Introduction to Deep Reinforcement Learning
From Theory to Applications

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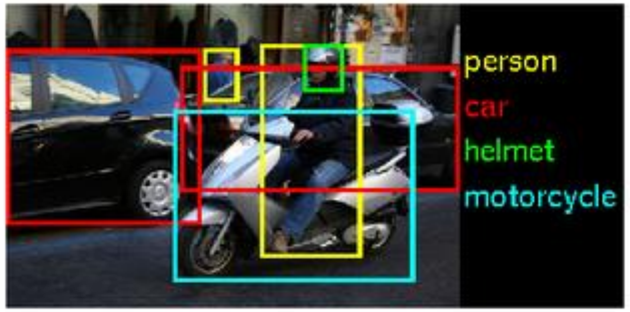
Background

- Deep learning methods are making major advances in solving many low-level perceptual tasks.



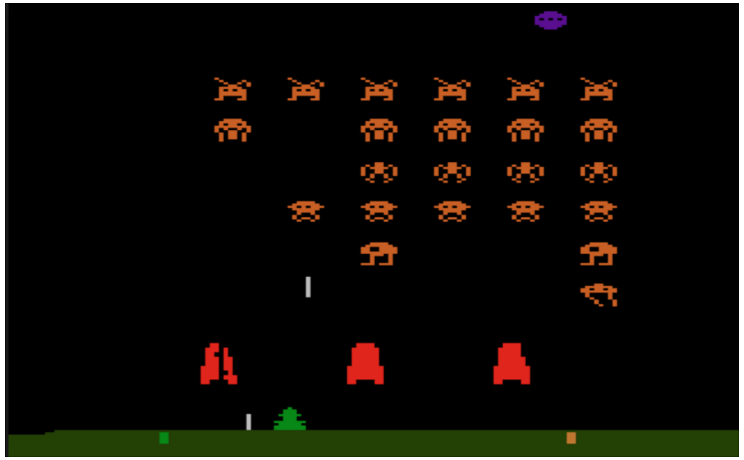
See (visual object recognition)

Read (text understanding)

Hear (speech recognition)

Background

- More sophisticated tasks that involve decision and planning require a higher level of intelligence.
- Real Artificial Intelligence system also requires the ability of reasoning, thinking and planning.



Playing Atari game

Robotic navigation

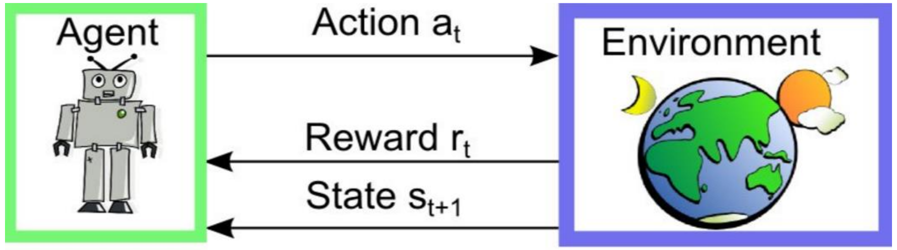
Limitations of Deep Learning

- Supervised learning assumptions
  - Training and testing instances are i.i.d(independent and identically distributed) random variables
  - Training data are labeled data with strong supervision

- Reality of most real-world tasks
  - Strong supervision is expensive and scarce
  - Sequential interactive process violates the i.i.d assumption

Reinforcement Learning (RL) in a nutshell

- RL is a general-purpose framework for decision making
  - RL is for an agent to act with an environment
  - Each action influences the agent's future state
  - Feedback is given by a scalar reward signal
  - Goal: select actions to maximize future reward



Deep RL: Deep Learning + RL

- Traditional RL approaches have been limited to domains with low-dimensional state spaces or handcrafted features
- By combining deep learning and RL, we want to embrace both the representation power of deep learning and generalization ability from RL
  - RL defines the objective
  - Deep Learning learns the representation

Outline

- Introduction to Deep Learning
- Introduction to Reinforcement Learning (RL)
- Value-Based Deep RL
- Policy-Based Deep RL
- Other Deep RL Extensions
- Deep RL Applications

Deep Representations

- A deep representation is a composition of many functions, where each composition level is learning feature representation at different level of abstraction

$$\begin{tikzcd}
x \arrow[r] & h_1 \arrow[r] & \ldots \arrow[r] & h_n \arrow[r] & y \arrow[r] & l \\
& \mathbf{w_1} \arrow[u] & \ldots & \mathbf{w_n} \arrow[u] & &
\end{tikzcd}$$

- The weights are learned using backpropagation by chain rule

$$\begin{tikzcd}
\frac{\partial l}{\partial x} & \frac{\partial l}{\partial h_1} \arrow[l, "\frac{\partial h_1}{\partial x}"'] \arrow[d, "\frac{\partial h_1}{\partial w_1}"'] & \ldots \arrow[l, "\frac{\partial h_2}{\partial h_1}"'] & \frac{\partial l}{\partial h_n} \arrow[l, "\frac{\partial h_n}{\partial h_{n-1}}"'] \arrow[d, "\frac{\partial h_n}{\partial w_n}"'] & \frac{\partial l}{\partial y} \arrow[l, "\frac{\partial y}{\partial h_n}"'] \\
& \frac{\partial l}{\partial \mathbf{w_1}} & \ldots & \frac{\partial l}{\partial \mathbf{w_n}} &
\end{tikzcd}$$

Deep Neural Network

- A deep neural network typically consists of:
  - Linear transformations

$$h_{k+1} = Wh_k$$

  - Nonlinear activation functions

$$h_{k+1} = \sigma(h_k)$$

$$\sigma(\cdot) = \tanh(\cdot), \frac{1}{1+\exp(\cdot)}, \cdots$$

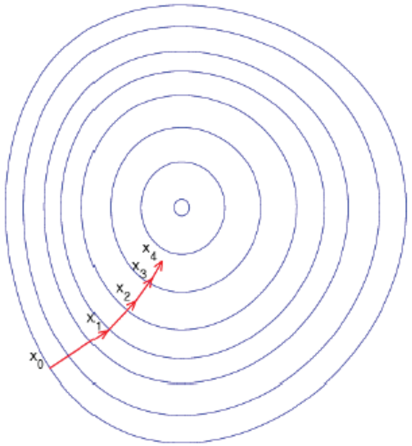
  - Loss function on the output
    - Mean squared error: $l = ||y - y^*||^2$
    - Log likelihood: $l = \log P(y^*)$

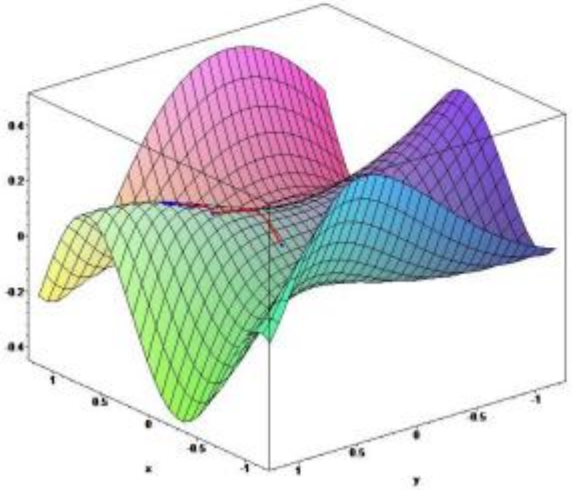
Training by Stochastic Gradient Descent

- Sample gradient of expected loss $L(\mathbf{w}) = \mathbb{E}[l]$ (better efficiency for large data)

$$\frac{\partial l}{\partial \mathbf{w}} \sim \mathbb{E}\left[\frac{\partial l}{\partial \mathbf{w}}\right] = \frac{\partial L(\mathbf{w})}{\partial \mathbf{w}}$$

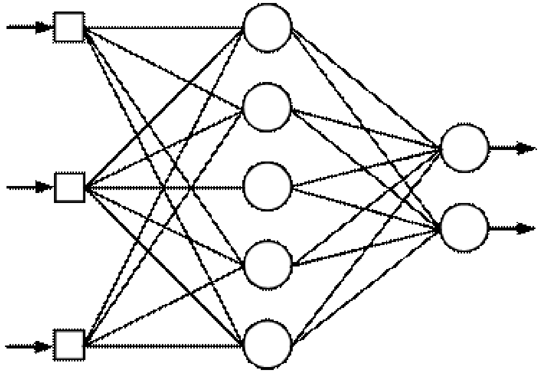
- Adjust $\mathbf{w}$ down the sampled gradient



