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MA225: End Semester Assignment for B.Tech. II year

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Answers

1. Given,

X, Y are independent random variables, distributed as one-parameter exponential RV with same mean 2

Let the parameter for pdf of X, Y be β

$$\Rightarrow \boxed{X \sim exp(\beta)} \text{ and } \boxed{Y \sim exp(\beta)}$$

Hence, $\boxed{f_X(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, \; x \in (0,\infty)}$ and $\boxed{f_Y(y) = \frac{1}{\beta} e^{-\frac{y}{\beta}}, \; y \in (0,\infty)}$

$$\text{Mean of } X = \int_0^\infty x f_X(x) dx = \int_0^\infty \frac{x}{\beta} e^{-\frac{x}{\beta}} dx = \frac{1}{}$$

$$= \frac{1}{\beta}\left[\cancel{-\beta x e^{-\frac{x}{\beta}} \Big|_0^\infty} \to 0 + \int_0^\infty \frac{e^{-\frac{x}{\beta}}}{\frac{1}{\beta}} dx \right]$$

$$= \frac{1}{\cancel{\beta}} \cdot \cancel{\beta} \int_0^\infty e^{-\frac{x}{\beta}} dx = \frac{e^{-\frac{x}{\beta}}}{-\frac{1}{\beta}} \Big|_0^\infty = \beta$$

$\Rightarrow$ Mean of $X = \beta = 2$ (given)

$$\therefore \boxed{\beta = 2}$$

Hence pdf of X, Y are.

$$f_X(x) = \frac{1}{2}e^{-\frac{x}{2}} \qquad f_Y(y) = \frac{1}{2}e^{-\frac{y}{2}}$$

$$x \in (0,\infty) \qquad y \in (0,\infty)$$

cd

$F_X(x)$ CDF of X, $F_X(x) = P(X \leq x) = \int_0^x \frac{1}{2}e^{-\frac{x}{2}}dx$

$$= \frac{1}{2} \times \frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\Big|_0^x = 1 - e^{-\frac{x}{2}}$$

$$\Rightarrow \boxed{F_X(x) = 1 - e^{-x/2}}$$

Similarly $\boxed{F_Y(y) = 1 - e^{-y/2}}$

Let $Z = \min(X, Y)$,

$$F_Z(z) = P(Z \leq z) = 1 - P(Z > z)$$

$$= 1 - P(\min(X,Y) > z)$$ [if $\min(X,Y) > z$, then both of X and Y must be greater than z]

$$= 1 - P(X > z, Y > z)$$

$$= 1 - P(X > z)P(Y > z)$$ [$\because$ X, Y are independent]

$$= 1 - [1 - P(X \leq z)][1 - P(Y \leq z)]$$

$$= 1 - [1 - F_X(z)][1 - F_Y(z)]$$

$$= 1 - [1 - (1 - e^{-z/2})][1 - (1 - e^{-\frac{z}{2}})]$$

$$= 1 - e^{-\frac{z}{2}} \cdot e^{-\frac{z}{2}}$$

$$\Rightarrow \boxed{F_Z(z) = 1 - e^{-z}}$$

PDF of $Z = \min(X,Y)$ is

$$f_Z(z) = \frac{d}{dz} F_Z(z) = \frac{d}{dz}(1-e^{-z})$$

$$\Rightarrow \boxed{f_Z(z) = e^{-z} \quad z \in (0,\infty)}$$

$\therefore$ Expected value of $\min(X,Y)$,

$$E[\min(X,Y)] = E[Z] = \int_0^\infty z f_Z(z)\,dz$$

$$= \int_0^\infty z e^{-z}\,dz = \cancel{-ze^{-z}\Big|_0^\infty}\,{}^{0} + \int_0^\infty e^{-z}\,dz$$

$$= \frac{e^{-z}}{-1}\Big|_0^\infty = 1$$

$$\Rightarrow \boxed{E[\min(X,Y)] = 1}$$

$\therefore$ $\boxed{\text{Expected value of } \min(X,Y) \text{ is } 1}$

2. Given,

$$X, Y \overset{iid}{\sim} N(0,1)$$

hence, $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad -\infty < x < \infty$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \quad -\infty < y < \infty$$

