Acceptance by Empty Stack

$a, z_0/az_0$

$b, a/\varepsilon$

$a, a/aa$

$b, a/\varepsilon$

$\varepsilon, z_0/\varepsilon$

$aabb\varepsilon$

z_0

pop

Ⓐ|

Ⓑ| $\delta(q_1, \varepsilon, z_0) \longrightarrow (q_1, \varepsilon)$

Acceptance   PDA

final state   Empty

$\delta(q_1, \varepsilon, z_0) \longrightarrow (q_f, z_0)$ $\delta(q_1, \varepsilon, z_0) \longrightarrow (q_1, \varepsilon)$

Equivalent in Power
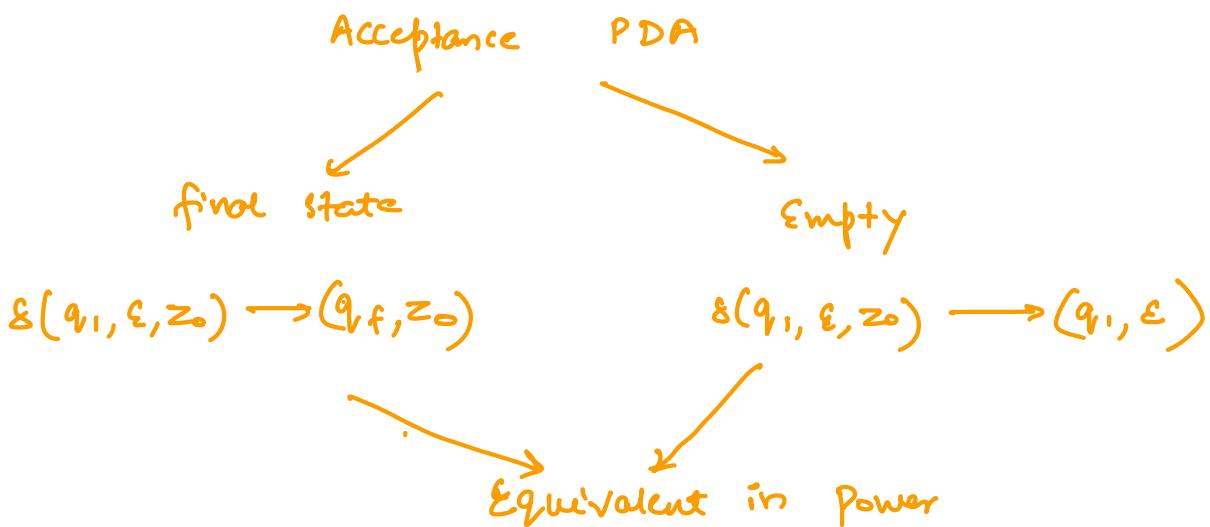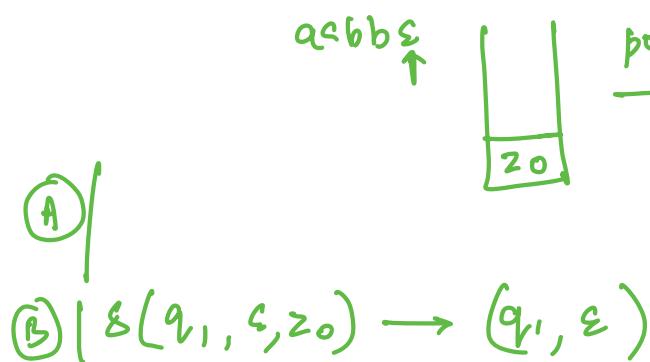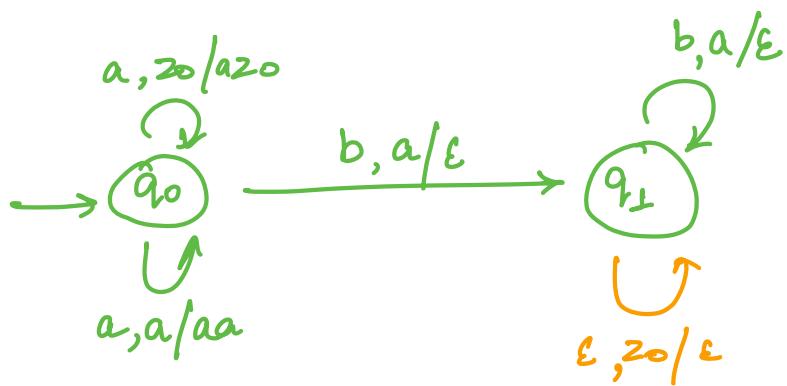
Eg:

$w \mid n_a(w) = n_b(w)$

Logic:

first time : push a, push b

top a, input b   pop

$\dfrac{b}{\boxed{\begin{matrix} a \\ z_0 \end{matrix}}}$

top b , input a   pop

$a, z_0 / a z_0$
$b, z_0 / b z_0$

$\to (q_0)$ $\quad$ $a, a / a a$
$\quad\quad\quad b, b / b b$

$a, b / \varepsilon$
$b, a / \varepsilon$

$\xrightarrow{\varepsilon, z_0 / z_0}$ (q_f)
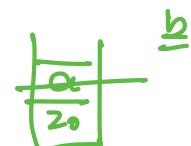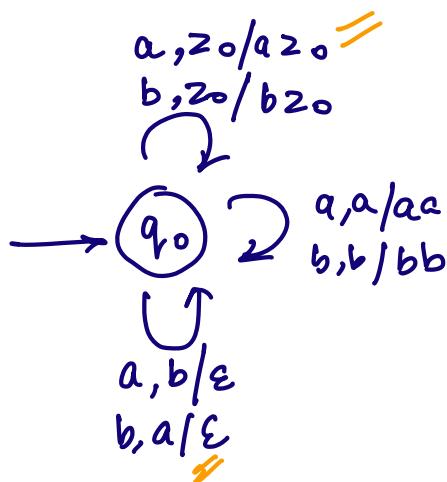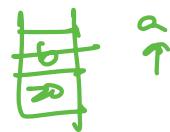
why not a
new state for pop?