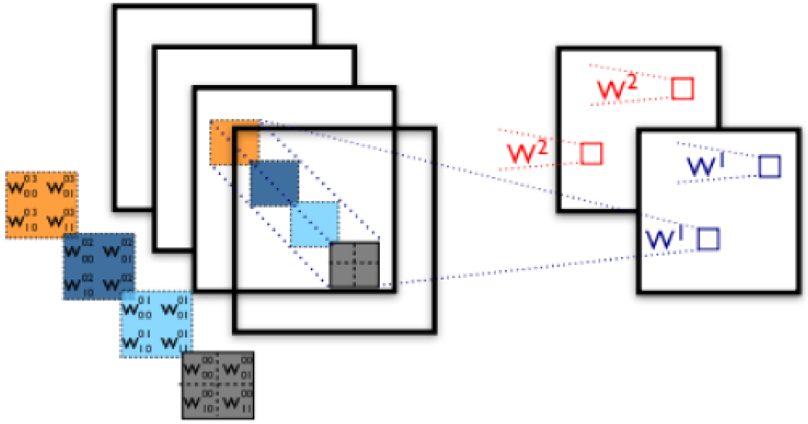


Deep Learning Models

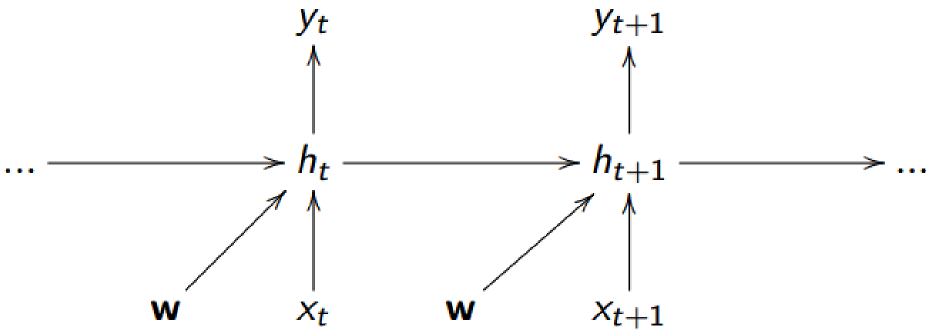
- Multilayer perceptrons (MLPs)
  - Fully-connected



- Convolutional neural networks (CNNs)
  - Weight sharing between local regions



- Recurrent neural networks (RNNs)
  - Weight sharing between time-steps



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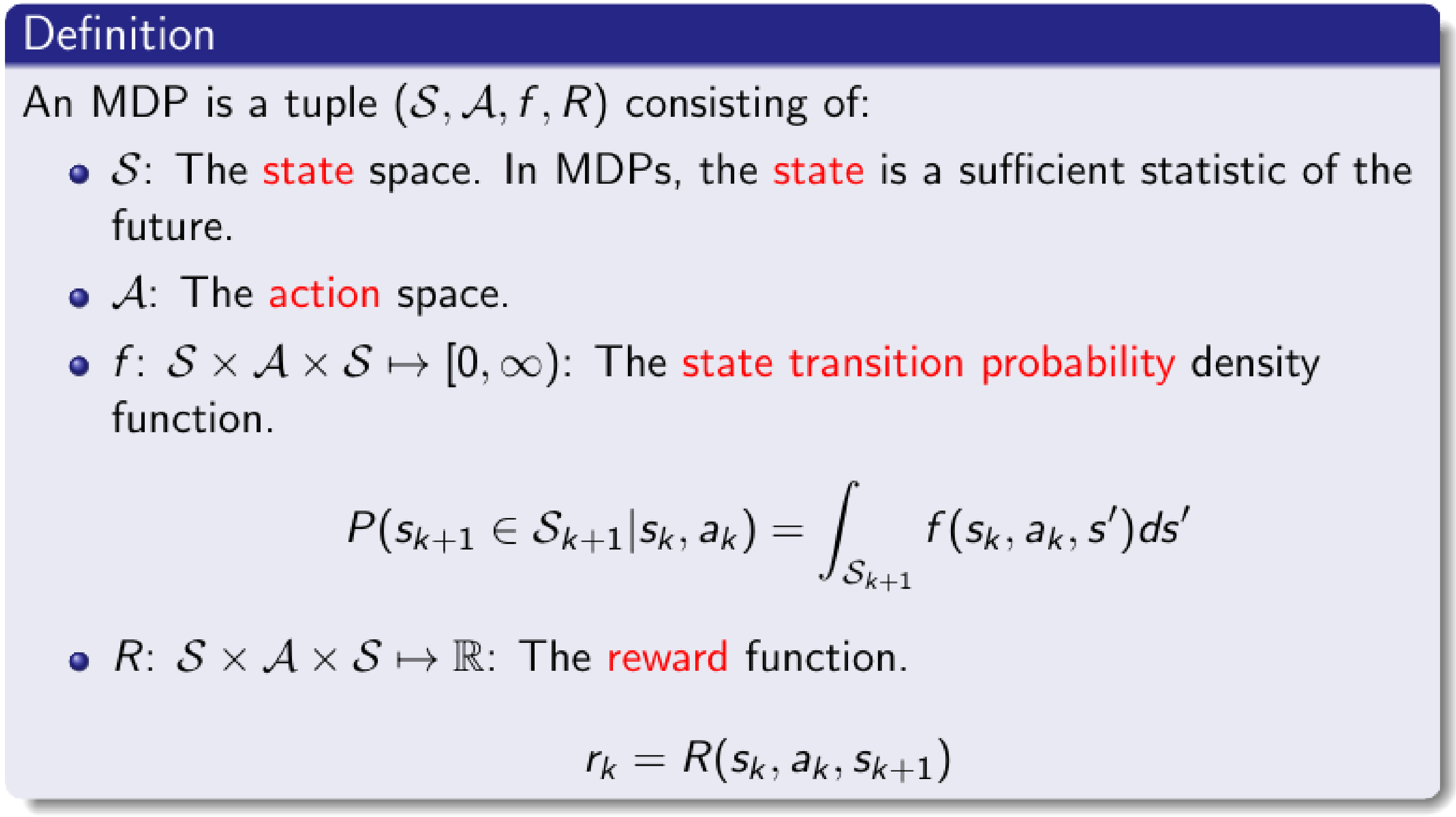
Markov Decision Processes (MDPs)

- MDPs formally describe an environment for RL, where the environment is fully observable



Definition

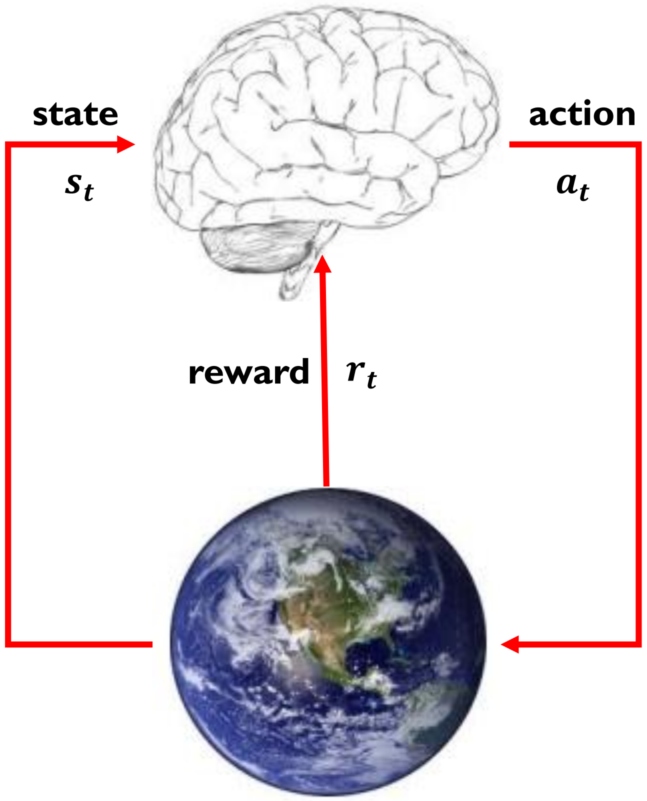
An MDP is a tuple $(\mathcal{S}, \mathcal{A}, f, R)$ consisting of:

- $\mathcal{S}$: The state space. In MDPs, the state is a sufficient statistic of the future.
- $\mathcal{A}$: The action space.
- $f \colon \mathcal{S} \times \mathcal{A} \times \mathcal{S} \mapsto [0, \infty)$: The state transition probability density function.

$$P(s_{k+1} \in \mathcal{S}_{k+1} | s_k, a_k) = \int_{\mathcal{S}_{k+1}} f(s_k, a_k, s')ds'$$

- $R \colon \mathcal{S} \times \mathcal{A} \times \mathcal{S} \mapsto \mathbb{R}$: The reward function.

$$r_k = R(s_k, a_k, s_{k+1})$$



Policy

A policy is the behavior of the agent.

