11

DC-DC Converters

> "It has today occurred to me that an amplifier using semiconductors rather than vacuum is in principle possible." William Shockley in 1939. Shockley, Brattain, and Bardeen, all of Bell Labs, were awarded the Noble Prize for Physics in 1956 for the earlier invention of the semiconductor transistor.

In this chapter, the reader is introduced to power electronic power converters. The commonly used buck (step-down) and boost (step-up) topologies are analyzed for the continuous, boundary, and discontinuous conduction modes (CCM, BCM, and DCM, respectively) of operation. Power semiconductors are briefly introduced, and the power losses of the insulated-gate bipolar transistor (IGBT) and diode are estimated. Topics such as the sizing of passive components and the benefits of interleaving are covered. Examples are reviewed based on HEV and FCEV dc-dc converters. The concepts of root-mean-squared (rms) and direct-current (dc) quantities are reviewed as they are key to learning about the various converters.

11.1 Introduction

Many of the advances in the evolving automotive powertrain are due to the advent of power electronics and power semiconductors. By utilizing power electronic converters, voltages of a given magnitude and frequency can be converted to voltages of virtually any magnitude and frequency. One of the major functions of power electronics, therefore, is to provide the electrical power conversion which can achieve maximum functionality from electromechanical devices, energy sources, and electric loads. A second factor driving power electronics is efficiency. Efficient power conversion typically results in reduced material, cooling, power demand, cost, and volume while increasing reliability for many electronic products such as laptop computers and smartphones.

Power electronic converters enable electrical power conversion by periodically switching an available power source in and out of a circuit. These converters use semiconductor devices to act as switches, and feature energy storage devices, such as inductors and

Electric Powertrain: Energy Systems, Power Electronics and Drives for Hybrid, Electric and Fuel Cell Vehicles,
First Edition. John G. Hayes and G. Abas Goodarzi.

Companion website: www.wiley.com/go/hayes/electricpowertrain

capacitors, to store energy and filter the sharp-edged waveforms created by the fast switchings. Transformers are used for isolation and safety in addition to voltage and current conversions. These converters are known as **switch-mode power converters**. Significant electrical noise, known as **electromagnetic interference** (**EMI**), can be generated by the fast switching. EMI can affect the operation of other electronic and electrical equipment and is regulated by agencies around the world, such as the Federal Communications Commission (FCC) in the United States.

Electric power conversion can be described as falling into two categories, alternating current (ac) and direct current (dc). Thus, there are four possible types of electrical power conversion: ac-dc, dc-dc, dc-ac, and ac-ac.

Electronic circuits typically require dc voltage sources in order to function. Ac voltages are used for bulk power generation, transmission, and local distribution. Thus, an ac-dc converter is used to interface the ac grid to the dc battery.

Dc-dc converters are power electronics circuits which convert a dc voltage to a different dc voltage level. For example, the voltage of a fuel cell can drop significantly with increased power demand, and a dc-dc converter is required to step up the fuel cell voltage to a higher level for a more efficient powertrain.

The electric traction motor used in the modern electric powertrain requires ac voltages, rather than the dc of the battery. A **dc-ac inverter** is a power electronics circuit which converts dc to ac. Power inverters are the topic of Chapter 13.

Ac-dc rectifiers are power electronics circuits which convert ac to dc. Rectifiers are discussed in depth in Chapter 14.

Ac-ac power conversion is often required for electrical machines and loads at very high voltages, and is usually not considered for automotive applications.

The circuit diagram for the power stages of the 2010 Toyota Prius is shown in Figure 11.1. The vehicle uses a dc-dc converter to step up the battery voltage from about 200 V to a level between 200 V and 650 V in order to operate the powertrain most efficiently. Dc-ac inverters are used to interface the high-voltage dc link to the electric motor and generator. These types of dc-dc converters and dc-ac inverters enable a bidirectional flow of power. Thus, the dc-dc converter can take power from the battery when motoring and can recharge the battery when power is available from the generator or from the traction motor during regenerative braking.

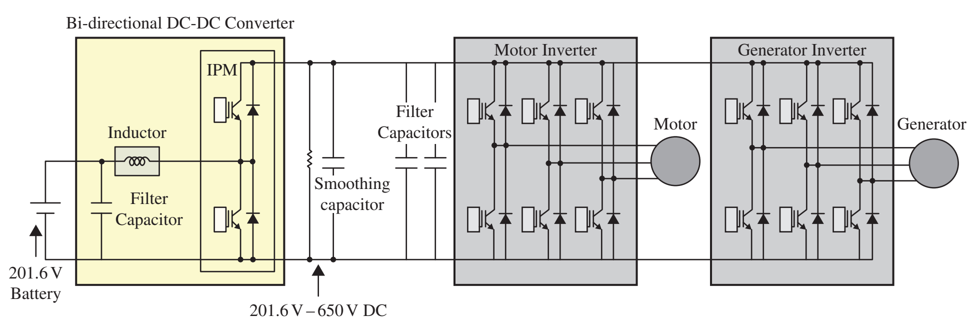


Figure 11.1 Power controller diagram for 2010 Toyota Prius [1]. (Courtesy of Oak Ridge National Laboratory, US Dept. of Energy.)

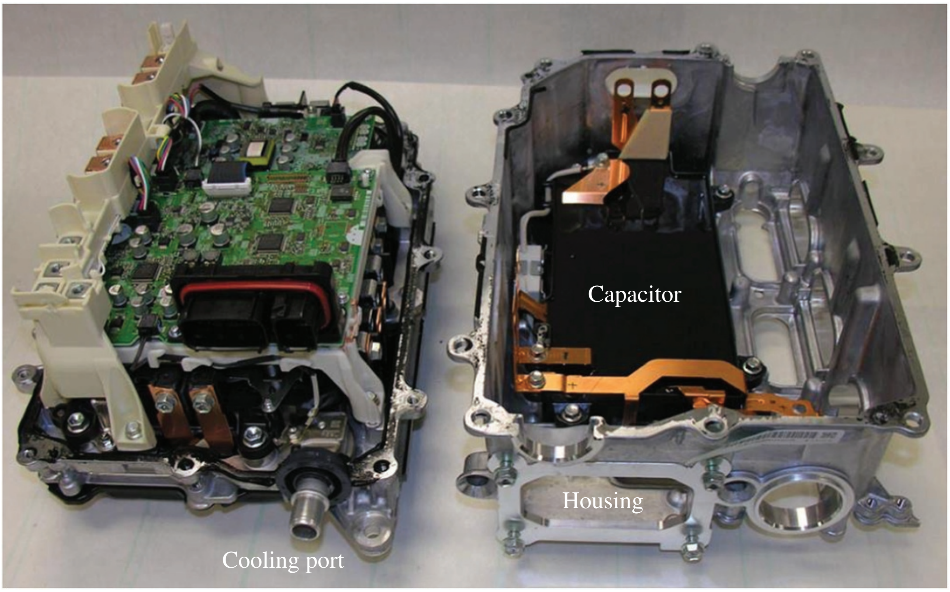


Figure 11.2 Views of the 2010 Toyota Prius power control unit [1]. (Courtesy of Oak Ridge National Laboratory, US Dept. of Energy.)

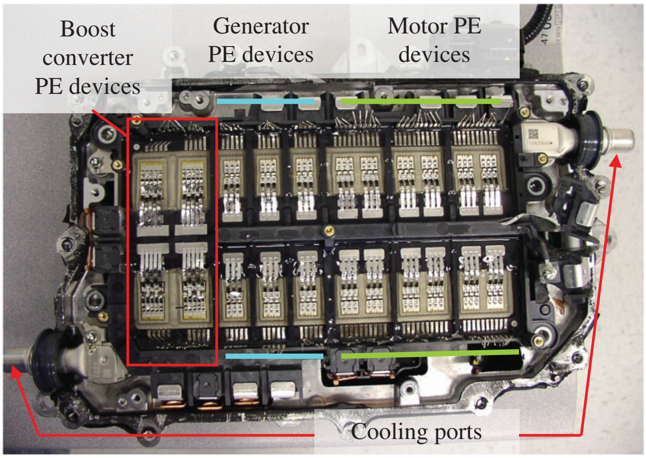
The power electronic converters and controllers for the 2010 Toyota Prius vehicle are integrated into a single box which is known as the power control unit (PCU). Views of the PCU are shown in Figure 11.2 and Figure 11.3 [1]. The PCU housing itself is made from aluminum and features an integrated heat sink. A cooling system is required as the power converters can generate significant power loss. The opened PCU is shown in Figure 11.2. The large capacitor assembly is noted, as is the cooling port for the liquid coolant.

Views of the power semiconductors for the dc-dc converter and the two dc-ac inverters are shown in Figure 11.3(a). The semiconductors are integrated into a single module which is mounted onto the PCU heat sink. A side view of the semiconductors is shown in Figure 11.3(b), where we can see the multiple thin bond wires from the connectors to the silicon dies and the thin silicon dies themselves.

The earliest generations of power converters were based on vacuum tubes, a technology which has almost been made obsolete with the invention of the semiconductor by William Shockley and others at Bell Labs in the early 1950s. The first power semiconductor device was a diode, which was later followed by the bipolar junction transistor (BJT) and the silicon-controlled rectifier (SCR), also known as the thyristor. Power diodes and thyristors continue to play a significant role in power conversion. The diode is ubiquitous, while the thyristor is commonly used for ac-dc and ac-ac power conversion.

Modern switch-mode power converters were first commercialized in the 1970s using the BJT as the main switching device. The metal-oxide-semiconductor field-effect transistor (MOSFET) took over in the late 1970s and is still the semiconductor switch of choice for many applications, especially for operating voltages less than 400 V.

(a)



(b)

Figure 11.3 Views of boost and inverter silicon for the 2010 Toyota Prius [1]. (Courtesy of Oak Ridge National Laboratory, US Dept. of Energy.)

The MOSFET surpassed the BJT as it is more efficient and easier to control. The invention of the IGBT pushed the semiconductor to achieve dominance at higher voltages where greater efficiencies can be achieved [2]. The IGBT dominates for high-power automotive power converters due to its efficiency and short-circuit capability. Wide-band-gap semiconductor technologies are shifting from niche applications into the mainstream of power conversion – based on the efficiency advantages of gallium nitride and silicon carbide.

The IGBT is paired with the silicon diode in this chapter in order to estimate the semiconductor power loss, whereas the MOSFET is paired with the silicon carbide diode in Chapter 14 for the power-factor-correction boost stage.

11.2 Power Conversion – Common and Basic Principles

Switch-mode technology has significant advantages over the competitive linear technology. Consider the simple example of an automotive power conversion going from a relatively high voltage of 14 V from the alternator on a vehicle to a lower voltage of 5 V in

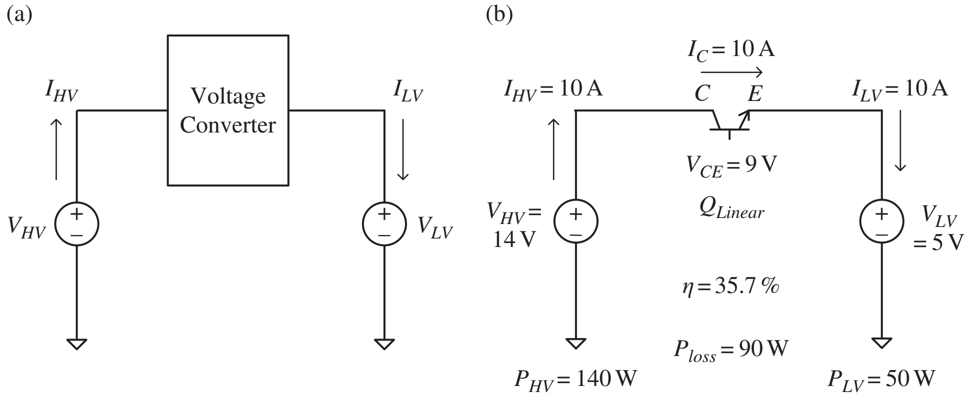


Figure 11.4 (a) Voltage conversion from 14 V to 5 V and (b) linear converter.

order to run vehicle loads. For consistency in dealing with the various voltages on the vehicle, the higher and lower voltage levels are represented by V_{HV} and V_{LV}, respectively, as shown in Figure 11.4(a). If a **linear power converter** is used to supply the low-voltage load from the high-voltage source, then the circuit will have a form similar to Figure 11.4 (b). For the linear power converter, a bipolar transistor, shown here as an NPN transistor, but typically a Darlington pair, is operated in the linear region, such that the desired output voltage V_{LV} is supplied to the load, and the voltage difference between the source and the load is dropped across the transistor. This method of converting electrical power has the advantages of simplicity, very high dynamic performance, fast response to any voltage or load changes, and the generation of little electrical noise.

The disadvantages of linear power converters are the large power loss, poor efficiency, and the resulting large physical size and mass required to dissipate the significant heat generated by the transistor's operation in the linear region. For the simple example shown, let us assume that the current to the load I_{LV} is 10 A; therefore, the load power consumption P_{LV} is 5 V times 10 A, which equals 50 W. In a linear converter, the current flowing from the source I_{HV} is also the transistor collector current I_C and is also the load current. Thus, the power pulled from the high-voltage source P_{HV} is 14 V times 10 A, which equals 140 W, and the difference between the source and load powers of 90 W is dissipated as heat from the linear transistor. The efficiency of the converter η is the ratio of load power to the source power and is given as follows:

$$\eta = \frac{P_{LV}}{P_{HV}} = \frac{50}{140} \times 100\% = 35.7\%$$

Switch-mode power converters operate by pulsing energy from the source to the load at a very high frequency, as shown in Figure 11.5. Similar to the linear power converter, the switch-mode power converter also uses a semiconductor device. However, in switch-mode converters, the switch is not operated in the linear region but is operated in a very efficient conduction mode, such that the switch is turned fully on with a minimal voltage drop across the device. A switch-mode converter providing the voltage conversion would be expected to have efficiencies in the mid-to-high 90s, in sharp contrast to the calculated efficiency of 35.7% for the preceding example.

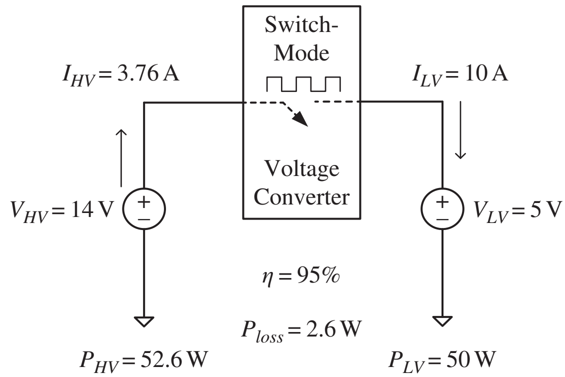


Figure 11.5 Switch-mode power converter.

Again, let us consider the voltage conversion from 14 V to 5 V at 50 W output. If the switch-mode converter efficiency is 95%, then the converter only pulls 3.76 A from the source, a significant reduction from the case of the linear converter. The power loss in the converter is only 2.6 W, resulting in a significant reduction in the size and weight of the switch-mode converter compared to the linear converter.

Although the switching power converter is more efficient and compact than the linear power converter, the switching power converter is more complex, has a lower bandwidth, and generates unwanted EMI.

11.2.1 The Basic Topologies

The next step is to realize a switch-mode power converter. There are three basic converters, all of which use a switch Q, a diode D, and an inductor L, in various configurations. These are the buck or step-down converter, the boost or step-up converter, and the buck-boost or step-down/step-up converter, as shown in Figure 11.6(a), (b), and (c), respectively.

The **buck converter** converts power from a high-voltage source to a lower voltage.

The **boost converter** converts power from a low-voltage source to a higher voltage.

The **buck-boost converter** converts power from an input voltage source to an output voltage and can step down or step up the output voltage.

The buck converter is initially analyzed, and the method is then applied to the boost converter. The buck-boost converter is briefly considered in the Appendix at the end of this chapter.

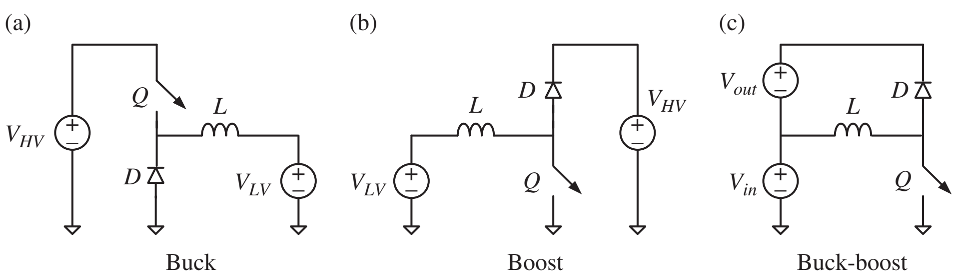


Figure 11.6 Buck, boost, and buck-boost converters.

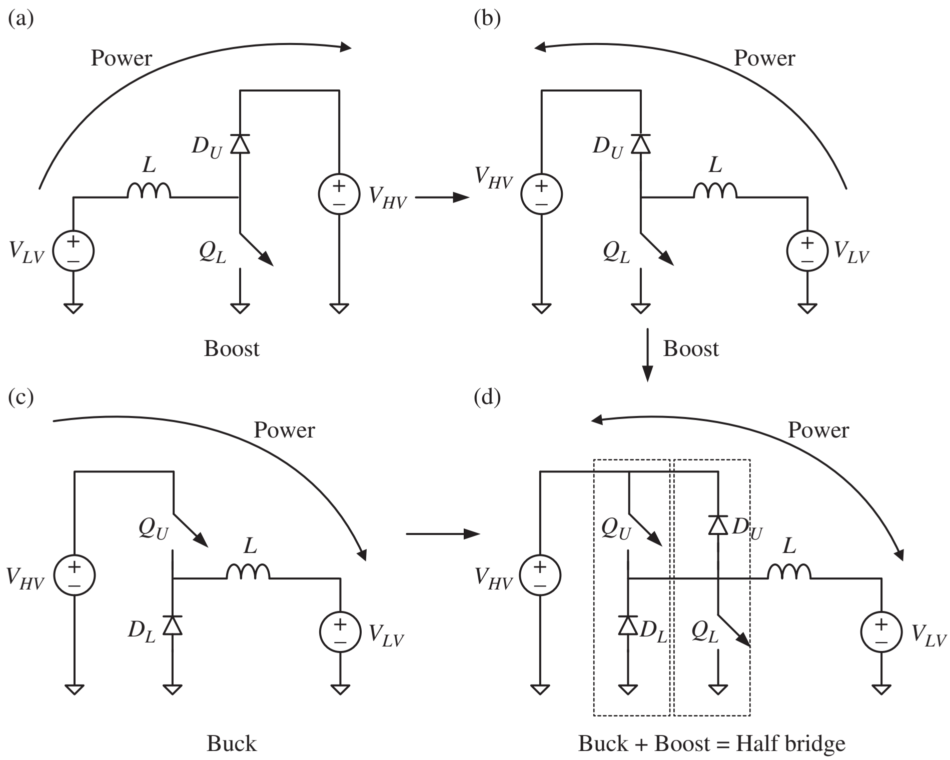


Figure 11.7 Half-bridge converter.

