1. The Coriolis force is $2m\omega v$ eastward.
But $v \approx gt$.

$\therefore$ Eastward acceleration is $2\omega gt$.

On integration, eastward speed $= \omega g t^2$.

This eastward speed produces a Coriolis force directed radially outward with magnitude

$$2m\omega(\omega g t^2) = 4m\omega^2\left(\frac{gt^2}{2}\right)$$

$$= 4m\omega^2 d.$$

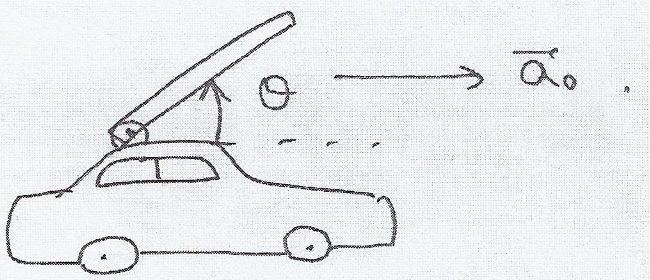
$\therefore$ Total correction to g_{eff} is

$$= \omega^2 d - 4\omega^2 d$$

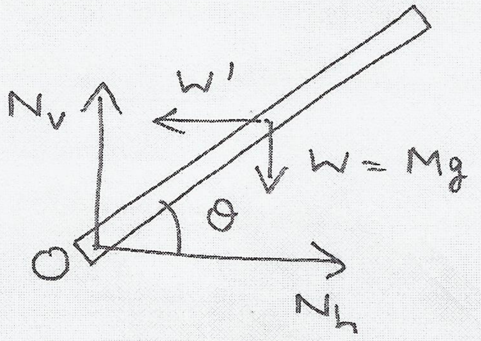
$$= -3\omega^2 d.$$

[Note: Negative sign $\Rightarrow$ g_{eff} is smaller by this amount].

2.



In accelerating system the force diagram



$\overline{W}'$ is the fictitious force

s.t., $\quad \overline{W}' = -M\overline{a_0}$.

Eq. of motion:

$$N_v + W = 0 \ , \quad N_h + W' = 0 .$$

(a.) Torque about pivot O is

$$\tau_0 = \frac{L}{2}(\cos\theta)W - \frac{L}{2}(\sin\theta)W' .$$

For eqbm. in the accl^{ng} system, $\tau_0 = 0$.

$$\Rightarrow 0 = \frac{L}{2}(\cos\theta)W - \frac{L}{2}(\sin\theta)Ma_0 .$$

$$\Rightarrow \tan\theta = \frac{g}{a_0} .$$

3

For equilibrium in any system, the torque about any point must vanish.

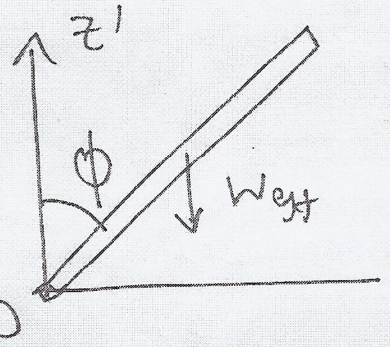
In particular, about center of mass,

$$\Rightarrow \tau_{cm} = \frac{L}{2}(\cos\theta) N_v - \frac{L}{2}(\sin\theta) N_h$$

$$= -\frac{L}{2}(\cos\theta) W + \frac{L}{2}(\sin\theta) W'$$

$$= 0.$$

(b.)



