Generalisation to n-dimensional Random variable

① Joint Probability Mass function:-

Let $(X_1, X_2 — X_n)$ be a discrete n-dimensional r.v, assuming discrete values, in some region, say R^n of n-dim space. Then the joint p.m.f of $(X_1, X_2 — X_n)$ is defined as

$$p_{X_1, — X_n}(x_1, x_2 —, x_n) = P[X_1 = x_1, X = x_2, — X = x_n]$$
$$= P[\cancel{x} \bigcap_{i=1}^{n} (X_i = x_i)].$$

where (i) $p(x_1, x_2 — x_n) \geq 0 \ \forall (x_1, x_2, — x_n) \in R^n$

(ii) $\sum_{x_1, x_2 — x_n} p(x_1, x_2 —, x_n) = 1.$

Marginal p.m.f :-

Summing $p(x_1, x_2 — x_n)$, over all values of other variables except X_i i.e.

$$P_{X_i}(x_i) = \sum_{\substack{(x_1, x_2 — x_n) \\ \text{except } x_i}} p(x_1, — x_n).$$

Special case of three r.v.s :-

Let $p(x_1, x_2, x_3)$ is the joint p.m.f of three r.v.s $X_1 X_2$ and X_3, then the marginal p.m.f of X_1 is given by

$$P_{X_1}(x_1) = \sum_{x_2, x_3} p(x_1, x_2, x_3),$$ and so on.

→ Independency of r.v.'s :- The r.v.'s $X_1, X_2, \ldots X_n$ are independent if and only if their joint p.m.f is equal to the product of their marginal p.m.f's. i.e.

$$p_{X_1, X_2 \ldots X_n}(x_1, x_2 \ldots x_n) = p_{X_1}(x_1) \cdot p_{X_2}(x_2) \cdot \ldots \, p_{X_n}(x_n).$$

(2) Joint and marginal probability Density function

Let $(X_1 \ldots X_n)$ be n-dim continuous r.v. assuming all the values in some region, say R_1^n of the n-dimensional space. Then the joint p.d.f of $(X_1, X_2, \ldots X_n)$ is given by:

$$f_{X_1, X_2 \ldots X_n}(x_1, x_2 \ldots x_n) = \lim_{\substack{dx_1 \to 0,\, dx_2 \to 0 \\ \ldots\, dx_n \to 0}} \frac{P[x_1 < X_1 < x_1 + dx_1 \ldots \ldots x_n < X_n < x_n + dx_n]}{dx_1 \ldots dx_n}$$

$$= \lim_{\substack{dx_1 \to 0 \ldots \\ \ldots dx_n \to 0}} \frac{P[\bigcap_{i=1}^{n} (x_i < X_i < x_i + dx_i)]}{dx_1 \cdot dx_2 \ldots \cdot dx_n}$$

where

(i) $f_{X_1, X_2 \ldots X_n}(x_1, x_2 \ldots x_n) \geq 0 \quad \forall (x_1, x_2 \ldots x_n) \in R_1^n$

(ii) $\underbrace{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{n\text{-times}} f(x_1, x_2 \ldots x_n)\, dx_1\, dx_2 \ldots dx_n = 1.$

→ The marginal p.d.f of any variable say x_i, is obtained on integrating the joint p.d.f over the range of all the variables except x_i.

$$f_{X_i}(x_i) = \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{(n-1)\text{ times}} f(x_1, x_2, \cdots x_n)\, dx_1 \cdots dn$$

(except x_i)

→ In particular, for three r.v.'s $X_1, X_2,$ and X_3 with joint p.d.f $f(x_1, x_2, x_3)$

