STANDARD RESULTS

1. $\frac{d}{dx}(x^n) = nx^{n-1}$

2. $\frac{d}{dx}(a^x) = a^x \log_e a$

3. $\frac{d}{dx}(e^x) = e^x$

4. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

5. $\frac{d}{dx}(\log_{10} x) = \frac{1}{x} \log_{10} e$

6. $\frac{d}{dx}(\sin x) = \cos x$

7. $\frac{d}{dx}(\cos x) = -\sin x$

8. $\frac{d}{dx}(\tan x) = \sec^2 x$

9. $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$

10. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

11. $\frac{d}{dx}(\sec x) = \sec x \tan x$

12. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

13. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

16. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

17. $\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

18. $\sinh x = \frac{e^x - e^{-x}}{2}$

19. $\cosh x = \frac{e^x + e^{-x}}{2}$

20. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

21. $\cosh^2 x - \sinh^2 x = 1$, $\text{sech}^2 x + \tanh^2 x = 1$, $\coth^2 x = 1 + \text{cosech}^2 x$

22. $\cosh^2 x + \sinh^2 x = \cosh 2x$

23. $\sinh^{-1} x = \log(x + \sqrt{x^2+1})$, $\cosh^{-1} x = \log(x + \sqrt{x^2-1})$

24. $\frac{d}{dx}(\sinh x) = \cosh x$

25. $\frac{d}{dx}(\cosh x) = \sinh x$

26. $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$

27. $\frac{d}{dx}(\coth x) = -\text{cosech}^2 x$

28. $\frac{d}{dx}(\text{sech}\, x) = -\text{sech}\, x \tanh x$

29. $\frac{d}{dx}(\text{cosech}\, x) = -\text{cosech}\, x \coth x$

30. Product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

31. Quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

32. $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$ if $y = f_1(t)$ and $x = f_2(t)$

33. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$

34. $\tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1} a - \tan^{-1} b$, $\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1} a + \tan^{-1} b$

35. $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x$

36. $\sin 3x = 3\sin x - 4\sin^3 x$, $\cos 3x = 4\cos^3 x - 3\cos x$, $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\sin 2x = 2\sin x \cos x$, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$,

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$

37. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$, $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$; $|x| < 1$ $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$ $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$

38. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$, $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$, $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$,

39. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$, $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$, $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

40. $\sin(A+B) = \sin A\cos B + \cos A\sin B$, $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$, $\cos(A-B) = \cos A\cos B + \sin A\sin B$

41. $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$, $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$, where $|x| < 1$, $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{x^2-1}$, where $|x| > 1$

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$, $\frac{d}{dx}(\operatorname{cosech}^{-1} x) = -\frac{1}{x\sqrt{x^2+1}}$.

42. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$

43. $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta - \tan^2\theta = 1$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

44.

θ	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	-1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0

45.

θ	$90° - \theta$	$90° + \theta$	$\pi - \theta$	$\pi + \theta$
$\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$
$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$
$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$

46. sine formula: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$; cosine formula: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

47. Area of triangle $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

48. ${}^nC_r = \dfrac{n!}{r!\,n-r!}$

49. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c;\ n \neq -1$

$\int \dfrac{1}{x}\,dx = \log_e x + c;\ \int e^x\,dx = e^x + c;\ \int a^x\,dx = \dfrac{a^x}{\log_e a} + c$

$\int \sin x\,dx = -\cos x + c;\ \int \cos x\,dx = \sin x + c$

$\int \tan x\,dx = \log \sec x + c;\ \int \cot x\,dx = \log \sin x + c$

$\int \sec x\,dx = \log(\sec x + \tan x) + c = \log \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right) + c$

$\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x) + c = \log \tan \dfrac{x}{2} + c$

$\int \sec x \tan x\,dx = \sec x + c;\ \int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

50. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + c;\ \int \dfrac{-dx}{\sqrt{a^2 - x^2}} = \cos^{-1}\left(\dfrac{x}{a}\right) + c$

$\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + c;\ \int \dfrac{-dx}{a^2 + x^2} = \dfrac{1}{a}\cot^{-1}\left(\dfrac{x}{a}\right) + c$

$\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\log\left(\dfrac{a+x}{a-x}\right) + c;\ \int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\left(\dfrac{x-a}{x+a}\right) + c$

$\int \dfrac{dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\left(\dfrac{x}{a}\right) + c;\ \int \dfrac{-dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\operatorname{cosec}^{-1}\left(\dfrac{x}{a}\right) + c$

