Test of Hypothesis

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In many problems of Engineering or Business, In general, we have to decide which of two competing claims or statements about some parameters is true. These statements are called hypothesis, and the procedure of decision-making is called hypothesis testing. Moreprécisely,

Statistical Hypothesis :- To arrive at a decision about the population on the basis of sample information we make assumptions about the population parameters involved such assumption is called a statistical hypothesis.

Testing of Hypothesis :- It is an assumption or supposition and the decision making procedure about the assumption whether to accept or reject is called hypothesis testing.

Null hypothesis :- A null hypothesis which asserts that there is no significant difference between the statistics and the population parameter and whatever the observed difference is that, there is merely due to fluctuations in sampling from the sample population. It is denoted by

H_0.

Ex:1 Suppose, there are two different manufacturer companies of sleeping drugs, company A claims that its P drug is superior to that of B company's Q drug. It is desired to test which is a superior drug P or Q?

To formulate the statistical hypothesis let X be a random variable which denotes the additional hours of sleep gained by an individual when P is given and let the random variable Y denote for Q drug.

Let us suppose that X and Y

follow the probability distribution with mean μ_x & μ_y respectively.

Here the Null hypothesis would be that there is no difference between the effects of two drugs.

Symbolically,

$$H_0 : \mu_x = \mu_y.$$

Alternative Hypothesis :- Any Hypothesis which contradict the null hyp. is called an alternative hypo.

It is denoted by H_1

These two hypothesis H_0 and H_1 are such that if one is true, the other is false and vice-versa.

Ex2 :- In Ex 1; all other possibilities than equality, will be considered under alternative hypothesis, like

$H_1 : \mu_x \neq \mu_y$ or

$\mu_x < \mu_y$ or

$\mu_x > \mu_y$.

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Level of Significance :- The level of significance is denoted by 'α' is the confidence with which we accept or reject the null hypothesis.

→ It is generally specified before a test procedure, which can be either 5% (0.05), 1% (0.01) etc, which means that there are about 5 chances out of 100 that we would reject the null hypothesis H_0 in '0.05' case and the remaining 95% confidient that we would accept the null hypothesis H_0.

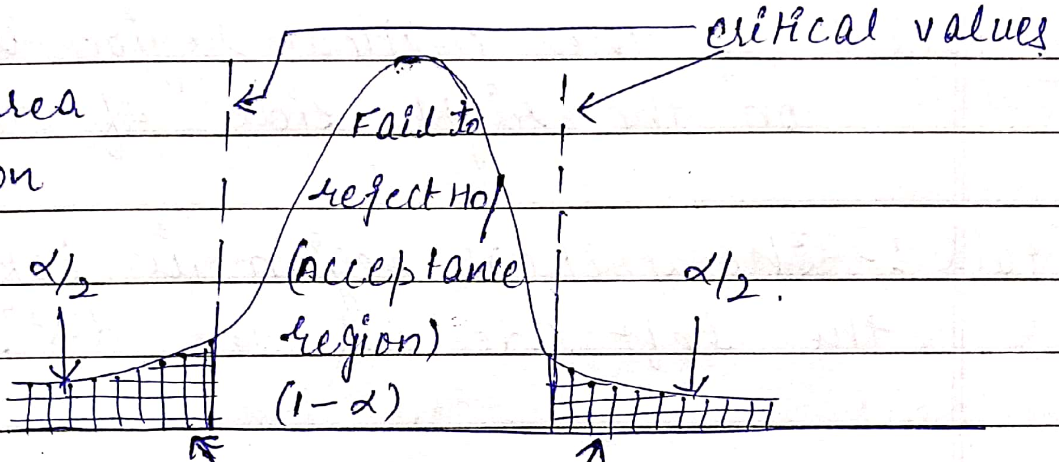
Similarly, it is applicable for different level of significance.

Test of statistic :- There are several tests of significance like z, t, and F etc. Depending upon the nature of the information given in the problem we have to select the right test and construct the test criterion and appropriate probability distribution.

