

(12) Longest subarray with given sum (positive)

Longest subarray with sum 'K' (positive case)

eg:  $arr[] = [1, 2, 3, 1, 1, 1, 1, 9, 2, 3]$

Sum of subarray means.

with eg.  $[1, 2]$  is a subarray its sum is 3 but its length is 2.

$[3]$  is a subarray its sum is 3 but its length is 1.

but  $[1, 1, 1]$  is also subarray its sum is 3 and length is 3.

So one of them '3' is the length so this subarray is answer.

a subarray defined as ~~from~~ late element from array, contiguous element. otherwise it is not array.

Brute force approach:

generate the every possible subarray from that array and find the each length which is max and that follow condition ( $max == sum$ ) then that subarray is answer.

How do you generate subarray.

By taking 2 pointer.

arr =  $\begin{matrix} & i & & & & & & & & \\ 1 & 2 & 3 & 1 & & & & & & \\ \uparrow & \uparrow & \uparrow & \uparrow & & & & & & \\ j & j & j & j & & & & & & \end{matrix}$   $i \leq j \rightarrow$  stay where that is a subarray  
=  $[1], [1, 2], [1, 2, 3], [1, 2, 3, 1]$

these are subarray and it is first iteration, if 'j' will go to last of array then we move again.

arr  $\rightarrow \begin{matrix} & i & & & & & & & & \\ 1 & 2 & 3 & 1 & & & & & & \\ & \uparrow & \uparrow & \uparrow & & & & & & \\ & j & j & j & & & & & & \end{matrix}$  find sum btw i & j  
=  $[2], [2, 3], [2, 3, 1]$  this  $\frac{j-i+1}{}$  for loop  
all other subarray.

```

i = 0
for (i = 0; i < n; i++) {
    for (j = i; j < n; j++) {
        // we are finding sum of generated subarray and calculate
        S = 0
        for (k = i; k <= j; k++) {
            S = S + arr[k];
        }
        if (S == K) return j - i + 1;
    }
}
return 0;
    
```

Time complexity is near about  $O(n^3)$  because i & k are reducing in next iteration.

Space and space is not used so space complexity is  $O(1)$

# ~~optimized solution~~

# reduced brute force method (time  $O(n^2)$ ).  
we don't need to run extra loop that is 'k'  
simple we can calculate sum after moving 'j'

for (i = 0; i < n; i++)

s = 0

for (j = i; j < n; j++)

s += a[j];

~~if (s == k) return i;~~

i = Math.max(i, j - 1);

length  
update

~~if (s == k) return i;~~

here time

Complexity =  $O(n^2)$

Space =  $O(1)$

Better solution: Naïve

and prefix sum pattern:

Let's learn first about this algorithm / pattern

eg arr = [1, 3, -2, 4, -1, 5]

It is simple and powerful technique that allows  
to perform fast calculation on the sum of elements  
in a given range (called contiguous segment of arr)

a[i] = [1, 4, 2, 6, 5, 10]

that is prefix sum stored  
in same array

we sum prefix element and  
current element from start to end

Not taken extra space  
But use  $O(n)$  time

// we always start iterating loop from next index (not 0, but at 1)

Step wise

a = [1, 3, -2, 4, -1, 5]

[1, 4, -2, 4, -1, 5]

[1, 4, 2, 6, -1, 5]

[1, 4, 2, 6, 5, 5]

[1, 4, 2, 6, 5, 10]

a = [1, 4, 2, 6, 5, 10]

we are doing, for (i = 1; i < n)

a[0] = 1, a[1] = a[0] + a

a[2] = a[2] + a[1] = 4 + (-2) = 2

so for (i = 1; i < n; i++)

a[i] = a[i] + a[i-1]



Application: (cases)

If we have array  $\Rightarrow A[] = [6, 9, -2, 9, -1, 0, -5]$

prefix sum =  $A[] = [6, 9, 7, 11, 10, 10, 5]$

If i start from 0 to n  
for (i=0; i<n; i++)  
a[i] =

① Calculate the sum between range  $[0, 4]$ ?

then we can simply take element at '4' index without calculating again, using prefix sum. So ans 10

So if we have ~~the~~ given  $[0, i]$  and say find sum b/w both range so we can take element at index i. we can easily access i<sup>th</sup> index element in  $O(1)$  time.

