Ayush Sahni Roll no 21111019

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Ques 1

(a) $P(R) = P(R \mid \text{Type-1 bag})\,P(\text{Type-1 bag}) + P(R \mid \text{Type-2 bag})\,P(\text{Type-2 bag})$

$R \to$ red ball

$$= \frac{5}{25} * (0.6) + \frac{15}{25} * (0.4)$$

$$= \frac{1}{5} * 0.6 + \frac{3}{5} * 0.4$$

$$= \frac{0.6}{5} + \frac{1.2}{5} = \frac{1.8}{5}$$ Ans

(b) ~~Type-1~~ Bag having 5 red & 20 green balls is Type-1 bag

$G \to$ green ball

$P(\text{Type-1 bag} \mid G)$

$$= \frac{P(G \mid \text{Type-1 bag})\,P(\text{Type-1 bag})}{P(G)}$$

$$= \frac{\frac{20}{25}(0.6)}{\frac{20}{25}(0.6) + \frac{10}{25}(0.4)}$$

$$= \frac{4/5 * 0.6}{4/5 * 0.6 + 2/5 * 0.4} = \frac{2.4/5}{\frac{2.4}{5} + \frac{0.8}{5}} = \frac{2.4/5}{3.2/5}$$

$$= \frac{24}{32} = \frac{3}{4}$$ Ans

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Ques 2

(a) $P\left[\left(X > \frac{3}{4} \;\middle|\; X > \frac{1}{2}\right)\right]$

$$= \frac{P\left[\left(X > 3/4\right) \cap \left(X > 1/2\right)\right]}{P\left[\left(X > 1/2\right)\right]}$$

$$= \frac{P\left[\left(X > 3/4\right)\right]}{P\left[\left(X > 1/2\right)\right]}$$

$$= \frac{\int_{3/4}^{1} 2x\,dx}{\int_{1/2}^{1} 2x\,dx} = \frac{\left[x^2/2\right]_{3/4}^{1}}{\left[x^2/2\right]_{1/2}^{1}}$$

$$= \frac{\left[x^2\right]_{3/4}^{1}}{\left[x^2\right]_{1/2}^{1}} = \frac{1 - 9/16}{1 - \frac{1}{4}}$$

$$= \frac{7/16}{3/4} = \frac{7}{16} \times \frac{4}{3} = \frac{7}{12}$$ Ans

(b) $E[X^{-1}] = \int_x x^{-1} f_X(x)\,dx$

$$= \int_0^1 x^{-1}(2x)\,dx$$

$$= \int_0^1 2\,dx = 2[x]_0^1 = 2[1-0]$$

$$= 2$$ Ans

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(c) CDF of $Y = \frac{1}{X}$ = $F_Y(y)$

$$F_Y(y) = P(Y \leq y)$$

$$= P\left(\frac{1}{X} \leq y\right)$$

$$= P\left(X \geq \frac{1}{y}\right)$$

$$= 1 - P(X < 1/y)$$

$$= 1 - F_X(1/y)$$ {F_X is the CDF of R.V X}

$$= 1 - \int_0^{1/y} 2x$$

$$= 1 - 2\left[\frac{x^2}{2}\right]_0^{1/y} = 1 - [x^2]_0^{1/y}$$

$$= 1 - \left[\frac{1}{y^2} - 0\right] = 1 - \frac{1}{y^2}$$ Ans

PDF of $Y = 1/X$ = $f_Y(y)$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy}\left(1 - \frac{1}{y^2}\right)$$

$$= -\frac{d}{dy}(y^{-2}) = -(-2y^{-3})$$

$$= \frac{2}{y^3}$$ Ans

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Ques (3)

(a) Let $f_Z(z)$ be pdf of $Z = X+Y$

$f_Z(z) =$ Probability that Z takes value z

= Probability that R.V. X takes value x & prob. that R.V Y takes value y.

$= f_{X,Y}(x,y)$ {Convolution law}

$= 2e^{-x-y}$

$= 2e^{-z}$ → pdf of Z

~~(b) CDF of Z = $F_Z(z)$~~

~~$F_Z(z) = \int_z 2e^{-z} dz$~~

(c) $f_X(x) = \int_Y f_{X,Y}(x,y)\,dy$

$= \int_{y=x}^{\infty} 2e^{-x-y}\,dy$

$= 2\int_{y=x}^{\infty} e^{-x} e^{-y}\,dy$

$= 2e^{-x}\int_{y=x}^{\infty} e^{-y}\,dy$

$= 2e^{-x}\left[-e^{-y}\right]_x^{\infty}$

$= 2e^{-x}\left[-e^{-\infty} + e^{-x}\right]$

$= 2e^{-2x}$ Ans

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(d) $E[Y \mid X=x] = \int_Y y \, f_{Y|X}(y \mid X=x) \, dy \quad \text{—} (1)$

$$f_Y(y) = \int_X f_{X,Y}(x,y) \, dx$$

$$= \int_0^y 2e^{-x-y} \, dx$$

$$= 2e^{-y} \int_0^y e^{-x} \, dx$$

$$= 2e^{-y} \left[-e^{-x}\right]_0^y$$

$$= 2e^{-y}\left[e^0 - e^{-y}\right]$$

$$= 2e^{-y}\left[1 - e^{-y}\right]$$

~~Now, from eqn (1) :-~~

$$\cancel{E[Y \mid X=x] = \int y \, 2e^{-y}\left[1-e^{-y}\right] dy}$$

$$f_{Y|X}(y \mid X=x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$= \frac{2e^{-x-y}}{2e^{-2x}} = e^{-x-y+2x} = e^{x-y}$$

Now, from eqn (1) :-

$$E[Y \mid X=x] = \int_x^\infty y \, e^{x-y} \, dy$$

$$= e^x \int_x^\infty y e^{-y} \, dy \qquad \underline{\text{Ans}}$$

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(b) CDF of Z

$$F_Z(z) = \int_z f_Z(z)\,dz$$

$$= \int_0^{z} 2e^{-z}\,dz$$

$$= 2\left[-e^{-z}\right]_0^{z}$$

$$= \cancel{2\left[e^{0} - e^{\infty}\right]}$$

$$= 2\left[-e^{-z} + e^{0}\right]$$

$$= 2\left[1 - e^{-z}\right]$$

Ans