Given transformations are,

$$U = X + Y \qquad V = X - Y$$

These transformations are one-one

X in terms of U,V

$$U+V = (X+Y)+(X-Y) = 2X$$

$$\Rightarrow \boxed{X = \frac{U+V}{2}}$$

Y in terms of U,V

$$U-V = (X+Y)-(X-Y) = 2Y$$

$$\Rightarrow \boxed{Y = \frac{U-V}{2}}$$

Jacobian for the given transformation, $U = X+Y$ and $V = X-Y$,

$$J = \begin{vmatrix} \frac{\partial X}{\partial U} & \frac{\partial X}{\partial V} \\ \frac{\partial Y}{\partial U} & \frac{\partial Y}{\partial V} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix}$$

$$\Rightarrow J = -\frac{1}{4} - \frac{1}{4} \Rightarrow \boxed{J = -\frac{1}{2}}$$

Joint PDF of (U,V) is given by,

$$f_{U,V}(u,v) = f_{X,Y}\left(x(u,v), y(u,v)\right) |J|$$

$$= f_{X,Y}\left(\frac{u+v}{2}, \frac{u-v}{2}\right) \left|-\frac{1}{2}\right|$$

$$= f_X\left(\frac{u+v}{2}\right) f_Y\left(\frac{u-v}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{u+v}{2}\right)^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{u-v}{2}\right)^2} \cdot \frac{1}{2}$$

$$\Rightarrow f_{U,V}(u,v) = \frac{1}{4\pi} e^{-\frac{1}{2}\left[\left(\frac{u+v}{2}\right)^2+\left(\frac{u-v}{2}\right)^2\right]}$$

$$= \frac{1}{4\pi} e^{-\frac{1}{\cancel{2}}\left[\frac{1}{4}\cdot \cancel{2}(u^2+v^2)\right]} = \frac{1}{4\pi} e^{-\frac{1}{4}(u^2+v^2)}$$

$$\Rightarrow \boxed{f_{U,V}(u,v) = \frac{1}{4\pi} e^{-\frac{1}{4}(u^2+v^2)} \quad \begin{array}{l} -\infty < u < \infty \\ -\infty < v < \infty \end{array}}$$

is the joint probability density of (U,V)

Marginal probability density of U,

$$f_U(u) = \int_{-\infty}^{\infty} f_{U,V}(u,v)\,dv = \int_{-\infty}^{\infty} \frac{1}{4\pi} e^{-\frac{1}{4}(u^2+v^2)}\,dv$$

$$= \int_{-\infty}^{\infty} \frac{1}{4\pi} e^{-\frac{1}{4}u^2} \cdot e^{-\frac{1}{4}v^2}\,dv = \frac{1}{4\pi} e^{-\frac{1}{4}u^2} \int_{-\infty}^{\infty} e^{-\frac{1}{4}v^2}\,dv$$

Let $\frac{v}{\sqrt{2}} = p \Rightarrow dv = dp\sqrt{2}$ and $2p^2 = v^2$

$$\Rightarrow f_U(u) = \frac{1}{4\pi} e^{-\frac{1}{4}u^2} \cdot \int_{-\infty}^{\infty} e^{-\frac{p^2}{2}} \cdot dp\sqrt{2} = \frac{1}{2\sqrt{2}\pi} e^{-\frac{1}{4}u^2} \int_{-\infty}^{\infty} e^{-p^2/2}\,dp$$

$$\Rightarrow f_U(u) = \frac{1}{2\sqrt{\pi}} e^{-\frac{u^2}{4}} \left[\underline{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{p^2}{2}}\,dp} \right]$$

$$\Rightarrow \boxed{f_U(u) = \frac{1}{2\sqrt{\pi}} e^{-\frac{u^2}{4}}}$$

[This is equal to $\Phi(\infty)$, where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$ is the CDF of a standard normal distribution]

Similarly, marginal pdf of V

$$f_V(v) = \int_{-\infty}^{\infty} f_{U,V}(u,v)\,du = \int_{-\infty}^{\infty} \frac{1}{4\pi} e^{-\frac{1}{4}(u^2+v^2)}\,du$$

$$\Rightarrow \boxed{f_V(v) = \frac{1}{2\sqrt{\pi}} e^{-\frac{v^2}{4}}}$$

$\therefore$ Joint probability density of (U,V), $\boxed{f_{U,V}(u,v) = \frac{1}{4\pi} e^{-\frac{1}{4}(u^2+v^2)}}$

