Congratulations! You passed!

TO PASS 80% or higher

Keep Learning

GRADE
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Quiz 1

LATEST SUBMISSION GRADE

100%

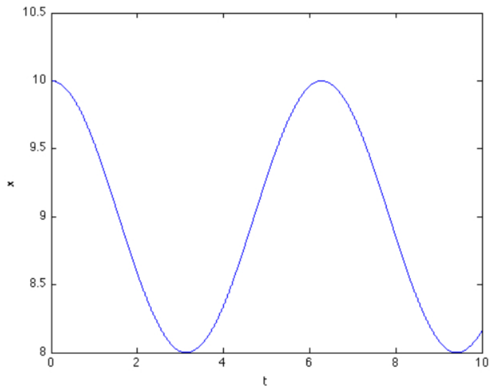
1. One way of getting a general feeling for what a differential equation is up to is to look at the sign and magnitude of the derivative at different points for different values of x. Use this idea for the dynamics

   1 / 1 point

   $\dot{x}(t) = -x(t)^3.$

   Which one of the plots below (where t is on the ``x-axis" and $x(t)$ is on the ``y-axis") was generated by this system? Note that $x(0) = 10$.

   ○



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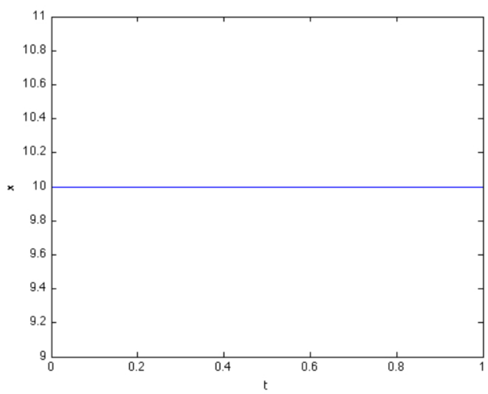

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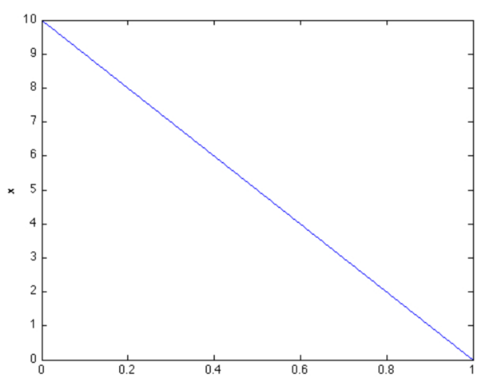

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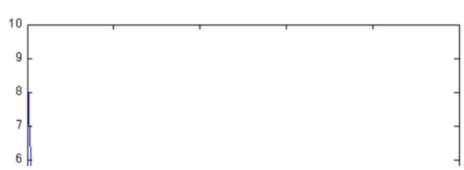

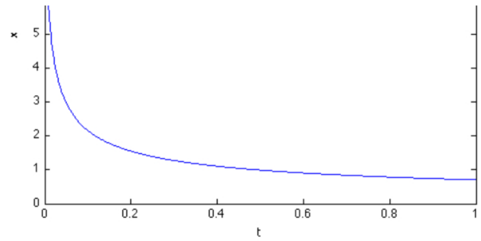

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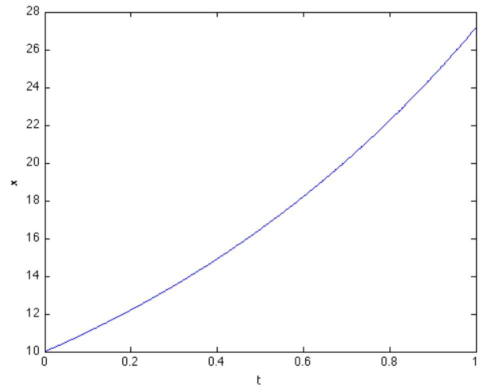

✓ **Correct**

The first thing to check is what the axes and the initial condition for each plot is; in our case, these are plots of $x(t)$ that start at $x_0 = 10$. Thus, the correct plot will (at least initially) have a negative rate of change ($\dot{x}(0) = -1000$ to be exact). Working through using process of elimination we find the correct plot.

2. One way of modeling epidemics is to describe how the fraction of infected individuals evolves over time. Let I be that fraction, with the model being

1 / 1 point

$$\dot{I} = \beta I(1 - I) - \rho I.$$

Here, the constants β and ρ are the infection and recovery rates, respectively.

What are the possible equilibrium points to this system (values for I when the fraction of infected individuals is not changing)?

- ○ Only when $I = 0$
- ○ Only when $I = (\beta - \rho)/\beta$
- ○ Only when $I = (1 - \beta)/\rho$
- ○ When $I = 0$ or $I = (1 - \beta)/\rho$
- ◉ When $I = 0$ or $I = (\beta - \rho)/\beta$

✓ **Correct**

We need to see what happens when $I(t)$ does not change, i.e., when $\dot{I} = 0$. So, solve $\beta I(1 - I) - \rho I = 0$. This is a 2nd-order polynomial equation, which means it has two solutions (although the two solutions may be the same).

3. If someone gives you a possible solution to a differential equation, one of the checks needed to see if this is indeed a solution is by taking the required number of derivatives and seeing if the proposed solution does in fact satisfy the differential equation.

1 / 1 point

Let

$$\ddot{x}(t) = -\omega^2 x(t).$$

Which of the following options is *not* a possible solution to this equation?

- ○ $x(t) = \cos(\omega t)$
- ◉ $x(t) = e^{-\omega t}$
- ○ $x(t) = \sin(\omega t)$
- ○ $x(t) = \omega \sin(\omega t) - \cos(\omega t)$
- ○ $x(t) = 0$

✓ **Correct**

Plugging in each function and differentiating twice we get a second derivative that is not the same as the function in the question for one of the choices.

4. We saw that the model of a cruise-controller could be given by

$\dot{x} = \frac{c}{m}u - \gamma x,$

where u is the input, x is the speed of the car, and c, m, γ are constant parameters.

If there was no wind resistance in the cruise-control model $(\gamma = 0)$, what would the steady-state values be for the velocity x when using a pure D-regulator, i.e., when $u = k\dot{e} = k(\dot{r} - \dot{x}) = -k\dot{x}$ (since r is constant)?

1 / 1 point

- ○ $x(\infty) > r$
- ○ $x(\infty) = 0$
- ◉ Impossible to say
- ○ $x(\infty)$ less than r
- ○ $x(\infty) = r$

✓ **Correct**

Plugging in our choice of controller $u = -k\dot{x}$ and $\gamma = 0$ as well we get: $\dot{x} = -k\frac{c}{m}\dot{x}$, which is no longer a differential equation. Think in terms of discrete time: $x_{k+1} = x_k + dt``\dot{x}"$ but what is $\dot{x}$ now? And how can we determine x now?

5. Let a discrete-time system be given by

$x_{k+1} = \max\{0, 5 - x_k\}.$

If this system starts at $x_0 = 10$, what happens to the state of the system?

1 / 1 point

- ○ It keeps growing from 10 up to ∞
- ○ It jumps down to $x_1 = 0$ and remains at 0 for ever
- ◉ It keeps switching between 0 and 5
- ○ It keeps switching between 0 and -5
- ○ It jumps down to $x_1 = 0$ and increases up by one until $x_5 = 5$ and then jumps back to 0 again (and the process repeats)

✓ **Correct**

Start by plugging in $x_0 = 10$. Find x_1. Keep plugging in, using this discrete update rule, until you see the pattern start to repeat. Be careful -- it's easy to make simple mistakes in your addition!