Total No. of Pages **1**

V-SEMESTER

MID TERM EXAMINATION

Roll No.........

B.Tech.(CO)

Sept-2024

CO-303 THEORY OF COMPUTATION

Time: 1:30 Hours **Max. Marks: 25**

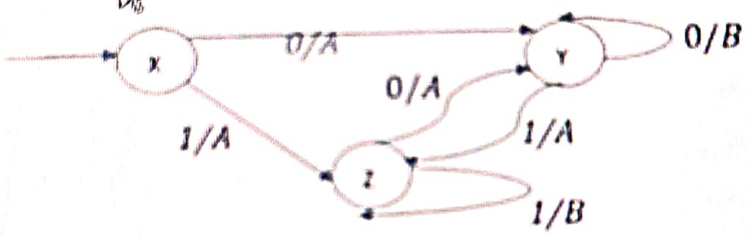
Note: Attempt all questions. Assume suitable missing data, if any

Q.No. 1 *[5] CO1*

Explain equivalence of two finite Automata with suitable example and Design a DFA for strings of 0's and 1's where all binary strings are divisible by 3.

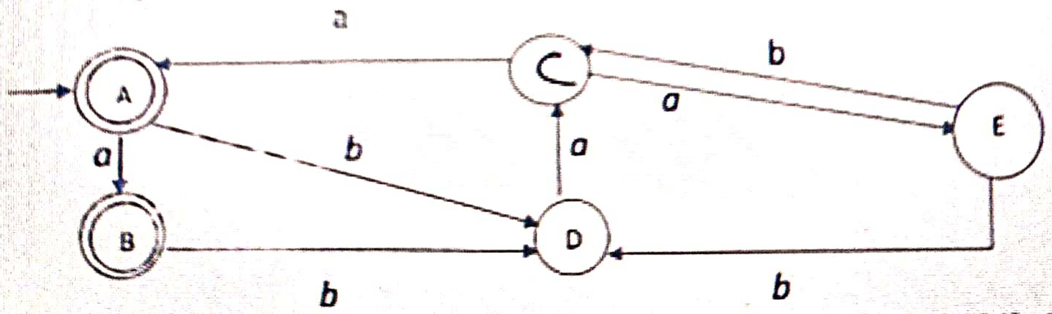
Q.No. 2 *[5] CO1*

Construct a Moore machine equivalent to the following mealy machine



Q.No. 3 *[5] CO2*

What is Arden's theorem? Find a regular expression (RE) corresponding to the following FA using Arden's theorem.



Q.No. 4 *[5] CO2*

What is chomsky's classification for the grammar? Design a Context Free Grammar (CFG) which accepts the language $L = \{0^i 1^j 0^k | j > i + k\}$.

Q.No. 5 *[5] CO3*

What is pumping lemma for regular expression? Show that the language $L = \{0^i 1^i | i >= 1\}$ is not regular.

CO303

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2K22/CO/133

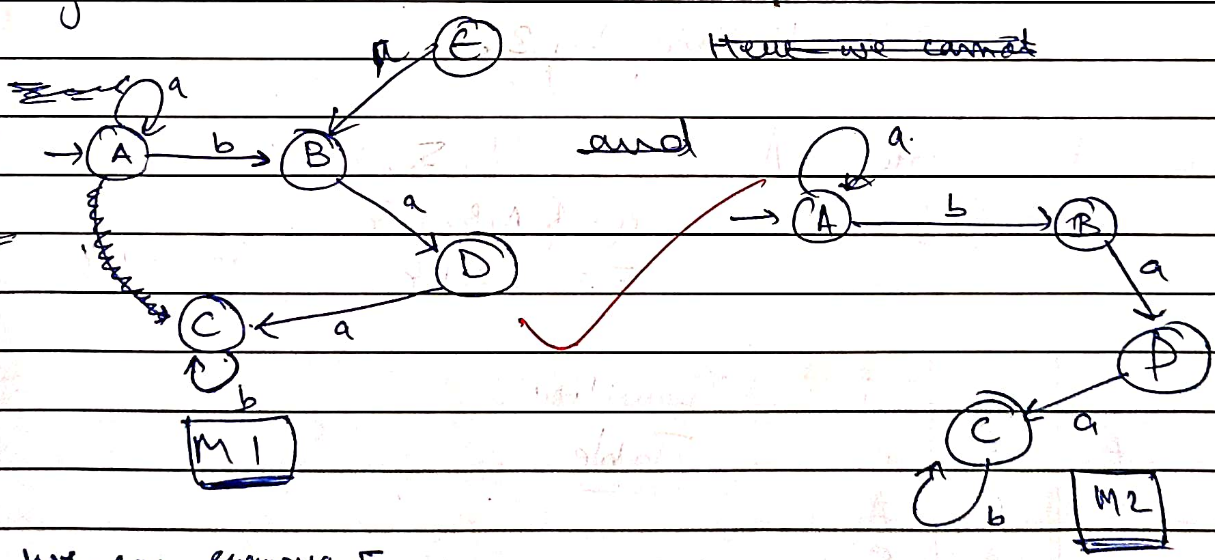
Q.1) Two finite automata (FA) are said to be equivalent if both accept same set of strings for all in Σ.
If The minimal DFA of two FA are same, then two finite automata are equivalent to each other.