- Stochastic policy π: $\mathcal{S} \times \mathcal{A} \mapsto [0, \infty)$.

$$P(a|s) = \pi(s, a)$$

- Deterministic policy π: $\mathcal{S} \mapsto \mathcal{A}$.

$$a = \pi(s)$$

Expected Return

- The goal of RL is to find the policy which maximizes the expected return

$$J(\pi) = \mathbb{E}\{g(r_0, r_1, \ldots)|\pi\}.$$

- In most cases, the function g is either the discounted sum of rewards or the average reward
- Discounted reward

$$g(r_0, r_1, \ldots) = \sum_{k=0}^{\infty} \gamma^k r_k$$

- Average reward

$$g(r_0, r_1, \ldots) = \lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} r_k$$

Value Function

- Consider the discounted reward case

$$J(\pi) = \mathbb{E}\left\{ \sum_{k=0}^{\infty} \gamma^k r_k \middle| d_0, \pi \right\}$$

$$= \int_{\mathcal{S}} d^\pi(s) \int_{\mathcal{A}} \pi(s,a) \int_{\mathcal{S}} f(s,a,s') R(s,a,s') ds' da ds$$

$$d^\pi(s) = \sum_{k=0}^{\infty} \gamma^k p(s_k = s | d_0, \pi)$$

- A value function is the prediction of the above expected return
- Two definitions exist for the value function
  - State value function
  - State-action value function

$$V^\pi(s) = \mathbb{E}\left\{ \sum_{k=0}^{\infty} \gamma^k r_k \middle| s_0 = s, \pi \right\}$$

$$\Updownarrow \quad V^\pi(s) = \mathbb{E}\left\{ Q^\pi(s,a) | a \sim \pi(s, \cdot) \right\}$$

$$Q^\pi(s,a) = \mathbb{E}\left\{ \sum_{k=0}^{\infty} \gamma^k r_k \middle| s_0 = s, a_0 = a, \pi \right\}$$

Bellman Equation and Optimality

- Value functions decompose into Bellman equations, i.e., the value functions can be decomposed into immediate reward plus discounted value of successor state

$$V^{\pi}(s) = \mathbb{E}\left\{R(s, a, s') + \gamma V^{\pi}(s')\right\}$$

$$Q^{\pi}(s, a) = \mathbb{E}\left\{R(s, a, s') + \gamma Q^{\pi}(s', a')\right\}$$

- An optimal value function is the maximum achievable value.

$$V^*(s) = \max_{\pi} V^{\pi}(s) \quad Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a)$$

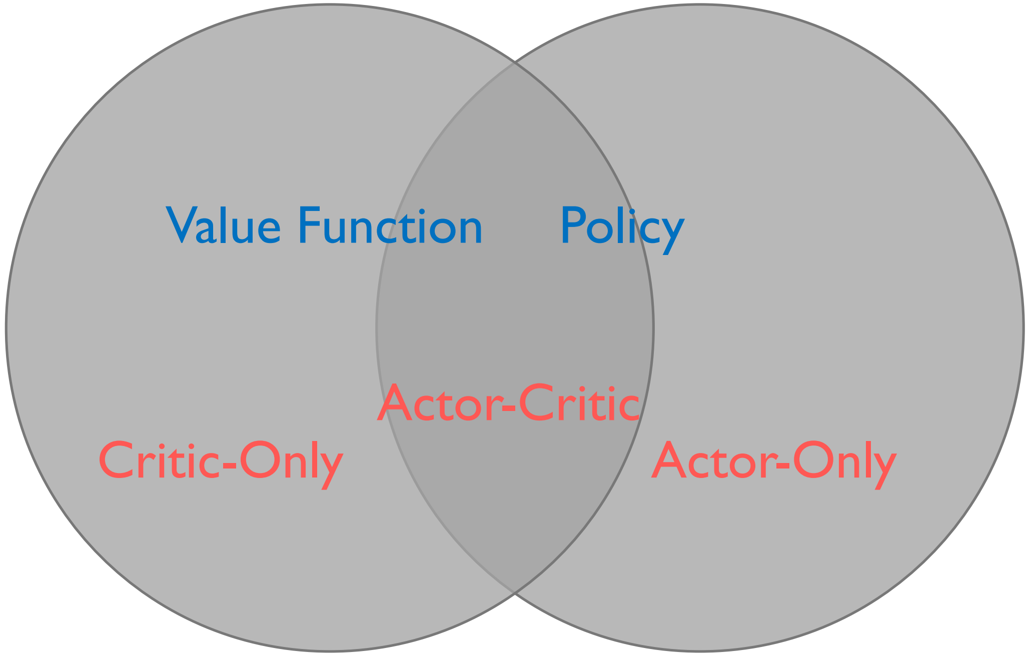
- Optimality for value functions is governed by the Bellman optimality equations.

$$V^*(s) = \max_{a} \mathbb{E}\left\{R(s, a, s') + \gamma V^*(s')\right\}$$

$$Q^*(s, a) = \mathbb{E}\left\{R(s, a, s') + \max_{a'} \gamma Q^*(s', a')\right\}$$

Approaches to RL

- Critic-Only Methods
    - Learn value function
    - Implicit policy
- Actor-Only Methods
    - No value function
    - Learnt policy
- Actor-Critic Methods
    - Learn value function
    - Learn policy



Actor and *critic* are synonyms for the *policy* and *value* function.

Critic-Only

- A value function defines an optimal policy.
- In critic-only methods, policy can be derived by selecting *greedy actions*

$$\pi^*(s) = \arg\max_a \mathbb{E}\left\{R(s,a,s') + \gamma V^*(s')\right\}$$

$$\pi^*(s) = \arg\max_a Q^*(s,a).$$

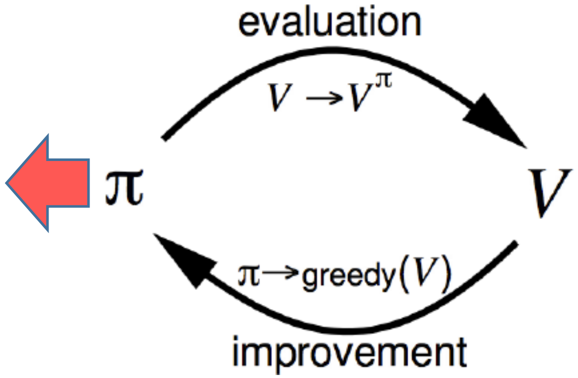
- Finding optimal policy from state-action value function is direct.
- Finding optimal policy from state value function is more complicated (requires the knowledge of transition model)

Critic-Only

- Dynamic programming-based methods
  - Policy iteration
  - Value iteration
- Monte Carlo (MC) methods
- Temporal difference (TD) learning methods
  - TD(λ)
  - Q-learning
  - SARSA

Dynamic Programming (DP)

- DP methods require a model of the state transition density function f and the reward function R to calculate the state value function
- DP is *model-based*
- Policy evaluation: updates the value function for the current policy
- Policy improvement: improve the policy by acting according to the current value function
- Typical methods:
    - Policy iteration
        - Alternates between policy evaluation and policy improvement
    - Value iteration
        - No explicit policy
        - Directly update value function



Monte Carlo Methods

- MC methods learn directly from episodes of experiences
- MC is *model-free*: no knowledge of transitions/rewards
- MC learns from complete episodes.
  - All episodes must terminate
- MC uses simple idea: empirical mean return
- The estimates are unbiased but have high variance.

Temporal Difference (TD) Learning

- TD methods learn directly from episodes of experiences
- TD is *model-free*: no knowledge of transitions/rewards
- TD learns from incomplete episodes, it can learn online after every step
- TD uses temporal errors to update value function

$$V'(s) = V(s) + \alpha\left(R(s,a,s') + \gamma V(s') - V(s)\right)$$

$$Q'(s,a) = Q(s,a) + \alpha\left(R(s,a,s') + \gamma Q(s',a') - Q(s,a)\right) \quad \Leftarrow \quad \text{SARSA}$$

$$Q'(s,a) = Q(s,a) + \alpha\left(R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)\right) \quad \Leftarrow \quad \text{Q-learning}$$

- The estimates are biased but have low variance.

Actor-Only

- Critic-only methods do not scale well to high-dimensional or continuous action spaces, since selecting greedy actions is computationally intensive.
- Actor-only methods work with a parameterized family of policies over which optimization procedures can be used directly.
- Advantages
  - Better convergence properties
  - Effective in high-dimensional or continuous action spaces
  - Can learn stochastic policies
- Disadvantages
  - Evaluating a policy is inefficient and of high variance

Policy Gradient

- Given policy π_θ with parameters θ, the goal is find best θ to maximize the expected return
- Can use gradient descent

$$\theta_{k+1} = \theta_k + \alpha_{a,k} \nabla_\theta J(\theta_k)$$

- Policy gradient

$$\nabla_\theta J(\theta) = \frac{\partial J}{\partial \pi_\theta} \frac{\partial \pi_\theta}{\partial \theta}$$

- How to estimate the policy gradient?
  - Finite-difference methods
  - Likelihood ratio methods

Finite-Difference Methods

- Idea is simple, i.e., to vary the policy parameterization by small increments and evaluate the cost by rollouts
- Estimate k-th partial derivative of object function

$$\frac{\partial J(\theta)}{\partial \theta_k} \approx \frac{J(\theta + \epsilon u_k) - J(\theta)}{\epsilon}$$

- Simple, works even if policy is not differentiable
- Noisy, inefficient

Likelihood Ratio Methods (REINFORCE)

- We can formulate the expected return from the view of trajectories generated by rollouts

$$\tau \sim p(\tau|\theta) \quad J^{\tau} = \sum_{k=0}^{H} \gamma^k r_k \quad J(\theta) = \int_{\mathbb{T}} p(\tau|\theta) J^{\tau} d\tau$$

- Use likelihood ratios to compute the policy gradient

$$\begin{aligned}\nabla_\theta J(\theta) &= \int_{\mathbb{T}} \nabla_\theta p(\tau|\theta) J^{\tau} d\tau \\ &= \int_{\mathbb{T}} p(\tau|\theta) \nabla_\theta \log p(\tau|\theta) J^{\tau} d\tau \\ &= \mathbb{E}\left\{\underbrace{\nabla_\theta \log p(\tau|\theta)}\, J^{\tau}\right\}.\end{aligned}$$