Critical Region :- The region of rejection of Ho when ~~it~~ Ho is true or a region where Ho is rejected if the sample points falls in that region is called the critical region.

→ 'α' is known as size of critical region.

▦ : shaded area is rejection area.



critical region / Rejection region with size 'α'

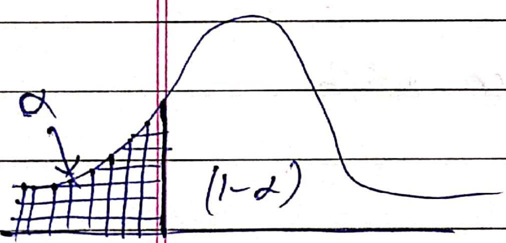
① **Two-tail :-** A two-tailed test, is a method in which the critical area of a distribution is two sided and tests whether a sample is greater than or less than a certain range of value. since the critical region is split into two parts, often having equal probabilities (especially in case of Normal distn; due to symm.)

placed in each tail of the distribution of the test statistics

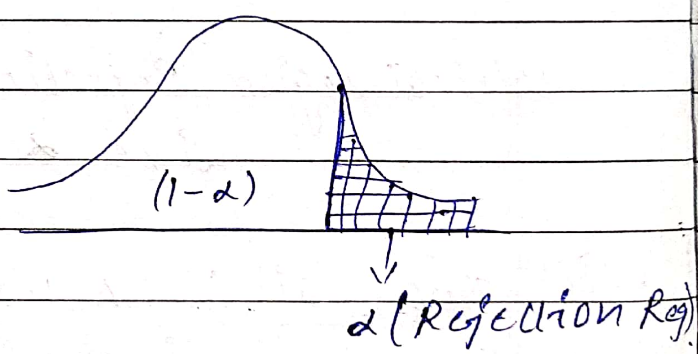
One-sided / one tailed test :- The critical region under the curve is distributed on one side of the test statistics.

Right tail :- If the critical region is taken on the right side of the dist^n.

Left tail :- The critical region is taken on the left side of the dist^n.



Left one-tailed



Right one-tailed.

Ex :- Recall example '1'.

Null hyp. H_0 : $\mu_x = \mu_y$

Alter. hyp. H_1 : (i) (Two-tailed) $\mu_x \neq \mu_y$

(ii) $\mu_x < \mu_y$

(iii) $\mu_x > \mu_y$

$\left.\begin{array}{l} \text{(ii)} \\ \text{(iii)} \end{array}\right\}$ one-tailed.

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Errors of Sampling :- While drawing conclusions for population parameters on the basis of sample results, we may encounter with the following two types of Errors :-

1. **Type I Error :-** Reject H_0 when it is true, i.e.; If null hypothesis H_0 is true but it is rejected by test procedure.

2. **Type II Error :-** Accept H_0 when it is false, i.e. If null hypothesis is false but it is accepted by test procedure.

Decision Table

	H_0 is accepted	H_0 is rejected
H_0 is true	NO Error	Type I Error
H_0 is false	Type II Error	NO Error

Probability of Type I Error :-

$$\alpha = P(\text{type I Error})$$
$$= P(\text{reject } H_0 \text{ when } H_0 \text{ is true}).$$

→ Type II Error :-

$$\beta = P(\text{type II Error})$$
$$= P(\text{accept } H_0 \text{ when } H_0 \text{ is false})$$

→ Power of the test :-

"The power of a statistical test is the propability of rejecting the null hypothesis H_0 when H_0 is false."

Power = $1-\beta$

NOTE :- Power : Ability of a test to detect the difference.
i.e. where equality (H_0) is rejected.

→ P-value :- The smallest level of significance that would lead to rejection of the null hypothesis H_0 with given data.

