<u>Tutorial No.1</u>

1.1 Mean Value Theorems:

1. Is Rolle's Theorem applicable to the following functions?

i) $\log\left[\dfrac{x^2+6}{5x}\right]\ on\ [2,3]$

Ans: $c=\sqrt{6}$

ii) $1-\sqrt[3]{(x-1)^2}\quad on\ [0,2]$

Ans: Not applicable

iii) $f(x)=\begin{cases} x^2-2 & -1\le x\le 0 \\ x-2 & 0\le x\le 1 \end{cases}$

Ans: Not applicable

iv) $\cos^2 x\quad on\ \left[\dfrac{-\pi}{4},\dfrac{\pi}{4}\right]$

Ans: $c=0$

v) $(x-1)(x-3)e^{-x}\quad on\ [1,3]$

Ans: $c=3-\sqrt{2}$

vi) $|\cos x|\quad on\ [0,\pi]$

Ans: Not applicable

2. Verify Rolle's Theorem for $f(x)=e^{-x}(\sin x-\cos x)\ in\ \left[\dfrac{\pi}{4},\dfrac{5\pi}{4}\right]$.

Ans: Theorem is verified and $c=\dfrac{\pi}{2}$.

3. Examine the validity of the conditions and the conclusion of Lagrange's Mean Value theorem
for the functions:

i) $x^{2/3}\ on\ [-2,2]$ Ans: Not applicable

ii) $x+\frac{1}{x}$ $on \left[\frac{1}{2},3\right]$ Ans: $c=\sqrt{\frac{3}{2}}$

iii) $2x^2-7x+10$ $on\ [2,5]$ Ans: $c=\frac{7}{2}$

iv) $(x-1)(x-2)$ $on\ [0,4]$ Ans: $c=2$

4. Show that for the curve $y=x^2+2k_1x+k_2$, the chord joining the points $x=a$ and $x=b$ is parallel to the tangent at $x=\frac{a+b}{2}$.

5. At what point is the tangent to the curve $y=x^n$ parallel to the chord joining $(0,0)$ *and* (a,a^n)?

Ans: $at\ x=\frac{a}{n^{1/n-1}}$

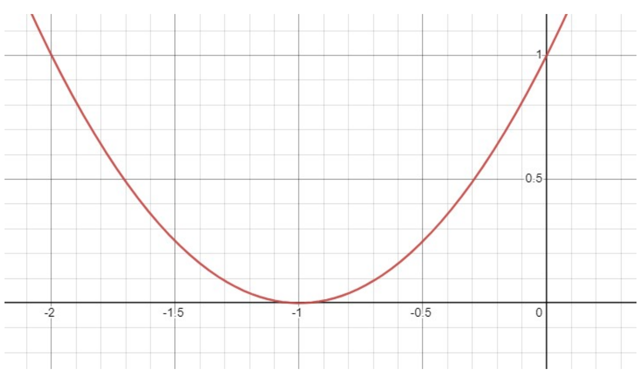
6. If $f(x)$ and $g(x)$ are respectively $\sqrt{x}$ and $\frac{1}{\sqrt{x}}$ then prove that c of Cauchy's mean value theorem is the geometric mean between a and b , $a>0, b>0$.

7. Prove that $\frac{\sin b-\sin a}{\cos a-\cos b}=\cot c,\ \ a<c<b.$ Putting $a=0, b=x$ deduce that $c=\frac{x}{2}$.

8. Verify Cauchy's mean value theorem for

i) $f(x)=3x+2, g(x)=x^2+1 \quad on\ \ 1\le x\le 4$

9. The following is the graph of $y=x^2+2x+1$ in the interval [-2,0].



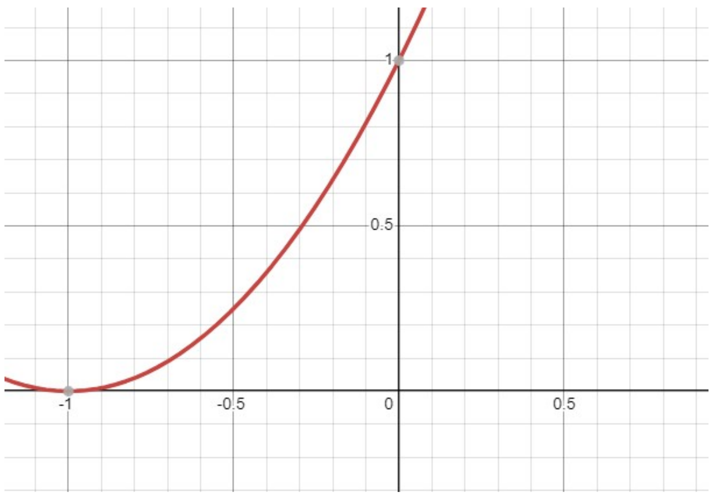
a) Is Rolle's theorem applicable to the above function?

b) From the graph, observe at what point the tangent to the graph is parallel to the x-axis?

c) Verify your answer of 'b' part by applying Rolle's theorem to the given function.

