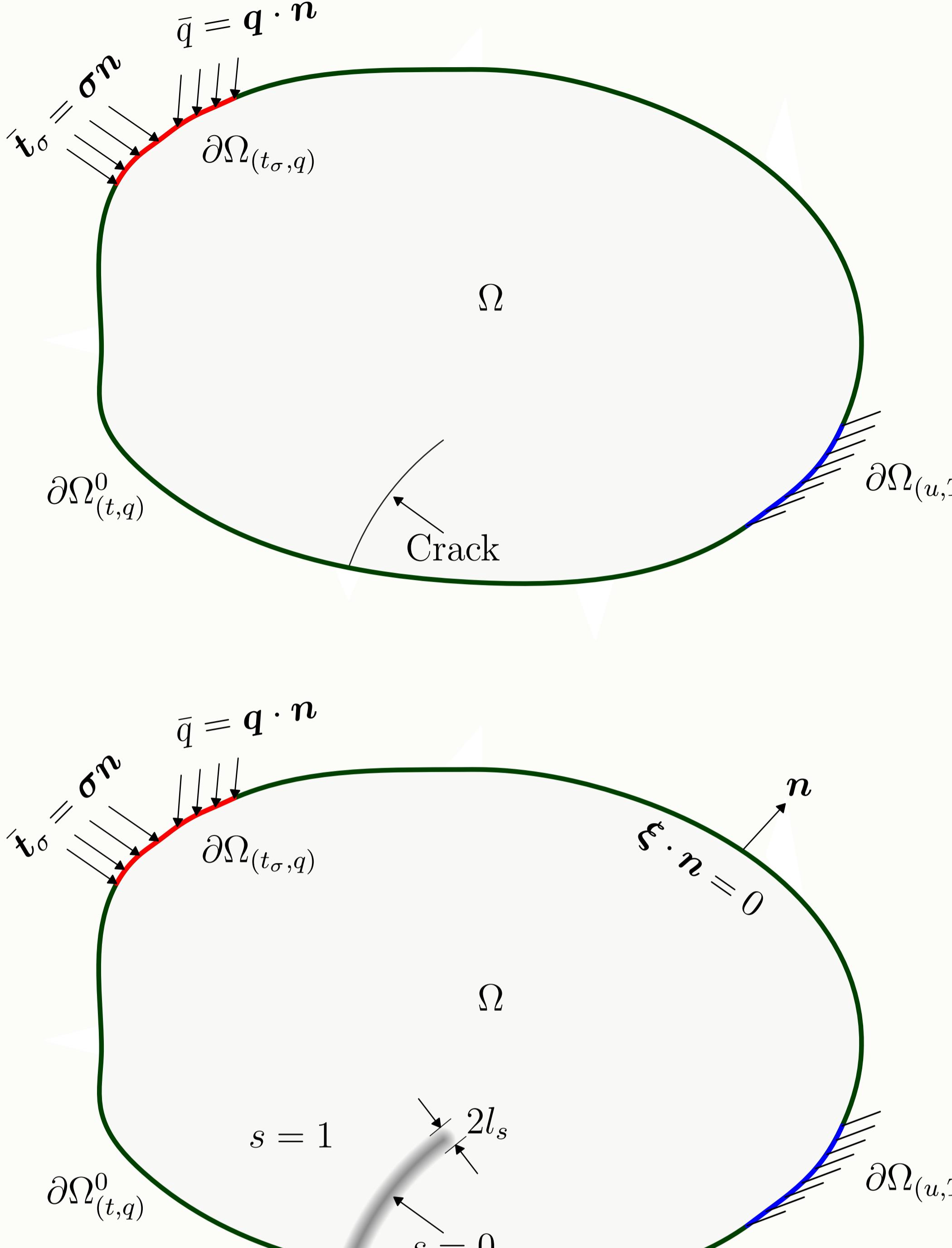


Proposed fatigue model for FGM

Sharp and Diffused crack representation with a regularized crack topology using a scalar phase-field s and a characteristic length scale l_s .