Marginal probability density of U, $\boxed{f_U(u) = \frac{1}{2\sqrt{\pi}} e^{-\frac{u^2}{4}}}$

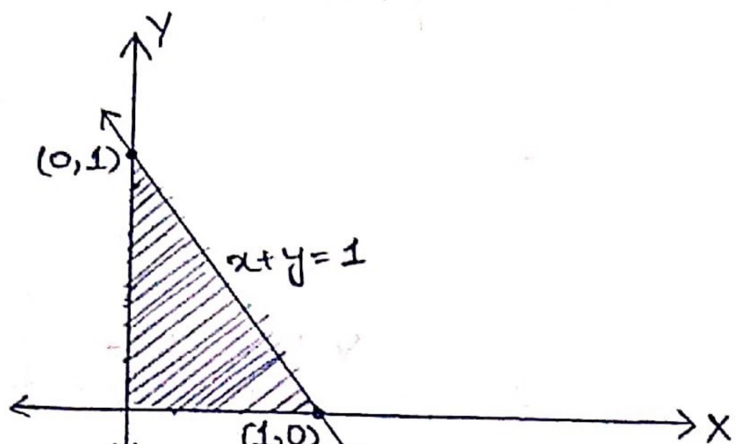
Marginal probability density of V, $\boxed{f_V(v) = \frac{1}{2\sqrt{\pi}} e^{-\frac{v^2}{4}}}$

3. Given,

X, Y are two random variables with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 6x & x>0,\ y>0,\ 0<x+y\leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Region in which PDF is not zero



Correlation Coefficient between X, Y is

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

where cov(X,Y) is covariance between X, Y

$$\boxed{cov(X,Y) = E(XY) - E(X)E(Y)}$$

σ_X is standard deviation of X

σ_Y is standard deviation of Y

$$E(XY) = \iint_{x,y \in R} xy f_{X,Y}(x,y)\,dx\,dy = \int_0^1 \int_0^{1-y} xy \cdot 6x\,dx\,dy$$

$$= \int_0^1 \int_0^{1-y} 6x^2 y \, dx\,dy = \int_0^1 \left(\int_0^{1-y} 6x^2 dx \right) y\,dy$$

$$= \int_0^1 \cancel{6}^{2} \frac{x^3}{\cancel{3}} \Big|_0^{1-y} y\,dy = \int_0^1 2(1-y)^3 y\,dy$$

$$= 2\int_0^1 (1-y^3-3y+3y^2)y\,dy = 2\int_0^1 (y-y^4-3y^2+3y^3)\,dy$$

$$= 2\left[\frac{1}{2}-\frac{1}{5}-\frac{3}{3}+\frac{3}{4}\right] = 2\left[\frac{3}{10}-1+\frac{3}{4}\right]$$

$$\boxed{E(XY) = \frac{1}{10}}$$

$$E(X) = \iint_{x,y \in R} x f_{X,Y}(x,y)\,dx\,dy = \int_0^1 \int_0^{1-y} x \cdot 6x\,dx\,dy$$

$$= \int_0^1 \int_0^{1-y} 6x^2 dx\,dy = \int_0^1 2x^3 \Big|_0^{1-y} dy = \int_0^1 2(1-y)^3 dy$$

$$\Rightarrow E(X) = \int_0^1 2(1-y^3-3y+3y^2)\,dy = 2\left[1-\frac{1}{4}-\frac{3}{2}+1\right]$$

$$\Rightarrow \boxed{E(X) = \frac{1}{2}}$$

$$E(Y) = \iint_{x,y\in R} y\, f_{X,Y}(x,y)\,dx\,dy = \int_0^1\int_0^{1-y} y\cdot 6x\,dx\,dy$$

$$= \int_0^1\int_0^{1-y} 6x\,dx\cdot y\,dy = \int_0^1 3x^2\Big|_0^{1-y} y\,dy = \int_0^1 3(1-y)^2 y\,dy$$

