Congratulations! You passed!

TO PASS 80% or higher

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GRADE
80%

Quiz 3

LATEST SUBMISSION GRADE

80%

1. This question investigates how to obtain linearized models of non-linear dynamical systems.

1 / 1 point

A controlled pendulum with friction can be described by

$$m\ell\ddot{\phi} = -mg\sin\phi - k\ell\dot{\phi} + \tau,$$

where ϕ is the angle of the pendulum and τ is the input torque applied at the base of the pendulum. m is the mass of the pendulum, ℓ its length, g is the gravitational constant, and k is a friction coefficient.

Linearize this model around the two equilibrium points, i.e., when the pendulum is hanging straight down ($\phi = 0$) and balancing straight up ($\phi = \pi$). Let $x_1 = \phi, x_2 = \dot{\phi}$, and $u = \tau$, giving the linearized system $\dot{x} = Ax + Bu$, where

$$B = \begin{bmatrix} 0 \\ \frac{1}{m\ell} \end{bmatrix}.$$

But what is A? Which of the following A-matrices give the correct linearizations around the two equilibrium points?

- [x] $\phi = 0: \ A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{\ell} & -\frac{k}{m} \end{bmatrix}$
  $\phi = \pi: \ A = \begin{bmatrix} 0 & 1 \\ \frac{g}{\ell} & -\frac{k}{m} \end{bmatrix}$
- [ ] $\phi = 0: \ A = \begin{bmatrix} 1 & 0 \\ -\frac{g}{\ell} & -\frac{k}{m} \end{bmatrix}$
  $\phi = \pi: \ A = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{k}{m} \end{bmatrix}$
- [ ] $\phi = 0: \ A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{g}{\ell} \end{bmatrix}$
  $\phi = \pi: \ A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}$
- [ ] $\phi = 0: \ A = \begin{bmatrix} 1 & 0 \\ -\frac{k}{m} & -\frac{g}{\ell} \end{bmatrix}$
  $\phi = \pi: \ A = \begin{bmatrix} 1 & 0 \\ -\frac{k}{m} & 0 \end{bmatrix}$
- [ ] $\phi = 0: \ A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{k}{m} \end{bmatrix}$
  $\phi = \pi: A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{\ell} & -\frac{k}{m} \end{bmatrix}$

Correct
Follow the procedure given in the lecture to compute A and B. Hint: Confirm your linearization against the given B matrix, for a sanity check!

2. Which of the following statements is correct?

1 / 1 point

- [ ] The unicycle model has a useful linearization.
- [x] The stability properties of $\dot{x} = Ax$ are determined by A's eigenvalues.
- [ ] Output-feedback is more general than state-feedback.
- [ ] Most systems are already linear so linearizations are not really needed.
- [ ] Stability is not a control objective when designing controllers.

Correct

This question tests your understanding of the vocabulary introduced so far. If you are having trouble with the question, see what words you don't understand and go back and find them in the lectures and/or notes.

3. This question involves the computation of eigenvalues and how these eigenvalues relate to the stability properties of the system. **1 / 1 point**

Given a linear system

$$\dot{x} = \begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix} x.$$

Which of the options below captures all the values of α and β for which the system is asymptotically stable?

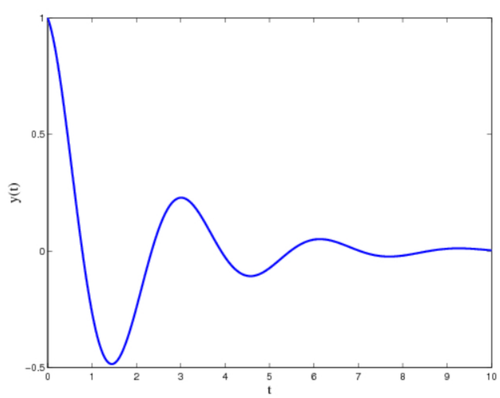
Hint: The following fact might come in handy: The determinant of a 2×2 matrix is given by $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

- ( ) α whatever, $\beta < 0$
- ( ) $\alpha < 0,\ \beta < 0$
- ( ) Impossible to say -- not enough information provided.
- ( ) $\alpha < 0,\ \beta > 0$
- (•) $\alpha < 0,\ \beta$ whatever

Correct

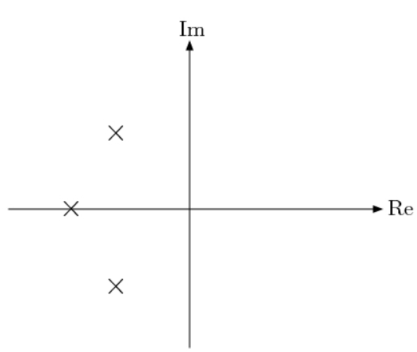
The keys to this question are to compute the eigenvalues of the general matrix given, and then to relate these to the criteria for an asymptotically stable matrix.