Schematic representation of a material domain Ω with a smooth boundary $\partial\Omega$. Dirichlet conditions are prescribed on $\partial\Omega_{(u,T)}$. Neumann-boundary conditions—namely, the applied traction $\bar{t}_\sigma = \sigma\mathbf{n}$ and heat flux $\bar{q} = \mathbf{q} \cdot \mathbf{n}$ —are enforced on $\partial\Omega_{(t,q)}$. The remainder of the boundary, $\partial\Omega^0_{(t,q)}$, is subjected to homogeneous Neumann conditions, implying vanishing traction and heat flux. A natural boundary condition for the phase-field variable is imposed as $\xi \cdot \mathbf{n} = 0$ on the entire boundary $\partial\Omega$.

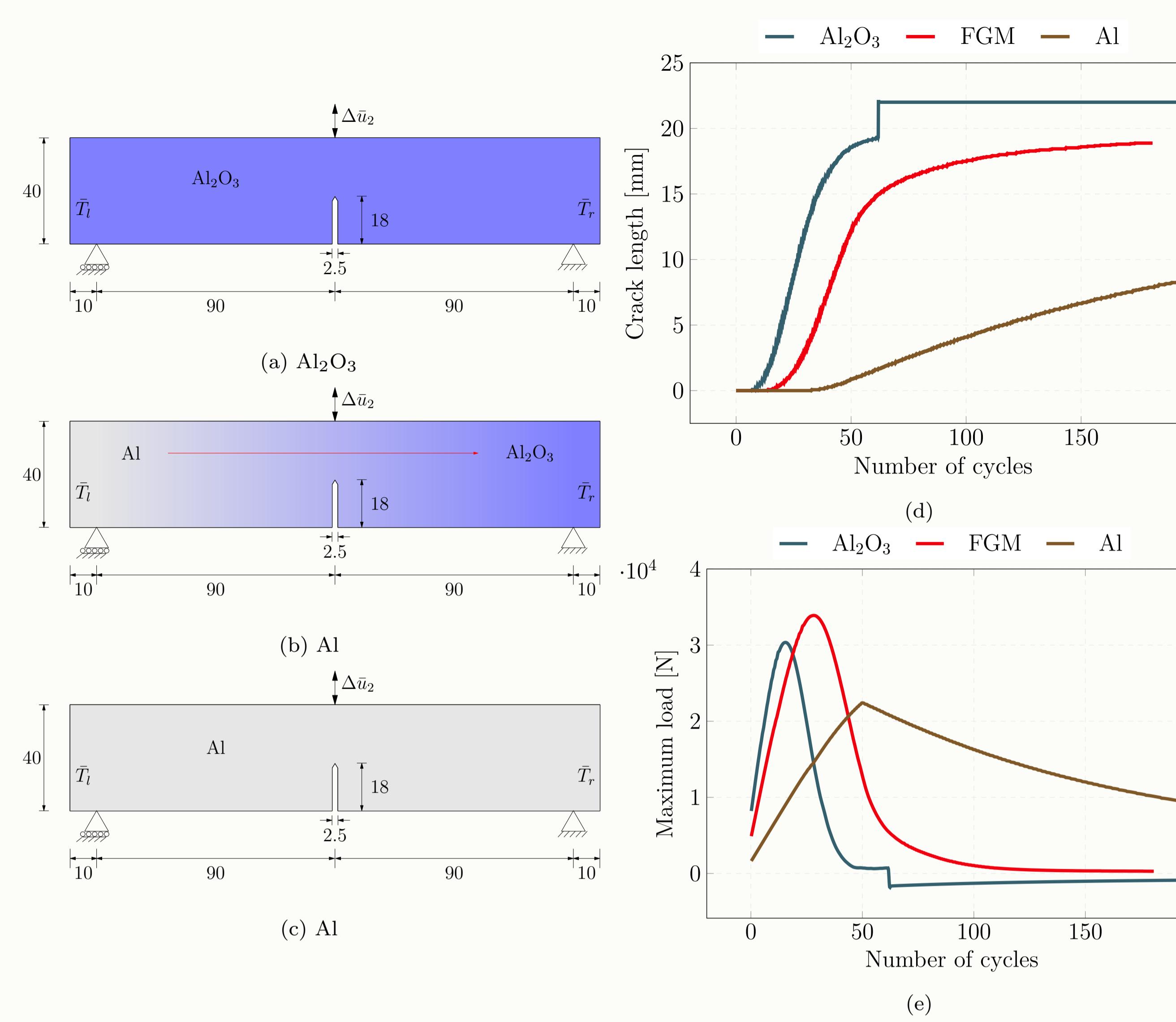
Strong form equations

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma} &= \mathbf{0}, \\ \rho c \dot{T} - \nabla \cdot (k \nabla T) &= Q_h \quad \text{in } \Omega, \\ \nabla \cdot \left(\frac{2G_c l_s}{c_0} \nabla s \right) - g'(s) \frac{\mathcal{H}(\mathcal{E})}{f(\bar{\alpha})} - \frac{G_c}{c_0 l_s} (\alpha'_g(s)) &= 0 \end{aligned}$$

