Total No. of Pages 2

FIFTH SEMESTER

Roll No. 2K22/CO/6…

B.Tech. [CSE]

MID SEMESTER EXAMINATION ***September-2024***

CO325 Probability and Statistics

Time: 1:30 Hours ***Max. Marks: 25***

Note:	Answer ***ALL*** questions. Assume suitable missing data, if any. CO# is course outcome related to question

1[a] The odds that person X speaks the truth are 3:2 and the odds that person Y speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point. [2]

[b] The density function of a random variable X is given by [3]

$$f(x) = a + bx \qquad 0 \le x \le 2,$$
$$0 \qquad otherwise$$

We also know that E(X) = 7/6

(i) Compute a and b. (ii) Compute Var(X).

2[a] At a gas station, the daily demand for regular gasoline is normally distributed with a mean of 1,000 gallons and a SD of 100 gallons. The station manager has just opened the station for business and notes that there is exactly 1,100 gallons of regular gasoline in storage. The next delivery is scheduled later today at the close of business. The manager would like to know the probability that he will have enough regular gasoline to satisfy today's demands [2]

[b] Suppose that X is a continuous random variable has probability density function given by: [3]

$$f(x) = \frac{k}{1+x^2}, \qquad 0 \le x \le \infty$$

(i) What is the value of k?

(ii) Find the probability distribution of X.

3[a] Given a Gaussian random variable (Normal R.V.), find the value of k,

(i) P(Z > k) = 0.3015

(ii) Find P(k < Z < −0.18) = 0.4197. [3]

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[b] In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? [2]

4. [a] A computer network experiences packet losses that follow a Poisson process with an average of 2 losses per minute. What is the probability of experiencing no packet losses in a 30-second interval? [3]

[b] If X is uniformly distributed with mean 1 and variance 4/3, find P(X < 0). [2]

5. [a] In a game, you roll a fair six-sided die. If you roll a 6, you win immediately. If you roll any other number, you must roll again, but now you win only if you roll the number that is one higher than your previous roll (with 6 wrapping around to 1). What is the probability of winning this game? [3]

[b] In a standard 52-card deck, you draw 3 cards without replacement. What is the probability of drawing 3 cards of the same suit? [2]

K_2 0.52 1st

$P(K < Z < 1.87)$ = 0.4138 − 0.4285 = 0.0088

Ex8:- The odds that person X speaks the truth are 3:2 and the odds that person Y speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point.

Sol^n:- Let us define the events:

A = X speaks the truth; $\bar{A}$ = tell a lie

B = Y speaks the truth; $\bar{B}$ = tell a lie

Given:- $P(A) = \frac{3}{3+2} = \frac{3}{5}$, $P(\bar{A}) = \frac{2}{5}$

$P(B) = \frac{5}{5+3} = \frac{5}{8}$, $P(\bar{B}) = \frac{3}{8}$.

The event E = X & Y contradict

Contradictions happen as:

(i) X speaks truth & Y tells a lie

$= A \cap \bar{B}$

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(ii) Y speaks truth and X tells a lie

$= \bar{A} \cap B$

(multiplicative)
By addition laws, the required probability is

$$P(E) = P(A \cap \bar{B}) \text{ or } P(\bar{A} \cap B)$$

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B) \quad (\because A \,\&\, \bar{B} \text{ and } \bar{A} \,\&\, B \text{ are independent})$$

$$= \frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{5}{8} = \frac{19}{40}$$

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Q1 (b) Density function:

① 2, 3
② 2, 3
③ 3, 2
④ 3, 2
⑤ 3, 2

$$f(x) = \begin{cases} a+bx, & 0 \le x \le 2 \\ 0, & o.w. \end{cases}$$

$E(X) = \frac{7}{6}$,

$$\Rightarrow \int_0^2 (a+bx)\,dx \Rightarrow \left[ax + \frac{bx^2}{2}\right]_0^2 = 1$$

$$\Rightarrow 2a + \frac{\cancel{4}b}{\cancel{2}} - 0 = 1, \Rightarrow 2a + 2b = 1 \quad \text{——— (1)}$$

$$\Rightarrow E(X) = \int_0^2 x\,(a+bx)\,dx$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^2$$

3

$$= \frac{4a}{2} + \frac{8b}{3}$$

$$\frac{7}{6} = 2a + \frac{8b}{3}$$

$$\Rightarrow 12a + 16b = 7 \quad \text{——— (2)}$$

using (i) and (ii)