Do not need to compute the system dynamics

$$\Leftarrow \quad \nabla_\theta \log p(\tau|\theta) = \sum_{k=0}^{H} \nabla_\theta \log \pi_\theta(s_k, a_k)$$

$$\nabla_\theta J(\theta) = \mathbb{E}\left\{\left(\sum_{k=0}^{H} \nabla_\theta \log \pi_\theta(s_k, a_k)\right) J^{\tau}\right\}$$

Likelihood Ratio Methods (REINFORCE)

- The above computation of policy gradient can be further reduced by replacing the trajectory return by the state-action value function

$$\nabla_\theta J(\theta) = \mathbb{E}\left\{\sum_{k=0}^{H} \nabla \log \pi_\theta(s_k, a_k) Q^\pi(s_k, a_k)\right\}$$

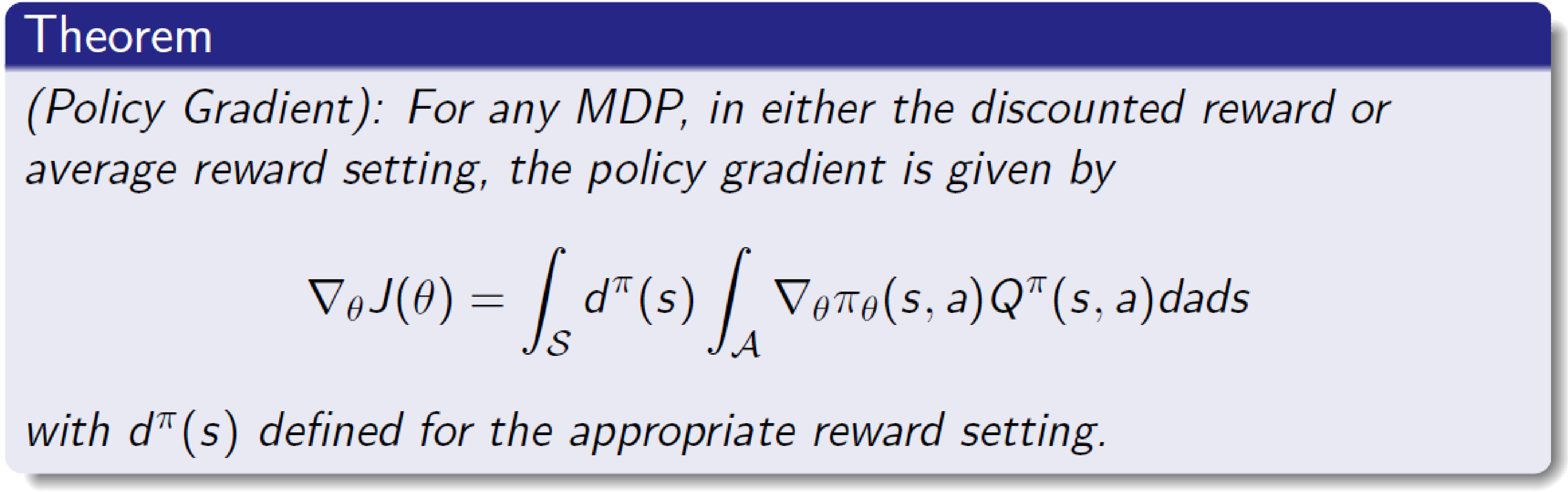
- The trajectory return or the state-action value can be estimated by the return v_t obtained from Monte Carlo rollouts
- Thus the estimated policy gradient may have large variance
- In practice, subtracting a baseline from the trajectory return or the state-action value helps a lot

Actor-Critic

- Critic-only
  - Pros: low variance
  - Cons: difficult for continuous action domains
- Actor-only
  - Pros: easy to handle continuous actions
  - Cons: high variance
- Actor-critic combines the advantages of actor-only and critic-only methods
  - Actions are generated by the parameterized actor
  - The critic supplies the actor with low variance gradient estimates

Policy Gradient Theorem

- Actor-critic methods rely on the following policy gradient theorem



Theorem

(Policy Gradient): For any MDP, in either the discounted reward or average reward setting, the policy gradient is given by

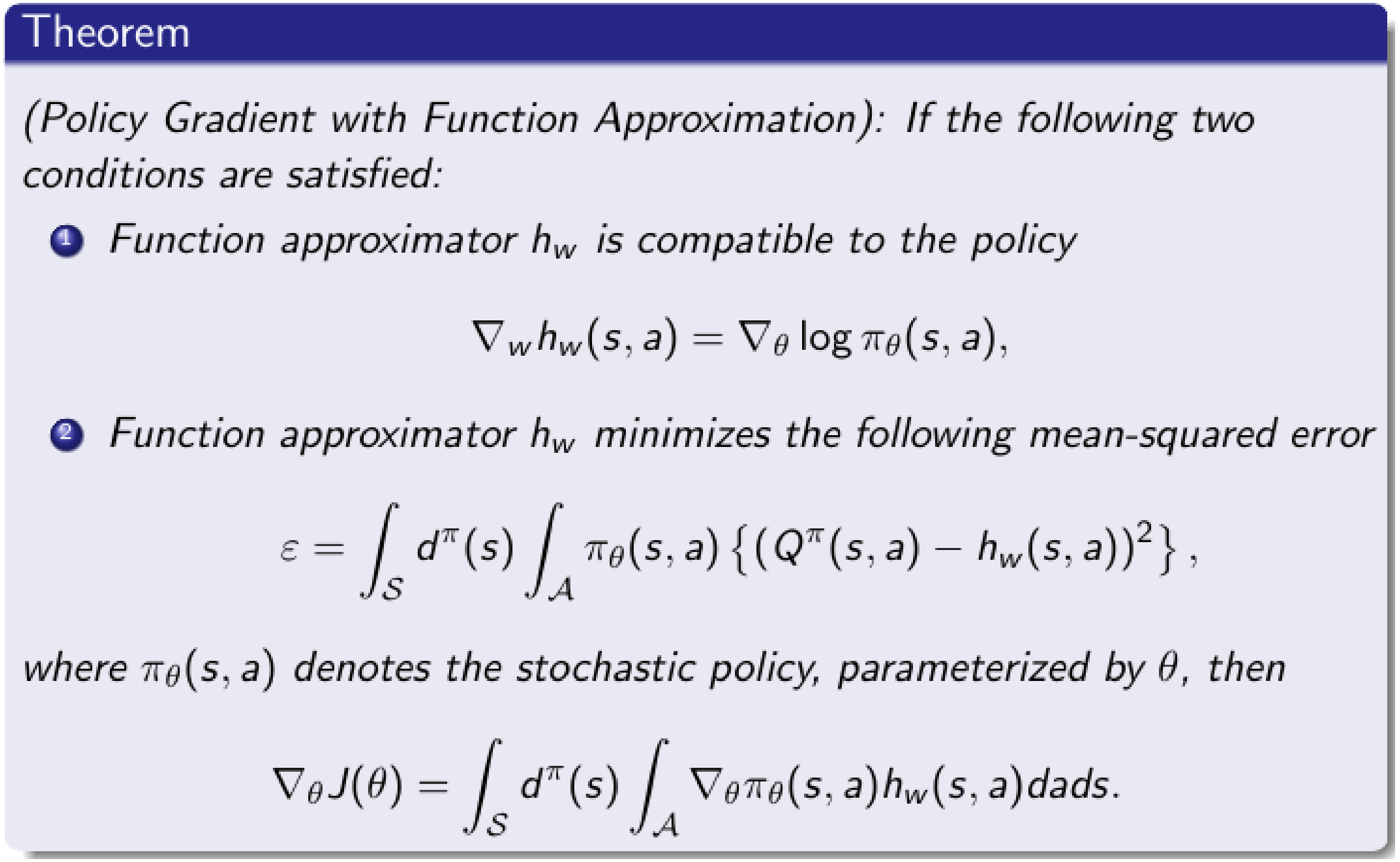
$$\nabla_\theta J(\theta) = \int_S d^\pi(s) \int_{\mathcal{A}} \nabla_\theta \pi_\theta(s, a) Q^\pi(s, a) dads$$

with $d^\pi(s)$ defined for the appropriate reward setting.

- The above theorem shows the relationship between the policy gradient and the exact critic function.