11.2.2 The Half-Bridge Buck-Boost Bidirectional Converter

The buck and boost converters can be integrated to create a bidirectional buck-boost converter. This converter is commonly used for hybrid electric vehicles as it enables discharge of the low-voltage battery to a higher voltage during motoring using a boost and charging of the low-voltage battery from the high-voltage link using a buck converter. This integration of a buck converter and a boost converter is often known as a **half–bridge converter**. The half-bridge can be explained as follows. The boost converter of Figure 11.7(a) can be redrawn as Figure 11.7(b). The switch and diode of the buck converter of Figure 11.7(c) are then integrated with the boost components of Figure 11.7(b) to create the buck-boost half-bridge of Figure 11.7(d). The subscripts U and L are used to designate the upper and lower switches and diodes.

11.3 The Buck or Step-Down Converter

The buck or step-down converter produces a lower average output voltage V_{LV} than the input voltage V_{HV}. The converter has three basic power components to perform the voltage conversion. First, a controlled power semiconductor switch, shown as Q in Figure 11.8, is pulsed on and off at a high frequency in order to transfer energy from

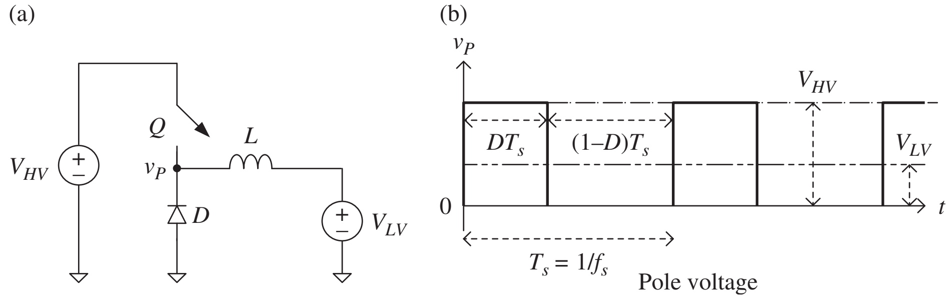


Figure 11.8 Buck or step-down (a) converter and (b) pole voltage.

the source V_{HV} to the load V_{LV}. The output from the switch is a controlled pulse train of a controlled switching frequency f_s, or switching period T_s, and a controlled conduction time within that period $D\ T_s$, where D is termed the **duty cycle** of the switch, or the portion of time during the period for which the switch is conducting. The output voltage is known as the pole output voltage v_p and is shown in Figure 11.8(a). The pole output voltage waveform $v_p(t)$ is as shown in Figure 11.8(b). As can be seen, the pole voltage is pulsing between 0 V and the source voltage V_{HV}.

Note that the switching frequency f_s is simply the inverse of the period T_s:

$$f_s = \frac{1}{T_s} \tag{11.1}$$

A dc or constant voltage is required at the load, and so a second component, the inductor L is required in order to remove the ac component of the pole voltage.

The inductor functions in the circuit by storing energy. The stored energy within the inductor cannot instantly flow from the source to the load. Current conduction must be maintained through the inductor when the switch Q is pulsed on and off. A third component, known as the **inverse diode**, is required as an uncontrolled switch in order to provide a conduction path for the inductor current when the switch Q is not conducting.

Note that D is used in this text both as a label to describe a diode and as a variable to describe the duty cycle. The terms are obviously not interchangeable, and the meaning should be obvious from the context.

In general, from a power perspective, dc-dc converters require additional filtering components. Typically, the on-vehicle voltage sources, such as the battery, fuel cell, or generator, are modeled to include the voltage source itself and the internal inductance of the voltage source and the required cabling. In addition, components such as the battery or fuel cell should be filtered such that they supply dc only, as the ac components generate electrical noise and additional heating, reducing lifetimes for these critical components. There are the three essential components of the buck converter plus the discrete filter capacitors on both voltage links and internal or added inductances between the dc links and the sources, as shown in Figure 11.9. Thus, the high-voltage source, shown as $V_{S(HV)}$, has the internal inductance L_{HV} and the dc link capacitance C_{HV}, while the low-voltage source, shown as $V_{S(LV)}$, has the internal inductance L_{LV} and the dc link capacitance C_{LV}.

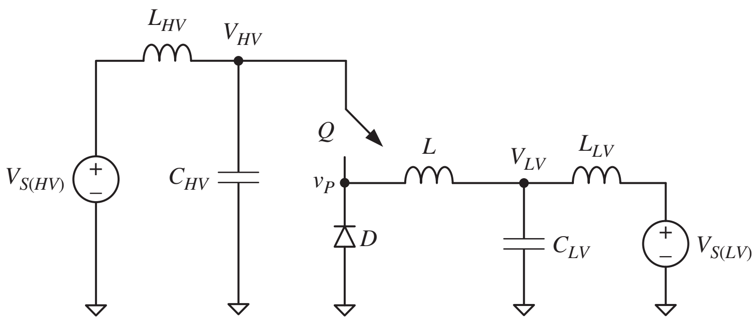


Figure 11.9 A buck converter with source and load filter components.

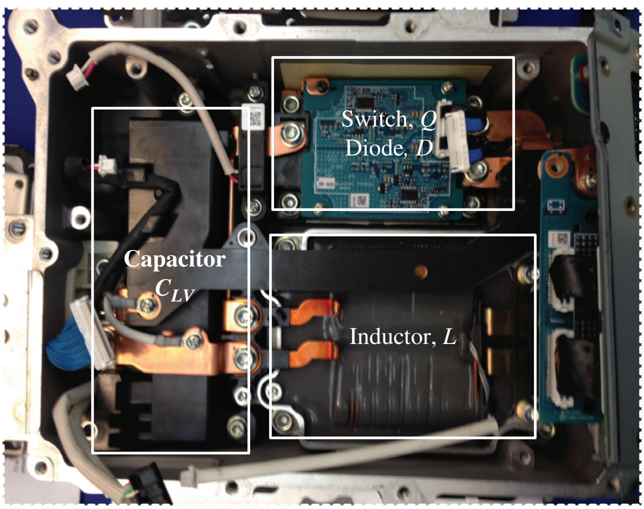


Figure 11.10 HEV bidirectional dc-dc converter.

The additional components can be of significant physical size and mass. An automotive dc-dc converter is shown in Figure 11.10. The approximate areas for the various components are outlined.

11.3.1 Analysis of Voltage Gain of Buck Converter in CCM

In CCM, the inductor of the buck converter is always conducting current. This mode of operation is relatively easy to analyze. First, the relationship between the voltage gain of the converter and the duty cycle must be determined.

Neglecting parasitics, in a buck converter, the dc or average output voltage equals the average of the pole voltage. If we ignore the switch and diode voltage drops, then the pole voltage is pulsing from the dc return or 0 V to the high-voltage dc link voltage V_{HV} as shown in Figure 11.8(b), and again in Figure 11.11(a). The pole is at V_{HV} for the time

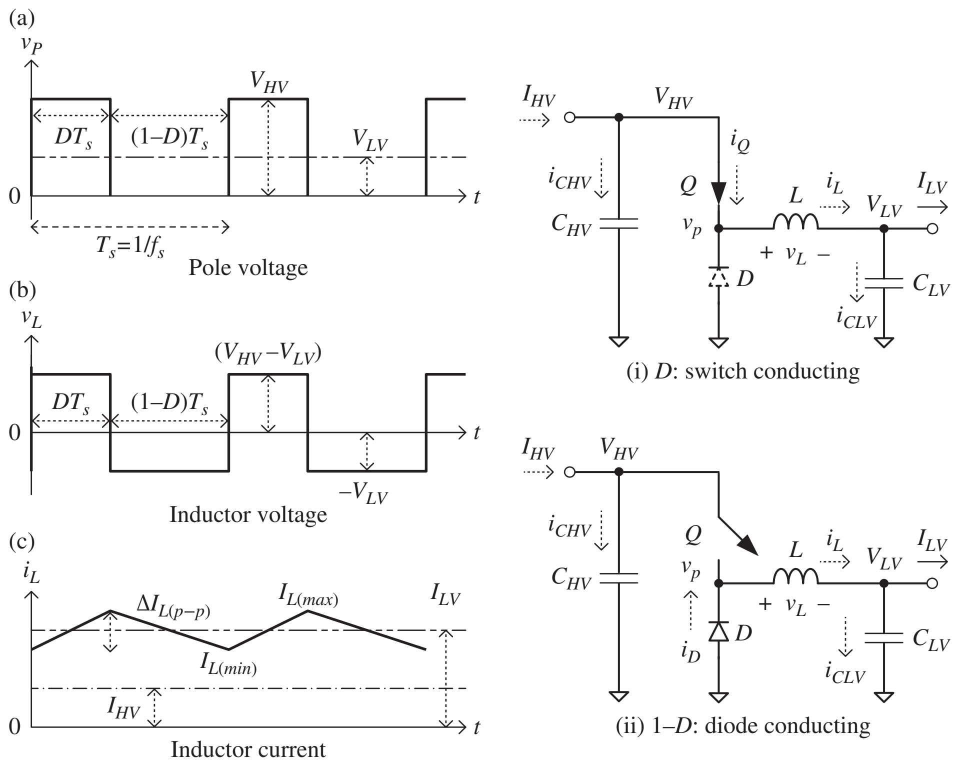


Figure 11.11 Buck CCM (a) pole voltage, (b) inductor voltage, and (c) inductor current.

period DT_s and at 0 V for the time period $(1–D)\ T_s$. The circuits are shown in Figure 11.11 (i) and (ii) for the two operating modes.

The dc output voltage is given by

$$V_{LV} = \frac{1}{T_s}[V_{HV}DT_s + 0 \times (1-D)T_s] = DV_{HV} \tag{11.2}$$

Thus, the duty cycle of a buck converter represents the voltage gain of the buck converter and is expressed as

$$D = \frac{V_{LV}}{V_{HV}} \tag{11.3}$$

It is noted here that the parasitic voltage drops of the various passive and active components are not considered. This is reasonable for high-voltage dc-dc converters, but the derivation of the duty cycle should be expanded to consider parasitics for low-voltage converters.

This relationship can also be shown in integral form as

$$V_{LV} = \frac{1}{T_s}\int_0^{T_s} v_p(t)dt = \frac{1}{T_s}\int_0^{DT_s} V_{HV}dt + \frac{1}{T_s}\int_{DT_s}^{T_s} 0\,dt = DV_{HV} \tag{11.4}$$

It is also productive to derive the voltage relationships by considering the inductor voltage and current. The inductor voltage and current are illustrated in Figure 11.11 (b) and (c), respectively. The inductor current increases linearly while the switch is closed during DT_s as a voltage of (V_{HV}–V_{LV}) appears across the inductor. When the switch opens and the diode is conducting during (1–D) T_s, the inductor current linearly decreases as there is a negative voltage of $-V_{LV}$ across the inductor.

Applying Faraday's law to determine the back emf across the inductor yields:

$$v_L(t) = v_p(t) - V_{LV} = L\frac{di_L(t)}{dt} \tag{11.5}$$

Rearranging the preceding equation in terms of the change in inductor current $i_L(t)$, we get

$$di_L(t) = \frac{1}{L}\left[v_p(t) - V_{LV}\right]dt. \tag{11.6}$$

Assuming that the inductor current is changing linearly in the steady-state operation, the change in inductor current from peak to peak can be described by $\Delta I_{L(p\text{-}p)}$.

During time DT_s:

$$\Delta I_{L(p-p)} = \frac{1}{L}(V_{HV} - V_{LV})DT_s = \frac{1}{f_s L}(V_{HV} - V_{LV})D \tag{11.7}$$

During time (1–D)T_s:

$$\Delta I_{L(p-p)} = \frac{V_{LV}(1-D)T_s}{L} = \frac{V_{LV}(1-D)}{f_s L} \tag{11.8}$$

Equating the two expressions for the change in current $\Delta I_{L(p\text{-}p)}$, we get

$$\Delta I_{L(p-p)} = \frac{1}{f_s L}(V_{HV} - V_{LV})D = \frac{V_{LV}(1-D)}{f_s L} \tag{11.9}$$

Rearranging the preceding equation again yields the earlier expression, Equation (11.3), for the voltage gain.

11.3.1.1 Analysis of Buck Converter in CCM

It is necessary to know the average, rms, minimum, and maximum currents in each component in order to design and specify the components of a buck converter.

The inductor current can be seen to have two current components: (1) a dc, or constant, or average component which flows to the load and (2) an ac or time-varying periodic component which is shunted from the load by the low-voltage capacitor due to the low ac impedance of the capacitor. The dc current is shown as I_{LV} in Figure 11.12(a), while the ac current is represented by the periodic triangular waveform oscillating between $I_{L(min)}$ and $I_{L(max)}$ for a total amplitude within the cycle of $\Delta I_{L(p\text{-}p)}$.

The inductor current is conducted by the switch as i_Q during D, as shown in Figure 11.12(b), and by the diode as i_D during (1-D), as shown in Figure 11.12(c).

The high-voltage and low-voltage capacitor currents can be determined by simply subtracting the respective dc currents from the switch and inductor currents, respectively, as shown in Figure 11.12(d) and (e).

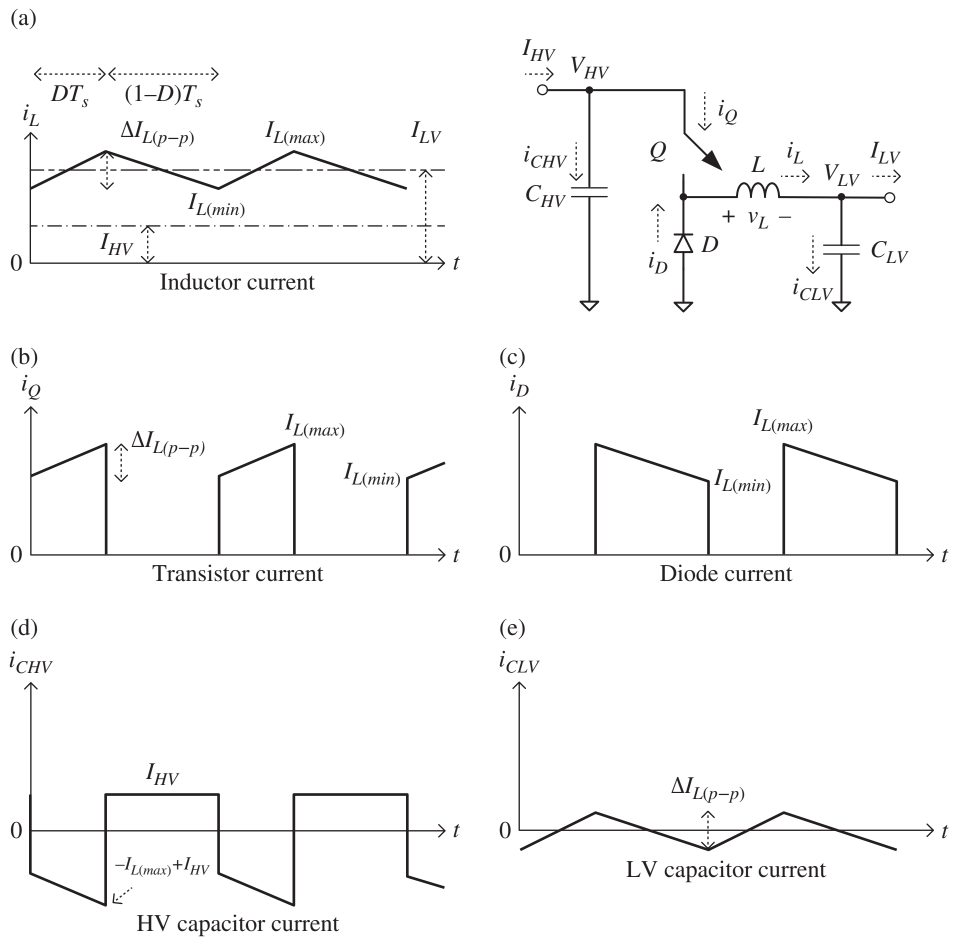


Figure 11.12 Currents for (a) inductor, (b) transistor, (c) diode, (d) high-voltage capacitor, and (e) low-voltage capacitor.

11.3.1.2 Determining Low-Voltage Capacitor RMS Current

The rms component of the low-voltage capacitor current $I_{CLV(rms)}$ is of initial interest as it is the rms of a triangular waveform. The rms value is given by

$$I_{CLV(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{CLV}(t)^2 dt} \tag{11.10}$$

From an examination of the current waveform in Figure 11.12(e), it can easily be shown that the low-voltage capacitor current i_{CLV} has the following two straight line segments:

During time DT_s:

$$i_{CLV}(t) = \frac{\Delta I_{L(p-p)}}{DT_s}t - \frac{\Delta I_{L(p-p)}}{2} \tag{11.11}$$

During time $(1-D)T_s$:

$$i_{CLV}(t) = -\frac{\Delta I_{L(p-p)}}{(1-D)T_s}t + \frac{\Delta I_{L(p-p)}}{2}\frac{(1+D)}{(1-D)} \tag{11.12}$$

Substituting Equation (11.11) and Equation (11.12) into Equation (11.10) gives

$$I_{CLV(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{CLV}(t)^2 dt}$$

$$= \sqrt{\frac{1}{T_s}\int_0^{DT_s}\left(\frac{\Delta I_{L(p-p)}}{DT_s}t - \frac{\Delta I_{L(p-p)}}{2}\right)^2 dt + \int_{DT_s}^{T_s}\left(-\frac{\Delta I_{L(p-p)}}{(1-D)T_s}t + \frac{\Delta I_{L(p-p)}}{2}\frac{(1+D)}{(1-D)}\right)^2 dt} \tag{11.13}$$

Although this equation appears complex, it reduces to a simple expression for the rms value of a triangular waveform with no average component:

$$I_{CLV(rms)} = \frac{\Delta I_{L(p-p)}}{\sqrt{12}} \tag{11.14}$$

Knowing the dc load current and the capacitor rms current, it can similarly be derived that the rms value of the inductor $I_{L(rms)}$ is given by

$$I_{L(rms)} = \sqrt{I_{L(dc)}{}^2 + I_{CLV(rms)}{}^2} = \sqrt{I_{LV}{}^2 + \frac{\Delta I_{L(p-p)}{}^2}{12}} \tag{11.15}$$

where $I_{L(dc)} = I_{LV}$ is the dc current flowing through the inductor to the low-voltage output.