4. Let the output of a third order system ($\dot{x} = Ax,\ y = Cx$) be given in the figure below: **1 / 1 point**



The options below show the possible placements of the eigenvalues to the A matrix, with the axis being the real parts and the imaginary parts of the eigenvalues. Which option corresponds to the system used to generate the figure?

(•)



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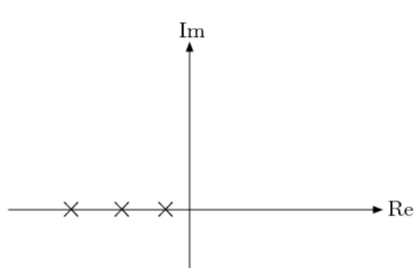

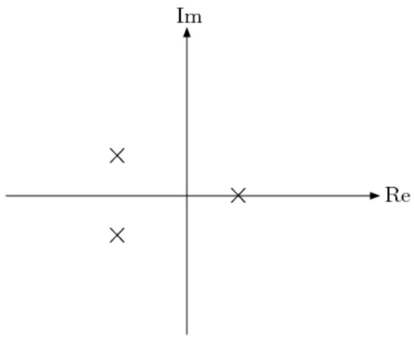

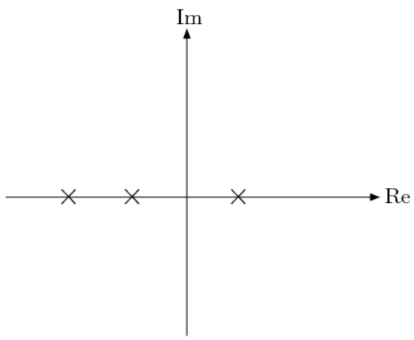

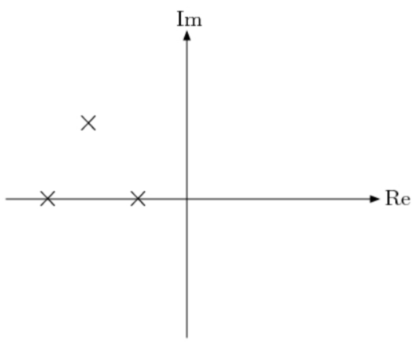

Correct

Notice whether the system behavior plotted in the question exhibits stability and/or oscillations. Then, remember the relationship between the location (or value) of poles of a linear system in the complex plane to stability and oscillations in the resulting system behavior. Then, process of elimination yields one correct option.

5. Recall the quadrotor model from Module 1

0 / 1 point

$$\ddot{h} = cv - g,$$

where h is the altitude of the quadrotor and v is the input. The purpose of this question is to revisit PID design but in the context of state-feedback to see that there really isn't anything magical/special about PID. So, please keep reading and follow along in the math...

First, if we directly compensate for gravity by introducing a new control input $v = u + g/c$ we get the much simpler system $\ddot{h} = cu$.

Now, let $x_1 = h,\ x_2 = \dot{h}$ and the resulting linear system is

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ c \end{bmatrix} u.$$

As seen, a PD regulator is $u = K_P e + K_D \dot{e} = K_P(r - h) + K_D(\dot{r} - \dot{h})$ where r is the reference signal. In other words

$$u = K_{PD} \begin{bmatrix} r \\ \dot{r} \end{bmatrix} - K_{PD} x,$$

where $K_{PD} = [K_P, K_D]$,

i.e., it is just a state-feedback controller, and the closed-loop system becomes

$$\dot{x} = (A - BK_{PD})x + BK_{PD} \begin{bmatrix} r \\ \dot{r} \end{bmatrix}.$$

So doing state-feedback is exactly the same as PD control for this system.

But what about the I-part? This requires some more work. For this we actually need to introduce another, extra state. Let the new state be

$$x_3(t) = \int_0^t x_1(\tau) d\tau.$$

The resulting system has three states instead of two and if we let

$$\tilde{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

we get the closed-loop PID dynamics

$$\dot{\tilde{x}} = (\tilde{A} - \tilde{B}K_{PID})\tilde{x} + \tilde{B}K_{PID} \begin{bmatrix} r \\ \dot{r} \\ \int r \end{bmatrix},$$

where $K_{PID} = [K_P, K_D, K_I]$ and where

$$\tilde{B} = \begin{bmatrix} 0 \\ c \\ 0 \end{bmatrix}.$$

But, which of the options below give the correct $\tilde{A}$? (Write out the dynamics of the new, third order system in order to figure out the new A-matrix.)

- ○ $\tilde{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
- ○ $\tilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- ○ $\tilde{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
- ○ $\tilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
- ◉ $\tilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

! **Incorrect**