Policy Gradient With Function Approximation

- The following theorem shows that the state-action value function can be approximated with a certain function, without affecting the unbiasedness of the policy gradient estimate



Theorem

(Policy Gradient with Function Approximation): If the following two conditions are satisfied:

1. *Function approximator h_w is compatible to the policy*

$$\nabla_w h_w(s,a) = \nabla_\theta \log \pi_\theta(s,a),$$

2. *Function approximator h_w minimizes the following mean-squared error*

$$\varepsilon = \int_{\mathcal{S}} d^\pi(s) \int_{\mathcal{A}} \pi_\theta(s,a) \left\{ (Q^\pi(s,a) - h_w(s,a))^2 \right\},$$

where $\pi_\theta(s,a)$ denotes the stochastic policy, parameterized by θ, then

$$\nabla_\theta J(\theta) = \int_{\mathcal{S}} d^\pi(s) \int_{\mathcal{A}} \nabla_\theta \pi_\theta(s,a) h_w(s,a) dads.$$

Reducing Variance Using a Baseline

- The policy gradient theorem generalizes to the case where a state-dependent baseline function is taken into account. This can reduce variance, without changing expectation

$$\begin{aligned}\mathbb{E}_{\pi_\theta}\left\{\nabla_\theta \pi_\theta(s,a) b(s)\right\} &= \int_{\mathcal{S}} d^{\pi}(s) \int_{\mathcal{A}} \nabla_\theta \pi_\theta(s,a) b(s) da ds \\ &= \int_{\mathcal{S}} d^{\pi}(s) b(s) \nabla_\theta \int_{\mathcal{A}} \pi_\theta(s,a) da ds = 0\end{aligned}$$

$$\nabla_\theta J(\theta) = \int_{\mathcal{S}} d^{\pi}(s) \int_{\mathcal{A}} \nabla_\theta \pi_\theta(s,a) \left[h_w(s,a) - b(s)\right] da ds$$

- A good baseline is the state value function
- The policy gradient can be formulated by both the Q function and the advantage function

$$A^{\pi_\theta}(s,a) = Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ A^{\pi_\theta}(s,a)\right]$$

Standard Actor-Critic Algorithms

- If both conditions in the above theorem are met, then the resulting algorithm is equivalent to the REINFORCE algorithm
- Practical actor-critic algorithms often relax the second condition: use TD learning to update the critic approximator.
- TD(0) actor-critic

$$\begin{aligned}\delta_k &= r_k + \gamma V_{w_k}(s_{k+1}) - V_{w_k}(s_k)\\ w_{k+1} &= w_k + \alpha_{c,k}\delta_k \nabla_w V_{w_k}(s_k)\\ \theta_{k+1} &= \theta_k + \alpha_{a,k}\delta_k \nabla_\theta \log \pi_\theta(s_k, a_k)\end{aligned}$$

- The TD error is actually an estimate of the advantage function

Practical Actor-Critic Variants

- The policy gradient has many equivalent forms

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ v_t\right] && \text{REINFORCE} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ Q^w(s,a)\right] && \text{Q Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ A^w(s,a)\right] && \text{Advantage Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ \delta\right] && \text{TD Actor-Critic}
\end{aligned}$$

Natural Policy Gradient

- The vanilla gradient is sensitive to policy parameterizations
- The natural policy gradient is parameterization independent
- It finds ascent direction that is closest to vanilla gradient, when changing policy by a small, fixed amount

$$\nabla_\theta^{nat} \pi_\theta(s, a) = G_\theta^{-1} \nabla_\theta \pi_\theta(s, a)$$

Fisher information matrix (FIM)

$$G_\theta = \mathbb{E}_{\pi_\theta} \left[\nabla_\theta \log \pi_\theta(s, a) \nabla_\theta \log \pi_\theta(s, a)^T \right]$$

- Natural policy gradient methods converges faster in most practical cases. However, estimating the FIM may induce large computation cost

Natural Actor-Critic

- Using compatible function approximation

$$\nabla_w A_w(s,a) = \nabla_\theta \log \pi_\theta(s,a)$$

- The natural policy gradient is then

$$\begin{aligned}\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a) A^{\pi_\theta}(s,a)\right] \\ &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a) \nabla_\theta \log \pi_\theta(s,a)^T w\right] \\ &= G_\theta w\end{aligned}$$

$$\nabla_\theta^{nat} J(\theta) = w$$

Deep Reinforcement Learning

- Use deep neural networks to represent
    - Value function
    - Policy

- Optimize loss function by stochastic gradient descent

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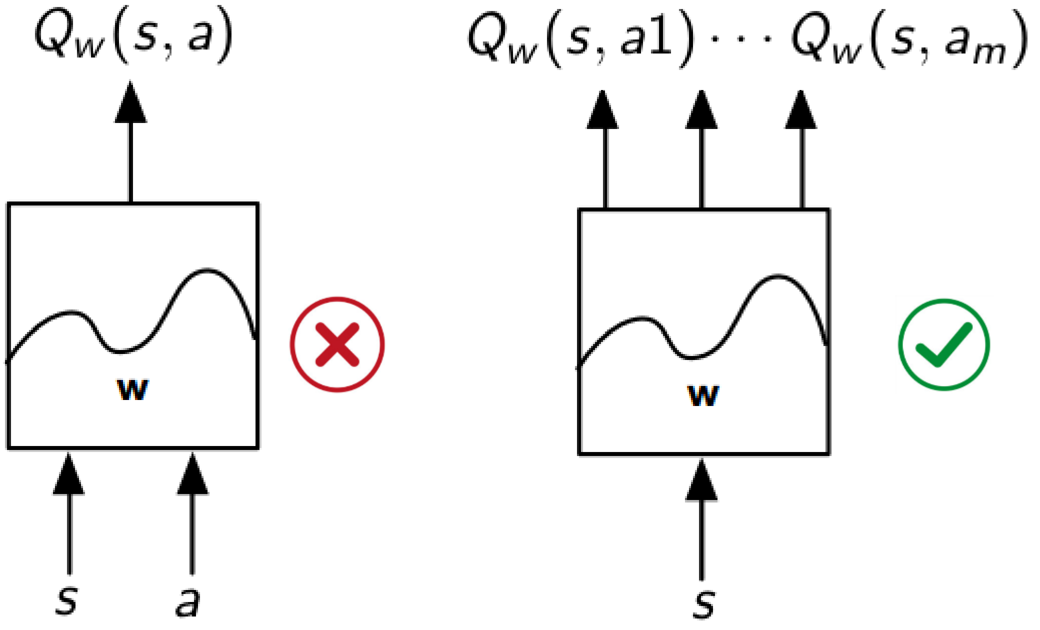
Q-Networks

- Represent the state-action value function by Q-network with weights w

$$Q_w(s,a) \approx Q^*(s,a)$$



Q-Learning

- Optimal Q-values obey Bellman equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left\{ \underbrace{r + \gamma \max_{a'} Q(s',a')} \middle| s, a \right\}$$

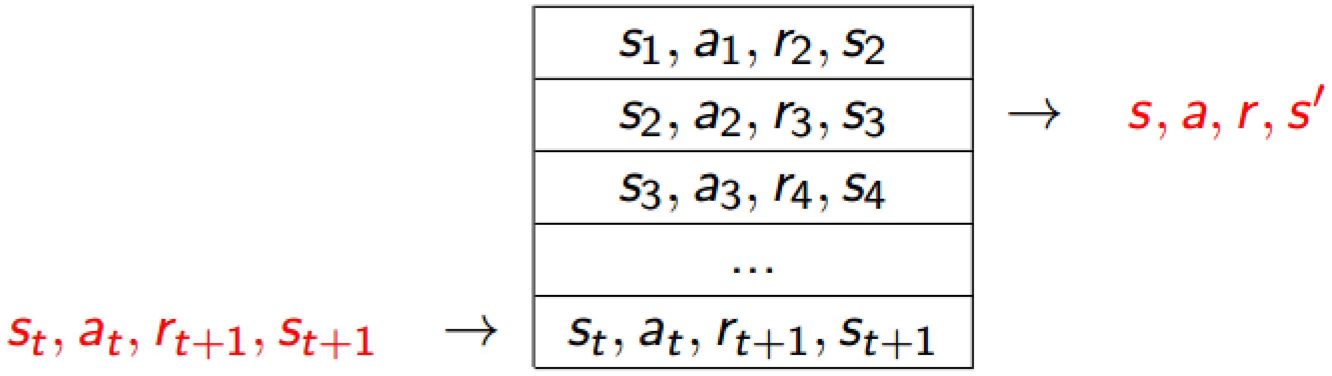
- Treat right-hand size as a target and minimize MSE loss by SGD

$$l = \left(r + \gamma \max_{a'} Q_w(s',a') - Q_w(s,a) \right)^2$$

- Convergence guarantee using table lookup representation
- But diverges using neural networks due to
  - Correlations between samples
  - Non-stationary targets

Deep Q-Networks (DQN)

- Experience replay
  - Build data set from agent’s own experience
  - Sample experiences uniformly from data set to remove correlations



$s_t, a_t, r_{t+1}, s_{t+1} \rightarrow$

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

$\rightarrow \quad s, a, r, s'$

- Target Network
  - To deal with non-stationarity, target parameters $\hat{w}$ are held fixed

$$l = \mathbb{E}_{(s,a,r,s')\sim U(D)}\left\{\left(r + \gamma \max_{a'} Q_{\hat{w}}(s', a') - Q_w(s, a)\right)^2\right\}$$

Double DQN

- Q-learning is known to overestimate state-action values
  - The max operator uses the same values to select and evaluate an action

$$Q^*(s, a) = \mathbb{E}_{s'}\left\{ r + \gamma \enclose{circle}{\max_{a'}} Q(s', a') | s, a \right\}$$

- The upward bias can be removed by decoupling the selection from the evaluation
  - Current Q-network is used to select actions
  - Older Q-network is used to evaluate actions

$$l = \mathbb{E}_{(s,a,r,s')\sim U(D)}\left\{ \left(r + \gamma \enclose{circle}{Q_{\hat{w}_i}}(s', \arg\max_{a'} \enclose{circle}{Q_{w_i}}(s', a')) - Q_{w_i}(s, a) \right)^2 \right\}$$