When designing or specifying the inductor, the maximum current is also of interest. The inductor peak current is simply given by

$$I_{L(max)} = I_{LV} + \frac{\Delta I_{L(p-p)}}{2} \tag{11.16}$$

The switch and diode currents can now be determined. The average currents of the switch and diode, designated $I_{Q(dc)}$ and $I_{D(dc)}$, respectively, are easily determined and are given by

$$I_{Q(dc)} = D\, I_{LV} \tag{11.17}$$

and

$$I_{D(dc)} = (1-D)\, I_{LV} \tag{11.18}$$

The rms currents of the switch and diode, designated $I_{Q(rms)}$ and $I_{D(rms)}$, respectively, can be determined by analysis and simply work out to be

$$I_{Q(rms)} = \sqrt{D}\, I_{L(rms)} \tag{11.19}$$

and

$$I_{D(rms)} = \sqrt{(1-D)}\, I_{L(rms)} \tag{11.20}$$

In addition to the dc and rms currents, the instantaneous turn-on and turn-off currents in the switch and diode are of interest:

$$I_{Q(off)} = I_{D(on)} = I_{L(max)} = I_{LV} + \frac{\Delta I_{L(p-p)}}{2} \tag{11.21}$$

and

$$I_{Q(on)} = I_{D(off)} = I_{L(min)} = I_{LV} - \frac{\Delta I_{L(p-p)}}{2} \tag{11.22}$$

where $I_{Q(on)}$ and $I_{Q(off)}$ and $I_{D(on)}$ and $I_{D(off)}$ are the turn-on and turn-off currents in the switch and diode, respectively, and $I_{L(min)}$ is the minimum current in the inductor for a given cycle.

The rms current in the high-voltage link capacitor $I_{CHV(rms)}$ can be determined by subtracting the dc current coming from the high-voltage source I_{HV} from the rms current in the switch $I_{Q(rms)}$:

$$I_{CHV(rms)} = \sqrt{I_{Q(rms)}{}^2 - I_{HV}{}^2} \tag{11.23}$$

Note that it is common to use the term **current-ripple ratio** $\boldsymbol{r_i}$, which is defined as

$$r_i = \frac{\Delta I_{L(p-p)}}{I_{L(dc)}} \tag{11.24}$$

and is often expressed as a percentage.

11.3.1.3 Capacitor Voltages

The voltage ripple on the dc capacitors is also of interest. From Gauss's law of electricity, the capacitor charge Q is equal to the product of the capacitance C and the capacitor voltage V_C:

$$Q = CV_C \tag{11.25}$$

The capacitor voltage changes over the switching period as the capacitor is charged and discharged. The peak-to-peak voltage ripple on the capacitor $\Delta V_{C(p-p)}$ is directly proportional to the change in stored charge ΔQ as the capacitor charges and discharges. Thus:

$$\Delta V_{C(p-p)} = \frac{\Delta Q}{C} \tag{11.26}$$

As the various component currents have already been determined, it is easy to determine ΔQ as it is simply the change in charge due to the capacitor current.

The high-voltage and low-voltage capacitor voltages and currents are shown in Figures 11.13(a) and (b).

Referring to Figure 11.13(a), the peak-to-peak voltage ripple on the high-voltage capacitor $\Delta V_{CHV(p-p)}$ is determined by calculating the change in charge ΔQ_{CHV} on the basis of the negative or positive current. Using the current-time area product of the current and the diode conduction time gives the following answer:

$$\Delta V_{CHV(p-p)} = \frac{\Delta Q_{CHV}}{C_{HV}} = \frac{I_{HV}(1-D)T_s}{C_{HV}} = \frac{I_{HV}(1-D)}{f_s C_{HV}} \tag{11.27}$$

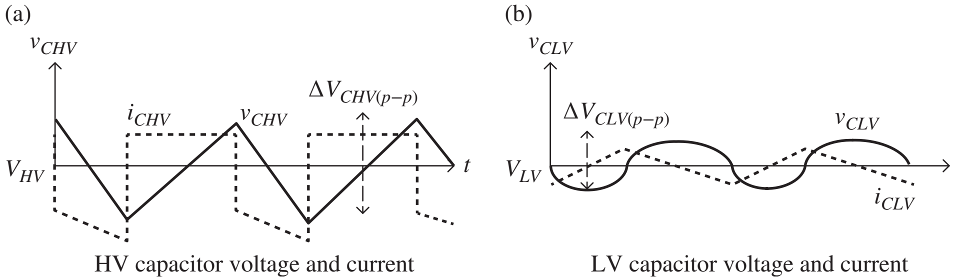


Figure 11.13 Capacitor voltages (solid) and currents (dashed).

Similarly, the peak-to-peak voltage ripple on the low-voltage capacitor $\Delta V_{CLV(p-p)}$ is estimated by determining the charge ΔQ_{CLV} on the basis of the negative or positive current. In this case, the current-time area product is based on a simple triangular current waveform, and the result is as follows:

$$\Delta V_{CLV(p-p)} = \frac{\Delta Q_{CLV}}{C_{LV}} = \frac{1}{2}\frac{\Delta I_{L(p-p)}}{2}\frac{T_s}{2}\frac{1}{C_{LV}} = \frac{\Delta I_{L(p-p)} T_s}{8C_{LV}} = \frac{\Delta I_{L(p-p)}}{8f_s C_{LV}} \tag{11.28}$$

At this point, all the current and voltages have been determined in order to analyze a given design, or design a converter based on a set of specifications, as shown in the next two examples.

We also use the term **voltage-ripple ratio r_v**, which is defined as

$$r_v = \frac{\Delta V_{C(p-p)}}{V} \tag{11.29}$$

and is often expressed as a percentage.

11.3.1.4 Example: Designing Buck Converter for CCM Operation

A hybrid electric vehicle requires a 20 kW bidirectional converter to generate a 500 V dc link voltage from the 200 V NiMH battery. The switching frequency is 10 kHz.

Determine the component parameters in order to have a 28% current-ripple ratio on the inductor and a 0.5% voltage-ripple ratio on the high- and low-voltage capacitors.

Determine the various component currents.

Assume ideal components, and ignore the power loss.

Solution:

The duty cycle is

$$D = \frac{V_{LV}}{V_{HV}} = \frac{200\,\text{V}}{500\,\text{V}} = 0.4$$

The average inductor current equals the output load current and is given by

$$I_{LV} = I_{L(dc)} = \frac{P}{V_{LV}} = \frac{20000}{200}\text{A} = 100\,\text{A}$$

The inductor ripple current is specified as

$$\Delta I_{L(p-p)} = r_i I_{L(dc)} = 0.28 \times 100\,\text{A} = 28\,\text{A}$$

The desired inductance is

$$L = \frac{(V_{HV} - V_{LV})D}{f_s \Delta I_{L(p-p)}} = \frac{(500-200)0.4}{10000 \times 28}\text{H} = 428.5\,\mu\text{H}$$

The rms current in the low-voltage capacitor is given by

$$I_{CLV(rms)} = \frac{\Delta I_{L(p-p)}}{\sqrt{12}} = \frac{28}{\sqrt{12}}\text{A} = 8.083\,\text{A}$$

The inductor rms current is

$$I_{L(rms)} = \sqrt{I_{L(dc)}{}^2 + I_{CLV(rms)}{}^2} = \sqrt{100^2 + 8.083^2}\text{A} = 100.3\,\text{A}$$

The inductor, switch, and diode maximum and minimum currents are

$$I_{L(\max)} = I_{Q(off)} = I_{D(on)} = I_{L(dc)} + \frac{\Delta I_{L(p-p)}}{2} = 114\,\text{A}$$

and

$$I_{L(min)} = I_{Q(on)} = I_{D(off)} = I_{L(dc)} - \frac{\Delta I_{L(p-p)}}{2} = 86\,\text{A}$$

The switch rms and average currents are

$$I_{Q(rms)} = \sqrt{D}I_{L(rms)} = 63.44\,\text{A}$$

and

$$I_{Q(dc)} = DI_{L(dc)} = 40\,\text{A}$$

The diode rms and average currents are

$$I_{D(rms)} = \sqrt{1-D}I_{L(rms)} = 77.69\,\text{A}$$

and

$$I_{D(dc)} = (1-D)I_{L(dc)} = 60\,\text{A}$$

The average high-voltage input current is

$$I_{HV} = \frac{P}{V_{HV}} = \frac{20000}{500}\text{A} = 40\,\text{A}$$

The rms current in the high-voltage link capacitor is

$$I_{CHV(rms)} = \sqrt{I_{Q(rms)}{}^2 - I_{HV}{}^2} = \sqrt{63.44^2 - 40^2}\text{A} = 49.24\,\text{A}$$

Finally, the capacitances can be determined by rearranging Equation (11.27) and Equation (11.28):

$$C_{HV} = \frac{I_{HV}(1-D)}{f_s \Delta V_{CHV(p-p)}} = \frac{I_{HV}(1-D)}{f_s r_v V_{HV}} = \frac{40 \times (1-0.4)}{10000 \times 0.005 \times 500}\text{F} = 960\,\mu\text{F}$$

$$C_{LV} = \frac{\Delta I_{L(p-p)}}{8 f_s \Delta V_{CLV(p-p)}} = \frac{\Delta I_{L(p-p)}}{8 f_s r_v V_{LV}} = \frac{28}{8 \times 10000 \times 0.005 \times 200}\text{F} = 350\,\mu\text{F}$$

11.3.2 BCM Operation of Buck Converter

As the load current and power drop in the buck converter for given input and output voltages, the same analysis can be applied for these part-load conditions until the minimum inductor current drops to zero. The CCM condition at which the current drops to zero is known as **BCM**. This is a common operating mode for many power converters, even at full load, as it has the significant advantage compared to CCM that the diode turns off at zero current, eliminating reverse-recovery losses in the silicon diode.

The inductor waveform is shown in Figure 11.14. The complete set of BCM voltage and current waveforms is shown in Figure 11.15.

As can be seen, the switch now turns on at zero current, and the diode turns off at zero current. Interestingly, there have been no changes in the inductor ripple current and the low-voltage capacitor ripple current as these currents are not a function of the load in BCM and CCM – assuming, of course, that the inductance value is constant with the load, which may not necessarily be correct as discussed in Chapter 16. The average and rms currents seen by the inductor, switch, diode, and input capacitor are a function of the load and can change significantly compared to the earlier CCM waveforms.

The converter enters BCM operation when the minimum inductor current equals 0 A. At this condition, the dc current going through the inductor, and into the load, is equal to one half the peak-to-peak inductor current. Thus, the load current at which the converter enters BCM is given by

$$I_{LV} = \frac{\Delta I_{L(p-p)}}{2} = \frac{V_{LV}(1-D)}{2f_s L} \tag{11.30}$$

Knowing the BCM level, the various component currents can be determined using the equations developed for the CCM analysis.

11.3.2.1 Example of Buck in BCM

Determine the power level at which the converter enters BCM for the converter voltages used in the previous example.

Solution:

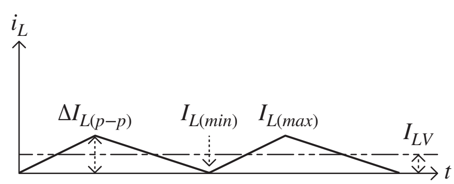
As before, the duty cycle is 0.4, and the inductor ripple current remains at

$$\Delta I_{L(p-p)} = 28\,\text{A}$$

From Equation (11.30), the average inductor current in BCM is

$$I_{LV} = I_{L(dc)} = \frac{\Delta I_{L(p-p)}}{2} = 14\,\text{A}$$

Figure 11.14 Buck BCM inductor current.



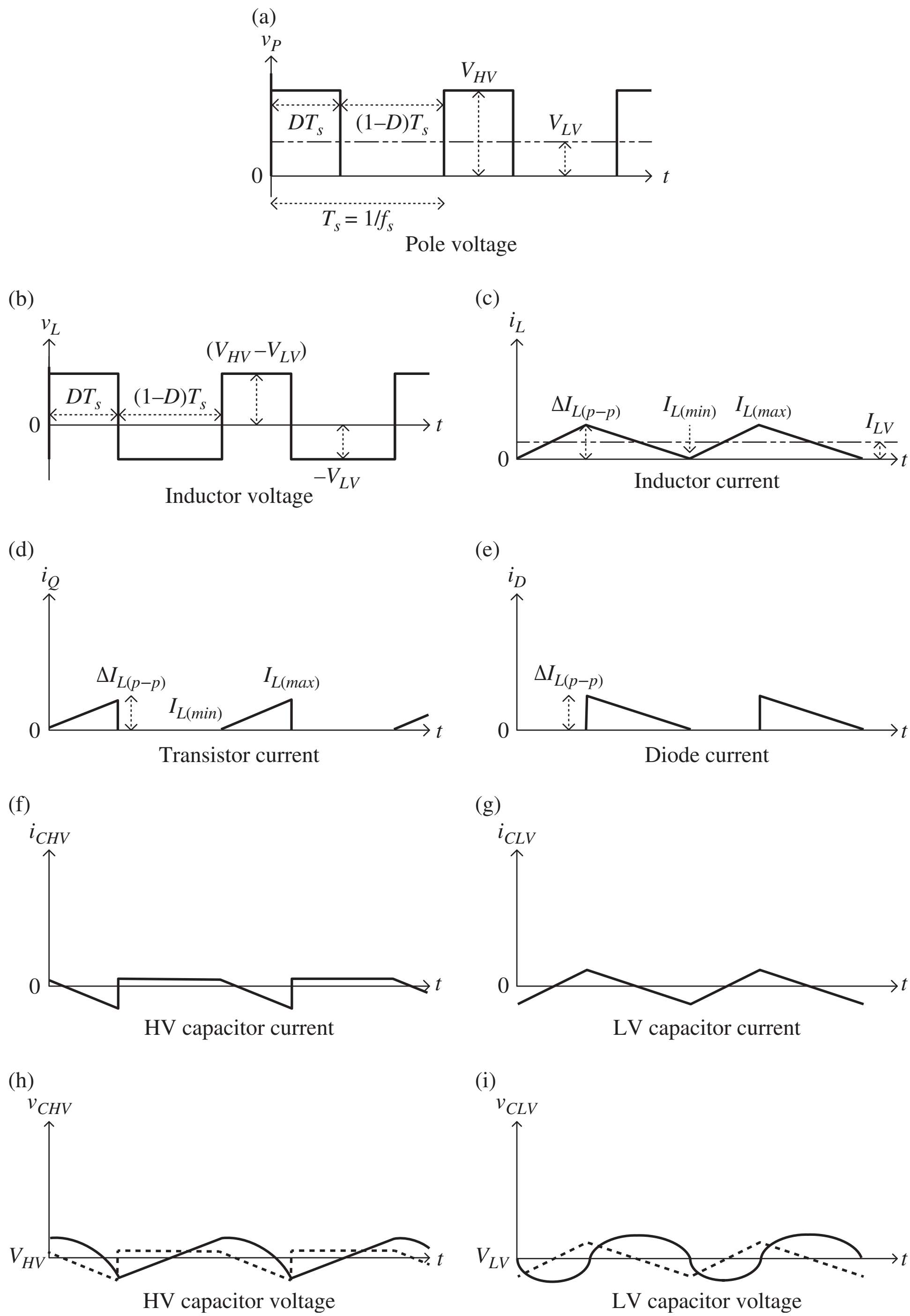


Figure 11.15 Buck BCM voltage and current waveforms.

The power at which this occurs is

$$P = V_{LV} I_{LV} = 200 \times 14\,\text{W} = 2.8\,\text{kW}$$

The low-voltage capacitor rms current remains

$$I_{CLV(rms)} = \frac{\Delta I_{L(p-p)}}{\sqrt{12}} = 8.083\,\text{A}$$

The inductor rms current reduces to

$$I_{L(rms)} = \sqrt{I_{LV}{}^2 + I_{CLV(rms)}{}^2} = \sqrt{14^2 + 8.083^2}\,\text{A} = 16.17\,\text{A}$$

The inductor, switch, and diode maximum and minimum currents are

$$I_{L(max)} = I_{Q(off)} = I_{D(on)} = I_{L(dc)} + \frac{\Delta I_{L(p-p)}}{2} = 28\,\text{A}$$

$$I_{L(min)} = I_{Q(on)} = I_{D(off)} = I_{L(dc)} - \frac{\Delta I_{L(p-p)}}{2} = 0\,\text{A}$$

The rms and average switch currents are

$$I_{Q(rms)} = \sqrt{D} I_{L(rms)} = \sqrt{0.4} \times 16.17\,\text{A} = 10.22\,\text{A}$$

and

$$I_{Q(dc)} = D I_{L(dc)} = 0.4 \times 14\,\text{A} = 5.6\,\text{A}$$

The rms and average diode currents are

$$I_{D(rms)} = \sqrt{1-D} I_{L(rms)} = \sqrt{(1-0.4)} \times 16.17\,\text{A} = 12.53\,\text{A}$$

and

$$I_{D(dc)} = (1-D) I_{L(dc)} = (1-0.4) \times 14\,\text{A} = 8.4\,\text{A}$$

The average high-voltage input current is

$$I_{HV} = \frac{P}{V_{HV}} = \frac{2800}{500}\,\text{A} = 5.6\,\text{A}$$

The rms current in the high-voltage link capacitor is

$$I_{CHV(rms)} = \sqrt{I_{Q(rms)}{}^2 - I_{HV}{}^2} = \sqrt{10.22^2 - 5.6^2}\,\text{A} = 8.55\,\text{A}$$

11.3.3 DCM Operation of Buck Converter

As the load current is reduced, the inductor current reduces in value and becomes discontinuous. The inductor current is no longer in CCM or BCM and is operating in DCM. This mode occurs commonly at light loads and has the advantages of zero-current turn-on and turn-off of the switch and diode, respectively, resulting in reasonably high efficiencies at light loads.

The pole voltage and inductor voltage and current waveforms for DCM operation are shown in Figure 11.16.