a b a
↓ ↓ ↓
z_0 ε z_0

baab ε
↑

bbaa ε
↑

baab ε
↑

bbaa ε
↑

baab ε
↑

bbaa ε
↑

baab ε
↑

bbaa ε
↑

$baab\varepsilon$
↑

$bbaa\varepsilon$
↑

b
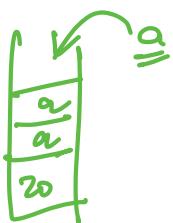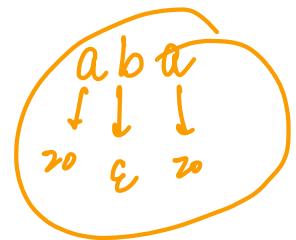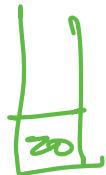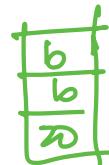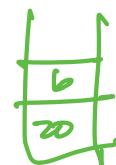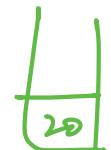
$aaa\,bbb$
↑↑
c

**Eg:**

$$a^n b^n c^m \mid n, m \geq 1$$

a : push
b : pop
at last | c → final state

(Push a on empty stack)

a, Z_0/aZ_0

b,a/ε (Pop)

c,Z_0/Z_0

→ q_0
  b,a/ε
  (Pop)
q_1
  c,Z_0/Z_0
  first c
q_f
→ more no of c's

a, a/aa
Push a's

$aabc$ $aabc$ $aabc$

q_0 q_0

DFA:

a → ⟨⟩ b   a ⟨⟩ b   a ⟨⟩ b
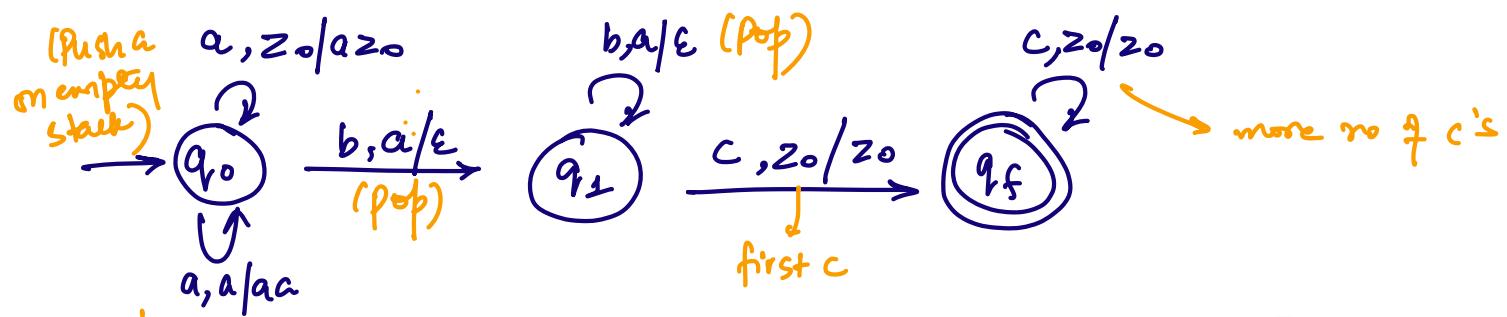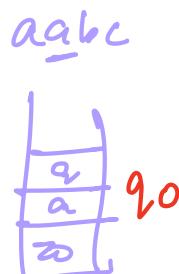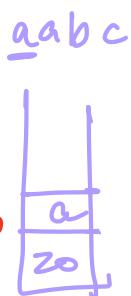
$aabc$ $aabc$
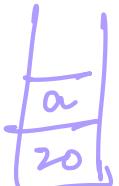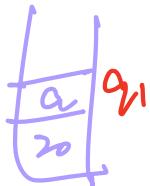
q_1 dead state
            no transitions available ✗

$$\delta(q_1, c\ a)\ ?$$
✗

Eg: $a^n b^m c^n \mid n, m \geqslant 1$

a: push
b: ignore
c: pop

push a's { $a, a/aa$
          $a, z_0/a z_0$ }

(ignore b's) $b, a/a$

(pop a's) $c, a/\varepsilon$

$\rightarrow (q_0) \xrightarrow{b, a/a} (q_1) \xrightarrow[\text{(pop a)}]{c, a/\varepsilon} (q_2) \xrightarrow{\varepsilon, z_0/z_0} ((q_f))$

$aabc\varepsilon$

push a's { $a, a/aa$
          $a, z_0/a z_0$ }

(ignore b's) $b, a/a$

(pop a's) $c, a/\varepsilon$

$\rightarrow (q_0) \xrightarrow{b, a/a} (q_1) \xrightarrow[\text{(pop a)}]{c, a/\varepsilon} (q_2) \xrightarrow{\varepsilon, z_0/z_0} ((q_f))$

