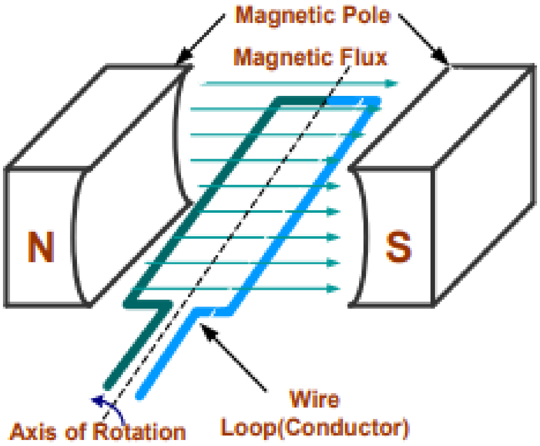
Single - Phase AC Circuits

1 Equation for generation of alternating induce EMF

- An AC generator uses the principle of Faraday's electromagnetic induction law. It states that when current carrying conductor cut the magnetic field then emf induced in the conductor.
- Inside this magnetic field a single rectangular loop of wire rotes around a fixed axis allowing it to cut the magnetic flux at various angles as shown below figure



Where,
N =No. of turns of coil
A = Area of coil (m^2)
ω=Angular velocity (radians/second)
ϕ_m= Maximum flux (wb)

Figure Generation of EMF

- When coil is along XX' (perpendicular to the lines of flux), flux linking with coil= ϕ_m. When coil is along YY' (parallel to the lines of flux), flux linking with the coil is zero. When coil is making an angle θ with respect to XX' flux linking with coil, $\phi = \phi_m \cos\omega t$ [$\theta = \omega t$].

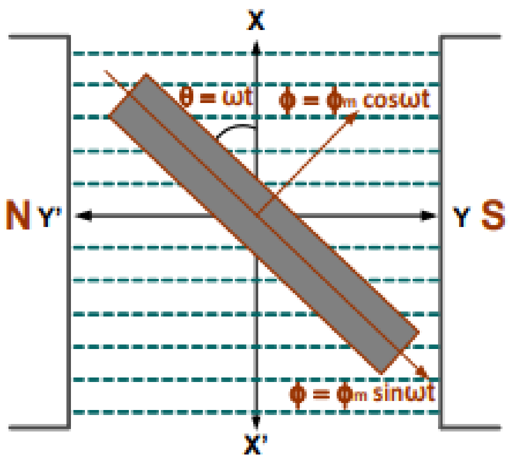


Figure Alternating Induced EMF

- According to Faraday's law of electromagnetic induction,

$$e = -N\frac{d\phi}{dt}$$

$$e = -Nd\frac{(\phi_m \cos\omega t)}{dt}$$

$$e = -N\phi_m(-\sin\omega t)\times\omega$$

$$e = N\phi_m\omega\sin\omega t$$

$$e = E_m \sin\omega t$$

Where,

$E_m = N\phi_m\omega$

$N =$ no. of turns of the coil

$\phi_m = B_m A$

$B_m =$ Maximum flux density (wb/m^2)

$A =$ Area of the coil (m^2)

$\omega = 2\pi f$

$$\therefore e = N B_m A 2\pi f \sin\omega t$$

- Similarly, an alternating current can be express as

  $i = I_m \sin\omega t$ Where, I_m = Maximum values of current

- Thus, both the induced emf and the induced current vary as the sine function of the phase angle $\omega t = \theta$. Shown in figure 2.3.

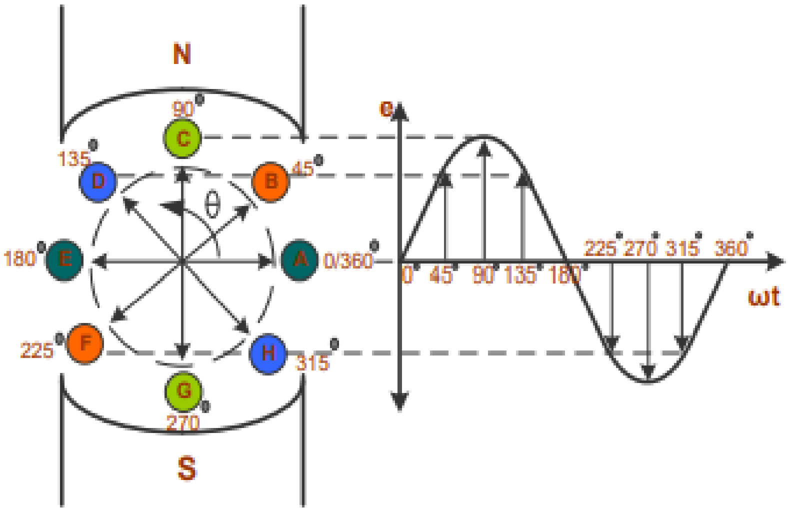


Figure 2.3 Waveform of Alternating Induced EMF

Phase angle	Induced emf $e = E_m \sin\omega t$
$\omega t = 0^0$	$e = 0$
$\omega t = 90^0$	$e = E_m$
$\omega t = 180^0$	$e = 0$
$\omega t = 270^0$	$e = -E_m$
$\omega t = 360^0$	$e = 0$

2.2 Definitions

➢ **Waveform**

It is defined as the graph between magnitude of alternating quantity (on Y axis) against time (on X axis).