Introducing a coordinate system with z' along the equilibrium position of the board,

$$g_{eff} = \sqrt{g^2 + a_0^2}.$$

$$\therefore W_{eff} = M g_{eff}.$$

For small displacements, the torque is $\tau = \frac{L}{2}(\sin\phi) M g_{eff} \simeq \frac{L}{2} M g_{eff}\, \phi.$

$$\Rightarrow I_a \frac{d^2\phi}{dt^2} = \frac{L}{2} M g_{eff} \phi$$

$$\Rightarrow \frac{1}{12} ML^2 \frac{d^2\phi}{dt^2} - \frac{L}{2} M g_{eff} \phi = 0.$$

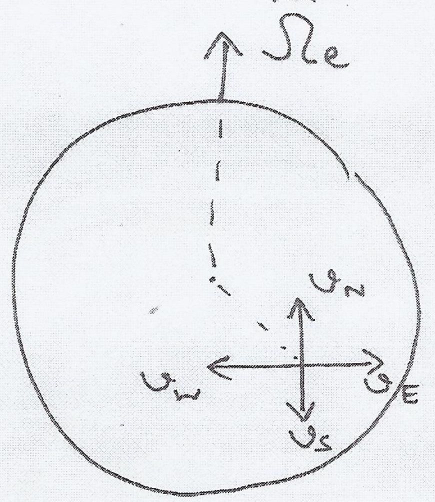
$$\therefore \quad \frac{d^2\phi}{dt^2} - 6\frac{g_{eff}}{L} \phi = 0.$$

$\therefore$ Motion is such that

$$\phi = \phi_0 e^{\pm \gamma t}.$$

$$\gamma = \sqrt{\frac{6\, g_{eff}}{L}}.$$

3. The velocity dependent fictitious force is given by



$$\vec{F}_f = -2m\vec{\Omega}_e \times \vec{v}.$$

The apparent change in gravity $\equiv$ component of $\frac{1}{m}\vec{F}_f$ normal to surface of earth.

Thus following cases result:

(a.) East:

$\vec{\Omega}_e \times \vec{v}_E$ points radially inward

$\therefore \frac{1}{m}\vec{F}_f$ is radially outward decreasing g

$$\therefore \frac{\Delta g}{g} = -\frac{F_f}{mg} = -\frac{2\Omega v_E}{g}.$$

$$\Omega_e = 7.27 \times 10^{-5} \text{ rad/s}$$

$$v_E = 200 \text{ mph} = 293 \text{ ft/s}$$

$$\frac{\Delta g}{g} = -\frac{2 \times 7.27 \times 10^{-5} \times 293}{32}$$

$$= 1.33 \times 10^{-3}.$$

(b.) West:

Sign reversal compared to case (a.).

$\therefore$ g is increased.

$$\Rightarrow \frac{\Delta g}{g} = +1.33 \times 10^{-3}.$$

(c.) South:

$\vec{\Omega}_e$ & $\vec{v}_s$ are antiparallel.

$$\therefore \Delta g = 0.$$

$$\Rightarrow \frac{\Delta g}{g} = 0.$$

(d.) North:

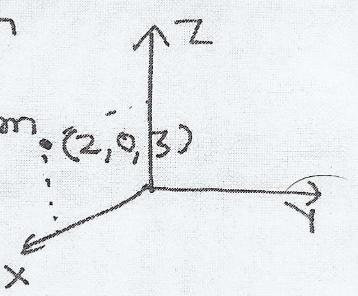
$\vec{\Omega}_e$ & $\vec{v}_N$ are parallel.

$$\therefore \Delta g = 0.$$

$$\Rightarrow \frac{\Delta g}{g} = 0.$$

Location $(2,0,3)$

4. (a.) $I_{xx} = m(y^2+z^2) = 9m$



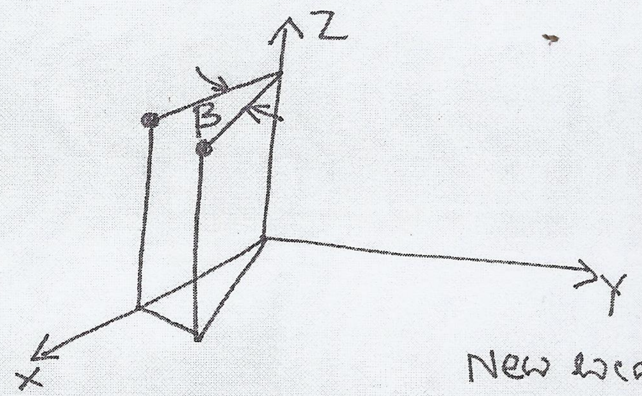
$I_{xy} = -mxy = 0$

$I_{xz} = -mxz = -6m.$

e.t.c.,

$$\overleftrightarrow{I} = m\begin{pmatrix} 9 & 0 & -6 \\ 0 & 13 & 0 \\ -6 & 0 & 4 \end{pmatrix}$$

(b.)