$$= 3\int_0^1 (y+y^3-2y^2)\,dy = 3\left(\frac{1}{2}+\frac{1}{4}-\frac{2}{3}\right)$$

$$\Rightarrow \boxed{E(Y) = \frac{1}{4}}$$

$$\therefore \operatorname{COV}(X,Y) = E(XY)-E(X)E(Y)$$

$$= \frac{1}{10}-\frac{1}{2}\cdot\frac{1}{4} = \frac{1}{10}-\frac{1}{8}$$

$$\Rightarrow \boxed{\operatorname{COV}(X,Y) = -\frac{1}{40}}$$

$$E(X^2) = \iint_{x,y\in R} x^2 f_{X,Y}(x,y)\,dx\,dy = \int_0^1\int_0^{1-y} x^2\cdot 6x\,dx\,dy$$

$$= \int_0^1\int_0^{1-y} 6x^3\,dx\,dy = \int_0^1 \frac{6x^4}{4}\Big|_0^{1-y} dy = \frac{3}{2}\int_0^1 (1-y)^4\,dy$$

$$= \frac{3}{2}\int_0^1 (1-4y+6y^2-4y^3+y^4)\,dy = \frac{3}{2}\left[\cancel{1}-\cancel{2}+\cancel{2}-\cancel{1}+\frac{1}{5}\right]$$

$$\Rightarrow \boxed{E(X^2) = \frac{3}{10}}$$

$$E(Y^2) = \iint_{x,y\in R} y^2 f_{X,Y}(x,y)\,dx\,dy = \int_0^1\int_0^{1-y} y^2 \cdot 6x\,dx\,dy$$

$$= \int_0^1\int_0^{1-y} 6x\,dx\; y^2 dy = \int_0^1 3x^2\Big|_0^{1-y} y^2 dy$$

$$= \int_0^1 3(1-y)^2 y^2 dy = 3\int_0^1 (y^2 - 2y^3 + y^4)dy = 3\left[\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right]$$

$$\Rightarrow \boxed{E(Y^2) = \frac{1}{10}}$$

$$V(X) = E(X^2) - (E(X))^2 = \frac{3}{10} - \left(\frac{1}{2}\right)^2 = \frac{3}{10} - \frac{1}{4} = \frac{1}{20}$$

$$\sigma_X = \sqrt{V(X)} \Rightarrow \boxed{\sigma_X = \frac{1}{\sqrt{20}}}$$

$$V(Y) = E(Y^2) - (E(Y))^2 = \frac{1}{10} - \left(\frac{1}{4}\right)^2 = \frac{1}{10} - \frac{1}{16} = \frac{3}{80}$$

$$\sigma_Y = \sqrt{V(Y)} \Rightarrow \boxed{\sigma_Y = \frac{\sqrt{3}}{2\sqrt{20}}}$$

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{-\frac{1}{40}}{\frac{\sqrt{3}}{\sqrt{20}\cdot 2\cdot\sqrt{20}}} = \frac{-\frac{1}{40}}{\frac{\sqrt{3}}{40}}$$

$$\Rightarrow \boxed{\rho_{X,Y} = \frac{-1}{\sqrt{3}}}$$

Hence, correlation co-efficient between X and Y is $\frac{-1}{\sqrt{3}}$

4. Given,

$$f_{X,Y}(x,y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$P(X > Y) = \int_0^1 \int_0^x \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\, dx$$

$$= \frac{6}{7} \int_0^1 x^2y + \frac{xy^2}{4} \Big|_0^x dx$$

$$= \frac{6}{7} \int_0^1 \left(x^3 + \frac{x^3}{4}\right) dx = \frac{6}{7} \int_0^1 \frac{5x^3}{4} dx$$

$$= \frac{6}{7} \cdot \frac{5}{4} \cdot \frac{1}{4} \Rightarrow \boxed{P(X>Y) = \frac{15}{56}}$$

$$P(Y>0.5 \mid X<0.5) = \frac{P(X<0.5,\ Y>0.5)}{P(X<0.5)}$$

$$P(X<0.5,\ Y>0.5) = \int_0^{0.5} \int_{0.5}^{2} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\, dx$$

