Set A - M2

Q1(a)(i) $\lim_{(x,y)\to(2,0)} \dfrac{x \sin y}{x^2+1}$

$= \dfrac{2\cdot \sin 0}{4+1} = \underline{\underline{0}}$ — ½ — ½.

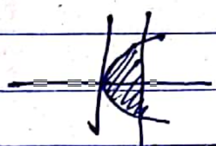
(ii) $u = 2xy$

$u_x = 2y$ $\quad$ $u_y = 2x$ $\Big\}$ continuous

$u_{xx} = 0$ $\quad$ $u_{yy} = 0$ $\Big\}$ p.d. — ½

$u_{xx} + u_{yy} = 0$ $\quad$ Harmonic. — ½

(b) ~~Area of the surface~~



$x = 2$.

$\cancel{= 2\pi \int_{x=a}^{b} 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx}$.

About the line $y=0$.

$\therefore p = 0$.

Volume $= \pi \int_a^b (y-p)^2\, dx$. — ½

$= \pi \int_{x=0}^{2} y^2\, dx$ — ½ $= \pi \int_0^2 x\, dx$ — ½

$= \pi \cdot \dfrac{x^2}{2}\Big]_0^2 = \underline{\underline{\pi \cdot 2}}$ — ½

Q2) $\int_0^2 x \cdot \sqrt[3]{8-x^3}\, dx$. $\quad$ Put $x = 2t$

$x^3 = 8t^3$

$= \int_0^1 2t\,(8-8t^3)^{1/3}\, 2\,dt$ — ½ $\quad$ $dx = 2\,dt$.

x	0	2
t	0	1

— ½

$= 4 \times 8^{1/3} \int_0^1 t\,(1-t^3)^{1/3}\, dt$. — ½ $\quad$ $t^3 = u$

$\cancel{dt}\ t = u^{1/3}$

$\cancel{= 8 \int_0^1 u^{1/2}(1-u)^{1/3} \frac{1}{2} u^{-1/2}\, du}$. $\quad$ $\cancel{dt = \frac{1}{2}u^{-1/2} du}$

$dt = \frac{1}{3}u^{-2/3} du$

$\cancel{= 4 \int u^0 (1-u)^{1/3} du = 4 \cdot \beta\left(1, \frac{4}{3}\right)}$ — ½

$$= \frac{8}{3}\int_0^1 u^{1/3}(1-u)^{1/3}u^{-2/3}\,du$$

$$= \frac{8}{3}\int_0^1 u^{-1/3}(1-u)^{1/3}\,du$$

$$= \frac{8}{3}\beta\left(\frac{2}{3}, \frac{4}{3}\right) \quad \text{— } 1/2$$

$$= \frac{8}{3}\,\frac{\Gamma\frac{2}{3}\,\Gamma\frac{4}{3}}{\Gamma 2}$$

$$= \frac{8}{3}\,\Gamma\frac{2}{3}\cdot\frac{1}{3}\,\Gamma\frac{1}{3}$$

$$= \frac{8}{9}\,\Gamma\frac{2}{3}\,\Gamma\frac{1}{3} \quad \text{— } 1/2$$

Q3. $\nabla \times F = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix} = 0 \quad \text{— } 1/2$$

$$\Rightarrow \hat{i}(c+1) - \hat{j}(4-a) + \hat{k}(b-2) = 0$$

$$\Rightarrow c=-1,\ a=4,\ b=2 \quad \text{— } 1/2$$

$$\therefore \vec{F} = \nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}.$$

$$\frac{\partial \phi}{\partial x} = x+2y+4z \Rightarrow \phi = \int x+2y+4z\,dx$$
$$= \frac{x^2}{2} + 2yx + 4xz$$
1/2

$$\frac{\partial \phi}{\partial y} = 2x-3y-z \Rightarrow \phi = \int 2x-3y-z\,dy$$
$$= 2xy - \frac{3y^2}{2} - yz$$
1/2

$$\frac{\partial \phi}{\partial z} = 4x-y+2z \Rightarrow \phi = \int 4x-y+2z\,dz$$
$$= 4xz - yz + z^2$$
1/2

$$\therefore \phi = \frac{x^2}{2} + 2xy + 4xz - \frac{3y^2}{2} - yz + z^2 + c$$
1/2

4) $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

$$\frac{\partial f}{\partial x} = 3x^2 + 3y^2 - 6x$$

$$\frac{\partial f}{\partial y} = 6xy - 6y$$

$$r = \frac{\partial^2 f}{\partial x^2} = 6x - 6$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = 6y$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6x - 6$$

(01)

For st. pts $\rightarrow$ $\left.\begin{matrix}\frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0\end{matrix}\right\} \Rightarrow$ $3x^2 + 3y^2 - 6x = 0$
$6y(x-1) = 0$
$y = 0, x = 1$
1/2

$y=0 \Rightarrow \quad 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x=0, \; x=2$

$x=1 \Rightarrow \quad 3 + 3y^2 - 6 = 0$

$3y^2 - 3 = 0$

$3y^2 = 3$

$y^2 = 1 \quad y = \pm 1.$

PTs are. $(0,0), (2,0), (1,1), (1,-1)$ — 1/2

St. pts	r	t	s	$rt - s^2$	
$(0,0)$	$-6<0$	-6	0	$36>0$	Max.
$(2,0)$	6	6	0	$36>0$	Min.
$(1,1)$	0	0	6	$-36<0$	Saddle pt
$(1,-1)$	0	0	-6	$-36<0$	"

