CS698C 2021 August Quiz 2

Anirudh Nanduri

TOTAL POINTS

46 / 100

QUESTION 1

2X-3Y 25 pts

1.1 Mean 9 / 9

+ **0 pts** Correct

✓ + **9 pts** **Correct expression**

1.2 Variance 12 / 16

+ **0 pts** Incorrect

+ **16 pts** Corrrect expression

+ **12** **Point adjustment**

QUESTION 2

Rademacher 20 pts

2.1 Mean 10 / 10

+ **0 pts** Incorrrect

✓ + **4 pts** **Linearity of Expectation**

✓ + **6 pts** **Variance**

2.2 Variance 10 / 10

+ **0 pts** Incorrect

✓ + **10 pts** **Correct expression**

QUESTION 3

LInear Combination 25 pts

3.1 Variance of Y 5 / 10

+ **0 pts** Incorrect

+ **8 pts** Correct variance expression

+ **2 pts** Write in terms of Sigma matrix

+ **5** **Point adjustment**

3.2 Covar(Y,Z) 0 / 15

✓ + **0 pts** **Incorrect**

+ **13 pts** Corrrect expression

+ **2 pts** Write in terms of Sigma matrix

Not attempted

QUESTION 4

Conditional Expectation 30 pts

4.1 Calculate a 0 / 15

✓ + **0 pts** **Incorrect**

+ **15 pts** Correct step off taking expectations

4.2 Calculate b 0 / 15

✓ + **0 pts** **Incorrect**

+ **15 pts** Correct step of taking expectation

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$Z = 3X - 2Y$

$E[3X-2Y]$

$= 3E[X] - 2E[Y]$

$= 3(1) - 2(4)$

$= 3 - 8$

$= -5$

$Var(Z) = Var[3X-2Y]$

$\rho = \frac{E[X-1][Y-4]}{\sigma_1 \sigma_2}$

$Var[3X-2Y] = E\left[(3X-2Y - E[3X-2Y])^2\right]$

$= E\left[(3X-2Y+5)^2\right]$

$= E[9X^2 + 4Y^2 + 25 - 12XY + 30X - 20Y]$

$= 9E[X^2] + 4E[Y^2] + 25 - 12E[XY] + 30E[X] - 20E[Y]$ —— ①

$Var(X) = E[(X-M)^2]$

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Var $\sigma_1^2 = E[X^2] - (E[X])^2$

$4 = E[X^2] - 1$

$E[X^2] = 5$

1.1 Mean 9 / 9

+ **0 pts** Correct

✓ **+ 9 pts** **Correct expression**

1 a) cont'd.

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$\sigma_2^2 = E[Y^2] - (E[Y])^2$

$6 = E[Y^2] - 16$

$E[Y^2] = 22$

$\rho = \frac{E[XY] - \mu_X \mu_Y}{\sigma_1 \sigma_2}$

$\frac{1}{2} = \frac{E[XY] - 4}{\sigma_1 \sigma_2}$

~~$\sigma_1 \sigma_2 = 2\sqrt{6}$~~

$E[XY] = 4 + \frac{1}{2} = \frac{9}{2}$

Substituting in (1).

$Var(3X - 2Y) = 9(5) + 4(22) + 25 - 12\left(\frac{9}{2}\right)$

$+ 30(1) - 20(4)$

$= 45 + 88 + 25 - 54 + 30 - 80$

$= 54$

1
45 80
88 128
25 134
30
188
134
60
54

1.2 Variance **12 / 16**

+ **0 pts** Incorrect
+ **16 pts** Corrrect expression

+ 12 Point adjustment

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(3)

