21111014
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(1) 25 balls in each bags.
60% bag → 5 red, 20 green
40% → 15 R, 10 G.

(a) : $\frac{60}{100} \times \frac{5}{25} + \frac{40}{100} \times \frac{15}{25}$

(b) $P(G \mid (5R, 20G))$

$$\Rightarrow \frac{\left(\frac{60}{100}\right) \cdot \frac{20}{25}}{\frac{60}{100}\cdot\frac{20}{25} + \frac{40}{100}\cdot\frac{10}{25}}$$

(2) $f(x) = 2x \quad 0 < x < 1$

(a) $P[X \geq 3/4 \mid X \geq 1/2]$

$$= \frac{P[(X \geq 3/4) \cap (X \geq 1/2)]}{P[X \geq 1/2]}$$

$$= \frac{P[X \geq 3/4]}{P[X \geq 1/2]}$$

$$\Rightarrow \frac{1 - P[X < 3/4]}{1 - P[X < 1/2]}$$

$$\Rightarrow \frac{1 - \int_0^{3/4} 2x\,dx}{1 - \int_0^{1/2} 2x\,dx}$$

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(b) $E[1/X] = \int_{-\infty}^{\infty} \frac{1}{x} 2x\,dx = \int_{0}^{1} 2\,dx \Rightarrow 2$

(c) $Y = 1/X \qquad g(x) = \frac{1}{x} \qquad g^{-1}(y) = \frac{1}{y} = x$

$f_X(x) = 2x. \qquad \frac{dx}{dy} = \frac{-1}{y^2}$

$f_X(x) = P[X \le x]$

$F_Y(y) = P[Y \le y] = P[1/X \le y] = P[X \ge 1/y]$

$= \int_{1/y}^{\infty} 2x\,dx.$

$= \frac{1}{y^2}.$

$f_Y(y) = f_X(g^{-1}(y)) \left(\left|\frac{dx}{dy}\right|\right).$

$= 2 \cdot \left(\frac{1}{y}\right)\left(\frac{1}{y^2}\right) = \frac{2}{y^3}.$

(3) $f(x,y) = 2e^{-x-y}. \quad 0 < x < y < \infty.$

(a) $Z = X+Y$

~~$f_Z(z) = f_Z(x+y)$~~

~~let $t =$~~ let $t = X$

$\begin{bmatrix} Z \\ t \end{bmatrix} = \begin{bmatrix} X+Y \\ X \end{bmatrix}$ & $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} t \\ z-t \end{bmatrix}$

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$$f_z(z) = f_{x,y}(g^{-1}(x,y))\,|J|$$

$$= f_{x,y}(t, z-t)\,|J|$$

$$= 2e^{-(t)-(z-t)}\,|J| = 2e^{-z}$$

$$|J| = \begin{vmatrix} \frac{dx}{dz} & \frac{dy}{dz} \\ \frac{dx}{dt} & \frac{dy}{dt} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = 1$$

$$f_z(z) = 2e^{-2}$$

(b) $$F_z(z) = \int_{-\infty}^{z} 2e^{-2}\,dz.$$

$$= \left[2e^{-2}\right]_{-\infty}^{z} \Rightarrow +2e^{-2}$$

$$= 2e^{-2}$$