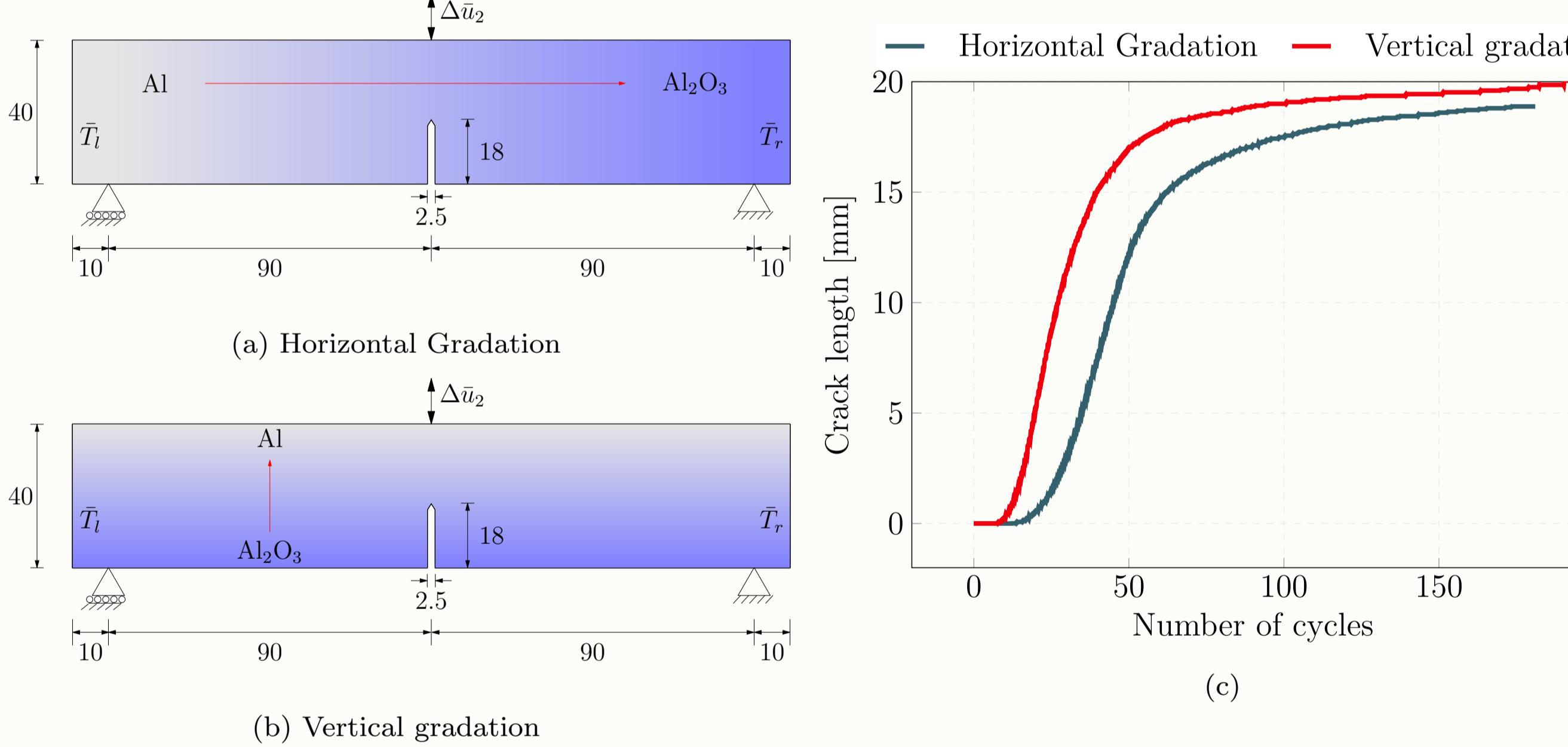
Fatigue related parameters

$$f(\bar{\alpha}) = \begin{cases} \left(\frac{2\alpha_T}{\alpha_T + \bar{\alpha}(t)} \right)^2, & \text{if } \bar{\alpha}(t) \geq \alpha_T, \\ 1, & \text{if } \bar{\alpha}(t) \leq \alpha_T, \end{cases} \quad \alpha_T = \frac{\psi_c}{k_f}$$

