LIMITS AND CONTINUITY

Properties:

1. $\lim_{x \to a}(f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a}(f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a}[f(x). g(x)] = \lim_{x \to a} f(x). \lim_{x \to a} g(x)$

4. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

Formulae:

1. $\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = n. a^{n-1}$

2. $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

3. $\lim_{x \to 0} \dfrac{\tan x}{x} = 1$

4. $\lim_{x \to 0} \dfrac{a^x - 1}{x} = \log a$

5. $\lim_{x \to 0}(1 + x)^{1/x} = e$

6. $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x = e$

7. $\lim_{x \to 0} \dfrac{\log(1+x)}{x} = 1$

Definition:

1. A function is said to be continuous at a point x=a of its domain if

(i) $\lim_{x \to a^-} f(x) \; exists.$

(ii) $\lim_{x \to a^+} f(x) \; exists.$

(iii) $f(a) \; exists$ and

(iv) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

2. If the function is not continuous then it is called discontinuous

3.If f and g are two functions defined on the same domain, then $k.f$, $k \in r, f \pm g, f.g$ are continuous functions.

4.Further $\frac{f}{g}$ is continuous at all points except those for which $g(x)$=0

5.The following types of functions are continuous at every number in their domains:

- Polynomial functions
- Rational functions
- Root functions
- Trigonometric functions
- Inverse Trigonometric functions
- Exponential functions
- Logarithmic functions
- Constant functions