

# MECHANICS

Page No.

FORCE → BODY → REACTION

RIGID

DEFORMABLE

NO

MOTION

object is considered  
as particle.

STATICS

(balanced  
force)

DYNAMICS

CAUSE OF  
MOTION

YES

KINEMATICS

v, a, s, t.

KINETICS

F, E

FORCE

Magnitude

↓

Direction

↓

Point of application

FORCE

equivalent

10KN

10KN

CONTACT

eg. push,  
pull, etc.

NON-CONTACT

eg. gravity,  
nuclear,  
electrostatic

PLANAR  
(2D)

NON-PLANAR.  
(3D)

CONCURRENT

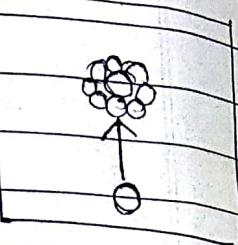
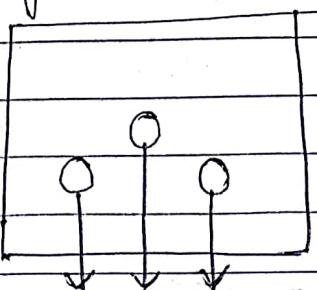
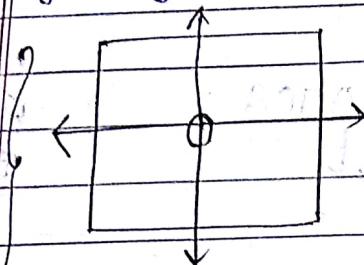
eg. acting at a point

PARALLEL

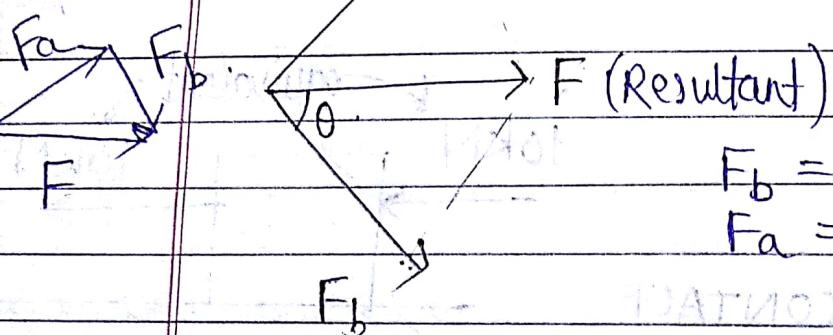
eg.

GENERAL

eg.

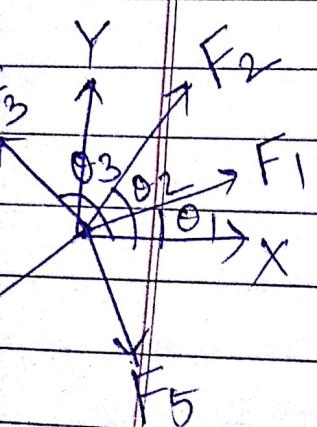
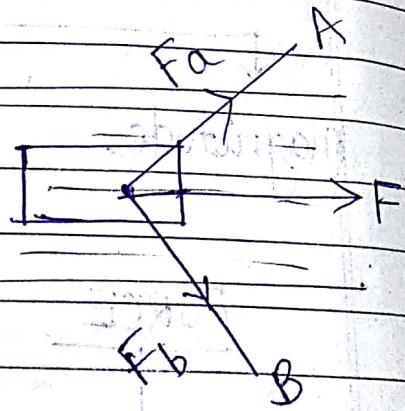


\* RESOLUTION :-



$$F_b = F \cos \theta$$

$$F_a = F \sin \theta$$



$$\sum F_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + \dots$$

$$\sum F_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + \dots$$

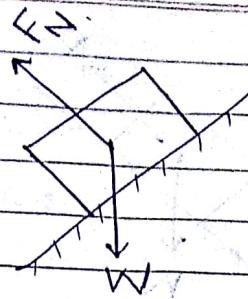
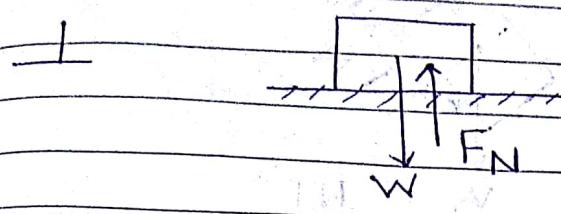
$$F = F_x \hat{i} + F_y \hat{j}$$

$$\bar{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

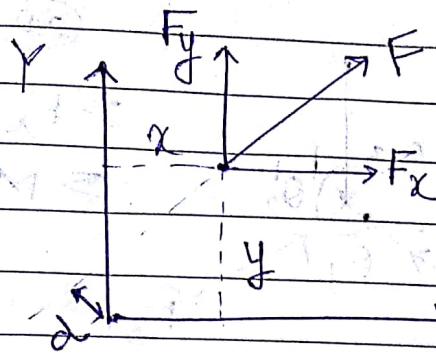
$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

$$F_x = \vec{F} \cdot \hat{i}, F_y = \vec{F} \cdot \hat{j}$$

\* NORMAL REACTION :-



\* Moment of force :-



$$M = Fd$$

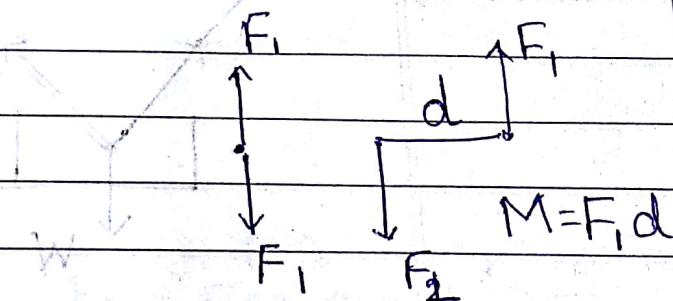
$$M = -F_x y + F_y x.$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

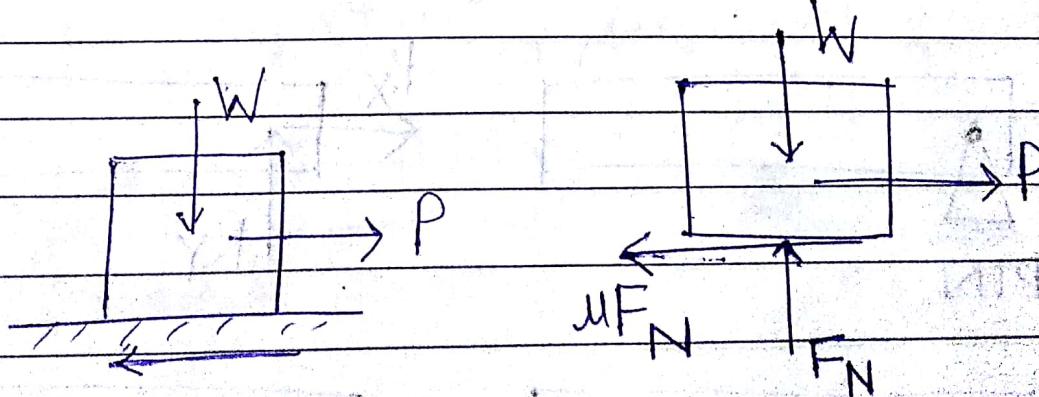
$$\vec{\gamma} = \gamma_x \hat{i} + \gamma_y \hat{j} + \gamma_z \hat{k}$$

$$\overline{M} = \vec{\gamma} \times \vec{F} = -\vec{F} \times \vec{\gamma}.$$

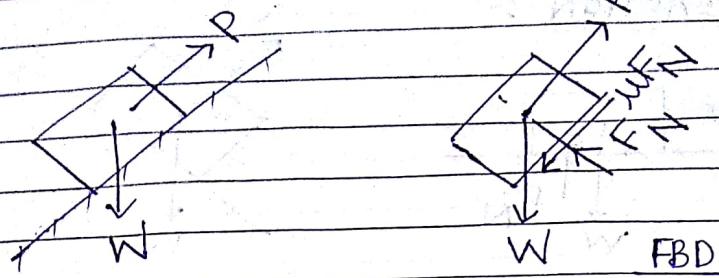
$$\overline{M} = \begin{vmatrix} i & j & k \\ \gamma_x & \gamma_y & \gamma_z \\ F_x & F_y & F_z \end{vmatrix}$$



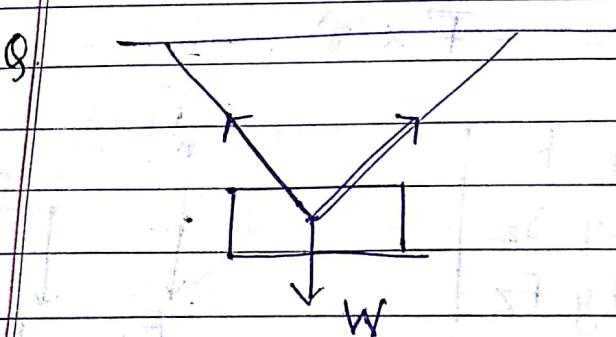
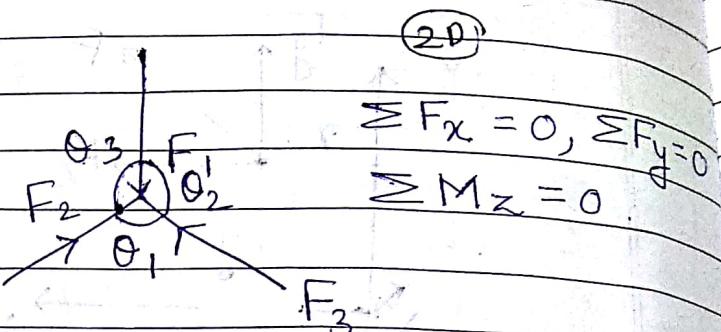
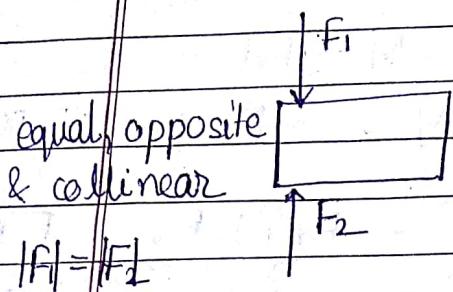
\* Free body diagram (FBD) :-



FBD of block



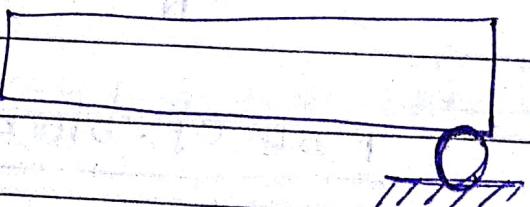
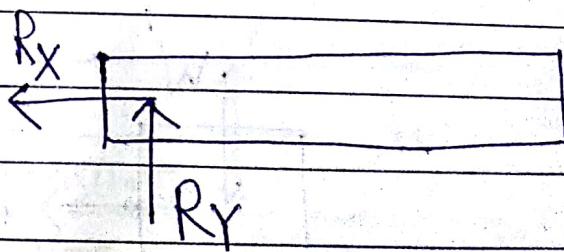
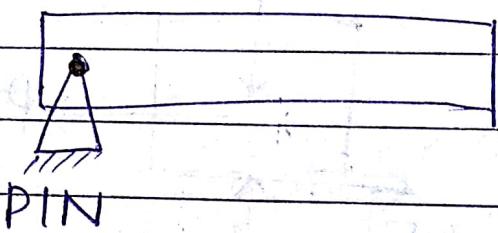
\* Equilibrium :-  $\sum F = 0$  &  $\sum M = 0$



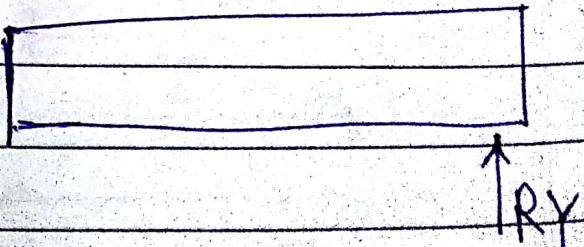
$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

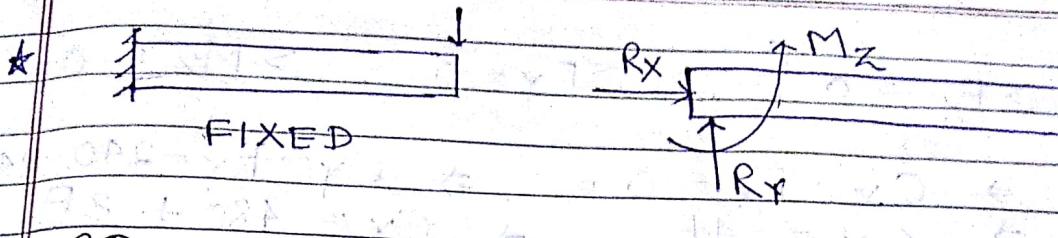
(3D)

$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$   
 $\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$ .

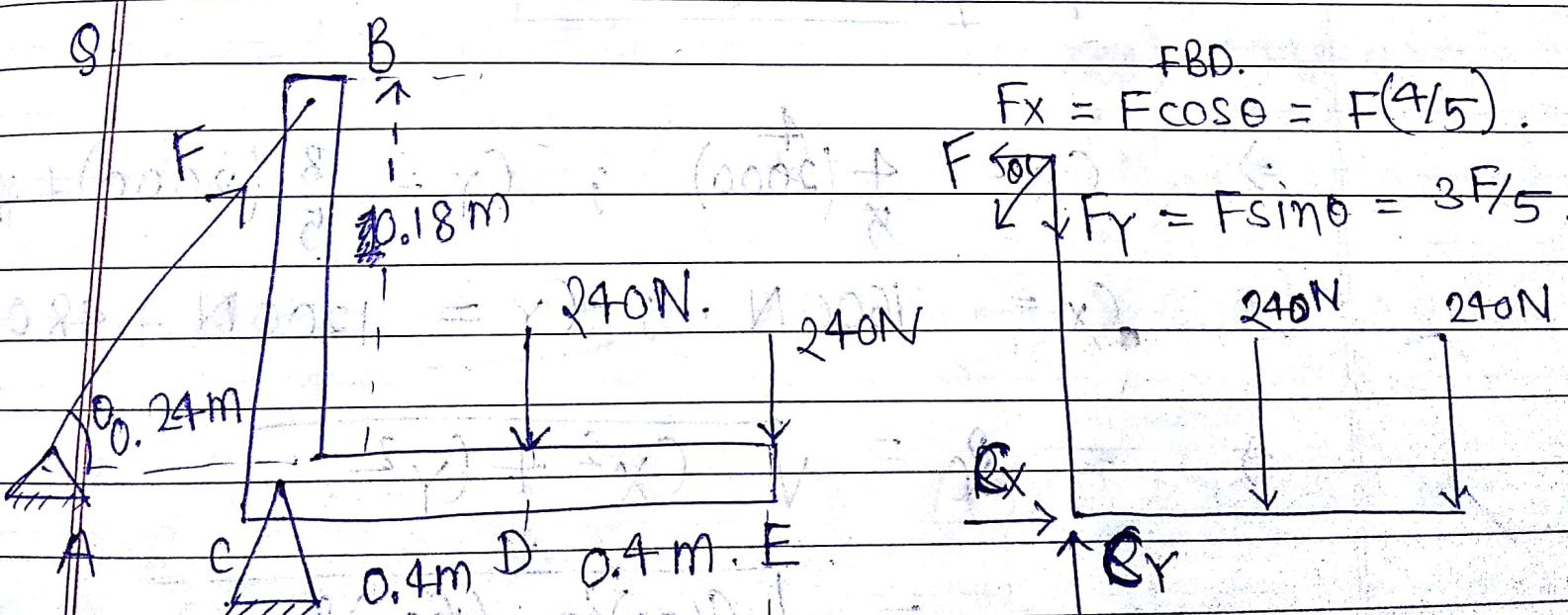
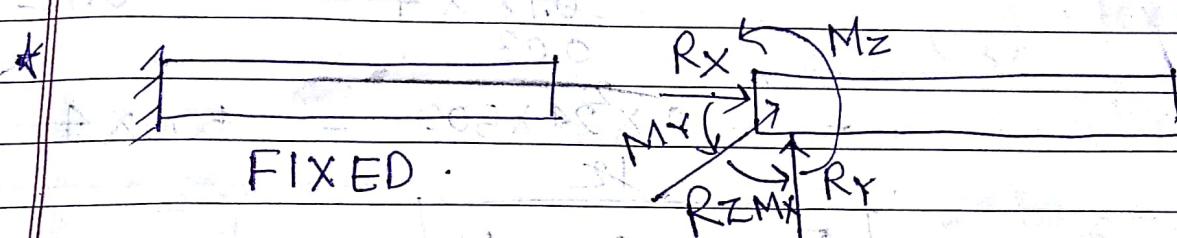
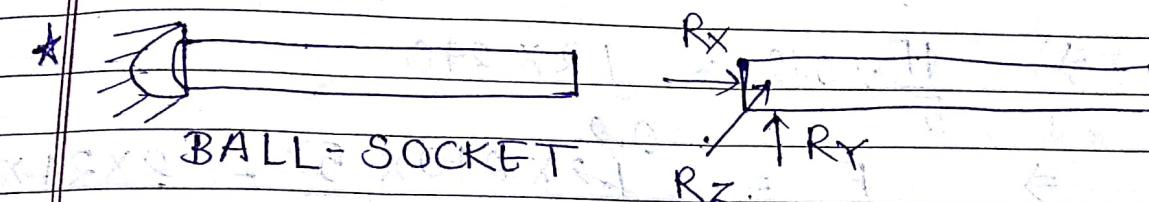
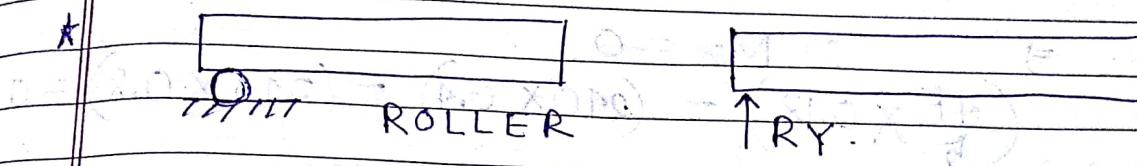


ROLLER





3D Joints



$$F = ?$$

$$R_c = ?$$

$$\sum F_x = 0 ; \quad \sum F_y = 0 ; \quad \sum M_z = 0$$

$$\Rightarrow C_x - F_x = 0 \Rightarrow C_x = \frac{4F}{5}$$

$$\Rightarrow C_y = 480 + 3F$$

$$\Rightarrow \sum M_z = 0$$

$$\left( \frac{4F}{5} \times 0.18 \right) - (240 \times 0.4) - (240 \times 0.8) = 0$$

$$\Rightarrow \frac{4F}{5} \times 0.18 = 1.2 \times 240$$

$$\Rightarrow F = \frac{1.2 \times 240 \times 5}{0.18 \times 4} = \frac{2 \times 24 \times 5}{0.12}$$

$$= \frac{2 \times 24 \times 500}{12} = 500 \times 4$$

$$\Rightarrow F = 2000 \text{ N}$$

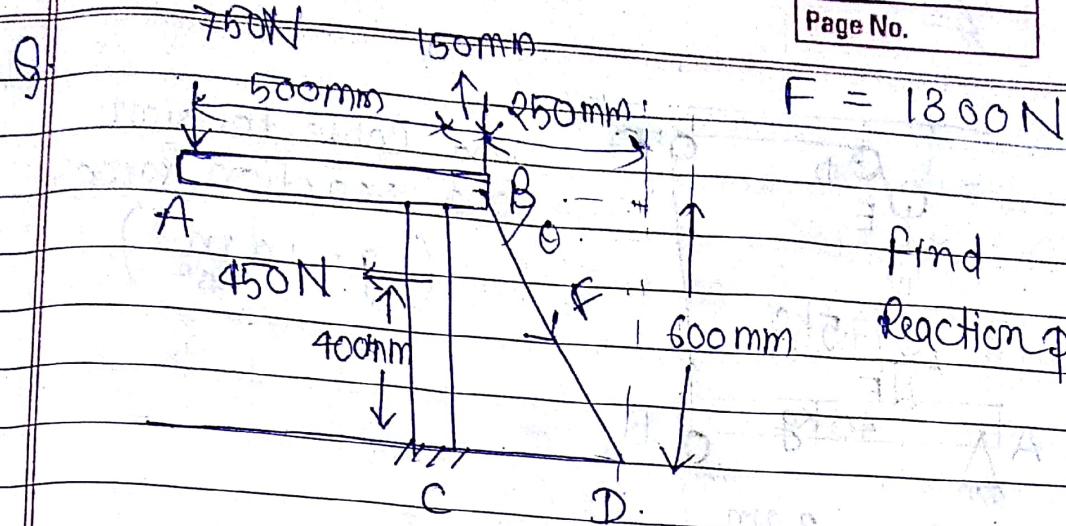
$$\Rightarrow C_x = \frac{4}{5}(2000) ; \quad C_y = \frac{3}{5}(2000) + 480$$

$$C_x = 1600 \text{ N} ; \quad C_y = 1200 \text{ N} + 480$$

$$R_c = \sqrt{C_x^2 + C_y^2}$$

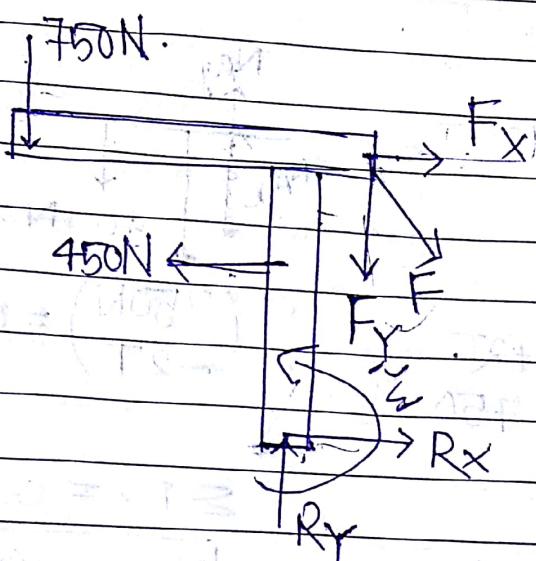
$$= \sqrt{(1600)^2 + (1200)^2}$$

$$R_c = \frac{100 \times 20}{2000} = 2320 \text{ N}$$



Find

Reaction point at C.



$$\sum F_x = 0$$

$$F_x + R_x - 450 = 0$$

$$F\left(\frac{5}{13}\right) + R_x - 450 = 0$$

$$R_x = -50 \text{ N}$$

$$\sum F_y = 0$$

$$\sum M_z = 0$$

$$R_y - F_y - 750 = 0$$

$$(450 \times 0.4) + (750 \times 0.5)$$

$$R_y = -F\left(\frac{12}{13}\right) - 750 = 0 \quad \Rightarrow \quad -\left(\frac{5F}{13} \times 0.6\right)$$

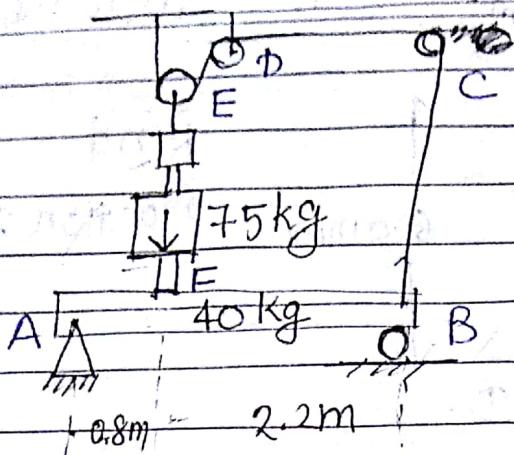
$$R_y = 1950 \text{ N}$$

$$-\left(\frac{12F}{13} \times 0.15\right) = 0$$

$$R_c = \sqrt{R_x^2 + R_y^2} \Rightarrow (45 \times 4) + 75 \times 5 + M_c = \frac{3F + 1.8F}{13}$$

$$\Rightarrow M_c = 480 - 375 - 180 = 300 - 375 \\ M_c = -75 \text{ Nm}$$

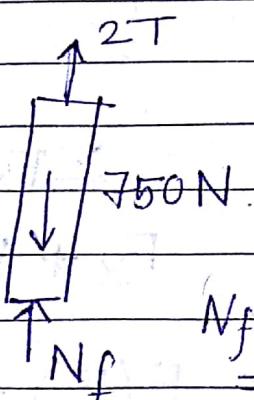
Q



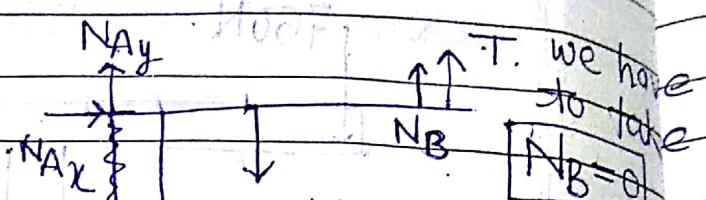
Find Cable tension  
and reaction forces

$$(g = 10 \text{ m/s}^2)$$

Ans →



$$N_f + 2T = 750$$



$$(750 - 2T) = N_f$$

$$\sum M_B = 0$$

$$\sum F_x = 0$$

$$N_Ax = 0$$

$$(400 \times 1.5) + ((750 - 2T) \times 2.2) - (N_Ay \times 3) = 0$$

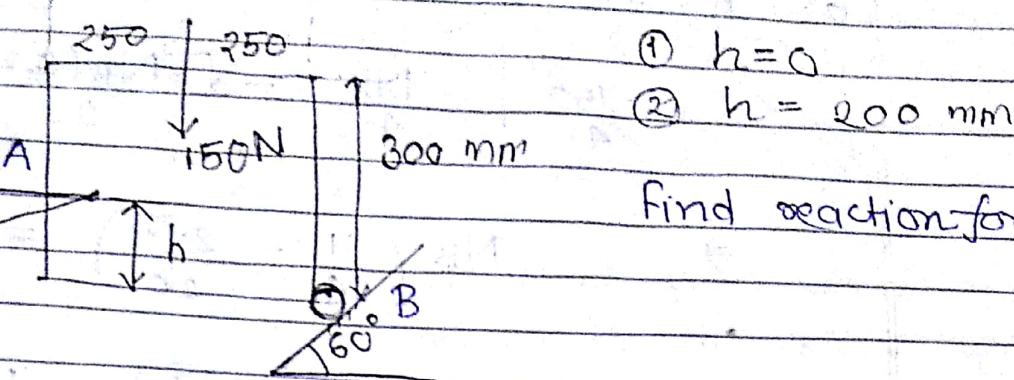
$$(400 \times 1.5) + ((750 - 2T) \times 2.2) = N_Ay$$

$$\sum F_y = 0$$

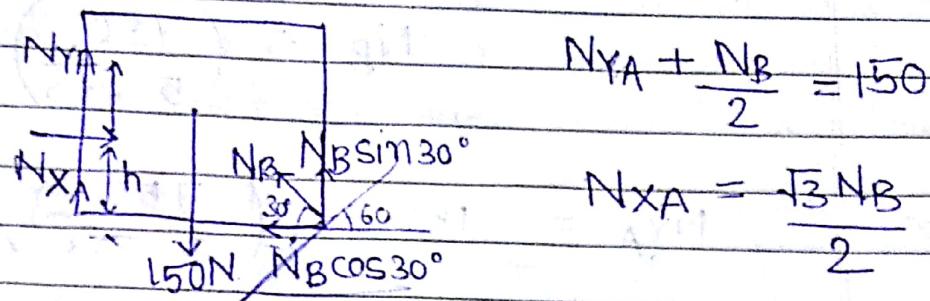
$$N_Ay + N_B + T - 400 - 750 + 2T = 0$$



Q.