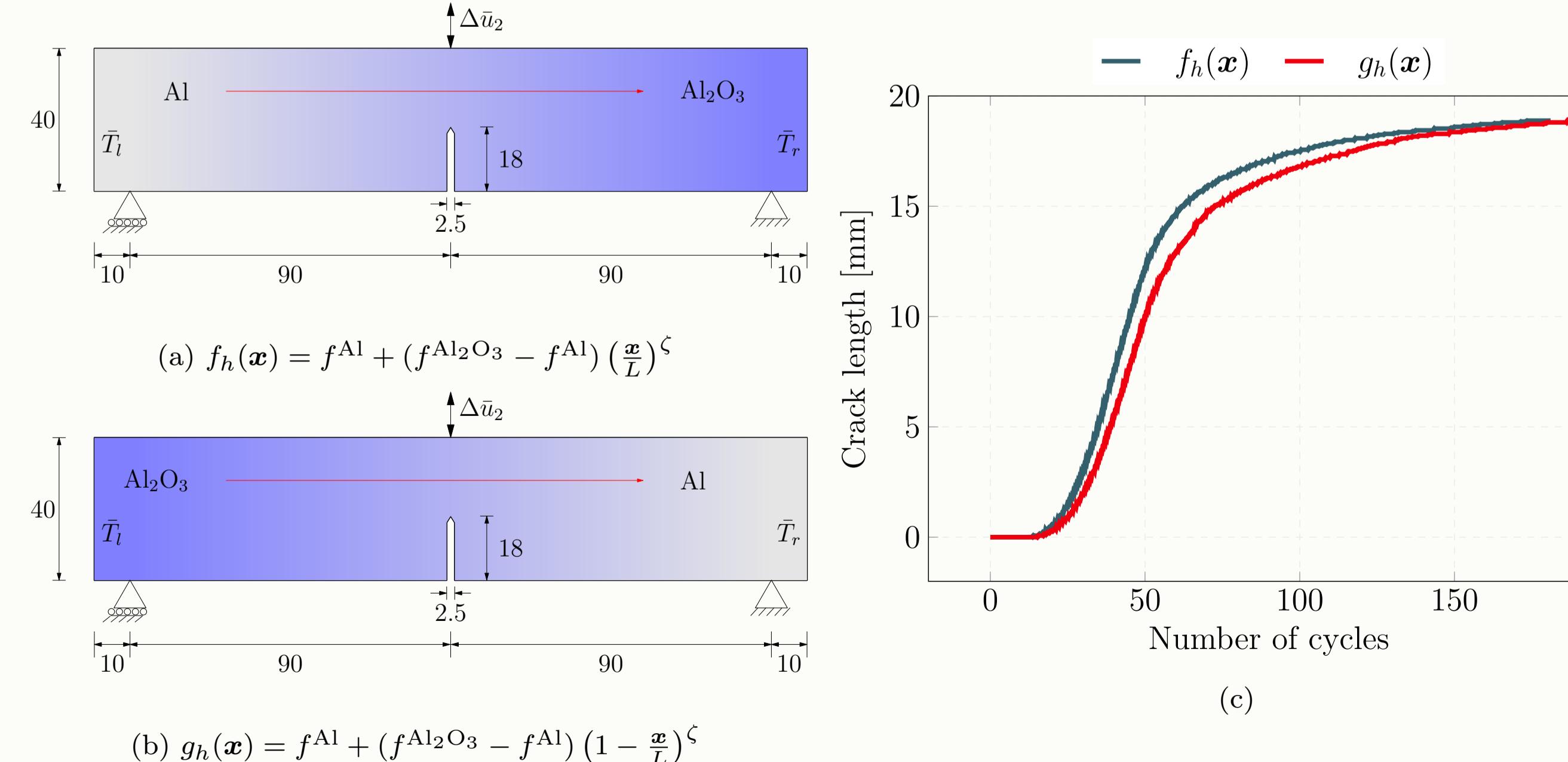
Fatigue response under different gradation



Comparison of the fatigue response from a homogeneous and functionally graded specimen.

