MDL Assignment 3, Part 2

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First Roll Number = 2019101004

Second Roll Number = 2019101024

Used Roll Number = 2019101004

LastFourDigitsOfRollNumber = 1004

x = 1 - (((LastFourDigitsOfRollNumber)%30 + 1) / 100) = 1 - 0.15 = 0.85

Success Reward = (RollNumber%90 + 10) = (2019101004%90 + 10) = 64

Few asusmptions:

- States are : [(A_i, A_j), (T_i, T_j), call] & sorted in ascending order.
- Whenever not specified we assume probabilities to be uniform.
- Discount factor is assumed to be 0.5.

Given Data

Positions :

- Possible Positions of an Agent or Target :

Pos_Grid	**col = 0**	**col = 1**	**col = 2**	**col = 3**
row = 0	(0, 0)	(0, 1)	(0, 2)	(0, 3)
row = 1	(1, 0)	(1, 1)	(1, 2)	(1, 3)

Target's Actions :

- Movement Probability Distribution of a Target is following:

Action	**STAY**	**UP**	**DOWN**	**LEFT**	**RIGHT**
Probability	0.6	0.1	0.1	0.1	0.1

- If Target try to move out of the grid world, it will remain at the same pos. with 0.1 probability.
- The calling functionality of Target is independent of its movement.

Action	**Call On**	**Call Off**
Probability	0.5	0.1

Transition probabilities for the agent :

- Since, x = 0.85 in our case. Hence, the following table:

Action	STAY	UP	DOWN	LEFT	RIGHT
Success Probability	1	0.85	0.85	0.85	0.85
Failure Probability	0	0.15	0.15	0.15	0.15

- For, failure of `Non-STAY` action, the agent moves in the opposite direction.
- For either of success or failure, if Agent try to move outside the grid world it will stay at the same position with given success or failure prob. resp.

Possible Observations from the Grid World :

- All observation have 100% accuracy.

Observation	Target's Position w.r.t Agent's position
o1	Same
o2	Right
o3	Below
o4	Left
o5	Above
o6	Not in the 1 cell neighbourhood

Rewards :

- -1 for each step that Agent takes.
- (RollNumber%90 + 10) = 64 for reaching the target before the call is turned off.

Question 1

Target Cell : (1, 0)

Observation : O6 with 100% accuracy

Therefore, initial equi-probable possible positions of the Agent: `(0,1)` , `(0,2)` , `(0,3)` , `(1,2)` and `(1,3)` . Also, for each cell the agent is likely to be in, the target is equally likely to be or not to be on a call.

Thus,start states are following:-

S = { `(0,1,1,0,0)` , `(0,2,1,0,0)` , `(0,3,1,0,0)` , `(1,2,1,0,0)` , `(1,3,1,0,0)` , `(0,1,1,0,1)` , `(0,2,1,0,1)` , `(0,3,1,0,1)` , `(1,2,1,0,1)` , `(1,3,1,0,1)` } and all of them are equally likely.

Clearly, `|s| = 10` .

Therefore, belief state `b` i.e. probability distribution over our set of states will be:

```
b(s) = 0.1 ∀ s ∈ S
otherwise b(s) = 0.
```

Initial Belief State:

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.1, 0, 0, 0, 0, 0, 0]

NOTE : The optimal policy file for the POMDP taking into account the obtained initial belief state `b` is `2019101004_2019101024.policy`.

Question 2

Agent Cell : (1, 1)

As target is in one cell neighbourhood of agent & not making call. Hence, possible equi-probable positions of the target are : `(0,1)`, `(1,0)`, `(1,1)` and `(1,2)`.

And target is not making call. Thus,start states are following:-

S = { `(1,1,0,1,0)`, `(1,1,1,0,0)`, `(1,1,1,1,0)`, `(1,1,1,2,0)` } and all of them are equally likely.

Clearly, `|s| = 4`.

