CS698C 2021 August Quiz 1

Anirudh Nanduri

TOTAL POINTS

67 / 100

QUESTION 1

Conditional probability 20 pts

1.1 a 10 / 10

+ **0 pts** Incorrect or not attemped
+ **6 pts** Identify all 6 events correctly
+ **4 pts** Correct expression for Red ball
+ **0 pts** Click here to replace this description.
+ **10 Point adjustment**

1.2 b 9 / 10

+ **0 pts** Incorrect or not attempted
+ **6 pts** Identify all events correctly
+ **4 pts** Correct expression for required posterior
+ **9 Point adjustment**

mismatch of bag1 and bag2 in calculation

QUESTION 2

conditional prob. and cdf, pdf of functions of one variable 30 pts

2.1 a 10 / 10

+ **0 pts** Incorrect or not attempted
✓ + **8 pts** **Correct expression**
✓ + **2 pts** **Correct evaluation of integral**

2.2 10 / 10

+ **0 pts** Incorrect
✓ + **8 pts** **Correct expression**
✓ + **2 pts** **Correct evaluation**

2.3 10 / 10

+ **0 pts** Incorrect
✓ + **8 pts** **Correct expression for pdf**
✓ + **2 pts** **Correct calculation for pdf**

QUESTION 3

function of two variables, marginal and conditional expectations. 50 pts

3.1 a 0 / 10

✓ + **0 pts** **Incorrect or not attemped**
+ **2 pts** Correct expression for P(X+Y <=z)
+ **3 pts** Correct integration limits
+ **3 pts** Correct integration
+ **2 pts** Correct pdf

3.2 b 0 / 10

✓ + **0 pts** **Incorrect or not attemped**
+ **6 pts** Correct expression for CDF
+ **4 pts** Correct Evaluation

3.3 10 / 10

+ **0 pts** InCorrect or not attemped
✓ + **3 pts** **Correct expression**
✓ + **4 pts** **Correct Integration limit**
✓ + **3 pts** **Correct evaluation**

3.4 8 / 20

+ **0 pts** Incorrect or not attemped
✓ + **4 pts** **First basic expression**
✓ + **4 pts** **Expresion to integration**
+ **8 pts** Evaluate integration
+ **4 pts** Replace with the random variable

Aniruddh Nandurkar
B111011

1)

a) $Pr(R) = 0.6 \times \frac{5}{25} + 0.4 \times \frac{15}{25}$

$= \frac{3+6}{25}$

$= \frac{9}{25}$

Probability (Red) $= \frac{9}{25}$

1b) Let Bag2 be the type of bag containing 5 Red and 20 Black balls event.

$Pr(\text{Bag2} / \text{Green})$

$= \frac{Pr(\text{Bag2})\, Pr(\text{Green} / \text{Bag2})}{Pr(\text{Bag2})\, Pr(\text{Green} / \text{Bag2}) + Pr(\text{Bag1})\, Pr(\text{Green} / \text{Bag1})}$

$= \frac{0.4 \times \frac{20}{25}}{0.4 \times \frac{20}{25} + 0.6 \times \frac{10}{25}}$

$= \frac{0.4 \times \frac{4}{5}}{0.4 \times \frac{4}{5} + 0.6 \times \frac{2}{5}} = \frac{1.6}{1.6+1.2} = \frac{1.6}{2.8} = \frac{4}{7}$

1.1 a **10 / 10**

+ **0 pts** Incorrect or not attemped
+ **6 pts** Identify all 6 events correctly
+ **4 pts** Correct expression for Red ball
+ **0 pts** Click here to replace this description.

+ 10 Point adjustment

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$= \frac{0.4 \times \frac{20}{25}}{0.4 \times \frac{20}{25} + 0.6 \times \frac{10}{25}}$

$= \frac{0.4 \times \frac{4}{5}}{0.4 \times \frac{4}{5} + 0.6 \times \frac{2}{5}} = \frac{1.6}{1.6+1.2} = \frac{1.6}{2.8} = \frac{4}{7}$

1.2 b **9 / 10**

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+ **6 pts** Identify all events correctly
+ **4 pts** Correct expression for required posterior
+ **9 Point adjustment**

mismatch of bag1 and bag2 in calculation

Anirudh Nanduri
21111011

2) pdf $f(x) = 2x \quad 0 < x < 1$

a) $\Pr_x\left(X \geq \frac{3}{4} \mid x \geq \frac{1}{2}\right)$.

$$= \frac{\Pr_x\left(x \geq \frac{3}{4} \cap x \geq \frac{1}{2}\right)}{\Pr_x\left(x \geq \frac{1}{2}\right)}$$

$$= \frac{\Pr_x\left(x \geq \frac{3}{4}\right)}{\Pr_x\left(x \geq \frac{1}{2}\right)} \quad \text{---- (1)}$$

$$\Pr\left(x \geq \frac{3}{4}\right) = \int_{3/4}^{1} 2x\,dx = 2\left(\frac{x^2}{2}\right)_{3/4}^{1}$$

$$= 1 - \frac{9}{16} = \frac{7}{16}$$

$$\Pr\left(x \geq \frac{1}{2}\right) = \int_{1/2}^{1} 2x = (x^2)_{1/2}^{1}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}.$$

$$\frac{\Pr\left(x \geq \frac{3}{4}\right)}{\Pr\left(x \geq \frac{1}{2}\right)} = \frac{\frac{7}{16}}{\frac{3}{4}} = \frac{7}{12}$$