10. A particle is travelling along the path $f(t) = -t^2 + 2t$, $0 \le t \le 2$. Apply Rolle's theorem to *f(t)*, and find at what point of time the velocity of the particle is zero.

11. The following is the graph of $y = x^2 + 2x + 1$ in the interval [-1,0].



a) Find the slope of the line joining (-1,0) and (0,1).

b) Use LMVT to find a point 'c' such that the tangent at 'c' to this graph is parallel to the line joining (-1,0) and (0,1).

12. A trucker travels 163 miles on a toll road with a speed limit of 70 miles per hour. He took 2 hours for completing the journey. Apply Mean Value Theorem to check whether the trucker should be issued a speeding ticket.

1.2 Convergence of Sequences and series:

1) Test the convergence of following sequences:

1. $a_n = 2^n$ (dgt)
2. $a_n = 3 + (-1)^n$ (dgt)
3. $a_n = \left(n + (-1)^n\right)^{-1}$ (cgt)

4. $a_n = \left(\dfrac{n}{n-1}\right)^2$ (cgt)

5. $a_n = 1 + \dfrac{(-1)^n}{n}$ (cgt)

2) Test the convergence of following series:

Using geometric test:

1. $1 - \dfrac{1}{3} + \dfrac{1}{3^2} - \dfrac{1}{3^3} + \dfrac{1}{3^4} - \cdots$ (cgt)

2. $\sum_{n=0}^{\infty} \dfrac{2^{3n}}{3^{2n}}$ (cgt)

Using ratio test:

1. $\sum_{n=1}^{\infty} \dfrac{n^2}{3^n}$ (cgt)

2. $1 + \dfrac{2!}{2^2} + \dfrac{3!}{3^3} + \dfrac{4!}{4^4} + \cdots$ (cgt)

3. $\sum_{n=1}^{\infty} \dfrac{n!}{2^n}$ (dgt)

Using Root test

1. $\sum_{n=1}^{\infty} (\log n)^{-2n}$ (cgt)

2. $\sum_{n=1}^{\infty} \left(\dfrac{3n+1}{4-2n}\right)^{2n}$ (dgt)

Tutorial No.2

1.3 Taylor's Series and Maclaurin's series

1. Expand $\log(\cos x)$ about $\frac{\pi}{3}$.

2. Prove that $\frac{1}{1-x}=\frac{1}{3}+\frac{(x+2)}{3^2}+\frac{(x+2)^2}{3^3}+\frac{(x+2)^3}{3^4}+\ldots$

3. Expand $\tan^{-1}x$ in powers of $\left(x-\frac{\pi}{4}\right)$.

$$\textbf{Ans: } \tan^{-1}x=\tan^{-1}\frac{\pi}{4}+\left(x-\frac{\pi}{4}\right)\cdot\frac{1}{\left(1+\frac{\pi^2}{16}\right)}-\left(\frac{\pi}{4}\right)\cdot\left(x-\frac{\pi}{4}\right)^2\cdot\frac{1}{\left(1+\frac{\pi^2}{16}\right)^2}+\cdots$$

4. Show that $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots.$

5. Prove that $a^x=1+x\log a+\frac{x^2}{2!}(\log a)^2+\cdots$

6. Show that $\frac{1}{1+x}=1-x+x^2-x^3+\ldots$

7. Show that $\frac{1}{1-x}=1+x+x^2+x^3+\ldots$

1.4 Indeterminate Forms and L'Hospital's rule:

1. Evaluate $\lim\limits_{x\to1}\frac{1+\cos\pi x}{x^3-3x+2}$ Ans: $\frac{\pi^2}{6}$

2. Evaluate $\lim\limits_{x\to0}\frac{x^2+2\cos x-2}{x\sin x}$ Ans: 0

3. Evaluate $\lim\limits_{x\to0}\frac{\log(1-x^2)}{\log(\cos x)}$ Ans: 2

4. Evaluate $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{\frac{1}{2}}-1}$. Ans. $2\log 2$.

5. Evaluate $\lim_{x\to 0}\frac{\log\sin 2x}{\log\sin x}$. Ans: 1.

6. Evaluate $\lim_{x\to a}\log\left(2-\frac{x}{a}\right)\cot(x-a)$. Ans: $\frac{-1}{a}$.

7. $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{e^x-1}\right]$. Ans: $\frac{1}{2}$

8. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$. Ans: $(abc)^{\frac{1}{3}}$.

9. Evaluate $\lim_{x\to 0}(\cot x)^{\sin x}$. Ans: 1

10. Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{2\sin x}$ **Ans:**1