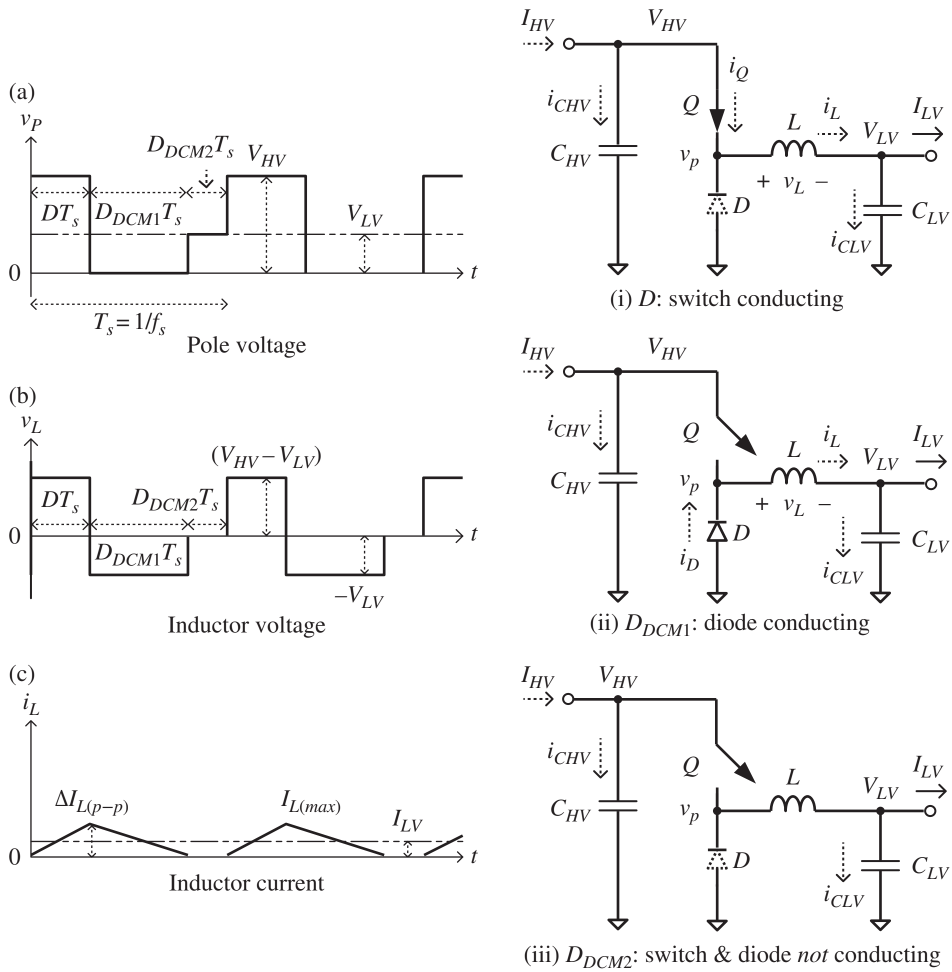


Figure 11.16 Buck DCM pole voltage and inductor voltage and current.

There are three time periods of interest within the switching period. During the first period DT_s, the switch is closed, and a positive voltage is applied across the inductor, causing the current to increase. During the time period $D_{DCM1}T_s$, the switch is open, and the diode is conducting, resulting in a negative voltage across the inductor and a decrease in the inductor current to zero. During the time period $D_{DCM2}T_s$, the inductor current is zero, and the load current is provided by the low-voltage capacitor. Note that during $D_{DCM2}T_s$ the inductor voltage and current are both zero, and the pole voltage equals the output voltage.

The rms and average values of the various waveforms are relatively straightforward to determine as the critical currents are discontinuous.

First, let us determine the relationship between the dc output current I_{LV} and the duty cycle during DCM. Two of the three time periods contribute to the average current.

$$
\begin{aligned}
I_{LV} &= I_{L(dc)} = \frac{1}{T_s}\int_0^{T_s} i_L(t)dt \\
&= \frac{1}{T_s}\left[\int_0^{DT_s} i_L(t)dt + \int_{DT_s}^{DT_s+D_{DCM1}T_s} i_L(t)dt + \int_{DT_s+D_{DCM1}T_s}^{T_s} i_L(t)dt\right] \\
&= \frac{1}{T_s}\left[\frac{1}{2}\Delta I_{L(p-p)}DT_s + \frac{1}{2}\Delta I_{L(p-p)}D_{DCM1}T_s + 0\times D_{DCM2}T_s\right] \\
&= \frac{\Delta I_{L(p-p)}}{2}(D + D_{DCM1})
\end{aligned}
\tag{11.31}
$$

The relationship between the duty cycle of the switch D and the duty cycle of the diode D_{DCM1} is easily determined. In the steady state, the inductor current increases by $\Delta I_{L(p-p)}$ during DT_s, and decreases by $\Delta I_{L(p-p)}$ during $D_{DCM1}T_s$.

Thus, assuming a linear current change, we get

$$\Delta I_{L(p-p)} = \frac{(V_{HV} - V_{LV})DT_s}{L} = \frac{V_{LV}D_{DCM1}T_s}{L} \tag{11.32}$$

This equation can be rearranged to show that

$$D_{DCM1} = \frac{(V_{HV} - V_{LV})}{V_{LV}}D \tag{11.33}$$

The three duty cycles sum to unity:

$$D + D_{DCM1} + D_{DCM2} = 1 \tag{11.34}$$

Thus, the third period of time is then easily determined and can be simplified to

$$
\begin{aligned}
D_{DCM2} &= 1 - D - D_{DCM1} = 1 - D - \frac{(V_{HV} - V_{LV})}{V_{LV}}D \\
&= 1 - \frac{V_{HV}}{V_{LV}}D
\end{aligned}
\tag{11.35}
$$

An expression for the duty cycle in terms of the dc input and output levels can be determined by rearranging Equation (11.31) to get

$$D + D_{DCM1} = \frac{2I_{LV}}{\Delta I_{L(p-p)}} \tag{11.36}$$

Substituting in Equation (11.32) and Equation (11.33) yields

$$D + \frac{(V_{HV} - V_{LV})}{V_{LV}}D = \frac{2LI_{LV}}{(V_{HV} - V_{LV})DT_s} \tag{11.37}$$

Rearranging the preceding equation gives an expression for D:

$$D = \sqrt{\frac{2V_{LV}}{V_{HV}(V_{HV} - V_{LV})}f_sLI_{LV}} \tag{11.38}$$

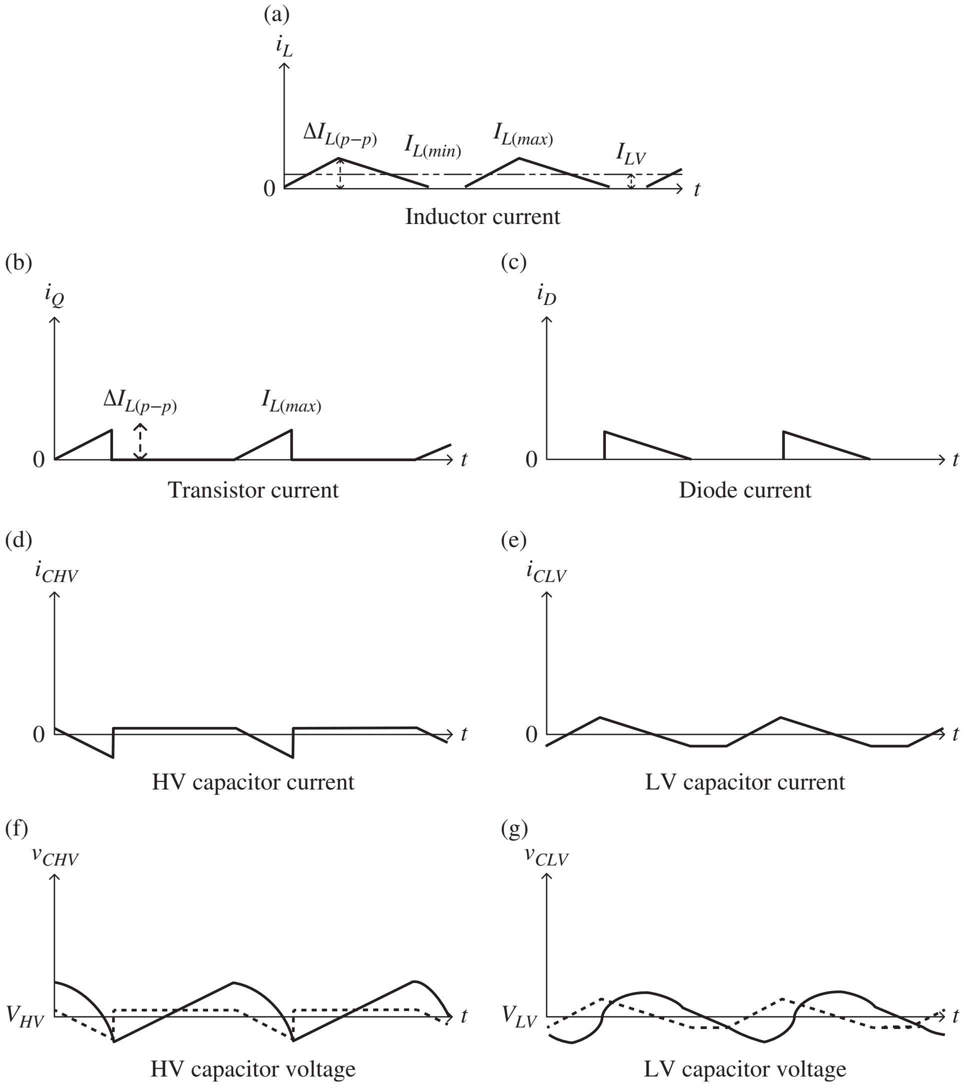


Figure 11.17 Buck DCM operation voltage and current waveforms.

Once the various duty cycles are known, the current values are easily be determined for a load condition.

The various converter waveforms in DCM are shown in Figure 11.17.

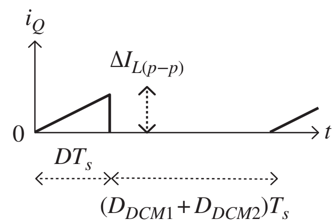
It is useful to note the following relationships between the peak, average, and rms currents for various waveforms. The various waveforms can be simply deconstructed in order to determine the rms and average components.

The switch current of Figure 11.17 is shown again in Figure 11.18.

The current in the switch is a ramp and is described by

$$i_Q(t) = \frac{\Delta I_{L(p-p)}}{DT_s} t \tag{11.39}$$

Figure 11.18 Switch current in DCM.



The rms value of the switch current is simply given by

$$\begin{aligned} I_{Q(rms)} &= \sqrt{\frac{1}{T_s}\int_0^{T_s} i_Q(t)^2 dt} \\ &= \sqrt{\frac{1}{T_s}\int_0^{DT_s}\left(\frac{\Delta I_{L(p-p)}}{DT_s}t\right)^2 dt + \int_{DT_s}^{T_s} 0^2 dt} \\ &= \Delta I_{L(p-p)} \times \sqrt{\frac{1}{D^2 T_s^{\,3}}\left[\frac{t^3}{3}\right]_0^{DT_s}} \\ &= \sqrt{\frac{D}{3}}\Delta I_{L(p-p)} \end{aligned} \tag{11.40}$$

The rms value of the diode current is similarly given by

$$I_{D(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_D(t)^2 dt} = \sqrt{\frac{D_{DCM1}}{3}}\Delta I_{L(p-p)} \tag{11.41}$$

The rms value of the inductor current is

$$I_{L(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_L(t)^2 dt} = \sqrt{\frac{(D + D_{DCM1})}{3}}\Delta I_{L(p-p)} \tag{11.42}$$

The rms value of the low-voltage capacitor current is

$$I_{CLV(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{CLV}(t)^2 dt} = \sqrt{I_{L(rms)}{}^2 - I_{LV}{}^2} \tag{11.43}$$

The rms value of the high-voltage capacitor current is

$$I_{CHV(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{CHV}(t)^2 dt} = \sqrt{I_{Q(rms)}{}^2 - I_{HV}{}^2} \tag{11.44}$$

Similarly, the average currents of the switch and diode are

$$I_{Q(dc)} = \frac{1}{T_s}\int_0^{T_s} i_Q(t)dt = \frac{1}{T_s}\int_0^{DT_s}\left(\frac{\Delta I_{L(p-p)}}{DT_s}t\right)dt + \int_{DT_s}^{T_s} 0dt = \frac{D}{2}\Delta I_{L(p-p)} \tag{11.45}$$

$$I_{D(dc)} = \frac{1}{T_s}\int_0^{T_s} i_D(t)dt = \frac{D_{DCM1}}{2}\Delta I_{L(p-p)} \tag{11.46}$$

11.3.3.1 Example: Buck Converter in DCM Operation

Knowing from the previous example that the converter operates in DCM below 2.8 kW for the given voltage conditions, determine the various converter currents when operating in DCM at 2 kW.

Solution:

The dc current in the low-voltage dc link is

$$I_{LV} = \frac{P}{V_{LV}} = \frac{2000}{200}\text{A} = 10\,\text{A}$$

The average high-voltage input current is

$$I_{HV} = \frac{P}{V_{HV}} = \frac{2000}{500}\text{A} = 4\,\text{A}$$

The duty cycle at this condition is

$$D = \sqrt{\frac{2V_{LV}}{V_{HV}(V_{HV} - V_{LV})} \times f_s L I_{LV}}$$

$$= \sqrt{\frac{2 \times 200}{500 \times (500 - 200)} \times 10000 \times 428.5 \times 10^{-6} \times 10} = 0.338$$

and

$$D_{DCM1} = \frac{(V_{HV} - V_{LV})}{V_{LV}}D = \frac{(500 - 200)}{200} \times 0.338 = 0.507$$

The inductor ripple current is

$$\Delta I_{L(p-p)} = \frac{(V_{HV} - V_{LV})D}{f_s L} = \frac{(500 - 200) \times 0.338}{10000 \times 428.5 \times 10^{-6}}\text{A} = 23.66\,\text{A}$$

The various rms currents are

$$I_{Q(rms)} = \sqrt{\frac{D}{3}}\Delta I_{L(p-p)} = \sqrt{\frac{0.338}{3}} \times 23.66 = 7.94\,\text{A}$$

$$I_{D(rms)} = \sqrt{\frac{D_{DCM1}}{3}}\Delta I_{L(p-p)} = \sqrt{\frac{0.507}{3}} \times 23.66\,\text{A} = 9.73\,\text{A}$$

$$I_{L(rms)} = \sqrt{\frac{(D + D_{DCM1})}{3}} \Delta I_{L(p-p)} = \sqrt{\frac{(0.338 + 0.507)}{3}} \times 23.66\text{A} = 12.56\text{A}$$

$$I_{CLV(rms)} = \sqrt{I_{L(rms)}^2 - I_{LV}^2} = \sqrt{12.56^2 - 10^2}\,\text{A} = 7.6\,\text{A}$$

and

$$I_{CHV(rms)} = \sqrt{I_{Q(rms)}^2 - I_{HV}^2} = \sqrt{7.94^2 - 4^2}\,\text{A} = 6.86\,\text{A}$$

The various dc currents are

$$I_{Q(dc)} = \frac{D}{2} \Delta I_{L(p-p)} = \frac{0.338}{2} \times 23.66\,\text{A} = 4\,\text{A}\left(= I_{HV}\,\text{as expected}\right)$$

$$I_{D(dc)} = \frac{D_{DCM1}}{2} \Delta I_{L(p-p)} = \frac{0.509}{2} \times 23.66\,\text{A} = 6\,\text{A}$$

and

$$I_{L(dc)} = \frac{D + D_{DCM1}}{2} \Delta I_{L(p-p)} = \frac{0.338 + 0.509}{2} \times 23.66\,\text{A}$$
$$= 10\text{A}\left(= I_{Q(dc)} + I_{D(dc)}\ \text{as expected}\right)$$

11.4 The Boost or Step-up Converter

The **boost** or **step-up** converter produces a higher average output voltage V_{HV} than the input voltage V_{LV}. As with the buck converter, the boost converter has three basic power components to perform the voltage conversion. First, a controlled power semiconductor switch, shown as Q in Figure 11.19(a), is pulsed on and off at a high frequency in order to transfer energy from the source V_{LV} to the load V_{HV}. The output from the switch is a controlled pulse train of a controlled switching frequency f_s, or switching period T_s, and a controlled conduction time within that period DT_s, where D is termed the duty cycle or proportional on-time of the switch. The pole voltage waveform $v_p(t)$ is as shown in Figure 11.19(b). As can be seen, the pole voltage is pulsing between 0 V and V_{HV}. The input voltage V_{LV} is the average value of the pole voltage.

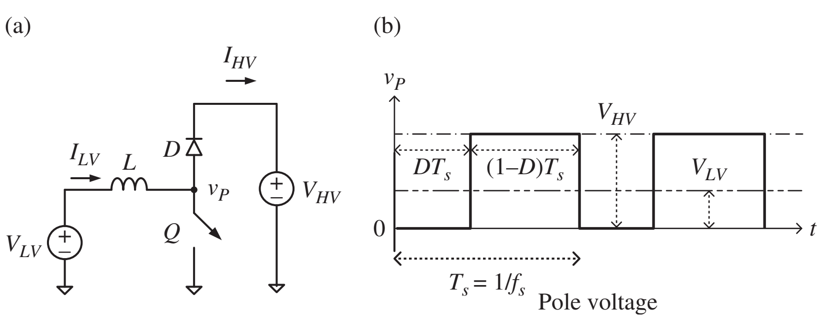


Figure 11.19 Boost or step-up converter.

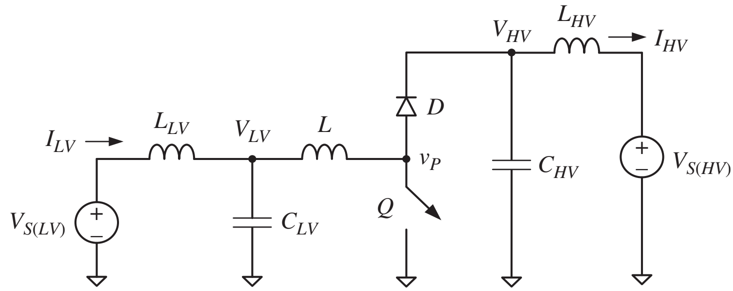


Figure 11.20 A boost converter with source and load filter components.

As with the buck converter, both voltage links feature internal or added inductances between the dc links and the sources, as shown in Figure 11.20. Thus, the high-voltage source, shown as $V_{S(HV)}$, has the internal inductance, L_{HV}, and the dc link capacitance C_{HV}, while the low-voltage source, shown as $V_{S(LV)}$, has the internal inductance L_{LV} and the dc link capacitance C_{LV}.

