1. If $T(n) = \Theta(1)$, for $n < 5$, write the solutions to the following recursions. (2 marks)

   (a) $T(n) = T(n/2024) + 1$.
   (b) $T(n) = T(n/2024) + n$.
   (c) $T(n) = 2024T(n/2024) + 1$.
   (d) $T(n) = 2024T(n/2024) + n$.

2. Show that the solution to the following recursions is linear (2 marks).

   (a) $T(n) = 5T(n/9) + 2T(n/5) + n$.
   (b) $T(n) = n + 1/n \sum_{k=0}^{k=n-1} T(k)$

3. Given a linked list containing a cycle, write a linear time algorithm to delete the cycle. (2 marks)

4. You are given a string containing capital letters , A to Z. Write a linear time algorithm to find the length of the shortest sub string in which all the letters appear at least once. (3 marks)

5. Let us consider an array of n integers, $a_0, a_1, \ldots a_{n-1}$. For $i \leq j$, let $Max(i, j)$ be the maximum among $a_i, a_{i+1}, \ldots a_j$. Design a linear time algorithm to compute (3 marks)

$$\sum_{i=0}^{n-1} \sum_{j=i}^{n-1} Max(i, j)$$

Example: Given $3, 1, 2, 4$, the answer is 30.

6. Given an array A of n integers and another integer $X, X > 0$. Write an $O(n \log n)$ time algorithm to compute the following function (3 marks).

```
Compute(A, n, X){
    max = 0;
        for(i = 0; i < n; + + i){
        c = 0;
        for(j = 0; j < n; + + j)
            if(abs(A[i] − A[j]) < X)  c + +;

        if(c > max) max = c; }

return max;
}
```

Here, $abs(x)$ returns the absolute value of x.

Example: Given $4, 3, 9, 8, 1$ and $X = 5$, the answer is 4.