51. $\int \text{sech}^2 x\, dx = \tanh x + c$, $\int \text{cosech}^2 x\, dx = -\coth x + c$

$\int \sinh x\, dx = \cosh x + c$, $\int \cosh x\, dx = \sinh x + c$

$\int \text{sech}\, x \tanh x\, dx = -\text{sech}\, x + c$, $\int \text{cosech}\, x \coth x\, dx = -\text{cosech}\, x + c$

52. $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a} + c$

$\int \sqrt{a^2 + x^2}\, dx = \frac{1}{2} x \sqrt{a^2 + x^2} + \frac{1}{2} a^2 \log (x + \sqrt{a^2 + x^2}) + c$

$\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \log (x + \sqrt{x^2 - a^2}) + c$

$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \left(\frac{x}{a}\right) + c$; $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \left(\frac{x}{a}\right) + c$

53. $\int_a^b f(x)\, dx = \int_a^b f(y)\, dy$; $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$; $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

$$\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx, & \text{if } f(x) \text{ is even function} \\ 0, & \text{if } f(x) \text{ is odd function} \end{cases}$$

$$\int_0^{2a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx, & \text{if } f(2a - x) = f(x) \\ 0, & \text{if } f(2a - x) = -f(x) \end{cases}$$

54. Leibnitz rule for differentiation under the integral sign

$$\frac{d}{dx} \int_{\phi(\alpha)}^{\psi(\alpha)} f(x, \alpha)\, dx = \int_{\phi(\alpha)}^{\psi(\alpha)} \frac{\partial}{\partial \alpha} \{f(x, \alpha)\}\, dx + f\{\psi(\alpha), \alpha\} \frac{d\psi(\alpha)}{d\alpha} - f\{\phi(\alpha), \alpha\} \frac{d\phi(\alpha)}{d\alpha}$$

55. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ and $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

56. $\overrightarrow{AB}$ = position vector of B-position vector of A = $\overrightarrow{OB} - \overrightarrow{OA}$

57. $\vec{a} \cdot \vec{b} = |\vec{a}|\, |\vec{b}| \cos \theta$; work done = $\int_c \vec{F} \cdot \vec{dr}$

58. $\vec{a} \times \vec{b} = |\vec{a}|\, |\vec{b}| \sin \theta\, \hat{n}$

59. Area of parallelogram = $\vec{a} \times \vec{b}$, Moment of force = $\vec{r} \times \vec{F}$

60. $\vec{a}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (\vec{a}\times\vec{b})\cdot\vec{c}$

where $\vec{a} = \Sigma a_1\hat{i}$, $\vec{b} = \Sigma b_1\hat{i}$ and $\vec{c} = \Sigma c_1\hat{i}$

If $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$, then $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

61. $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c}$

62. $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$

63. $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\,\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\,\vec{d}$

64. $A\,(\text{Adj. } A) = |\,A\,|\,I$

65. $AA^{-1} = I = A^{-1}A$

66. $AI = A = IA$

67. $(ABC)' = C'B'A'$

68. $(AB)C = A(BC)$; $A(B + C) = AB + AC$

69. $A + B = B + A$; $A + (B + C) = (A + B) + C$

70. $(AB)^{-1} = B^{-1}A^{-1}$

71. **Walli's formula**

$$\int_0^{\pi/2} \sin^n\theta\, d\theta = \int_0^{\pi/2} \cos^n\theta\, d\theta = \begin{cases} \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\cdots\dfrac{3}{4}\cdot\dfrac{1}{2}\cdot\dfrac{\pi}{2} & \text{if } n \text{ is even} \\ \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\cdots\dfrac{4}{5}\cdot\dfrac{2}{3} & \text{if } n \text{ is odd} \end{cases}$$

72. $\int e^{ax}\sin bx\, dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$

$\int e^{ax}\cos bx\, dx = \dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

73. $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(-1/2) = -2\sqrt{\pi}$

74. $\log(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \dfrac{x^5}{5} - \dfrac{x^6}{6} + \cdots\cdots$

$\log(1-x) = -x - \dfrac{x^2}{2} - \dfrac{x^3}{3} - \dfrac{x^4}{4} - \dfrac{x^5}{5} - \dfrac{x^6}{6} - \cdots\cdots$

75. $\sin n\pi = 0$; $\cos n\pi = (-1)^n$, $\sin\left(n+\dfrac{1}{2}\right)\pi = (-1)^n$; $\cos\left(n+\dfrac{1}{2}\right)\pi = 0$, where $n \in I$

76. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.