11.4.1 Analysis of Voltage Gain of Boost Converter in CCM

In CCM, the inductor of the boost converter always conducts current. First, the relationship between the voltage gain of the converter and the duty cycle is determined. Neglecting parasitics, in a boost converter, the dc or average input voltage equals the average of the pole voltage. Ignoring the switch and diode voltage drops, the pole voltage pulses from the dc return or 0 V to the high-voltage dc link voltage, $V_{HV.}$ The pole is at V_{HV} for the time period (1–D) T_s and at 0 V for the time period DT_s. Thus, the dc output voltage is given by

$$V_{LV} = \frac{1}{T_s}\int_0^{T_s} v_P(t)dt = \frac{1}{T_s}\int_0^{DT_s} 0dt + \frac{1}{T_s}\int_{DT_s}^{T_s} V_{HV}dt = (1-D)V_{HV} \tag{11.47}$$

Thus, the duty cycle of a boost converter is

$$D = 1 - \frac{V_{LV}}{V_{HV}} \tag{11.48}$$

The voltage gain of the converter can be expressed as

$$V_{HV} \Big/ V_{LV} = \frac{1}{1-D} \tag{11.49}$$

It is also productive to derive the voltage relationships by considering the inductor current. As can be seen in Figure 11.21, the inductor current increases while the switch is closed during DT_s, and a voltage of V_{LV} appears across the inductor. When the switch opens and the diode conducts during (1–D) T_s, the inductor current decreases as there is a negative voltage of (V_{LV}–V_{HV}) across the inductor.

During time DT_s:

$$\Delta I_{L(p-p)} = \frac{V_{LV}}{L}DT_s \tag{11.50}$$

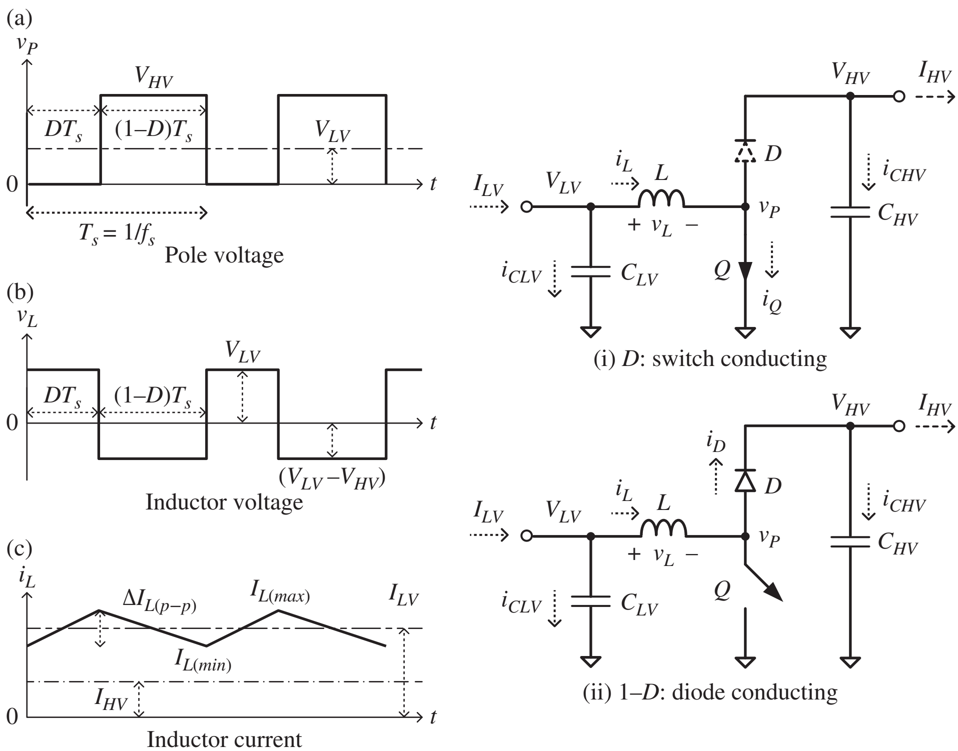


Figure 11.21 Boost CCM pole voltage and inductor voltage and current.

During time $(1–D)T_s$:

$$\Delta I_{L(p-p)} = \frac{-(V_{LV} - V_{HV})(1-D)T_s}{L} \tag{11.51}$$

Solving the two equations results in the earlier expression, Equation (11.48), for the voltage gain.

11.4.1.1 Analysis of Boost Converter in CCM

In order to design and specify the components of a boost converter, it is necessary to know the average, rms, minimum, and maximum currents in each component, the waveforms of which are shown in Figure 11.22. The inductor current $i_L(t)$ has the dc component, $I_{L(dc)} = I_{LV}$, flowing from the source and an ac component flowing into the low-voltage capacitor as $i_{CLV}(t)$, as shown in Figure 11.22(e).

As before for the CCM buck converter, the rms component of the capacitor triangular current is given by

$$I_{CLV(rms)} = \frac{\Delta I_{L(p-p)}}{\sqrt{12}} \tag{11.52}$$

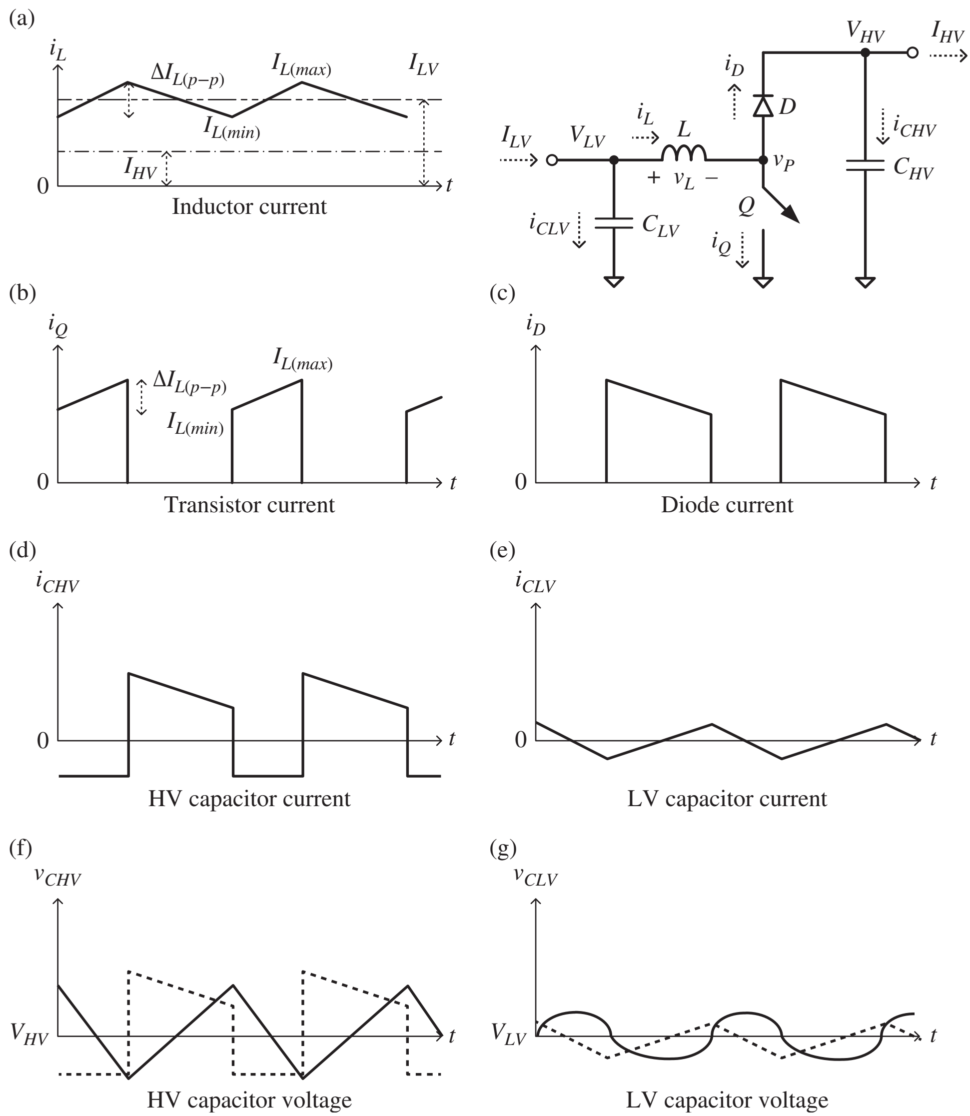


Figure 11.22 Boost CCM voltage and current waveforms.

Knowing the dc input current and the capacitor rms current, the rms value of the inductor $I_{L(rms)}$ is

$$I_{L(rms)} = \sqrt{I_{L(dc)}{}^2 + I_{CLV(rms)}{}^2} = \sqrt{I_{LV}{}^2 + \frac{\Delta I_{L(p-p)}{}^2}{12}} \tag{11.53}$$

The inductor peak current is simply given by

$$I_{L(max)} = I_{LV} + \frac{\Delta I_{L(p-p)}}{2} \tag{11.54}$$

The average currents of the switch and diode are given by

$$I_{Q(dc)} = D\,I_{LV} \tag{11.55}$$

$$I_{D(dc)} = (1-D)\,I_{LV} \tag{11.56}$$

The rms currents work out to be

$$I_{Q(rms)} = \sqrt{D}\,I_{L(rms)} \tag{11.57}$$

$$I_{D(rms)} = \sqrt{(1-D)}\,I_{L(rms)} \tag{11.58}$$

The instantaneous turn-on and turn-off currents in the switch and diode are

$$I_{Q(off)} = I_{D(on)} = I_{L(max)} = I_{LV} + \frac{\Delta I_{L(p-p)}}{2} \tag{11.59}$$

$$I_{Q(on)} = I_{D(off)} = I_{L(min)} = I_{LV} - \frac{\Delta I_{L(p-p)}}{2} \tag{11.60}$$

The rms current in the high-voltage link capacitor $I_{CHV(rms)}$ can be determined by subtracting the dc current I_{HV} from the rms current in the diode $I_{D(rms)}$:

$$I_{CHV(rms)} = \sqrt{I_{D(rms)}{}^2 - I_{HV}{}^2} \tag{11.61}$$

Referring to Figure 11.22(d) and (f), the peak-to-peak voltage ripple on the high-voltage capacitor $\Delta V_{CHV(p-p)}$ is determined by calculating the charge based on the negative or positive current. Using the current-time area product of the negative current and the diode conduction time gives the answer as follows:

$$\Delta V_{CHV(p-p)} = \frac{\Delta Q_{CHV}}{C_{HV}} = \frac{I_{HV}\cdot(1-D)T_s}{C_{HV}} \tag{11.62}$$

Similarly, the peak-to-peak voltage ripple on the high-voltage capacitor $\Delta V_{CLV(p-p)}$ is estimated by calculating the charge based on the negative or positive current. In this case, the current-time area product is the product of ½, ½ of the peak-to-peak inductor ripple current, and ½ of the period, because of the triangular waveform:

$$\Delta V_{CLV(p-p)} = \frac{\Delta Q_{CLV}}{C_{LV}} = \frac{1}{2}\frac{\Delta I_{L(p-p)}}{2}\frac{T_s}{2}\frac{1}{C_{LV}} = \frac{\Delta I_{L(p-p)}T_s}{8C_{LV}} \tag{11.63}$$

At this point, all the currents and voltages for the CCM boost converter have been determined in order to analyze a given design or design a converter based on a set of specifications.

11.4.1.2 Example: Analyzing Boost for CCM Operation

The vehicle in the earlier buck example is now operating in **motoring** mode, and the bidirectional converter is required to act as a **boost** at full power.

Determine the currents in the various components. Assume ideal components.

Solution:

The duty cycle of the boost converter is

$$D = 1 - \frac{V_{LV}}{V_{HV}} = 1 - \frac{200\,\text{V}}{500\,\text{V}} = 0.6$$

The average inductor current equals the input current and is given by

$$I_{LV} = I_{L(dc)} = \frac{P}{V_{LV}} = \frac{20000}{200}\text{A} = 100\,\text{A}$$

The inductor ripple current is

$$\Delta I_{L(p-p)} = \frac{V_{LV}D}{f_s L} = \frac{200 \times 0.6}{10000 \times 428.5}\text{A} = 28\,\text{A}$$

The rms current in the low-voltage capacitor is given by

$$I_{CLV(rms)} = \frac{\Delta I_{L(p-p)}}{\sqrt{12}} = 8.083\,\text{A}$$

The inductor rms current is

$$I_{L(rms)} = \sqrt{I_{L(dc)}{}^2 + I_{CLV(rms)}{}^2} = \sqrt{100^2 + 8.083^2}\text{A} = 100.3\,\text{A}$$

The inductor, switch, and diode maximum and minimum currents are

$$I_{L(max)} = I_{Q(off)} = I_{D(on)} = I_{L(dc)} + \frac{\Delta I_{L(p-p)}}{2} = 114\,\text{A}$$

$$I_{L(min)} = I_{Q(on)} = I_{D(off)} = I_{L(dc)} - \frac{\Delta I_{L(p-p)}}{2} = 86\,\text{A}$$

The switch rms and average currents are

$$I_{Q(rms)} = \sqrt{D}I_{L(rms)} = 77.69\text{A}$$

and

$$I_{Q(dc)} = DI_{L(dc)} = 60\,\text{A}$$

The diode rms and average currents are

$$I_{D(rms)} = \sqrt{1-D}I_{L(rms)} = 63.44\,\text{A}$$

and

$$I_{D(dc)} = (1-D)I_{L(dc)} = 40\,\text{A}$$

It can be seen that many of the currents are the same for the buck and boost examples and that the current values for the switch and diode are swapped for the same power and voltage levels.

11.4.2 BCM Operation of Boost Converter

As the load current and power drop in the boost converter for given input and output voltages, the converter enters **BCM,** when the CCM inductor current drops to zero. A number of boost converters operate in this mode as it can result in low semiconductor losses and compact inductors. Some designs of lower-power power-factor-correction boost converters operate in BCM in order to eliminate the reverse recovery of the silicon diode.

The earlier waveforms in Figure 11.22 have been modified to those of Figure 11.23 such that the converter is in BCM. As can be seen, the switch now turns on at zero current, and

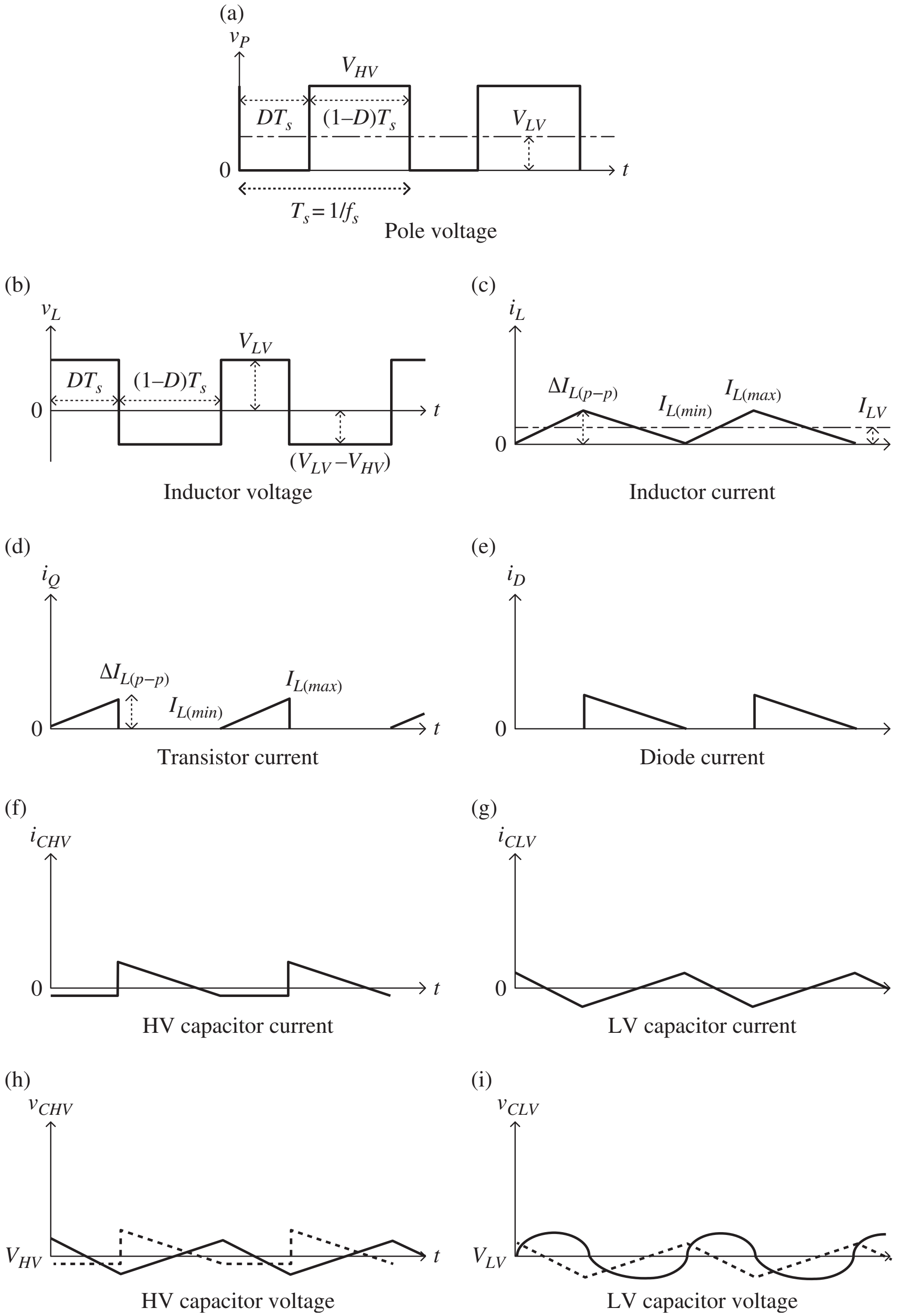


Figure 11.23 Boost BCM voltage and current waveforms.

the diode turns off at zero current. The converter enters BCM operation when the minimum inductor current equals 0 A. At this condition, the dc current going through the inductor is equal to ½ the peak-to-peak inductor current. Thus, the load current at which the converter enters BCM is given by

$$I_{LV} = \frac{\Delta I_{L(p-p)}}{2} = \frac{V_{LV}D}{2f_sL} \tag{11.64}$$

In BCM, the various component currents can be determined using the equations developed for the CCM analysis.

11.4.2.1 Example: Boost Converter in BCM

Determine the power level at which the boost converter enters BCM for the converter voltages used in the previous example.

Solution:

As before, the duty cycle is

$$D = 1 - \frac{V_{LV}}{V_{HV}} = 1 - \frac{200}{500} = 0.6$$

The inductor ripple current remains

$$\Delta I_{L(p-p)} = 28\,\text{A}$$

The average inductor current in BCM is

$$I_{LV} = \frac{\Delta I_{L(p-p)}}{2} = 14\text{A}$$

The power at which this occurs

$$P = V_{LV}I_{LV} = 200 \times 14\,\text{W} = 2.8\,\text{kW}$$

Again, these current and power values are the same as for the buck converter.