2.1 a **10 / 10**

+ **0 pts** Incorrect or not attempted

✓ **+ 8 pts** **Correct expression**

✓ **+ 2 pts** **Correct evaluation of integral**

Aniruddh Nanduri
2111101

2)

b) $E[1/x]$

$$= \int_0^1 \frac{1}{x} \cdot 2x \, dx$$

$$= 2(1-0)$$

$$= 2$$

2) c) cdf and pdf of $Y = \frac{1}{X}$

$f_Y(y = \frac{1}{x})$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$g^{-1}(x) = \frac{1}{y}$

$$\left|\frac{dx}{dy}\right| = \left|\frac{1}{y^2}\right| = \frac{1}{y^2}$$

$$f_Y(y = \frac{1}{x}) = f_Y(g^{-1}(x)) \left|\frac{dx}{dy}\right|$$

$$= \frac{1}{y}\left(\frac{1}{y^2}\right) = \frac{1}{y^3}$$

Pdf of $Y = \frac{1}{X} = \frac{1}{y^3}$

for $0 < x < 1$

$1 < y < \infty$

$$\text{cdf} = \int_1^y \frac{1}{y^3} \, dy$$

$$= \left(\frac{y^{-3+1}}{-2}\right)_1^y = -\frac{1}{2}\left(y^{-2}\right)_1^y = -\frac{1}{2}\left(\frac{1}{y^2} - 1\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{y^2}\right)$$

2.2 **10 / 10**

+ **0 pts** Incorrect

✓ **+ 8 pts** **Correct expression**

✓ **+ 2 pts** **Correct evaluation**

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Avinash Nandwani
2111191

2c)

$$cdf = \frac{1}{2}\left(1 - \frac{1}{y^2}\right)$$

3) $f(x,y) = 2e^{-x-y} \quad 0 < x < y < \infty$

a) ~~$z = x + y$~~

~~pdf~~

$x = z - y$

$y = z - x$

3c) marginal distribution $f_x(x)$

$$f_x(x) = \int_x^\infty 2e^{-x-y}\, dy$$

$$= 2e^{-x} \int_x^\infty e^{-y} dy$$

$$= 2e^{-x} - (e^{-y})_x^\infty$$

$$= 2e^{-x} - (0 - e^{-x})$$

$$= 2e^{-x} e^{-x}$$

$$= 2e^{-2x}$$

2.3 **10 / 10**

+ **0 pts** Incorrect

✓ **+ 8 pts** **Correct expression for pdf**

✓ **+ 2 pts** **Correct calculation for pdf**

3 a continue

Aniruddh Nandur,
2111101

$$= e^{x}\left[e^{-y}(1-y)\right]_{x}^{\infty}$$

$$= e^{x}\left[0 - e^{-x}(1-x)\right].$$

$$= x-1$$

3 a) $z = x+y$.

$f(x,y) = 2e^{-x-y}$.

$= 2e^{-(x+y)}$.

$f(z,z) = 2e^{-z}|J|$

$$J = \begin{vmatrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$x = z-y$.

$y = z-x$.

$f_z(x+y) = 2e^{-(x+y)}$

$= 2e^{-z}\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right)$.

$= 4e^{-z}$.

3.1 a **0 / 10**

✓ **+ 0 pts** **Incorrect or not attemped**

+ 2 pts Correct expression for P(X+Y <=z)

+ 3 pts Correct integration limits

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Aniruddh Nandur,
2111101

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3.2 b **0 / 10**

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Avinash Nandwani
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~~pdf~~

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$$= 2e^{-x} e^{-x}$$

$$= 2e^{-2x}$$

3.3 **10 / 10**

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211110111

3d). $E[Y|x]$.

$E[Y|x] =$

$\Rightarrow E[Y|x=x]$.

$= \int y \, f_{Y|x}(y|x=x)\, dy$

$= \int_x^\infty y \, \frac{f(x,y)}{f_x(x)}\, dy$

$f_x(x) = 2e^{-2x}$

$= \int_x^\infty y \, \frac{2e^{-x-y}}{2e^{-2x}}\, dy$

$= \int_x^\infty y \, e^{x-y}\, dy$

$= e^x \int_x^\infty y e^{-y}\, dy$

using $\int uv \quad u = y, \; v = e^{-y}$.

$= u\int v - \int \frac{du}{dy} \int v$.

$= e^x \left[-ye^{-y} - \int -e^{-y} \right]$

$= e^x \left[-ye^{-y} + e^{-y} \right]_x^\infty$

3.4 8 / 20

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