New location is

$(2\cos\beta, 2\sin\beta, 3)$

To first order in β, it is,

$(2-\beta^2, 2\beta, 3)$.

$$I' = m\begin{pmatrix} 9+4\beta^2 & -4\beta & -6+3\beta^2 \\ -4\beta & 13-4\beta^2 & -6\beta \\ -6+3\beta^2 & -6\beta & 4 \end{pmatrix}$$

$\rightarrow I$ as $\beta \ll 1$.

e.g., $I'_{zz} = m\left[(2-\beta^2)^2 + (2\beta)^2\right]$

$\approx m\left[4 - 4\beta^2 + 4\beta^2\right]$

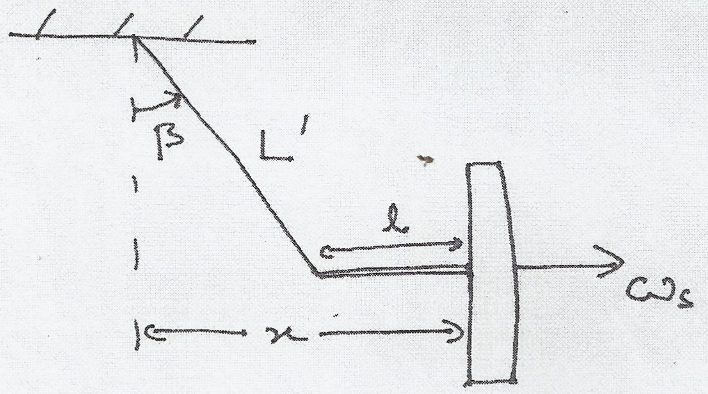
$= 4m.$

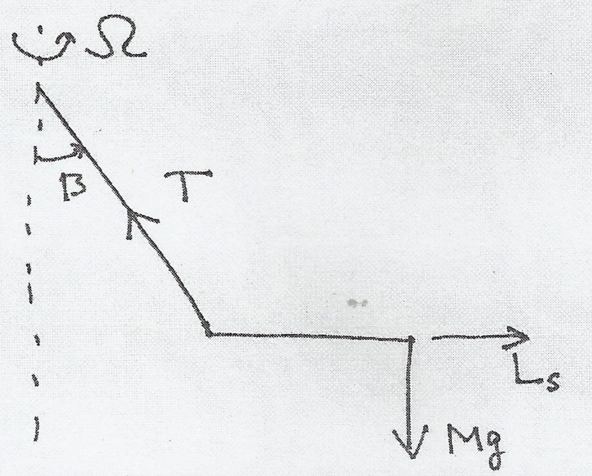
5. In this case both β & Ω are unknown.

As β is small,

$$\sin\beta \approx \beta$$
$$\cos\beta \approx 1$$

$$\Rightarrow x = l + L'\sin\beta \approx l + L'\beta.$$





$$Mg = T\cos\beta \approx T.$$

$$Mx\Omega^2 = T\sin\beta \approx T\beta. \Rightarrow \Omega^2 = \frac{T\beta}{Mx} = \frac{g\beta}{l + L'\beta}.$$

Torque : $\dot{L}_s = Tl = \Omega I_0 \omega_s$

$$\Rightarrow \Omega = \frac{Mgl}{I_0 \omega_s}.$$

$\therefore$ Thus,

$$\Omega^2 \equiv \left(\frac{Mgl}{I_0\omega_s}\right)^2 = \frac{g\beta}{1+L'\beta}.$$

$$\Rightarrow \beta\left(1-\frac{M^2gl^2L'}{I_0^2\omega_s^2}\right) = \frac{M^2gl^3}{I_0^2\omega_s^2}.$$

$$\therefore \boxed{\beta = \frac{M^2gl^3}{I_0^2\omega_s^2\left(1-\frac{M^2gl^2L'}{I_0^2\omega_s^2}\right)}}$$

6. We will assume that

$$I_1 > I_2 > I_3.$$

Conservation of L^2 & E tells us that

$$I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2 = L^2 \quad \text{—(I.)}$$

$$\& \quad I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2 = 2E \quad \text{—(II.)}$$

are constant.

