

Calculation of Constant Power Lithium Battery Discharge Curves

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Abstract: Standard battery testing procedure consists of discharging the battery at constant current. However, for battery powered aircraft application, consideration of the cruise portion of the flight envelope suggests that power should be kept constant, implying that battery characterization should occur over a constant power discharge. Consequently, to take advantage of existing battery discharge curves it would be useful to have a methodology that can extract a constant power discharge curve from a constant current discharge curve. The development of such a methodology for lithium batteries is described in this article.

Keywords: battery discharge curve; lithium battery; constant power discharge curve; battery powered aircraft

1. Introduction

Battery powered unmanned aerial vehicles have experienced consistent growth in use and application. Like any other class of flight vehicle, accurate estimation of range and endurance is paramount during the design and selection phase. Methods to estimate range and endurance for battery powered aircraft [1–4] are not as well established as those for fossil fuel powered craft [5–7]. In [8], a method is developed to estimate the range and endurance of battery powered aircraft with validation of the approach presented in [9]. To implement the method and approach of [8,9], battery discharge curves are required at constant power, where the battery voltage and current vary. This is atypical from the usual method of battery performance characterization, where the current is fixed and power and voltage are variable. Consequently, to use the method in [8], battery data is required for a constant power discharge, or a means is required to convert constant current discharge curves to constant power discharge curves. Such a technique is the focus of this article.

2. Theoretical Development

Working with lead acid batteries, Peukert [10,11] established that their rate of discharge affected their effective capacity; for discharge rates greater than a 1C, the capacity would reduce (and *vice versa*). An increase in the discharge current of the battery may decrease the effective capacity due to a decline of the reactivity of the battery's active materials. Mathematically, this is expressed as:

$$K = i^P t \quad (1)$$

where P is the Peukert constant, i is current and K is a constant. For lithium batteries, which are at present the “standard” due to their high power density, the impact of the Peukert effect is not well established [12]. A value of $P \approx 1.05$ has been suggested [12]. The greatest impact on lithium battery capacity stems from their sensitivity to temperature; low temperatures severely attenuate the rated battery capacity C [13,14]. Lithium batteries are also sensitive to the number of charging and

discharging cycles; the greater the number of cycles the less the capacity due to a loss of active material within the cell and primarily loss of lithium inventory [15].

In order to transform a constant current discharge curve, see Figures 1a–3a for examples of typical discharge profiles [16,17], it would be useful to correlate the curves by establishing their dependence on discharge rate or current. The data in Figures 1–3 was obtained from digitization of the manufacturer's charts. Higher discharge currents are seen to reduce the voltage during discharge because the polarization voltage in the electrode reactions increases [14]. It was found heuristically within the spirit of Equation (1) that the constant current discharge curves for a given battery collapse when the voltage V during the discharge is multiplied by the current raised to the power n for a given discharged capacity (while this collapse is seen for high power cells used in aircraft application, high energy cells may not show such a correlation):

$$i^n V(D) = \text{Constant}(D) \quad (2)$$

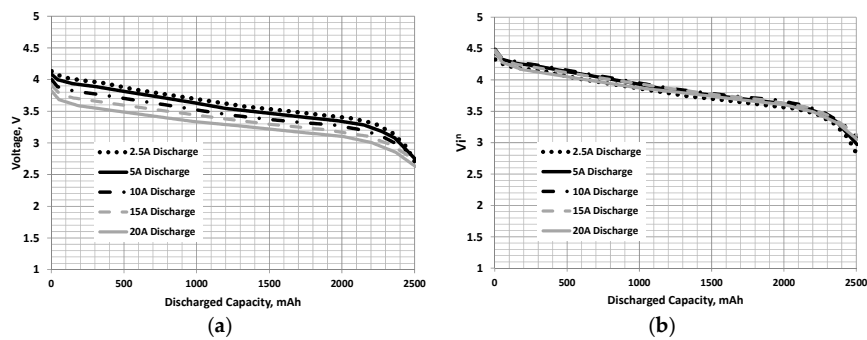


Figure 1. INR 18650 [16]: (a) constant current and (b) correlated discharge curves. $n = 0.05$, and $C = 2500$ mAh.

