

# Gradient Descent in Practice II: Learning Rate

Multivariate Linear Regression

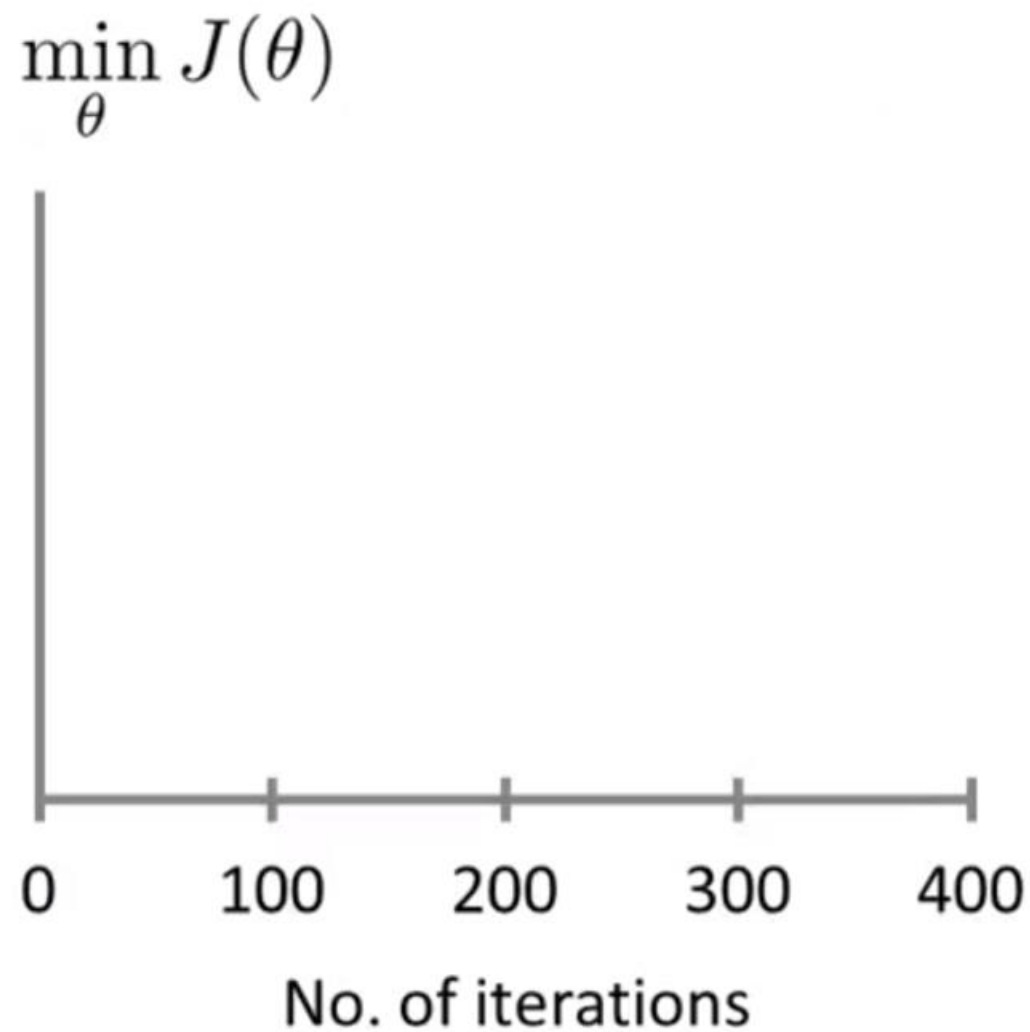
*Linear Regression with Multiple Variables*

## Gradient descent

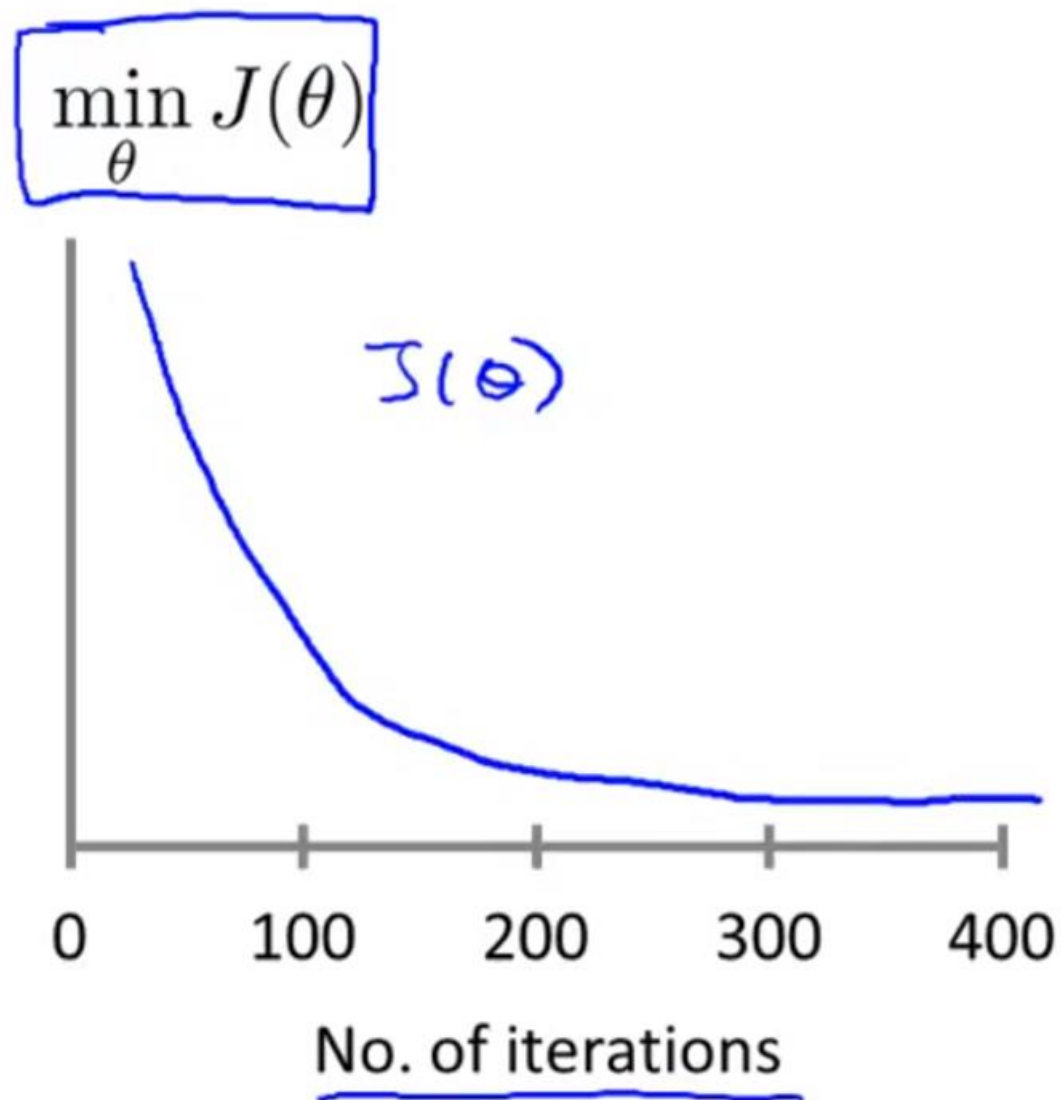
$$\rightarrow \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