Ans →



$$\sum M_B = 0$$

$$(150 \times 0.25) - (N_YA \times 0.5) - (N_{XA} \times h) = 0.$$

$$\frac{150}{4} - \left( \left( 150 - \frac{\sqrt{3}}{2} N_B \right) \times \frac{1}{2} \right) - \left( \frac{\sqrt{3} N_B h}{2} \right) = 0$$

$$\frac{150}{4} - 75 + \frac{N_B}{4} - \frac{\sqrt{3} N_B h}{2} = 0.$$

①  $h = 0$

$$\frac{N_B}{4} = 75 - \frac{150}{4} = \frac{150}{4} \Rightarrow N_B = 150$$

$$N_{YA} = 150 - \frac{150}{2} = 75 \text{ N}$$

$$N_{XA} = \frac{\sqrt{3}}{2} (150) = 75\sqrt{3} \text{ N.}$$

$$N_A = \sqrt{(75)^2 + (\sqrt{3}75)^2} = \boxed{150 \text{ N.}}$$

$$\textcircled{2} \quad h = 200 \text{ mm} = 0.2 \text{ m.}$$

$$\frac{-150}{4} + \frac{N_B}{4} - \frac{-\sqrt{3}N_B(0.2)}{2} = 0.$$

$$\Rightarrow N_B \left( \frac{1}{4} - \frac{2\sqrt{3}}{20} \right) = \frac{150}{4}$$

$$\Rightarrow N_B \left( 1 - \frac{2\sqrt{3}}{5} \right) = 150$$

$$\Rightarrow N_B = \left( \frac{750}{5-2\sqrt{3}} \right) \text{ N}$$

$$N_{YA} = 150 - \left( \frac{750}{5-2\sqrt{3}} \right) / 2$$

$$= \left( 150 - \frac{375}{5-2\sqrt{3}} \right) \text{ N}$$

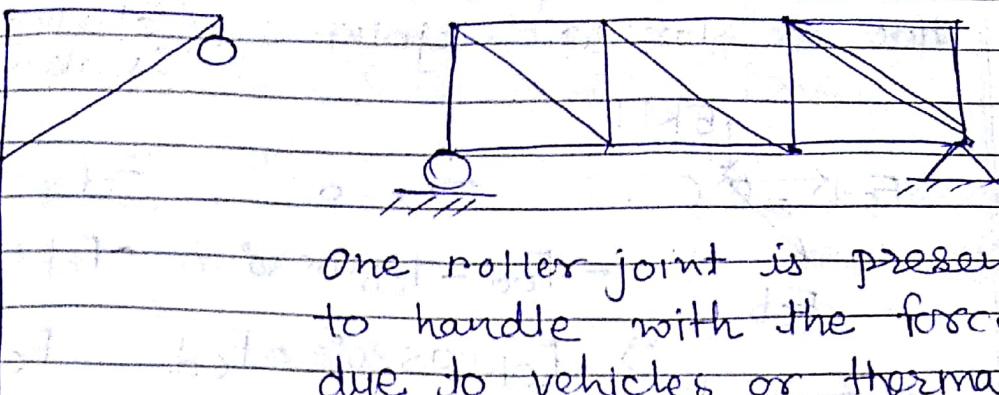
$$N_{XA} = \frac{\sqrt{3}}{2} \left( \frac{750}{5-2\sqrt{3}} \right) \text{ N.}$$

$$\underline{m = 2j - 3}, \quad m \Rightarrow \text{no. of members} \\ j \Rightarrow \text{no. of joints}$$

-Transparent

Date:  
Page No.

TRUSS

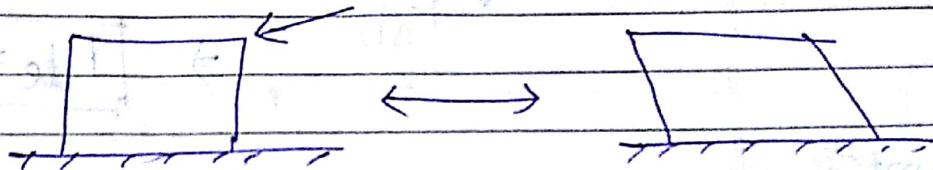


One roller joint is present to handle with the force applied due to vehicles or thermal stress.

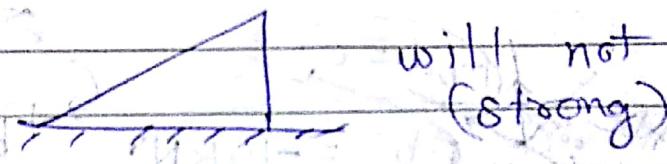
**Truss** → A framework typically supporting a load, bridge or other structure.

- A basic unit of triangle truss is a triangle

if rectangle

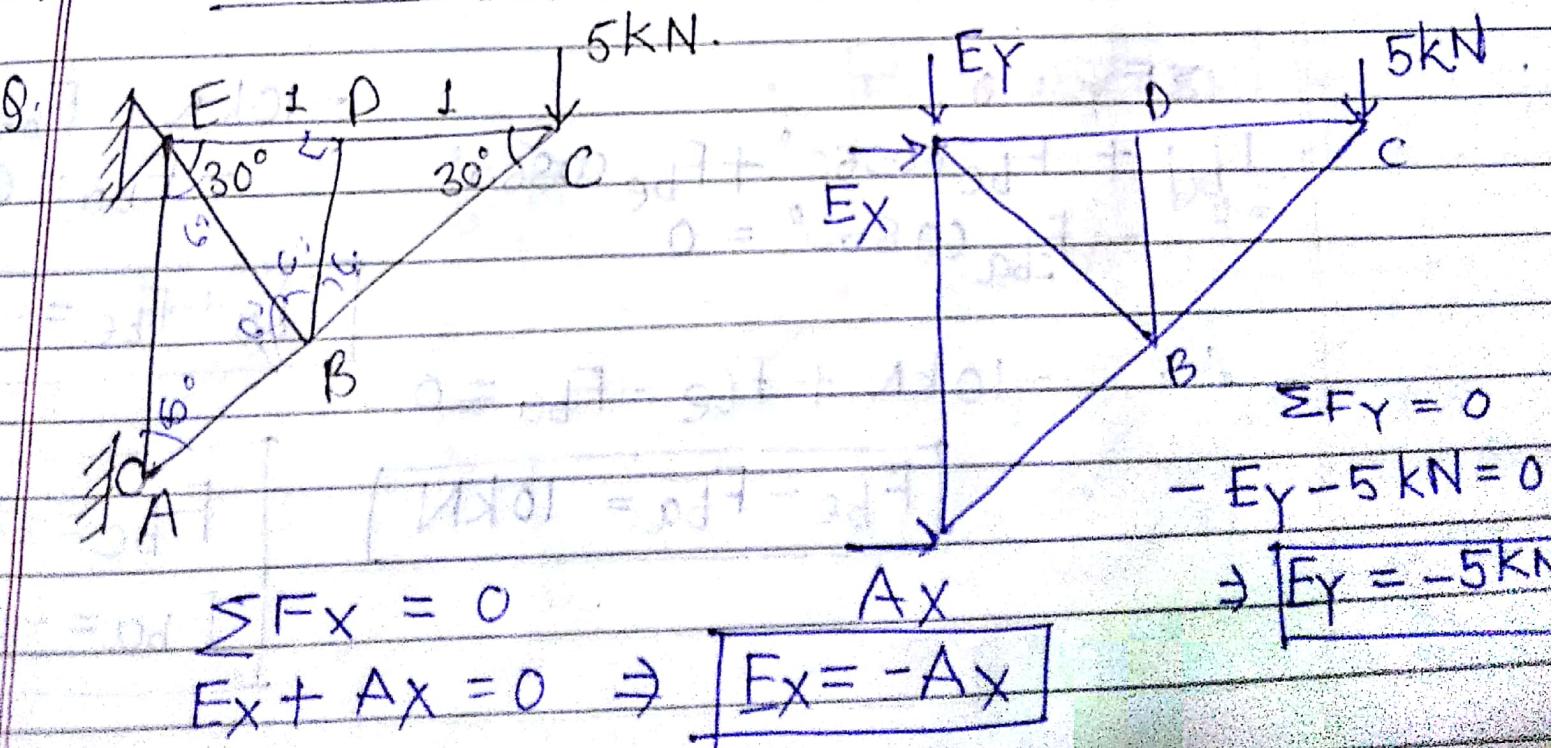


if triangle



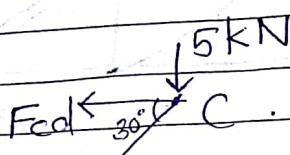
will not bend  
(strong)

## Method of Joints for solving TRUSS problem



Start with the joint which has max two unknowns  
Here, we start with joint C (starting with A) is also possible

Joint C.



$$\sum F_x = 0 ; \sum F_y = 0$$

$$F_{cd} - F_{cb} \cos 30^\circ = 0 \Rightarrow F_{cb} \cos 60^\circ = 5 \text{ kN}$$

$$\Rightarrow F_{cb} \cos 30^\circ = -F_{cd} \quad F_{cb} = -5 \text{ kN}$$

$\cos 60^\circ = \frac{1}{2}$

$$\Rightarrow F_{cd} = 5\sqrt{3} \text{ kN} \Rightarrow F_{cd} = -(10 \text{ kN})\sqrt{3}$$

$$F_{cb} = -10 \text{ kN}$$

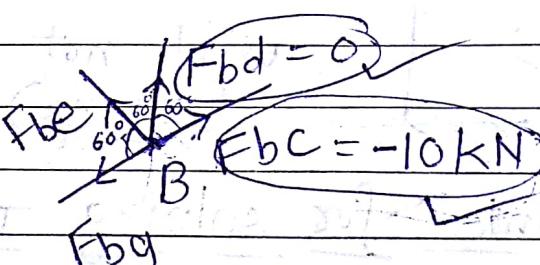
Joint D.

$$F_{de} \leftarrow \rightarrow F_{dc} \quad F_{de} = F_{dc} ; \quad F_{db} = 0$$

$$F_{db}$$

$$\Rightarrow F_{de} = 5\sqrt{3} \text{ kN}$$

Joint B.



$$\sum F_x = 0 ; \sum F_y = 0$$

$$F_{bc} \cos 30^\circ + F_{be} \cos 30^\circ$$

$$-F_{ba} \cos 30^\circ = 0$$

$$\sum F_y = 0$$

$$F_{bd} + F_{bc} \cos 60^\circ + F_{be} \cos 60^\circ - F_{ba} \cos 60^\circ = 0$$

$$-10 \text{ kN} + F_{be} - F_{ba} = 0$$

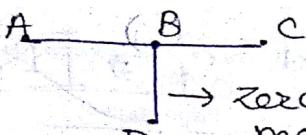
$$F_{ba} + F_{be} = -10 \text{ kN}$$

$$-10 \text{ kN} + F_{be} - F_{ba} = 0$$

$$F_{be} - F_{ba} = 10 \text{ kN}$$

$$F_{be} = 0$$

$$F_{ba} = -10 \text{ kN}$$



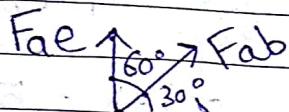
If A, B, C are collinear & zero force BD is  $\perp$  them, A, C are member in equilibrium then BD is transparent.

& There is no external force at B & no reaction force.

JOINT A:

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$A_x + F_{ab} \cos 30^\circ - F_{ae} + F_{ab} \cos 60^\circ = 0$$



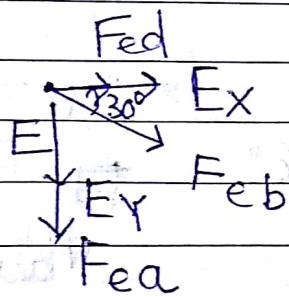
$$\checkmark A_x = 5\sqrt{3} \text{ kN}$$

$$F_{ae} = 5 \text{ kN}$$

JOINT E:

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$F_{ed} + F_{ex} + F_{eb} \cos 30^\circ - F_y - F_{eq} = 0 \quad - F_{eb} \cos 60^\circ = 0 \quad = 0 = 0$$



$$\checkmark \boxed{Ex = -5\sqrt{3} \text{ kN}}$$

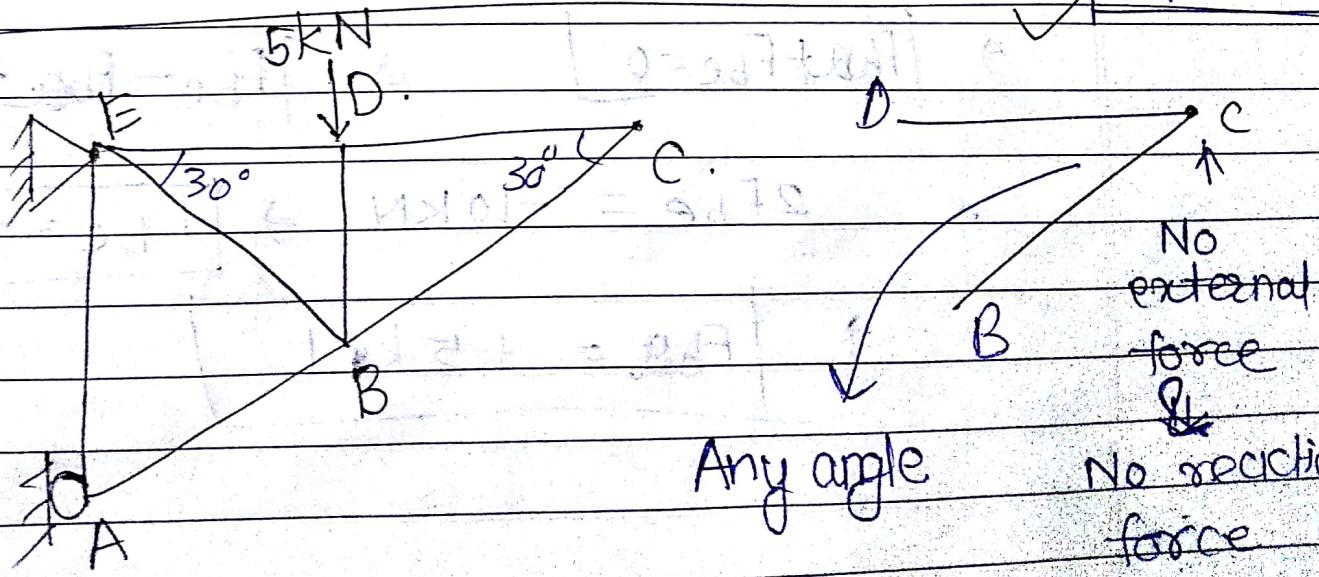
$$(\because Ex = -Ax) \quad F_{ea} = -E_y$$

$$\therefore F_{ed} = -Ex$$

$$\boxed{F_{eq} = 5 \text{ kN}}$$

$$\therefore \boxed{F_{ed} = 5\sqrt{3} \text{ kN}}$$

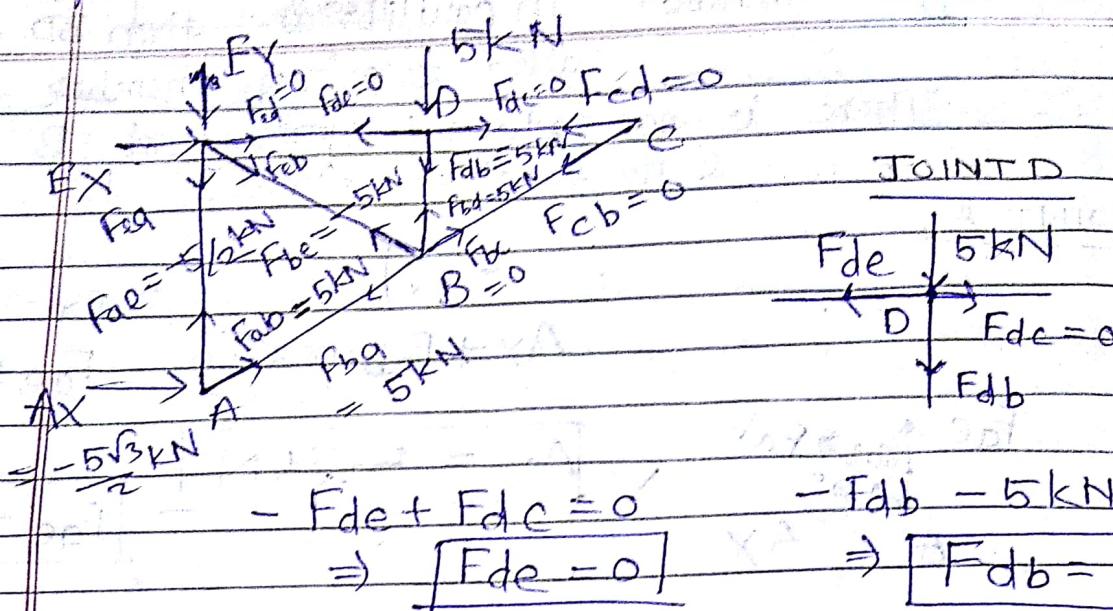
$$\checkmark \boxed{E_y = -5 \text{ kN}}$$



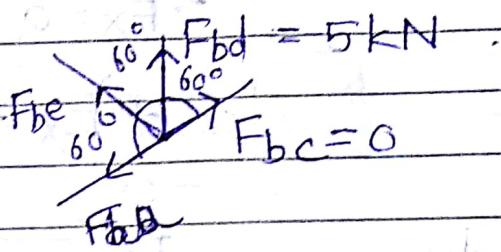
DC & BC are zero force members

$$Ex + Ax = 0$$

$$-Fy - 5 \text{ kN} = 0 \rightarrow Fy = -5 \text{ kN}$$



JOINT B



$$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$

$$-F_{be} \cos 30^\circ - F_{ba} \cos 30^\circ = 0 \quad \& \quad F_{bd} + F_{be} \cos 60^\circ - F_{ba} \cos 60^\circ = 0$$

$$\Rightarrow F_{ba} = -F_{be}$$

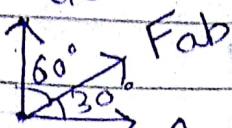
$$\Rightarrow 5 \text{ kN} + \frac{F_{be}}{2} - \frac{F_{ba}}{2}$$

$$\Rightarrow F_{bd} + F_{be} = 0$$

$$\Rightarrow F_{be} - F_{bd} + 10 \text{ kN} = 0$$

$$\Rightarrow 2F_{be} = -10 \text{ kN} \Rightarrow F_{be} = -5 \text{ kN}$$

$$\Rightarrow F_{ba} = +5 \text{ kN}$$

JOINT A $F_{ae}$  $A_x$ 

$\sum F_x = 0$

$\sum F_y = 0$

$A_x + F_{ab} \cos 30^\circ - F_{ae} - F_{ab} \cos 60^\circ = 0$

$A_x = -F_{ab} \cos 30^\circ$

 $F_{ae}$ 

$\Rightarrow A_x = -\left(\pm 5 \text{ kN}\right) \frac{\sqrt{3}}{2}$

$= \mp \frac{F_{ab}}{2}$

$$\boxed{A_x = \pm \frac{5\sqrt{3}}{2} \text{ kN}}$$

$F_{ae} = -\frac{(+5 \text{ kN})}{2}$

$F_{ae} = -\frac{5\sqrt{3}}{2} \text{ kN}$

JOINT E $E_x$  $E_y$ 

$\cancel{E_x} + F_{ed} = 0$

$(\textcircled{E}_x) + F_{ed} + F_{eb} \cos 30^\circ = 0$

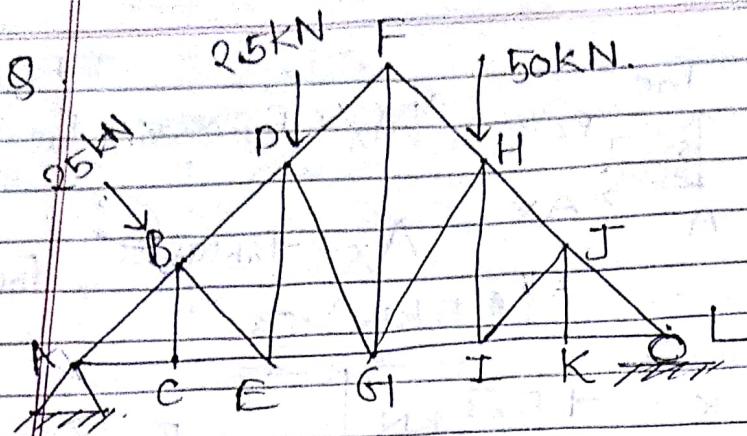
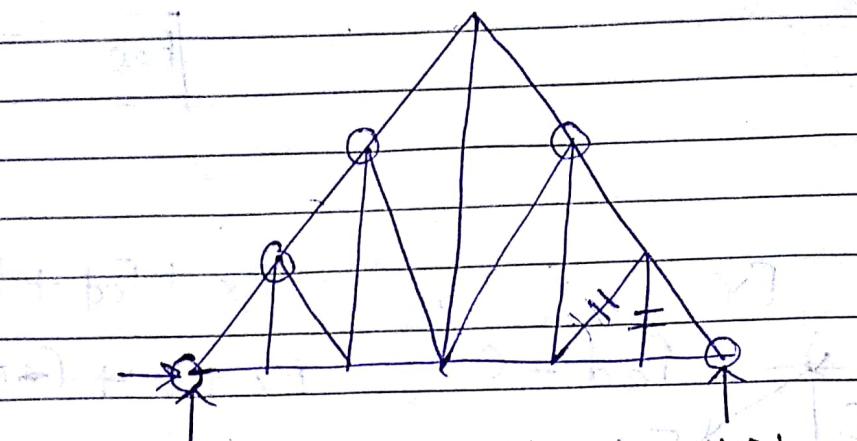
$E_x + 0 + \left(-5 \text{ kN} \frac{\sqrt{3}}{2}\right) = 0$

 $F_{eq}$ 

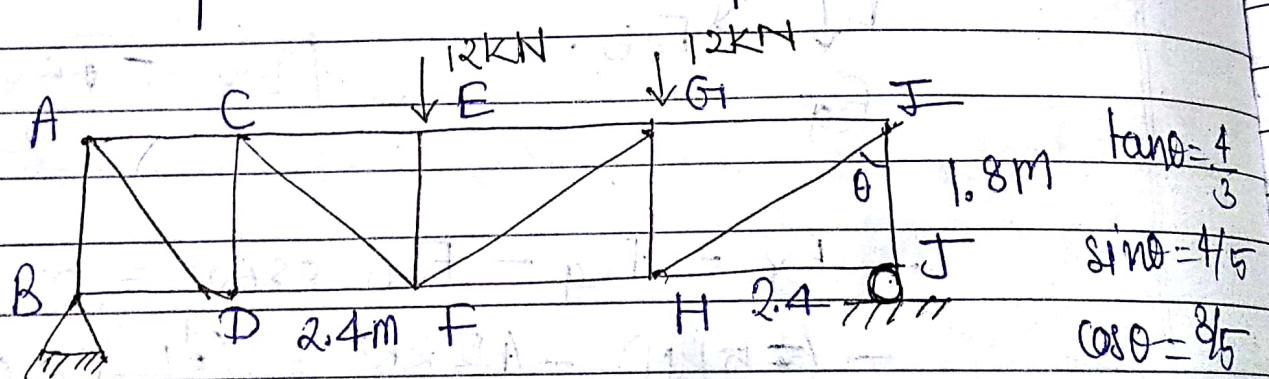
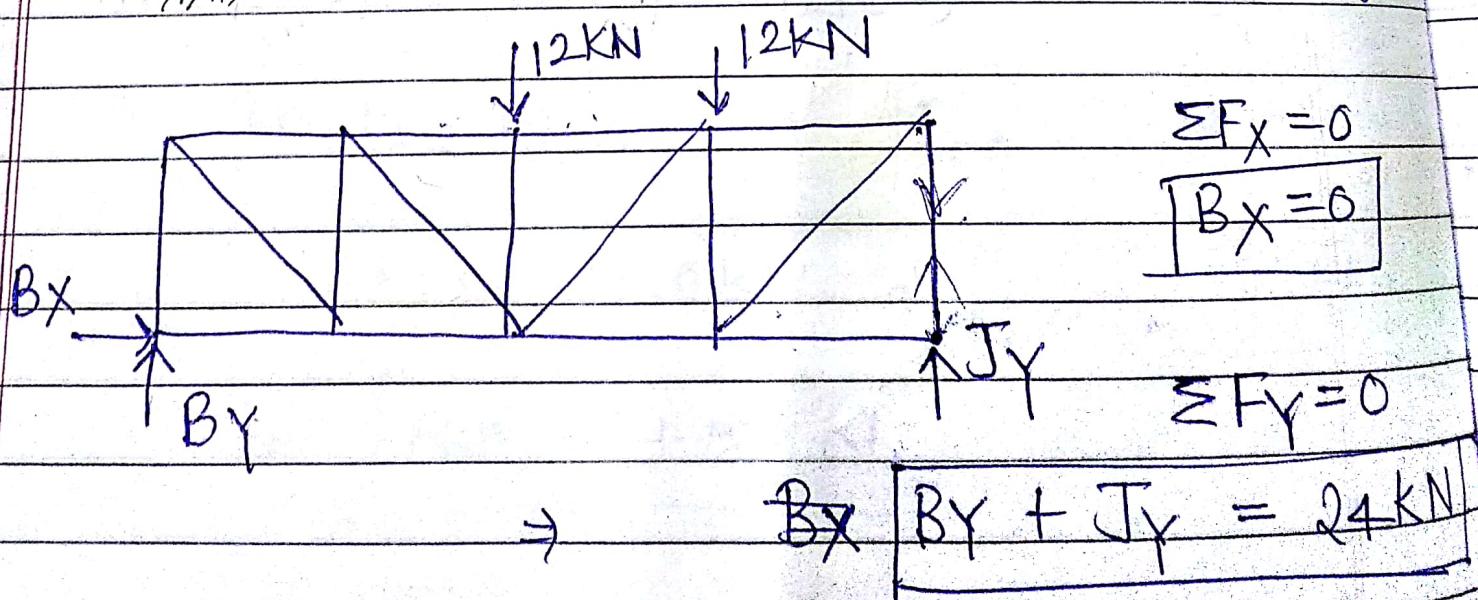
$\cancel{E_x} = \pm 5\sqrt{3} \text{ kN}$

$\cancel{E_y} - F_{eq} - F_{eb} \cos 60^\circ = 0$

$-\left(\pm 5 \text{ kN}\right) - \left(\frac{5}{2} \text{ kN}\right) - \left(\frac{5}{2}\right) = 0 \quad \checkmark$

Ans

Q.