→ Relation between Hypothesis test and confidence Intervals :-

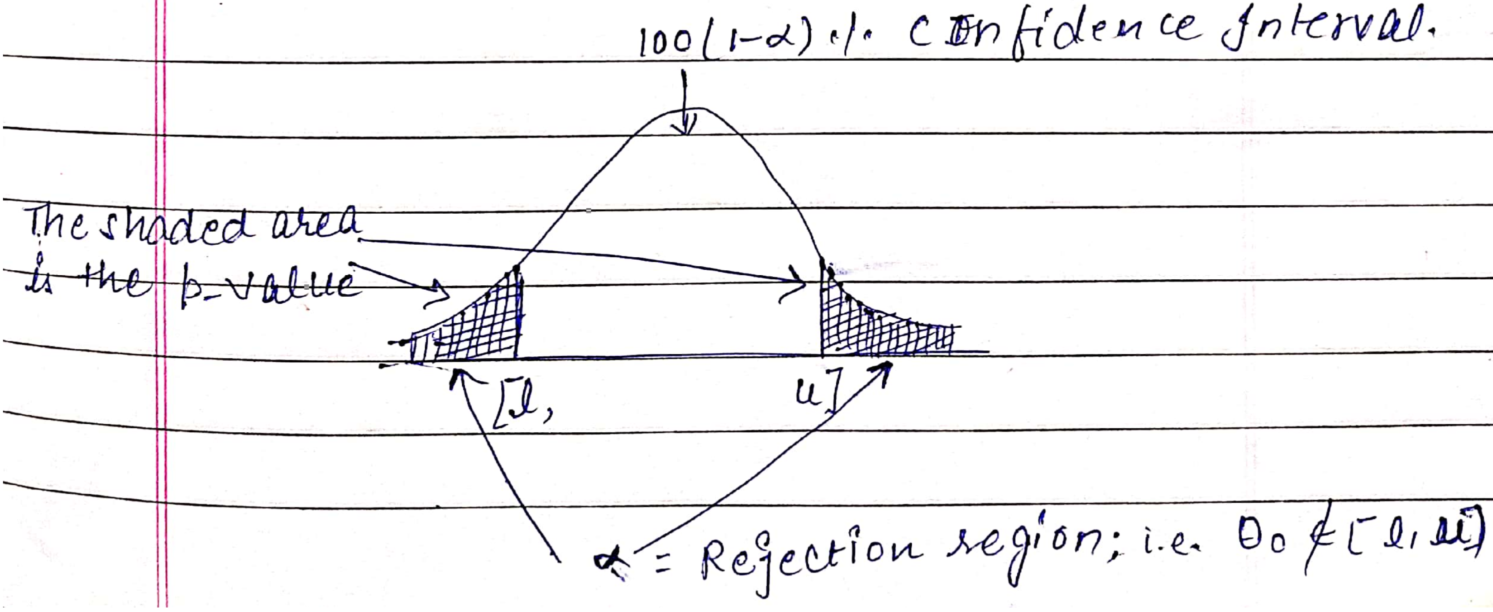
There is a very close relationship between the test of a hypothesis about any parameter, say θ, and the confidence interval for θ.

If $[l, u]$ is a $100(1-\alpha)\%$ confidence interval for the parameter θ. the test of size α of critical region of the hypothesis

$$H_0 : \theta = \theta_0$$
$$H_1 : \theta \neq \theta_0$$

will lead to rejection of H_0 iff θ_0 is not in $100(1-\alpha)\%$ confidence interval, i.e.

$$\theta_0 \notin [l, u].$$



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→ **Most Powerful Test :-** A statistical test that has the greatest power of all other tests with the same significance level.

"A test is said to be most powerful test which in addition to control 'α' at any desired low level has the minimum type II Error i.e 'β' or maximum power i.e. '1-β' compared to 'β' or '1-β' respectively of other tests having this 'α'."

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→ Steps in solving Testing of Hypothesis problem :-

1. choose the parameter of interest
2. setup the Null & alternative hypothesis
3. choose level of significance
4. Determine appropriate test statistics
5. set up the critical region and acceptance region for null hypothesis;
   a. Reject H_0 if $|t| > t_\alpha$
   b. Accept H_0 if $|t| < t_\alpha$.