- “Debugging”: How to make sure gradient descent is working correctly.
- How to choose learning rate  $\alpha$ .

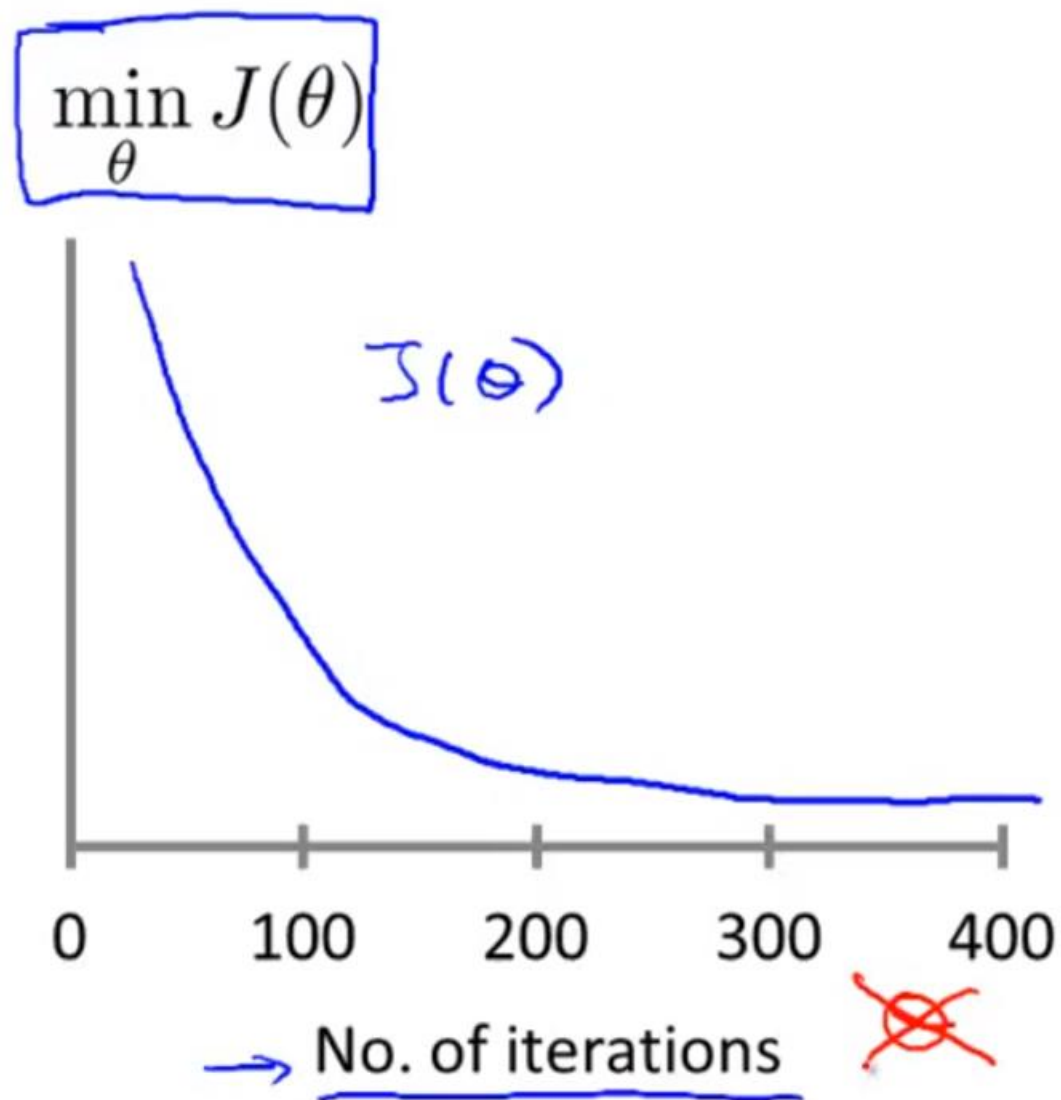
## Making sure gradient descent is working correctly.