	input	stack	state
IS:	a a bc ε	z_0	q_0
	aabcε	a / z_0	q_0
	aabcε	a / a / z_0	q_0

aabc‿ε

q_1

q_2

aabc‿ε

$\delta(q_2, \varepsilon, a) \underset{?}{} \times$
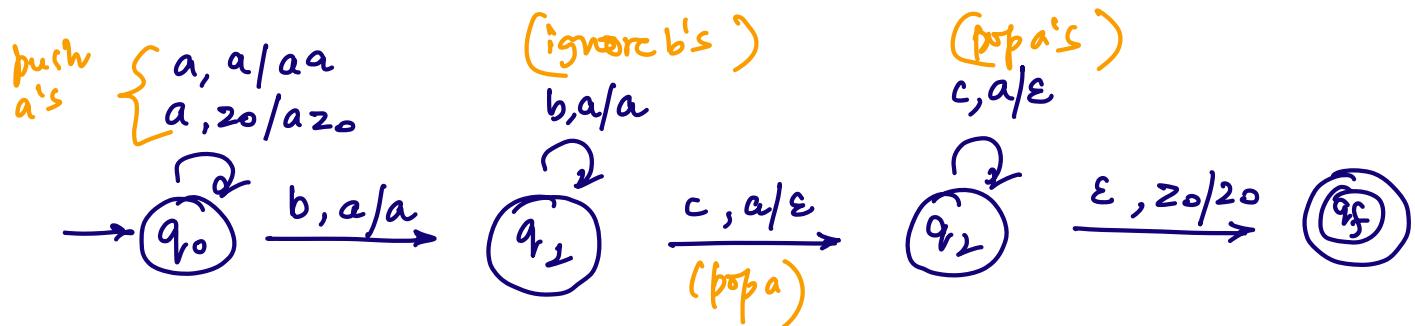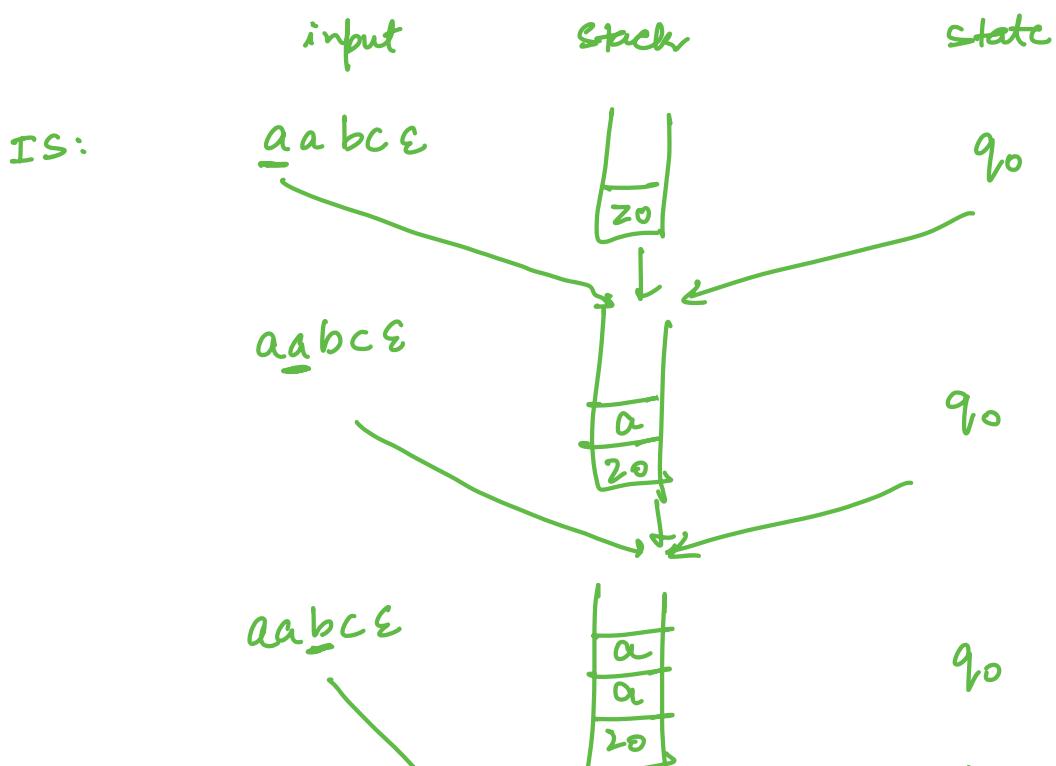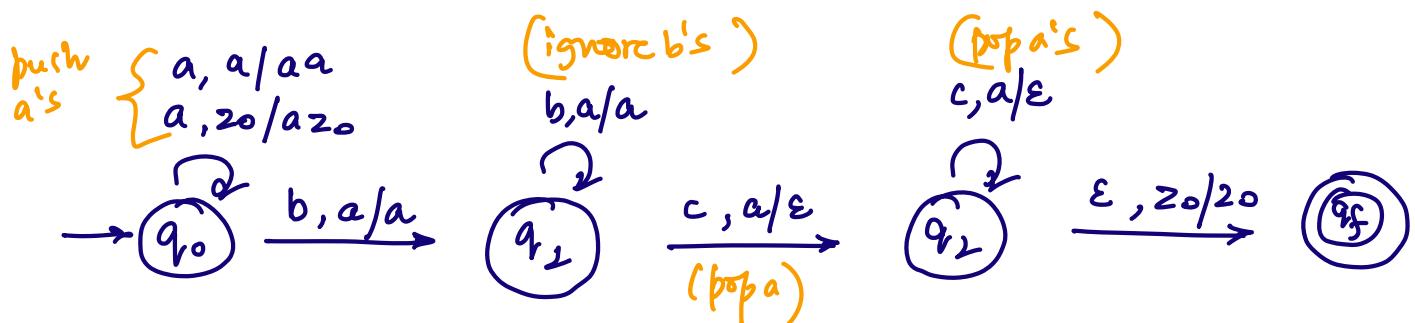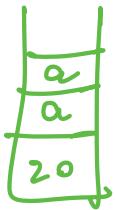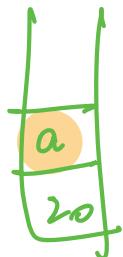
Dead State ⟶ String not accepted

Eq: $a^{m+n} \, b^m \, c^n \quad | \, n, m \geq 1$

a: push
b: pop
c: pop

aaaa bb ccc

Stack at
state q_2

$a,a/aa$
$a,z_0/az_0$

$b,a/\varepsilon$

q_0 $\xrightarrow{b,a/\varepsilon}$ q_1 $\xrightarrow{c,a/\varepsilon}$ q_2 $\xrightarrow{\varepsilon, z_0/z_0}$ $\boxed{q_f}$