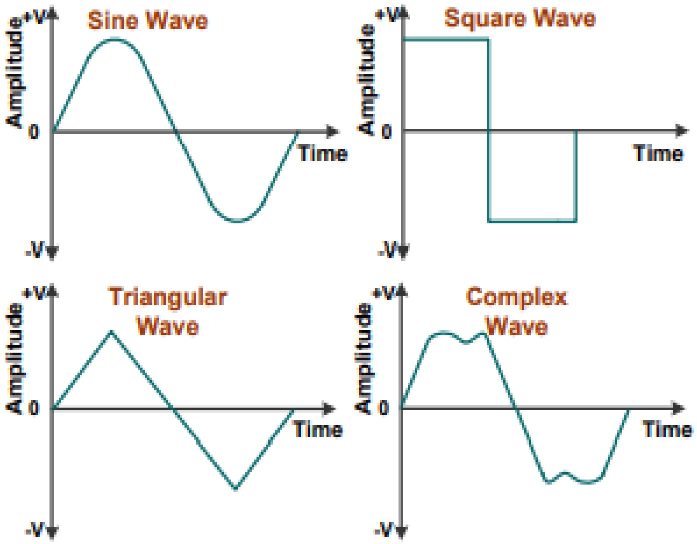


Figure 2.4 A.C. Waveforms

➢ **Cycle**

It is defined as one complete set of positive, negative and zero values of an alternating quantity.

- **Instantaneous value**

  It is defined as the value of an alternating quantity at a particular instant of given time. Generally denoted by small letters.

  e.g. i= Instantaneous value of current

  v= Instantaneous value of voltage

  p= Instantaneous values of power

- **Amplitude/ Peak value/ Crest value/ Maximum value**

  It is defined as the maximum value (either positive or negative) attained by an alternating quantity in one cycle. Generally denoted by capital letters.

  e.g. I_m= Maximum Value of current

  V_m= Maximum value of voltage

  P_m= Maximum values of power

- **Average value**

  It is defined as the average of all instantaneous value of alternating quantities over a half cycle.

  e.g. V_{ave} = Average value of voltage

  I_{ave} = Average value of current

- **RMS value**

  It is the equivalent dc current which when flowing through a given circuit for a given time produces same amount of heat as produced by an alternating current when flowing through the same circuit for the same time.

  e.g. V_{rms} =Root Mean Square value of voltage

  I_{rms} = Root Mean Square value of current

- **Frequency**

  It is defined as number of cycles completed by an alternating quantity per second. Symbol is f. Unit is Hertz (Hz).

- **Time period**

  It is defined as time taken to complete one cycle. Symbol is T. Unit is seconds.

- **Power factor**

  It is defined as the cosine of angle between voltage and current. Power Factor = pf = $\cos\phi$, where ϕ is the angle between voltage and current.

- **Active power**

  It is the actual power consumed in any circuit. It is given by product of rms voltage and rms current and cosine angle between voltage and current. ($VI\cos\phi$).

  Active Power= $P= I^2R = VI\cos\phi$.

  Unit is Watt (W) or kW.

- **Reactive power**

  The power drawn by the circuit due to reactive component of current is called as reactive power. It is given by product of rms voltage and rms current and sine angle between voltage and current (VI sinϕ).

  Reactive Power = Q= I^2X = VIsinϕ.

  Unit is VAR or kVAR.

- **Apparent power**

  It is the product of rms value of voltage and rms value of current. It is total power supplied to the circuit.

  Apparent Power = S = VI.

  Unit is VA or kVA.

- **Peak factor/ Crest factor**

  It is defined as the ratio of peak value (crest value or maximum value) to rms value of an alternating quantity.

  Peak factor = Kp = 1.414 for sine wave.

- **Form factor**

  It is defined as the ratio of rms value to average value of an alternating quantity. Denoted by K_f. Form factor K_f = 1.11 for sine wave.

- **Phase difference**

  It is defined as angular displacement between two zero values or two maximum values of the two-alternating quantity having same frequency.