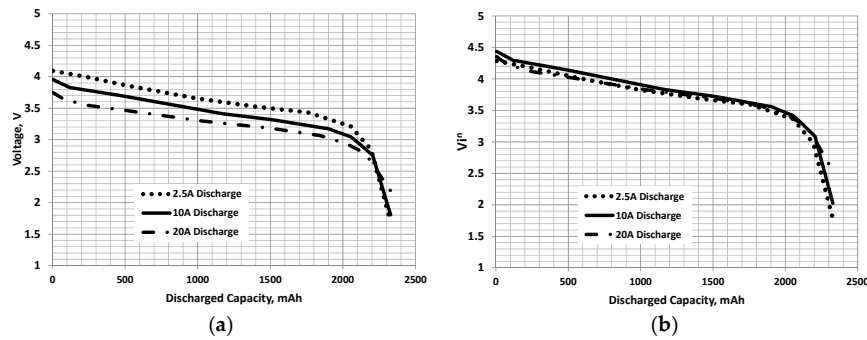


Figure 2. LG18B50HE2 [16]: (a) constant current and (b) correlated discharge curves. $n = 0.05$, and $C = 2500$ mAh.

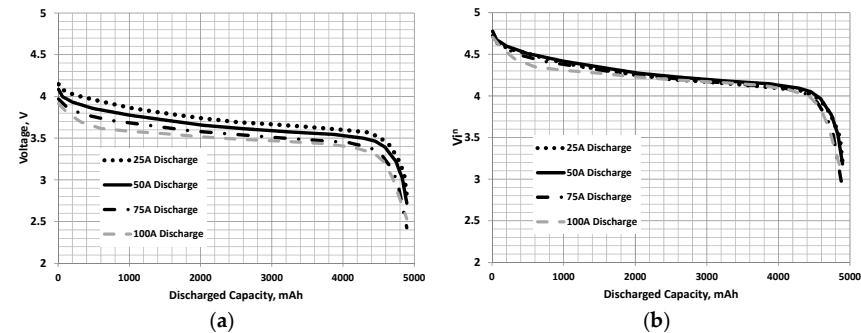


Figure 3. Thunderpower Extreme [17]: (a) constant current and (b) correlated discharge curves. $n = 0.05$, and $C = 5000$ mAh.

The discharged capacity, D is the total charge drawn from the battery at the time instant in which it is considered. This correlation is shown in Figures 1b–3b where data is presented for lithium batteries from various manufacturers. It was found that values of $n \approx 0.05$ (determined using a non-linear least squares minimization) showed the best collapse of the curves. As the discharge curves can be correlated, they may then be used to estimate a constant power discharge via the following procedural steps:

- (1) Using the available constant current discharge curves for the battery of interest, plot $i^n V$ as a function of the discharged capacity, D , see Figures 1b–3b for examples. Adjust n to afford the best collapse of the curves. Alternatively, n may be established using a non-linear least square minimization as follows. Let M = the number of test cases (discharges at different i for a given battery). The least square criterion is implemented as:

$$Sum = \sum_{j=1}^M [V_j i_j^n - \overline{V i^n}]^2 \quad (3)$$

where the overbar indicates an average for all j (for a discharge D). The counter j refers to the individual data sets. A non-linear least square solver (found in MS Excel, or most mathematical computational software) may be used to find the value of n that minimizes the summation.

- (2) Once n has been determined, the collapsed plots are curve fitted as a function of D . A function of the form:

$$V i^n(D) = (a + cD + eD^2) / (1 + bD + dD^2 + fD^3) \quad (4)$$

matches a battery discharge curve profile well and can be readily found using software such as TableCurve, MathCad or most statistical curve fitting packages. Naturally, other curve fit functions may also be used. Note that the non-linear least square minimization solver may be used directly to solve Equations (3) and (4) with the form of the curve fit expressed by Equation (4) prescribed. However, ill-conditioning and divergence may be an issue unless the initial estimates of the coefficients in Equation (4) are close to the optimal values.