In order to find if two FAs are equivalent, try finding minimal DFA of both FA. This can be done by:
(1) Removing states to which can never be reached.
(2) Finding n equivalent states by partitioning into final and non-final states.
(3) Making & Combining states to make minimal DFA.

Equivalence of Finite Automata will help to determine if language is regular or not.

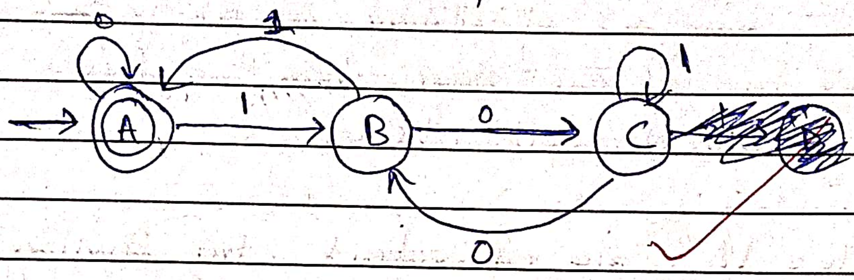
~~Here we cannot~~

eg



we can remove E as it is unreachable node,
removing E, we get M1 $\equiv$ M2. Hence, the two finite automata are equivalent.

$\Sigma = \{0,1\}$ Binary Strings divisible by 3.



For 3, 0, 011, 101, 1001, 1111 are divisible by 3,
we notice that,

if MSB = 1, then, no. of ones should be even.
and if MSB = 1, and bit representation has 0.
then even zeroes should be present
and should have LSB = 1.

~~if MSB = 0~~

$2^5+1 = 33$ is accepted but $2^6+1 = 65$ is rejected
and 33 is divisible by 3.