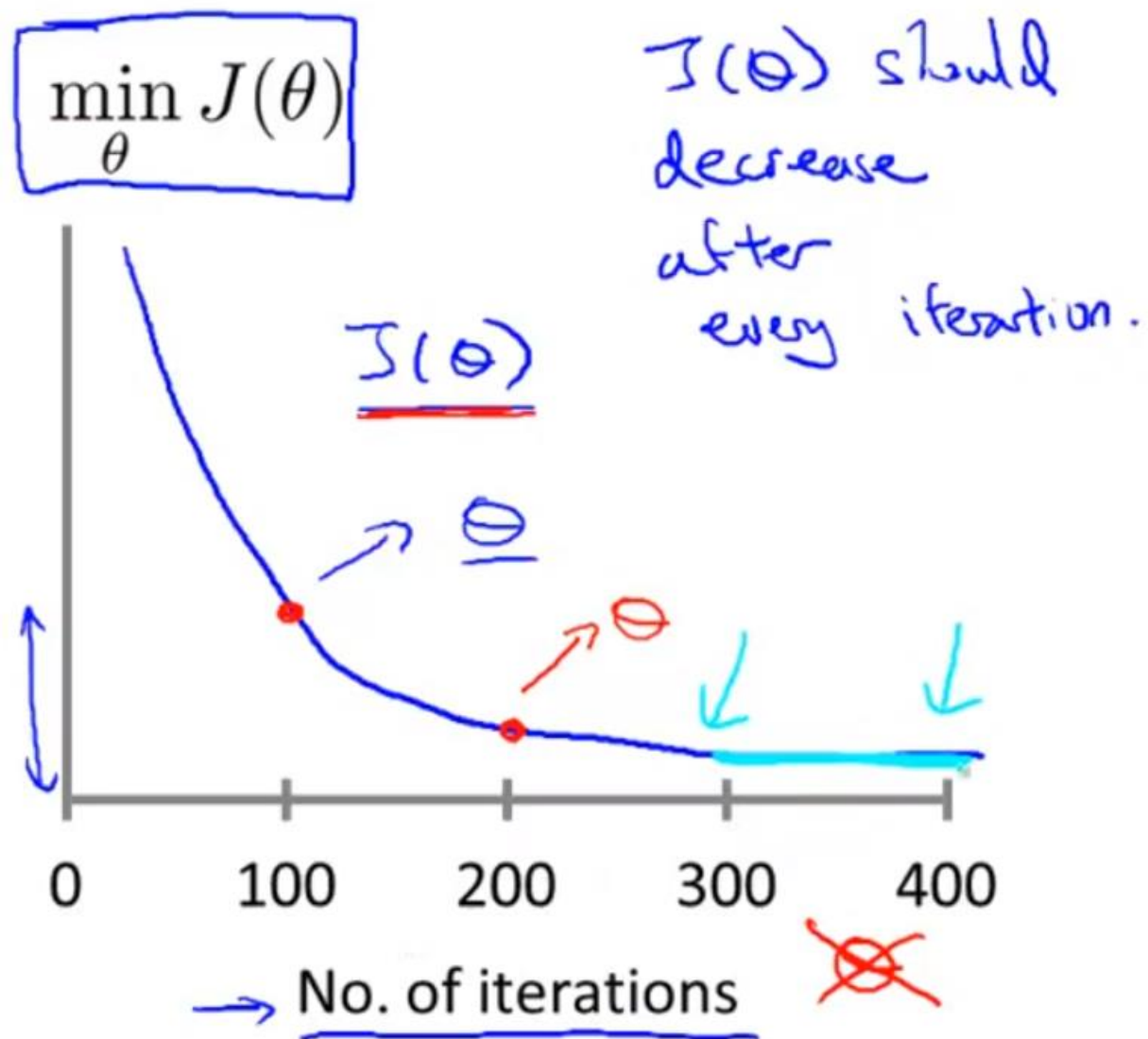
## Making sure gradient descent is working correctly.