$12a + 16b = 7$

$6(2a + 2b) = 1 \times 6$

$\Rightarrow \cancel{12a} + 16b = 7$

$\underline{\cancel{12a} + 12b = 6}$

$4b = 1$

$\boxed{b = 1/4}$

$2a = 1 - 2 \times \frac{1}{4}$

$\boxed{a = 1/4}$

(i) $a = 1/4,\ b = 1/4.$

(ii) $var(X) = E(X^2) - [E(X)]^2$

$var(X) = \int_0^2 x^2 (a + bx)\, dx - [E(X)]^2.$

$= \left[\frac{ax^3}{3} + \frac{bx^4}{4}\right]_0^2 - [E(X)]^2.$

$= \frac{8}{3}a + \frac{\cancel{16}}{\cancel{4}}b - \left(\frac{7}{6}\right)^2$

$= \frac{8}{3}a + 4b - \frac{49}{36}$

$= \frac{8}{3}\left(\frac{1}{4}\right) + \cancel{4}\left(\frac{1}{\cancel{4}}\right) - \frac{49}{36}$

$= \frac{2}{3} + 1 - \frac{49}{36} = \frac{24 + 36 - 49}{36} = \frac{11}{36}$

Q. (6)

Q2. (a)

Given: mean = 1000 gallons
Standard deviation = 100 gallons
current amount in
the storage = 1100 gallons

2

Find: the probability that the demand for regular gasoline does not exceed the current amount in the store = $P(X \le 1100)$

Let X be a random variable, which signifies the daily demand for regular gasoline

$$P(X \le K) = \int_{-\infty}^{K} f(x)\, dx$$ or use the standard normal distribution

(to avoid the tedious calculations of integration of e^{x^2}, which has not direct closed form of integration).

Now, using the standard normal variate $z = \frac{x - \mu}{\sigma}$, using standard normal variate 'z'

$$P(X < K) = P\left(\frac{X-\mu}{\sigma} < \frac{K-\mu}{\sigma}\right)$$

$$= P\left(Z < \frac{K-\mu}{\sigma}\right)$$

$$= P\left(Z < \frac{1100-1000}{100}\right)$$

$$= P(Z < 1)$$

using the z-table : (Row = 1.0, column = 0.0)

$P(Z<1) = .8413$, which is 84.13%

Q2 (b)

$\Rightarrow f(x) = \dfrac{k}{1+x^2} \quad 0 < x < \infty.$

(i)

$\Rightarrow \displaystyle\int_{-\infty}^{\infty} \frac{k}{1+x^2}\, dx = 1.$ 3

$\Rightarrow \displaystyle\int_{0}^{\infty} \frac{k}{1+x^2}\, dx = 1$

$\Rightarrow \cancel{k \times \frac{1}{\quad}}$

$\Rightarrow k\left(\tan^{-1} x\right)_0^{\infty} = 1$

$k\left(\tan^{-1}\infty - \tan^{-1} 0\right) = 1$

$k \cdot \dfrac{\pi}{2} = 1$

$\Rightarrow k = 2/\pi$

(ii) Distribution function:

$F(x) = \displaystyle\int_{-\infty}^{x} f(x).dx$

$= \displaystyle\int_{0}^{x} \frac{k}{1+x^2}\, dx$

$= \dfrac{2}{\pi}\left[\tan^{-1} x\right]_0^{x}$

$= \dfrac{2}{\pi} \tan^{-1} x \quad$ for $0 < x < \infty.$

Q3 (a) (i) Finding the value of k;

$P(Z > k) = 0.3015$

$\Rightarrow P(Z > k) = 1 - P(Z \leq k)$

3

$P(Z < k) = 1 - 0.3015$

$= 0.6985$

Row ; column

from z-table $k = (0.5 ; 0.02)$

$k = \underline{\underline{0.52}}$

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(11) $P(k < z < -0.18) = 0.4197$

$P(z < k) = P(z < -0.18) - P(k < z < -0.18)$

$= 0.4286 - 0.4197$

$= 0.0089$

using z-table

0.0089 value is for

$k = -2.37$

Q3(b)

Probability (bomb strikes the target) $= \frac{1}{2}$

(2)

$n =$ the no. of bombs required to ensure 99% chance or better

At least two strikes gives

$n \geq 2$

Probability (out of n bombs, meet the 99% chance or better of completely destroying the target with at least two strikes) $\geq 99\%$

$\Rightarrow P(X \geq 2) \geq \frac{99}{100}$

Let X be a random variable representing the no. of bombs

$X \sim B(n, p) = B(n, \frac{1}{2})$

with mass function

$P(X = x) = p(x) = {}^nC_x\, p^x (1-p)^{n-x}$

where $x = 0, 1, 2, \ldots n$.