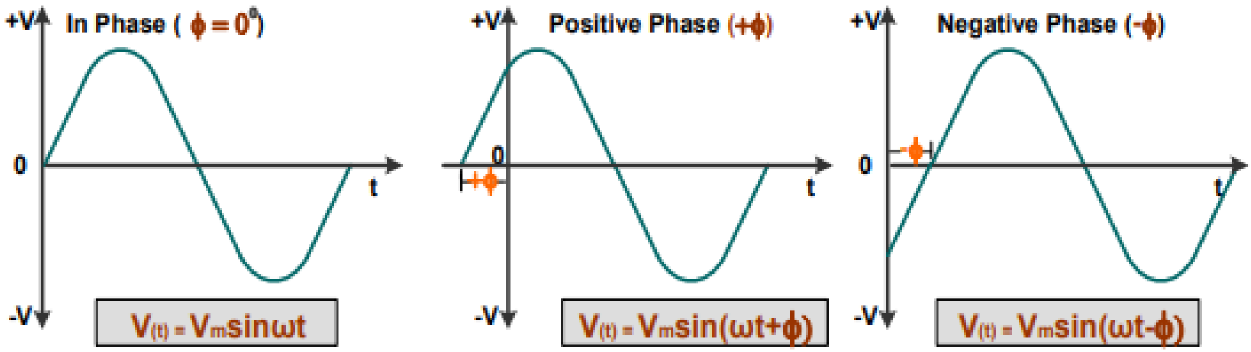


Figure 2.5 A.C. Phase Difference

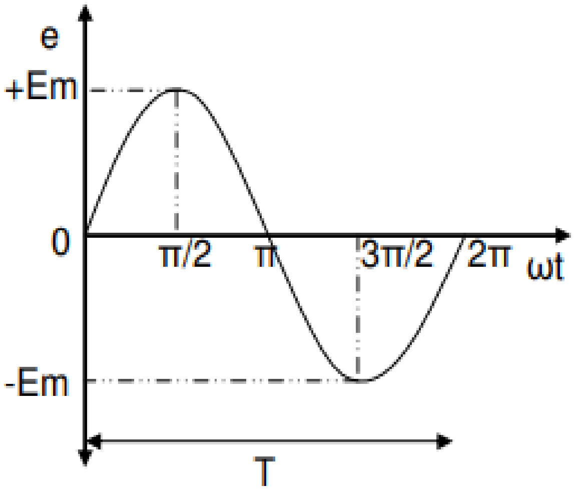
- **Leading phase difference**

  A quantity which attains its zero or positive maximum value before the compared to the other quantity.

- **Lagging phase difference**

  A quantity which attains its zero or positive maximum value after the other quantity.

Definition of Alternating Quantity



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time. Important terms associated with an alternating quantity are defined below.

1. Amplitude

   It is the maximum value attained by an alternating quantity. Also called as maximum or peak value

2. Time Period (T)

   It is the Time Taken in seconds to complete one cycle of an alternating quantity

3. Instantaneous Value

   It is the value of the quantity at any instant

4. Frequency (f)

   It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec. The relationship between frequency and time period can be derived as follows.

   Time taken to complete f cycles = 1 second

   Time taken to complete 1 cycle = 1/f second

$$T = 1/f$$

Angular Frequency (ω)

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Problem 1

An alternating current i is given by

i = 141.4 sin 314t

Find i) The maximum value

ii) Frequency

iii) Time Period

iv) The instantaneous value when t=3ms

i = 141.4 sin 314t

$$i = I_m \sin \omega t$$

i) Maximum value Im=141.4 V

ii) ω = 314 rad/sec

f = ω/2π = 50 Hz

iii) T=1/f = 0.02 sec

iv) i=141.4 sin(314x0.003) = 114.35A

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

$$\text{Average value} = \frac{\text{Area under one cycle}}{\text{Base}}$$

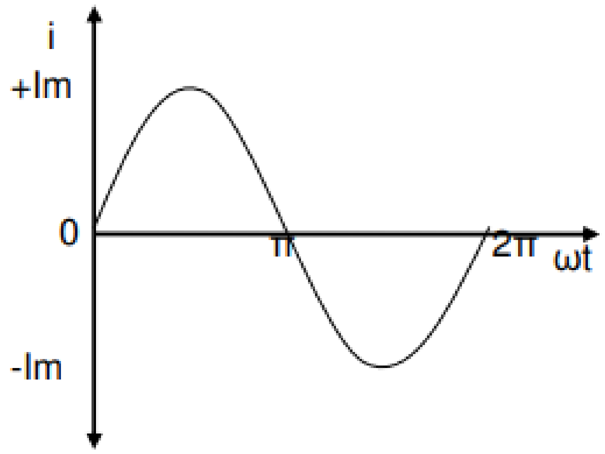
$$V_{av} = \frac{1}{2\pi}\int_0^{2\pi} v\,d(\omega t)$$

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$\text{Average value} = \frac{\text{Area under one half cycle}}{\text{Base}}$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\,d(\omega t)$$

Average value of a sinusoidal current



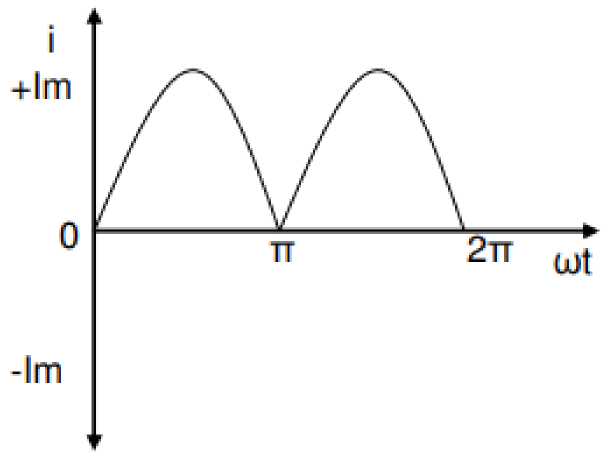
$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Average value of a full wave rectifier output



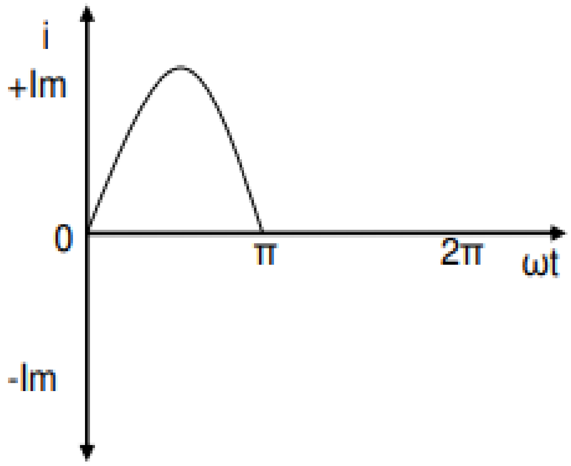
$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Average value of a half wave rectifier output