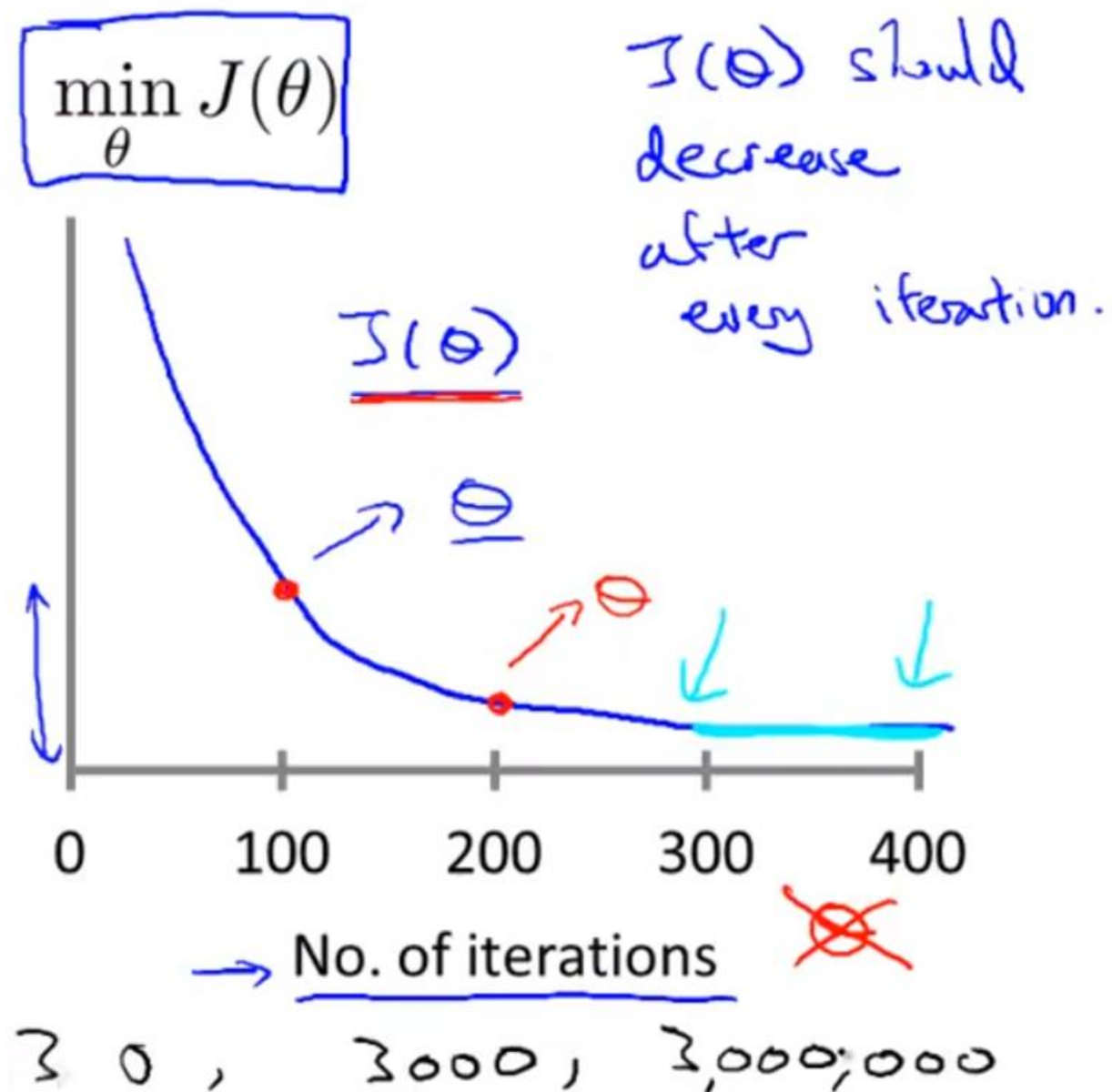
## Making sure gradient descent is working correctly.



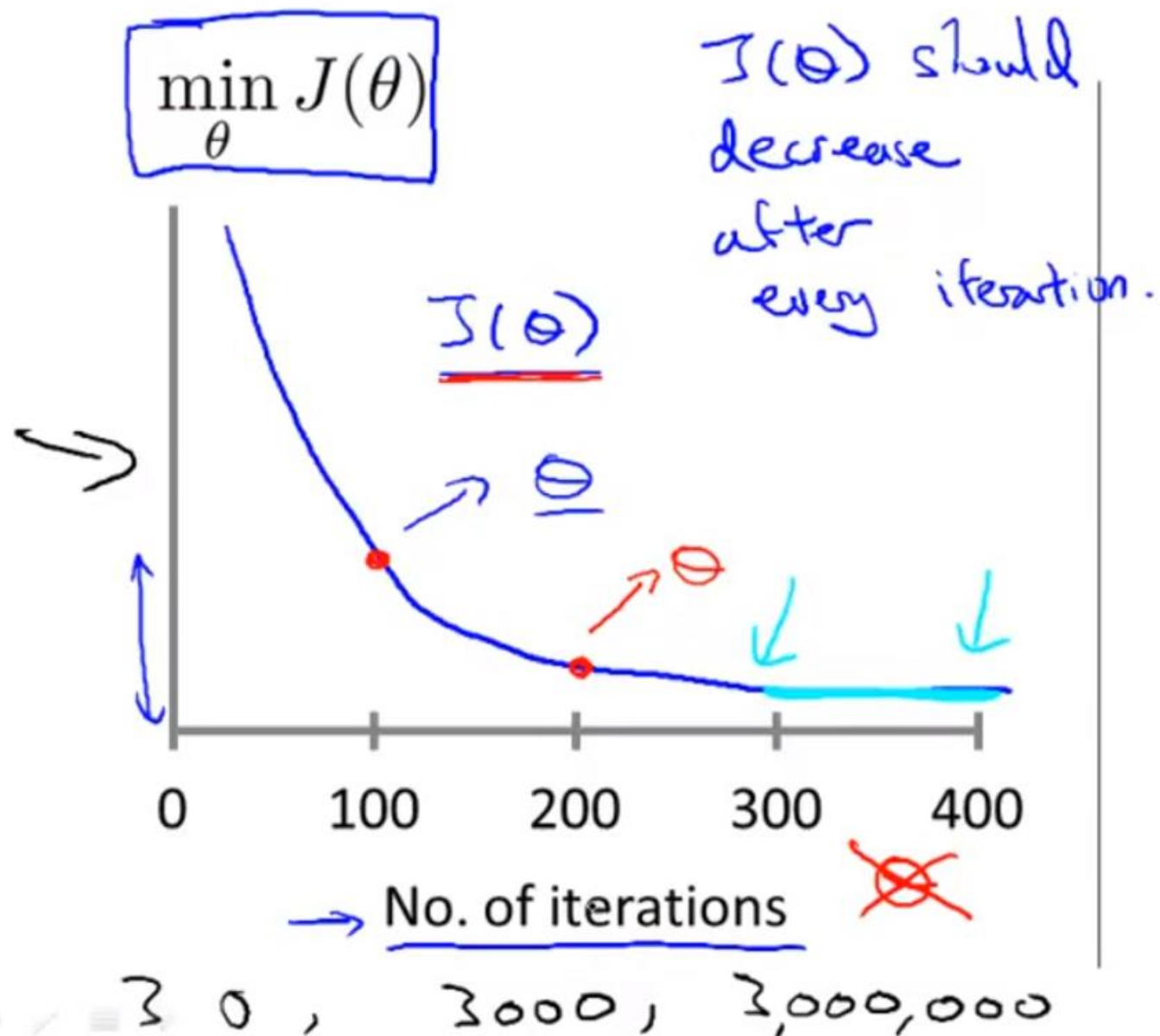
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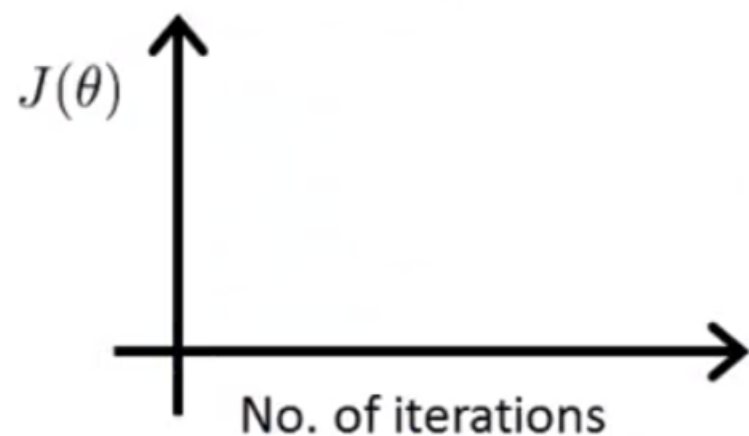
Example automatic convergence test:

Declare convergence if  $J(\theta)$  decreases by less than  $10^{-3}$  in one iteration.

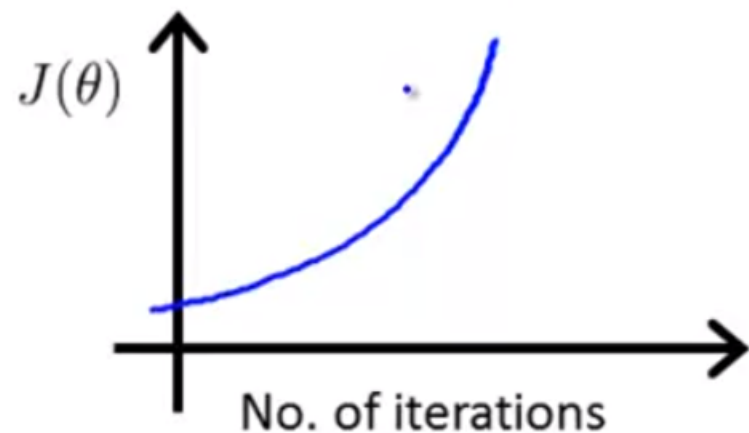
Epsilon



## Making sure gradient descent is working correctly.



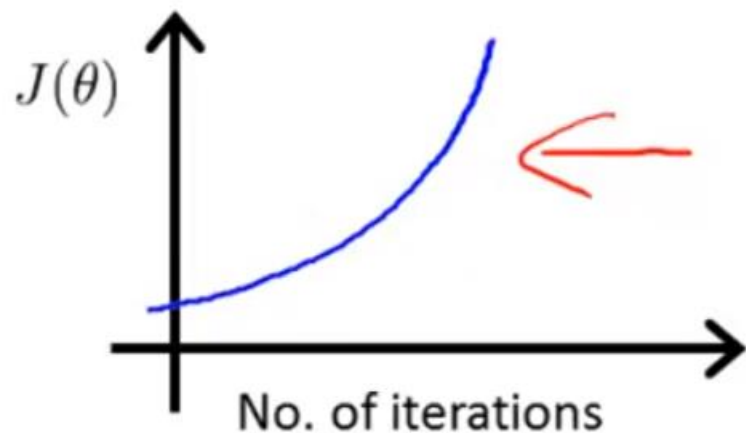
## Making sure gradient descent is working correctly.



Gradient descent not working.

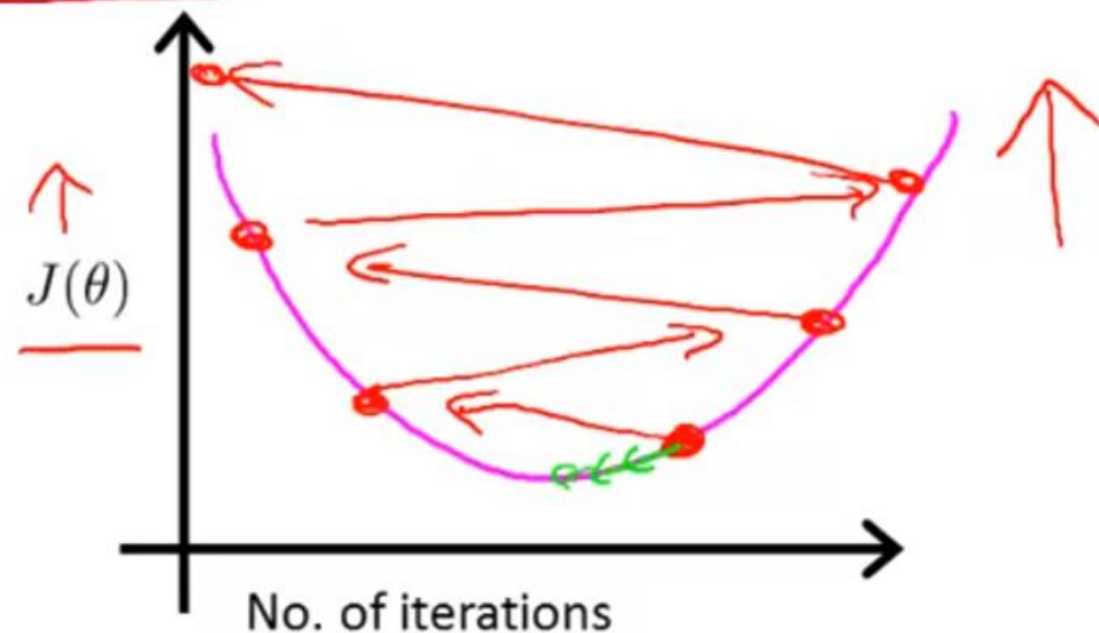
Use smaller  $\alpha$ .