— (01

Max. value $= f(0,0) = 4$ — 1/2

Min value $f(2,0) = 8 - 12 + 4$

$= 0$ — 1/2

Set B - M2

Q(1) (a) (i) $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$

$= e^0 \cdot \lim_{x\to 0} \frac{\sin x}{x}$ — 1/2

$= 1$. — 1/2

(ii) $u = xy$

$u_x = y$ $u_y = x$

$u_{xx} = 0$ $u_{yy} = 0$ — 1/2

$u_{xx} + u_{yy} = 0$ — 1/2

Harmonic

Q (2) $\int_0^\infty x^2 e^{-x^4} dx \int_0^\infty e^{-x^4} dx$. $x^4 = t$

$x = t^{1/4}$

$dx = \frac{1}{4} t^{-3/4} dt$ — 1/2

$= \frac{1}{4}\int_0^\infty t^{1/2} e^{-t} t^{-3/4} dt \cdot \int_0^\infty e^{-t} \frac{1}{4} t^{-3/4} dt$. — 1/2

$= \frac{1}{16} \int_0^\infty t^{-1/4} e^{-t} dt \int_0^\infty e^{-t} t^{-3/4} dt$.

$= \frac{1}{16} \cdot \Gamma\left(-\frac{1}{4}+1\right) \cdot \Gamma\left(-\frac{3}{4}+1\right)$ — 1/2 $= \frac{1}{16} \Gamma\frac{3}{4} \Gamma\frac{1}{4}$ — 1/2

$= \frac{1}{16} \cdot \Gamma\left(1-\frac{1}{4}\right) \Gamma\frac{1}{4}$

$= \frac{1}{16} \cdot \frac{\pi}{\sin\frac{\pi}{4}} = \frac{1}{16} \cdot \frac{\pi}{\frac{1}{\sqrt{2}}}$ — 1/2

$= \frac{\sqrt{2}\pi}{16}$ — 1/2

Q 3) $\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2+2x+3y & 3x+2y+z & y+2xz \end{vmatrix}$

$= \hat{i}(1-1) - \hat{j}(2z-2z) + \hat{k}(3-3)$

$= \underline{\underline{0}}$ — (01)

$\therefore$ Irrotational

$\therefore \vec{F} = \nabla\phi$

$= \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\frac{\partial\phi}{\partial x} = z^2+2x+3y \Rightarrow \phi = xz^2+x^2+3xy$ — 1/2

$\frac{\partial\phi}{\partial y} = 3x+2y+z \Rightarrow \phi = 3xy+y^2+yz$ — 1/2

$\frac{\partial\phi}{\partial z} = y+2xz \Rightarrow \phi = yz+xz^2$ — 1/2

$\therefore \phi = \underline{\underline{xz^2+x^2+3xy+y^2+yz+c}}$ — 1/2

Q4) $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$

$$\frac{\partial f}{\partial x} = 3x^2 + 3y^2 - 30x + 72$$

$$\frac{\partial f}{\partial y} = 6xy - 30y.$$

$$r = \frac{\partial^2 f}{\partial x^2} = 6x - 30$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = 6y$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6x - 30.$$

(01/2)

$\frac{\partial f}{\partial x} = 0$ $\Rightarrow$ St. pts

$\frac{\partial f}{\partial y} = 0$

$3x^2 + 3y^2 - 30x + 72 = 0$

$6y(x-5) = 0 \Rightarrow y = 0, \; x = 5$ — 1/2

$y = 0 \Rightarrow 3x^2 - 30x + 72 = 0 \Rightarrow x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$

$x = 4, 6.$

$x = 5 \Rightarrow 75 + 3y^2 - 150 + 72 = 0$

$3y^2 - 3 = 0 \Rightarrow y^2 = 1$

$y = \pm 1$

Pts are. $(4,0), (6,0), (5,1), (5,-1)$ — 1/2

Pts	r	s	t	$rt - s^2$	Conclusion.
(4,0)	-6	0	-6	$36 > 0$	Max
(6,0)	6	0	6	$36 > 0$	Min
(5,1)	0	6	0	$-36 < 0$	Saddle pts.
(5,-1)	0	-6	0	$-36 < 0$	Saddle pts.

— (01)

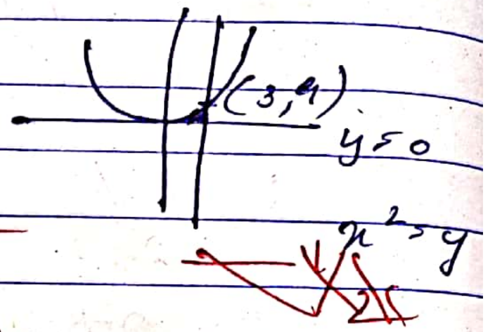
Max value $= f(4,0) = 64 - 240 + 288$

$= 112$ — $\frac{1}{2}$

Min value $= f(6,0) = 216 - 540 + 432$

$= 108$ — $\frac{1}{2}$

Q (1) (b)



$$V = \pi \int_a^b (y-p)^2\, dx$$ — $\frac{1}{2}$

$$= \pi \int_{x=0}^{3} y^2\, dx$$ — $\frac{1}{2}$

$$= \pi \int_0^3 x^4\, dx$$ — $\frac{1}{2}$

$$= \pi \left.\frac{x^5}{5}\right]_0^3 = \frac{243\pi}{5}$$ — $\frac{1}{2}$