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Transportation Model

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2.1 The Transportation Model

The transportation model is the special case of linear programming problem which deals with the transportation of a product available at several sources to a number of different destinations. The *transportation model* is an important example of a network optimization problem. It can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, personal assignment, and many others, so that the model is not really confined to transportation or distribution only.
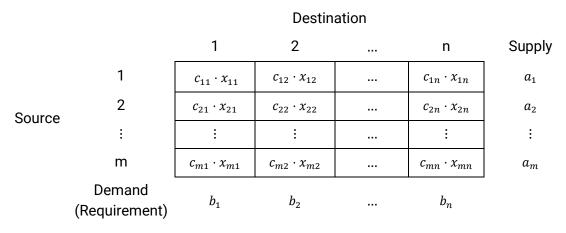
2.2 Assumptions in the Transportation Problem

1. The total quantity of items available at different sources is equal to the total requirement at different destinations.
2. Item can be transported conveniently from all sources to destinations.
3. The unit transportation cost of the item from all sources to destinations is certainly and precisely known.
4. The transportation cost on a given route is directly proportional to the number of units shipped on that route.
5. The objective is to minimize the total transportation cost for the organization as a whole and not for individual supply and distribution centres.

2.3 Mathematical/L.P. Model of the Transportation Problem

Transportation model deals with problems concerning as to what happens to te effectiveness function when we associate each of a number of origins (sources) with each of a possibly different number of destinations (jobs).

- Suppose that there are m sources and n destinations.
- Let a_i be the number of supply units available at source $i(i = 1, 2, 3, …, m)$ and let b_j be the number of demand units required at destination $j(j = 1, 2, 3, …, n))$.
- Let c_{ij} represent the unit transportation cost for transporting the units from source i to destination j.
- The objective is to determine the number of units to be transported from source i to destination j so that the total transportation cost is minimum. In addition, the supply limits at the sources and the demand requirements at the destinations must be satisfied exactly.
- The above information can be put in the form of a general matrix as shown below:

<div align="center">Destination</div>

Source	1	2	…	n	Supply
1	$c_{11} \cdot x_{11}$	$c_{12} \cdot x_{12}$	…	$c_{1n} \cdot x_{1n}$	a_1
2	$c_{21} \cdot x_{21}$	$c_{22} \cdot x_{22}$	…	$c_{2n} \cdot x_{2n}$	a_2
⋮	⋮	⋮	…	⋮	⋮
m	$c_{m1} \cdot x_{m1}$	$c_{m2} \cdot x_{m2}$	…	$c_{mn} \cdot x_{mn}$	a_m
Demand (Requirement)	b_1	b_2	…	b_n	

If x_{ij} $(x_{ij} \geq 0)$ is the number of units shipped from source i to destination j, then the equivalent linear programming model will be given as follows:

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Find $x_{ij}(i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n)$ in order to

$$Minimize\ (total\ cost)\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,3, \ldots \ldots, m,$$

and

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2,3, \ldots \ldots, n,$$

where

$$x_{ij} \geq 0$$

The two sets of constraints will be consistent i.e., the system will be in balance if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Note that the transportation problem will have a feasible solution if the above restriction is satisfied. Thus $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is necessary as well as sufficient condition for a transportation problem to have a feasible solution.

Problems that satisfy this condition are called balanced transportation problems. Techniques have been developed for solving *balanced or standard* transportation problems only. It follows that any non-standard problem in which the supplies and demands do not balance, must be converted to balanced transportation problem before it can be solved. This can be done by the use of a dummy source/destination.

2.4 The Solution of the Transportation Model

Following steps are involved in the solution of transportation model.

1. Balance the problem if it is not.
2. Obtain the initial feasible solution by
   a. North-West Corner Method (NWCM) or,
   b. Inspection Method / Least Cost Method (LCM) or,
   c. Penalty Method / Vogel's Approximation Method (VAM).
3. Test the optimality of the solution either by
   a. Stepping Stone Method or,
   b. Modified Distribution Method (MODI).

2.4.1 Methods for Finding Initial Solution

2.4.1.1 North-West Corner Method (NWCM)

It is the simplest procedure used to generate an initial feasible solution. It is so called because we begin with the north-west or upper-left corner cell of the transportation table. Various steps of this method can be summarized as follows:

1. Select the north-west (upper-left) corner cell of the transportation table and allocate as many units as possible (i.e. equal to the minimum between available supply and demand.)

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2. Adjust the supply and demand numbers in the respective rows and columns allocation.
3. If the supply for the first row is exhausted, then move down to the first cell in the second row and first column and go to step 2. **OR**
   If the demand for the first column is satisfied, then move horizontally to the next cell in the second column and first row and got to step 2.
4. If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.
5. Continue the procedure until the total available quantity is fully allocated to the cells as required.

2.4.1.2 Least Cost Method (LCM)

Various steps of this method can be summarized as under:

1. Select the cell with the lowest transportation cost among all the rows or columns of the transportation table.
   If the minimum cost is not unique, then select arbitrarily any cell with this minimum cost. For a better solution select the cell where the highest units can be allotted.
2. Allocate as many units as possible to the cell determined in step 1 and eliminate that row (or column) in which either supply is exhausted or demand is satisfied.
3. Repeat steps 1 and 2 for the reduced table until the entire supply at different factories is exhausted to satisfy the demand at different warehouses.

2.4.1.3 Vogel's Approximation Method (VAM) / Penalty Method

This method is preferred over the other two methods described above because the initial feasible solution obtained is either optimum or very much nearer to an optimum solution.

