archi gupta
21111014

Q1

X $\mu_1 = 1$ $\sigma_1^2 = 4$ $\rho = \frac{1}{2}$ (Independent r.v)
Y $\mu_2 = 4$ $\sigma_2^2 = 6$

$Z = 3X - 2Y$

$E[Z] = E[3X-2Y] = 3E[X] - 2E[Y]$
$= 3 \cdot 1 - 2 \cdot 4$
$\Rightarrow -5$

$Var[Z] = E[Z^2] - \mu^2$
$= Var[3X - 2Y]$
$= 9 \cdot Var[X] + 4 \cdot Var[Y]$
$= 9 \cdot 4 + 4 \cdot 6$
$\Rightarrow 60$

archi gupta
21111014

Q2 $P[r_i = 1] = P[r_i = -1] = \begin{cases} 1/2 \\ 0 & o.w \end{cases}$

$S = a_1 r_1 + \cdots + a_n r_n$

(a) $E[S] = E[a_1 r_1 + a_2 r_2 + \cdots a_n r_n]$.

$E[r_i] = 1 \cdot \frac{1}{2} + -1 \cdot \frac{1}{2} = 0$.

$E[S] = 0$.

(b) $Var[S] = Var[a_1 r_1 + a_2 r_2 + \cdots + a_n r_n]$

~~Var[a.~~ $Var[r_i] = E[r_i^2] - (\mu_{r_i})^2$
$= 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} - 0 = 1$

$Var[S] = \sum_{i=1}^{n} a_i^2 \cdot Var[r_i] = \sum_{i=1}^{n} a_i^2$

archi gupta
211110014

Q3 $X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$ $Y = a_1 X_1 + \cdots + a_n X_n + c$

$Z = b_1 X_1 + \cdots + b_n X_n + d$

$\Sigma = Cov(X) \Rightarrow$ variance-covariance matrix

$$Var[Y] = Var[a_1 X_1 + a_2 X_2 + \cdots + a_n X_n + c]$$
$$= \sum_{i=1}^{n} a_i^2 Var[X_i]$$
$$= \sum_{i=1}^{n} a_i^2 [\Sigma]_{ii}$$ (as ith diagonal entry in Σ is $Var[X_i]$)

$Cov[Y, Z]$

$$= Cov[a_1 X_1 + \cdots a_n X_n + c, \; b_1 X_1 + \cdots + b_n X_n + d]$$

$$= \sum_{i,j=1}^{n} a_i b_j \, cov(X_i, X_j)$$

$$= \sum_{i,j=1}^{n} a_i b_j [\Sigma]_{ij}$$

Q4 $X_1 \quad \mu_1 \quad \sigma_1^2 \qquad \rho = Cov(X, Y)$ archi gupta 21111044

$X_2 \quad \mu_2 \quad \sigma_2^2$.

$$E[(X_1 - \mu_1) \mid X_2 = x_2] = a + bx_2$$

(a) $\Rightarrow E[X_1 \mid X_2 = x_2] - E[\mu_1 \mid X_2 = x_2]$

$\Rightarrow E[X_1 \mid X_2 = x_2] - \mu_1 = a + bx_2$

$E[X_1 \mid X_2 = x_2] = a + bx_2 + \mu_1$

$E[E[X_1 \mid X_2 = x_2]] = E[a + bx_2 + \mu_1]$

$= E[X_1] = \mu_1 = a + bE[X_2] + \mu_1$

$\Rightarrow a + b\mu_2 = 0$.

$\Rightarrow a = -b\mu_2$

(b) $\cancel{\sigma_1^2 = E[(X_1 - \mu_1)^2]}$ $\quad \rho = \cancel{E[(X_1 - \mu_1)(X_2 - \mu_2)]}$

$\cancel{\dfrac{E[(X_1 - \mu_1)(Y - \mu_2)]}{\sqrt{\sigma_1^2 \sigma_2^2}}}$.

$\rho = \dfrac{Cov(X_1, X_2)}{\sigma_1 \sigma_2}$

$\rho\sigma_1\sigma_2 = cov(X_1, X_2)$ $\qquad E[(X_1 - \mu_1)(X_2 - \mu_2)$

$X_1X_2 - \mu_1 X_2$

$-\mu_2 X_1$

$= E[X_1X_2] - \mu_1\mu_2$

$= \cancel{E[X_2] \cdot E[X_1 \mid X_2] - \mu_1 \mu_2} + \mu_1\mu_2$

$= \mu_2 \cdot a$

$\int (x_2 - \mu_2)^2 f_{X_2}(x_2)$

$E[X_1 \mid X_2] = a + bx_2 + \mu_1$

$E[X_1 \mid x_2] =$

$\int x_1 f_{X_1 \mid X_2}$

$E[X_1X_2] = \iint x_1 x_2 \cdot f_{X_1 X_2}(x_1, x_2)\, dx_1\, dx_2$.

$= \iint x_1 x_2 \cdot f_{X_1 \mid X_2}(x_1 \mid X_2 = x_2) \cdot f_{X_2}(x_2)\, dx_1$

arshi gupta
21111014

$$\rho = \frac{\operatorname{Cov}(X_1, X_2)}{\sigma_1 \sigma_2}.$$

$$\rho\sigma_1\sigma_2 = \operatorname{cov}(X_1, X_2)$$

$$= E[X_1 X_2] - \mu_1 \mu_2$$

$$= E[(X_1 - \mu_1)(X_2 - \mu_2)]$$

$$= \iint (x_1 - \mu_1)(x_2 - \mu_2) \cdot f_{X_1, X_2}(x_1, x_2)\, dx_1\, dx_2$$

$$= \iint (x_1 - \mu_1)(x_2 - \mu_2)\, f_{X_1|X_2}(x_1 | x_2) \cdot f_{X_2}(x_2)\, dx_1 \cdot dx_2.$$

$$= \int (x_2 - \mu_2) \cdot f_{X_2}(x_2) \cdot \underbrace{\int (x_1 - \mu_1) \cdot f_{X_1|X_2}(x_1 | X_2 = x_2)\, dx_1}_{a + b x_2.}\, dx_2$$

$$= \int (x_2 - \mu_2) \cdot f_{X_2}(x_2) \cdot (a + b x_2)\, dx_2 .$$

$$= \int \cancel{b \cdot x_2^2} (x_2 - \mu_2) \cdot (b x_2 - b\mu_2 + \underbrace{b\mu_2 + a}_{0}) \cdot f_{X_2}(x_2)\, dx_2 .$$

$$= \int (x_2 - \mu_2) \cdot b(x_2 - \mu_2) \cdot f_{X_2}(x_2)\, dx_2$$

$$\Rightarrow b \int (x_2 - \mu_2)^2 f_{X_2}(x_2)\, dx_2$$

$$= b \cdot \sigma_2^2$$