## Making sure gradient descent is working correctly.

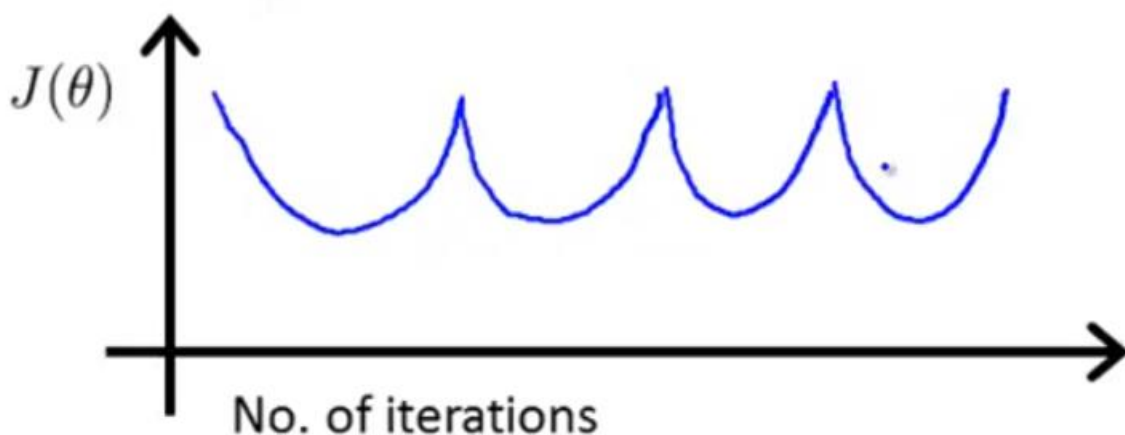
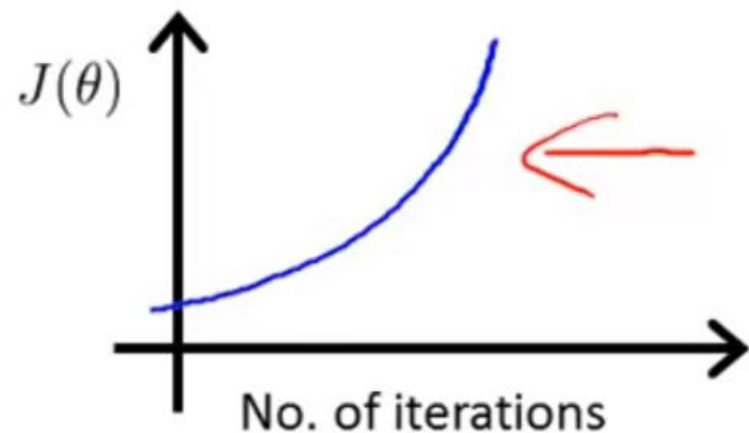


Gradient descent not working.

Use smaller  $\alpha$ .