Ans

U/I/I 0 KN T/HF = 20 KN.

Transparent

Date:

Page No.

$$\sum M_B = 0$$

$$\Rightarrow J_Y \times 9.6 - 12 \times 4.8 - 12 \times 7.2 = 0$$

$$\Rightarrow J_Y = \frac{12 \times 4.8^2 + 12 \times 7.2^2}{9.6 \cdot 4}$$

$$= \frac{60}{4} = 15 \text{ kN}$$

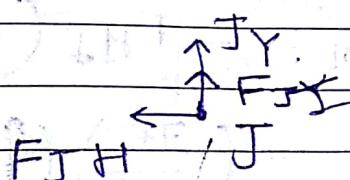
$$\Rightarrow J_Y = 15 \text{ kN}$$

$$\Rightarrow B_Y + 15 \text{ kN} = 24 \text{ kN}$$

$$\boxed{B_Y = 9 \text{ kN}}$$

$$\Rightarrow B_{\text{net}} = \sqrt{(0)^2 + (9)^2} = 9 \text{ kN}$$

Joint J



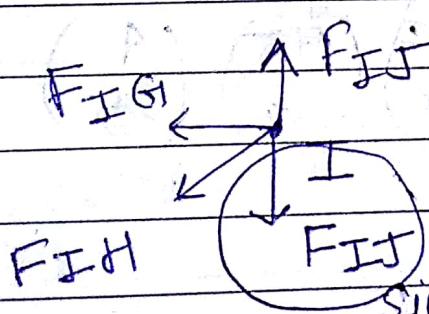
$$\boxed{F_{JH} = 0}$$

$$J_Y + F_{JZ} = 0$$

$$\Rightarrow (F_{JZ} = -15 \text{ kN})$$

we got  $-ve$  so  
it will be in opp. side

Joint I



$$-F_{IG} - F_{IH} \sin \theta + F_{IJ} - F_{IH} \cos \theta = 0$$

$$F_{IG} = -F_{IH} \frac{4}{5}$$

$$F_{IH} = +F_{IJ} \frac{\cos \theta}{\cos \theta}$$

$$F_{IG} = -\frac{4}{5}(F_{IH})$$

$$F_{IH} = +\frac{(15)(5)}{3}$$

$$F_{IG} = -\frac{4}{5} \left( -\frac{75}{3} \text{ KN} \right)$$

$$F_{IH} = +\frac{75}{3} \text{ KN}$$

$$\boxed{F_{TA} = 20 \text{ kN}}$$

$$\boxed{F_{IH} = 25 \text{ KN}}$$

$$20 - \frac{32}{3} = \frac{20}{3}$$

Transparent

Date:

Page No.

For joint G1

$$F_{G1J} - F_{G1E} - F_{G1F} \sin\theta = 0$$

$$(20 \text{ kN}) - F_{G1E} - \frac{4(4)}{3} \left(\frac{1}{3}\right) = 0$$

$$F_{G1E} \leftarrow \rightarrow F_{G1I} \quad \therefore F_{G1E} = \frac{28}{3} \text{ kN}$$

$$-12 \text{ kN} + F_{G1H} - F_{G1F} \cos\theta = 0$$

$$-12 \text{ kN} + (20 \text{ kN}) = F_{G1F} \left(\frac{3}{5}\right)$$

$$\text{=ve so on opp-side} \quad F_{G1F} = \frac{-4 \cancel{15}}{3} = \frac{40}{3}$$

for joint H

$$F_{HGI} + F_{HI} \cos\theta = 0$$

$$F_{HGI} \leftarrow \rightarrow -F_{HF} + F_{HJ} + F_{HI} \sin\theta = 0$$

$$F_{HGI} = -F_{HJ} \left(\frac{4}{5}\right)$$

$$F_{HG} = -\left(\frac{75}{3}\right)\left(\frac{4}{5}\right) = -20 \text{ kN}$$

$$-F_{HF} + F_{HI} \sin\theta = 0$$

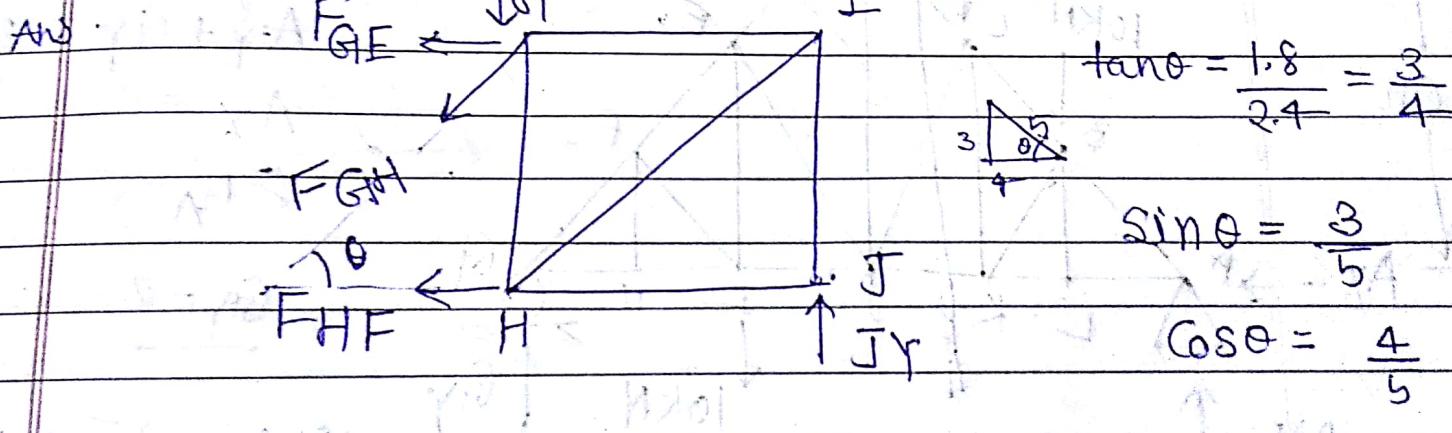
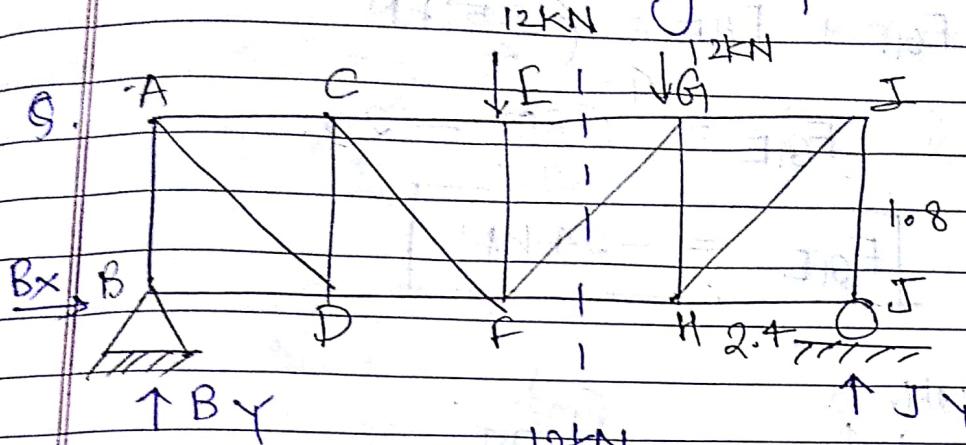
$$\Rightarrow F_{HF} = \left(\cancel{\frac{3}{5}}\right) \left(\frac{15}{3}\right) \left(\frac{4}{5}\right) = \cancel{12} \text{ kN}$$

~~20 kN~~

# METHOD OF SECTIONS

Select a part by

- ① Area of Interest.
- ② Parting line should pass through three members only.
- ③ Draw the FBD of area of interest (part) and solve using equilibrium equations.



$$\sum F_y = 0$$

$$\sum F_x = 0$$

$$-F_{GH} \sin\theta - F_{GE} 12kN + 15kN =$$

$$-F_{GE} - F_{GH} \cos\theta - F_{HF} = 0$$

$$\Rightarrow F_{GH} \sin\theta = 3kN$$

$$\Rightarrow F_{GE} + F_{HF} + 5kN = 0$$

$$\Rightarrow F_{GH} = \frac{3kN}{\sin\theta}$$

$$\Rightarrow F_{GE} + F_{HF} = -\frac{25}{4}kN$$

$F_{GH} = 5kN$

$$\sum M_A = 0 \Rightarrow (-F_{HF} \times 1.8) + (2.4 \times J_Y) = 0.$$

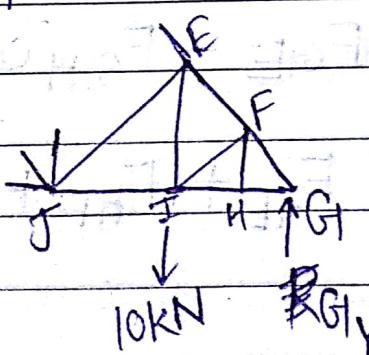
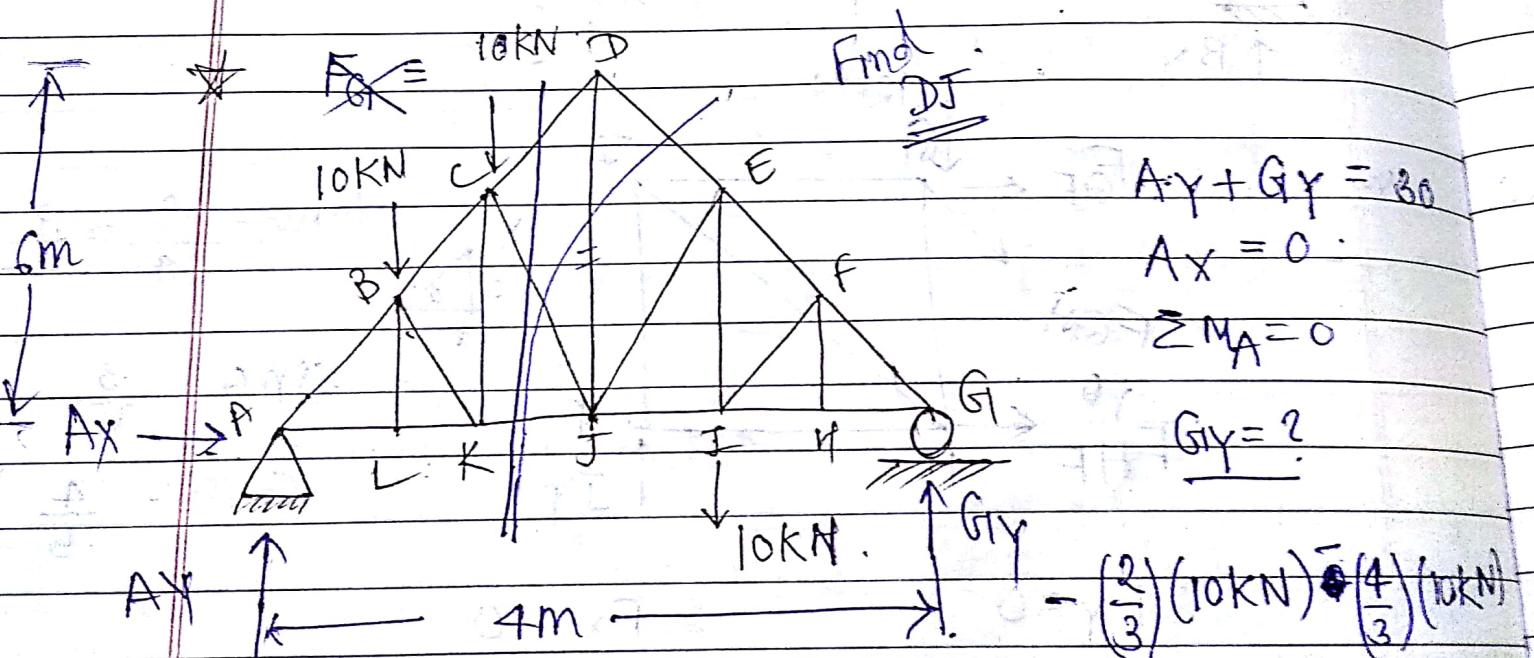
$$\Rightarrow F_{HF} = \frac{2.4 \times J_Y}{1.8} = \frac{4}{3} \times 15 \text{ kN}$$

$$F_{HF} = 20 \text{ kN}$$

$$\therefore F_{GE} + F_{HF} = -\frac{25}{4} \text{ kN}$$

$$\Rightarrow F_{GE} = -\frac{25}{4} - 20$$

$$F_{GE} = -24 \text{ kN}$$



$$= 0$$

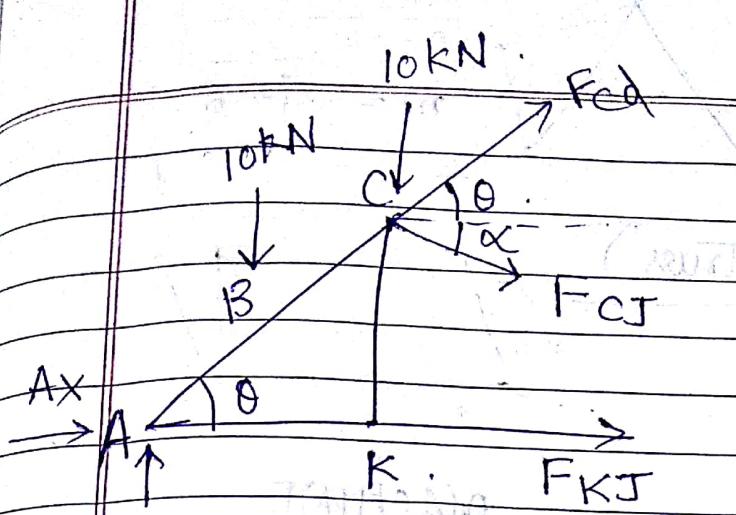
$$A G_y = \frac{14}{3} \times 10 \text{ kN}$$

$$\Rightarrow A_y = 30 - \frac{35}{3}$$

$$A_y = \frac{55}{3} \text{ kN}$$

$$G_y = \frac{35}{3} \text{ kN}$$

Date: \_\_\_\_\_  
Page No. \_\_\_\_\_



$$\sum F_x = 0$$

$$F_{CD} \cos \theta$$

$$+ F_{CJ} \cos \theta = 0$$

$$\sum F_y = 0$$

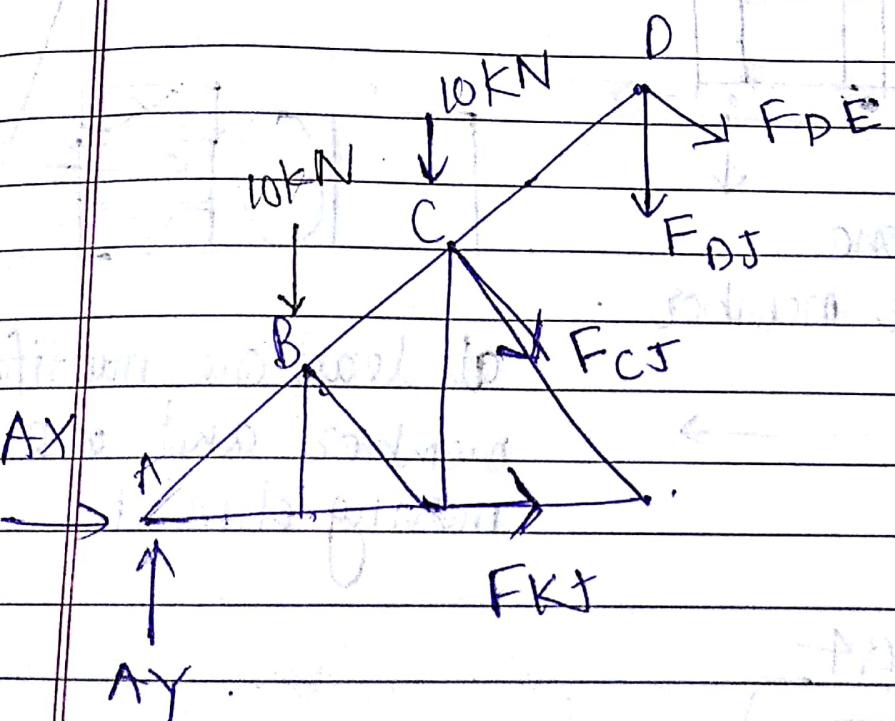
$$F_{CD} \sin \theta - F_{CJ} \sin \theta$$

$$-10 - 10 + 18.33 = 0$$

$$\sum M_J = 0$$

$$\tan \theta = \frac{6}{2} = 3.$$

$$\sin \theta =$$



$$m = 2j - 3$$

$$m = 3j - 6$$

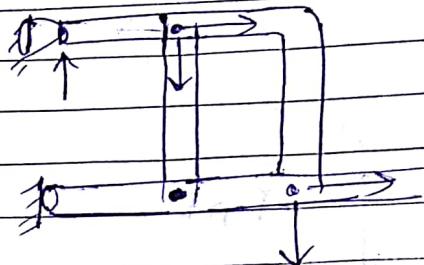
\* 3D Truss (Space Truss)

TRUSS

FRAME

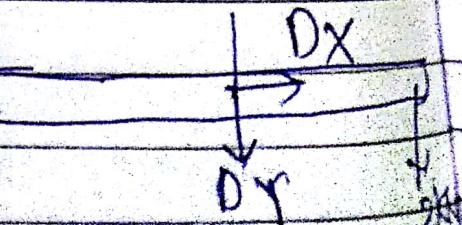
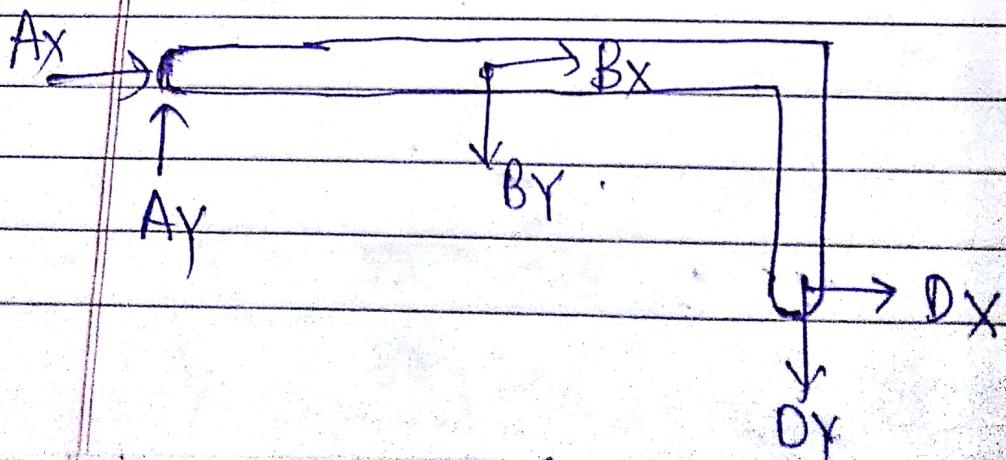
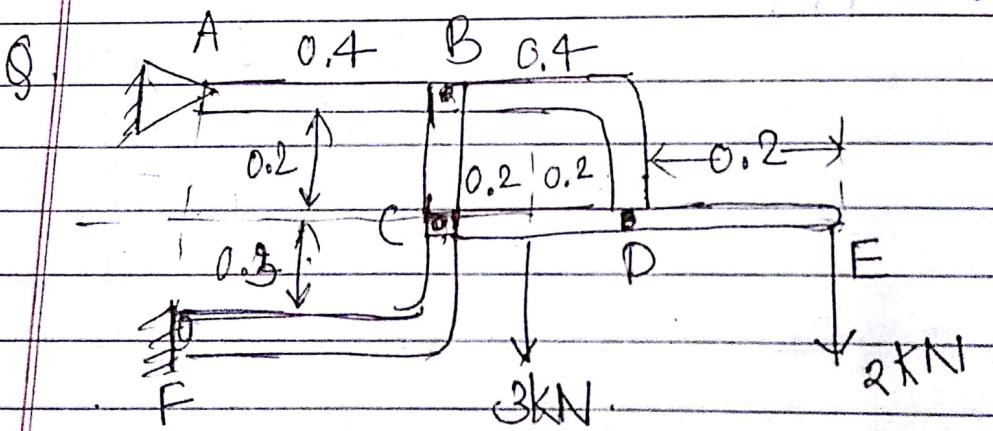
MACHINE

2 Force  
Member



at least one  
multiforce member

at least one multiforce  
member and one  
moving element

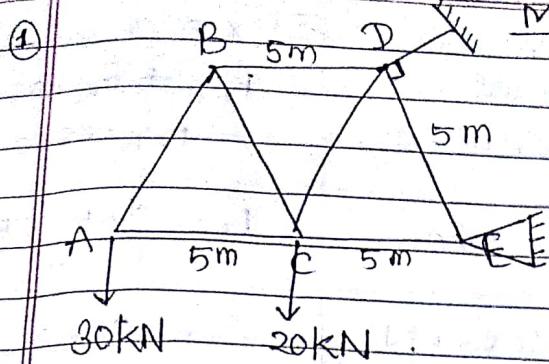


# Tutorial

Date: \_\_\_\_\_  
Page No. \_\_\_\_\_

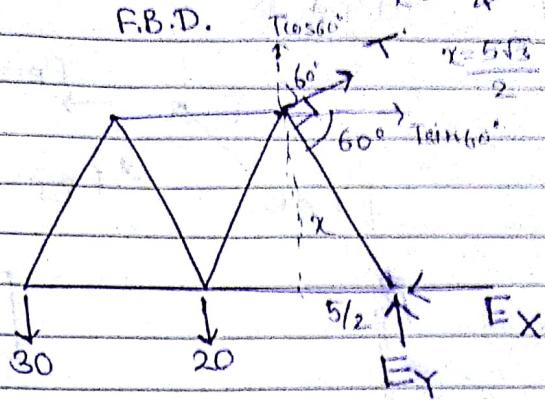
$$\left(\frac{5}{2}\right)^2 + x^2 = \frac{25}{4}$$

$$x^2 = \frac{75}{4}$$



METHOD OF JOINTS

F.B.D.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\therefore -Ex + Ts \sin 60^\circ = 0 \quad \therefore -30 - 20 + Ey + Tc \cos 60^\circ = 0$$

$$\Rightarrow Ex = Ts \sin 60^\circ \quad \text{--- (1)} \quad \Rightarrow Ey = 50 - Tc \cos 60^\circ \quad \text{--- (2)}$$

$$\sum M_E = 0$$

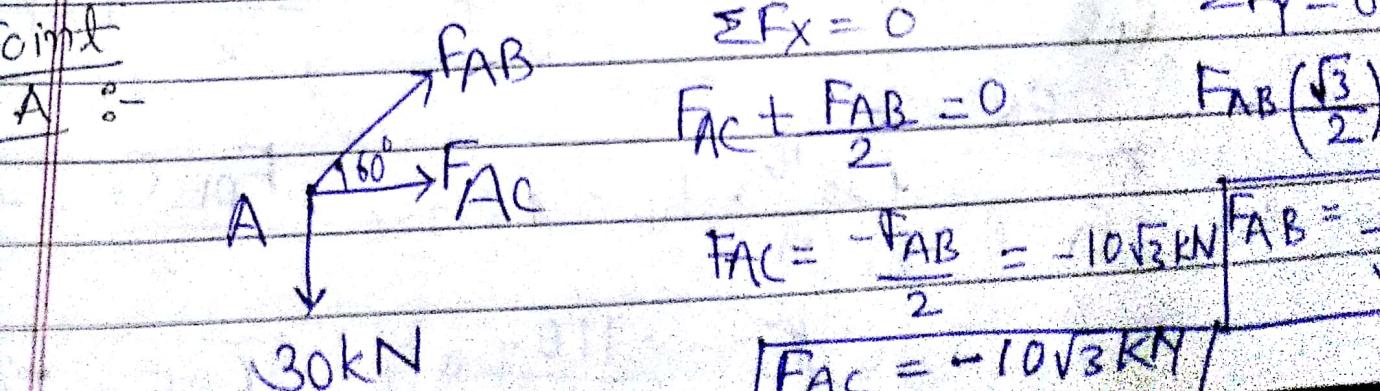
$$\Rightarrow (5 \times 20) + (10 \times 30) - \left( \frac{5\sqrt{3}}{2} Tc \cos 60^\circ \right) - \left( \frac{5\sqrt{3}}{2} Ts \sin 60^\circ \right) = 0$$

$$\Rightarrow 400 = \frac{5T}{4} + \frac{15T}{4}$$

$$\Rightarrow 5T = 400 \Rightarrow T = 80 \text{ kN}$$

$$\Rightarrow Ex = 80 \times \frac{\sqrt{3}}{2} \Rightarrow Ex = 40\sqrt{3} \text{ kN} \quad \Rightarrow Ey = 50 - (80) \left( \frac{1}{2} \right)$$

$$\Rightarrow Ex = 40\sqrt{3} \text{ kN} \quad \Rightarrow Ey = 10 \text{ kN}$$



$$\sum F_x = 0$$

$$F_{AC} + \frac{F_{AB}}{2} = 0$$

$$\sum F_y = 0$$

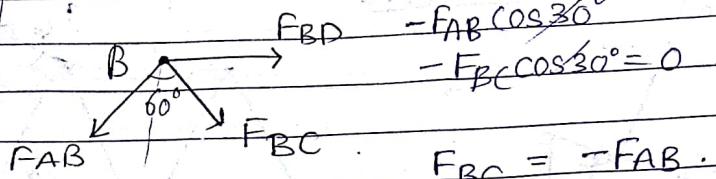
$$F_{AB} \left( \frac{\sqrt{3}}{2} \right) - 30 = 0$$

$$F_{AC} = -\frac{F_{AB}}{2} = -10\sqrt{3} \text{ kN}$$

$$F_{AB} = \frac{60}{\sqrt{3}} = 20 \text{ kN}$$

$$F_{AC} = -10\sqrt{3} \text{ kN}$$

Joint B



$$\sum F_y = 0$$

$$-F_{AB} \cos 30^\circ$$

$$-F_{BC} \cos 30^\circ = 0$$

$$\sum F_x = 0$$

$$F_{BD} + F_{BC} \cos 60^\circ$$

$$-F_{AB} \cos 60^\circ = 0$$

$$F_{BC} = -F_{AB}$$

$$F_{BD} = \frac{F_{AB} - F_{BC}}{2}$$

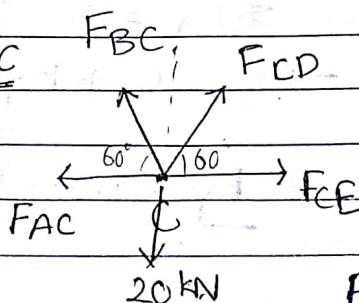
$$F_{BC} = -20\sqrt{3} \text{ kN}$$

$$= 20\sqrt{3} - (-20\sqrt{3})$$

$$F_{BD} = 20\sqrt{3} \text{ kN}$$

(2)