Therefore, belief state `b` i.e. probability distribution over our set of states will be:

```
b(s) = 0.25 ∀ s ∈ S
otherwise b(s) = 0.
```

Initial Belief State:

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.25, 0, 0, 0, 0, 0, 0.25, 0, 0.25, 0, 0.25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

Question 3

for Q1:

The expected utility for Q1 was generated using the commands:

```
    python3 code.py
    ./sarsop/src/pomdpconvert 2019101004_2019101024.pomdp
    ./sarsop/src/pomdpsol 2019101004_2019101024.pomdpx
    ./sarsop/src/pomdpsim --simLen 100 --simNum 1000 --policy-file out.policy
2019101004_2019101024.pomdpx
```

expected utility: 13.614
95% confidence interval: (12.9975, 14.2285)

Q1 output

```
Loading the model ...
  input file   : 2019101004_2019101024.pomdpx

Loading the policy ...
  input file   : out.policy

Simulating ...
  action selection :  one-step look ahead

-----------------------------------
 #Simulations  | Exp Total Reward
-----------------------------------
 100             13.0744
 200             13.6369
 300             13.6711
 400             13.9158
 500             13.6625
 600             13.7338
 700             13.4707
 800             13.5196
 900             13.5264
 1000            13.613
-----------------------------------

Finishing ...

-------------------------------------------------------------
 #Simulations  | Exp Total Reward | 95% Confidence Interval
-------------------------------------------------------------
 1000            13.613             (12.9975, 14.2285)
-------------------------------------------------------------
```

for Q2:

The expected utility for Q2 was generated using the commands:

```
    python3 code.py
    ./sarsop/src/pomdpconvert q2.pomdp
    ./sarsop/src/pomdpsol q2.pomdpx
    ./sarsop/src/pomdpsim --simLen 100 --simNum 1000 --policy-file out.policy
q2.pomdpx
```

expected utility: 26.1293
95% confidence interval: (25.4966, 26.762)

Q2 output

```
Loading the model ...
  input file   : q2.pomdpx

Loading the policy ...
  input file   : out.policy

Simulating ...
  action selection :  one-step look ahead

-----------------------------------
 #Simulations  | Exp Total Reward
-----------------------------------
 100            25.0127
 200            24.9856
 300            25.8228
 400            26.1573
 500            26.0896
 600            26.0014
 700            25.9998
 800            26.1
 900            26.1483
 1000           26.1293
-----------------------------------

Finishing ...

-------------------------------------------------------------
 #Simulations  | Exp Total Reward | 95% Confidence Interval
-------------------------------------------------------------
 1000           26.1293            (25.4966, 26.762)
-------------------------------------------------------------
```

statistics used: simLen: 100
simNum: 1000

Question 4

- Agent's Possible position & probability

State	(0,0)	(1,3)
Probability	0.4	0.6

- Targets's Possible position & probability. (Call of Target doesn't matter as no observation detects it.)

State	(0,1)	(0,2)	(1,1)	(1,2)
Probability	0.25	0.25	0.25	0.25

Positions (Agent, Target)	Possible Observation	Probability
[(0,0),(0,1)]	o2	0.1
[(0,0),(0,2)]	o6	0.1
[(0,0),(1,1)]	o6	0.1
[(0,0),(1,2)]	o6	0.1
[(1,3),(0,1)]	o6	0.15
[(1,3),(0,2)]	o6	0.15
[(1,3),(1,1)]	o6	0.15
[(1,3),(1,2)]	o4	0.15

Now we have,

Probability(observation) = $\sum^{n}_{i=0}$ { Probability (observation | state) × Probability (state) }

So calculating this probability for every observation,

P(o1) = Σ (P(o1|state)*P(state)) = 0

P(o2) = Σ (P(o2|state)*P(state)) = (P(o2|Agent in (0,0) and Target in (0,1))*P(Agent in (0,0) and Target in (0,1))) = 1 * 0.1 = 0.1

P(o3) = Σ (P(o3|state)*P(state)) = 0

P(o4) = Σ (P(o4|state)*P(state)) = { P (o4 | (Agent in (1,3) and Target in (1,2))) * P (Agent in (1,3) and Target in (1,2))} = 1 * 0.15 = 0.15

P(o5) = Σ (P(o5|state)*P(state)) = 0

P(o6) = Σ (P(o6|state)*P(state))

= (P(o6|Agent in (0,0) and Target in (0,2)) * P(Agent in (0,0) and Target in (0,2))) +

(P(o6|Agent in (0,0) and Target in (1,1)) * P(Agent in (0,0) and Target in (1,1))) +

(P(o6|Agent in (0,0) and Target in (1,2)) * P(Agent in (0,0) and Target in (1,2))) +

(P(o6|Agent in and (1,3) Target in (0,1)) * P(Agent in (1,3) and Target in (0,1))) +

(P(o6|Agent in and (1,3) Target in (0,2)) * P(Agent in (1,3) and Target in (0,2))) +

(P(o6|Agent in (1,3) and Target in (1,1)) * P(Agent in (1,3) and Target in (1,1)))

=1 * 0.1 + 1 * 0.1 + 1 * 0.1 + 1 * 0.15 + 1 * 0.15 + 1 * 0.15 = 0.1 + 0.1 + 0.1 + 0.15 + 0.15 + 0.15 = 0.75

Hence, O6 is clearly the most like observation, as it has the highest probability.