11.4.3 DCM Operation of Boost Converter

As the load current is reduced, the inductor current reduces in value and becomes discontinuous. The inductor current is no longer in CCM or BCM, but is operating in **DCM**. This mode occurs commonly at light loads and has the advantages of zero-current turn-on and turn-off of the switch and diode respectively, resulting in reasonably high efficiencies at light loads.

The waveforms for DCM operation are shown in Figure 11.24. There are three time periods of interest within the switching period. During the first period DT_s, the switch is closed, and a positive voltage is applied across the inductor, causing the current to linearly increase. During the time period $D_{DCM1}T_s$, the switch is open, and the diode is conducting, resulting in a negative voltage across the inductor and a linear decrease in the inductor current to zero. During the time period $D_{DCM2}T_s$, the inductor current is zero, and the load current is provided by the high-voltage capacitor.

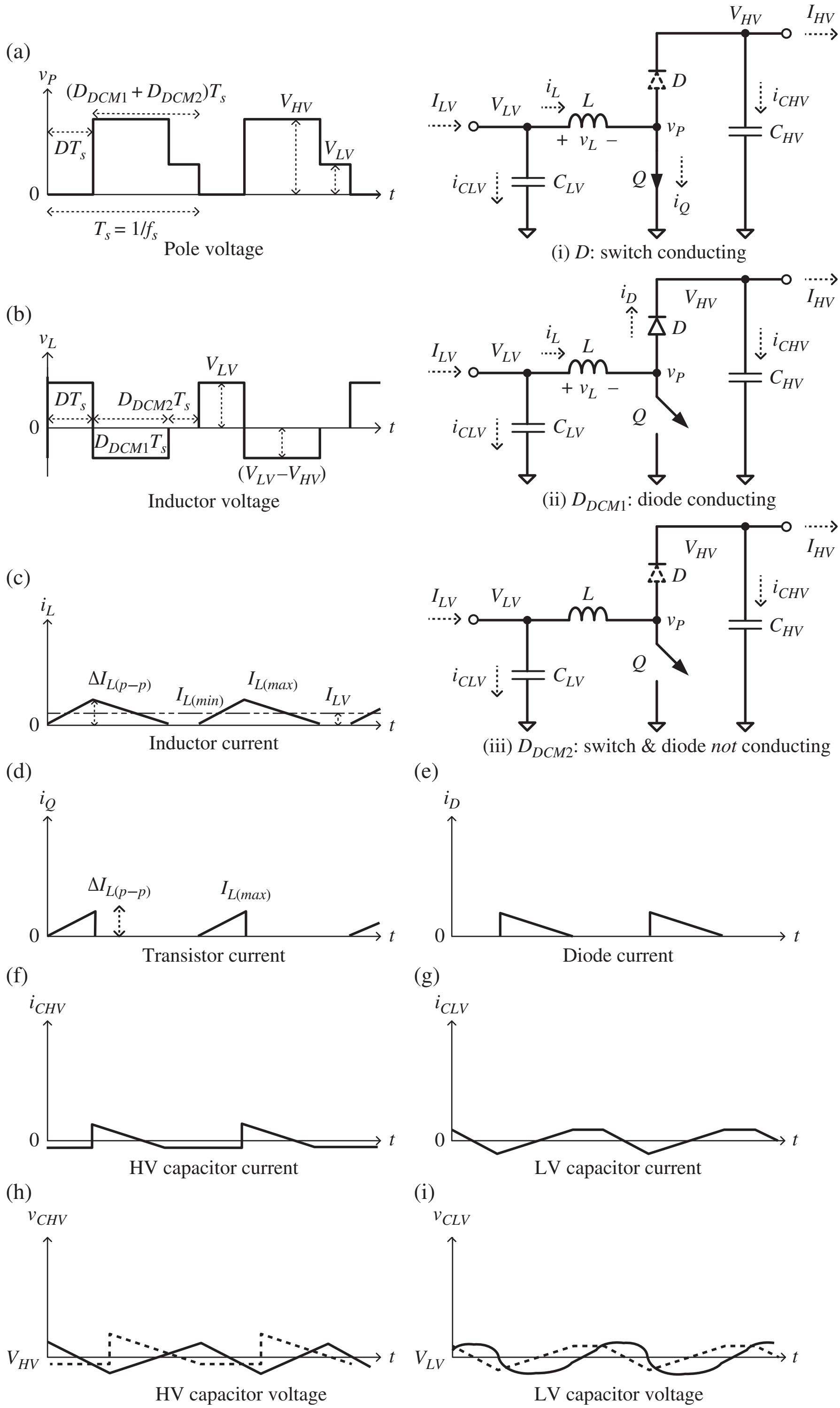


Figure 11.24 Boost DCM voltage and current waveforms.

The rms and average values of the various waveforms are relatively straightforward to determine as some of the currents are discontinuous.

First, the relationship between the dc input current and the duty cycle for the boost converter during DCM is determined as follows:

$$I_{L(dc)} = I_{LV} = \frac{1}{T_s}\int_0^{T_s} i_L(t)dt$$

$$= \frac{1}{T_s}\left[\int_0^{DT_s} i_L(t)dt + \int_{DT_s}^{DT_s + D_{DCM1}T_s} i_L(t)dt + \int_{DT_s + D_{DCM1}T_s}^{T_s} i_L(t)dt\right] \tag{11.65}$$

$$= \frac{1}{T}\left[\frac{1}{2}\Delta I_{L(p-p)}DT + \frac{1}{2}\Delta I_{L(p-p)}D_{DCM1}T + 0\cdot D_{DCM2}T\right]$$

$$= \frac{\Delta I_{L(p-p)}}{2}(D + D_{DCM1})$$

The relationship between the duty cycle of the switch D and the duty cycle of the diode D_{DCM1} is easily determined as the inductor current increases by $\Delta I_{L(p-p)}$ during DT_s, and decreases by $\Delta I_{L(p-p)}$ during $D_{DCM1}T_s$, while in the steady state. Thus:

$$\Delta I_{L(p-p)} = \frac{V_{LV}DT_s}{L} = \frac{(V_{HV} - V_{LV})D_{DCM1}T_s}{L} \tag{11.66}$$

resulting in the following relationships between the three duty cycles:

$$D_{DCM1} = \frac{V_{LV}}{(V_{HV} - V_{LV})}D \tag{11.67}$$

and

$$D_{DCM2} = 1 - \frac{V_{HV}}{V_{HV} - V_{LV}}D \tag{11.68}$$

Combining the above equations, the following expression can be determined for the duty cycle D in DCM:

$$D = \sqrt{\frac{(V_{HV} - V_{LV})}{V_{HV}V_{LV}}2f_sLI_{LV}} \tag{11.69}$$

Once the various duty cycles are known, the current values are easily determined for a load condition. The various currents can be derived as in Section 11.3.3.

The current in the switch is simply a ramp and is described by

$$i_Q(t) = \frac{\Delta I_{L(p-p)}}{DT_s}t \tag{11.70}$$

Thus, the rms value of the switch current is simply given by

$$I_{Q(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_Q(t)^2 dt} = \sqrt{\frac{1}{T_s}\left[\int_0^{DT_s}\left(\frac{\Delta I_{L(p-p)}}{DT_s}t\right)^2 dt + \int_{DT_s}^{T_s} 0^2 dt\right]} = \sqrt{\frac{D}{3}}\Delta I_{L(p-p)} \tag{11.71}$$

The rms value of the diode current is similarly given by

$$I_{D(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_D(t)^2 dt} = \sqrt{\frac{D_{DCM1}}{3}}\Delta I_{L(p-p)} \tag{11.72}$$

The rms value of the inductor current is

$$I_{L(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_L(t)^2 dt} = \sqrt{\frac{(D + D_{DCM1})}{3}}\Delta I_{L(p-p)} \tag{11.73}$$

The rms value of the low-voltage capacitor current is

$$I_{CLV(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{CLV}(t)^2 dt} = \sqrt{I_{L(rms)}{}^2 - I_{LV}{}^2} \tag{11.74}$$

The rms value of the high-voltage capacitor current is

$$I_{CHV(rms)} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{CHV}(t)^2 dt} = \sqrt{I_{D(rms)}{}^2 - I_{HV}{}^2} \tag{11.75}$$

Similarly, the average currents of the switch, diode, and inductor are

$$I_{Q(dc)} = \frac{1}{T_s}\int_0^{T_s} i_Q(t)dt = \frac{1}{T_s}\left[\int_0^{DT_s}\left(\frac{\Delta I_{L(p-p)}}{DT_s}t\right)dt + \int_{DT_s}^{T_s} 0\,dt\right] = \frac{D}{2}\Delta I_{L(p-p)} \tag{11.76}$$

$$I_{D(dc)} = \frac{1}{T_s}\int_0^{T_s} i_D(t)dt = \frac{D_{DCM1}}{2}\Delta I_{L(p-p)} \tag{11.77}$$

$$I_{L(dc)} = \frac{1}{T_s}\int_0^{T_s} i_L(t)dt = \frac{D + D_{DCM1}}{2}\Delta I_{L(p-p)} = I_{LV} \tag{11.78}$$

11.4.3.1 Example: Boost Converter in DCM Operation

Knowing from the previous example that the converter enters DCM at 2.8 kW for the given voltage conditions, determine the duty cycle and the various converter currents when operating in DCM at 2 kW.

Solution:

The dc current in the low-voltage dc link is

$$I_{LV} = \frac{P}{V_{LV}} = \frac{2000}{200}\,\mathrm{A} = 10\,\mathrm{A}$$

The average high-voltage input current is

$$I_{HV} = \frac{P}{V_{HV}} = \frac{2000}{500}\text{A} = 4\text{A}$$

The duty cycle at this condition is

$$D = \sqrt{\frac{(V_{HV} - V_{LV})}{V_{LV}V_{HV}} \cdot 2f_s L I_{LV}} = \sqrt{\frac{(500-200)}{200 \times 500} \times 2 \times 10000 \times 428.5 \times 10^{-6} \times 10} = 0.507$$

and

$$D_{DCM1} = \frac{V_{LV}}{(V_{HV} - V_{LV})} D = \frac{200}{(500-200)} \times 0.507 = 0.338$$

It is left as an exercise for the reader to calculate the various currents and note that they are similar to the values of the earlier example on the buck converter operating in DCM.

11.5 Power Semiconductors

Semiconductor technology is a key enabler of modern power converters. Millions of transistors can be integrated onto a piece of silicon to act as a single power switch – or can be integrated to operate as a digital controller. Power semiconductors have hugely benefited from the advances at the signal level, and many variations on power semiconductors exist. New diamond-like semiconductor materials, such as gallium nitride (GaN) and silicon carbide (SiC), are competing with traditional silicon materials to enable high-efficiency power conversion.

The modern automotive powertrain power converter is based on the **IGBT**. The IGBT is closely related to its sibling, the **MOSFET**. In effect, the IGBT has one additional silicon layer compared to the MOSFET. The effect of the additional layer is to make the IGBT look like a hybrid between the MOSFET and the BJT. The IGBT is the preferred device for high-voltage converters and inverters due to its lower power loss and its short-circuit capability.

From a semiconductor perspective, the IGBT is a minority-carrier device, similar to the BJT, thyristor, and *pn* diode, while the MOSFET is a majority-carrier device, similar to the junction-field-effect transistor (JFET) and Schottky diode. A characteristic of minority-carrier devices is that the device can generate significant additional power loss when it is turned off. This phenomenon is manifested as the **tail current** in the IGBT and as **reverse recovery** in the power diode. A **minority-carrier device** is a semiconductor in which the mobile hole or electron, or minority carrier, flows through semiconductor regions which are very conductive to carriers of the opposite polarity, the majority carrier. A **majority-carrier device** is a semiconductor in which the mobile hole or electron, or majority carrier, flows through semiconductor regions which are very conductive to the majority carrier.

The symbol for the IGBT is as shown in Figure 11.25(a). The IGBT has three terminals: the gate (G) for control, and the collector (C) and emitter (E) for power. The IGBT must be paired with a separate discrete inverse diode in order to function in a typical power converter. The vertically diffused power MOSFET, with the symbol shown in Figure 11.25(b), actually comes with a free inverse diode as part of its basic construction.

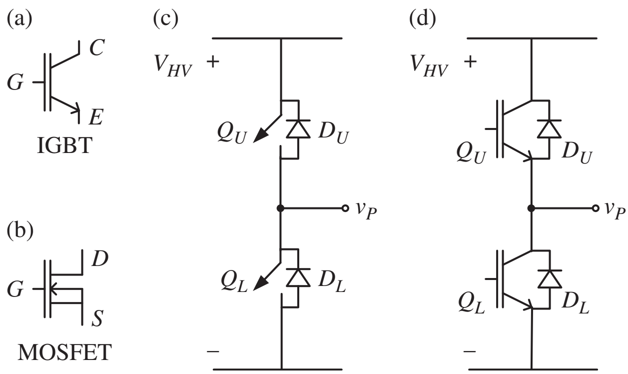


Figure 11.25 (a) IGBT, (b) MOSFET, and half-bridges with (c) generic switch and (d) IGBT switch.

The technical specifications for a power device are quite detailed. Typical specifications are the rated collector-emitter voltage and the continuous dc current. For example, a 600 V, 200 A IGBT can withstand a voltage of 600 V from collector to emitter when switched off, and can continuously conduct 200 A from collector to emitter when switched on. Of course, the IGBT generates a significant power loss which leads to device heating and temperature rise. For example, the hotspot of the IGBT at the semiconductor junction will be at its maximum temperature of 175 °C when the device is conducting 200 A, while the case of the module is at 80 °C. While 80 °C may appear to be a very high temperature, it is easily reached on a vehicle in the engine compartment. The device is switched on and off by applying gating signals across the gate and emitter. The maximum gate voltage that can be applied is quite low, at 20 V typically, in order to avoid over-voltaging the gate dielectric layer. It is important to keep the semiconductor junction hotspot relatively low as device reliability decreases and failure rates increase with rising temperature.

Two switches and two diodes are often arranged in a module as a half bridge, as shown in Figure 11.25(c) and (d). Such half-bridge modules can be used in high-power rectifiers, converters and inverters.

11.5.1 Power Semiconductor Power Loss

A power semiconductor device has two main power loss components. **Conduction loss** is the power loss due to the voltage drop within the device as it conducts current. **Switching loss** is the power loss within the device as the devices switch on and off. A third loss mechanism, known as **leakage loss**, results from the application of a voltage across a device and tends to be relatively small compared to the other two loss mechanisms. See References [3–5] for comprehensive discussions of semiconductor losses.

11.5.1.1 Conduction Losses of IGBT and Diode

The output characteristic of an IGBT is similar to that of a diode or BJT. The collector-emitter characteristic of a representative 600 V, 200 A IGBT is shown in Figure 11.26(a). While the device actually begins to conduct at the gate threshold voltage, typically about 4 to 5 V, the gate-emitter voltage must be increased significantly above the threshold, 15

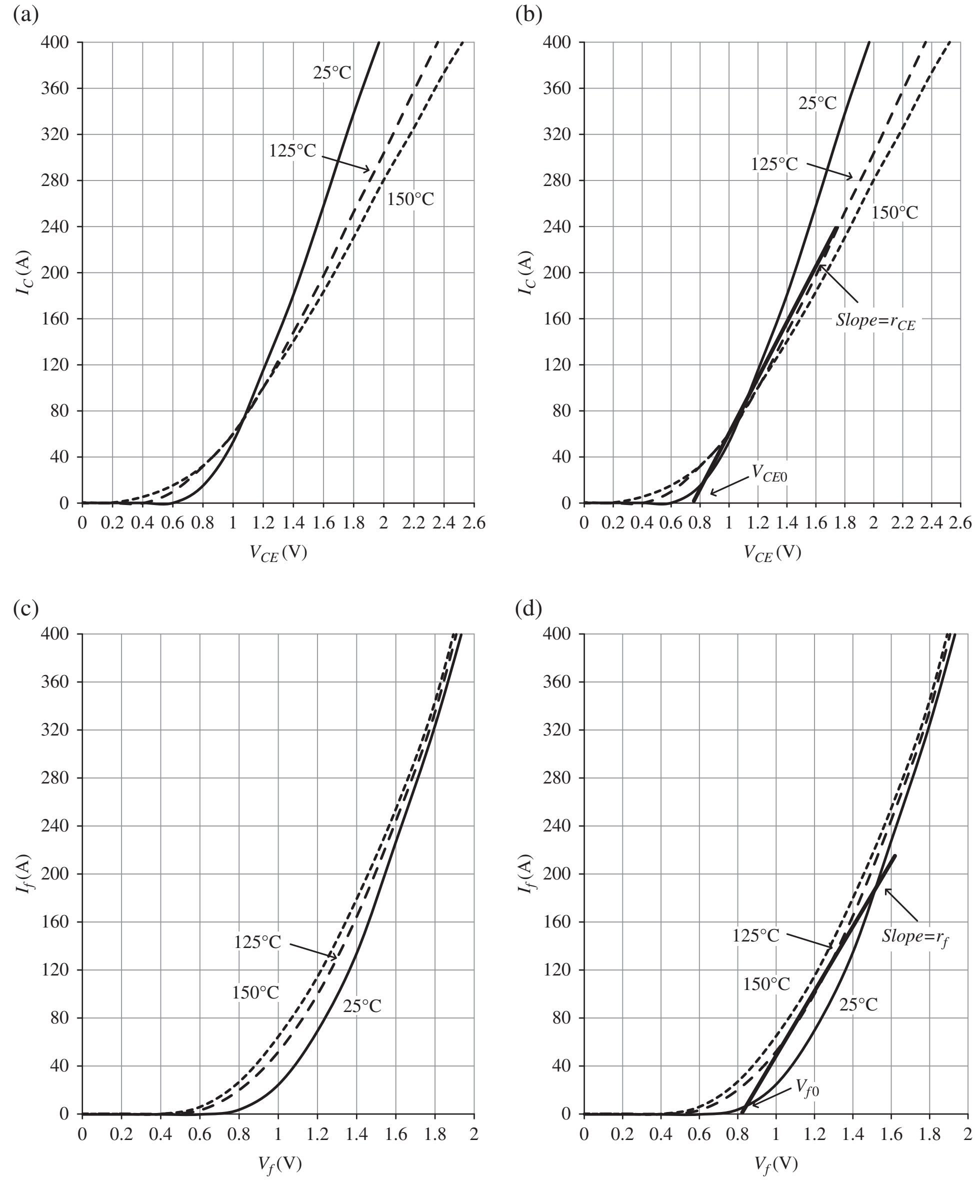


Figure 11.26 Output characteristics of 600 V, 200 A (a-b) IGBT and (c-d) diode.

