**SVKM's NarseeMonjee Institute of Management Studies**
**Mukesh Patel School of Technology Management & Engineering**

## Unit IV

**Tutorial No. 1**

1. Evaluate $\int_0^1 \int_{x^2}^{x} xy(x+y)dxdy$. Ans: $\frac{3}{56}$

2. Evaluate $\int_0^{a\sqrt{3}} \int_0^{\sqrt{a^2+x^2}} \frac{xdydx}{y^2+x^2+a^2}$ . Ans: $\frac{\pi a}{4}$.

3. Evaluate $\int_0^1 \int_{-\sqrt{y}}^{-y^2} xydxdy$. Ans: $-\frac{1}{12}$

4. Evaluate $\int_0^1 \int_0^{\sqrt{x^2+1}} \frac{1}{y^2+x^2+1} dxdy$. Ans: $\frac{\pi}{4}\left[\log\left(1+\sqrt{2}\right)\right]$

5. Evaluate $\iint_R ydxdy$ where R is the region between the parabola $x^2 = y$ and the line $x+y=2$.

Ans. $\frac{36}{5}$

6. Evaluate $\iint_R \frac{dydx}{\sqrt{1-x^2-y^2}}$ where R is the region of the first quadrant of ellipse $2x^2+y^2=1$.

Ans: $\frac{\pi}{4}$

7. Change the order of integration $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)dx.dy$ Ans. $\int_a^{2a} \int_{y^2/2a}^{2a} + \int_0^a \int_{y^2/2a}^{a+\sqrt{a^2-y^2}} + \int_0^a \int_{a+\sqrt{a^2-y^2}}^{2a}$

8.Change the order and Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy.dx$ Ans. $1-\frac{1}{\sqrt{2}}$

9. Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx.dy$ Ans. 1

10. Change the order and Evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\ dx\,dy$ Ans. $\frac{3a^4}{8}$

11. Change to polar co-ordinates and evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \left(x^2+y^2\right) dx.dy$ Ans. $\frac{3\pi}{4}$

12. Find the area bounded by $y^2 = 4-2x, x \geq 0, y \geq 0$ . Ans: $8/3$ sq.units.

13. Using double integration, find the area bounded between parabolas $x^2 = 4ay$ and

$x^2 = -4a(y - 2a)$. Ans: $[\frac{16a^2}{3}]$

14. Find by double integration the area common to the circle $x^2 + y^2 = 10$ and the parabola $y^2 = 9x$.

**Tutorial No. 2**

1. Evaluate $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dxdydz$. Ans: 48

2. Evaluate $\int_{0}^{2}\int_{0}^{x}\int_{0}^{2x+2y} e^{x+y+z}\,dxdydz$. Ans: $\frac{e^{12}}{18} - \frac{e^6}{9} - \frac{e^4}{2} + e^2 - \frac{4}{9}$

3. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x + y + z)\,dxdydz$. Ans: 0

4. Evaluate $\int_{0}^{1}\int_{0}^{1-x}\int_{0}^{1-x-y} \frac{dxdydz}{(1 + x + y + z)^3}$. Ans: $\frac{1}{2}\left[\log 2 - \frac{5}{8}\right]$

5. Evaluate $\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{dxdydz}{(1 + x^2 + y^2 + z^2)^2}$ using spherical coordinates. Ans: $\frac{\pi^2}{8}$

6. Evaluate $\iiint \frac{dxdydz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$ over the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

Ans: $\pi^2 a^2$.

8. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$
Ans: $16\pi$

9. Find the volume bounded by $y^2 = x, x^2 = y$ and the planes $z = 0$ *and* $x + y + z = 1$.
Ans: $[\frac{1}{30}]$

10. Find the volume of the tetrahedron bounded by the plane $x = 0, y = 0, z = 0$ and $6x + 3y + 2z = 6$ Ans. 1

11. Find the volume of the sphere $x^2 + y^2 + z^2 = 9$ in first octant. Ans. $36\pi$