$c,a/\varepsilon$

Eq: $a^n \, b^{n+m} \, c^m \quad | \, n, m \geq 1$

$a^n \; b^n \; b^m \, c^m$

push   pop   empty

$a^3 \, b^3 \, b^2 \, c^2$

a^n — $a, a/aa$; $a, z_0/a z_0$

b^n — $b, a/\varepsilon$

c^m — $c, b/\varepsilon$

$$\rightarrow q_0 \xrightarrow{\ b, a/\varepsilon\ } q_1 \xrightarrow{\ c, b/\varepsilon\ } q_2 \xrightarrow{\ \varepsilon, z_0/z_0\ } q_f$$

q_1 self loop: $b, z_0/b z_0$; $b, b/bb$ — b^m

$[z_0]$

Eg: $a^n b^m c^{n+m} \mid n, m \geqslant 1$

a^n — $a, a/aa$; $a, z_0/a z_0$

$$\rightarrow q_0 \xrightarrow{\ c, b/\varepsilon\ } q_1 \xrightarrow{\ \varepsilon, z_0/z_0\ } q_f$$

q_0 self loop (b^m): $b, a/ba$; $b, b/bb$

q_1 self loop: $c, b/\varepsilon$ and $c, a/\varepsilon$ — c^{n+m}

Eg: $a^n b^m c^n d^m \mid n, m \geqslant 1$

stack contents (bottom to top): a, a, a, b, b
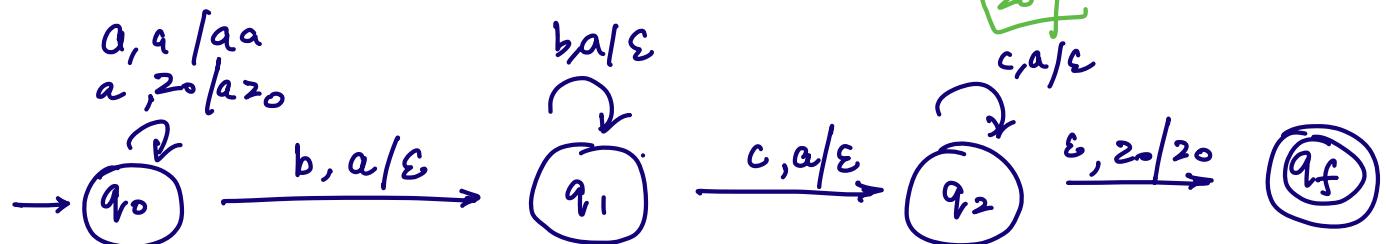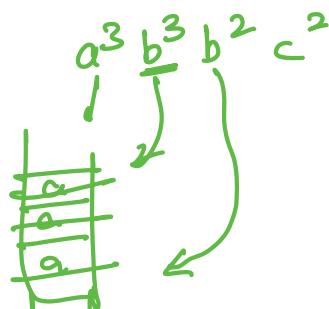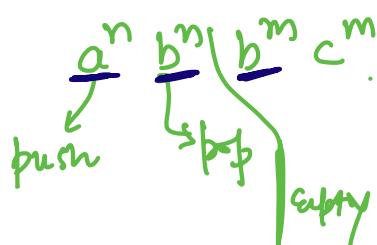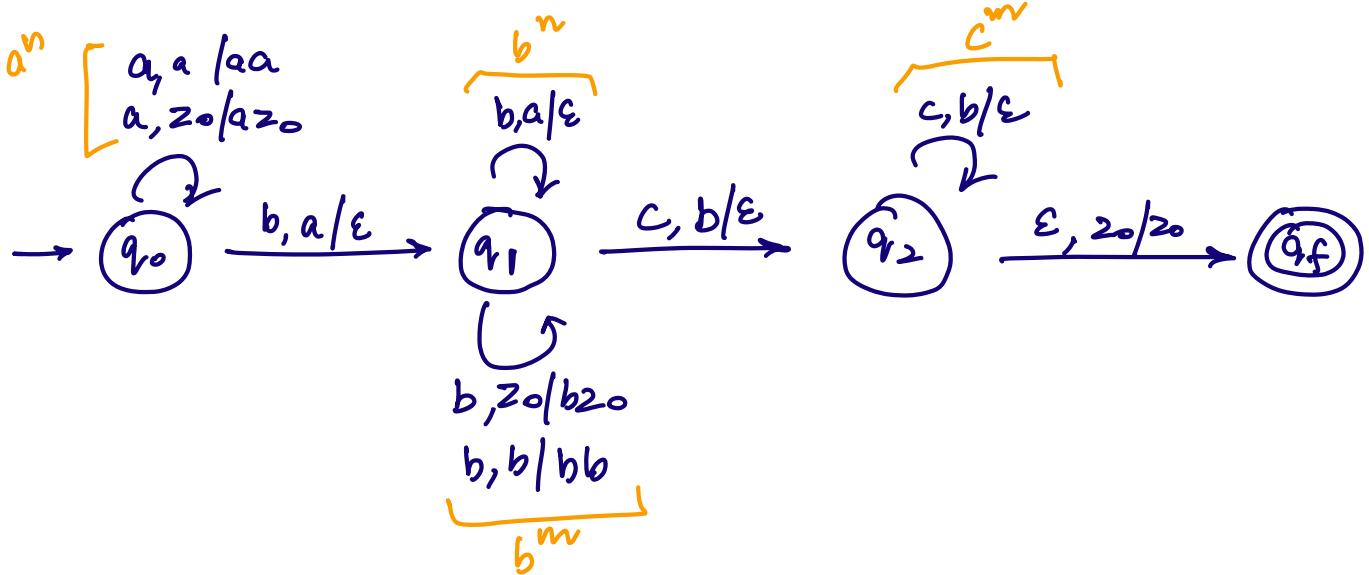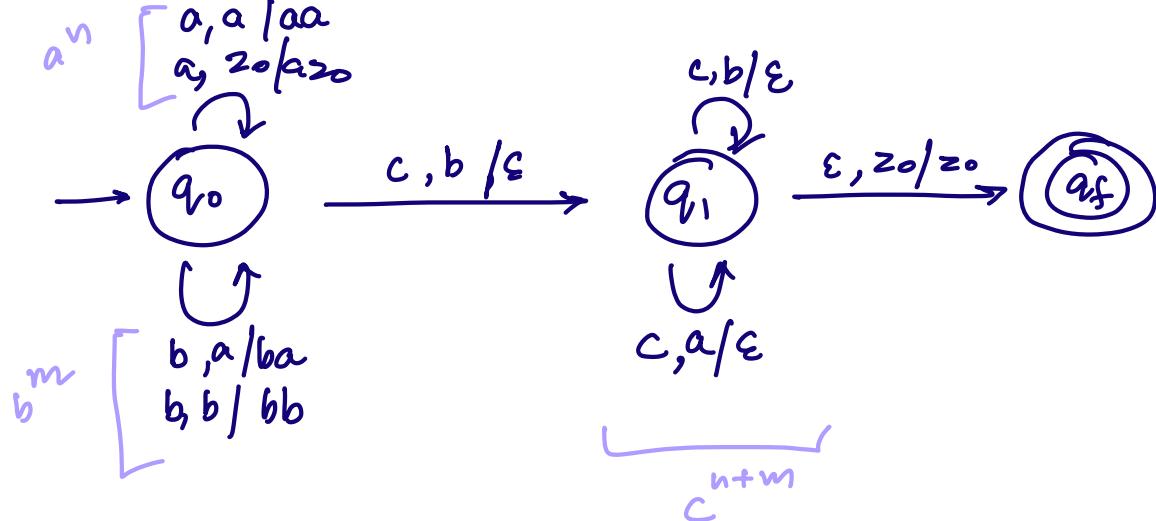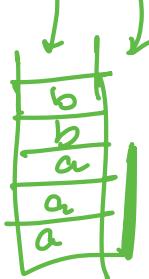
Single Stack PDA not possible