$$= \frac{6}{7} \int_0^{0.5} x^2y + \frac{xy^2}{4} \Big|_{0.5}^{2} dx$$

$$= \frac{6}{7} \int_0^{0.5} \left(1.5x^2 + \frac{3.75x}{4}\right) dx$$

$$= \frac{6}{7} \quad \frac{1.5x^3}{3} + \frac{3.75x^2}{8} \Big|_0^{0.5}$$

$$\boxed{P(X<0.5, Y>0.5) = \frac{6}{7} \cdot \frac{23}{128}}$$

$$P(X<0.5) = \int_0^{0.5}\int_0^{2} \frac{6}{7}\left(x^2+\frac{xy}{2}\right)dy\,dx = \frac{6}{7}\int_0^{0.5}\int_0^{2}\left(x^2+\frac{xy}{2}\right)dy\,dx$$

$$= \frac{6}{7}\int_0^{0.5} x^2y+\frac{xy^2}{4}\Big|_0^2 dx = \frac{6}{7}\int_0^{0.5}(2x^2+x)dx$$

$$= \frac{6}{7}\ \frac{2x^3}{3}+\frac{x^2}{2}\Big|_0^{0.5} = \frac{6}{7}\cdot\left(\frac{1}{12}+\frac{1}{8}\right)$$

$$= \frac{6}{7}\cdot\frac{5}{24}$$

$$P(Y>0.5\mid X<0.5) = \frac{P(X<0.5,\ Y>0.5)}{P(X<0.5)}$$

$$= \frac{\cancel{\frac{6}{7}}\cdot\frac{23}{128}}{\cancel{\frac{6}{7}}\cdot\frac{5}{24}} = \frac{23}{\cancel{128}_{16}}\times\frac{\cancel{24}^{3}}{5}$$

$$\boxed{P(Y>0.5\mid X<0.5) = \frac{69}{80}} \quad \text{and} \quad \boxed{P(X>Y) = \frac{15}{56}}$$

5. Given,

X, Y are random variables with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 1 & : 0\le x\le 2,\ 0\le y\le 1,\ 2y\le x \\ 0 & \text{elsewhere} \end{cases}$$

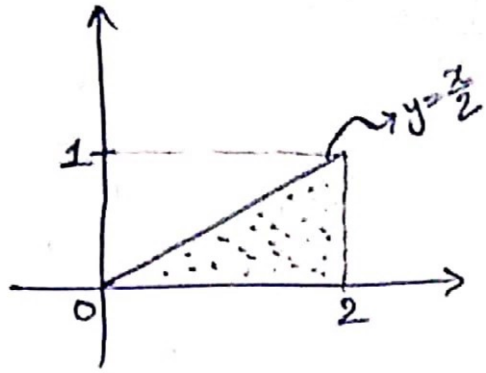
$Z = X+Y$ is another random variable, which is a function of X, Y

CDF of Z, $F_Z(z)$

$= P(Z \leq z)$

$= P(X+Y \leq z)$

$= P(X \leq z-y)$



$$\boxed{F_Z(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{z-y} f_{X,Y}(x,y)\,dx\,dy}$$

The probability density is non-zero in the following region, $0 \leq x \leq 2,\ 0 \leq y \leq 1,\ 2y \leq x$

To calculate the probability when $x \leq z-y$, the valid bounds for y should be found.

$0 \leq z-y \leq 2$ $0 \leq y \leq 1$ $2y \leq z-y$

$\Rightarrow -z \leq -y \leq 2-z$ $3y \leq z$

$\Rightarrow \boxed{z-2 \leq y \leq z}$ $\boxed{0 \leq y \leq 1}$ $\boxed{y \leq \frac{z}{3}}$

Therefore, y varies from $\max(0, z-2)$ and $\min(\frac{z}{3}, 1)$

i.e., $\boxed{\max(0, z-2) \leq y \leq \min\left(\frac{z}{3}, 1\right)}$

$$f_Z(z) = \frac{d}{dz}\left(F_Z(z)\right) = \frac{d}{dz}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{z-y} f_{X,Y}(x,y)\,dx\,dy\right)$$

$$= \int_{-\infty}^{\infty}\left(\frac{d}{dz}(z-y)\, f_{X,Y}(z-y,y) - 0\right)dy$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{z-y} \frac{\partial}{\partial z} f_{X,Y}(x,y)\,dx\,dy$$