6. Draw conclusions : Decide whether H_0 should be accepted or rejected.

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Testing of Hypothesis

Large sample — used tests

- z-test

Small samples

- t-test
- F-test

TOHyp.

One sample

→ Z-test
→ t-test
→

Two sample

→ Z-test
→ t-test
→ F-test
→ Chi-square test

(Large samples)

Z-test

one-sample | two-samples

→ Test of significance of a single mean

→ Test of significance for single proportion

→ Test of significance for difference of means

→ Test of significance for equality of two proportions (populations).

t - test (small sample)

one - sample | two samples

→ test for single mean (significance of the mean)

→ test for two means for independent samples (significance of difference of two means)

→ paired t-test
Test the significance of the difference between means of two dependent samples

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F - distribution

→ testing for equality of two variance

→ testing the equality of three or more means of indep-endent populat-ions

i.e ANOVA-test (Analysis of variance)

Note:- Utility dependence of t-test and F-test.

- To test the equality of means of two samples of small size, we use t-test, which is dependent on the assumption that the populations from which the samples are drawn have equal variance.
- Hence before applying t-test, it is necessary to test the equality

of population variances.
For this F-distribution is used.

Application of Z-test

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∴ Application of Large samples :-

1. Test of significance of a single Mean
2. Test of significance for difference of Means of two samples
3. Test of significance for single proportions
4. Test of Equality of two proportions (populations)

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Procedures for testing of Hypothesis :→

For Mean :-

Let a random variable sample of size n, $\bar{x}$ be the mean of of the sample and μ be the population

(I) Test for significance of a single mean:-

Steps :-

(1) Null Hypothesis :- H_0 : There is no significant difference in the given population mean value say μ_0.

i.e. $H_0 : \mu = \mu_0$

(2) Alternative hypothesis :- H_1 : There is some significant difference in the given population mean value.

i.e.

$H_1 : \mu \neq \mu_0$ (Two-tailed)
$H_1 : \mu > \mu_0$ (Right one-tailed)
$H_1 : \mu < \mu_0$ (Left one-tailed)

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③ Level of significance : set the value of 'α'.

④ Test statistics :

$$Z_{cal} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \Big/ \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

⑤ Decision / conclusion :-

If $Z_{cal} < Z_\alpha$ accept H_0

otherwise reject H_0.

Critical values of Z

α	1% (0.01)	5% (0.05)	10% (0.1)	
(T.T) $\mu \neq \mu_0$	$\lvert Z\rvert > 2.58$	$\lvert Z\rvert > 1.96$	$\lvert Z\rvert > 1.645$	$(Z_{\alpha/2})$
(R.T) $\mu > \mu_0$	$Z > 2.33$	$Z > 1.645$	$Z > 1.28$	(Z_α)
(L.T) $\mu < \mu_0$	$Z < -2.33$	$Z < -1.645$	$Z < -1.28$	(Z_α)

Remark :- Confidence limits for the mean of population corresponding to given sample:

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$$\mu = \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

(II) Test of significance for difference of means of two large samples :-

Let $\bar{x}_1$ and $\bar{x}_2$ be the means of the samples of two random size n_1 and n_2 drawn from two populations having means μ_1 and μ_2 and standard deviation's σ_1 and σ_2.

Steps :-

(i) Null hypo. :- $H_0 : \mu_1 = \mu_2$

(ii) Alternative hypo. :- $H_1 : \mu_1 \neq \mu_2$ (two-tailed)
$\mu_1 > \mu_2$ (Right one-t)
$\mu_1 < \mu_2$ (left one-t)

(iii) Level of sign. :- Set α

(iv) Test statistics :-

$$Z_{cal} = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{SD \text{ of } (\bar{x}_1 - \bar{x}_2)}$$

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where $\delta = \mu_1 - \mu_2$

$$Z_{cal} = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ is the standard deviation of the difference of two sample means.

(v) Decision: If $|Z_{cal}| < Z_\alpha$

accept Ho otherwise reject.