V being a common level, in order to minimize the collector-emitter voltage drop and the associated conduction loss.

The output characteristic of the IGBT is the plot of the collector current i_C against the collector-emitter voltage v_{CE}, and can be simply modeled using a straight line approximation as follows:

$$v_{CE} = V_{CE0} + r_{CE} i_C \tag{11.79}$$

where V_{CE0} is the knee voltage and r_{CE} is the equivalent resistance, as shown in Figure 11.26(b). Note that the straight line should be drawn tangentially to the average current within the switch or diode while conducting. Thus, there can be variations in the parameters with the current level.

The conduction power loss of the IGBT $P_{Q(cond)}$ can then be simply modeled as

$$P_{Q(cond)} = V_{CE0} I_{Q(dc)} + r_{CE} {I_{Q(rms)}}^2 \tag{11.80}$$

As the silicon power diode is also a minority-carrier device, the diode forward drop v_f, as a function of diode current i_f and power loss $P_{D(cond)}$ are similarly given by

$$v_f = V_{f0} + r_f i_f \tag{11.81}$$

and

$$P_{D(cond)} = V_{f0} I_{D(dc)} + r_f {I_{D(rms)}}^2 \tag{11.82}$$

where V_{f0} is the knee voltage, and r_f is the equivalent resistance of the diode. A plot of the diode forward drop is presented in Figure 11.26(c). The diode characteristic can also be modeled by a straight line drawn tangentially to the current, as shown in Figure 11.26(d).

11.5.1.2 Example: Boost IGBT Conduction Losses

Determine the conduction losses for the IGBT and diode in the earlier CCM boost of Section 11.4.1.2 From Figure 11.26(b) and (d), the IGBT and diode have the following parameters: $V_{CE0} = 0.75$ V, $r_{CE} = 4.6$ mΩ, $V_{f0} = 0.85$ V, and $r_f = 3.6$ mΩ.

Solution:

The IGBT and diode conduction losses are simply

$$P_{Q(cond)} = V_{CE0} I_{Q(dc)} + r_{CE} {I_{Q(rms)}}^2 = 0.75 \times 60\,\text{W} + 0.0046 \times 77.69^2\,\text{W} = 73\,\text{W}$$

and

$$P_{D(cond)} = V_{f0} I_{D(dc)} + r_f {I_{D(rms)}}^2 = 0.85 \times 40\,\text{W} + 0.0036 \times 63.44^2\,\text{W} = 48\,\text{W}$$

11.5.1.3 Switching Losses of IGBT and Diode

Every instance that the current transitions from the switch to the diode, and vice versa, results in an energy loss and resulting power dissipation and temperature rise within the device. The IGBT and diode suffer from tail-current and reverse-recovery losses, respectively, as they turn off. The turn-on losses of the diode can be ignored, whereas the turn-on losses of the switch can be significant, especially as the diode reverse recovery at diode turn-off also affects the switch at turn-on.

These losses are often specified for the IGBT and diode. The energy losses for the representative module are shown in Figure 11.27 (a) and (b) for the IGBT and diode, respectively. The speeds of the turn-on and turn-off, and the resulting energy losses, are dependent on the gate voltages and the gate-drive resistor. Thus, these values are typically shown in the datasheets for the test. Energies E_{on} and E_{off} are the turn-on and turn-off energies of the IGBT, while E_{rec} is the turn-off energy of the diode due to reverse recovery. It is noteworthy that low-inductance connections are critical to power loss. Excessive parasitic inductances can cause increased power loss and voltage spikes.

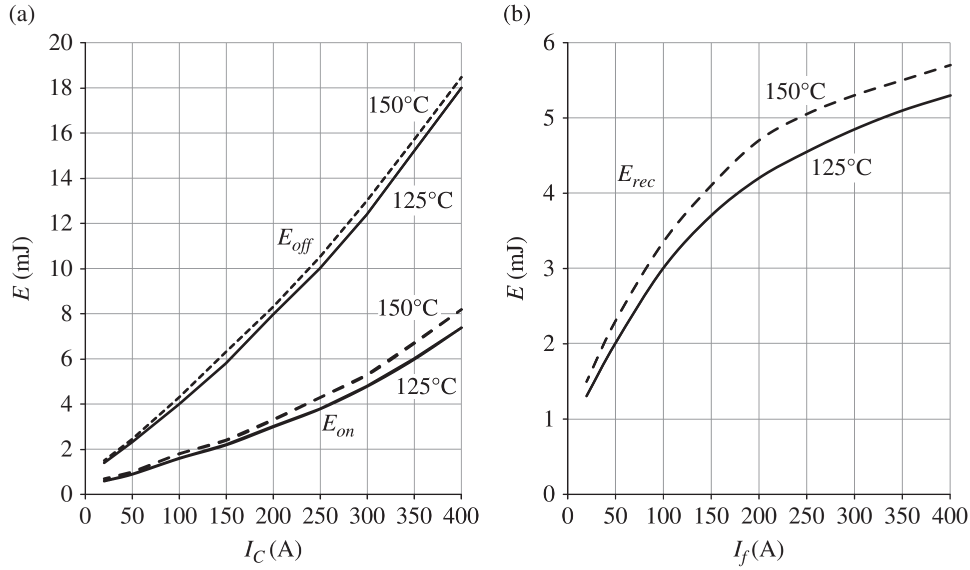


Figure 11.27 Switching losses for (a) IGBT and (b) diode at a test voltage of 300 V.

In order to determine the power loss, the various energy losses are simply added up and multiplied by the switching frequency. An additional caveat is that the test voltage may also need to be adjusted from that used in the test. For instance, the test voltage V_{test} in Figure 11.27 is 300 V. If we are testing at a different voltage, a reasonable assumption is to simply scale the energy loss linearly with voltage (or non-linearly as in [5]). Thus, the switching power loss $P_{Q(sw)}$ in the IGBT is equal to the sum of the voltage-adjusted turn-on and turn-off losses multiplied by the switching frequency:

$$P_{Q(sw)} = f_s \left(E_{on} + E_{off} \right) \frac{V_{HV}}{V_{test}} \tag{11.83}$$

The diode switching loss $P_{D(sw)}$ is similarly defined as

$$P_{D(sw)} = f_s E_{rec} \frac{V_{HV}}{V_{test}} \tag{11.84}$$

11.5.1.4 Example: Switching Losses of IGBT Module

Determine the switching losses in the IGBT and diode from the previous section. See Figure 11.27. Assume a junction temperature of 125 °C.

Solution:

The energy loss values are simply determined from the figures and substituted into the loss formulas. The earlier currents were

$$I_{L(max)} = I_{Q(off)} = I_{D(on)} = 114\,\text{A}$$

$$I_{L(min)} = I_{Q(on)} = I_{D(off)} = 86\,\text{A}$$

The approximate losses from the figures are as follows:

$$E_{on}\left(I_{Q(on)}\right) = E_{on}(86\,\mathrm{A}) \approx 1.6\,\mathrm{mJ}$$

$$E_{off}\left(I_{Q(off)}\right) = E_{off}(114\,\mathrm{A}) \approx 4.7\,\mathrm{mJ}$$

$$E_{rec}\left(I_{D(off)}\right) = E_{rec}(86\,\mathrm{A}) \approx 2.8\,\mathrm{mJ}$$

Thus, the power losses are as follows:

$$P_{Q(sw)} = f_s\left(E_{on} + E_{off}\right)\frac{V_{HV}}{V_{test}} = 10^4 \times (1.6 + 4.7) \times 10^{-3}\frac{500}{300}\,\mathrm{W} = 105\,\mathrm{W}$$

The diode switching loss is similarly defined:

$$P_{D(sw)} = f_s E_{rec}\frac{V_{HV}}{V_{test}} = 10^4 \times 2.8 \times 10^{-3} \times \frac{500}{300}\,\mathrm{W} = 47\,\mathrm{W}$$

11.5.2 Total Semiconductor Power Loss and Junction Temperature

The total semiconductor power loss is the sum of the switching and conduction power losses. Thus, the total IGBT power loss P_Q and the diode power loss P_D are as follows:

$$P_Q = P_{Q(cond)} + P_{Q(sw)} \tag{11.85}$$

and

$$P_D = P_{D(cond)} + P_{D(sw)} \tag{11.86}$$

Finally, a key semiconductor parameter is the hotspot temperature of the semiconductor junction. Excessive temperatures can result in semiconductor failure. Although the maximum temperature of the semiconductor is specified, the device is usually operated well below the maximum temperature in order to improve the reliability and lifetime of the semiconductor device.

The junction temperatures of the IGBT T_{JQ} and diode T_{JD} are given by

$$T_{JQ} = T_{HS} + R_{JQ-HS} \times P_Q \tag{11.87}$$

and

$$T_{JD} = T_{HS} + R_{JD-HS} \times P_D \tag{11.88}$$

where temperature T_{HS} is the temperature of the surface of the heat sink cooling the semiconductors, and $R_{JQ\text{-}HS}$ and $R_{JD\text{-}HS}$ are the thermal impedances of the IGBT and diode, respectively, in °C/W from the surface of the heat sink to the semiconductor junction. Note that the thermal impedance of the IGBT is typically less than that of the diode due to the larger die of the IGBT compared to the diode.

The above temperature relationship can be modeled by a simple electrical circuit as shown in Figure 11.28.

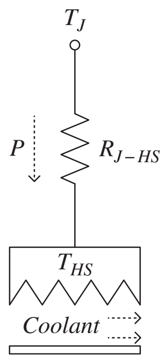


Figure 11.28 Thermal circuit model for heat flow from junction to heat sink.

11.5.2.1 Example: Total IGBT Module Loss and Die Temperatures

Determine the IGBT and diode power losses and their respective hotspot temperatures if the heat sink is maintained at 70 °C, and the thermal resistances of the IGBT and diode are 0.25 °C/W and 0.48 °C/W, respectively.

Solution:

The total losses are given by

$$P_Q = P_{Q(cond)} + P_{Q(sw)} = 73\text{W} + 105\text{W} = 178\text{W}$$

and

$$P_D = P_{D(cond)} + P_{D(sw)} = 48\text{W} + 47\text{W} = 95\text{W}$$

The junction temperatures are given by

$$T_{JQ} = T_{HS} + R_{JQ-HS} \times P_Q = 70°\text{C} + 0.25 \times 178°\text{C} = 114°\text{C}$$

and

$$T_{JD} = T_{HS} + R_{JD-HS} \times P_D = 70°\text{C} + 0.48 \times 95°\text{C} = 116°\text{C}$$

11.6 Passive Components for Power Converters

The inductors and capacitors can be a significant part of the cost and size of the dc-dc converter. The sizing of these components is covered in Chapter 16.

11.6.1 Example: Inductor Sizing

Determine the following inductor parameters for the buck converter of Example 11.3.1.4: area product, core area, and number of turns. Use the area product method, and assume that the core area equals the window area ($A_c = A_w$).

Let $k = 0.5$ be the copper fill factor, $J_{cu} = 6\text{A/mm}^2$ be the current density, and $B_{max} = 1.3$ T be the maximum core flux density.

Solution:

From Chapter 16, Section 16.3.7, the area product AP is given by

$$AP = A_c A_w = \frac{LI_{L(rms)} I_{L(max)}}{k_{cu} J_{cu} B_{max}} = \frac{428.5 \times 10^{-6} \times 100.3 \times 114}{0.5 \times 6 \times 10^{6} \times 1.3} \text{m}^4 = 1.256 \times 10^{-6} \text{m}^4 = 125.6 \text{cm}^4$$

The area is given by

$$A_c = A_w = \sqrt{AP} = 1.12 \times 10^{-3} \text{m}^2 = 11.2 \text{ cm}^2$$

The number of turns is given by

$$N = \frac{LI_{L(\max)}}{B_{max} A_c} = \frac{428.5 \times 10^{-6} \times 114}{1.3 \times 1.12 \times 10^{-3}} = 34 \text{ turns}$$

11.6.2 Capacitor Sizing

The size of a capacitor can depend on many factors. An estimate of the physical size of a film capacitor can be made based on some simple calculations and assumptions, as presented in Chapter 16, Section 16.6.1

11.6.2.1 Example: Capacitor Sizing

Determine the length of foil and the foil volume of the 500 V, 960 μF capacitor required for the buck converter of Example 11.3.1.4 if the foil has the following specifications: dielectric strength $DS = 150$ V/μm, relative permittivity $\varepsilon_r = 2.2$, and foil width $w = 5$ cm. Allow for a voltage overshoot $V_{OS} = 100$ V on the capacitor.

Solution:

The thickness of the dielectric is

$$d = \frac{V_{HV} + V_{OS}}{DS} = \frac{500\,\text{V} + 100\,\text{V}}{150\,\text{V}/\mu\text{m}} = 4\,\mu\text{m}$$

As the width is specified, the length l is given by

$$l = \frac{Cd}{\varepsilon_r \varepsilon_0 w} = \frac{960 \times 10^{-6} \times 4 \times 10^{-6}}{2.2 \times 8.84 \times 10^{-12} \times 0.05}\,\text{m} = 3949\,\text{m}$$

The high-voltage capacitor foil volume V is

$$V = l \times w \times d = 3949 \times 0.05 \times 4 \times 10^{-6}\text{m}^3 = 0.632 \times 10^{-3}\text{m}^3 = 790\,\text{cm}^3$$

11.7 Interleaving

It can be understood from the previous examples that a significant mass and volume may be required for the passive components of the power converter. Thus, the size of a high-power system required for a fuel cell vehicle can take up a significant volume on the vehicle as the dc-dc converter interfacing the fuel cell is required to process all the fuel cell power. For many applications, the size of a power converter can be minimized by building multiple parallel stages and interleaving the operation of the stages. A two-phase interleaved boost converter is presented in Figure 11.29.

The main waveforms for the converter are shown in Figure 11.30. The turn-on of phase 2 is delayed by 180° with respect to phase 1 as shown in Figure 11.30(a) and (b). This results in the two-phase currents being out of phase as shown in Figure 11.30(c) and (d). The two-phase currents summed together gives the input current as shown in Figure 11.30(e).

It can be shown (and is left as an exercise for the student) that the peak-to-peak ripple of the input current $\Delta I_{int(p\text{-}p)}$ of an interleaved two-stage boost converter is as follows, the formula being dependent on whether the duty cycle is greater than or less than 0.5:

$$0 \le D \le 0.5 \quad \Delta I_{int(p-p)} = \frac{V_{LV}}{fL}\frac{(1-2D)}{(1-D)}D \tag{11.89}$$

$$0.5 \le D \le 1.0 \quad \Delta I_{int(p-p)} = \frac{V_{LV}}{fL}(2D-1) \tag{11.90}$$

We can appreciate interleaving by noting that the input ripple current is zero when the duty cycle is 0.5.

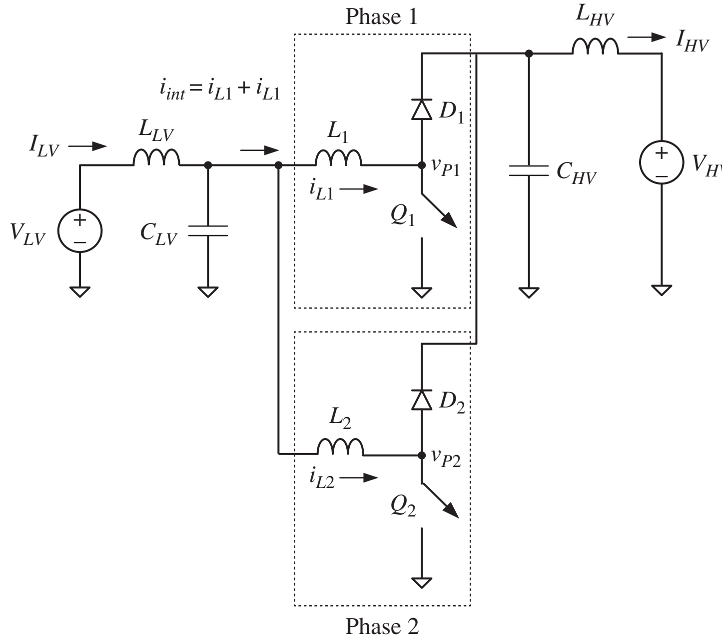


Figure 11.29 Interleaved two-phase boost converter.

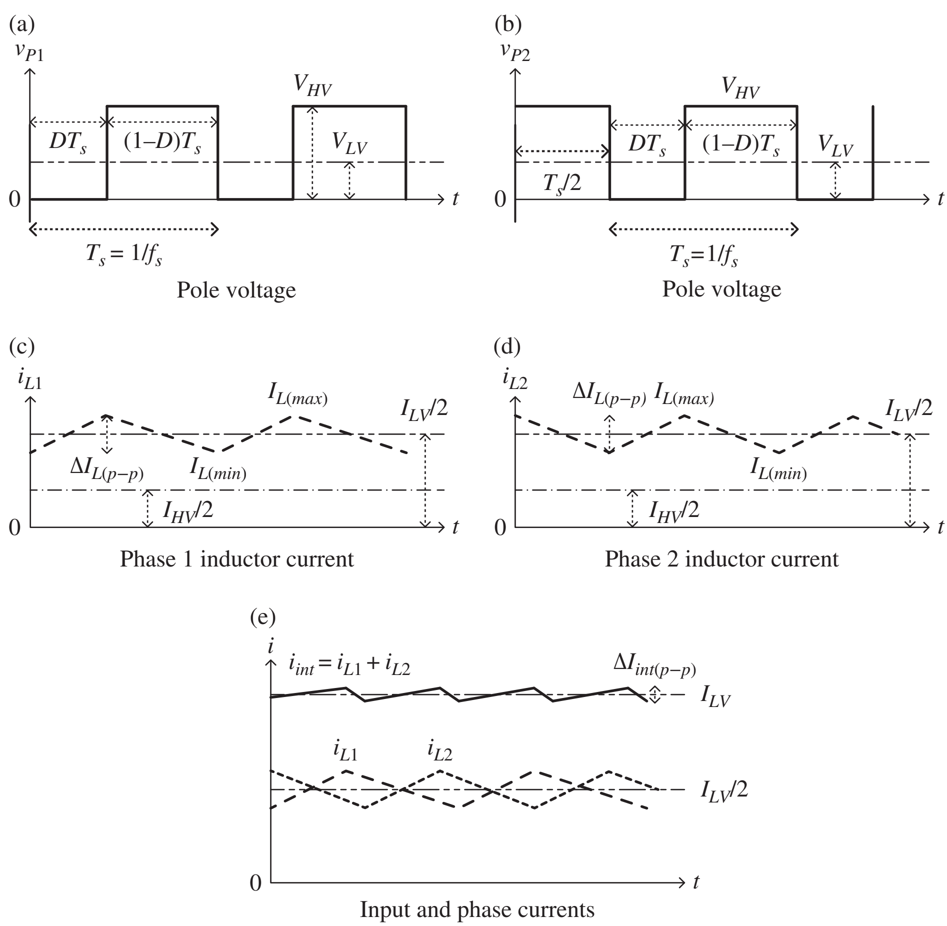


Figure 11.30 Waveforms for two-phase interleaved converter.