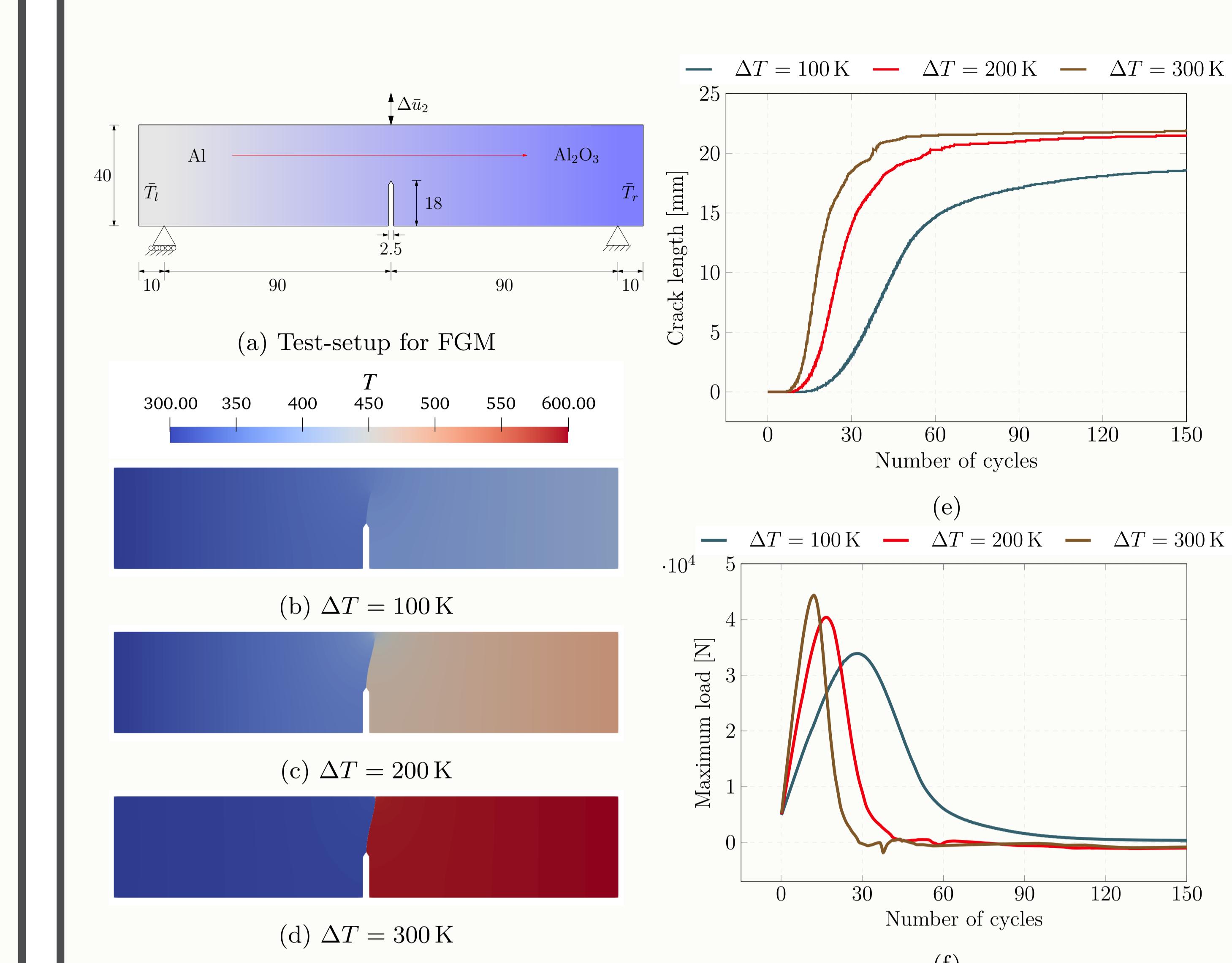


Comparison of the fatigue response from a FGM with horizontal and vertical orientations of material gradation.