Remark :- Confidence limit for difference of means

$$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2} \left(\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$

III Test of significance for single proportions :-

A random sample of size 'n' has a sample proportion 'p' of a population having proportion 'P'. To test the hypothesis that the proportion 'P' in population has a specified value 'P_0'.

Steps :-

(i) $H_0 : P = P_0$

(ii) $H_1 : P \neq P_0$ (two-t)

$P > P_0$ (right-t)

$P < P_0$ (left-t)

(iii) Set α

(iv) Test statistics :

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

, where p = sample prop.

P = population proportion

$\sqrt{\frac{PQ}{n}}$ is standard deviation

with $Q = 1 - P$.

(v) Decision : If

$|Z_{cal}| < Z_\alpha$, accept H_0

otherwise, reject H_0.

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Remark :- Confidence limits for population proportion.

$$P = p \pm Z_{\alpha/2} \sqrt{\frac{PQ}{n}}$$

(IV) Test for Equality of two proportions (population)

Let p_1 and p_2 be the sample proportions in two large random samples of sizes n_1 and n_2 drawn from two populations having proportions P_1 and P_2.

Steps :-

(i) $H_0 : P_1 = P_2$ (T.T)

(ii) $H_1 : P_1 \neq P_2$ (T.T)

$P_1 > P_2$ (R.T)

$P_1 < P_2$ (L.T)

(iii) Test Statistic :

$$Z_{cal} = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_1 Q_1}{n_2}}}$$

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If given only sample proportions

$$Z_{cal} = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \dfrac{x_1 + x_2}{n_1 + n_2}$

and $q = 1 - p$, $p_1 = \dfrac{X_1}{n_1}$, $p_2 = \dfrac{X_2}{n_2}$

(iv) get critical value of Z at specified 'α'.

(v) Decision : If $|Z_{cal}| < Z_\alpha$ accept H_0 otherwise reject H_0.

Remark :- Confidence limit for difference of population proportion

$$P_1 - P_2 = (p_1 - p_2) \pm Z_{\alpha/2} \sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}$$

Examples

Ex 1:- A sample of 900 mark-members has a mean 3.4 cms and s.d. 2.61 cms. Is the sample from a large population of mean 3.25 cms and s.d. 2.61 cms.? If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

Sol^n :> 95% confidence limit / Interval
i.e $\alpha = 0.05$

(a) 9% fiducial limits for the proportion mean μ are

Given: $\bar{x} = 3.40$ cm
s.d $= 2.61$
$\mu = 3.25$ cms
$\sigma = 2.61$ cms

$$\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) =$$

$$= 3.40 \pm 1.96 \frac{3.25}{\sqrt{900}}$$

$$= 3.40 \pm 0.1705$$

$$= 3.5705 \text{ and } 3.2295$$

$$\sim [3.2295, 3.5705]$$

where $\pm Z_{\alpha/2} = \pm Z_{0.025} = \pm 1.96$ are the critical (tabulated) values for two-tailed test at 5% level of significance.

(b) 98% fiducial limits for μ : $\alpha = 0.02$

$$\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \because Z_{0.01} = 2.33$$

$$= 3.40 \pm 2.33 \times \frac{2.61}{\sqrt{900}} = 3.40 \pm 0.2027$$

$$= 3.6027 \text{ and } 3.1973$$

Testing :-

(1) Null hypothesis :- The sample has been drawn from the population with mean $\mu = 3.25$ cms and S.D. $\sigma = 2.61$ cms

(2) H_1 : $\mu \neq 3.25$ (two-tailed)

(3) Test statistic :- Since n is large enough

$$Z_{cal} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$Z_{cal} = \frac{3.40 - 3.25}{\frac{2.61}{\sqrt{900}}} = 1.73$$

(4) Decision :- As $|Z_{cal}| < 1.96$, that means,

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data don't provide us any evidence against Ho, which implies,
Ho is accepted at 5% level of significance level. As well as for 2% L.O.S.

Ex 12:- The means of two single large samples of 1,000 and 2,000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches? (Test at 5% level of significance.)