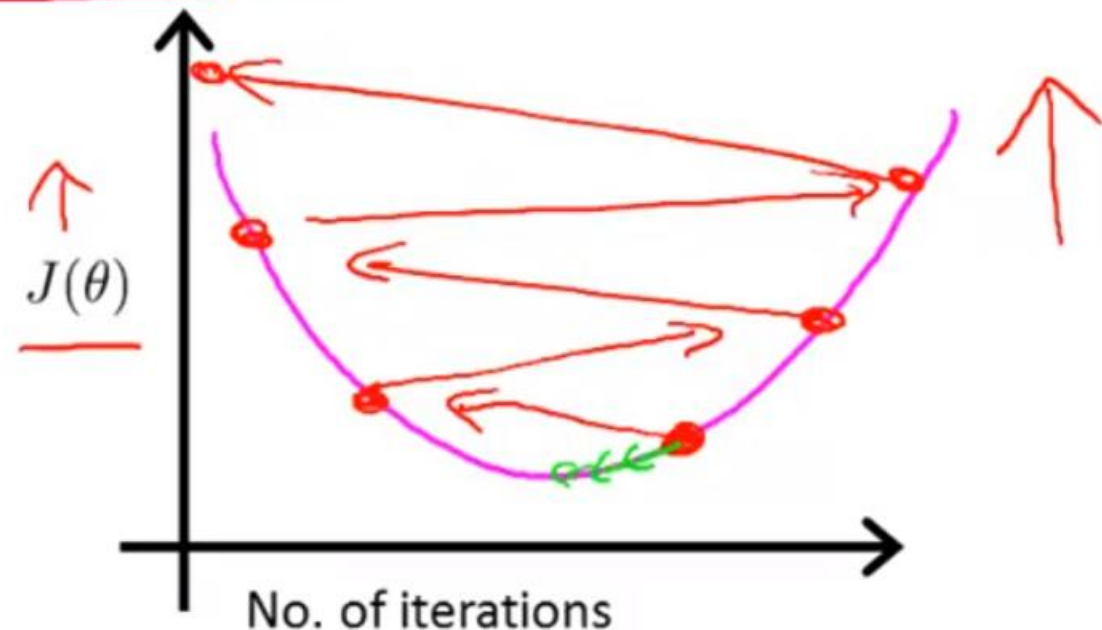


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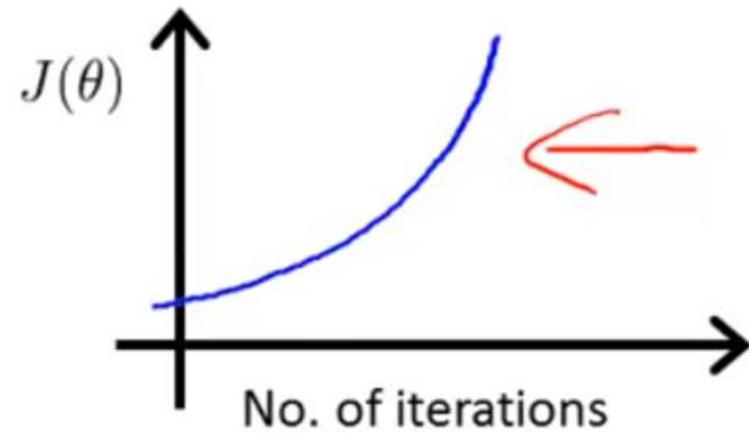


Gradient descent not working.

Use smaller  $\alpha$ .

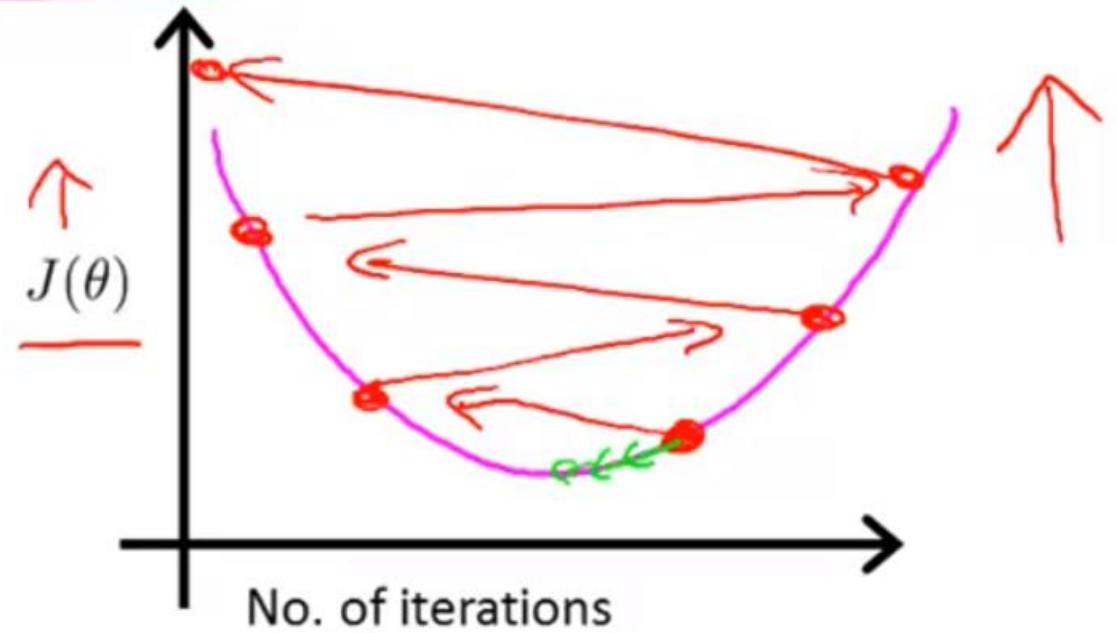
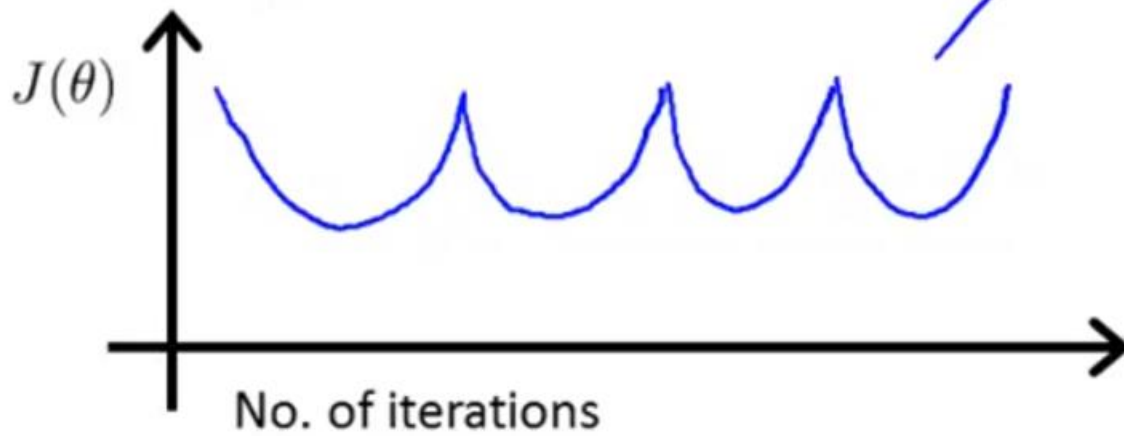