11.7.1 Example: Two-Phase Interleaved Boost Converter

A fuel cell vehicle features two interleaved 20 kW boost converters to provide a total output power of 40 kW. The converters are interleaved in order to reduce the ripple current coming from the source. The vehicle generates a 500 V dc link voltage when the fuel cell drops to 200 V at full power. Use the power stage of the previous boost example: $L = L_1 = L_2 = 428.5\ \mu H$.

i) Determine the peak-to-peak ripple on the input current.
ii) Determine the inductance and the area product which would be required if this converter was a single phase with the same low input ripple rather than a two-phase interleaved converter.

Solution:

i) As before, for a single-phase boost

$$D = 1 - \frac{V_{LV}}{V_{HV}} = 1 - \frac{200}{500} = 0.6$$

The per-phase current ripple remains

$$\Delta I_{L(p-p)} = \frac{V_{LV} D}{f_s L} = \frac{200 \times 0.6}{10000 \times 428.5} \text{A} = 28\,\text{A}$$

The input current ripple of the interleaved converter reduces to

$$\Delta I_{int(p-p)} = \frac{V_{LV}}{f_s L}(2D - 1) = \frac{200}{10^4 \times 428.5 \times 10^{-6}}(2 \times 0.6 - 1) = 9.33\,\text{A} \tag{11.91}$$

Thus, the input current ripple at 9.33 A is one third of the per-phase current ripple, and the input capacitance can be reduced correspondingly. The area product for the two interleaved phases is simply twice that of the single-phase value of 125.6 cm^4, or 251.2 cm^4.

ii) For the second part of the question, the converter is a single stage which is designed with the same low input ripple current of 9.33 A as of the interleaved two-phase option. Thus, the phase inductance of the single-phase option has to increase by a factor of three compared to the two-phase design.

The inductor size increases to

$$L = 428.5\,\mu\text{H} \times \frac{28}{9.33} = 1286\,\mu\text{H}$$

The inductor dc current for a single phase is

$$I_{LV} = I_{L(dc)} = \frac{P}{V_{LV}} = \frac{40000}{200} \text{A} = 200\,\text{A}$$

The rms current in the low-voltage capacitor is given by

$$I_{CLV(rms)} = \frac{\Delta I_{L(p-p)}}{\sqrt{12}} = \frac{9.33}{\sqrt{12}} \text{A} = 2.69\,\text{A}$$

The inductor rms current is

$$I_{L(rms)} = \sqrt{I_{L(dc)}{}^2 + I_{CLV(rms)}{}^2} = \sqrt{200^2 + 2.69^2}\,\text{A} = 200.0\,\text{A}$$

The inductor maximum current is

$$I_{L(max)} = I_{L(dc)} + \frac{\Delta I_{L(p-p)}}{2} = 204.7\,\text{A}$$

The revised area product for a single-phase converter is

$$AP = \frac{LI_{L(rms)}I_{L(max)}}{k_{cu}J_{cu}B_{\max}} = \frac{1286 \times 10^{-6} \times 200 \times 204.7}{0.5 \times 6 \times 10^6 \times 1.3}\text{m}^4 = 1125\,\text{cm}^4$$

The inductor area product has increased to 1125 cm^4, which is over four times larger than the combined value of 251.2 cm^4 for the two inductances in the interleaved two-phase design.

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Further Reading

1 N. Mohan, *Power Electronics A First Course*, John Wiley & Sons, 2012.
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3 B. C. Barry, J. G. Hayes, and M. S. Ryłko, "CCM and DCM operation of the interleaved two-phase boost converter with discrete and coupled inductors," *IEEE Transactions on Power Electronics*, **30**, pp. 6551–6567, December 2015.

Problems

11.1 A hybrid electric vehicle uses a 30 kW bidirectional converter to generate a 650 V dc link voltage from the 288 V NiMH battery. The bidirectional converter has an inductance of 245 μH and switches at 10 kHz.

The vehicle is operating in generating mode, and the bidirectional converter is required to act as a buck at full power.

i) Calculate the rms currents in the low-voltage capacitor and in the inductor.
ii) Calculate the maximum, minimum, rms, and average currents in the IGBT and diode.
iii) Calculate the rms current in the high-voltage capacitor.
iv) Determine the low-voltage and high-voltage capacitor values if the peak-to-peak voltage ripple is 0.5%.

Ignore component losses.

[Ans. $I_{CLV(rms)} = 18.9$ A, $I_{L(rms)} = 105.87$ A, $I_{L(max)} = 136.9$ A, $I_{L(min)} = 71.44$ A, $I_{QU(rms)} = 70.47$ A, $I_{QU(dc)} = 46.15$ A, $I_{DL(rms)} = 79.0$ A, $I_{DL(dc)} = 58.01$ A, $I_{CHV(rms)} = 53.26$ A, $C_{HV} = 791$ μF, $C_{LV} = 568$ μF]

11.2 At what power does the converter in the previous problem reach BCM at a dc link voltage of 650 V? Recalculate the various currents.

[Ans. 9.43 kW, $I_{CLV(rms)} = 18.9$ A, $I_{L(rms)} = 37.8$ A, $I_{L(max)} = 65.46$ A, $I_{L(min)} = 0$ A, $I_{QU(rms)} = 25.16$ A, $I_{QU(dc)} = 14.5$ A, $I_{DL(rms)} = 28.21$ A, $I_{DL(dc)} = 18.23$ A, $I_{CHV(rms)} = 20.56$ A]

11.3 Assuming the above power converter is in DCM at 5 kW, recalculate the various currents.

[Ans. $I_{CLV(rms)} = 15.82$ A, $I_{L(rms)} = 23.49$ A, $I_{L(max)} = 47.68$ A, $I_{L(min)} = 0$ A, $I_{QU(rms)} = 15.64$ A, $I_{QU(dc)} = 7.69$ A, $I_{DL(rms)} = 17.53$ A, $I_{DL(dc)} = 9.67$ A, $I_{CHV(rms)} = 13.62$ A]

11.4 Assume the converter in the Problem 11.1 is in boost mode at 30 kW. Recalculate the various currents.

[Ans. $I_{CLV(rms)} = 18.9$ A, $I_{L(rms)} = 105.87$ A, $I_{L(max)} = 136.9$ A, $I_{L(min)} = 71.44$ A, $I_{QL(rms)} = 79.0$ A, $I_{QL(dc)} = 58.01$ A, $I_{DU(rms)} = 70.47$ A, $I_{DU(dc)} = 46.15$ A, $I_{CHV(rms)} = 53.26$ A]

11.5 Assuming the above boost power converter is in DCM at 5 kW, recalculate the various currents.

[Ans. $I_{CLV(rms)} = 15.82$ A, $I_{L(rms)} = 23.49$ A, $I_{L(max)} = 47.68$ A, $I_{L(min)} = 0$ A, $I_{QL(rms)} = 17.53$ A, $I_{QL(dc)} = 9.67$ A, $I_{DU(rms)} = 15.64$ A, $I_{DU(dc)} = 7.69$ A, $I_{CHV(rms)} = 13.62$ A]

11.6 For the converter of Problem 11.1:

i) Determine the typical power losses and junction temperatures due to conduction and switching in the IGBT and diode for this full-power condition when using a 1200 V, 300 A half-bridge module. Let $V_{CE0} = 0.75$ V and $r_{CE} = 4.4$ mΩ, and $V_{f0} = 0.7$ V and $r_f = 3.8$ mΩ. Use the loss curves of Figure 11.31. The heat sink is maintained at 80 °C, and the thermal resistances of the IGBT and diode are 0.125 °C/W and 0.202 °C/W, respectively.
ii) Determine the following inductor parameters: area product, core area, and number of turns. Use the area product method, and assume that the core area equals the window area ($A_c = A_w$), and that $k = 0.5$ is the copper fill factor, $J_{cu} = 6$A/mm^2 is the current density, and $B_{max} = 1.3$ T is the maximum core flux density.

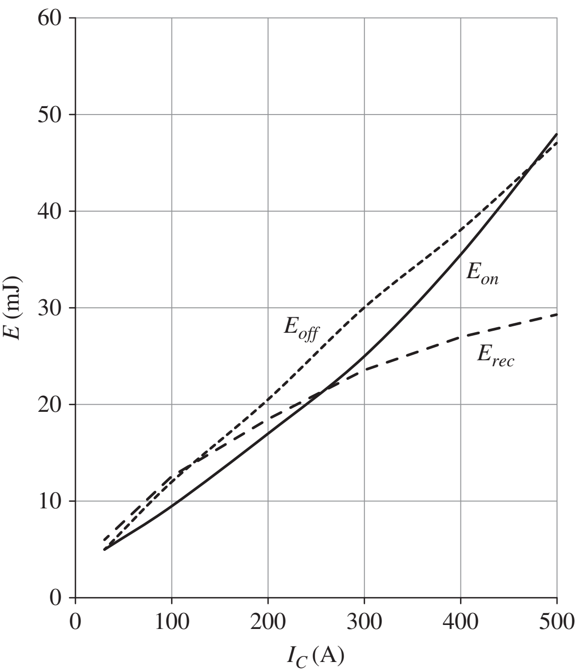


Figure 11.31 Energy losses at 125 °C for 1200 V, 300 A IGBT at a test voltage of 600 V.

iii) Determine the volume of the high-voltage capacitor, assuming a polypropylene film. Let the dielectric strength = 150 V/μm, relative permittivity = 2.2, and foil width = 5 cm. Allow for a 100 V design overshoot on the capacitor.

[Ans. P_Q = 300.3 W, P_D = 172.6 W, T_{JQ} = 117.5 °C, T_{JD} = 114.9 °C, AP = 91 cm^4, A = 9.5 cm^2, N = 27 turns, V_{CHV} = 1017 cm^3]

11.7 A fuel cell vehicle features two interleaved boost converters to provide a total output power of 72 kW. The converters are interleaved in order to reduce the ripple current coming from the source. The converters generate a 360 V dc link voltage when the fuel cell drops to 180 V at full power. Each converter has an inductance of 45 μH and switches at 16 kHz.

i) Determine the rms and peak currents in each phase inductor.
ii) Calculate the peak-to-peak ripple current from the fuel cell input capacitor.
iii) Determine the peak-to-peak current from the fuel cell input capacitor if the two phases were synchronized in phase rather than interleaved.

[Ans. $I_{L1(rms)} = I_{L2(rms)}$ = 203.2 A, $I_{L1(max)} = I_{L2(max)}$ = 262.5 A, $\Delta I_{int(p\text{-}p)}$ = 0 A, $\Delta I_{in(p\text{-}p)}$ = 250 A]

11.8 The half-bridge dc-dc converter of a HEV has an inductance of 428.5 μH and switches at 10 kHz when supplied by a 200 V NiMH battery.
The voltage on the dc link reduces proportionately with the motor power, and the dc link voltage drops to 350 V at 10 kW.

i) Calculate the switch rms and dc currents for this CCM generating/buck condition.
ii) Determine the typical power loss and junction temperatures at 125 °C due to conduction and switching in the IGBT for this partial-load condition when using the half-bridge module of Section 11.5.1.

Let $V_{CE0} = 0.65$ V and $r_{CE} = 5.9$ mΩ for the average current in the switch in this problem.
Use Figure 11.27 to estimate the switching loss for the module.

[Ans. $I_{Q(rms)} = 38.04$ A, $I_{Q(dc)} = 28.57$ A, $P_Q = 69$ W; $T_{JQ} = 97.3$ °C]

Assignments

11.1 The student is encouraged to experiment with circuit simulation software packages. The following simulation packages are among those which are commonly used.
i) Simetrix, www.simetrix.co.uk
ii) PSpice 9.1, available on various web sites.
iii) Matlab/Simulink Simscape Power Systems, http://www.mathworks.com/products/simpower/
iv) PSIM, www.powersimtech.com

11.2 Verify the waveforms and answers for the various examples and problems by simulation.

Appendix I

The simple Simulink schematic of Figure 11.32 simulates the buck CCM example. A resistive load is used rather than a battery to simplify the circuit. The resistor is set at 2 Ω, which is the equivalent resistive load of a 200 V battery pulling 100 A. Simulation waveforms are shown in Figure 11.33.

Appendix II: Buck-Boost Converter

The buck-boost converter is closely related to the buck and boost converters, and is the basis for the isolated flyback converter covered in Appendix II of Chapter 12. The relationship between voltage and the duty cycle can easily be determined for the buck-boost converter by applying the relationship already derived for the boost converter. The buck-boost converter is shown in Figure 11.34(a).

The voltage gain of the boost converter has been derived to be

$$\frac{V_{HV}}{V_{LV}} = \frac{1}{1-D}$$

The voltage gain for the buck-boost converter is easily derived if V_{in} is substituted for V_{LV} and $(V_{in} + V_{out})$ is substituted for V_{HV}:

$$\frac{V_{HV}}{V_{LV}} = \frac{V_{in} + V_{out}}{V_{in}} = \frac{1}{1-D} \tag{A11.II.1}$$

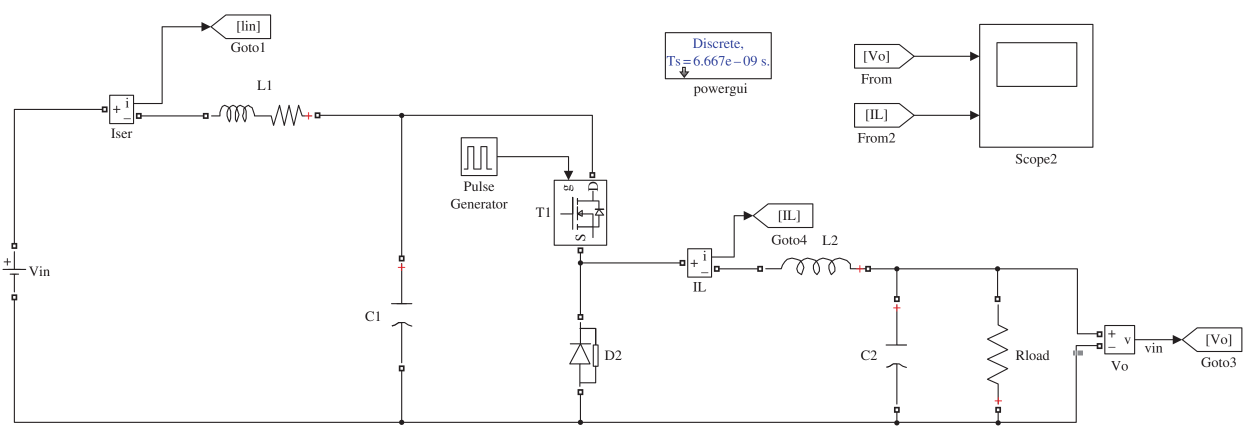
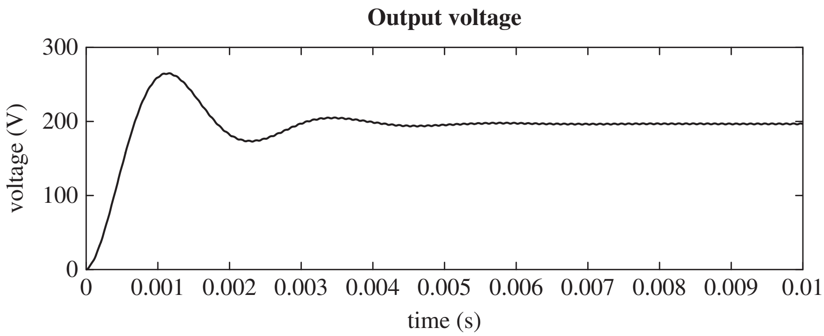


Figure 11.32 Simulink buck model.



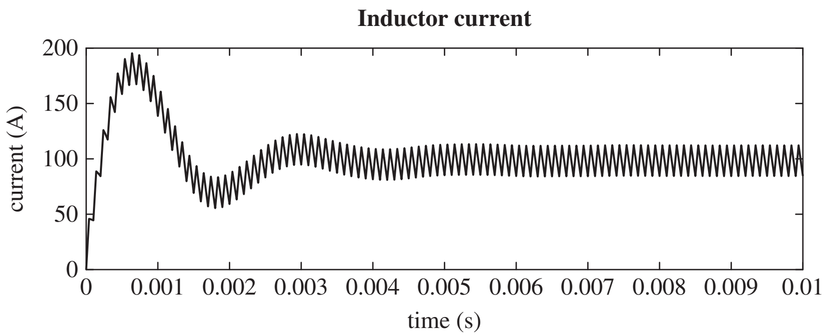


Figure 11.33 Simulation waveforms for a buck converter.

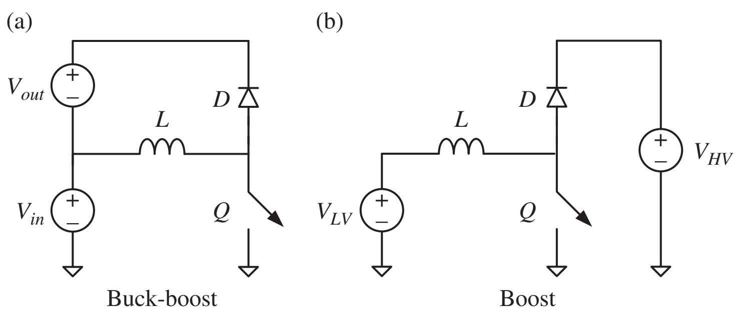
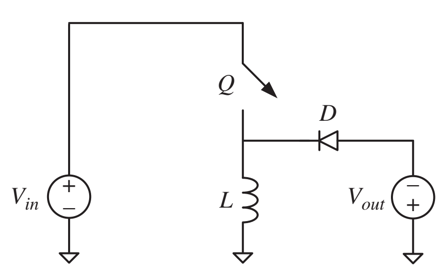


Figure 11.34 (a) Buck-boost and (b) boost converters.

Figure 11.35 Buck-boost converter.



By rearranging the preceding equation, the duty cycle for the buck-boost converter is derived as

$$D = \frac{V_{out}}{V_{in} + V_{out}} \tag{A11.II.2}$$

The voltage gain can be expressed as

$$\frac{V_{out}}{V_{in}} = \frac{D}{1-D} \tag{A11.II.3}$$

The buck-boost converter is often sketched as shown in Figure 11.35.