Sol^n:- Given data:-

$n_1 = 1000$ $\qquad$ $n_2 = 2000$

$\bar{x}_1 = 67.5$ inches $\qquad$ $\bar{x}_2 = 68.0$ inches

$\sigma = 2.5$ inches $\qquad$ $\sigma = 2.5$ inches

① Null Hypothesis: H_0: $\mu_1 = \mu_2$ and $\sigma = 2.5$ inch.
i.e. the sample has been drawn from the same population of standard deviation 2.5 inches.

② H_1: $\mu_1 \neq \mu_2$ (two-tailed)

③ $\alpha = 0.05$ →

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(4) Test statistics :-

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2 - E(\bar{x}_1 - \bar{x}_2)}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

under Ho hypothesis

i.e $E(\bar{x}_1 - \bar{x}_2) = E(\bar{x}_1) - E(\bar{x}_2)$

$= \mu_1 - \mu_2$

$= 0$

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

$$= \frac{67.5 - 68.0}{2.5\sqrt{\left(\frac{1}{1000} + \frac{1}{2000}\right)}}$$

$$= \frac{-0.5}{2.5 \times 0.0387} = -5.1$$

(5) Decision :- $|Z_{cal}| > 1.96$, ~~the~~ data does~~n't~~ provide ~~any~~ the significant evidence again Ho . i.e, there is highly significant difference.

So, we reject the null hypothesis and conclude that samples are certainly not from the same population.

Proportion :- **Probability of x successes in n trials :-**

If X is the number of success in n independent trials with constant probability 'P' of success for each trial, then follows binomial distribution i.e. $p(x) = {}^nC_x P^x Q^{n-x}$ $x = 0, 1, \dots n$

$E(X) = nP$, $V(X) = nPQ$; $Q = 1-P$ probability of failure.

Since for large 'n', binomial distⁿ tends to normal distⁿ

Hence $X \sim N(nP, nPQ)$ for $n \to \infty$

$$Z = \frac{X - E(X)}{\sqrt{V(X)}} = \frac{X - nP}{\sqrt{nPQ}} \sim N(0,1)$$

$\Rightarrow$ Observed proportion of successes $= \frac{X}{n} = p$ (say)

Here 'p' is the sample proportion

(i) $E(p) = E\left(\frac{X}{n}\right) = \frac{nP}{n} = P$

The expected value of sample proportion is 'P'
which implies
'p' is unbiased estimator of population proportion 'P'.

(ii) $Var(p) = V\left(\frac{X}{n}\right) = \frac{1}{n^2} V(X) = \frac{1}{n^2} nPQ$

$= \frac{PQ}{n}$

$S.D = \sqrt{\frac{PQ}{n}}$

(iii) Since $X \sim N(nP, nPQ)$

so $\frac{X}{n} \sim N\left(P, \frac{PQ}{n}\right)$

for standard normal variate

$$Z = \frac{p - E(P)}{\sqrt{\frac{PQ}{n}}} \sim N(0,1)$$

Note :- (i) 95% confidence limit for P

$= p \pm 1.96 \sqrt{\frac{pq}{n}}$

(ii) 99% confidence limit for P

$= p \pm 2.58 \sqrt{pq/n}$.

Ex 8 :- In a sample of 1,000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. Can we assume that rice and wheat are equally popular in this state at 1% level of significance?

Sol^n :- Given : $n = 1,000$

x = no. of rice eaters

$= 540$

p = sample proportion of rice eaters

$= \frac{x}{n} = \frac{540}{1000} = 0.54$

① H_0 : Both rice and wheat ~~eater~~ are equally popular in the state so that

P = population proportion of rice eaters in Maharashtra
= 0.5

Q = 0.5

(2) H_1 : $P \neq 0.5$ (two-tailed)

(3) Test-statistics :- Under H_0

$$Z_{cal} = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54-0.50}{\sqrt{\frac{0.5 \times 0.5}{1000}}}$$

$$= \frac{0.04}{0.0158}$$

$$= 2.532$$

(4) α = 1% LOS = 0.01
$Z_\alpha = 2.58$ for two-tailed.