Prioritized Replay

- Uniform experience replay samples transitions regardless of their significance
- Can weight experiences according to their significance
- Prioritized replay stores experiences in a priority queue according to the TD error

$$\left| r + \gamma \max_{a'} Q_{\hat{w}}(s', a') - Q_w(s, a) \right|$$

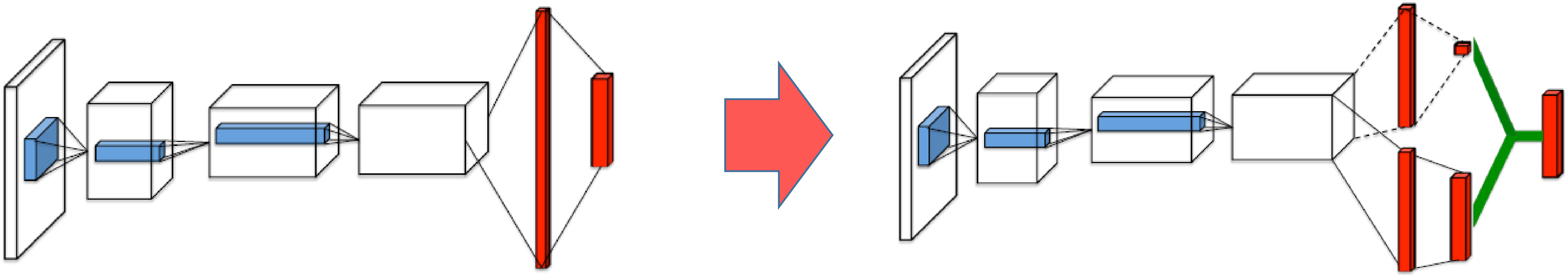
- Use stochastic sampling to increase sample diversity

$$P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$$

$$p_i = |\delta_i| + \epsilon$$

Dueling Network

- Dueling network splits Q-network into two channels
  - Action-independent value function $V(s)$
  - Action-dependent advantage function $A(s, a)$

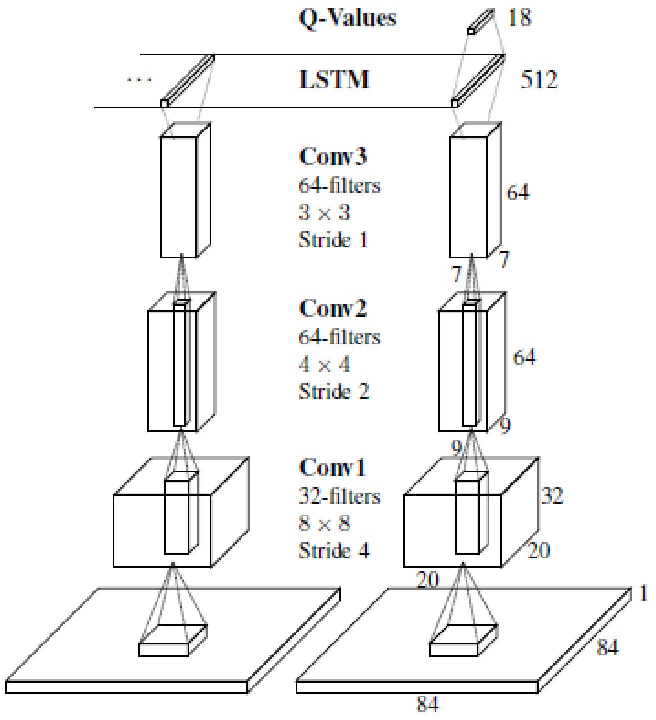


- The two stream are aggregated to get the Q function

$$Q(s, a; \beta, w_1, w_2) = V(s; \beta, w_1) + \left(A(s, a; \beta, w_2) - \max_{a'} A(s, a'; \beta, w_2) \right)$$

Deep Recurrent Q-Network (DRQN)

- DQNs learn a mapping from a limited number of past states.
- Most real world environments are Partially-Observable Markov Decision Processes (POMDPs)
- DRQN replaces DQN’s first fully connected layer with a LSTM (one variant of RNN)



Asynchronous Q-Learning Variations

- Asynchronous RL
  - Exploits multithreading of standard CPU
  - Execute many instances of agent in parallel
  - Parallelism decorrelates data
    - Thus an alternative to experience replay, which is memory inefficient
  - Network parameters shared between threads
- Asynchronous one-step Q-learning
- Asynchronous one-step SARSA

$$r + \gamma Q(s', a')$$

- Asynchronous n-step Q-learning

$$r_t + \gamma r_{t+1} + \cdots + \gamma^{n-1} r_{t+n-1} + \max_a \gamma^n Q(s_{t+n}, a)$$

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Asynchronous Advantage Actor-Critic (A3C)

- Estimate state value function by neural networks

$$V_w(s) \approx \mathbb{E}\{r_{t+1} + \gamma r_{t+2} + \cdots | s\}$$

- Q-value estimated by an n-step sample

$$q_t = r_{t+1} + \gamma r_{t+2} + \cdots + \gamma^{n-1} r_{t+n} + \gamma^n V_w(s_{t+n})$$

- Actor is updated by advantage policy gradient

$$\nabla_\theta J(\theta) = \nabla_\theta \pi_\theta(s_t, a_t)(q_t - V_w(s_t))$$

- Critic is updated by TD learning

$$l_v = (q_t - V_w(s_t))^2$$

Trust Region Policy Optimization (TRPO)

- Formulated as a trust region optimization problem, where each update of the policy is guaranteed to improve

$$\begin{aligned}&\max \ L_{\pi_{\theta_{\text{old}}}}(\pi_\theta)\\ &\text{subject to } \bar{D}_{KL}(\pi_{\theta_{\text{old}}}||\pi_\theta) \leq \delta\end{aligned}$$

➡

$$L_\pi(\tilde{\pi}) = J(\pi) + \int_{\mathcal{S}} d^\pi(s) \int_{\mathcal{A}} \tilde{\pi}(s,a) A^\pi(s,a)$$

- This provides a unifying perspective on a number of policy update schemes: standard policy gradient, natural policy gradient

$$\max_\theta \ \left[\nabla_\theta L_{\pi_{\theta_{\text{old}}}}(\pi_\theta)|_{\theta=\theta_{\text{old}}}(\theta - \theta_{\text{old}})\right]$$

⬅ First order approximation to the objective

$$\text{subject to } \frac{1}{2}(\theta_{\text{old}} - \theta)^T \mathbf{F}(\theta_{\text{old}})(\theta_{\text{old}} - \theta) \leq \delta$$

$$\mathbf{F}(\theta_{\text{old}})_{ij} = \frac{\partial}{\partial \theta_i}\frac{\partial}{\partial \theta_j}\mathbb{E}\{D_{KL}(\pi_{\theta_{\text{old}}}(s,\cdot)||\pi_\theta(s,\cdot))\}$$

Natural policy gradient

$$\text{subject to } \frac{1}{2}||\theta - \theta_{\text{old}}||^2 \leq \delta$$

Standard policy gradient

Practical TRPO Algorithm

- Use the same approximation schemes as the natural policy gradient
- TRPO enforces the constraint by line search
    - Increases stability in practice

- Use a conjugate gradient algorithm to compute the natural gradient direction
    - Makes it practical for deep neural network policies

Deep Deterministic Policy Gradient (DDPG)

- Deterministic policy gradient

$$\nabla_\theta J(\pi_\theta) = \int_{\mathcal{S}} d^\pi(s) \nabla_a Q^\pi(s,a)|_{a=\pi_\theta(s)} \nabla_\theta \pi_\theta(s) ds$$

- DDPG is the continuous analogue of DQN
  - Experience replay
  - Critic estimates value of current policy as in DQN

$$l_w = \left(r + \gamma Q_{\hat{w}}(s', \pi_{\hat{\theta}}(s')) - Q_w(s,a)\right)^2$$

  - Actor updates policy in the deterministic policy gradient direction

$$\nabla_a Q_w(s,a)|_{s=s_t, a=\pi_\theta(s_t)} \nabla_\theta \pi_\theta(s)|_{s=s_t}$$

  - The critic provides loss function for the actor

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Continuous Q-Learning

- General Q function parameterizations are difficult to find the maximum
- Specific Q function parameterizations can have analytic solution on the maximum

$$A(s, a; w^A) = -\frac{1}{2}(a - \mu(s; w^\mu))^T \mathbf{P}(s; w^P)(a - \mu(s; w^\mu))$$