Eg: $a^n b^{2n} \mid n \geqslant 1$

Way 1 — See a: push z_0's

Way 2 — See a: push $1a$

$a^3 b^6$

a a a b b b b b b

Way 1:

$a, a/aaa$
$a, z_0/aaz_0$

$b, a/\varepsilon$

$b, a/\varepsilon$

$\varepsilon, z_0/z_0$

$q_0 \quad q_1 \quad q_f$

Way 2:

$a, a/aa$
$a, z_0/az_0$

(1st b)
$b, a/a$

(2nd b)
$b, a/\varepsilon$

$b, a/a$
(1st b)

$\varepsilon, z_0/z_0$

$q_0 \quad q_1 \quad q_2 \quad q_f$

Eg: $a^n b^{2n+1} \quad | n \geq 1$

$\downarrow$

$a^n \cdot b^{2n} \cdot b$

Way 3:

$a, a / aa a$
$a, z_0 / aa z_0$

$b, a / \varepsilon$

q_0 $\xrightarrow{b, a / \varepsilon}$ q_1 $\xrightarrow{b, z_0 / z_0}$ q_2

$\varepsilon, z_0 / z_0$

q_f

way2:

$a, a / aa$
$a, z_0 / a z_0$

(1st b)
q_0 $\xrightarrow{b, a / a}$ q_1 $\xrightarrow[b, a / \varepsilon]{(2^{nd} b)}$ q_2

$b, a / a$
(1st b)