- (3) The voltage during the discharge may be found using $V_j = \frac{i^n V(D)_{j-1}}{i_j^n}$ which after substitution of $i_j = \frac{P_e}{V_j}$ yields:

$$V_j = \left(\frac{i^n V(D)_{j-1}}{P_e^n} \right)^{1/(1-n)} \quad (5)$$

note that for the calculations to not be circular, V_j cannot be a function of $i^n V(D)_j$. It is thus assumed that $i^n V(D)_j = i^n V(D)_{j-1}$ for each successive step, not for successive steps. What this implies is that $i^n V(D)$ does not change rapidly between each iteration. As long as the time step is small relative to the total discharge time this is a reasonable assumption. If the required P_{req} or mechanical power is known, then $P_e = P_{req} / \eta_{tot}$ where η_{tot} is the efficiency of the propulsion system including the propeller, electronic speed controller and motor.

- (4) The corresponding current at this time step is:

$$i_j = \frac{P_e}{V_j} \quad (6)$$

- (5) The discharged capacity then follows as:

$$D_j = i_j \times (\Delta t) + D_{j-1} \quad (7)$$

- (6) Calculate the voltage, current and discharged capacity at the next time step t ($t(h) = j/N$ where $j = 1, 2, 3, \dots$ and N is an arbitrary constant defining the time step increment. In this article $N = 180$ giving time increments Δt of 20 s). The time step should be small compared to the discharge duration to ensure the validity of Equation (5).
- (7) Repeat Steps 3–5 until the discharged capacity D_j is approximately equal to C .

Note that the discharge curves in Figures 1–3 were determined at temperatures between 20 °C and 25 °C. The validity of Equation (2) at very low temperatures needs to be established.

Consequently, Figure 4 presents data for a lithium polymer battery developed at the Jet Propulsion Laboratory (JPL) for application at low temperatures [18]. The cell was tested at -20 °C at various discharge rates. As shown in Figure 4a, a rate dependence on capacity is clearly evident with higher discharge rates reducing the effective capacity. Correlation of the data sets using Equation (2) is presented in Figure 4b. As seen, Equation (2) only affords correlation of the three curves during the initial segment of the discharge. It may thus be inferred that Equation (2) is not representative at very low temperatures.

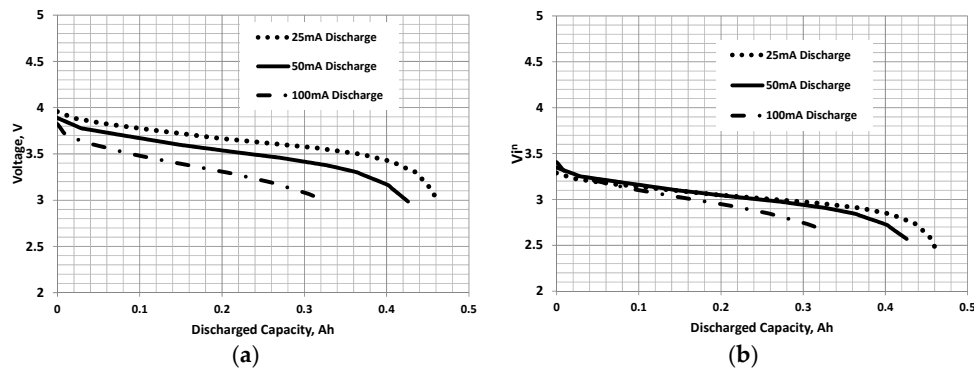


Figure 4. Jet Propulsion Laboratory (JPL)/WGL [18] lithium-ion cell discharged at -20 °C: (a) constant current and (b) correlated discharge curves. $n = 0.05$, and $C = 500$ mAh.