Question 5

The number of policy tree obtained is equal to $|A|^{N}$; where

$N = \sum_i |o|^i$
for all $i \in \{ 0, 1, 2, \dots , T-1 \}$

$\Rightarrow N = [|O|_T - 1] / [|O| - 1]$

Where

$|A|$ denotes the number of actions,

$|O|$ denotes the number of observations,

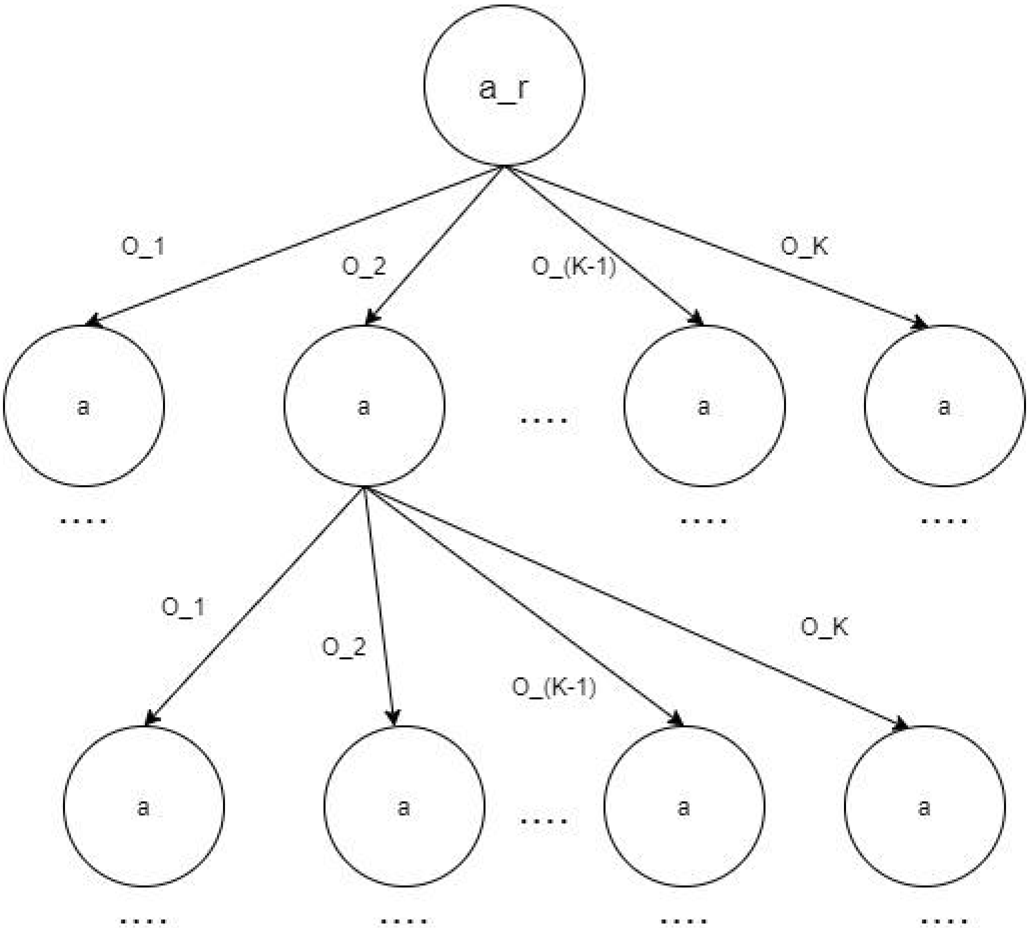
And T denotes the time horizon.

For our given case:-

$|A| = 5$ and $|O| = 6$ and T is the time horizon - unknown variable.

Thus, $N = [6^T - 1] / [6 - 1] = [6^T - 1] / [5]$

Therefore, total number of policy trees are $= |A|^N = 5^N$



The number of policy trees would increase exponentially as the time horizon increases because the convergence of nodes becomes more difficult as the time horizon increases. Hence, we get such a large number of policy trees.