$b, z_0 / z_0$

$\varepsilon, z_0 / z_0$
$q_f \xleftarrow{\varepsilon, z_0 / z_0} q_3$
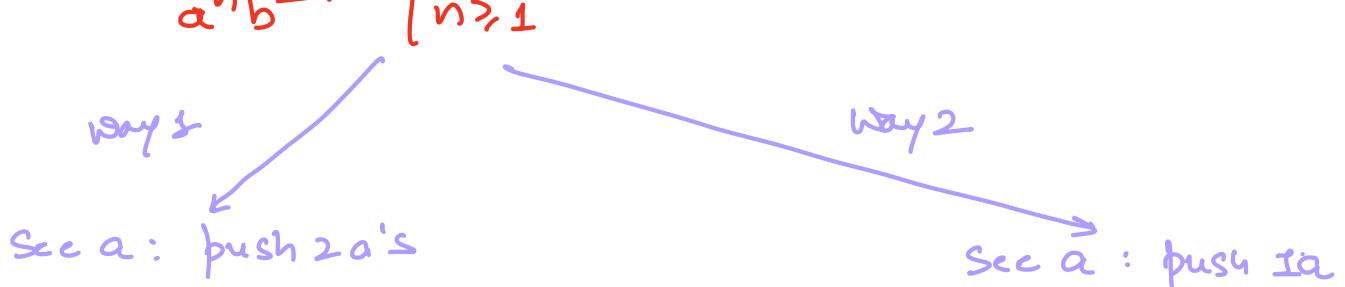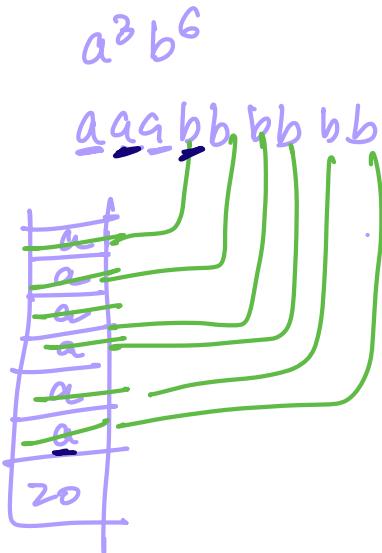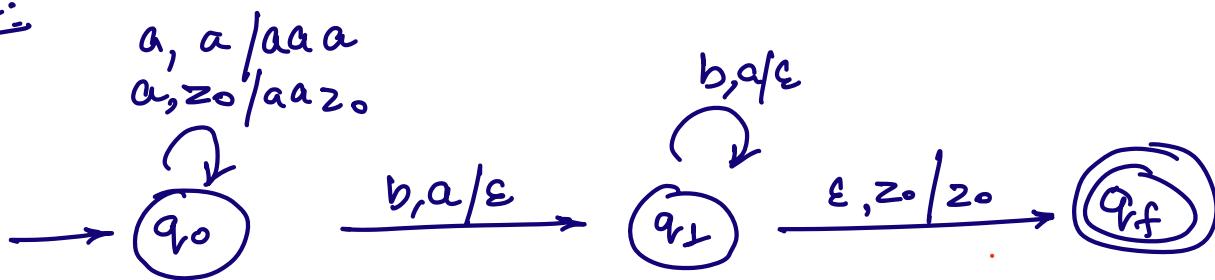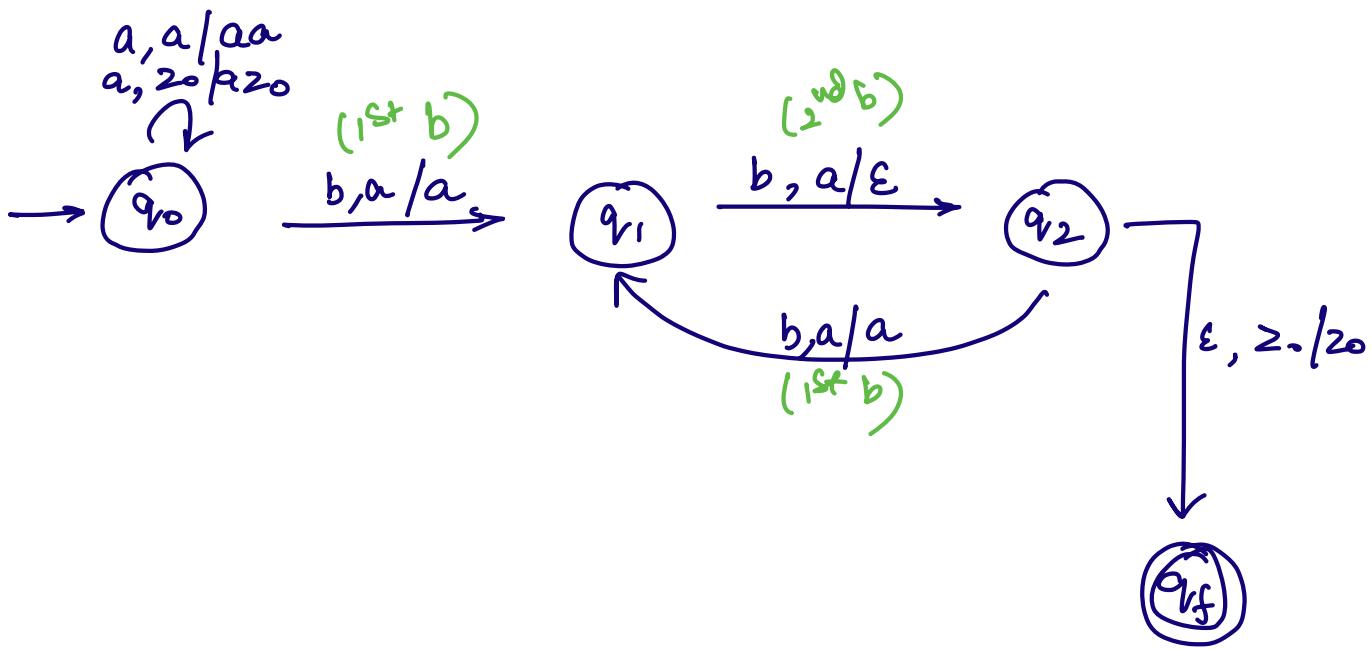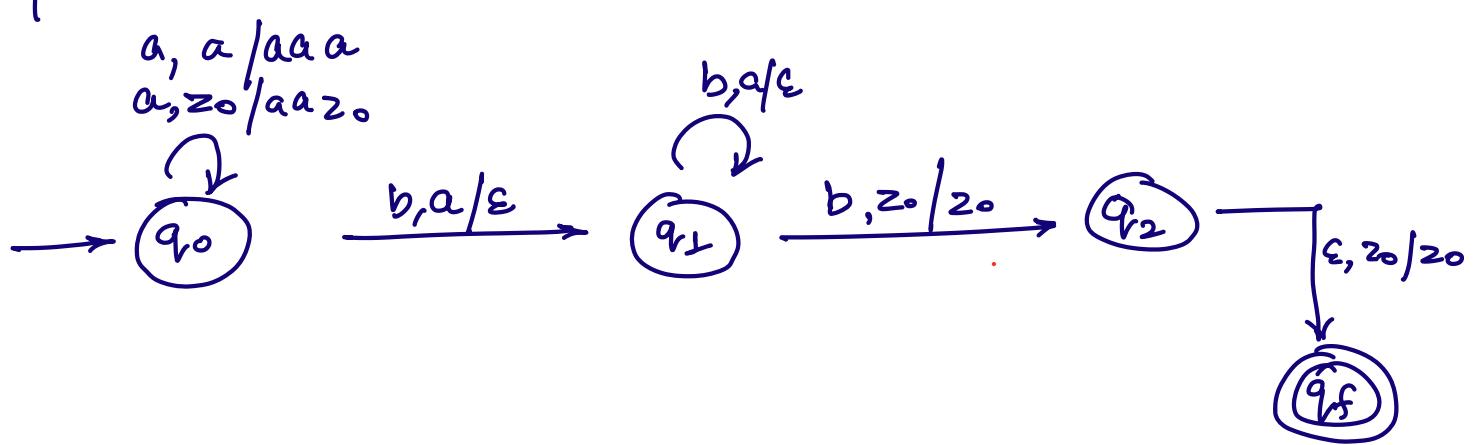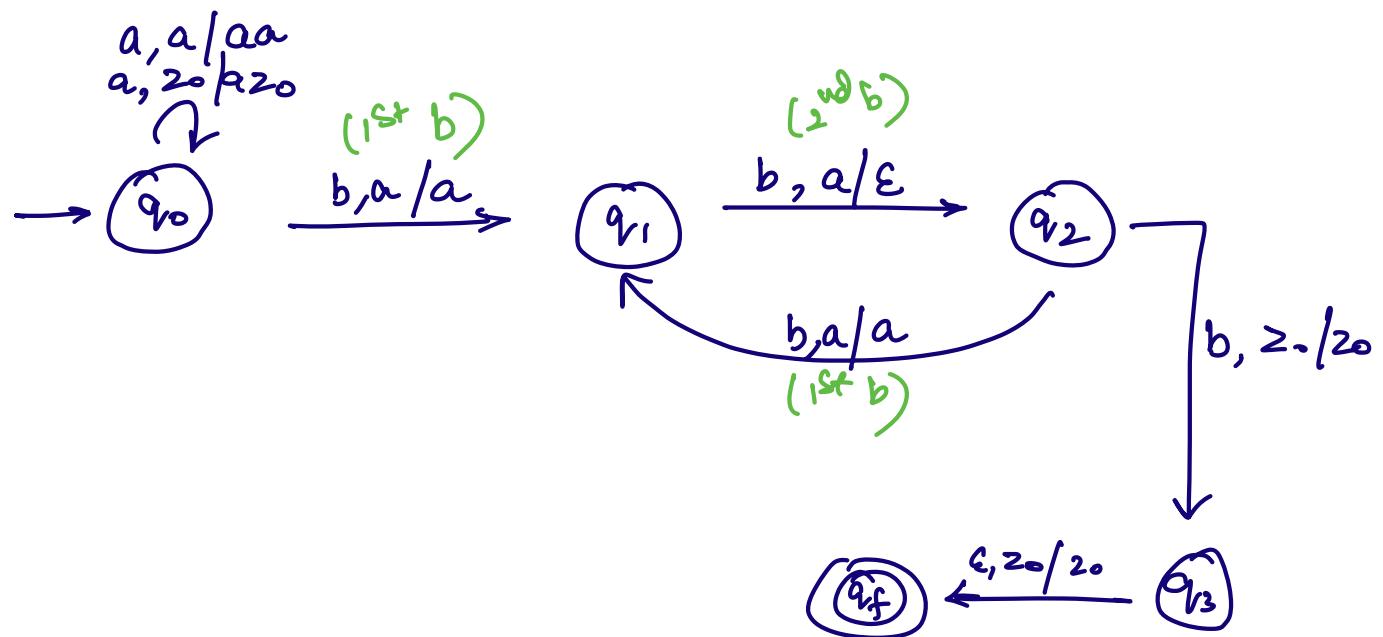
Eg: $a^n b^n c^n \mid n \geq 1$
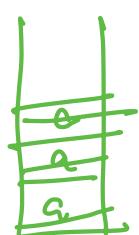
PDA Not Possible