$\Rightarrow P(X \geq 2) \geq 0.99$

$\Rightarrow [1 - P(X < 2)] \geq 0.99$

$\Rightarrow 1 - p(0) - p(1) \geq 0.99$

$\Rightarrow 1 - {}^nC_0 (\frac{1}{2})^n - {}^nC_1 (\frac{1}{2})^n \geq 0.99$

$\Rightarrow 1 - (1 + n)(\frac{1}{2})^n \geq 0.99$

$\Rightarrow 0.01 \geq (1+n)/2^n$

$\Rightarrow \quad 0.01 \geq \dfrac{(1+n)}{2^n}$

$\Rightarrow \quad 2^n \geq 100(1+n)$

By hit and trial method

Let $n = 10$

$100(1+10) \leq 2^{10}$

$1100 \leq 1024$ False

$n = 11$

$100(1+11) \leq 2^{11}$

$1200 \leq 2048$ True.

Hence the values is

$\underline{\underline{n = 11}}$

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Q4 (a) Given :- (1) The average rate of packet loss is 2 losses per minute

3

Find :- probability of no packet losses in 30-sec.

$\Rightarrow$ $\lambda = 2$ Losses per minute

$= 2 \times \frac{30}{60} = 1$ loss in 30 seco.

$\boxed{\lambda = 1}$

Probability for k events occuring in an interval with average rate λ is

$= P(k;\lambda)$

$$P(k;\lambda) = \frac{e^{-\lambda}\lambda^{k}}{k!}$$

where

$k = 0, \lambda = 1$

$$P(k;\lambda) = \frac{e^{-1}1^{0}}{0!} = \frac{1}{e}$$

$$P(0;1) = \frac{1}{2.7182} \Rightarrow \underline{0.3679}$$

Q4 (b) Given: mean = 1
variance = 4/3 (2)

$P(X < 0) = ?$

$X \sim U(a, b)$

$p.d.f = \frac{1}{b-a}$; $a < x < b$

→ $Mean = (b+a)/2$

$1 = (b+a)/2 \Rightarrow a+b = 2$ — (1)

→ $V(X) = \frac{(b-a)^2}{12}$

$\frac{4}{3} = \frac{(b-a)^2}{12}$

$\Rightarrow (b-a)^2 = 4^2$

$\Rightarrow b-a = \pm 4$ — (2)

By (1) & (2)

$a+b = 2$	$a+b = 2$
$-a+b = 4$	$-a+b = -4$
$2b = 6$	$2b = -2$
$\boxed{b = 3}$	$\boxed{b = -1}$
$\boxed{a = -1}$	$\boxed{a = 3}$

Since $a < b$

The values of (a, b) is $(-1, 3)$

$f(x) = \frac{1}{4}$; $-1 < x < 3$,

$P(X < 0) = \int_{-1}^{0} f(x).dx = \frac{1}{4}\int_{-1}^{0} 1.dx$

$= \frac{1}{4}[x]_{-1}^{0} = \frac{1}{4}$.

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Q5 (a) Rolling a fair six-sided die.

Given: (1) Rolling a 6, immediately win

(2) Rolling other than 6, roll again
→ win if second roll comes with a number that is one higher than ~~of~~ previous roll (with 6 wrapping around to 1).

Probability of winning on the first roll (if 6 comes)

$= \frac{1}{6}$

Probability of winning on the second roll

$= P(\text{appearing of } 1) \times P(\text{— of } 2)$
$+ P(\text{" } 2) \times P(\text{" } 3) +$

$= P(1) \cdot P(2) + P(2) \cdot P(3) + P(3) \cdot P(4)$
$+ P(4) \cdot P(5) + P(5) \cdot P(1)$

$= \left(\frac{1}{6}\right)^2 \times 5$

3

$= \frac{5}{36}$

Probability of winning of game

$P(\text{win}) = P(\text{win on first roll}) + P(\text{win on second roll})$

$= \frac{1}{6} + \frac{5}{36} = \frac{11}{36}$

Q5 (b)

Probability of drawing 3 cards of the same suit without replacement

= P(3 draws from hearts)

+ P (all 3 draws from diamonds)

+ P (from clubs)

+ P (spades)

$= 4 \times \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{6864}{132600}$ 2

$= 0.0517$