(5) Conclusion :- $|Z_{cal}| < 2.58$

There is not any significant evidence against H_0. Thus H_0 is accepted. Conclusion is that [the null hypothesis] rice and wheat are equally popular in Maharashtra state.

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Ex 4:- Before an increase in excise duty on tea, 800 persons out of a sample of 1,000 persons were found to be tea drinkers. After an increase in ~~ex~~. duty, 800 people were tea drinkers in a sample of 1,200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty?

Sol^n:- Given $n_1 = 1{,}000$, $n_2 = 1{,}200$

$$p_1 = \frac{800}{1000} = 0.80$$

$$p_2 = \frac{800}{1200} = 0.67$$

(I) H_0 : $P_1 = P_2$ i.e. no significant diff. between before and after increase of (tea consumption) excise duty.

(II) H_1 : $P_1 > P_2$ (Right-tailed)

(III) $\alpha = 0.05 \rightarrow Z_\alpha = 1.645$ (Right tailed)

$\alpha = 0.01 \rightarrow Z_\alpha = 2.33$ (Right tailed)

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(4) Test statistic :- Under H_0

$$Z_{cal} = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

where $\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{800 + 800}{1000 + 1200}$

$= \frac{16}{22}$

$\hat{Q} = 1 - \hat{P} = \frac{6}{22}$

$$Z_{cal} = \frac{0.80 - 0.67}{\sqrt{\frac{16}{22} \times \frac{6}{22}\left(\frac{1}{1000} + \frac{1}{2000}\right)}}$$

$= \frac{0.13}{0.19} = 6.842$

(5) Decision :- $Z_{cal} > 1.645$ (5% LOS)
and $Z_{cal} > 2.33$ (1% LOS)
highly significance
$\Rightarrow$ H_0 is rejected, H_1 is accepted
That is, There is a significant decrease

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in the consumption of tea after increase in the excise duty.

Small size samples

(t - test)

① To test the significance of the sample mean (population variance is not given):-

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

where $$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

→ Confidence limits for mean 'μ'

$$= \bar{x} \pm t_\alpha \cdot s/\sqrt{n} \quad / \quad \bar{x} \pm t_\alpha \cdot s/\sqrt{n-1}$$

② To test the significance of the difference between means of the two independent samples:-

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} - \frac{1}{n_2}\right)}}$$

where $\bar{x}_1 = \frac{\sum x_1}{n_1}$, $\bar{x}_2 = \frac{\sum x_2}{n_2}$

$$s^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_1-\bar{x}_1)^2 + \sum(x_2-\bar{x}_2)^2\right]$$

Remarks :- Confidence Limit

$$\mu_1-\mu_2 = (\bar{x}_1-\bar{x}_2) \pm t_\alpha \sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

(3) Paired t-test :-

$$t = \frac{\bar{d}}{s/\sqrt{n}} \qquad \bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(d_i-\bar{d})^2$$

$$d_i = x_i - y_i \; ; \; i = 1, 2, \ldots n$$

Ex!5:- The mean weekly sales of soap bars in departmental stores was ~~14.66~~ 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

Sol^n:- Given $n = 22$ $\bar{x} = 153.7$

$s = 17.2$

(i) $H_0 : \mu = 146.3$; campaign is not successful

(ii) $H_1 : \mu > 146.3$ (Right-tailed)

(iii) $\alpha = 0.05$; $t = 1.72$ at 21 d.o.f. for single test

(iv) T.S :- Under H_0

$$t = \frac{153.7 - 146.3}{\sqrt{(17.2)^2 / 21}} = \frac{7.4 \times \sqrt{21}}{17.2}$$

$= 9.03$

(v) Decision :- $Z_{cal} > 1.72$

$\Rightarrow$ H_0 is rejected

$\Rightarrow$ campaign is successful.