1. Calculate penalties for each row and column by taking the difference between the smallest and the next smallest unit transportation cost in the same row and column respectively.
2. Identify the row or column with the largest penalty and allocate the maximum possible quantity to the lowest cost cell in that selected row or column so as to exhaust either supply or demand. If there is a tie in the penalty or minimum cost select that row or column which will have maximum possible assignments.
3. Reduce the row supply or column demand by the amount assigned to the cell.
4. If the row supply is now zero, eliminate the row and if the column demand is zero, eliminate the column.
5. Recompute the row and column differences for the reduced transportation table, omitting rows and columns crossed out in the preceding step.
6. Repeat the above procedure until the entire supply at factories is exhausted to satisfy demand at different warehouses.

2.4.2 Methods for Finding an Optimal Solution

Once an initial solution has been found, the next step is to check its optimality.

2.4.2.1 Stepping Stone Method

In this method, we calculate the net cost change that can be obtained by introducing any of the unoccupied cells into the solution.

The stepping stone method can be summarized as under:

1. Determine an initial basic feasible solution using any of the three methods.
2. Make sure that the no. of occupied cells is exactly equal to m+n-1, where m is no. of rows and n is no. of columns.

3. Evaluate the cost effectiveness goods via transportation roots not currently in solution by the following five steps:
   a. Select an unoccupied cell, where a shipment should be made.
   b. Starting from this cell, formed a closed path (or loop) through at least three occupied cells and eventually returning to the same occupied cell. In the loop formulation only right angle turn is allowed and, therefore, skips all other cells which are not at the turning points. (Right angle turns in this path are permitted only at occupied cells)
   c. Assigning (+) and (-) alternatively on each corner cell of the closed path just traced, starting with a (+) sign at the unoccupied cell to be evaluated.
   d. Compute the net change in the cost along the closed path by adding together the unit transportation cost containing (+) and then subtracting unit cost containing (-).
4. Check the sign of each of the net changes. If all the net changes computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution.
5. Select the unoccupied cell having the highest negative net cost change and determine the maximum no. of units that can be assigned to a cell marked with a minus sign on the closed path corresponding to this cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on the closed path marked with a minus sign.
6. Go to step 2 and repeat the procedure until we get an optimal solution.

2.4.2.2 Modified Distribution Method (MODI Method)

The modified distribution method has a pattern similar to the stepping stone method.

1. Determine an initial basic feasible solution considering of (m+n-1) allocations in independent positions using any of the three methods.

2. Determine a set of numbers for each row and each column. To compute u_i (i = 1, 2, 3, …., m) for each row and v_j (j = 1, 2, 3, ….., n) for each column, set $c_{ij} = u_i + v_j$ for each of the m+n-1 occupied cells.

3. Compute the opportunity cost by $\Delta_{ij} = c_{ij} - (u_i + v_j)$ for each of the unoccupied cells.

4. Check the sign of opportunity cost. If one or more unoccupied cell has a negative opportunity cost, the given solution is not optimum.

5. Select the unoccupied cell with the largest negative opportunity cost as the cell to be included in the next solution.

6. Draw a closed path for unoccupied cell selected in step 5.

7. Assign alternate + and − signs on the corner point of the closed path.

8. Determine the maximum number of units that should be shipped to this unoccupied cell.

9. Repeat the whole procedure until an optimal solution is attained.

10. Finally, obtain the total transportation cost for the new solution.

2.5 Special Variations in the Transportation Model

2.5.1 Prohibited and Preferred Routes

In many practical situations, some of the routes in transportation are prohibited for some external reasons such as unexpected floods, weight or size conditions, transportation strikes and road construction, etc.

Such restrictions (or prohibitions) can be handled in the transportation problem by assigning a very high cost (say M or ∞) to the prohibited routes to ensure that these routes will not be included in the optimal solution and then the problem is solved in the usual manner.

2.5.2 Maximization Transportation Problem

When origins are related to destinations by a profit function instead of cost, the objective is to maximize instead of minimize.

The most convenient way to deal with the maximization case is to transform the profits to relative costs. The transformation is conducted by subtracting the profit associated with each cell from the largest profit in the matrix. In this way, the largest profit shows a zero relative cost. The solution procedures for relative costs are identical to those already described.

After an optimal solution to the transformed minimization problem is determined, relative costs in occupied cells are restated in their original profit functions, and the total profit is calculated according to quantities in the optimal distribution pattern.

2.6 Degeneracy in the Transportation Problem

An initial feasible solution to m origins and n destinations transportation problem consists of $(m + n - 1)$ basic variables, i.e., the number of occupied cells is less than $(m + n - 1)$. But if the number of occupied cells is less than $(m + n - 1)$ at any stage of the solution, then the problem is said to have a degenerate solution.

Degeneracy can occur at two stages: either (i) at the initial solution, or (ii) during testing of the optimal solution.

Case 1: Degeneracy occurs at the initial solution:

To resolve degeneracy at the initial stage, we make use of an artificial quantity, denoted by the Greek letter ε *(epsilon)* in one or more of the unoccupied cells so that the number of occupied cells is $(m + n - 1)$. The quantity ε is so small that it does not affect the supply & demand constraints. The ε value of which is assumed to be zero when actually used in the movements of goods from one cell to another is generally placed in the unoccupied cell with the lowest transportation cost and that cell is considered to be occupied.

Case 2: Degeneracy occurs during the testing of the optimal solution:

Degeneracy during the solution stage occurs, when the inclusion of the unoccupied cells with maximum negative opportunity cost results in vacating of two or more occupied cells simultaneously. This type of degeneracy is resolved by allocating ε to one or more of the recently vacated cells to complete the required $(m + n - 1)$ number of occupied cells. The problem is then solved in the usual manner.

2.7 References

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