2. $P[r_i = 1] = P[r_i = -1] = \frac{1}{2}$

$S = a_1 r_1 + a_2 r_2 + \cdots + a_n r_n$

a) $E[S]$

$E[r_1] = 1 \, P[r_1 = 1] + (-1) P[r_1 = -1]$

$= 1 \left(\frac{1}{2}\right) + (-1)\left(\frac{1}{2}\right) = 0$

$\Rightarrow E[r_i]_{i=1 \text{ to } n} = 0$

$E[S] = E[a_1 r_1 + a_2 r_2 + \cdots + a_n r_n]$

$= a_1 E[r_1] + a_2 E[r_2] + \cdots + a_n E[r_n]$

$= 0$

Let

$A = [a_1 \; a_2 \; a_3 \cdots a_n]$

$R = [r_1 \; r_2 \cdots r_n]$

$S = A^T R$

b) $\text{Var}(S)$

$= E[(S - E(S))^2]$

$= E[S^2] - (E[S])^2 = E[S^2]$

$E[S^2] = \text{Var}(S) = \text{Var}(A^T R)$

$= (A^T)^2 \text{Var}(R)$

$= (A^T)^2 [\text{Var}(r_1) \; \text{Var}(r_2) \cdots \text{Var}(r_n)]$

$\text{Var}(r_1) = 1^2 P[r_1 = 1] + (-1)^2 P[r_1 = -1] - 0$

$= 1$

$\text{Var}(aX) = a^2 \text{Var}(X)$

2.1 Mean 10 / 10

+ **0 pts** Incorrrect

✓ **+ 4 pts** **Linearity of Expectation**

✓ **+ 6 pts** **Variance**

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(3)

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$= a_1 E[r_1] + a_2 E[r_2] + \cdots + a_n E[r_n]$

$= 0$

Let

$A = [a_1 \; a_2 \; a_3 \cdots a_n]$

$R = [r_1 \; r_2 \cdots r_n]$

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b) $\text{Var}(S)$

$= E[(S - E(S))^2]$

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$E[S^2] = \text{Var}(S) = \text{Var}(A^T R)$

$= (A^T)^2 \text{Var}(R)$

$= (A^T)^2 [\text{Var}(r_1) \; \text{Var}(r_2) \cdots \text{Var}(r_n)]$

$\text{Var}(r_1) = 1^2 P[r_1 = 1] + (-1)^2 P[r_1 = -1] - 0$

$= 1$

$\text{Var}(aX) = a^2 \text{Var}(X)$

$Var(Y_2) = Var(Y_3) = \cdots = 1$

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(4).

$$Var(S) = \begin{bmatrix} a_1^2 & a_2^2 & a_3^2 & \cdots & a_n^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$= a_1^2 + a_2^2 + a_3^2 + \cdots a_n^2$$

$$= \sum_{i=1}^{n} a_i^2$$

2.2 Variance 10 / 10

+ **0 pts** Incorrect

✓ **+ 10 pts** **Correct expression**

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(5)

3) $Y = a_1x_1 + \cdots + a_nx_n + c$

$Z = b_1x_1 + \cdots + b_nx_n + d$

$$Y = \underbrace{[a_1 \cdots a_n]}_{A^T}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}^{X} + c \qquad Y = A^TX + c$$

$$Y - c = [a_1 \cdots a_n]\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$E[Y] = A^T E[X] + c$$

$$Var[Y] = E[(Y - E[Y])^2]$$

$$= E[Y - A^T E[X] - c]^2$$

$$E[Y^2] = \cancel{A^2 E[X^2] +} \; (A^T)^2 E[X^2] + 2A^T E[X]c + c^2$$

$$E[Y^2] - (E[Y])^2 = (A^T)^2 E[X^2] + 2A^T E[X] + c^2 - (A^T E[X])^2 - c^2 - 2A^T E[X]$$

$$= (A^T)^2 \, Var(X)$$

$$= (A^T)^2 \, trace(\text{covariance matrix})$$

a) $Variance(Y) = (A^T)^2 tr(\Sigma)$

3.1 Variance of Y **5 / 10**

+ **0 pts** Incorrect
+ **8 pts** Correct variance expression
+ **2 pts** Write in terms of Sigma matrix

+ 5 Point adjustment

3.2 Covar(Y,Z) 0 / 15

✓ **+ 0 pts Incorrect**

+ 13 pts Corrrect expression

+ 2 pts Write in terms of Sigma matrix

Not attempted

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4) $\rho = cov(X,Y)$.

$E[(X_1 - \mu_1) \mid X_2 = x_2] = a + bx_2$.

a)

$E[(X_1 - \mu_1) \mid X_2 = \mu_2] = a + b\mu_2$

$= E[X_1 \mid X_2 = \mu_2] - \mu_1$

$E[X_1 \mid X_2 = \mu_2] = E(X_1) = \mu_1$

$= \mu_1 - \mu_1$

$= 0$

$\Rightarrow a + b\mu_2 = 0$

b)

4.1 Calculate a **0 / 15**

✓ **+ 0 pts Incorrect**

+ 15 pts Correct step off taking expectations

4.2 Calculate b **0 / 15**

✓ **+ 0 pts Incorrect**

+ 15 pts Correct step of taking expectation