$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\, d(\omega t)$$

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t\, d(\omega t)$$

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

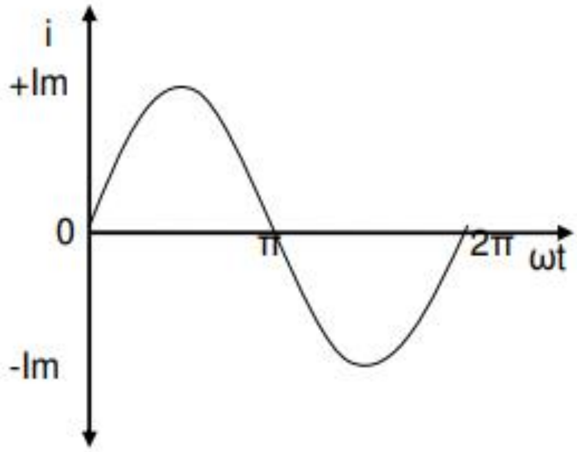
RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$RMS = \sqrt{\frac{\text{Area under squared curve}}{base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 d(\omega t)}$$

RMS value of a sinusoidal current



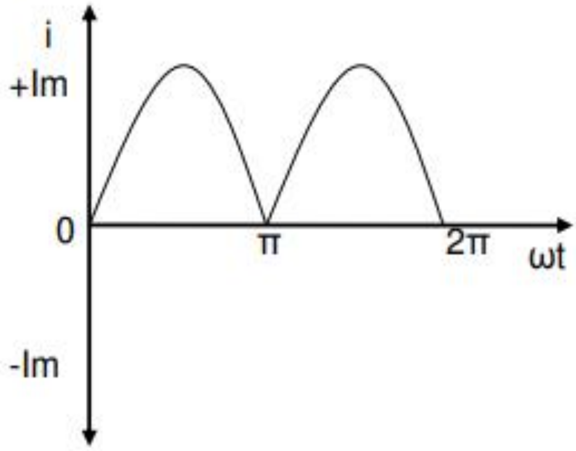
$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

RMS value of a full wave rectifier output



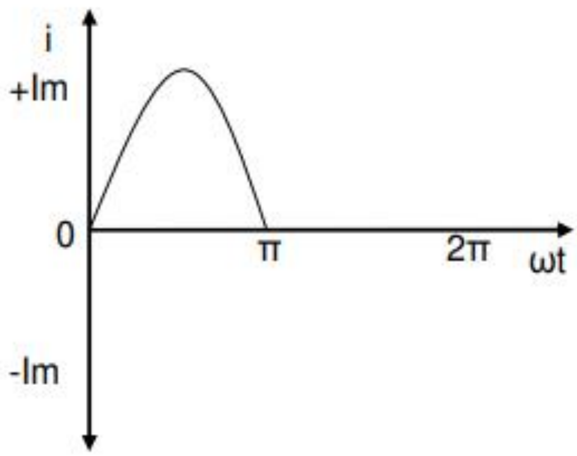
$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

RMS value of a half wave rectifier output



$$i = I_m \sin \omega t$$

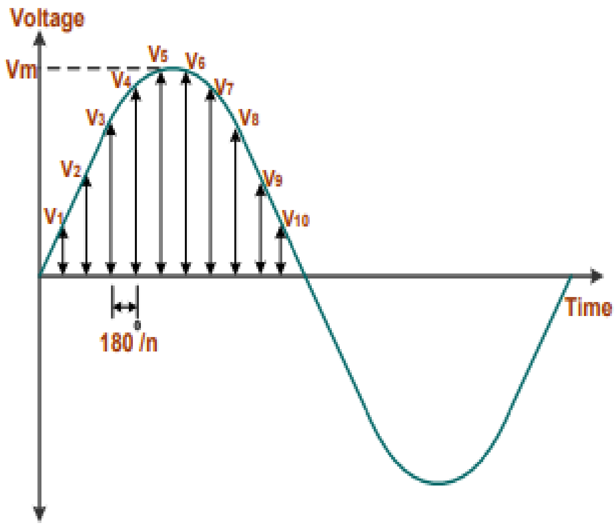
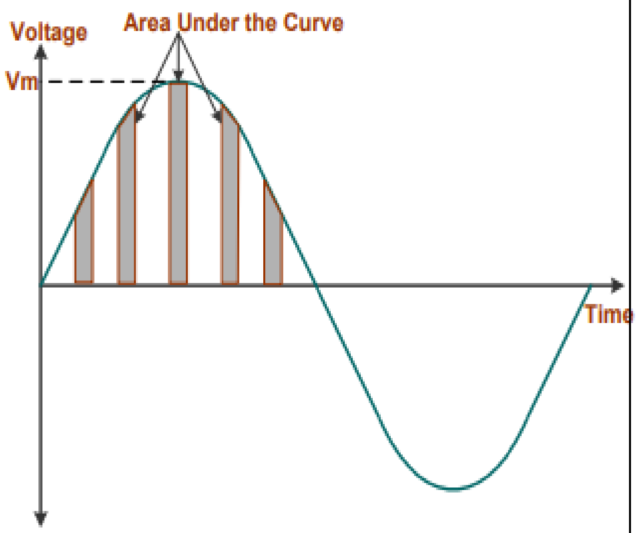
$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{2} = 0.5 I_m$$