F-test

(1) F-test for Equality of two population variance :-

$$F = \frac{Sx^2}{Sy^2}$$

where $Sx^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_i - \bar{x})^2$

and $Sy^2 = \frac{1}{n_2 - 1} \sum_{j=1}^{n_2} (y_j - \bar{y})^2$

with dof $(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$.

Ex :- In one sample of 8 observations the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in other sample of 10 observations it was 102.60. Test whether this difference is significant at 5 percent level, given that the 5 percent point of F for $n_1 = 7$ and $n_2 = 9$ degrees of freedom is 3.29.

Sol^n :- Given $n_1 = 8, \quad n_2 = 10$

$$\sum (x - \bar{x})^2 = 84.4$$

$$\sum (y - \bar{y})^2 = 102.6$$

$$S_x^2 = \frac{1}{n_1 - 1} \cdot \sum (x - \bar{x})^2 = \frac{84.4}{7} = 12.057$$

$$S_y^2 = \frac{1}{n_2 - 1} \sum (y - \bar{y})^2 = \frac{102.6}{9} = 11.4$$

(i) $H_0 : \sigma_x^2 = \sigma_y^2 = \sigma^2$

(ii) Test statistics :

$$F = \frac{S_x^2}{S_y^2} = \frac{12.057}{11.4} = 1.057$$

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Tabulated $F_{0.05}$ for $(7, 9)$ d.o.f.
is 3.29

$F_{cal} < F_{0.05}$; H_0 may be accepted at 5% LOS.

Chi-Square Test

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① Chi-square test as a test of goodness of fit :-

χ^2- test enables us to ascertain how well the theoretical distⁿ such as binomial, Poisson, normal etc., fit the distⁿ obtained from sample data. If the calculated value of χ^2 is less than the table value at a specified level of (generally 5%) significance, the fit is considered to be good.

If the calculated value of χ^2 is greater than the table value then the fit is considered to be ~~good~~. poor.

(i) H_0 : There is no difference in given values and calculated value

(ii) H_1 : some difference.

(iii) Test statistics :- $\chi^2_{cal} = \sum_{i=1}^{n} \left(\frac{(fi - ei)^2}{ei} \right)$

where $\sum_{i=1}^{n} fi = \sum_{i=1}^{n} ei$,

follows chi-square dist^n with $(n-1)$ d.o.f.
$\{f_i\}$ is a set of observed frequencies and $\{e_i\}$ is the corresponding set of expected frequencies.

(iv) LOS

(v) Accept H_0 if $\chi^2_{cal} < \chi^2_{\alpha(n-1)} / \chi^2_{tab}$

reject otherwise.

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Ex:- The demand for a particular spare part in a factory was found to vary from day-to-day. In a semple study the following Information was obtained:

Days	M	T	W	T	F	S
No. of parts demanded	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the days of the week.
(Given : chi-square value at 5 d.o.f. is 11.07).

Solⁿ:- H_0: no. of parts demanded doesn't depend on the day of week.
Under the null hypothesis, the expected frequencies of the spare parts demanded on each ~~day~~ of the six days would be:

$$\frac{1124 + 1125 + 1110 + 1120 + 1126 + 1115}{6}$$

$$= 1120.$$

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Days	observed (f_i)	Expected e_i	$(f_i-e_i)^2$	$\frac{(f_i-e_i)^2}{e_i}$
M	1124	1120	16	0.014
T	1125	1120	25	0.022
W	1110	1120	100	0.089
Th.	1120	1120	0	0.000
F	1126	1120	36	0.032
S	1115	1120	25	0.022
Total	6720	6720		0.179

$$\chi^2_{cal} = \frac{\Sigma (f_i-e_i)^2}{e_i} = 0.179$$

$$d.f. = 6-1 = 5$$

$$\chi^2_{0.05} \text{ for } 5 \, d.f. = 11.07$$

$$\chi^2_{cal} < \chi^2_{0.05}$$

H_0 is accepted at 5% LOS.

Hence we conclude that the number of parts demanded are same over the 6-day period.

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