Joint C



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$F_E - F_{AC} + \frac{F_{CD}}{2} - \frac{F_{BC}}{2} = 0 \quad -20 + \frac{(F_{BC} + F_{CD})\sqrt{3}}{2} = 0$$

$$F_E - (-10\sqrt{3})$$

$$+ \left( \frac{100}{\sqrt{3}} / 2 \right) - \left( \frac{-20\sqrt{3}}{2} \right) = 0 \Rightarrow (-20\sqrt{3} + F_{CD})\sqrt{3} = 40$$

$$\Rightarrow -20(3) + \sqrt{3}F_{CD} = 40$$

$$F_E + 10\sqrt{3} + \frac{50}{\sqrt{3}} + 10\sqrt{3} = 0.$$

$$\Rightarrow \sqrt{3}F_{CD} = 100$$

$$F_E = (-20\sqrt{3} - 50/\sqrt{3})$$

$$F_{CD} = \frac{100}{\sqrt{3}} \text{ kN}$$

$$F_E = \frac{-60 - 50}{\sqrt{3}} = \frac{-110}{\sqrt{3}} \text{ kN}$$

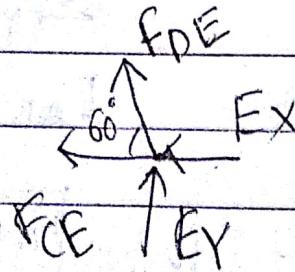
JOINT E

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$-\frac{F_{DE}}{2} - F_{CE} - F_x = 0$$

$$\frac{F_{DE}\sqrt{3}}{2} + F_y = 0$$



$$\frac{+20\sqrt{10}}{\sqrt{3}(2)} - F_{CE} - 40\sqrt{3} = 0$$

$$\frac{F_{DE}\sqrt{3}}{2} = -F_y$$

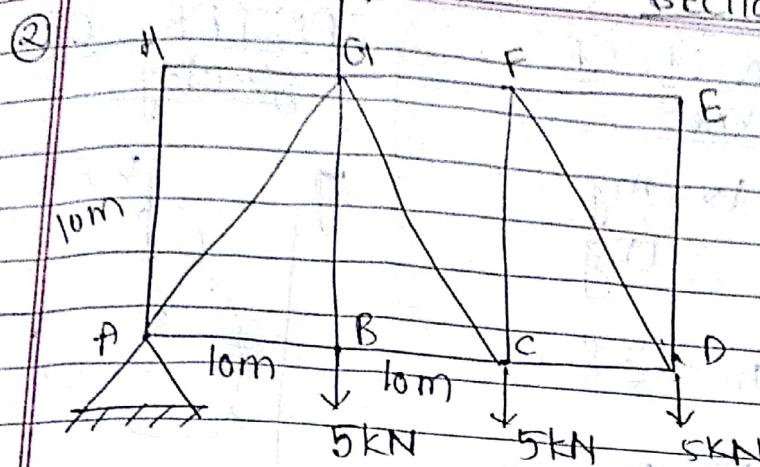
$$\Rightarrow F_{CE} = -\frac{110}{\sqrt{3}} \text{ kN}$$

$$F_{DE} = \frac{-20}{\sqrt{3}} \text{ kN}$$

## METHOD OF SECTION

Date: \_\_\_\_\_  
Page No. \_\_\_\_\_

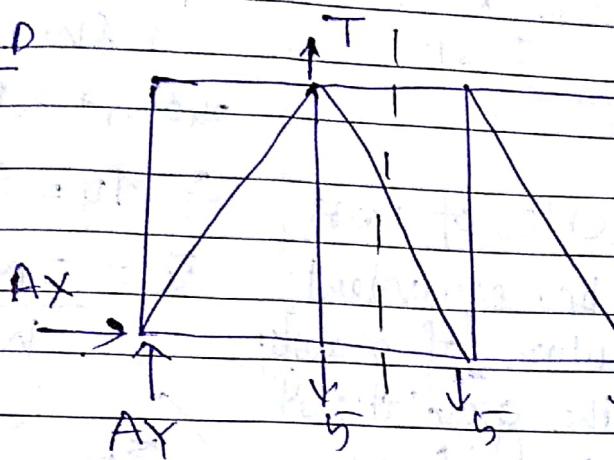
#### Transport



Find  $F_{CG}$  = ?

Use method of  
section

FBD



$$\sum F_x = 0$$

$$\Rightarrow Ax = 0$$

$$\Rightarrow Ax - 15 + T = 0$$

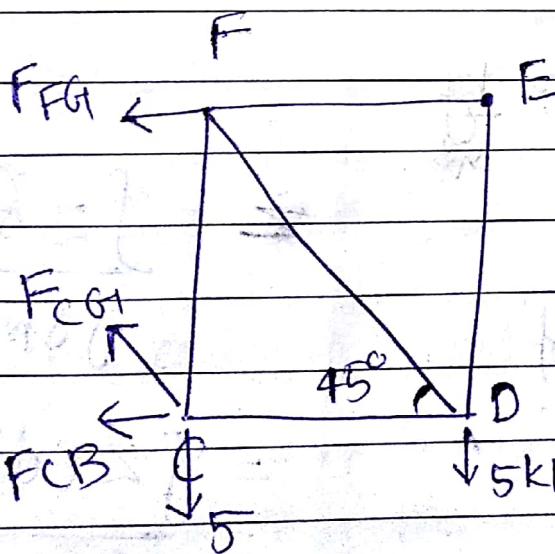
$$Ay = 15 - T$$

$$\Rightarrow M \cdot T \times 10 - 5 \times 10 - 5 \times 20 - 5 \times 30 = 0 \quad \Rightarrow AY = 15 - 30$$

$$10T = 5(60)$$

$$A_y = -15 \text{ kN}$$

$$T = 30 \text{ kN}$$



$$\sum M_C = 0$$

$$F_{EG} \times 10 - 5 \times 10 = 0$$

$$\Rightarrow F_{FG} = 5 \text{ kN}$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$-\frac{F_{FG1}}{\sqrt{2}} - \frac{FCG_1}{\sqrt{2}} + \frac{F_{CB}}{\sqrt{2}} = 0$$

$$-5 - 10 = FCB$$

$$F_{CB} = 15 \text{ kN}$$

$$F_{CG} = 10\sqrt{2} \text{ kN}$$

$$F_{CG} = 10\sqrt{2} \text{ kN}$$

## CENTROID

Geometrical centre  
of the body.

$$V = \int dV$$

$$\bar{x} = \frac{\sum x \delta V}{\sum \delta V}$$

So, when density  
is uniform. Then

centroid and cent  
er of mass will  
be same.

## CENTER OF MASS

$$W = mg  
= \rho V g$$

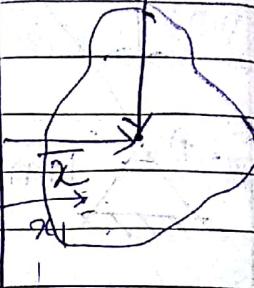
$$m = \sum \delta m$$

$$\bar{x} = \frac{\sum x \delta m}{\sum \delta m}$$

So, center of mass  
will be equivalent  
to center of gravity  
when the gravitational  
field is uniform.

## CENTER OF Gravity

$$g \quad W$$



$$n \delta W = W = \int \delta w$$

$$\pi_1 \delta w_1 + \pi_2 \delta w_2 +$$

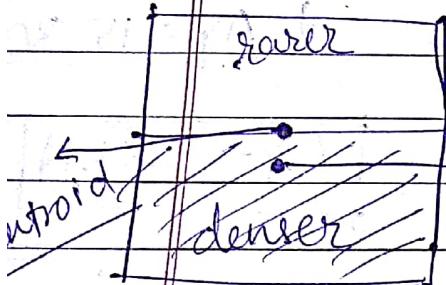
$$\sum M_y = \sum x \delta w$$

$$\bar{x} = \frac{\sum M_y}{\sum \delta w} = \frac{\sum x \delta w}{\sum \delta w}$$

$$\sum F = 0$$

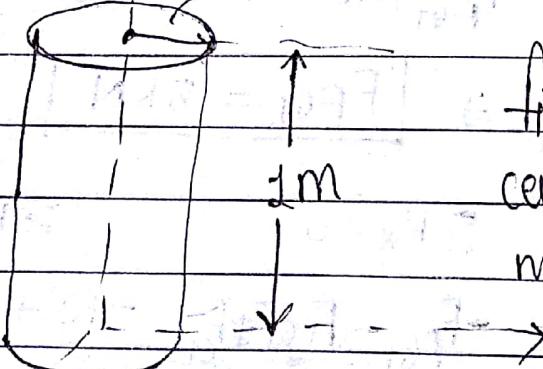
$$\sum M = 0$$

$$\bar{x} = \frac{\sum x \delta m g}{\sum \delta m g}$$



center of mass

$$* \quad Y \rightarrow 0.5m \quad l = 200 \text{ kg/m}^3$$



find

center of

mass

$$\bar{x} = \int z \delta m$$

$$8m$$

$$= \int z \delta v$$

$$\int \delta v$$

and -

as it is symmetric body so center will  
lie on Z-axis only  $\Rightarrow \bar{x} = 0, \bar{y} = 0$

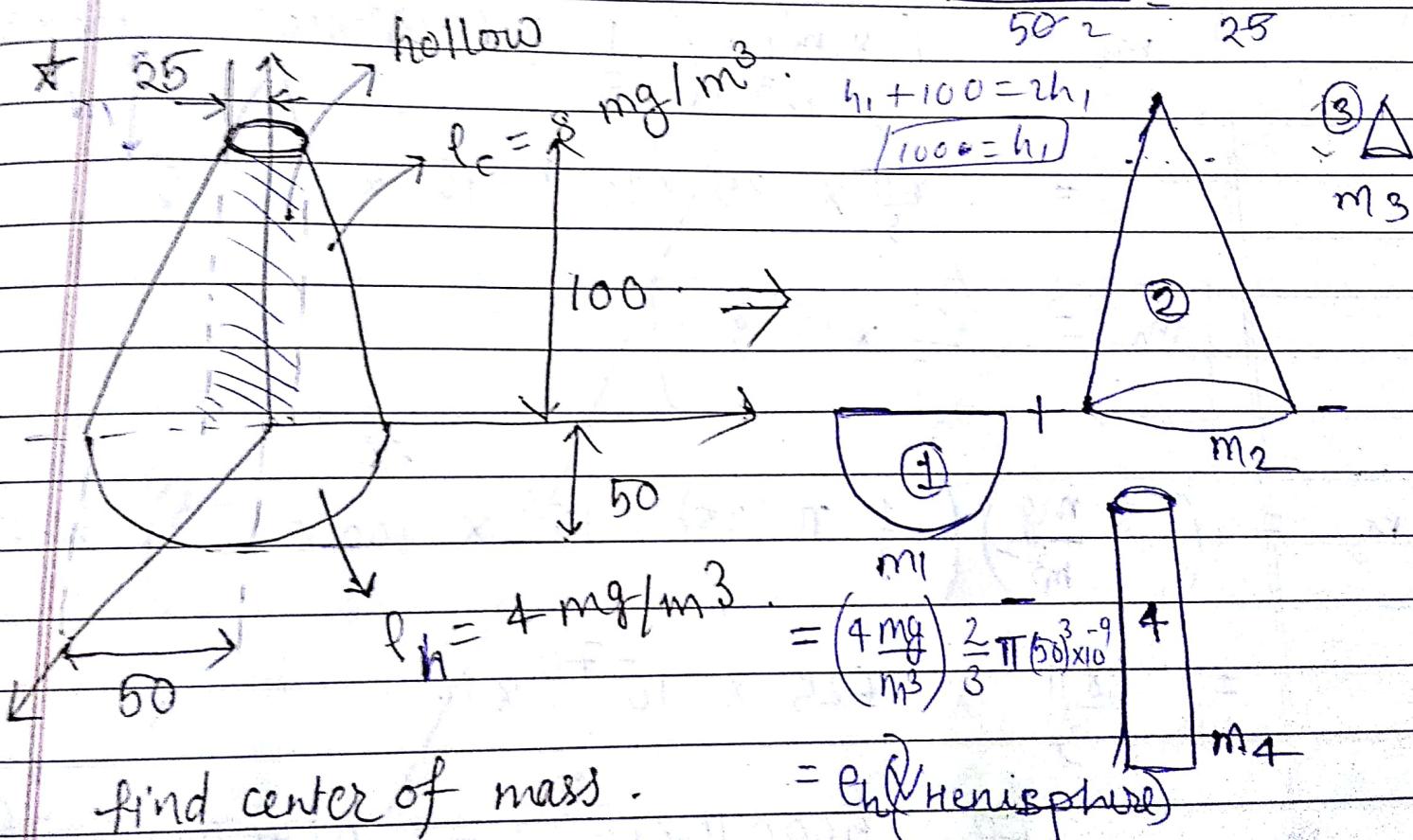
$$\Rightarrow \bar{z} = \frac{\int 200z^2 dV}{\int 200z dV} \quad V = \pi r^2 h$$

$$= \frac{\int z^2 (\cancel{\pi}) dz}{\int z (\cancel{\pi}) dz} \quad V = \pi (0.5)^2 (z) \\ dV = \frac{\pi}{4} dz$$

$$\bar{z} = \frac{(z^3)_0^1}{(z^2)_0^1} = \frac{V_3}{V_2} = \frac{2}{3}$$

center of mass =  $(0, 0, \frac{2}{3}m)$

~~$\frac{2}{3}R$~~   $\frac{h_1 + 100}{50} = \frac{h_1}{25}$



$$m = m_1 - m_2$$

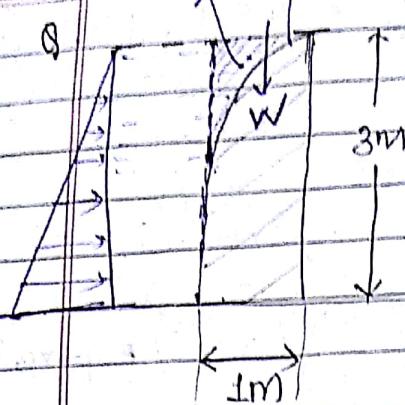
$$m = m_1 + m_2 - m_3 - m_4$$

$$m = m_1 - m_2$$

$$\bar{x} = \frac{xm_1 - xm_2}{m_1 - m_2}$$

PARABOLA

$$L = 5 \text{ m}, \rho = 1020 \frac{\text{kg}}{\text{m}^3}$$

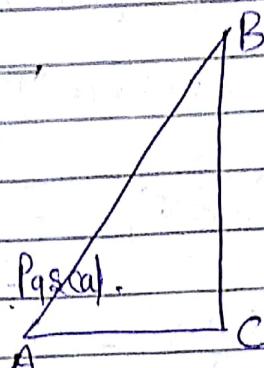


Find the net force acting on parabolic dam.

$$A = \frac{1}{2} AC \cdot BC$$

$$F_h = \frac{1}{2} \times 150.1 \times 3$$

$$P = \rho gh = 1020 \times 9.8 \times 3 \text{ Pa} \text{ (at)}.$$



$$F_h = 225.1 \text{ kN}$$

$$F_L = 1020 \times 9.8 \times 3 \times 5 \text{ N/m}$$

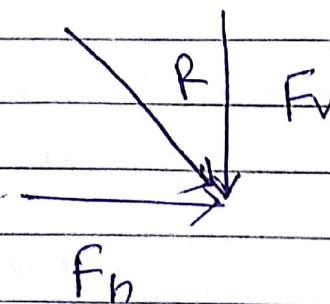
$$W = F_V = Aw \times L \times \rho_w g$$

$$F_V = 50 \text{ kN}$$

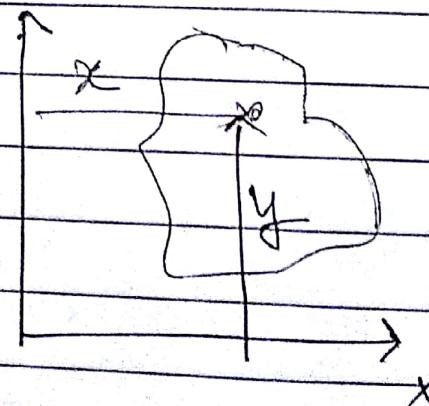
$$F_L = 150.1 \text{ kN}$$

$$R = \sqrt{F_h^2 + F_V^2}$$

$$= 231 \text{ kN}$$



\* Area Moment of Inertia :- (second moment)



$$I_y = \sum x^2 dA$$

$$I_x = \sum y^2 dA$$

$$I_{xy} = \sum xy dA$$

SI unit

 $\text{m}^2$  $\text{m}^4$  $\text{m}^6$

$$I_y = \pi r^2 dA$$

$$\frac{a^2 b dA}{4}$$

$$x^2 b dA$$

$$\frac{a^2}{4} b$$

$$\int x^2 dx$$

$$0$$

$$b \int x^2 dx$$

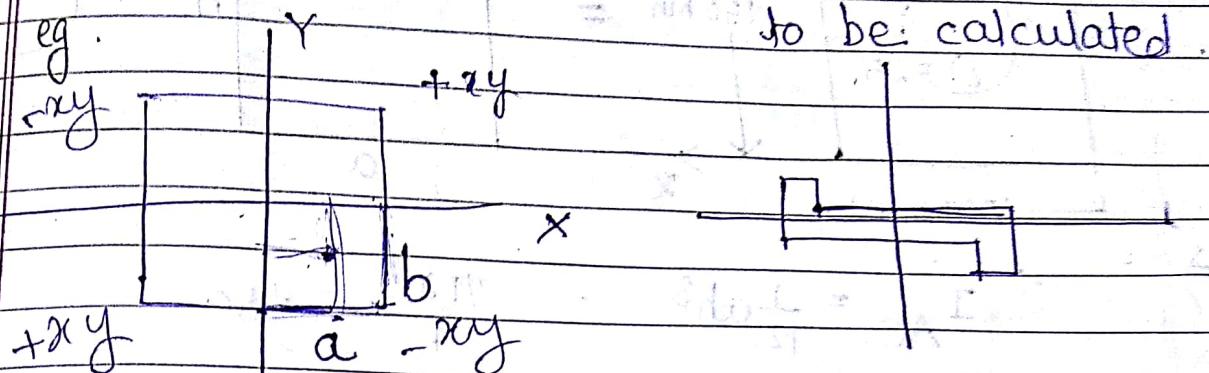
$$-\frac{a^2}{2}$$

Transparent

Date:

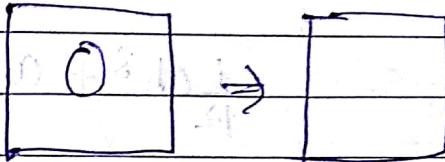
Page No.

For any symmetric body if axes are passing through centroid then Area moment of Inertia will be zero.



$$I_x = \frac{1}{12} ab^3$$

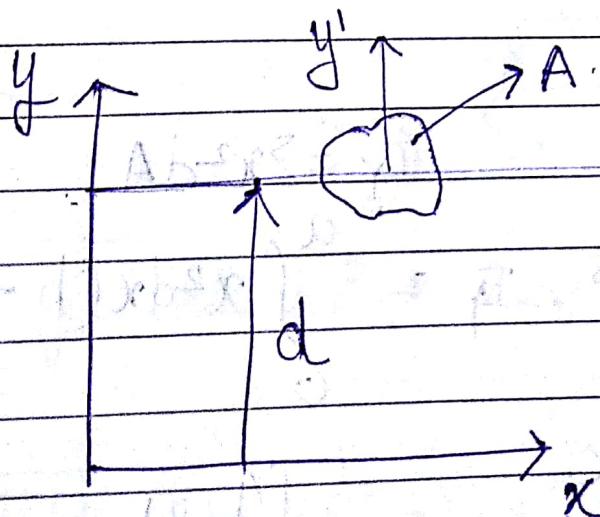
$$I_y = \frac{1}{12} ba^3$$



$$-0.$$

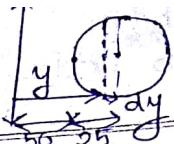
$$I_{xy} = 0.$$

## \* PARALLEL AXIS THEORY



$$I_x = I_{x'} + Ad^2$$

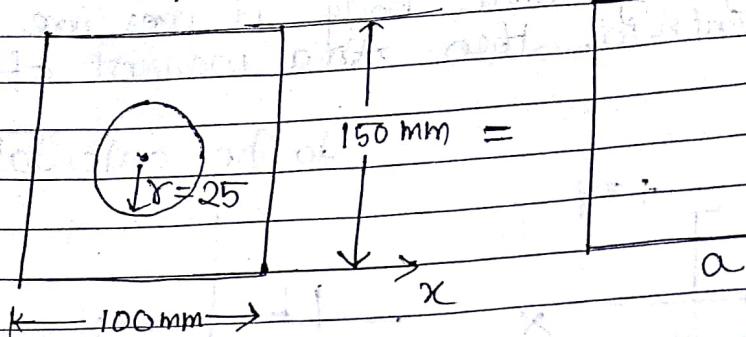
$$I_x = \sum y^2 dy$$



Date:  
Page No.