Eg: $\underline{w c w^R} \mid w \in (a, b)^+$

St of all palindromes of odd length

$\underline{aab} \; \textcircled{c} \; \underline{baa}$
w w^R pop

$bba \; c \; abb$

push

empty

$a, z_0 / a z_0$
$b, z_0 / b z_0$

after c

$b, b / \varepsilon$
$a, a / \varepsilon$

$$c, b / b$$
$$c, a / a$$

$\varepsilon, z_0 / z_0$

q_0 → q_1 → q_f

before c
part
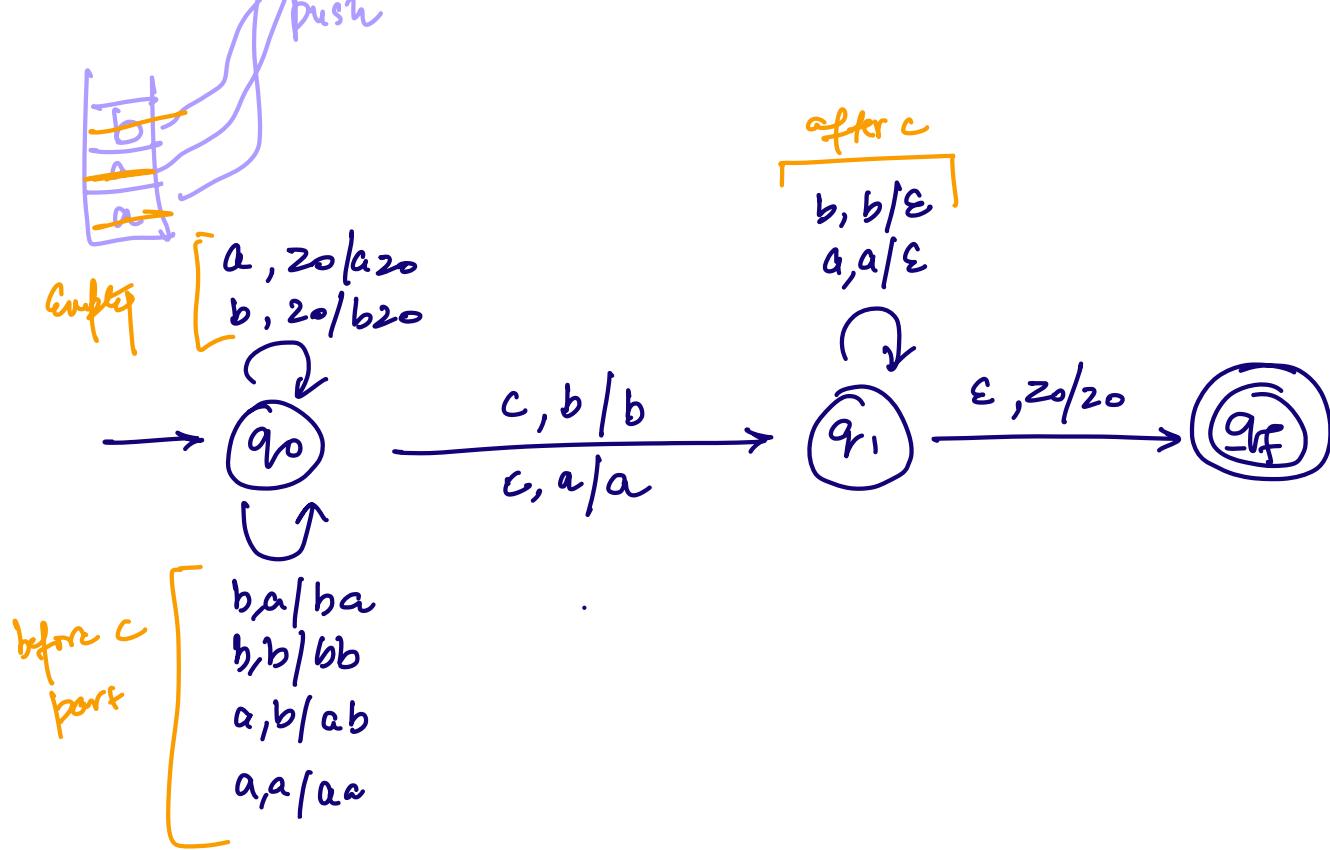
$b, a / b a$
$b, b / b b$
$a, b / a b$
$a, a / a a$