3. Materials and Methods

To validate the procedure enumerated above, a test was undertaken using an Electrify 1300 mAh 30C lithium polymer battery (which consists of three cells in series and is rated at 11.1 V with a capacity of 1300 mAh, Great Planes, Champaign, IL, USA). The battery was discharged at different constant current rates (1.45 A, 4.8 A and 7.5 A), while the voltage, power and current were monitored using a Watt's Up Watt Meter and Power Analyzer (Denver, CO, USA). The ambient temperature during testing was maintained at 20 °C. Measurements from this Watt meter and power analyzer were compared against a Fluke 175 multi-meter (Everett, WA, USA). The difference between readings was within 0.3% for voltage and current (up to 10 A—limited by the range of the Fluke meter). After each test, the battery was allowed to cool for at least 2 h so that it was at ambient temperature before recharging. A Great Planes AC/DC Triton EQ Charger Balancer was used to recharge the battery. During discharge either power or current was held constant by monitoring the output of the Watt's Up meter and manually adjusting the batteries power or current output. This method was viable without implementing a control system as changes in power or current were slow. Using this approach, current could be maintained within 0.06 A and power within 0.24 W. Consequently, the worst case scenario for the presented data gives an error in the current of 2.4% and power of 0.7% respectively.

4. Discussion

The resulting discharge curves are shown in Figure 5a. For reference, also included in this plot is an experimental constant power ($P_e = 34$ W) discharge curve where both the voltage and current varied.

Correlation (application of Step 1) of the constant current discharge curves in Figure 5a yields Figure 5b with $n = 0.05$. As seen for the other data sets in Figures 1–3, collapse of the curves is good. Data for a 7.5 A discharge is included for completeness; however, after approximately 960 mAh discharge, the battery was no longer capable of maintaining power. We fitted the function defined by Equation (4) to the collapsed curves in Figure 5b then defined $i^n V$ as a function of D (with $a = 12.3063$, $b = -0.000328$, $c = -0.008112$, $d = -4.7809 \times 10^{-7}$, $e = -7.7835 \times 10^{-7}$, and $f = 1.4086 \times 10^{-10}$). Implementation of the constant power discharge procedure yielded the estimate shown in Figure 5c, presented in conjunction with the experimentally measured curve. In order to clarify the application procedure, Table 1 contains data for the first two iterations.

