

Halting TM Problem \longrightarrow $\left. \begin{array}{l} \text{Rec languages} \\ \text{RE languages} \end{array} \right\} \text{undecidable.}$

In CFG, ambiguity problem is undecidable.

Halting TM Problem \longrightarrow Ambiguity Problem of CFG

Halting TM Problem \longrightarrow Post Correspondence Problem (PCP) \longrightarrow Ambiguity (CFG)

in this entire chain, all problems are undecidable.

Post Correspondence Problem:

Given 2 sequences of n strings on some alphabet Σ say $A = w_1 w_2 w_3 \dots w_n$ and $B = v_1 v_2 \dots v_n$

we say there exist a PC solution for pair (A, B)

if there is a non empty sequence of integers i, j, k such that

$$w_i w_j \dots w_k = v_i v_j \dots v_k$$

$n=3$

A

w_1	w_2	w_3
a	ab	bba

B

v_1	v_2	v_3
baa	aa	bb

You need to find out some seq. of integers in such a way that $w_1 w_2 \dots w_k = v_1 v_2 \dots v_k$

Seq: 3 2 3 1 PC solution

$$w_3 w_2 w_3 w_1 = v_3 v_2 v_3 v_1$$

$$bba ab bba a = bb aa bbbaa$$

If u are able to find a seq then it is called as PC solution

PC P is to device an algorithm that will tell us for any (A, B) whether or not there exist a PC solution.

$$w_3 w_2 w_3 w_1 = v_3 v_2 v_3 v_1$$

$$bba ab bba a = bb aa bbbaa$$

Relate to ambiguity problem ?
→ String

You can derive this string in 2 ways from start symbol in such a way final string is same but intermediate steps are different.

PC problem is converted to ambiguity problem

and PCP is undecidable, so, ambiguity problem will also be undecidable.

Modified PCP

first string from A & first string B has to be present at starting of solution.

$$w_1 w_i w_j \dots w_k = v_1 v_i v_j \dots v_k$$

Decidability Table:

Problem	RL	DCFL	CFL	CSL	Recursive Language	REL
1. Does $w \in L$? (Membership Problem)	D	D	D	D	D	UD
2. Is $L = \phi$? (Emptiness Problem)	D	D	D	UD	UD	UD
3. Is $L = \Sigma^*$? (Completeness Problem)	D	UD	UD	UD	UD	UD
4. Is $L_1 = L_2$? (Equality Problem)	D	UD	UD	UD	UD	UD
5. Is $L_1 \subseteq L_2$? (Subset Problem)	D	UD	UD	UD	UD	UD

6. Is $L_1 \cap L_2 = \emptyset$

D UD UD UD UD UD

7. Is L finite or
not?
(finiteness)

D D D UD UD UD

8. Is complement of
 L a language of
same type or not?

D D UD D D UD

9. Is intersection of
two languages of
same type

D UD UD UD UD UD

10. Is L regular
language.

D D UD UD UD UD