2.3 Derivation of average value and RMS value of sinusoidal AC signal

➢ **Average Value**

Graphical Method	**Analytical Method**
 *Figure 2.6 Graphical Method for Average Value*	 *Figure 2.7 Analytical Method for Average Value*
$V_{ave} = \dfrac{Sum\ \ of\ \ All\ \ Instantaneous\ \ Values}{Total\ \ No.\ \ of\ \ Values}$	$V_{ave} = \dfrac{Area\ \ Under\ \ the\ \ Curve}{Base\ \ of\ the\ \ Curve}$
$V_{ave} = \dfrac{v_1 + v_2 + v_3 + v_4 + v_5 + ... + v_{10}}{10}$	$V_{ave} = \dfrac{\int_0^{\pi} V_m\ Sin\ \omega t\ \ d\omega t}{\pi}$ $V_{ave} = \dfrac{V_m}{\pi}\left(-\cos\omega t\right)_0^{\pi}$ $V_{ave} = -\dfrac{V_m}{\pi}\left(\cos\pi - \cos 0\right)$ $V_{ave} = \dfrac{2V_m}{\pi}$ $V_{ave} = 0.637\ V_m$

- **RMS Value**

Graphical Method

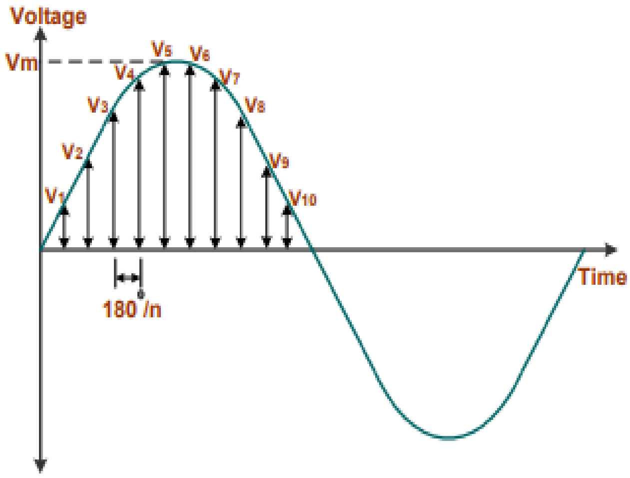


Figure 2.8 Graphical Method for RMS Value

$$V_{rms} = \sqrt{\frac{Sum\ \ of\ \ all\ \ sq.\ of\ instantaneous\ values}{Total\ \ No.\ \ of\ \ \ Values}}$$

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + \ldots\ldots + v_{10}^2}{10}}$$

Analytical Method

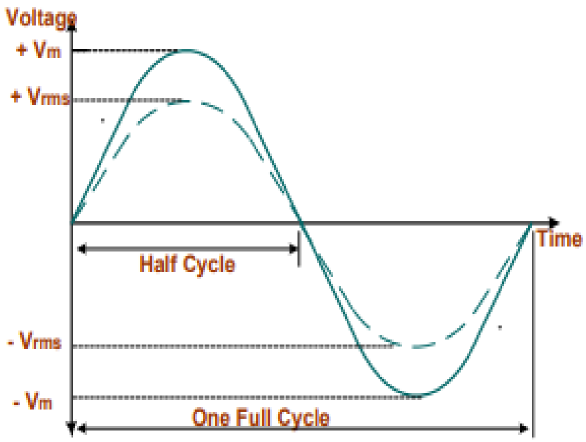


Figure 2.9 Analytical Method for RMS Value

$$V_{rms} = \sqrt{\frac{Area\ \ under\ \ the\ sq.\ curve}{Base\ of\ the\ curve}}$$

$$V_{rms} = \sqrt{\frac{\int_0^{2\pi} V_m^2\, Sin^2\, \omega t\ \ d\omega t}{2\pi}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \frac{(1-\cos 2\omega t)}{2} d\omega t}$$

$$V_{rms} = \sqrt{\frac{V^2{}_m}{4\pi}\left[[\omega t]_0^{2\pi} - \left[\frac{(\sin 2\omega t)}{2}\right]_0^{2\pi} \right]}$$

$$V_{rms} = \sqrt{\frac{V_m}{4\pi}(2\pi - 0)}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_{rms} = 0.707\ \ V_m$$

Form Factor

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMSValue}{AverageValue}$$

Peak Factor or Crest Factor

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

$$PF = \frac{MaximumValue}{RMSValue}$$

For a sinusoidal waveform

$$I_{av} = \frac{2I_m}{\pi} = 0.637I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.707I_m}{0.637I_m} = 1.11$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.707I_m} = 1.414$$

For a Full Wave Rectifier Output

$$I_{av} = \frac{2I_m}{\pi} = 0.637I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.707I_m}{0.637I_m} = 1.11$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.707I_m} = 1.414$$

For a Half Wave Rectifier Output

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

$$I_{rms} = \frac{I_m}{2} = 0.5 I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.5 I_m}{0.318 I_m} = 1.57$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.5 I_m} = 2$$