Here Final state = A $(Q, \Sigma, \delta, q_0, F)$

$F = \{A\}$

$Q = \{A, B, C\}$

$\Sigma = \{0,1\}$

$q_0 = \{A\}$

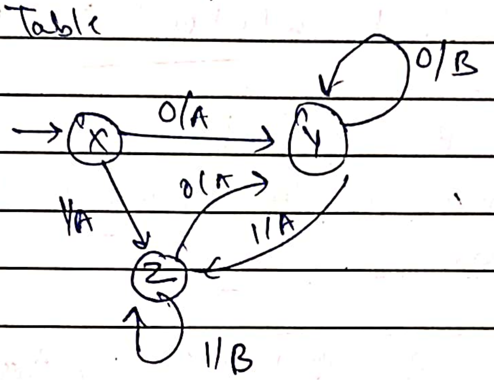
	0	1
A	A	B
B	C	A
C	B	C

Transition Table

5

Q.20) Moore machine For ⌉

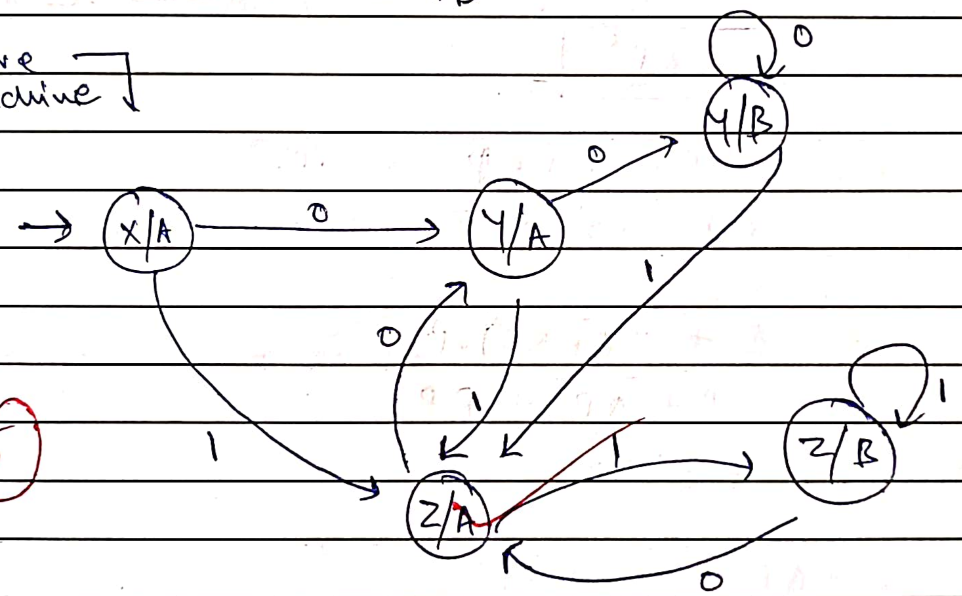
Mealy Machine Table



Mealy Table:

		O/P
(X,0)	→ Y	A
(X,1)	→ Z	A
(Y,0)	→ Y	B
(Y,1)	→ Z	A
(Z,0)	→ Y	A
(Z,1)	→ Z	B

Moore machine ⌉



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X is assigned arbitrary output out of A, B as it is initial state.

Q.3.) Arden Theorem is used to find regular expression for finite automata.
It is used to find solution to state equation of form

$$R = Q + RP$$

where P is not empty string, Q, P are regular expressions.

the solution for $R = Q + RP$ by Arden's theorem is

$$\boxed{R = QP^*}$$

Proof consider $R = Q + RP$ — (1)

put $R = Q + RP$ in (1)

$$\Rightarrow R = Q + (Q + RP) \cdot P$$

$$R = Q + QP + RP^2$$

again put $R = Q + RP$.

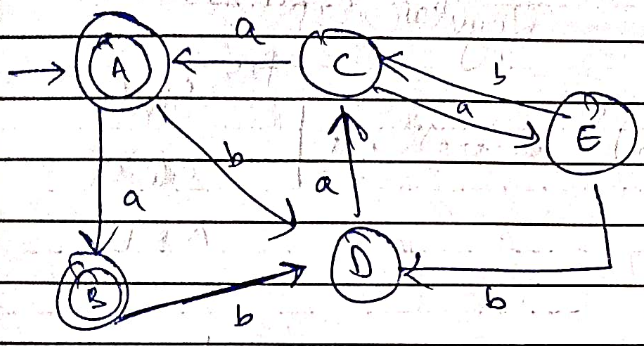
$$\Rightarrow R = Q + QP + QP^2 + RP^3$$

going by this we can write

$$R = Q(\epsilon + P + P^2 + P^3 + \ldots)$$

$$\Rightarrow \boxed{R = QP^*}$$

given $(P \neq \epsilon)$



write state equations

$\Rightarrow A = C.a$ — ①

$\Rightarrow B = A.a$ — ②

$\Rightarrow C = Da + Eb$ — ③

$\Rightarrow D = Ab + E.b + Bb$ — ④

$\Rightarrow E = C.a$ — ⑤

$\boxed{A = E}$

put ⑤ in ③

$\cancel{C = b.aE + a.D}$

$C = D.a + C(b.a)$ → it is of form $R = Q + RP$.

so

$\boxed{C = D.a(b.a)^*}$ $\cancel{E = D.a}$

we get

$\boxed{A = D.a(b.a)^*.a}$

substituting in ④

$\Rightarrow D = D.a(b.a)^*.a + D.a(b.a)^*.a.a.b$

$\Rightarrow D = \epsilon + D[a(b.a)^*a + a.(b.a)^*a.ab]$

$\Rightarrow D = \epsilon + D[a.(b.a)^*a.ab]$ $\{\epsilon + ab = ab\}$

Again by Arden's Theorem.