252

$$I_x = ?$$



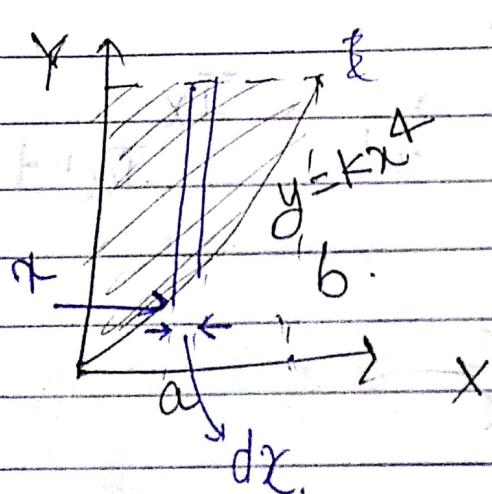
$$\Rightarrow I_{AC} = \frac{1}{12} ah^3 - \frac{\pi r^4}{4} = I_{BC}$$

$$I_{AX} = I_{AC} + A_A d_A^2 \quad J_{BX} = I_{BC} + A_B d_B^2$$

$$= \frac{1}{12} ah^3 + ah(75)^2 \quad = \frac{\pi}{4} r^4 + \pi r^2 (75)^2$$

$$I_x = I_{AX} - J_{BX}$$

$$= 101 \times 10^6 \text{ mm}^4$$



$$J_y = \sum x^2 dA$$

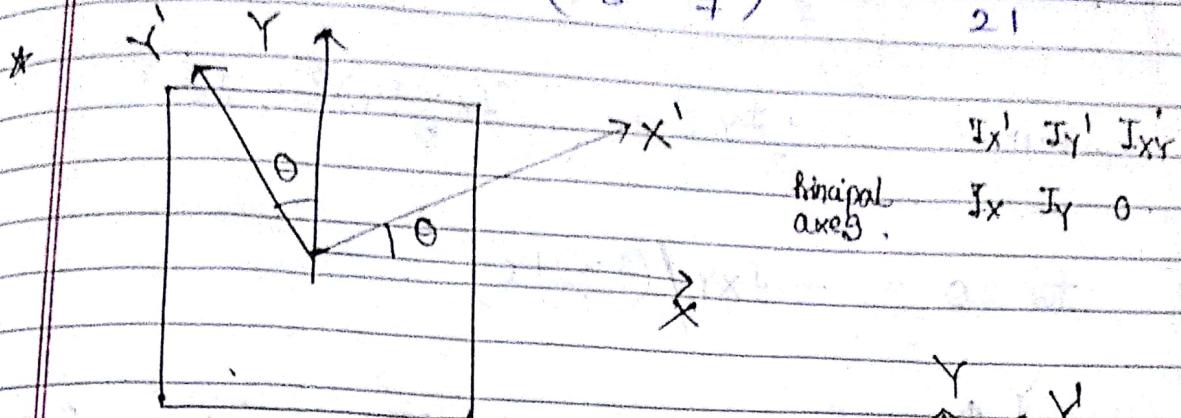
$$J_y = \int_0^a x^2 dx (b - kx^4)$$

$$= \int_0^a (bx^2 dx - kx^6 dx)$$

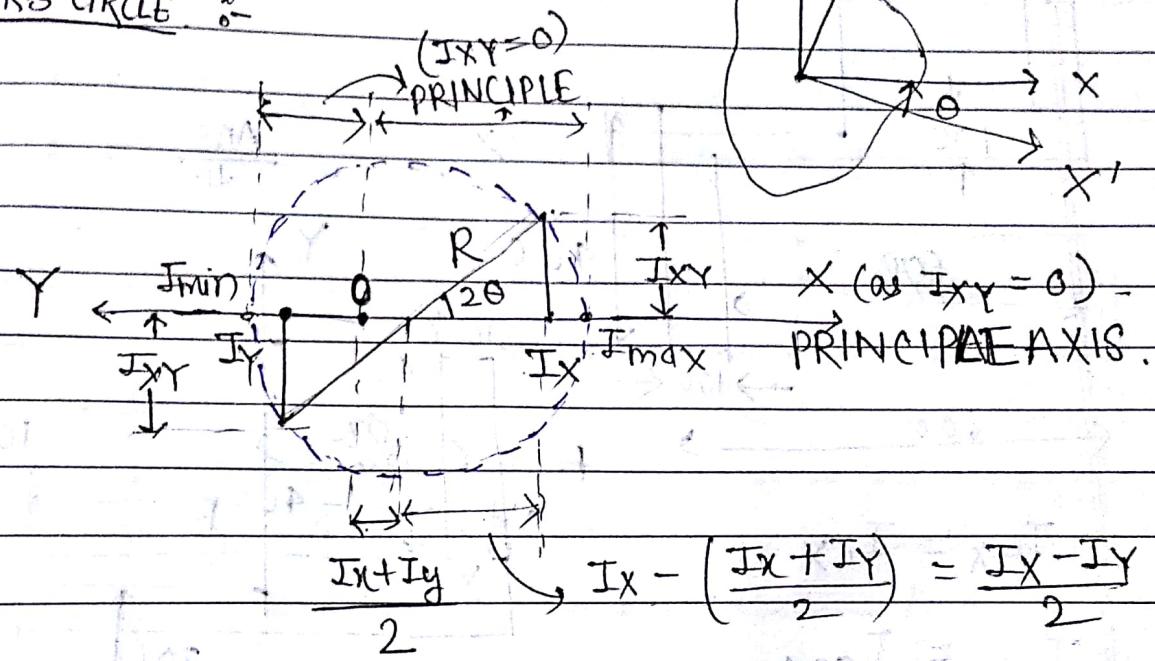
$$= b(x^3)_0^a - k(x^7)_0^a$$

$$= \frac{ba^3}{3} - \frac{ka^7}{7} = a^3 \left( \frac{b}{3} - \frac{ka^4}{7} \right)$$

$$I_Y = \frac{ab}{3} \left( \frac{b}{3} + \frac{b}{7} \right) = \frac{103b}{21}$$



\* MORRIS CIRCLE :-



$$R = \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + (I_{XY})^2}$$

$$\text{PRINCIPLE} = \frac{I_X + I_Y}{2} + R = \frac{I_X + I_Y}{2} + \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + (I_{XY})^2}$$

and

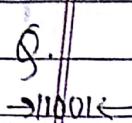
$$= R - \left( \frac{I_X + I_Y}{2} \right) \quad \cancel{R}$$

~~$$= R - \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + (I_{XY})^2}$$~~

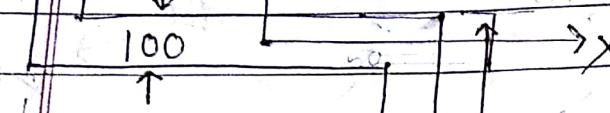
$$J_{\max} = \frac{J_x + J_y}{2} + \sqrt{\left(\frac{J_x - J_y}{2}\right)^2 + J_x J_y}$$

$$J_{\min} = \frac{J_x + J_y}{2} - \sqrt{\left(\frac{J_x - J_y}{2}\right)^2 + J_{xy}^2}$$

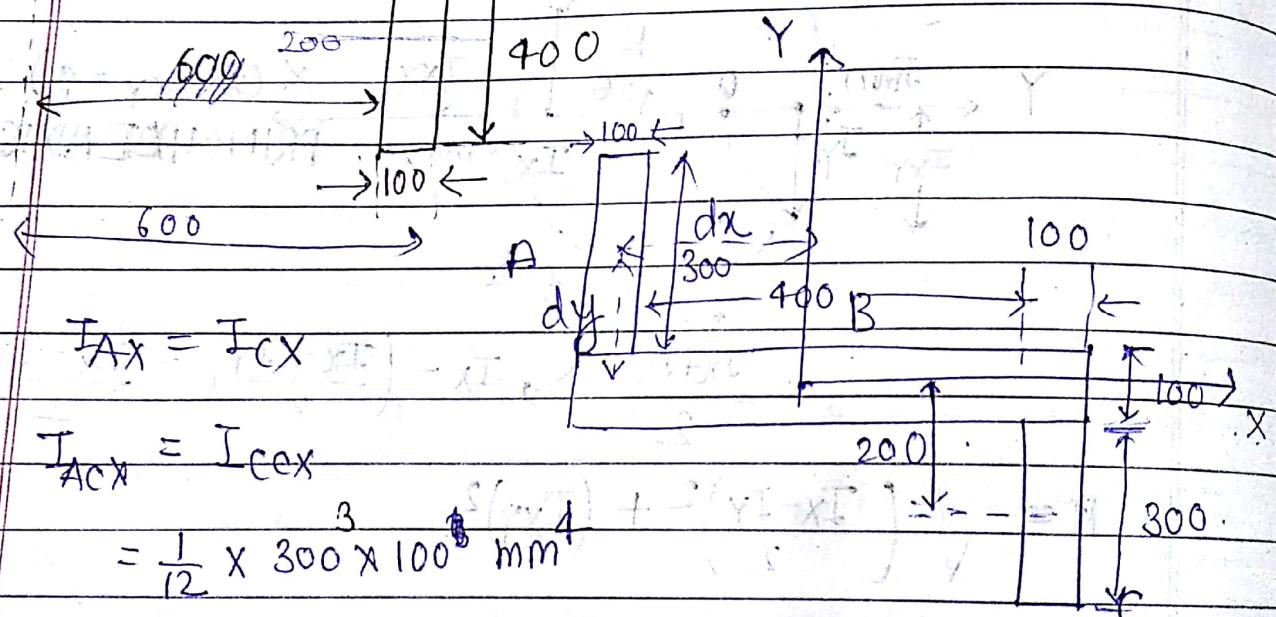
$$\tan 2\theta = - I_{XY} / \left( \frac{I_X - I_Y}{2} \right)$$



Find  $J_{\max}, J_{\min}, \theta = ?$



ANS



$$\mathcal{I}AX = \mathcal{I}CX$$

$$I_{\text{ACx}} = I_{\text{cex}}$$

$$= \frac{1}{12} \times 300 \times 100^3 \text{ mm}^4$$

$$T_{AX} = T_{ACX} + A_A(200)^2 = 1.425 \times 10^9 \text{ mm}^4$$

$$I_{BCX} = I_{BX} = \frac{1}{12} (100)^3 (600) \text{ mm}^4 = 0.05 \times 10^9 \text{ mm}^4$$

$$I_x = I_{Ax} + I_{Bx} + I_{Cx}$$

$$\therefore I_x = 2.9 \times 10^9 \text{ mm}^4$$

$$I_{AY} = I_{CY}$$

$$I_{AY} = I_{CCY}$$

$$= \frac{1}{12} \times 300 \times (100)^3 \text{ mm}^4$$

$$I_{AY} = I_{ACY} + A_A (250)^2$$

$$I_{BCY} = I_{BY}$$

$$I_Y = 5.6 \times 10^9 \text{ mm}^4$$

$$I_{XYAC} = 0$$

$$; I_{XXA} = I_{XYAC} + Adx dy$$

$$= 0 + (300 \times 100) (-250)(200)$$

$$= -150 \times 10^7 \text{ mm}^4 = -1.5 \times 10^9 \text{ mm}^4$$

$$I_{XYB} = 0 ; I_{XYC} = 0 + (300 \times 100) (250) (-200)$$

$$= -1.5 \times 10^9 \text{ mm}^4.$$

$$\therefore J_{XY} = I_{XXA} + I_{XYB} + I_{XYC}$$

$$= -3 \times 10^9 \text{ mm}^4.$$

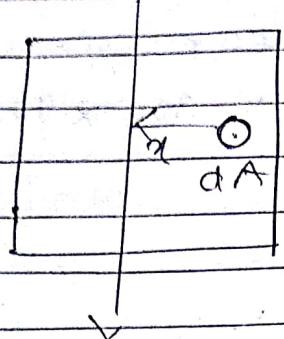
$$I_X = 2.9 \times 10^9 \text{ mm}^4$$

$$J_Y = 5.6 \times 10^9 \text{ mm}^4$$

$$J_{XY} = -3 \times 10^9 \text{ mm}^4$$

Calculate  $I_{max}$ ,  $I_{min}$ ,

$$= 7.5 \times 10^9 = 6.9 \times 10^9 = -65.8^\circ$$

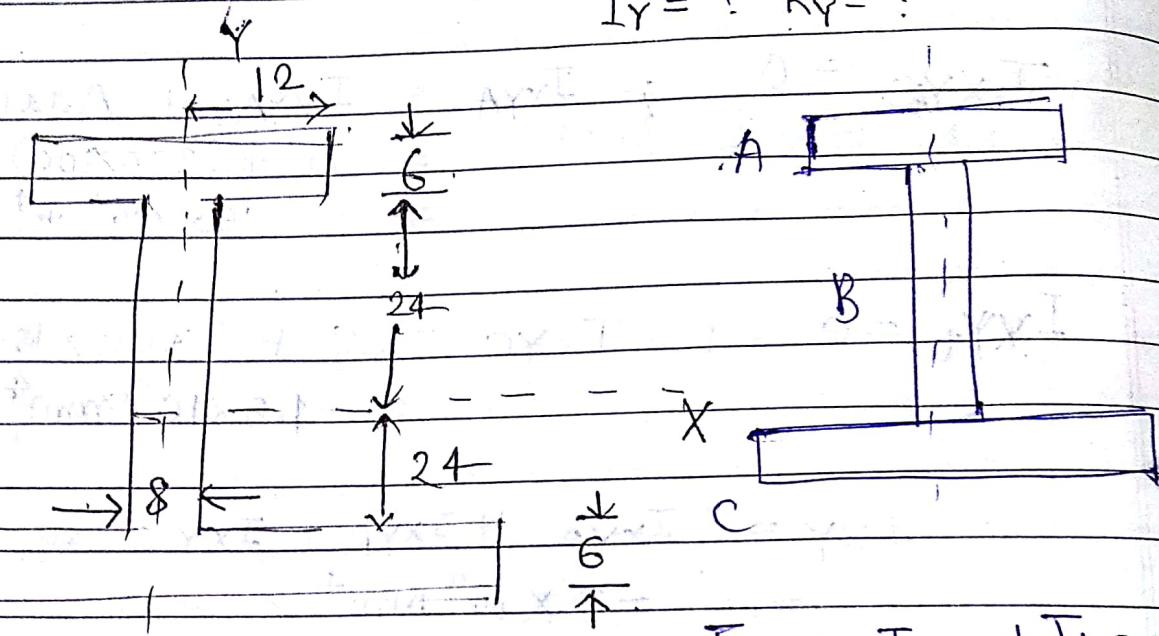


$$I_Y = \sum x^2 dA$$

$$= K_Y^2 A$$

$$K_Y = \sqrt{\frac{I_Y}{A}}$$

$$I_Y = ? \quad K_Y = ?$$



$$I_Y = I_{YA} + I_{YB} + I_{YC}$$

$$K_Y = \sqrt{\frac{I_Y}{A}}$$

$$A = A_A + A_B + A_C$$

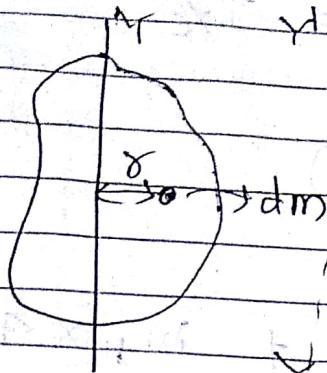
$$A = (6 \times 12) + (8 \times 24)$$

$$I_{YA} = \frac{1}{12} (6)(24)^3 + (6 \times 48)$$

$$I_{YB} = \frac{1}{12} (8)(8)^3$$

$$I_{YC} = \frac{1}{12} (6)(48)^3$$

**A MASS MOMENT of Inertia**



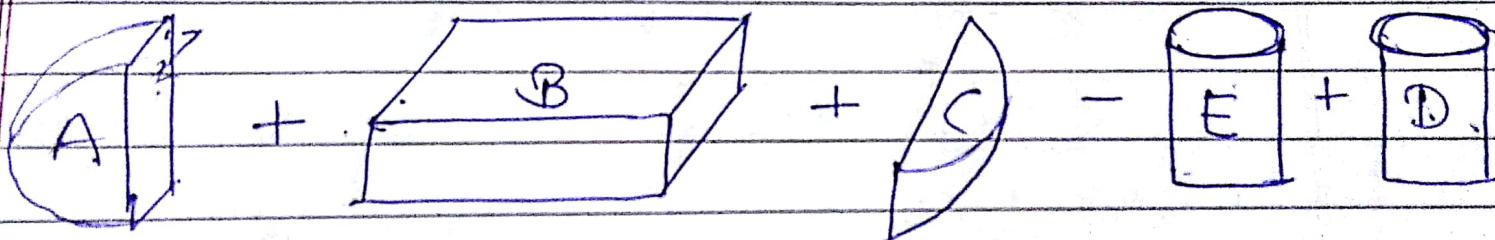
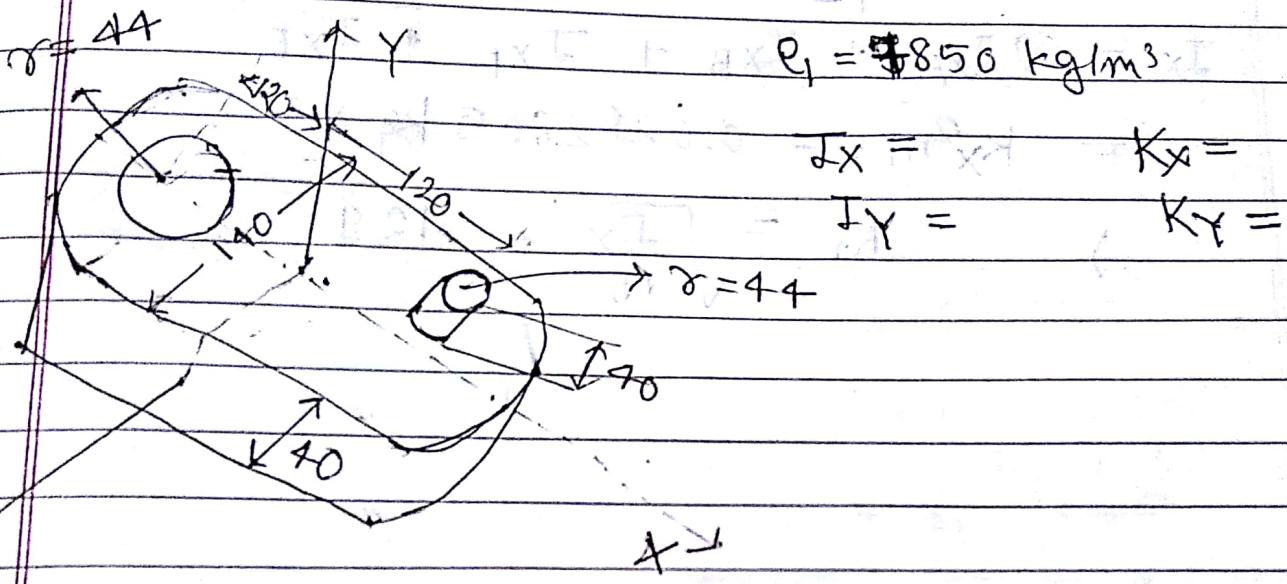
$$I_y = \sum r^2 dm$$

$$= \sum r^2 \rho dV$$

$$= \rho \sum r^2 dV$$

$$I_y = I_y = d^2 m$$

$$I = k^2 m$$



$$m_A = m_C ; \quad m_E = m_D ; \quad m_B .$$

$$m_A = m_C = \frac{\pi}{2} (40)^2 \times 20$$

$$m_E = m_D = \pi (44)^2 \times 40 \times 8$$

$$m_B = (40 \times 140 \times 240) 8$$

$$I_x = ?$$

$$I_{xA} = \frac{m_A}{12} (3r^2 + h^2) = I_x c$$

$$I_{xB} = \frac{m_B}{12} (b^2 + c^2)$$

$$I_{xD} = \frac{m_d}{12} (3r^2 + h^2) + m_p \times 40$$

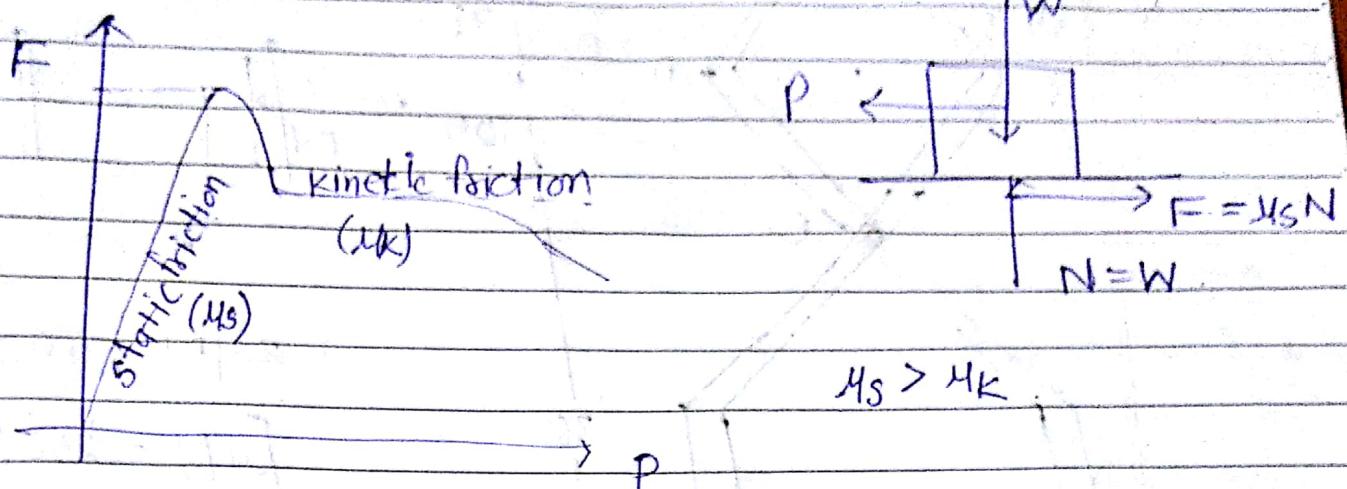
$$I_x = 2I_{xA} + I_{xB} + I_{xD} + I_{xE}$$

$$= k_x^2 m = 0.0282605 \text{ kg} \cdot \text{m}^2$$

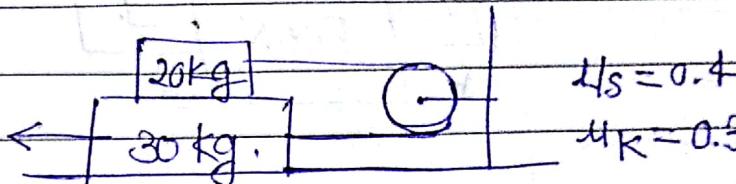
$$\Rightarrow k_x = \sqrt{\frac{I_x}{m}} = 42.9$$

# FRICITION

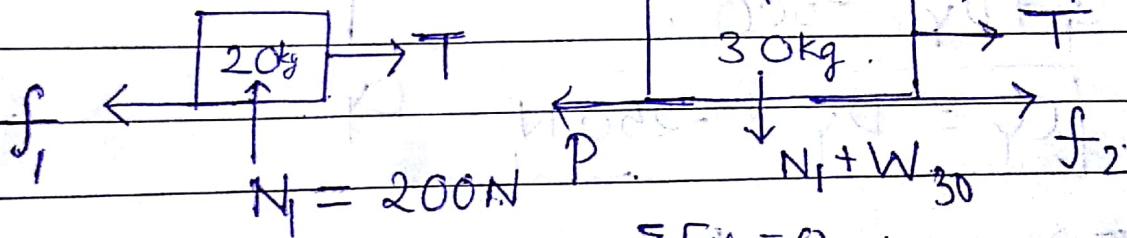
Date: \_\_\_\_\_  
Page No. \_\_\_\_\_



\* Find the minimum force  $P$  applied on a  $30\text{ kg}$  mass to just move it.



Given,  $g = 10 \text{ m/s}^2$



$$\sum F_x = 0$$

$$\sum f_x = 0 \quad f_1 = T \quad f_1 + T + f_2 = P$$

$$f_1 = \mu_s N_1$$

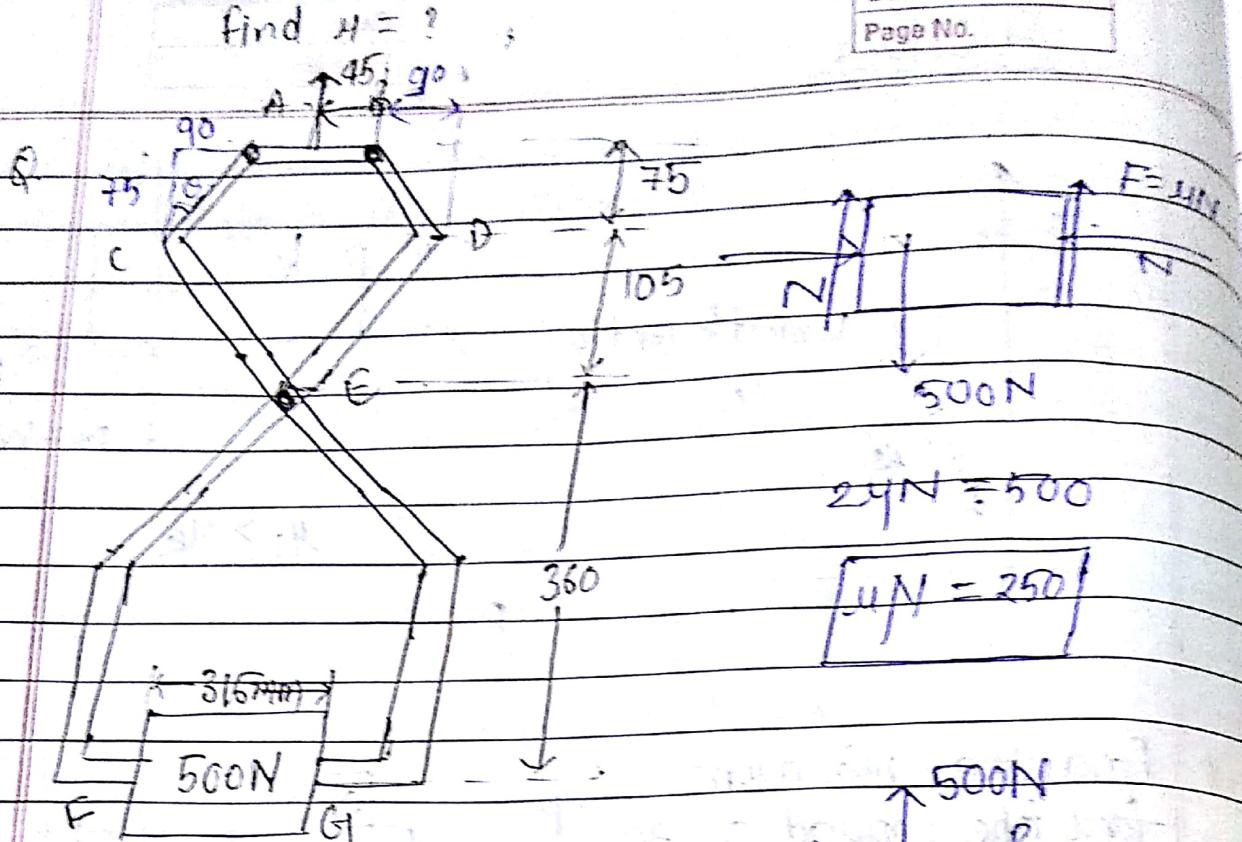
$$= (0.4)(200)$$

$$f_1 = 80 \text{ N}$$

$$T = 80 \text{ N}$$

$$\Rightarrow 80 + 80 + (0.4)(500) = P$$

$$P = 360 \text{ N}$$



$$24N = 500$$

$$\boxed{4N = 250}$$

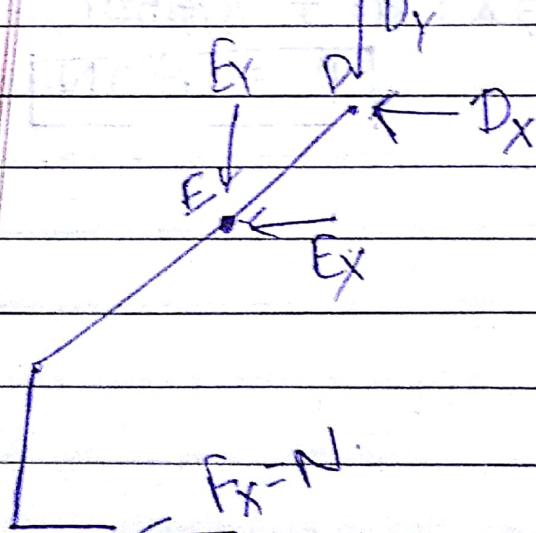
$$\sum F_y = 0$$

$$C_y + D_y = -500$$

$$\boxed{C_y = D_y = -250N}$$

$$\sum F_x = 0 \rightarrow \boxed{C_x = D_x} \quad \text{As } C_x = -300N$$

$$\boxed{D_x = -300N}$$



$$\boxed{F_x = 250}$$

$$-(D_y) \times (135) + (D_x) \times (105)$$

$$+ -(360 \times N) + \left(\frac{315}{2} \times 250\right)$$

$$-(50 \times 135) + (-300 \times 105)$$

$$-(360 \times N) + (315 \times 135) = 0$$

$$\tan \theta = \frac{49.6}{49.5} \quad \text{Ans}$$

P.F.C.