Phasor Representation

An alternating quantity can be represented using

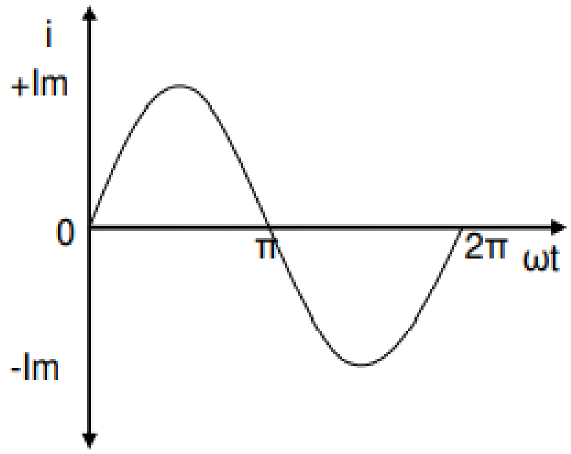
(i) Waveform

(ii) Equations

(iii) Phasor

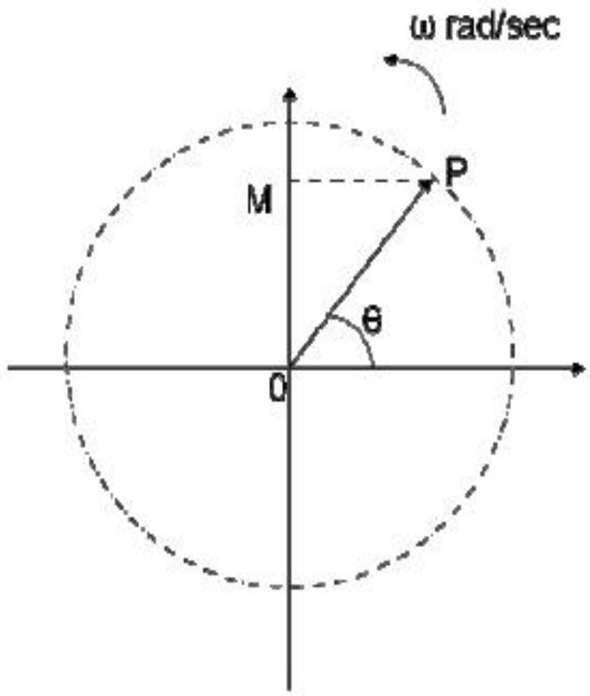
A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.

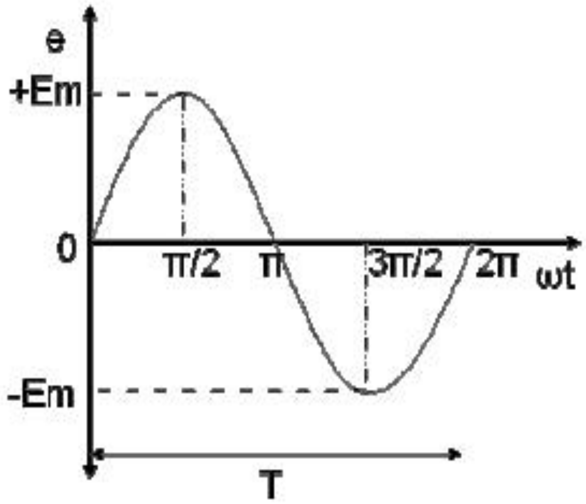


$$i = I_m \sin \omega t$$

Draw a line OP of length equal to I_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by $OM = OP\sin\theta = I_m \sin\omega t$. Hence the line OP is the phasor representation of the sinusoidal current



Phase

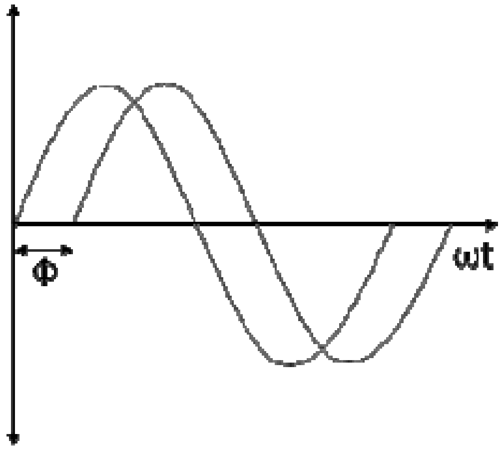


Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference

Phase of $+E_m$ is $\pi/2$ rad or T/4 sec

Phase of $-E_m$ is $3\pi/2$ rad or 3T/4 sec

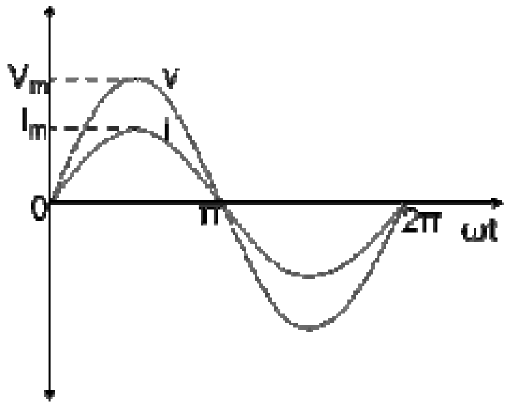
Phase Difference



When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

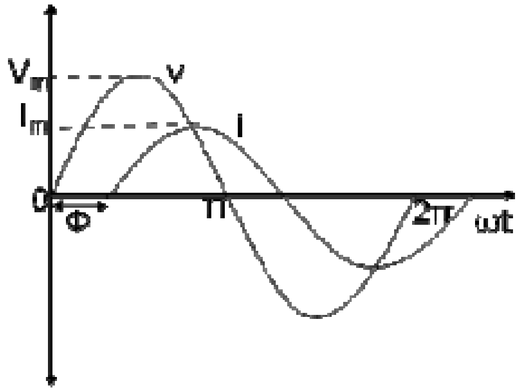


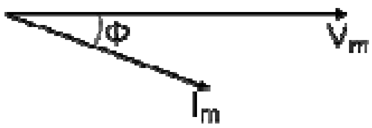
$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.

