Computational thinking :- (Lecture : 1)

① Sol^n : Let the person be A & B. Assume A moves x_1 moves to right & B moves x_2 moves/steps to right. So, their resp. left steps will be $N-x_1$ and $N-x_2$.

So, probability for A to moves x_1 steps to right out of N = $P_A(x)$

$$\Rightarrow P_A(x_1) = {}^N C_{x_1} \left(\frac{1}{2}\right)^{x_1} \left(\frac{1}{2}\right)^{N-x_1}$$; Prob. of making steps left & right are equal.

Similarly for B ;

$$P_B(x_2) = {}^N C_{x_2} \left(\frac{1}{2}\right)^{x_2} \left(\frac{1}{2}\right)^{N-x_2}$$

~~Ans~~

Now, displacement of A to right $= x_1 - (N - x_1)$

$= 2x_1 - N$

" " B " " $= x_2 - (N - x_2)$

$= 2x_2 - N$

As both meet after N steps So, $2x_1 - N = 2x_2 - N$

$\Rightarrow x_1 = x_2$.

i.e. both moves equal right steps.

thus ;

$$P_A(x) = P_B(x) = {}^N C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{N-x} = {}^N C_x \left(\frac{1}{2}\right)^N$$

where x is the number of steps to right by A & B both.

So, Prob. they meet after N steps again $\Rightarrow$

$$\sum_{x=0}^{N} P_A(x) \cdot P_B(x)$$

$$\Rightarrow \sum_{x=0}^{N} \left({}^N C_x\right)^2 \left(\frac{1}{2}\right)^{2N}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{2N} \sum_{x=0}^{N} \left({}^N C_x\right)^2$$

$$\Rightarrow \left(\frac{1}{2}\right)^{2N} \left({}^{2N} C_N\right)$$

$$\Rightarrow \frac{(2N)!}{\left(2^N \cdot (N!)\right)^2} .$$

Related Questions :-

(1) What is the prob. for a drunk to be at the origin after taking N steps.

⇒

We know prob. of taking x steps to right from N steps $= P(x)$

$$\Rightarrow P(x) = {}^N C_x \left(\frac{1}{2}\right)^N$$; equal prob. for left & right

So, if N is odd then prob. to return to origin $= 0$ because $(N/2)$ will not be zero & obviously not possible.

Now, N = even then

req. prob. $= P(x = N/2)$

$$= {}^N C_{(N/2)} \left(\frac{1}{2}\right)^N = \frac{N!}{((N/2)!)^2}\left(\frac{1}{2}\right)^N$$

(2) MEAN Displacement of the Drunk ?

(3) MEAN Sq. Displacement of Drunk ?

(2) Mean Displacement of the drunk i.e. expected displacement ?

Let the person walks x out of N steps to right. So, displacement $= d = x-(N-x)$

$\Rightarrow d = 2x - N$

Mean displacement $= E[d]$ i.e. expected value of d.

$\langle d \rangle = E[2x-N]$

$\langle d \rangle = 2E[x] - N$; due to linearity

We know for given N, $x \in R_X = \{0,1,--N\}$

So, $E[x] = \sum_{x=0}^{N} x P(x)$; $P(x)$ is prob. of x right steps out of N.

$E[x] = \sum_{x=0}^{N} x \, {}^N C_x \left(\frac{1}{2}\right)^N$

$E[x] = \left(\frac{1}{2}\right)^N \sum_{x=0}^{N} x \, {}^N C_x = \left(\frac{1}{2}\right)^N \left(N 2^{N-1}\right)$

$= N/2$

So, $\langle d \rangle = 2 \cdot \left(\frac{1}{2}\right)^N \left(N 2^{N-1}\right) - N = 0$.

(3) Mean Sq. displacement of the drunk i.e. variance of 'd'

$\text{VAR}(d) = E[d^2] - (\cancel{E[d]})^2$ (general definition of variance)

$$E[d^2] = E[(2x-N)^2] = E[4x^2 - 4xN + N^2]$$

$$E[d^2] = 4E[x^2] - 4NE[x] + N^2$$

; due to linearity

We already know $E[x] = N/2$

Finding $E[x^2]$;

$$E[x^2] = \sum_{x=0}^{N} x^2 \cdot P(x)$$; $P(x)$ is prob. of x right steps out of N.

$$= \sum_{x=0}^{N} x^2 \, {}^N C_x \left(\frac{1}{2}\right)^N$$

$$= \left(\frac{1}{2}\right)^N \sum_{x=0}^{N} x^2 \, {}^N C_x$$

$$= \left(\frac{1}{2}\right)^N \left[N \cdot 2^{N-1} + N \cdot (N-1)\, 2^{N-2}\right]$$

So, $$E[d^2] = 4\left(\frac{1}{2}\right)^N \left[N \cdot 2^{N-1} + N(N-1)\, 2^{N-2}\right] - 4N\left(\frac{N}{2}\right) + N^2$$

$$Var(d) = E[d^2] = \left[2N + N(N-1)\right] - 2N^2 + N^2$$

$$= N + N^2 - N^2$$

$$Var(d) = N.$$