$$F_{xP} + C_x = 0 \quad C_x + 645.69 = 0$$

$$F_{yP} = -C_y \quad C_y + -\frac{645.69}{\cos \theta} = 0$$

$$C_y = C_y \tan \theta$$

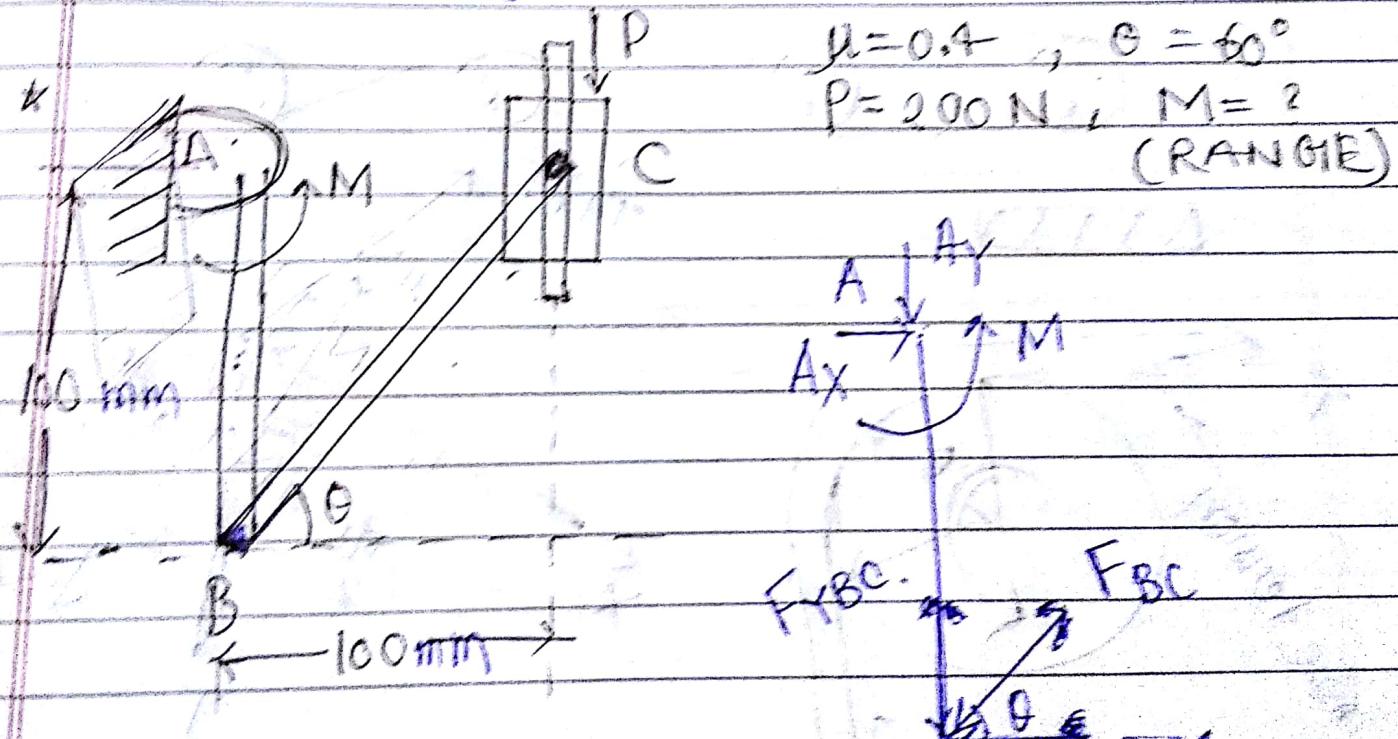
$$= (-250) \left(\frac{6}{5}\right)$$

$$260\%N = (315 \times 125) - (250 \times P_c) \quad C_x = -300 \text{ N}$$

$$= (300 \times 105) -$$

$$N = \frac{(315 \times 125) - (250 \times 135) - (300 \times 105)}{360}$$

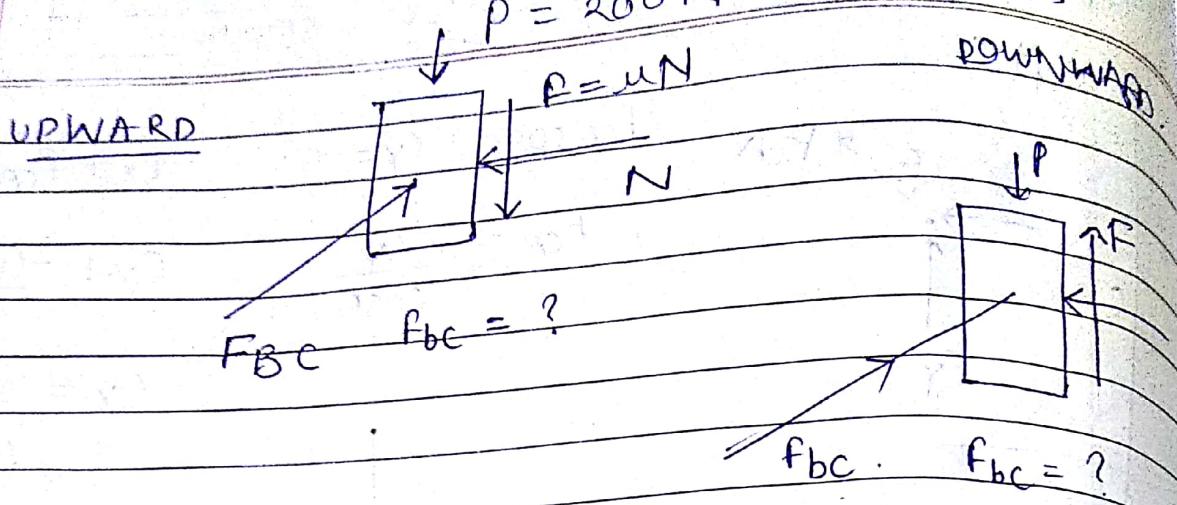
$$= -71.875$$



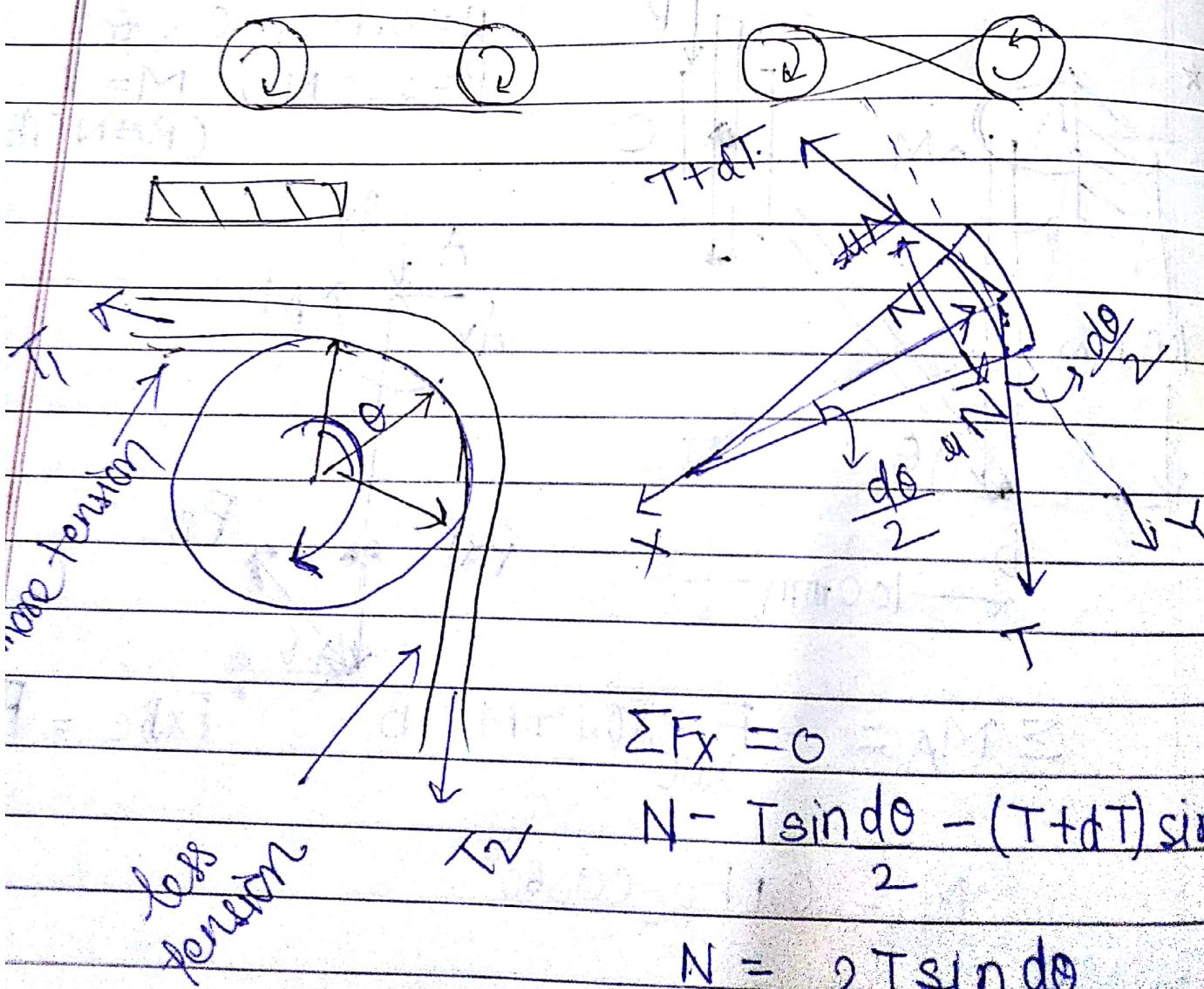
$$\sum MA = -F_{BC} \times 0.1 + M \quad B$$

$$F_{BC} = F_{BC} \cos 60^\circ$$

$$M = 0.1 F_{BC} \cos 60^\circ$$



\* Flat Belt :-  $P = T \cdot l$



$$\Rightarrow N = \frac{XT.d\theta}{2} \Rightarrow N = Td\theta \quad \text{--- (1)}$$

$$\sum F_y = 0 \Rightarrow$$

$$T \cos \frac{d\theta}{2} + uN - (T + dT) \cos \frac{d\theta}{2} = 0.$$

$$[uN = dT] \quad \text{--- (2)}$$

$$uT d\theta = dT.$$

$$\Rightarrow \frac{T_2 f dT}{T_1} = \int_0^\theta u d\theta.$$

$$\rightarrow \frac{\ln T_2}{T_1} = u\theta.$$

$$[T_2 = T_1 e^{u\theta}].$$

$$\Rightarrow N = 2T \frac{d\theta}{2} \Rightarrow N = T d\theta \quad \text{--- (1)}$$

$$\sum F_y = 0 \Rightarrow$$

$$T \cos \frac{d\theta}{2} + uN - (T + dF) \cos \frac{d\theta}{2} = 0.$$

$$uN = dT \quad \text{--- (2)}$$

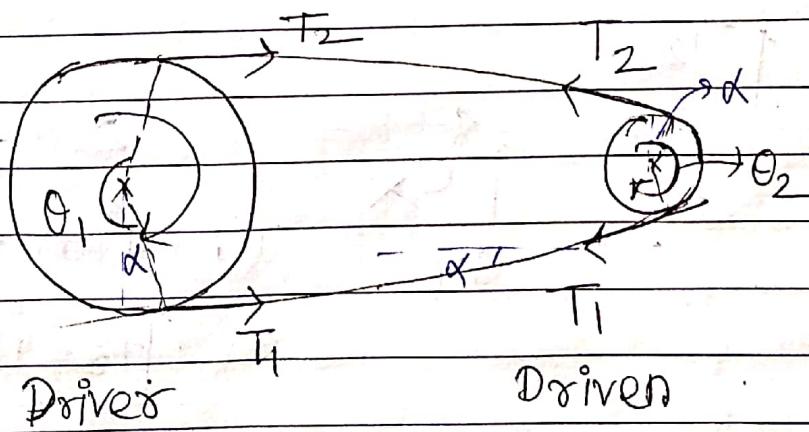
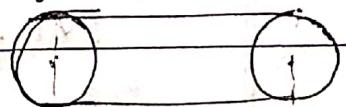
$$uT d\theta = dT.$$

$$\Rightarrow \int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\theta d\theta.$$

$$\rightarrow \frac{\ln T_2}{T_1} = u\theta.$$

$$T_2 = T_1 e^{u\theta}.$$

$$\theta = 180^\circ \quad \theta = 180^\circ$$



$$T_1 > T_2$$

$$\theta_1 = \pi + 2\alpha$$

$$\theta_2 = \pi - 2\alpha.$$

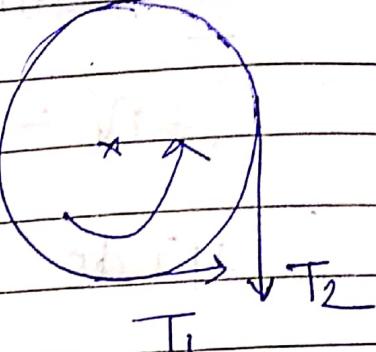
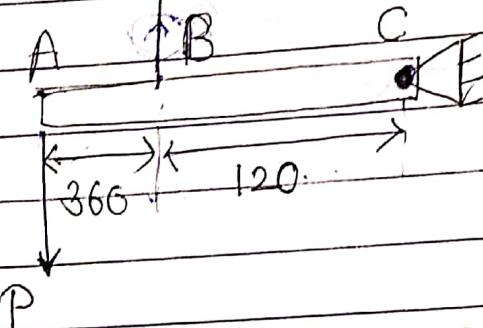
~~DEALER IDLER PULLEY.~~  $T_2 = T_1 e^{u\theta}$   
 $\theta = \text{Angle of Bite.}$

$T = M = -T_2 \delta + T_1 \gamma = (T_1 - T_2) \gamma.$   
 $\uparrow$  Torque.

$$P = TW.$$

$$\theta = 270^\circ$$

$$\mu_s = 0.3 ; \mu_k = 0.25 \\ P = 45 N ; M = ?$$



$$T_2 > T_1$$

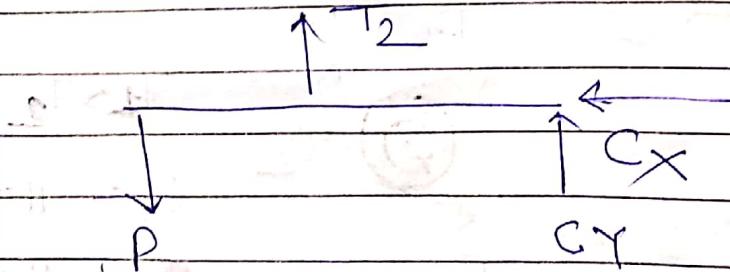
$$\Rightarrow T_2 = T_1 e^{\mu \theta}$$

$$T_2 = T_1 e^{\mu 3\pi/2}$$

$$M = (T_2 - T_1) r$$

$$= (T_1 e^{\mu 3\pi/2} - T_1) r$$

$$M = \frac{T_1 (e^{\mu 3\pi/2} - 1)}{r} r$$



$$\sum M_C = 0$$

$$(P \times 480) = T_2 \times 120$$

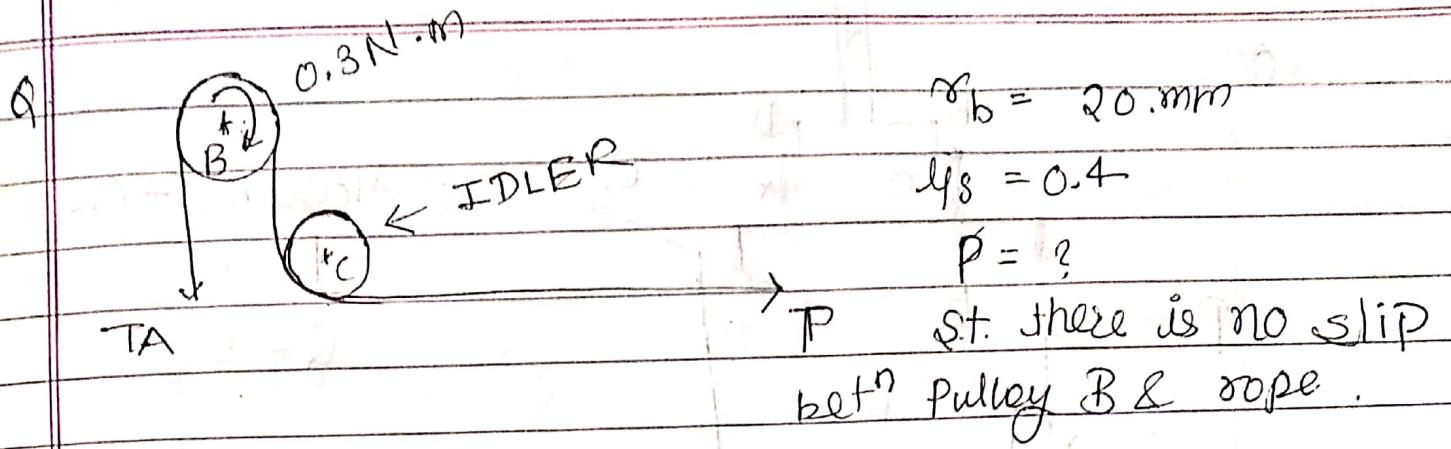
$$T_2 = 180 N$$

$$M =$$

$$\boxed{\mu = \mu_k}$$

$$\frac{T_2}{T_1} = e^{\mu \theta}$$

$$; M = \gamma (T_2 - T_1)$$



$\Rightarrow$

Free body diagram of pulley B shows tension  $T_A$  and force  $P$ .

$$T_A = P e^{4\pi} \quad M = \gamma (T_A - P)$$

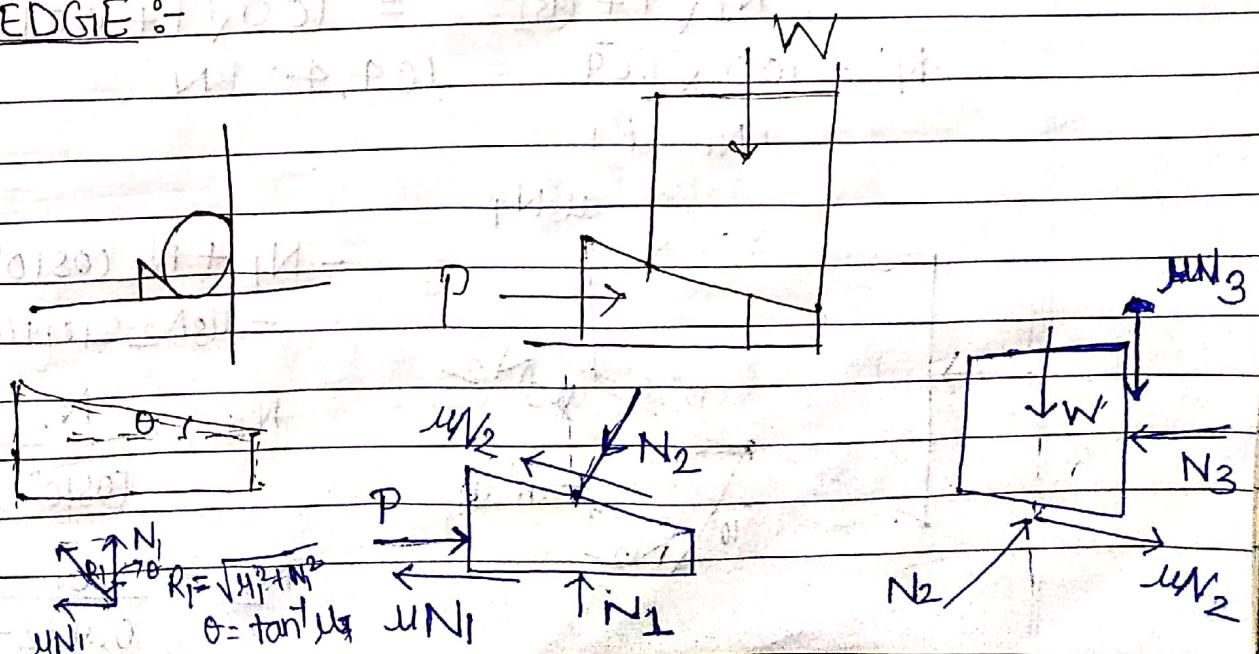
$$P = \frac{T_A}{e^{4\pi}} \quad M = \gamma (P) (e^{4\pi} - 1)$$

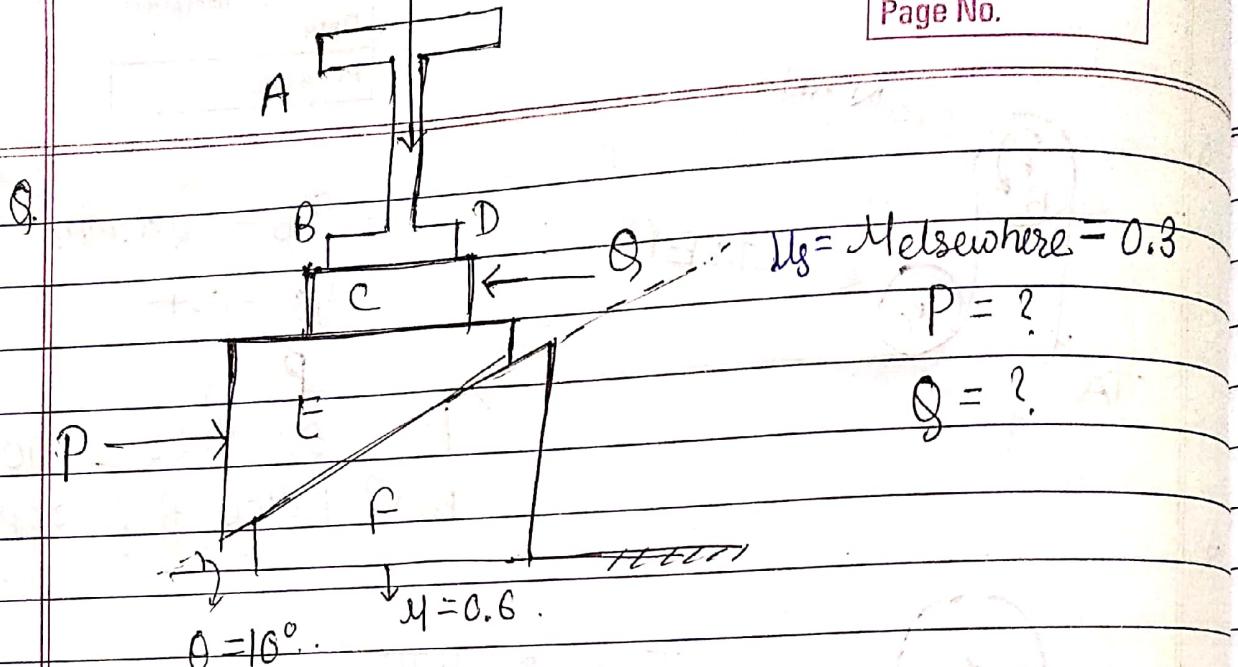
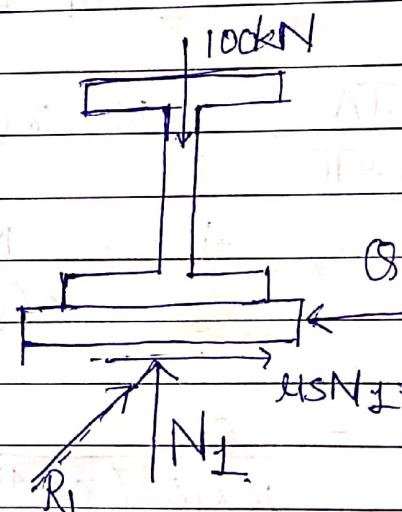
$$\Rightarrow P = \frac{M}{\gamma (e^{4\pi} - 1)}$$

$$T_A > P.$$

$$\Rightarrow P = \frac{0.3}{20 \times 10^{-3} (e^{0.4\pi} - 1)} = 15 = \frac{15}{2.5135} = 5.967 \text{ N.}$$