Eliminating ω_1 from (I.) & (II.),

$$\Rightarrow I_2(I_2-I_1)\omega_2^2 + I_3(I_3-I_1)\omega_3^2 = L^2 - 2I_1E. \quad \text{—(III.)}$$

$\because \; I_1 > I_2 > I_3$,

$\therefore$ both coeff. on left hand side of eq. (III.) are negative.

Multiplying throughout by -1,

$\Rightarrow \; A\omega_2^2 + B\omega_3^2 = C$

where $A, B > 0$ (hence, $C > 0$).

$\therefore$ We have an ellipse in the ω_2-ω_3 plane.

Hence, ω_2 & ω_3 are bounded.

i.e., if the both start small, then they must always remain small.

Similarly, if we eliminate ω_3 from eqn (I.) & (II.), we obtain an ellipse in the ω_1-ω_2 plane.

However, if we eliminate ω_2, we obtain

$$I_1(I_1 - I_2)\omega_1^2 + I_3(I_3 - I_2)\omega_3^2 = L^2 - 2I_2E.$$

Two coefficients on the L.H.S. are now

of opposite signs.

$\therefore$ We have a hyperbola in the $\omega_1 - \omega_3$ plane.

$\therefore$ ω_1 & ω_3 are free to become large!.

$\Downarrow$

"Unstable" to perturbations.

7. CM is halfway between B & D.

$\therefore$ Relative to CM,

$$\vec{L} = \vec{r} \times \vec{p} = (2a, -a, 0) \times (0, 0, -P).$$

$$= P(a, 2a, 0).$$

Principal moments of inertia are,

$$I_x = (3m)a^2 + 3(ma^2) = 6ma^2.$$

$$I_y = 2 \cdot m(2a)^2 = 8ma^2.$$

I_z does not matter here!

$\therefore$ Angular velocity right after the blow is,

$$\omega = \left(\frac{L_x}{I_x}, \frac{L_y}{I_y}, 0\right) = \left(\frac{Pa}{6ma^2}, \frac{2Pa}{8ma^2}, 0\right)$$

$$= \frac{P}{12ma}(2, 3, 0).$$

The velocities relative to the CM

are,

$$\vec{u}_A = \vec{\omega} \times \vec{r}_A = \frac{P}{12ma} a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 0 \\ -2 & -1 & 0 \end{vmatrix} = \frac{P}{12m}(0,0,4).$$

$$\vec{u}_B = \vec{\omega} \times \vec{r}_B = \frac{P}{12ma} a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \frac{P}{12m}(0,0,-2).$$

$$\vec{u}_C = \vec{\omega} \times \vec{r}_C = \frac{P}{12ma} a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 0 \\ 2 & -1 & 0 \end{vmatrix} = \frac{P}{12m}(0,0,-8).$$

$$\vec{u}_D = \vec{\omega} \times \vec{r}_D = \frac{P}{12ma} a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \frac{P}{12m}(0,0,2).$$

Adding to these the velocity of CM,
which is $\left(\frac{P}{6m}\right)(0,0,-1)$,

$$\Rightarrow \quad \vec{v}_A = \frac{P}{6m}(0,0,1).$$

$$\vec{v}_B = \frac{P}{6m}(0,0,-2).$$

$$\vec{v}_C = \frac{P}{6m}(0,0,-5)$$

$$\vec{v}_D = \frac{P}{6m}(0,0,0).$$

$\therefore$ Initially D does not move.