(Lebnitz Rule)

$$\boxed{f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(z-y,y)\,dy}$$

$$f_Z(z) = \begin{cases} \int_{-\infty}^{\infty} 1\cdot \mathbb{I}\left(0<y<\frac{z}{3}\right)dy = \int_0^{z/3} dy = \frac{z}{3} \\ \int_{-\infty}^{\infty} 1\cdot \mathbb{I}(0<y<1)\,dy \\ \int_{-\infty}^{\infty} 1\cdot \mathbb{I}\left(z-2<y<\frac{z}{3}\right)dy = 2-\frac{2z}{3} \\ \int_{-\infty}^{\infty} 1\cdot \mathbb{I}(z-2<y<1)\,dy = 3-z \end{cases}$$

$\frac{z}{3} \le 1 \Rightarrow z \le 3$
and
$z-2 \le 0 \Rightarrow z \le 2$
$\therefore \boxed{z \le 2}$

$0 \ge z-2 \Rightarrow z \le 2$
and
$1 \le \frac{z}{3} \Rightarrow z \ge 3$
$\boxed{\text{Not Possible}}$

$z-2 \ge 0 \Rightarrow z \ge 2$
and
$\frac{z}{3} \le 1 \Rightarrow z \le 3$
$\therefore \boxed{2 \le z \le 3}$

$z-2 \ge 0 \Rightarrow z \ge 2$
and
$1 \le \frac{z}{3} \Rightarrow z \ge 3$
$\therefore \boxed{z \ge 3}$

$$\Rightarrow f_Z(z) = \begin{cases} \frac{z}{3} & z \le 2 \\ 2 - \frac{2z}{3} & 2 \le z \le 3 \\ 3 - z & z \ge 3 \end{cases}$$

But, we know that $0 \le x \le 2$ and $0 \le y \le 1$ and $Z = X + Y$

possibly $0 \le Z \le 3$

$$\therefore \boxed{f_Z(z) = \begin{cases} \frac{z}{3} & 0 \le z \le 2 \\ 2 - \frac{2z}{3} & 2 \le z \le 3 \end{cases}}$$

This is the probability density function of Z

6. Poisson Process:

A Poisson Process is a stochastic process that is used for modelling the times at which arrivals enter a system. The arrivals occur at arbitrary times, and we are interested in the total no. of arrivals upto a specified time.

Examples of a poisson process:

No. of telephone calls received by a callcenter,
No. of customers arriving at a shop,
No. of defective products in a lot,
No. of errors printed in a book, etc.

Now, in the event of customers arriving at a shop,

Let $X(t)$ denote the no. of customers arrived in a time interval of length t,

We have to evaluate,

$$P(X(t)=n) = P_n(t) \qquad n = 0, 1, 2, 3, \dots$$

under the following assumptions.

Assumptions taken for a Poisson Process:

① The number of events occured during disjoint time intervals are independent.

For e.g., No. of customers arrived in time interval $[t, t+h]$ is independent of the number arrived in $[0, t]$ for all $h>0$

② The probability of exactly one event occuring in a short interval of time is proportional to the length of the interval.

For. e.g., Probability that only one customer arrives at the shop in the interval $(t, t+h]$ is λh, where λ is the rate of arrival of customers

③ The probability of 2 or more events occuring in a small interval is negligible.

From ②, $\boxed{P(X(h)=1) = P_1(h) = \lambda h}$

From ③, $P(X(h) \geq 2) = P_2(h) + P_3(h) + \dots = O(h)$

$$\boxed{P(X(h) \geq 2) \longrightarrow 0 \text{ as } h \to 0}$$

Also, $P_0(h) + \underbrace{P_1(h)}_{\lambda h} + \underbrace{P_2(h) + P_3(h) + \dots}_{O(h)} = 1$

$$\Rightarrow \boxed{P_0(h) = 1 - \lambda h - O(h)}$$

With, these assumptions, it turns out that $X(t)$ is follows a Poisson dist with parameter λt.

$$P(X(t)=n) = P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \qquad n = 0, 1, 2, 3, \dots$$