$$Q(s, a; w^A, w^V) = A(s, a; w^A) + V(s; w^V)$$

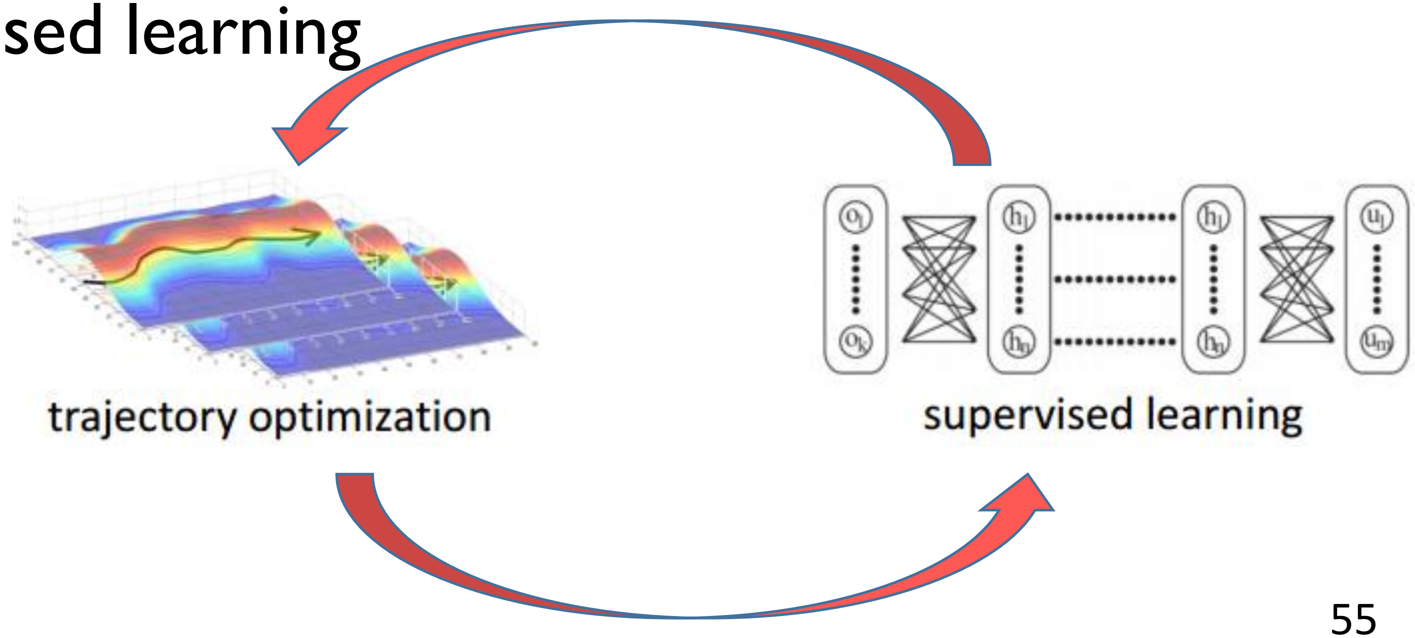
- The action that maximizes the Q function is always $\mu(s; w^\mu)$
- The parameterization can then be trained by DQN

Q-Learning with Model-Based Acceleration

- Use model-based methods to generate exploration behavior for Q-learning
  - In practice it often brings very small or no improvement
  - Off-policy model-based exploration is too different from the Q learning policy
- Imagination rollouts
  - Generate synthetic experiences under a learned model by model-based methods
  - Adding synthetic samples to replay buffer
  - Increases sample efficiency in practice

Guided Policy Search (GPS)

- GPS converts policy search into supervised learning
- Basically a model-based trajectory optimization algorithm generates training data for supervised learning where the neural network policy is trained by supervised learning
- To enforce convergence, GPS alternates between trajectory optimization and supervised learning
- GPS is data efficient



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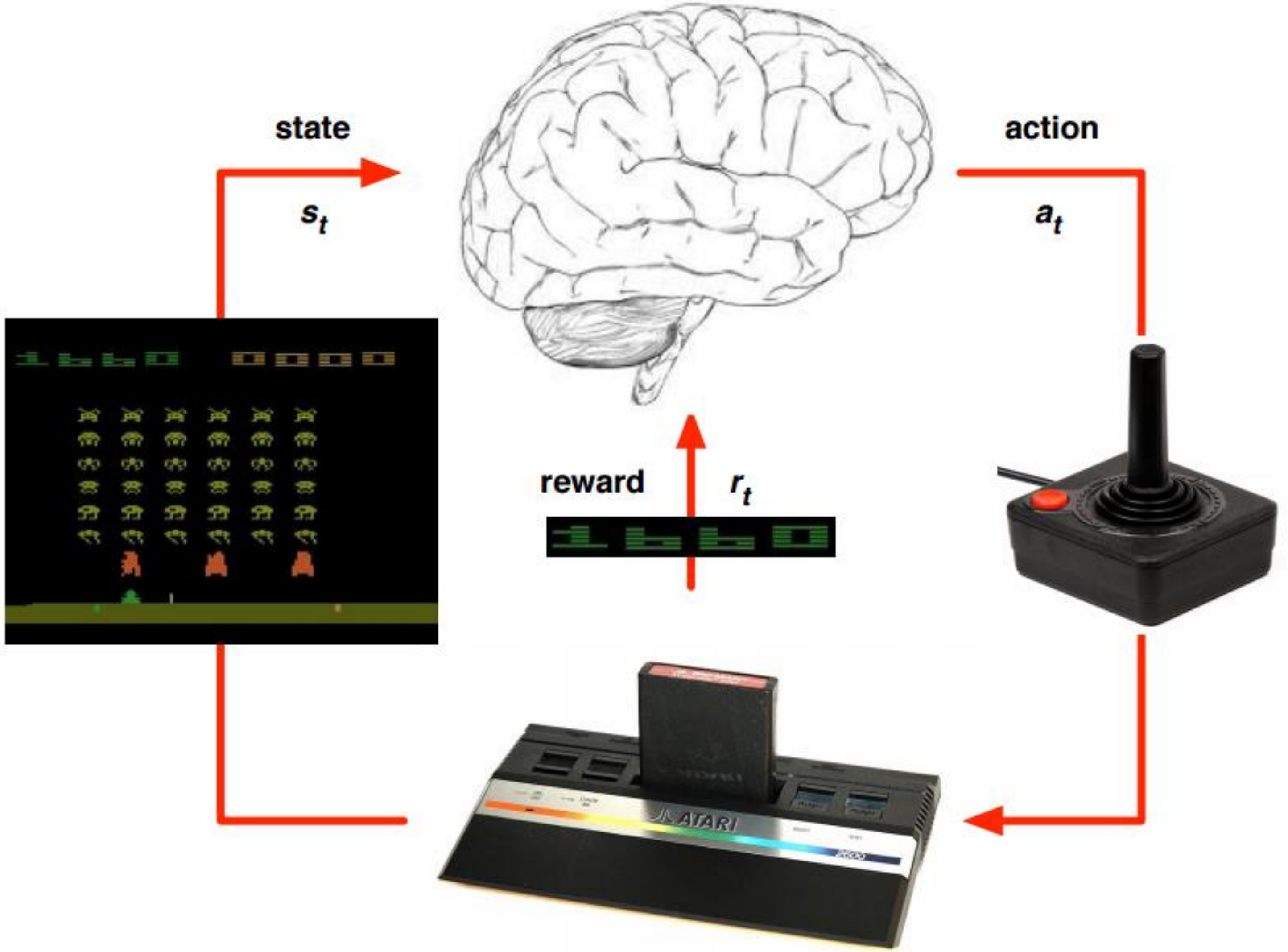
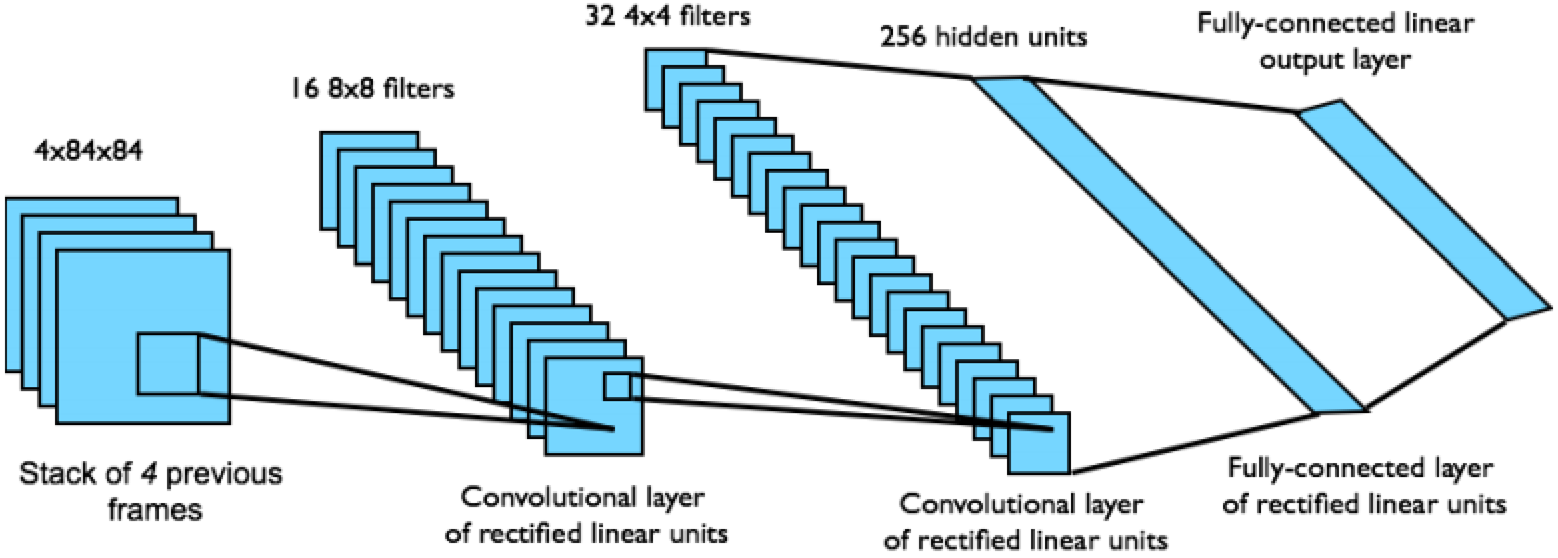
Deep RL in Atari