Eg: $\underline{w w^R} \mid w \in (a, b)^+$

Even length palindrome strings

$\underline{\text{Push}} \quad \underline{\text{Pop}}$

$\underline{a b b} \; \underline{b b a}$
$\quad w \qquad w^R$

1. $a b b | b b a$
    ←——|——→

2. $\boxed{} \boxed{a}$ ←—— $\underline{a} \; \underline{b} | \underline{b} \; b \; b \; a$:

Non deterministic PDA

Center
push Pop

not a center
push push

$b, z_0 / b z_0$
$a, z_0 / a z_0$

$a, a / \varepsilon$
$b, b / \varepsilon$

q_0

$a, a / \varepsilon$
$b, b / \varepsilon$

q_1

$\varepsilon, z_0 / z_0$

q_f

no way it can be a center
- $b, a / ba$
- $a, b / ab$

not a center
- $a, a / aa$
- $b, b / bb$

State   String   Stack

$(q_0, \underline{a}aaa\varepsilon, z_0)$

push a

$(q_0, \underline{a}aa\varepsilon, a z_0)$

no center   Center

push a   pop
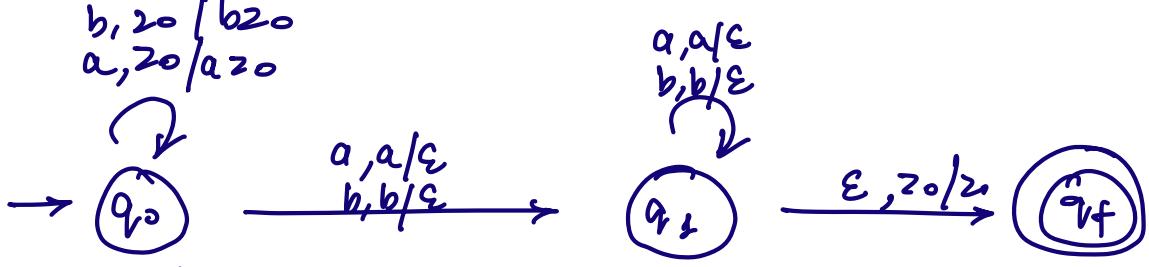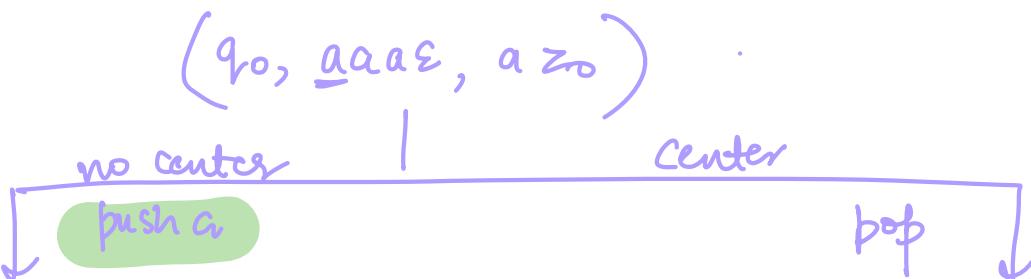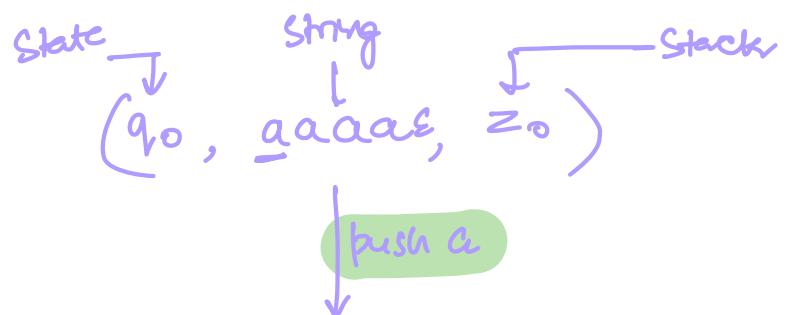
$(q_0, \underline{a}a\varepsilon, aa z_0)$ $(q_1, aa\varepsilon, z_0)$

Stuck D.S.

no center   center

pop

$(q_0, a\varepsilon, aaa z_0)$ $(q_1, a\varepsilon, a z_0)$

no center   center   pop

$(q_0, \varepsilon, aaaa z_0)$ $(q_1, \varepsilon, aa z_0)$ (q_1, ε, z_0)

DS   DS

q_f