

Numerical Solution of Diffusion equation

1-D Eq. & Solution

$$\left(\frac{\partial P(x, t)}{\partial t} \right) = D \left(\frac{\partial^2 P(x, t)}{\partial x^2} \right)$$

Solving above diffusion equation computationally Constraint:

$$(-L \leq x \leq L) \text{ \& \& } (0 \leq t \leq T)$$

$$P(x, 0) = 1 \text{ for } x=0 \text{ but } 0 \text{ otherwise}$$

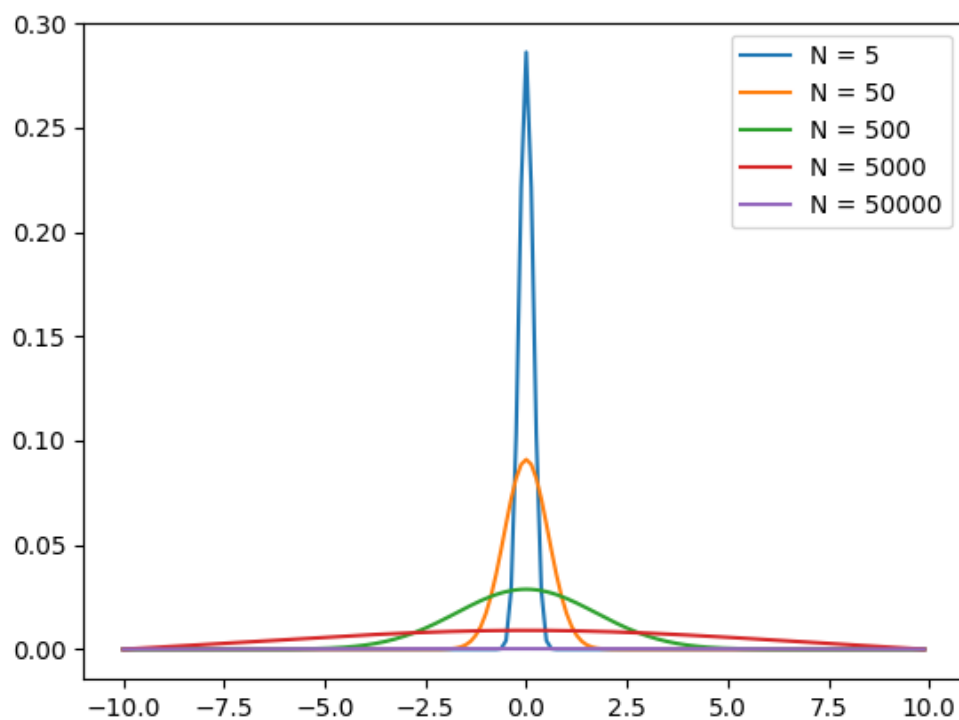
$$P(-L, t) = P(L, t) = 0$$

dt => time_step & dx => step_length

Using Formula:-

$$P(i, n+1) = P(i, n) + \left(D \Delta t / (\Delta x)^2 \right) [P(i+1, n) - 2P(i, n) + P(i-1, n)]$$

Graph:-



2-D Eq. & Solution

$$\left(\frac{\partial P(x, y, t)}{\partial t} \right) = D_x \left(\frac{\partial^2 P(x, y, t)}{\partial x^2} \right) + D_y \left(\frac{\partial^2 P(x, y, t)}{\partial y^2} \right)$$

Solving above diffusion equation computationally Constraint:

$(-L \leq x \leq L) \ \& \ (-L \leq y \leq L) \ \& \ (0 \leq t \leq T)$

$P(x, y, 0) = 1 \text{ for } x=y=0 \text{ but } 0 \text{ otherwise}$

$P(-L, 0, t) = P(L, 0, t) = P(0, -L, t) = P(0, L, t) = 0$

$dt \Rightarrow \text{time_step} \ \& \ dx \Rightarrow \text{x_step_length} \ \& \ dy \Rightarrow \text{y_step_length}$

Using Formula:-

$$a = (Dx * \Delta t / (\Delta x) * (\Delta x)) [P(i+1, j, n) - 2P(i, j, n) + P(i-1, j, n)]$$
$$b = (Dy * \Delta t / (\Delta y) * (\Delta y)) [P(i, j+1, n) - 2P(i, j, n) + P(i, j-1, n)]$$
$$c = (Dy * \Delta t / (\Delta x) * (\Delta x) * (\Delta y) * (\Delta y)) [$$

$$P(i+1, j+1, n) +$$
$$P(i-1, j-1, n) +$$
$$P(i-1, j+1, n) +$$
$$P(i+1, j-1, n) -$$
$$4 * P(i, j, n)$$

$$]$$
$$P(i, n+1) = P(i, n) + a + b + c$$

2D density plots

N (timestep)	Dx = 1 & Dy = 1	Dx = 3 & Dy = 1	Dx = 1 & Dy = 3
10			
100			
1000			
10000			