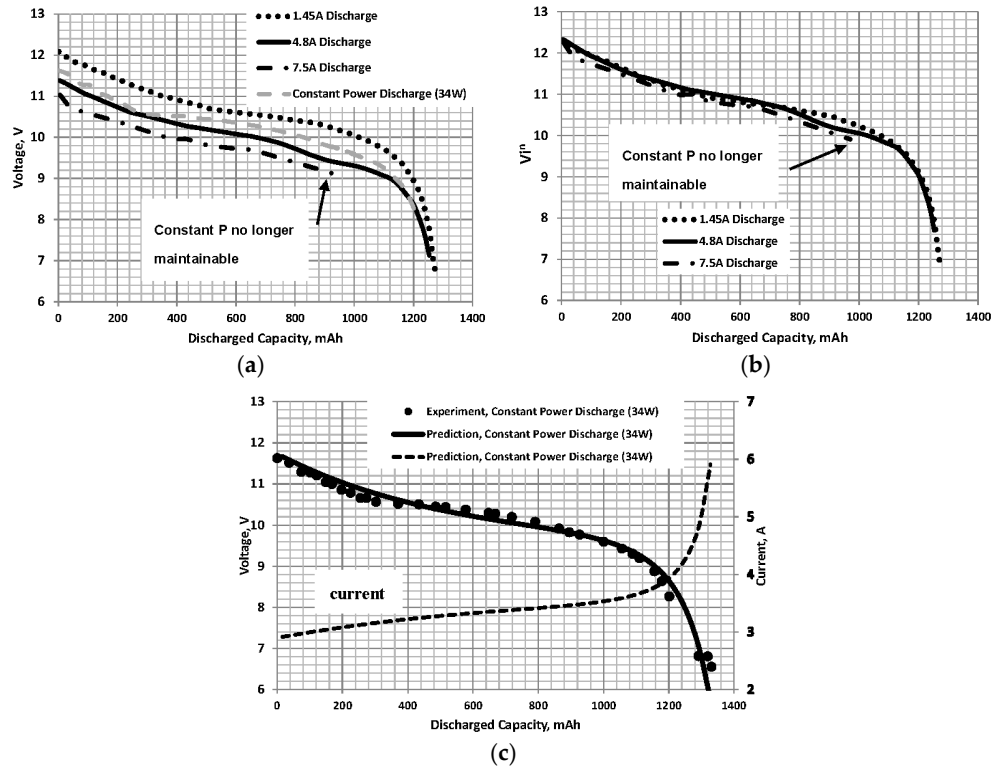


Figure 5. Electrify lithium polymer (LiPo) extreme: (a) constant current; and (b) correlated discharge curves. $n = 0.05$, and $C = 1300$ mAh. (c) Comparison of prediction and experiment for 34 W discharge.

Table 1. Initial calculations for 34 W discharge.

j	i_j (A)	V_j (V)	D (mAh)	t (h)
1	2.914	11.670	16.202	0.0056
2	2.931	11.601	32.498	0.0112

The steps for populating the table follow as:

- The time increment is given by $1/180 = 0.0056$ h (20 s);
- From Equation (5) with $j = 1$: $V_1 = \left(\frac{12.3063}{34^{0.05}} \right)^{1/(1-0.05)} = 11.67$ V; and
- Equation (6) gives: $i_1 = \frac{34}{11.67} = 2.914$ A;
- The discharged capacity follows from Equation (7) as: $D_1 = i_1 \Delta t + D_0 = 2.914 \times 10^3 \times (0.00556) + 0 = 16.202$ mAh;
- Advancing $j = 2$ and evaluating Equations (5)–(7) yields $V_2 = 11.601$ V, $i_2 = 2.931$ A and $D_2 = 32.498$ mAh.

It must be mentioned that published data for lithium batteries is not consistent in indicating the presence of capacity dependence on the rate of discharge (especially for high power cell technology). For the data sets presented in this article, with battery capacities and discharge rates consistent with that of a small unmanned aerial vehicle, significant rate dependence (which depends on the anode and cathode material as well as the electrolyte in the battery) is not evident. This result was also noted by Chang and Yu [19]. A rate dependency (or Peukert type effect) may only manifest at high discharge current values that would not be used if flying for maximum range or endurance. High discharge rates ($>20C$) may reduce the data collapse afforded by Equation (2). Consequently, if the application requires discharge rates above $20C$, the proposed method should be used cautiously as it may not be representative.

5. Conclusions

Estimates of range and endurance for battery powered aircraft are less well established than those for fossil fuel powered vehicles. A recently developed method to estimate these parameters (range and endurance) requires the variation of battery voltage at constant power as the battery is discharged. However, standard testing procedure for batteries involves discharge at constant current. Consequently, a procedure is developed to estimate constant power discharge curves for lithium batteries using information from constant current discharge data. The method is valid for high power cells and may not be applicable for high energy cells. Constant current discharge curves are shown to correlate when the voltage during the discharge is multiplied by the current raised to a power (with a value of ≈ 0.05). Curve fitting the correlation then allows for the determination of the voltage variation during discharge for a set power. Comparison of a prediction made using the correlation method and an experimental constant power discharge showed close accord. The correlation method is not shown to be applicable at very low temperatures and does require that the effective capacity is not affected by the discharge rate.

Conflicts of Interest: The author declares no conflict of interest.

Abbreviations

C	Capacity
D	Discharged capacity
e	Electrical
i	Current
j	Counter
K	Constant
M	Number of test cases
N	Time step increment
n	Exponent
P	Power
t	Time
V	Voltage
η_{tot}	Total efficiency of propulsion system

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