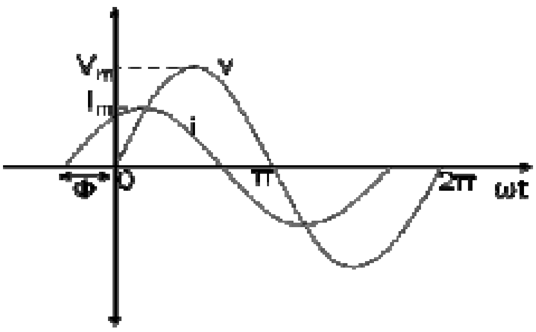


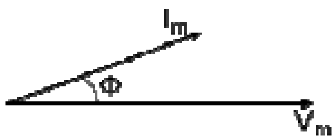


$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t - \Phi)$$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.





$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t + \Phi)$$

* Phasor Representation Alternating Quanties:-

→ An alternating quanity is also known as a phasor or rotating Vector.. It is a Complex Quantity, so it has a Real and imaginary Component. Real and imaginary Component is seperated by operator "J".

$\therefore \; \bar{a} = x \pm jy.$

$\bar{a} \rightarrow$ Phasor or Rotating vector.

$x \rightarrow$ Real. Component.

$y \rightarrow$ imaginary component.

The "J" indicate the Rotation of Phasor by 90° in anticlock wise direction.

$j = \sqrt{-1}$

The power of "j" indicate the number of times the phasor should be Rotated by 90° in anticlock wise direction.

(i) Rectangular Form:-

→ $\bar{A} = a \pm jb$

majnitude of Phasor $|\bar{A}| = \sqrt{a^2+b^2}$, Phase angle $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

(2) Polar Form:-

→ $\bar{A} = |\bar{A}| \angle \pm \phi$

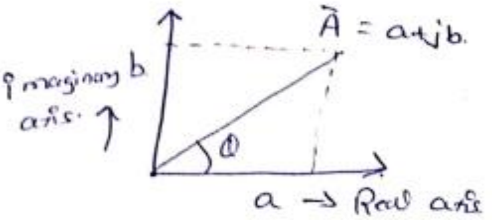
(3) Trigonometric form

$\bar{A} = \bar{A}\,(\cos\phi \pm j\sin\phi)$ ⇒ $a = |\bar{A}|\cos\phi$ → Real. Component.

$b = |\bar{A}|\sin\phi$ = imaginary Component.

(4) Exponential Form:-

$\bar{A} = |\bar{A}| e^{\pm j\phi}$ Where $e^{\pm j\phi} = \cos\phi + j\sin\phi$



* Arithmatic operation of Phasor Quantity:-

(1) Addition of Two Phasor quantity:-

$\bar{A}_1 = a_1+jb_1$ $\bar{A}_2 = a_2+jb_2$

$\bar{A}_1 + \bar{A}_2 = (a_1+a_2)+j(b_1+b_2)$

(2) Substraction:-

$\bar{A}_1 = a_1+jb_1$ $\bar{A}_1 - \bar{A}_2 = (a_1-a_2)+j(b_1-b_2)$

$\bar{A}_2 = a_2+jb_2$

(3) Multiplication:- → For multi, convert into polar form first.

$\bar{A}_1 = a_1+jb_1$ $\therefore |\bar{A}_1| = \sqrt{a_1^2+b_1^2}$

$\bar{A}_2 = a_2+jb_2$ $\phi_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

$\bar{A}_2 = \sqrt{a_2^2+b_2^2}$

$\theta_2 = \tan^{-1}\left(\frac{b_2}{a_2}\right)$

$$\boxed{\bar{A}_1 \cdot A_2 = |\bar{A}_1||\bar{A}_2| \angle \theta_1+\theta_2}$$

* Division:-

$\bar{A}_1 = a_1+jb_1$, $\bar{A}_2 = a_2+jb_2$

$\bar{A}_1 + A_2$ → Convert into polar form.

$$\therefore \boxed{\frac{\bar{A}_1}{\bar{A}_2} = \frac{|\bar{A}_1|}{|\bar{A}_2|} \angle \theta_1 - \theta_2}$$

Addition of Alternating Quantities

Alternating voltages and currents are phasors. They are added in the same manner as forces are added. Only phasors of the same kind may be added. Common sense tells us that we should not try to add volts to amperes. Addition of alternating currents or voltages can be accomplished by one of the following methods :

1. Parallelogram method 2. Method of components

1. Parallelogram method. This method is used for the addition of two phasors at a time. The two phasors are represented in magnitude and direction by the adjacent sides of a parallelogram. Then the diagonal of the parallelogram represents the maximum value of the resultant.

Consider two alternating currents i_1 and i_2 flowing in the two branches of a circuit [See Fig. (*i*)]. Let they be represented by :

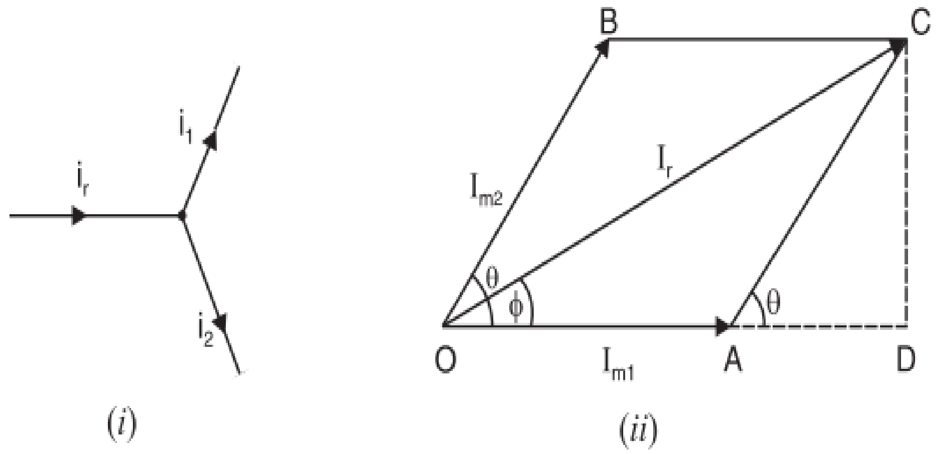


Fig.