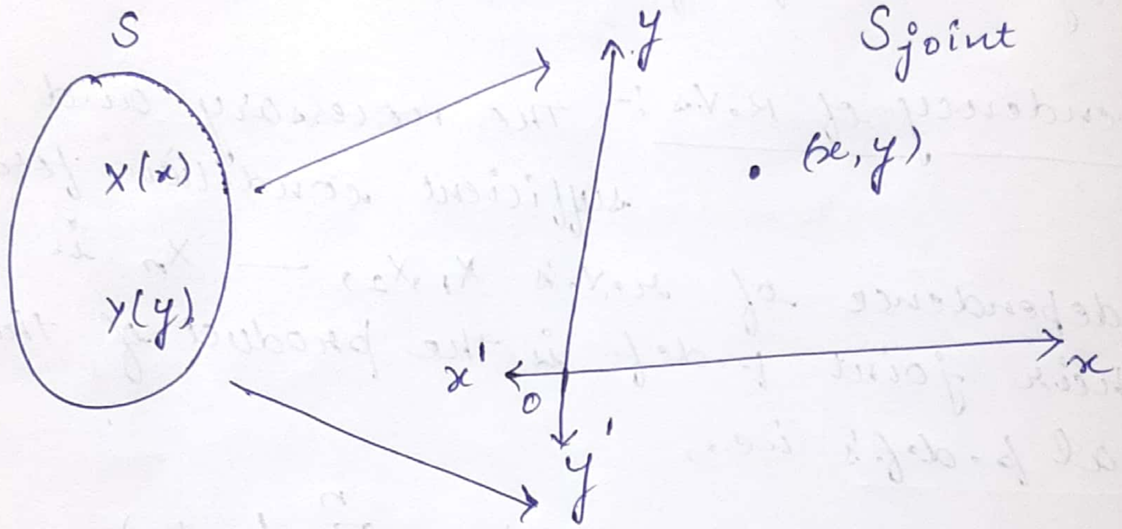
Independency of R.V.s :- The necessary and sufficient condition for the independence of r.v.'s $X_1, X_2, \cdots X_n$ is that their joint p.d.f is the product of their marginal p.d.f.'s i.e.,

$$f_{X_1, X_2 \cdots X_n}(x_1, x_2 \cdots, x_n) = \prod_{i=1}^{n} f_{X_i}(x).$$

Scanned with OKEN Scanner

Vector Random Variable

Let X and Y are ~~defined as~~ two random variables defined over a sample space S, with their own specific values x and y respectively. Then the ordered pair (x, y) is a specific value of, say, random point in the xy-plane, of a random vector or vector random variable.



: A mapping from S to the joint sample space S_{joint}.

Distribution and Density of a sum of Random variables :-

Here, we assumed that all the taken random variables are statistically independent random variables :-

Sum of two variables :-

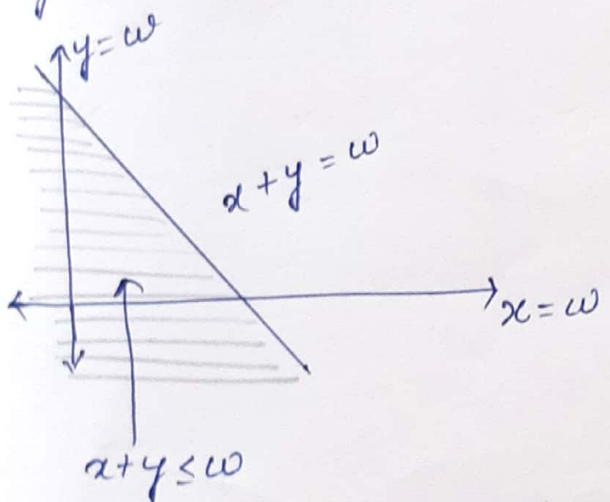
Let W be a r.v. equal to the sum of two independent random variables X and Y, i.e.

$$W = X + Y,$$

then the distribution function can be defined by

$$P[W \leq \omega] = P(X + Y \leq \omega).$$

→ The graphical representation of $[W = x + y \leq \omega]$ is



: Region in the xy-plane, where $x + y \leq \omega$.

The probability distribution $F_W(\omega) / F_{X+Y}(\omega)$, when '$f_{X,Y}(x,y)\,dx\,dy$' represents the elemental area, can be written as;

$$F_W(\omega) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{\omega - y} f_{X,Y}(x,y)\,dx\,dy.$$