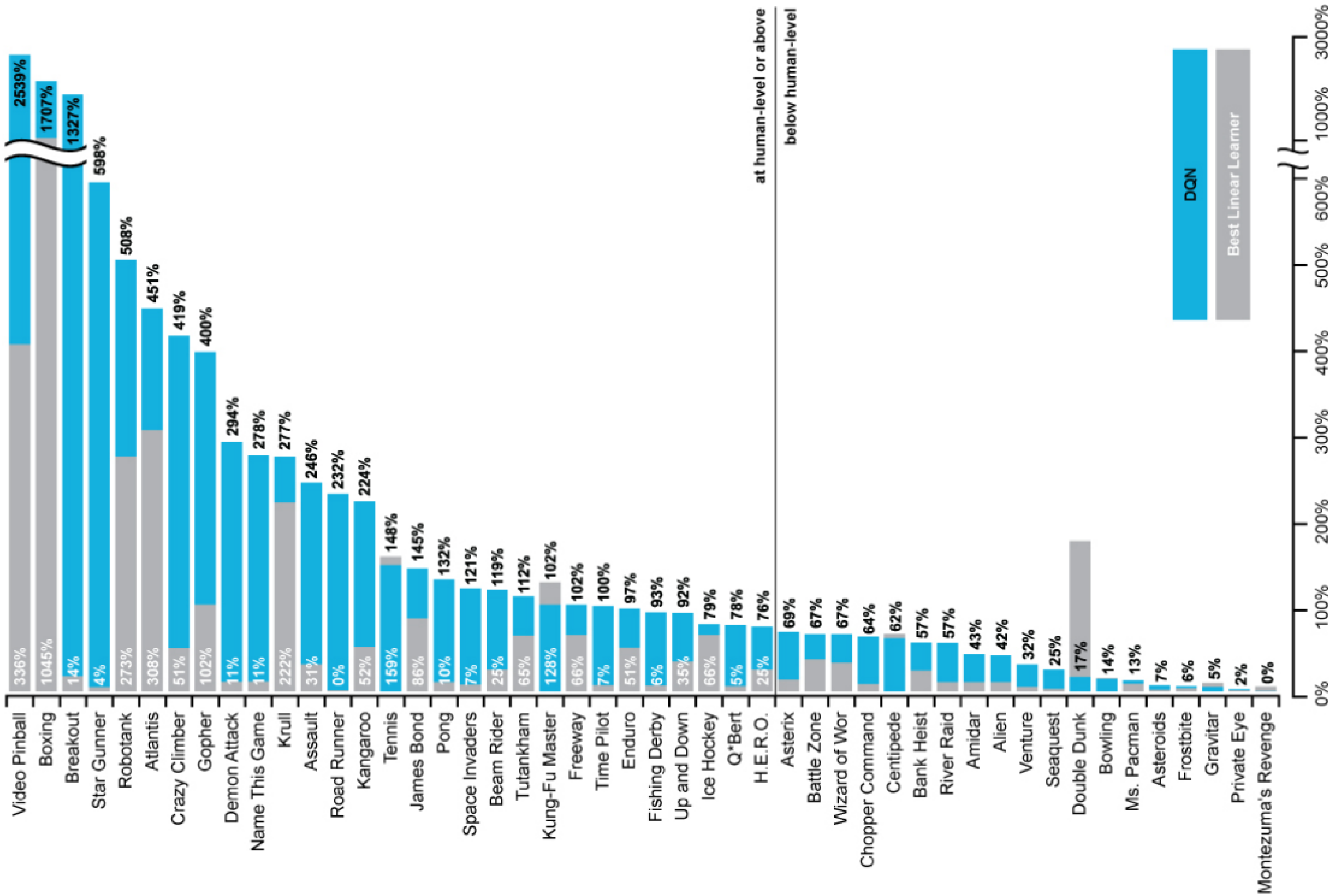


DQN in Atari

- End-to-end learning of state-action values from raw pixels
- Input state is stack of raw pixels from last 4 frames
- Output are state-action values from all possible actions
- Reward is change in score for that step

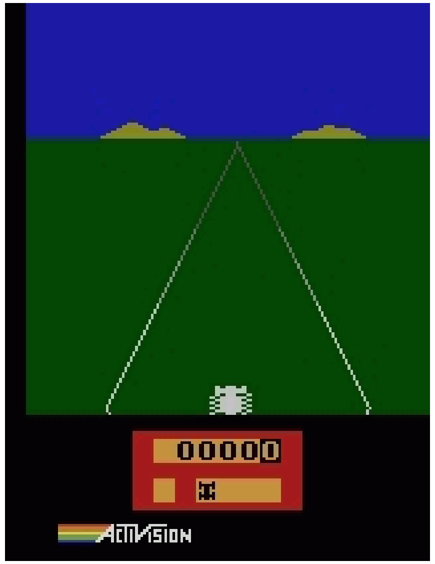


DQN Results in Atari



DQN Demos in Atari

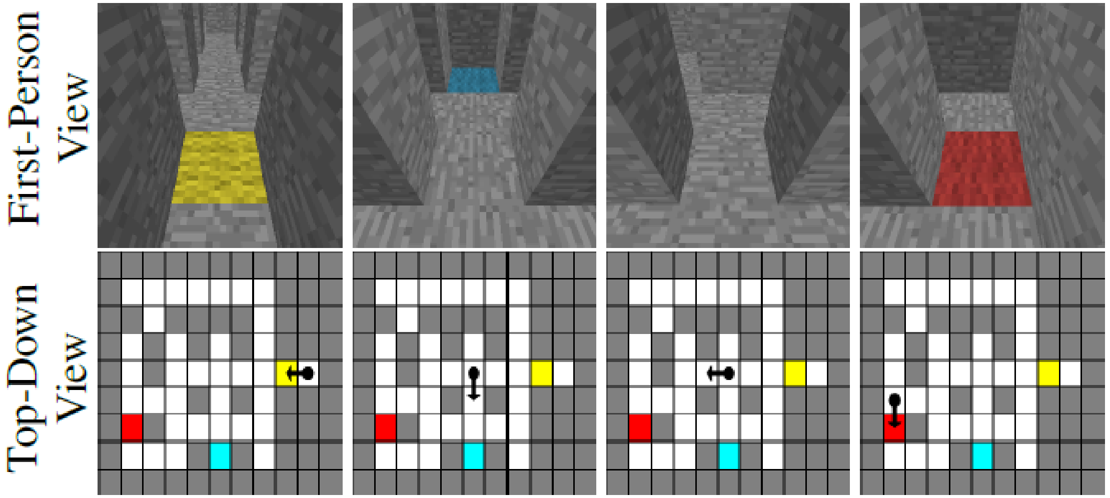
breakout



enduro

DQN Variants in Minecraft

- Challenges in environments
  - Partial observability (first-person visual observations)
  - Delayed rewards
  - High-dimensional perception



- Combine DQN with memory network to solve this kind of task

FPS Games

- Visual Doom AI Competition

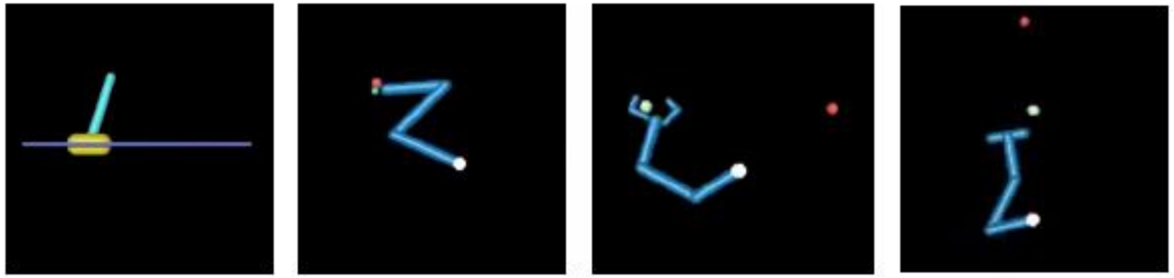
A DRQN Agent

Super Mario Games

Deep RL in Go

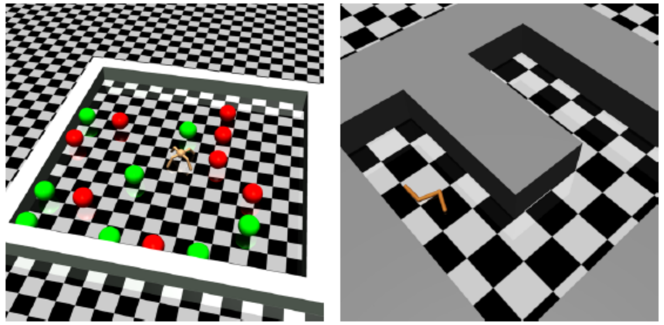
- Use supervised learning followed by deep RL
- Falls in the category of actor-critic framework
  - Use policy network to select moves
  - Use value network to evaluate board positions
- The learning approach is further combined with Monte Carlo search
- AlphaGo beats the human world champion

Deep RL for Classic Control Tasks



Simple tasks

Locomotion tasks



Hierarchical tasks

Suitable for benchmarking various algorithms

DDPG Demos for Classic Control

Deep RL in Real-World Robotics