## Making sure gradient descent is working correctly.

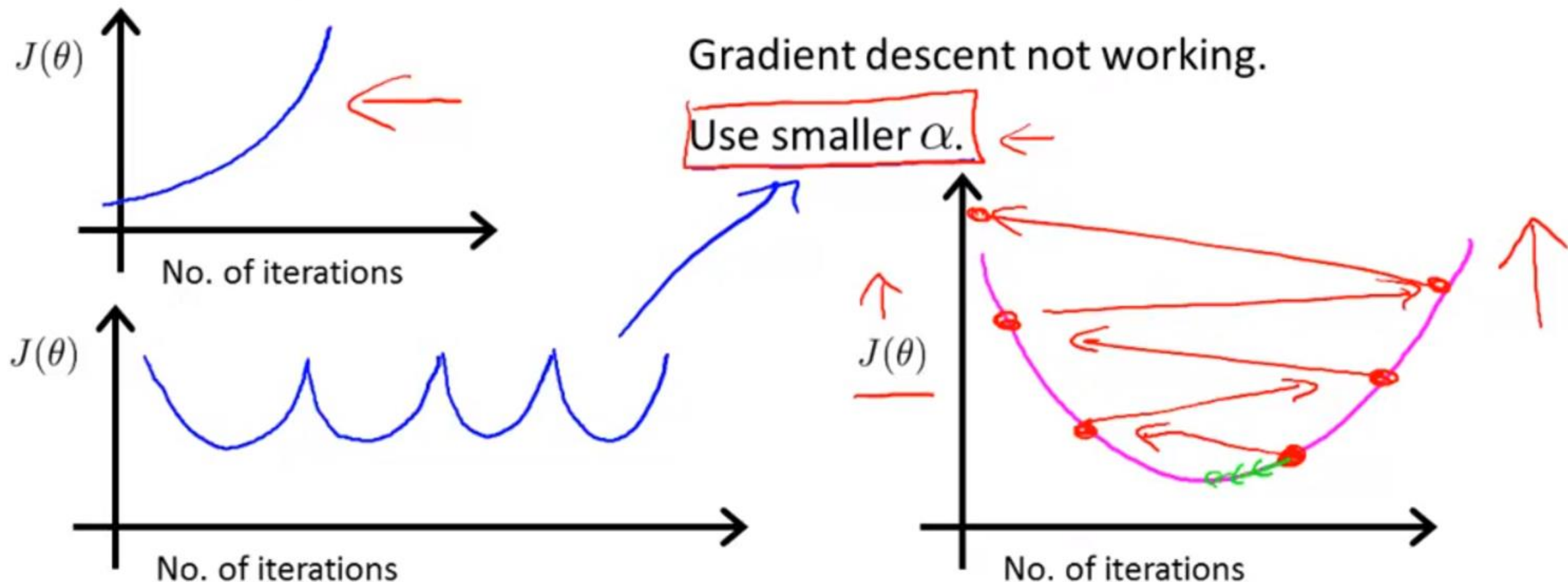


Gradient descent not working.

Use smaller  $\alpha$ .



## Making sure gradient descent is working correctly.



- For sufficiently small  $\alpha$ ,  $J(\theta)$  should decrease on every iteration.
- But if  $\alpha$  is too small, gradient descent can be slow to converge.

# Exercise

- Suppose a friend ran gradient descent three times, with

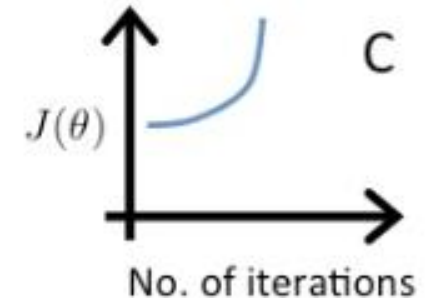
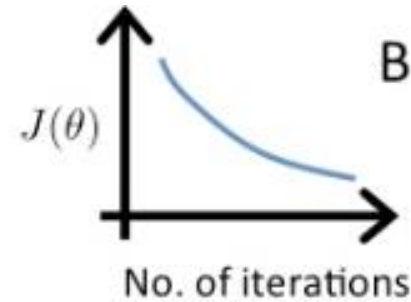
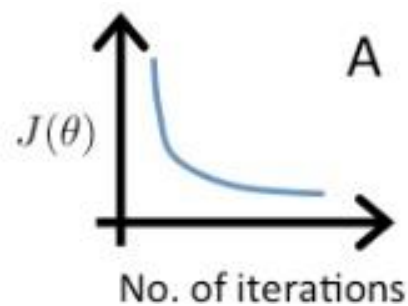
- $\alpha=0.01$ ,

- $\alpha=0.1$ ,

- $\alpha=1$ ,

- and got the following three plots (labeled A, B, and C):

- Which plots corresponds to which values of  $\alpha$ ?



	A	B	C
A	0.01	0.1	1
B	0.1	0.01	1
C	1	0.1	0.01
D	1	0.01	0.1

## Summary:

- If  $\alpha$  is too small: slow convergence.
- If  $\alpha$  is too large:  $J(\theta)$  may not decrease on every iteration; may not converge.



## Summary:

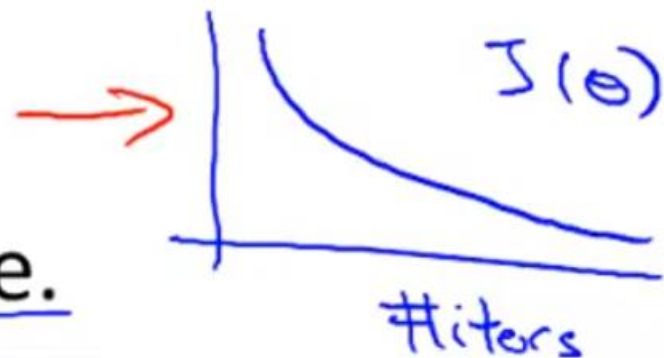
- If  $\alpha$  is too small: slow convergence.
- If  $\alpha$  is too large:  $J(\theta)$  may not decrease on every iteration; may not converge.

To choose  $\alpha$ , try

..., 0.001, ..., 0.01, ..., 0.1, ..., 1, ...

## Summary:

- If  $\alpha$  is too small: slow convergence.
- If  $\alpha$  is too large:  $J(\theta)$  may not decrease on every iteration; may not converge. (Slow converge also possible.)



To choose  $\alpha$ , try

..., 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1, ...

Curved arrows indicate a sequence of values, with labels  $\approx 3\times$  and  $\approx 1\times$  written below the arrows.