\* WEDGE :-



Ans =

$$N_1 = 100 \text{ kN}$$

$$Q = \mu_s N_1$$

$$= (0.3)(100) \text{ kN}$$

$$\sqrt{Q} = 30 \text{ kN}$$

~~$R_1 = 100$~~

~~$\cos(\tan^{-1}\mu)$~~

$$R_1 = 100 \sqrt{1.09} = 104.40 \text{ kN}$$

$$N_1 \cdot R_1$$

$$\downarrow \quad \leftarrow \mu s N_1$$

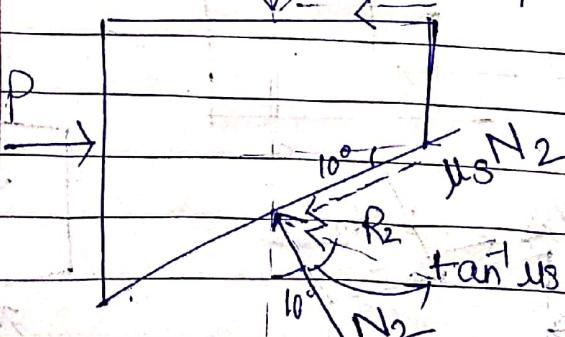
$$-N_1 + N_2 \cos 10^\circ$$

$$-\mu s N_2 \sin 10^\circ = 0$$

$$N_2 = \frac{N_1}{\cos 10^\circ - \mu s \sin 10^\circ}$$

$$= \frac{100}{\cos 10^\circ - 0.3 \sin 10^\circ} = 107.7 \text{ kN}$$

$$N_2 = \frac{100}{0.9327} = 107.7 \text{ kN}$$

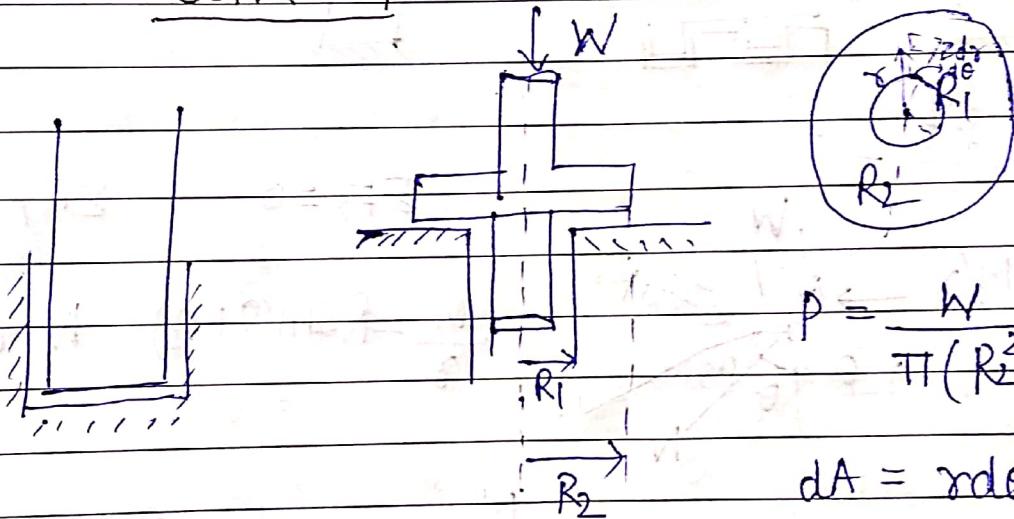


$$P - \mu_s N_1 - \mu_s N_2 \cos 10^\circ + N_2 \sin 10^\circ = 0$$

$$P = \mu_s (N_1 + N_2 \cos 10^\circ) - N_2 \sin 10^\circ$$

$$\checkmark P = (0.3)(205.58) - 18.61 \\ P = 43.064 \text{ KN}$$

### \* BEARING



$$P = \frac{W}{\pi(R_2^2 - R_1^2)}$$

$$dA = r d\theta dr \\ dN = P dA$$

$$\Rightarrow dF = \mu P dA$$

$$dF = \mu \frac{W}{\pi(R_2^2 - R_1^2)} r d\theta dr$$

$$\Rightarrow F = \int \mu P dA$$

$$dT = df \propto$$

$$= \int_{R_1}^{R_2} \int_0^{2\pi} \mu \frac{W}{\pi(R_2^2 - R_1^2)} r d\theta dr \quad T = \int_{R_1}^{R_2} \int_0^{2\pi} \mu \frac{W}{\pi(R_2^2 - R_1^2)} r^2 d\theta dr$$

$$= \frac{2\mu W}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} r dr$$

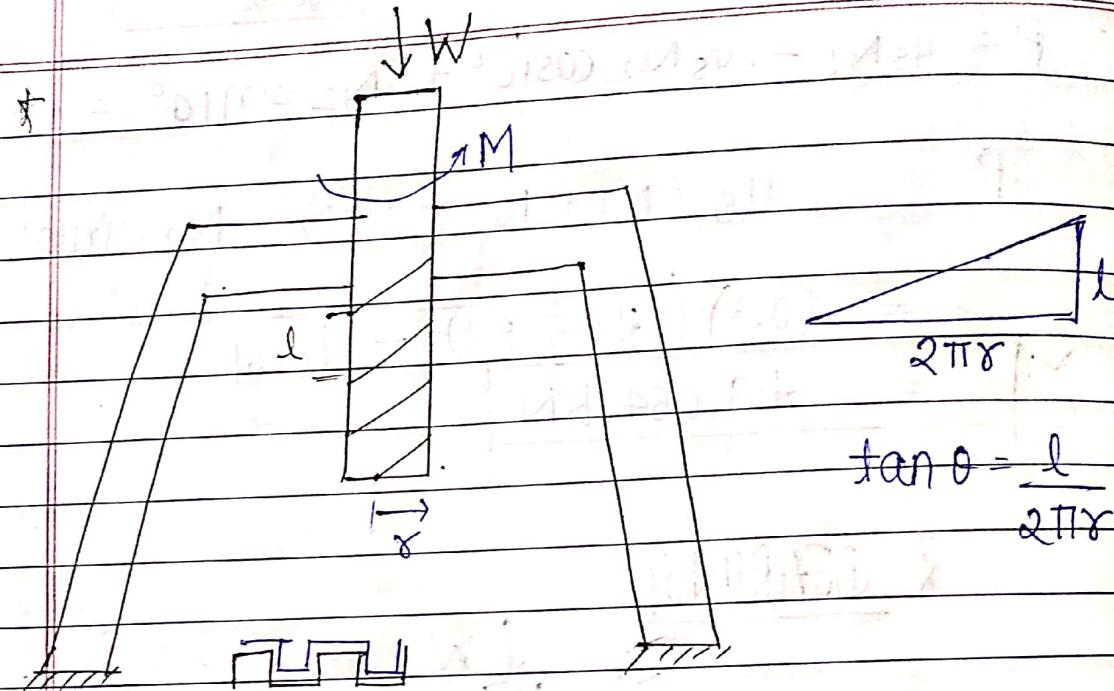
$$T = \frac{2\mu W}{(R_2^2 - R_1^2)} \left( \frac{R_2^3 - R_1^3}{3} \right)$$

$$= \frac{2\mu W}{(R_2^2 - R_1^2)} \left( \frac{R_2^2 - R_1^2}{2} \right) = \mu W$$

KN

# SCREW JACK

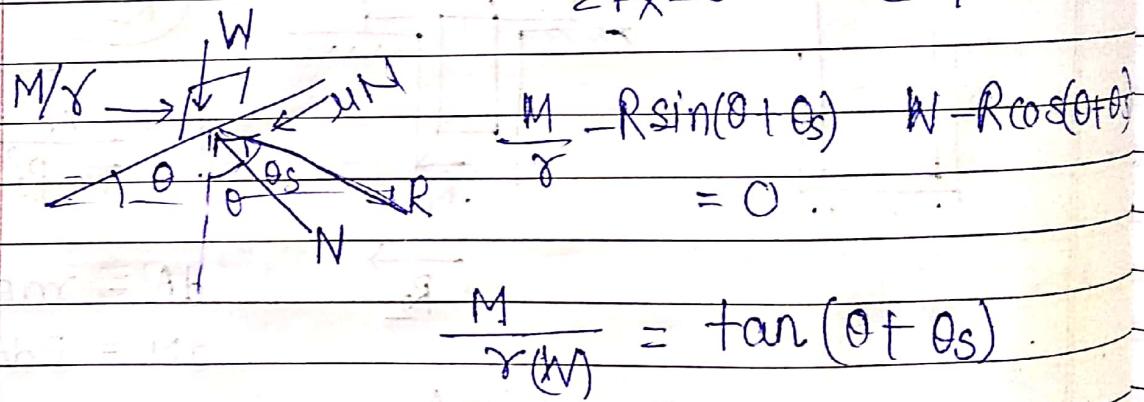
Date: \_\_\_\_\_  
Page No. \_\_\_\_\_



$$\tan \theta = \frac{l}{2\pi r}$$

Torque to lift up.

$$\sum F_x = 0 \quad \sum F_y = 0$$



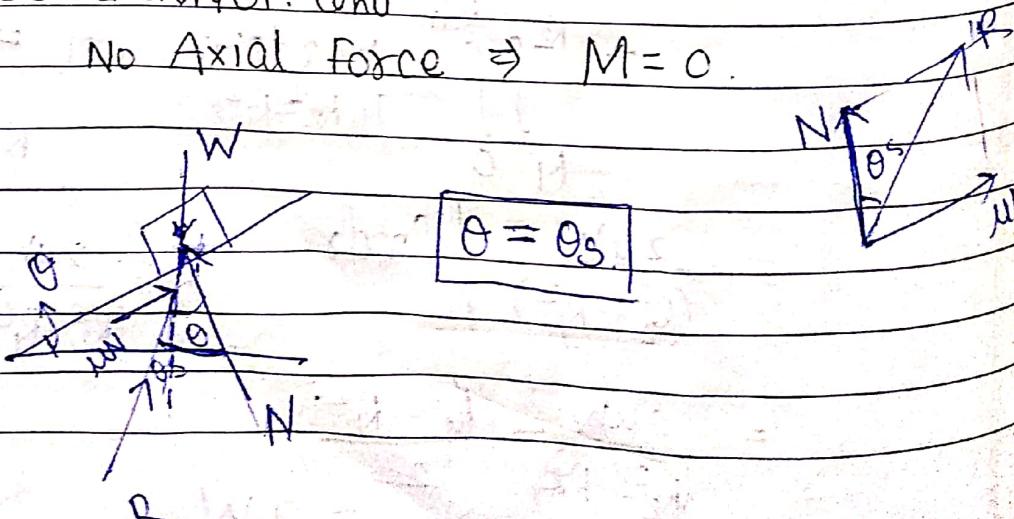
$$\frac{M}{r} = R \sin(\theta + \theta_s)$$

$$M = W r \tan(\theta + \theta_s) \quad M \propto W, r$$

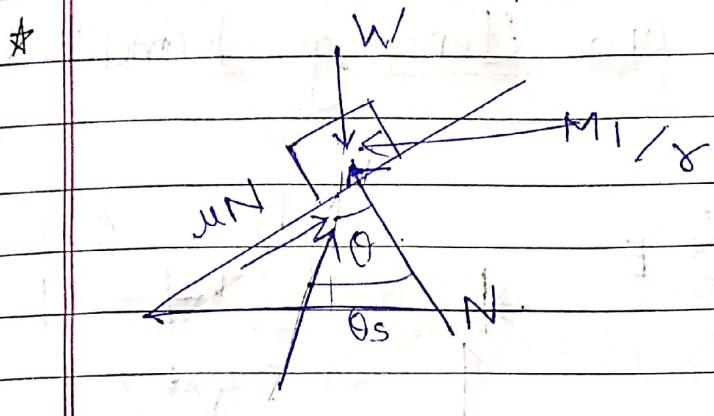
$$\tan \theta = \frac{l}{2\pi r} ; \tan \theta_s = \mu$$

\* SELFLOCKING COND.

No Axial Force  $\Rightarrow M = 0$ .



Minimum torque to bring down



$$M_1 = W\gamma \tan(\theta_s - \theta)$$

### ★ TURNBUCKLE

$$\gamma = 5 \text{ mm}$$

$$l = 2 \text{ mm}$$

$$\mu = 0.25$$

$$M = ?$$

$$\tan \theta = \frac{l}{2\pi\gamma}$$

$$= 2 \text{ mm}$$

$$2\pi(5) \text{ mm}$$

$$= \frac{1}{5\pi}$$

$$M = 2W\gamma \tan(\theta + \theta_s) = \frac{1}{4}$$

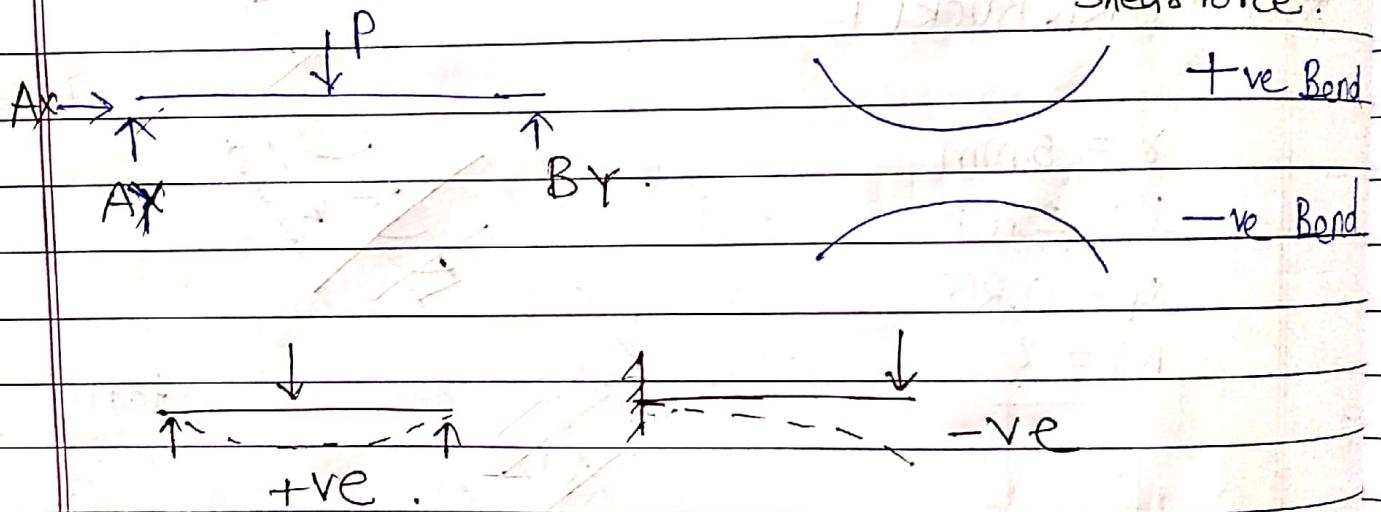
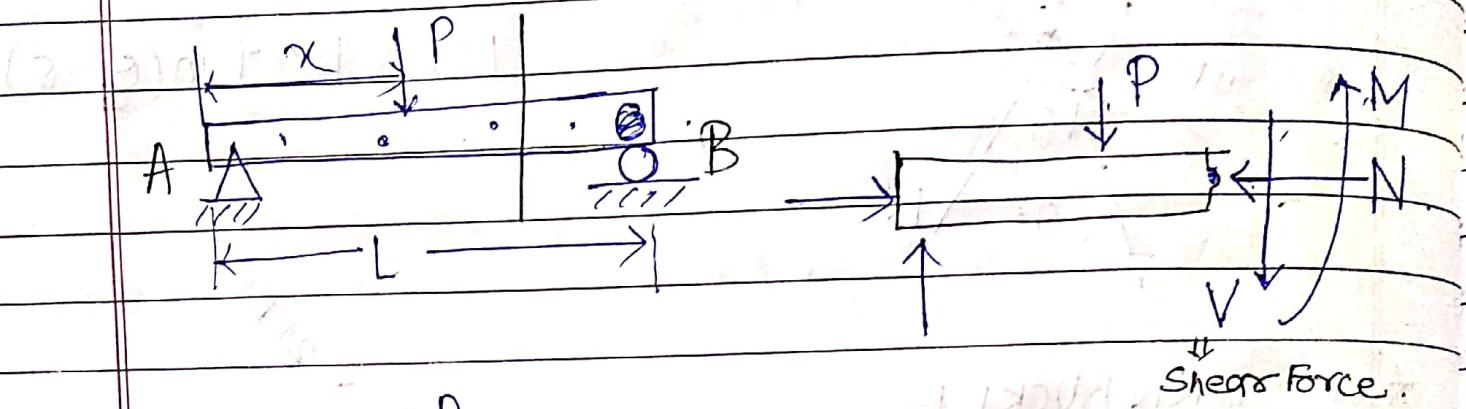
$$= 2 \times (2 \text{ kN}) (5 \text{ mm}) \frac{1}{5\pi} + \frac{1}{4}$$

$$= 20 \times \left( \frac{4\pi + 5\pi}{20\pi - 1} \right)$$

⇒

\*

# Shearing Force and Bending Moment:



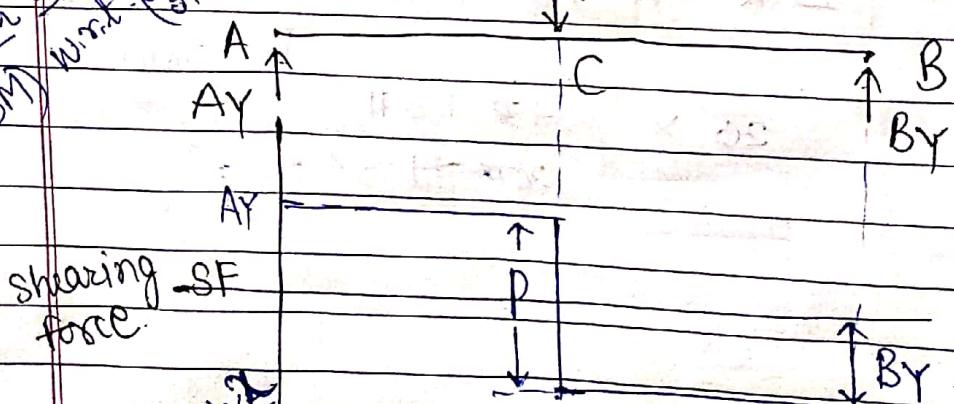
$$\sum F_x = 0 \quad \sum F_y = 0$$

$$A_x = 0 \quad A_y + B_y = P$$

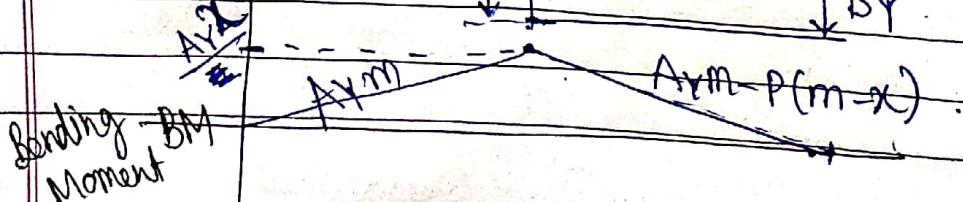
$$\sum M_B = 0 \quad A_y L = P(L-x)$$

$$A_y = \frac{P(L-x)}{L}$$

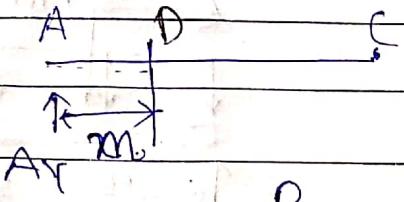
\* Graph of variation of SF or BM w.r.t. length of AB



Shearing SF force



AC



$$\sum M_D = Aym$$

Ay

P

CB

$\leftarrow x \rightarrow$

$\uparrow +$

$m$

B

$\sum ME$

$$= Aym - P(m-x)$$

\* Bending Moment at the end is equal to zero.

\* Shearing Force at the end is equal to normal reaction at that point

(if no other moments and forces are acting at that point).

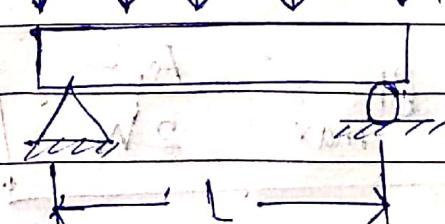
\* When  $SF = 0$ ;  $BM = \text{max}$ .

Distributed load.

$\downarrow \downarrow \downarrow \downarrow$   $W/m$

Total Load

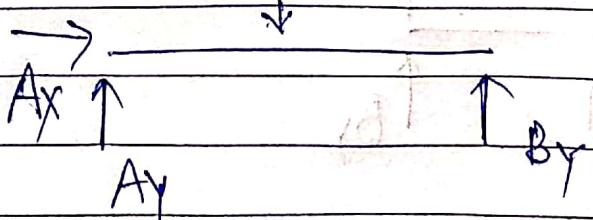
$= WL$



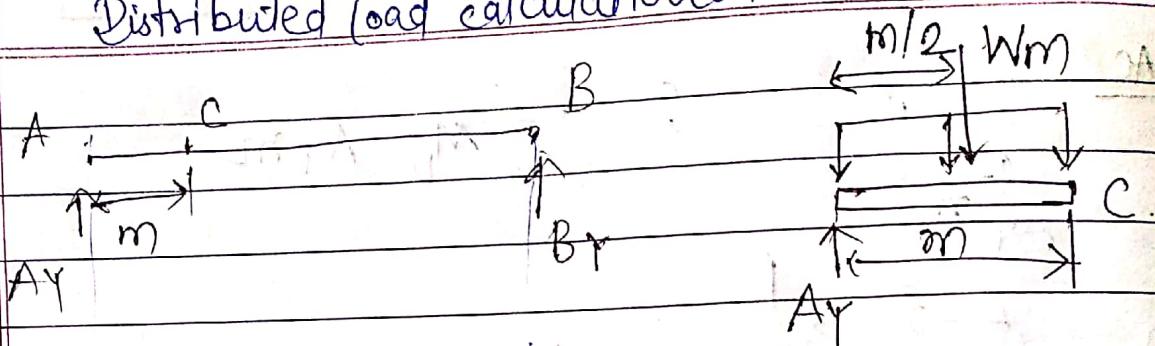
$\Downarrow$  can be considered

and solved like

the previous one.



## Distributed load calculations

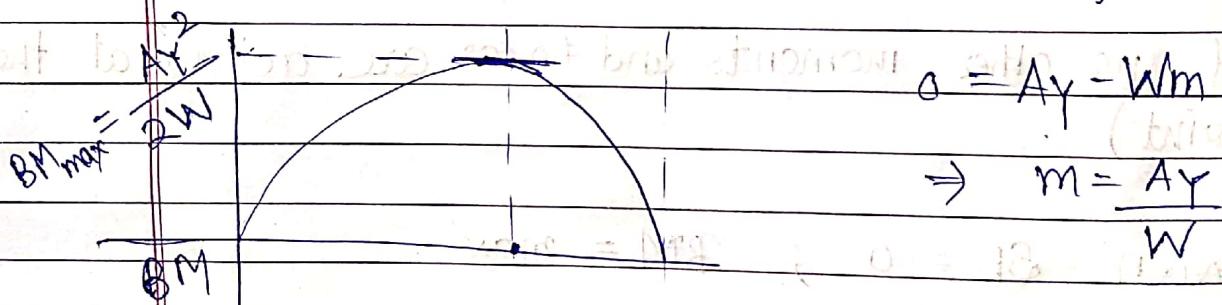


$$SF = Ay - Wm$$

$$BM = Ay \cdot m - \frac{(Wm)m}{2}$$

$$BM = Ay \cdot m - \frac{Wm^2}{2}$$

$SF = 0$  for max BM

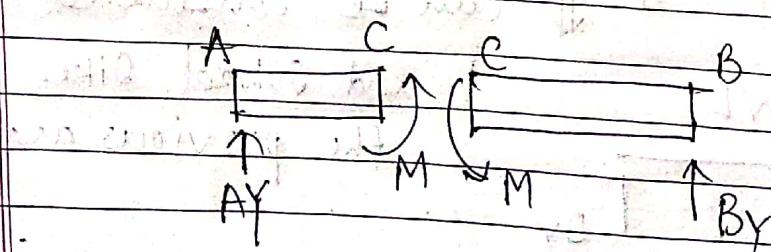


$$0 = Ay - Wm$$

$$\Rightarrow m = \frac{Ay}{W}$$

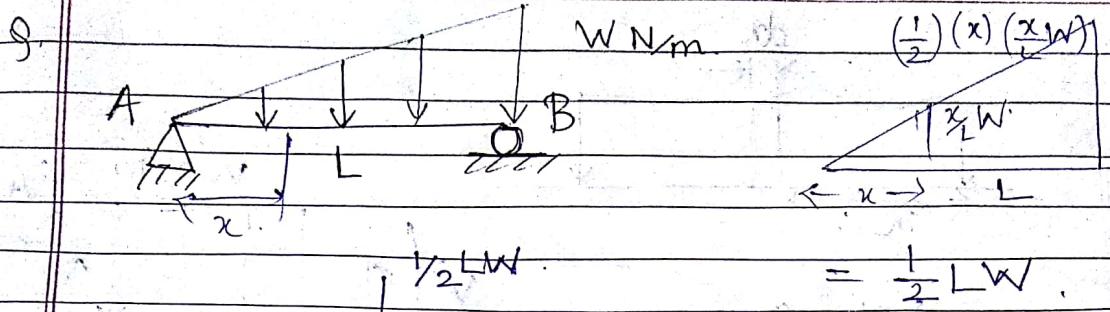
$$BM = Ay \left( \frac{Ay}{W} \right) - W \left( \frac{Ay^2}{W^2} \right) \frac{1}{2}$$

$$BM_{\max} = \frac{Ay^2}{2W}$$



# Uniformly Varying Distributed Load

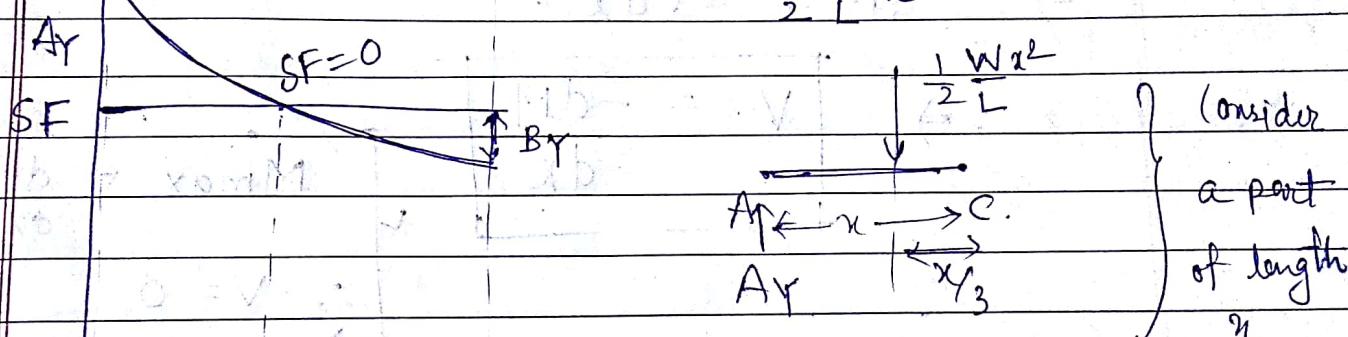
Date: \_\_\_\_\_  
Page No. \_\_\_\_\_



Area of distributed load will be total amount of load

∴ load at  $x$  from A

$$SF = \frac{1}{2} \frac{W}{L} x^2$$



$$\sum M_C = A_y x - \left( \frac{1}{2} \frac{W}{L} x^2 \right) \left( \frac{x}{3} \right)$$

$$BM_x = A_y x - \frac{1}{6} \frac{W}{L} x^3$$

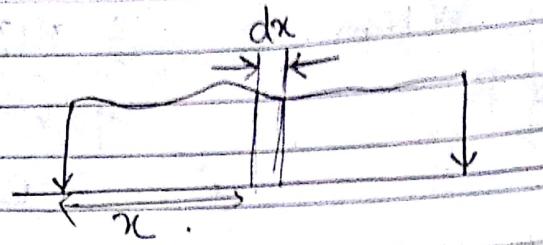
$$SF \text{ at } x = A_y - \frac{W x^2}{2L}$$

$$\text{for max: } 0 = A_y - \frac{W x^2}{2L}$$

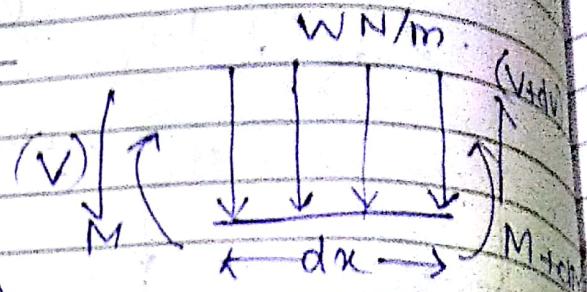
$$BM; \Rightarrow x^2 = \frac{2LAy}{W}$$

$$\Rightarrow x = \sqrt{\frac{2LAy}{W}}$$

8.