② Calculate the sum b/w range  $[2, 6]$ ?

$A[] = [6, 9, 7, 11, 10, 10, 5]$

find this  $\downarrow$  and find this  $\downarrow$

$[0, 6] - [0, 1] \Rightarrow (a[6] - a[1])$   
 $5 - 9 = -4$  Ans -4

formula =  $A[i, j] = A[j] - A[i-1]$

generalized form

$[0, i] - [0, i-1]$

complexity  $O(1)$ , and calculate prefix  $O(n)$  so overall  $O(n)$  worst case

If we have only positive integer ~~an~~ element in array and ~~we~~ we want to find subarray of sum  $x$ , then if u use hashmap, it ~~can~~ will create problem that is unnecessary, because if +ve integer have in an and want to find prefix sum and store it as a key and give its value index by checking if it is present or not, ~~at~~ at allocate key the sum of prefix will be increasing all the time because it have only +ve ~~is~~ element, it ~~can~~ will use more space not useful create problem like redundant check.

So if we have element combination of +ve & -ve both then this hashmap technique work efficiently. I will write down the pseudo code on next page.

```

→ Map <Integer, Integer> pm = new HashMap<>();
int currentSum = 0;
int max = 0;
for (int i = 0; i < arr.length; i++) {
    currentSum = currentSum + arr[i];
    if (currentSum == k) { // If subarray find current sum
        // at first & that is ans-
        max = i + 1;
    }
    if (pm.containsKey(currentSum - k)) {
        max = Math.max(max, i - pm.get(currentSum - k));
        // If (currentSum - k) exist in the map then
    }
    if (!pm.containsKey(currentSum)) {
        pm.put(currentSum, i);
    }
}
return max;

```

find sum  
subarray for  
+ve & -ve  
sum = k

Q. Let's dry run

$a = [1, -1, 1]$   
 $k = 0$   
 $CS = 1, -1, 1$   
 $max = 0$   
 $i = 0, 1, 2$   
 $CS - k = 1 - 0 = 1$   
 $-1 - 0 = -1$   
 $1 - 0 = 1$   
 $max = 1$   
 $i = 2$

optimal approach  
time c:  $O(n)$  because  
we are using one, and each  
operation with the hashmap  
is  $O(1)$   
space comp. is  $O(1)$   
&  $O(n)$  in the worst case  
we store each cumulative sum  
in the map

Q. In full dry run [6, 3, -2, 4, -1, 0, 5]



But if array just containing ~~just~~ +ve & zeros!

then the optimal approach: why: Two pointer or greedy approach / Slid window

$a[] = [1, 2, 1, 1, 1, 1, 1, 3, 3]$

$sum = 1+2+1+1+1+1+1+3+3$   
length = 9

$K=6$

$sum == K$

length find.

if  $sum > K$

$sum = sum - a[i]$

if  $sum \leq K$

then we add 'j'

into sum  $j++$

$sum = sum + a[j]$

we have to maintain always  $sum = 6$  or 10.  
if increase 'j' and add into sum if it increasing / greater  $> 6$  then we subtract (from) from the left that is i finally from sum

Time Complexity  
 $\rightarrow O(2n)$  Because j is not return

while i is here

Algorithm / Pseudo code:

when  $j > \text{length}$  then we will exceed.

$i=0, j=0,$

int  $sum = a[0];$

and  $max = 0, n = a.length;$

~~while~~ while ( $j < n$ ) {

while ( $sum > K$  &&  $i \leq j$ ) {

$sum = sum - a[i];$

$i++;$

if ( $sum == K$ ) {

$max = \max(max, j-i+1);$

}

~~i++;~~

if ( $sum \leq K$  &&  $j < n$ ) {

$j++;$

$sum = sum + a[j];$

}

return max;

}

this work only for +ve contain value

what is the time and space complexity?

Time complexity:

$\rightarrow O(2n)$  Because

not inner while loop run for each value of 'j', so it will consider as  $O(2n)$ ;

and space is not why ~~more~~ here any there so space

Complexity is  $O(1)$

Quite simple.

OPTIMISEZ APPROACH WITHOUT USING EXTRA SPACE SOLVED FOR +VE ELEMNT ONLY