$$i_1 = I_{m1} \sin \omega t \quad ; \quad i_2 = I_{m2} \sin (\omega t + \theta)$$

The two phasors I_{m1} and I_{m2} are represented by the adjacent sides *OA* and *OB* respectively of the parallelogram *OACB* [See Fig. 11.49 (*ii*)]. The phase difference between the phasors is θ° ; I_{m2} leading I_{m1}. The maximum value of resultant is I_r. It is represented by the diagonal *OC* and leads the phasor I_{m1} by φ°.

$$OC = \sqrt{(OD)^2 + (CD)^2} = \sqrt{(I_{m1} + I_{m2} \cos \theta)^2 + (I_{m2} \sin \theta)^2}$$

$$\therefore \qquad I_r = \sqrt{I^2_{m1} + I^2_{m2} + 2 I_{m1} I_{m2} \cos \theta}$$

Also

$$\tan \phi = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{I_{m2} \sin \theta}{I_{m1} + I_{m2} \cos \theta}$$

The instantaneous value of resultant current is given by ;

$$i_r = I_r \sin (\omega t + \phi)$$

This method is convenient only when we have to add two phasors. However, if the number of phasors to be added is more than two, this method becomes quite inconvenient.

2. Method of Components. This method provides a very convenient means to add two or more phasors. Each phasor is resolved into horizontal and vertical components. The horizontals are summed up algebraically to give the resultant horizontal component X. The verticals are likewise summed up algebraically to give the resultant vertical component Y.

Then, Resultant $= \sqrt{X^2 + Y^2}$

Phase angle of resultant, $\tan\phi = Y/X$

The previously considered currents are represented as phasors in Fig. 11.50.

$$X = I_{m1} + I_{m2}\cos\theta$$

$$Y = 0 + I_{m2}\sin\theta$$

$$\therefore \quad \text{Resultant, } I_r = \sqrt{(I_{m1} + I_{m2}\cos\theta)^2 + (I_{m2}\sin\theta)^2}$$

$$= \sqrt{I_{m1}^2 + I_{m2}^2 + 2I_{m1}I_{m2}\cos\theta}$$

which is the same as derived by parallelogram method.

$$\tan\phi = \frac{Y}{X} = \frac{I_{m2}\sin\theta}{I_{m1} + I_{m2}\cos\theta}$$

$$i_r = I_r \sin(\omega t + \phi)$$

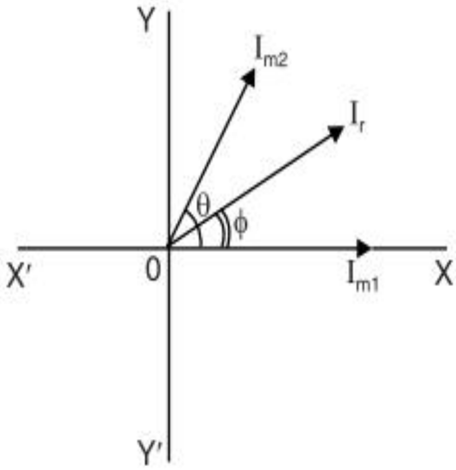
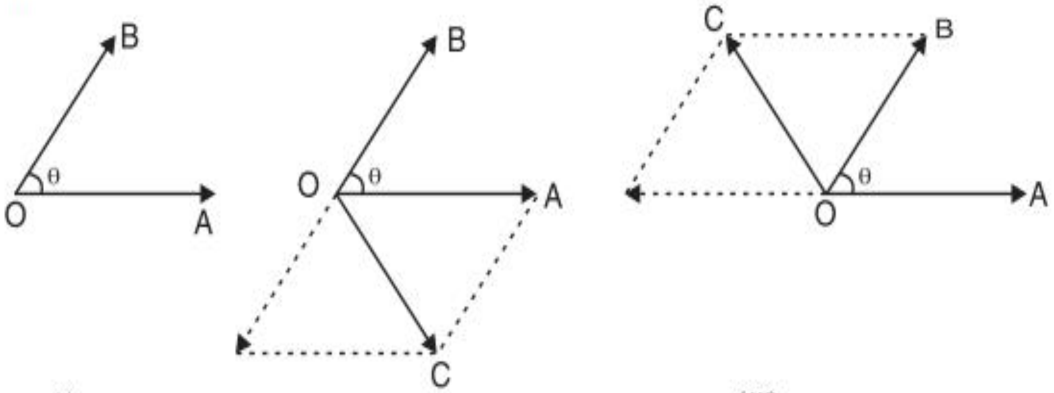


Fig.

Subtraction of Alternating Quantities

If difference of two phasors is required, then one of the phasors is reversed and this reversed phasor is then compounded with the other phasor using parallelogram method or method of components.



(*i*) (*ii*) (*iii*)

Fig.

Consider two phasors OA and OB representing two alternating quantities of the same kind [See Fig. (*i*)]. The phasor OB leads the phasor OA by θ. If it is required to subtract the phasor OB from OA, then OB is reversed and is compounded with phasor OA as shown in Fig. (*ii*). The phasor difference $OA–OB$ is given by the phasor OC. In Fig. (*iii*), phasor OC represents the phasor difference $OB–OA$.