Ans

$$\sum M = 0$$



$$\Rightarrow -M + (M+dm) + dx(V+dV) + \frac{wdx \cdot dx}{2} = 0$$

$$\Rightarrow dM + dxV + dx(dV) = 0$$

$$+ \frac{dVdm}{2} \approx 0$$

$$\Rightarrow dM = -Vdx$$

$$V + wdx - (V+dV) = 0$$

$$\Rightarrow W = \frac{dV}{dx}$$

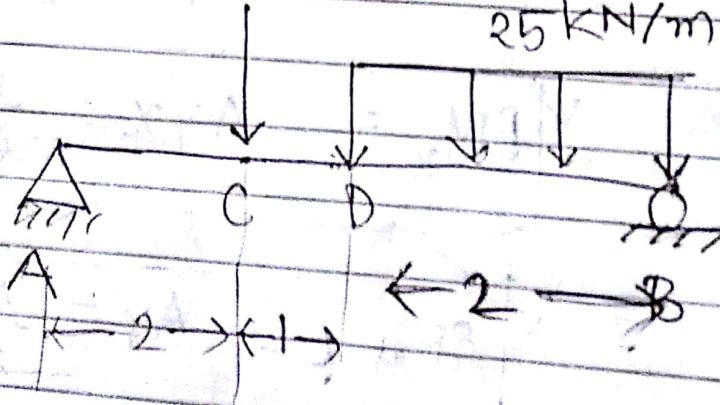
$$\Rightarrow V = -\frac{dM}{dx}$$

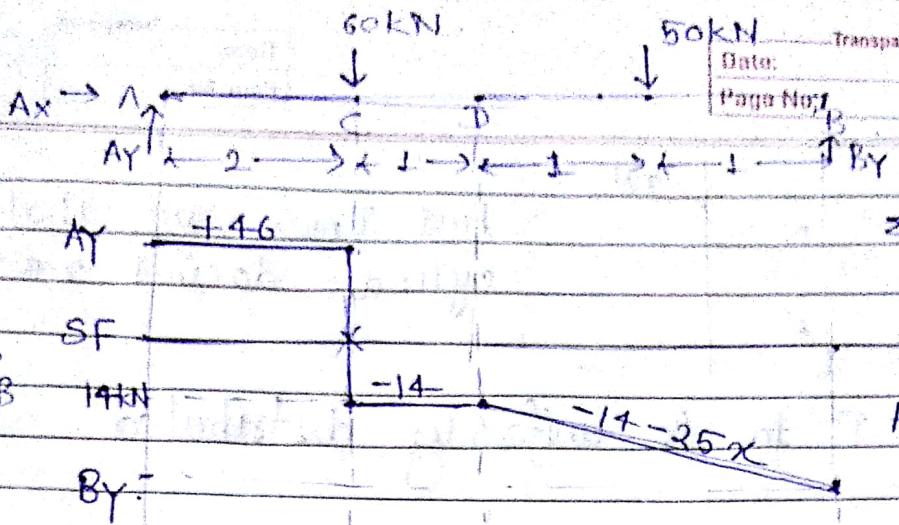
$$\begin{aligned} & M_{max} \Leftrightarrow \frac{dM}{dx} = 0 \\ & \therefore V = 0 \end{aligned}$$

9.

60 kN

25 kN/m





$$\sum F_x = 0$$

$$Ax = 0$$

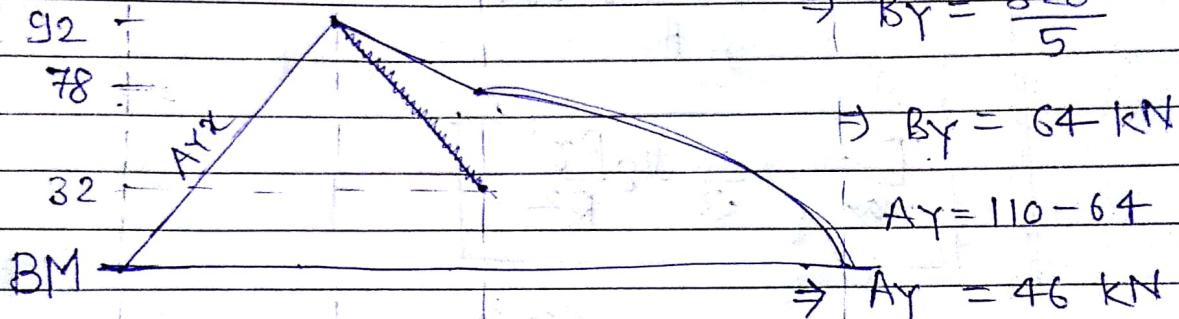
$$\sum F_y = 0$$

$$AY + BY = 110$$

$$\sum M_A = 0$$

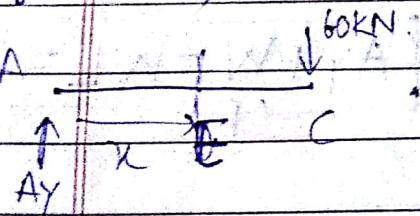
$$(60)(2) + 4(50) \\ = BY(5) \\ \Rightarrow BY = \frac{320}{5}$$

$$\max(AYx) = 92 + \\ 78 + \\ 32$$



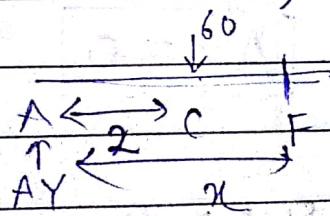
### BIM calculations.

For AC,



$$M_E = AYx$$

for CD,

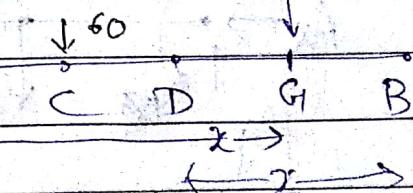


$$M_F = AYx - 60(x-2)$$

$$= 120 - 14x$$

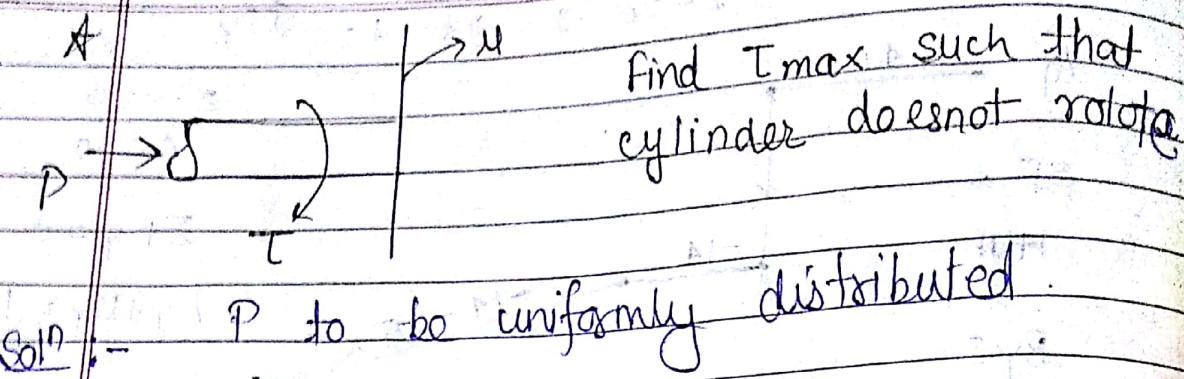
$$= 78$$

for DB.  $25(kN^{-3})$



$$MG_1 = AYx - 60(x-2) \\ = BY(5-x)$$

$$- 25(x-3) \left( \frac{x-3}{2} \right)$$



Soln:-  $P$  to be uniformly distributed.

$$dT_{\max} = \mu_s \frac{2Pr}{R^2} dr$$



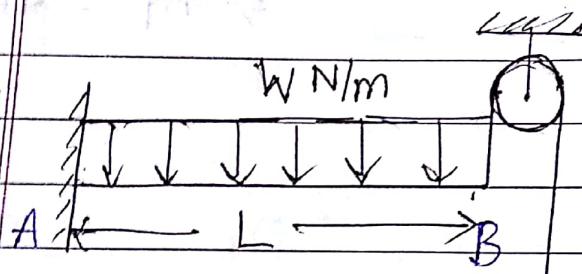
$$dN = \left( \frac{P}{2\pi R^2} \right) 2\pi r dr$$

$$dT = \mu_s \frac{2Pr}{R^2} : r dr$$

$$= \frac{2Prdr}{R^2}$$

$$T_{\max} = \frac{2}{3} \mu_s \frac{Pr^3}{R^2}$$

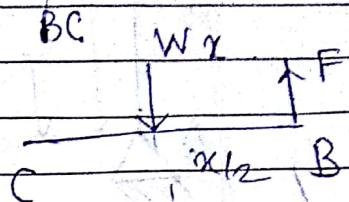
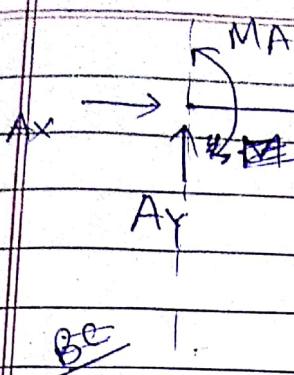
Q. Find SF, BM.



①  $M_{\max} = ?$

②  $w$  is removed

$$\begin{aligned} & \Rightarrow M_A \\ & Ax \rightarrow \cdot \quad Ay \rightarrow \cdot \\ & C \quad B \quad D \\ & \uparrow W = F \quad \uparrow WL \\ & \sum F_x = 0 \quad \sum F_y = 0 \\ & Ax = 0 \quad Ay + W - WL = 0 \\ & \quad (F) \\ & \leftarrow \frac{L}{2} \rightarrow \quad \sum F_n = 0 \\ & \Rightarrow +M_A - WL \left( \frac{L}{2} \right) + L(W) = 0 \\ & \quad (F) \\ & \boxed{M_A = -WL + \frac{WL^2}{2}} = \underline{\underline{WL^2}} - FL \end{aligned}$$



$$M_C = F_x \cdot x - Wx \cdot \frac{x}{2}$$

$$(BM) = F_x \cdot x - \frac{Wx^2}{2}$$

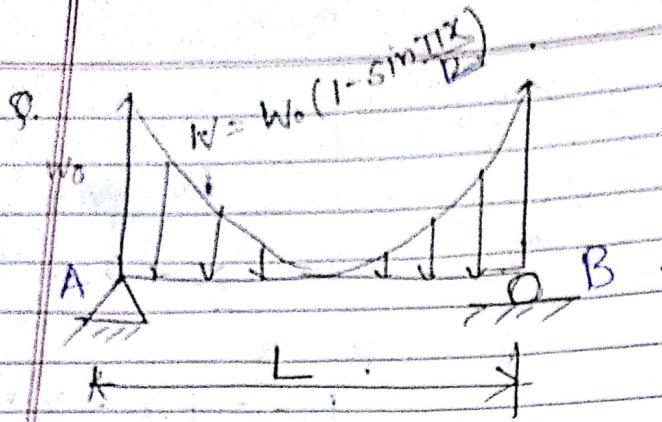
$$(BM)_A = FL \cdot \frac{WL^2}{2}$$

$$\frac{d(BM)}{dx} = 0$$

$$F - Wx = 0$$

$$x = F/W$$

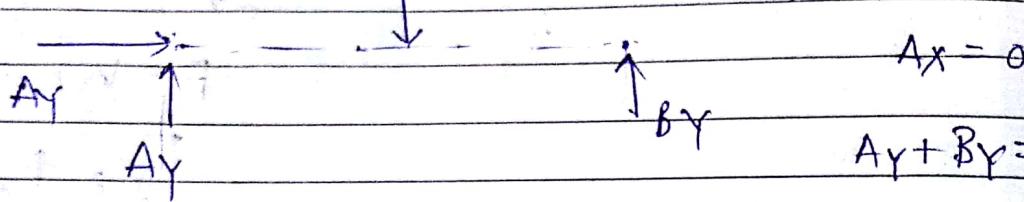
$$(F - Wx) \cdot x = Fx - Wx^2 = Fx - \frac{Fx^2}{2}$$



$$F = \int_0^L w dx = \int_0^L w_0 \left(1 - \sin \frac{\pi x}{L}\right) dx$$

$$= w_0 \left(x + \frac{\cos \frac{\pi x}{L}}{\frac{\pi}{L}}\right) \Big|_0^L = w_0 L \left(1 - \frac{1}{\pi}\right)$$

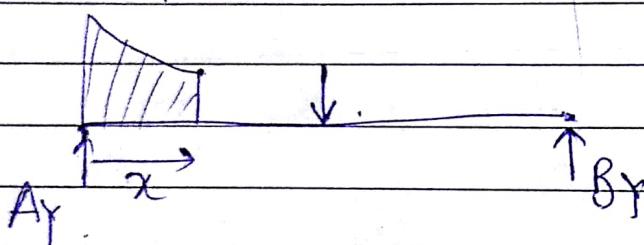
$$w_0 L \left(1 - \frac{1}{\pi}\right)$$



$$Ax = 0$$

$$Ay + By = w_0 L \left(1 - \frac{1}{\pi}\right)$$

$$\Rightarrow Ay = By = \frac{w_0 L}{2} \left(1 - \frac{1}{\pi}\right)$$



$$V = SF = Ay - \int w dx$$

$$= \frac{w_0 L}{2} \left(1 - \frac{x}{\pi}\right) - w_0 \left(x + \frac{\cos \frac{\pi x}{L}}{\frac{\pi}{L}}\right) \Big|_0^L$$

$$V = SF = w_0 \left( \frac{L}{2} - x - \frac{1}{\pi} - \frac{1}{\pi} \cos \frac{\pi x}{L} \right)$$

$$V = SF = w_0 \left( \frac{L}{2} - x - \frac{1}{\pi} \cos \frac{\pi x}{L} \right)$$

$$\Rightarrow M = \cancel{\int V dx} .$$

$$\Rightarrow M = \int W_0 \left( \frac{L}{2} - x - \frac{L}{\pi} \frac{2 \cos \pi x}{2} + \frac{2}{\pi} \right) dx$$

$$= \frac{1}{2} W_0 \left( \frac{Lx}{2} - x^2 - \frac{2Lx}{\pi} \right)$$

$$\frac{dM}{dx} = V \Rightarrow dM = V dx$$

$$\rightarrow M = \int V dx$$

$$= \int W_0 \left( \frac{L}{2} - x - \frac{L \cos \pi x}{\pi} \right) dx$$

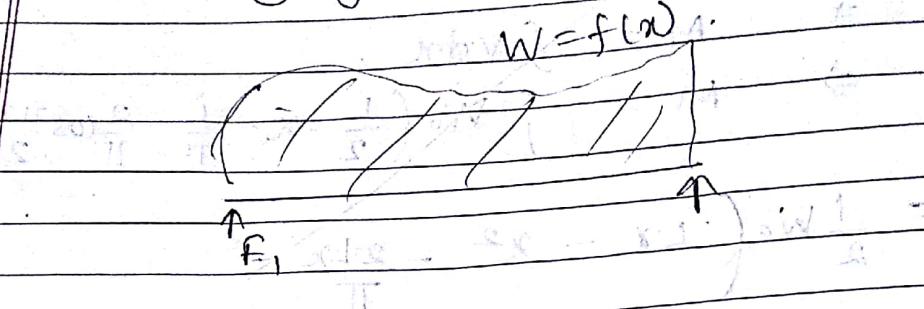
$$M = W_0 \left( \frac{Lx}{2} - \frac{x^2}{2} - \frac{L^2}{\pi^2} \frac{\sin \pi x}{L} \right)$$

$$\text{For } M_{\max}; \quad \frac{dM}{dx} = 0; \quad V = 0.$$

$$\Rightarrow W_0 \left( \frac{L}{2} - x - \frac{L \cos \pi x}{\pi L} \right) = 0$$

$$\Rightarrow \frac{L}{2} = x + \frac{L \cos \pi x}{2}$$

\* For any general power

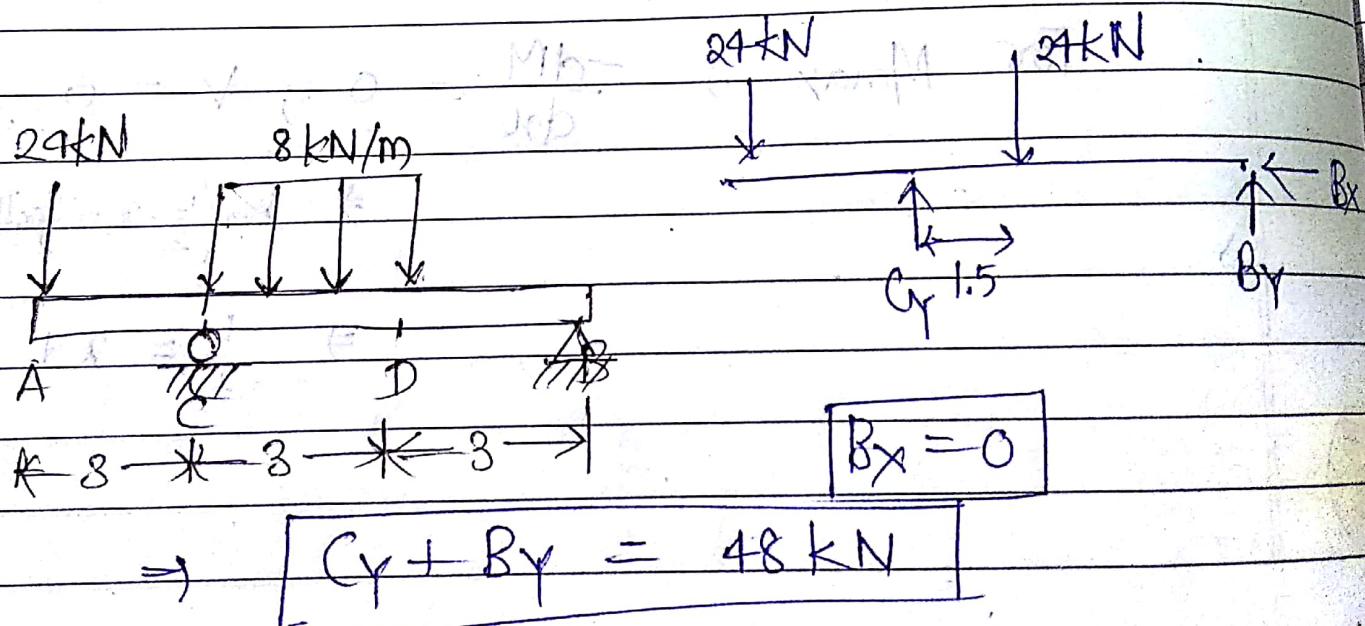


$$F_1 = \int W dx$$

$$V = F_1 - \int W dx ; \quad \frac{dM}{dx} = V$$

$$\Rightarrow dM = V dx \Rightarrow M = \int V dx$$

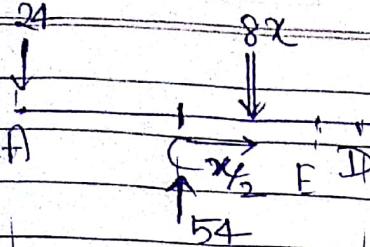
$$M_{\max} = \frac{dM}{dx} = 0 \Rightarrow V = 0 \text{ (corresponding SF)}$$



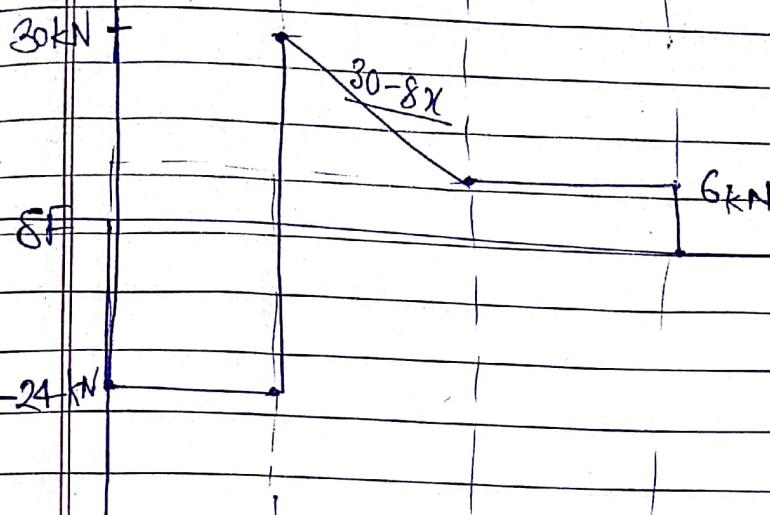
$$\Rightarrow [C_y + B_y = 48 \text{ kN}]$$

$$\sum M_B = 0 \Rightarrow \left( \frac{24 \times 9}{2} \right) + (24 \times 9) = C_y \times 8$$

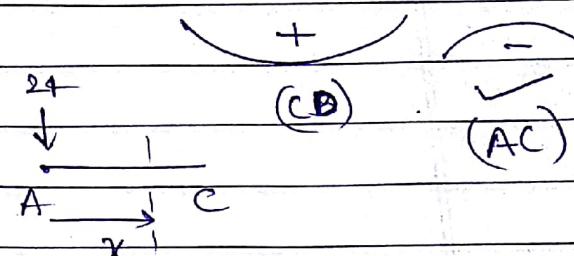
$$\Rightarrow [C_y = 54 \text{ kN}] \quad [B_y = 6 \text{ kN}]$$



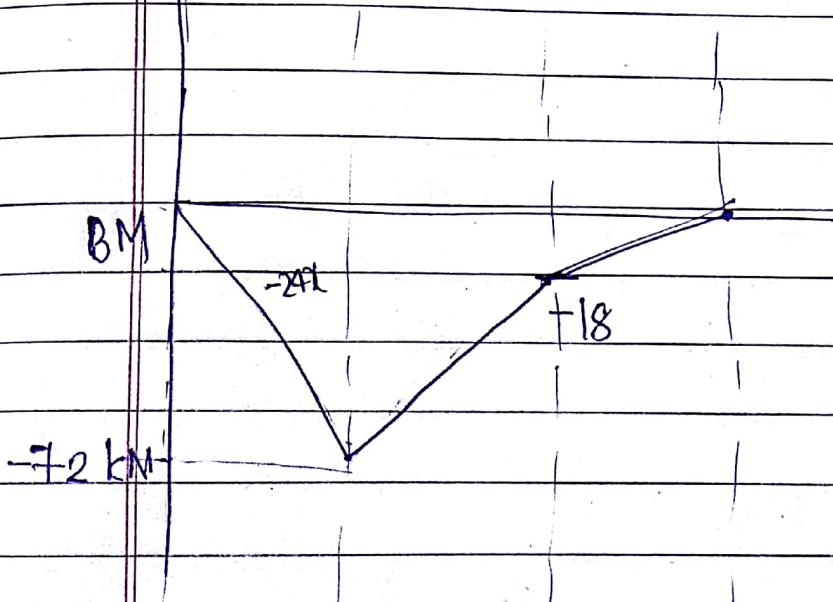
$$B_Y = -6 \text{ kN} = 30 - 8x$$



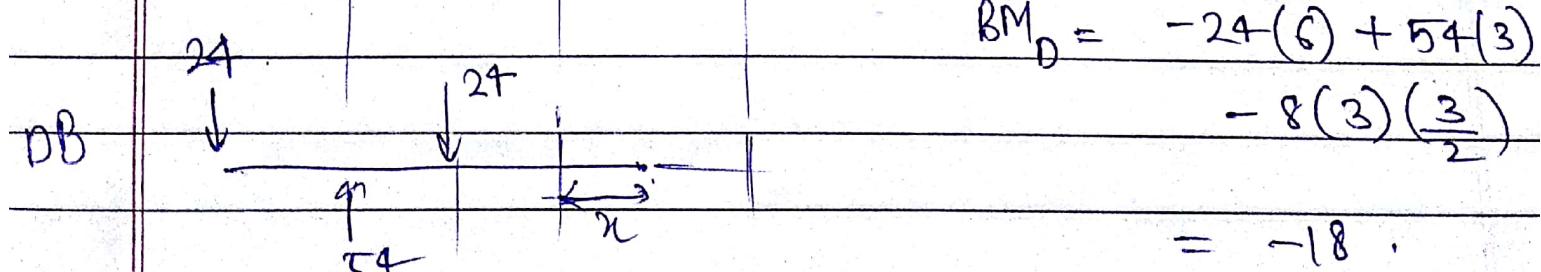
$$\begin{aligned} SF_E &= 30 - 8x \\ SF_D (x=3) &= 30 - 8 \times 3 \\ &= 6 \text{ kN} \end{aligned}$$



$$BM_{AC} = -24x$$



$$\begin{aligned} BM_{CD} &= -24(x+3) \\ &+ 54(x) \\ &- 8x(\frac{x}{2}) \end{aligned}$$



$$\begin{aligned} BM_D &= -24(6) + 54(3) \\ &- 8(3)(\frac{3}{2}) \\ &= -18 \end{aligned}$$

$$\begin{aligned} BM_{DB} &= -24(x+6) + 54(x+3) \\ &- 24(\frac{x+3}{2}) \end{aligned}$$