.stats as stats

      # Assuming T is your Chi-Square test statistic and DOF is the degrees of freedom
      p_value = stats.chi2.sf(T, DOF)
      print(f"p-value: {p_value}")
```

p-value: 3.932021483326344e-98

```
[54]: # Use library function to perform the Chi-Square Test
      from scipy.stats import chi2_contingency
```

```

spring_data = df_merged.iloc[:, 0].values
summer_data = df_merged.iloc[:, 1].values

import numpy as np
import scipy.stats as stats
data = np.array([spring_data, summer_data])

chi2, p, dof, expected = stats.chi2_contingency(data)

print("Chi-Square Statistic: ", chi2)
print("P-Value: ", p)
print("Degrees of Freedom: ", dof)

```

Chi-Square Statistic: 536.676606781657
P-Value: 3.932021483326344e-98
Degrees of Freedom: 24

```

[55]: print("Chi-Square Statistic: ", chi2)
      print("P-Value: ", p)
      print("Degrees of Freedom: ", dof)
      print("Expected: ", expected)

```

Chi-Square Statistic: 536.676606781657
P-Value: 3.932021483326344e-98
Degrees of Freedom: 24
Expected: [[578.51475725 445.45532428 318.02882464 214.25781004
139.17906551 151.78555715 311.4703832 636.32514653
1029.04422707 662.86569378 542.56482696 615.47321903
717.7582165 744.55947739 763.02630539 838.62495848
953.86828444 1188.03236553 1603.08341705 1314.90989843
1199.83768162 1163.26639225 1033.36842855 744.79973893
17910.1]
[801.63934903 617.26085929 440.68784203 296.89388078
192.8583741 210.32700324 431.59990666 881.74635105
1425.9313768 918.52319511 751.82406193 852.85214329
994.58695258 1031.72506368 1057.31427432 1162.0702106
1321.76118416 1646.23889051 2221.36899841 1822.05121268
1662.59734254 1611.92104737 1431.92335889 1032.05799054
24817.76086957]]

```
[ ]:
```