$\boxed{D = \epsilon.[a.(b.a)^*.a.a.b]^*}$ ← pure alphabet r.e.

put in C

$C = (a.(b.a)^* a.ab)^*.a.(b.a)^*$

we get final states

Regular Expression for final states A & B

$A = C \cdot a$

$$\boxed{A = (a \cdot (b \cdot a)^* \cdot aab)^* \cdot a (b \cdot a)^* \cdot a}$$

&

$B = A \cdot a$

$$\boxed{B = (a \cdot (b \cdot a)^* \cdot a \cdot a \cdot b)^* \cdot a \cdot (b \cdot a)^* \cdot a \cdot a}$$

5

$\boxed{E = A}$ so has same expression as A.

Q.4.) Chomsky Classifies grammar into 4 types

(1) Type 3 : Regular Grammar of form $A \to \alpha B \mid \beta$
where $\{A, B\} = V$ are $\alpha, \beta \in T^*$
Finite Automata

(2) Type 2 : Context free Grammar (CFG) $A \to \alpha$
generated by Push Down Automata
$A \in V$
$\alpha \in (V \cup T^*)$

(3) Type 1 : Context Sensitive Grammar. (CSG)

(4) Type 0 : Grammar generated by Turing Machine.

Context free Grammar (CFG)

language $L = \{ 0^i 1^j 0^k \mid j > i+k \}$

regular expression for this language.

~~$RE = (01)^* (1) (1)^* (10)^*$~~

string can be like $0011\,(1)(1)^*\,1100$

$T\{0,1\}$ $V = \{A, B, C, S\}$ always no of 1 > no of 0.

⇒
$$S \rightarrow ABC$$
$$B \rightarrow 1B \mid 1$$
$$A \rightarrow 0A1 \mid \epsilon$$
$$C \rightarrow 1C0 \mid \epsilon$$

$\underbrace{0011}_{A}\ \underbrace{(1)(1)^*}_{B}\ \underbrace{(1100)}_{C}$

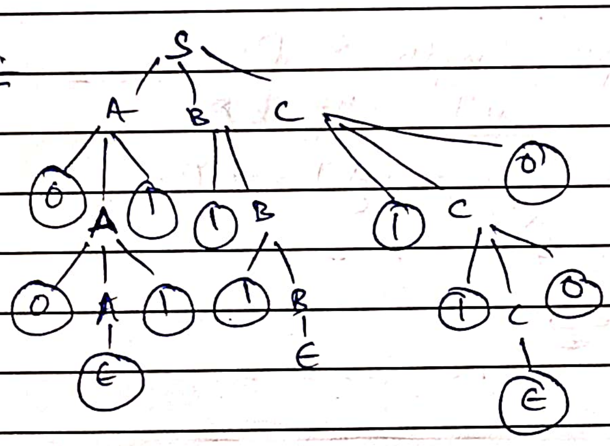
S $A \rightarrow 00--11$

$C \rightarrow 11---000$

Grammar for language

always by this language no of ones > no of zeros

eg:



001111 1100

hence

$0^i 1^j 0^k$

$i = 2,\ k = 2,\ j = 6$

$i + k = 4$

So $j > i+k$

Hence this Grammar

Q.5.) Pumping Lemma is a test which if a language falls is said to be non-regular.

Let w be combination of string from Σ^* for a language L $w \in L$ and let p be a number called as pumping length. Such that $|w| \geq p$ {length of word is greater than equal to p}. then assuming language is regular, we can divide $w = xyz$ into 3 parts.

if language L fails below 3 checks:

(1) $|xy| \leq p$

(2) $|y| > 0$ 5

(3) for $i \geq 0$ $xy^iz \in L$.

then it can be said language is not regular.

$$L = \{0^i 1^i \mid i \geq 1\}$$

assuming L is regular

let p be the pumping length $= p$.

~~& let w = 0^{2p}~~ ~~p+p = 4p > p~~

w has same no. of 1 and 0.

$w = (0 \ldots)^{2p} (1 \ldots)^{2p}$

let $w = \underbrace{000001}_{x}\underbrace{11}_{y}\underbrace{11}_{z}$

$y = 11$

if L is regular then $xy^iz \in L$ for $i \geq 0$

for $i=2$

$xy^2z = 00000111111$

here no. of Ones > no. of zeros.

hence. $xy^2z \notin L$.

~~cannot~~ pumping lemma test fails.

Since L Fails the test, then we can say that L is not regular.

Hence proved that L is not regular.