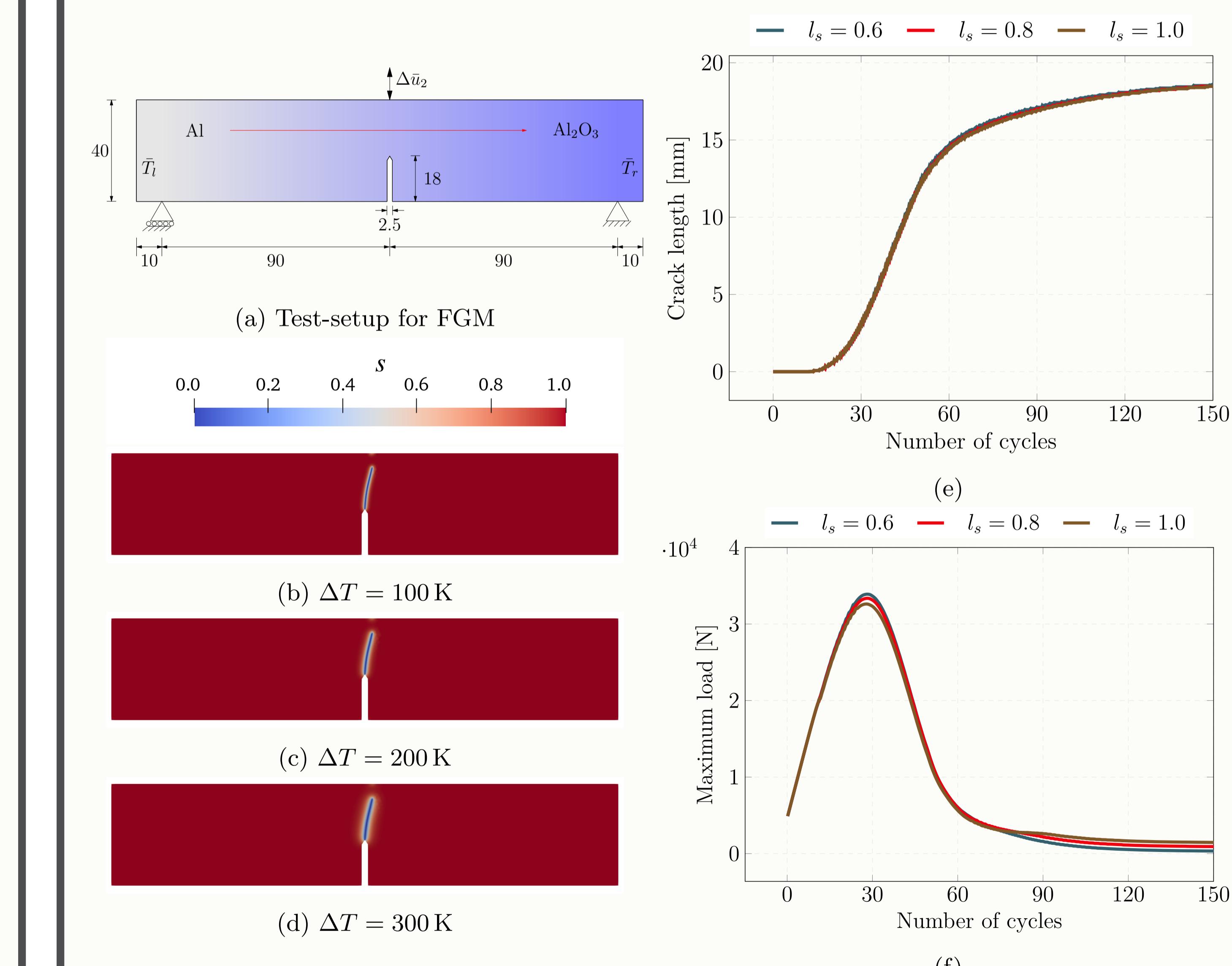


Comparison of the fatigue response from a FGM with two opposite orientations of material gradation.

Fatigue response under different temperature



Comparison of the fatigue response of a functionally graded specimen subjected to three-point bending with cyclic displacement loading for various temperature boundary conditions.



Comparison of the fatigue response of a functionally graded specimen subjected to three-point bending with cyclic displacement loading for various phase-field length scales (l_s).