SVKM's NarseeMonjee Institute of Management Studies
Mukesh Patel School of Technology Management & Engineering

Unit V

Tutorial No. 1

Gradient, directional derivatives, divergent, curl, scalar potential:

1. Find $grad\ f$ if $f(x,y,z)=x^2y+y^3+yz^2-xyz$ at (1,0,1).
2. Find $grad\left(\log\left(x^2+y^2+z^2\right)\right)$. Ans: $\dfrac{2(xi+yj+zk)}{\left(x^2+y^2+z^2\right)}$
3. Find $grad\ f$ if $f=e^{xy}-x\cos\left(yz^2\right)$

   $\left(Ans:\left(ye^{xy}-\cos yz^2\right)\vec{i}+\left(xe^{xy}+xz^2\sin yz^2\right)\vec{j}+\left(2xyz\sin yz^2\right)\vec{k}\right)$
4. Find directional derivative of $f = x^3 - xy^2 - z$ at $(1,1,0)$ in the direction of $2\vec{i}-3\vec{j}+6\vec{k}$.

   $\left(Ans:\dfrac{4}{7}\right)$
5. Find the directional derivative of the function $\emptyset = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q is the point (5, 0, 4).Ans: $\frac{28}{\sqrt{21}}$
6. What is the greatest rate of increasing $\emptyset = xyz^2$ at the point (1, 0, 3). Ans: 9
7. The temperature of points in space is given by $T(x.y,z) = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move. Ans: D.D.= $\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{3}$
8. In what direction from $(2,-1,2)$ is the directional derivative of $\phi=4xz^3-3x^2y^2z$ minimum? And what is the magnitude of minimum change? $\left(Ans:\dfrac{-2\vec{i}-12\vec{j}-21\vec{k}}{\sqrt{589}},4\sqrt{589}\right)$
9. Find $div\ \vec{F}$ and $curl\ \vec{F}$when $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.

Ans: $div\ \vec{F} = 6((x + y + z), curl\ \vec{F} = 0$

10. If $\overline{F}=x^2\hat{i}+xz\hat{j}+yz\hat{k}$ and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ find $div\left(\vec{F}\times\vec{r}\right)$and $curl\left(\vec{F}\times\vec{r}\right)$

    Ans: $div\left(\vec{F}\times\vec{r}\right)=z^2+xz-x^2$ and

    $curl\left(\vec{F}\times\vec{r}\right)=\left(2x^2-xy\right)\hat{i}+\left(4xz-2xy-y^2\right)\hat{j}+\left(3yz-2xz\right)\hat{k}$
11. Determine the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. Hence find the scalar potential $\emptyset$ s.t. $\vec{F} = grad\emptyset$.

Ans: a=4, b=2, c= -1, $\emptyset = \frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz = yz$

12. Show that the vector $\overrightarrow{F}=\left(y^2\cos x+z^3\right)\hat{i}+\left(2y\sin x-4\right)\hat{j}+\left(3xz^2+2\right)\hat{k}$ is irrotational and find its scalar potential. Ans: $\phi=y^2\sin x+z^3x-4y+2z$
13. Show that the vector $\overrightarrow{F}=\left(2x+z^2+3y\right)i+\left(2y+3x+z\right)j+\left(2xz+y\right)k$ is irrotational and find its scalar potential function ϕ such that $\overrightarrow{F}=\nabla\phi$ and $\phi\left(1,1,0\right)=4$.

    Ans: $\phi=x^2+y^2+z^2x+3xy+zy-1$
14. Show that $u=2xy+3y$ is harmonic function.
15. Show that $u=\cos x\left(\dfrac{e^y+e^{-y}}{2}\right)$ is a harmonic function.
16. If $v=3x^2y+6xy-y^3$, Show that v is harmonic function.