Mathematical Programming

$$\text{Solving the LP with} \begin{cases} \text{Excel} \\ \text{The Simplex Algoritm} \end{cases}$$

Example Problem I

Problem Statement:

Suppose that a gas processing plant receives a fixed amount of raw gas each week. The raw gas is processed into two grades of heating gas, regular and premium quality. These grades of gas are in high demand (that is, they are guaranteed to sell) and yield different profits to the company. However, their production involves both time and on-site storage constraints. For example, only one of the grades can be produced at a time, and the facility is open for only 80 hrs/week. Further, there is limited on-site storage for each of the products. All these factors are listed below (note that a metric ton is equal to 1000 kg):

	Product		
Resource	**Regular**	**Premium**	**Resource Availability**
Raw gas	7 m^3 /Ton	11 m^3 /Ton	77 m^3 /Week
Production time	10 hr /Ton	8 hr /Ton	80 hr /Week
Storage	9 Ton	6 Ton	
Profit	150 /Ton	175 /Ton	

Solving the LP (Prob. I)

Develop a linear programming formulation to maximize the profits for this operation.

Solution. The engineer operating this plant must decide how much of each gas to produce to maximize profits. If the amounts of regular and premium produced weekly are designated as X_1 and X_2, respectively, the total weekly profit can be calculated as

- Total profit = $150\ X_1 + 175\ X_2$
  $Z = 150\ X_1 + 175\ X_2$
- The constraints can be developed in a similar fashion.
  Total gas used = $7\ X_1 + 11\ X_2$
  That can not exceed 77 m^3 /Week
  $7\ X_1 + 11\ X_2 \leq 77$

Solving the LP (Prob. I)

- Remaining constrains can be developed in a similar fashion
  Total LP formulation given by

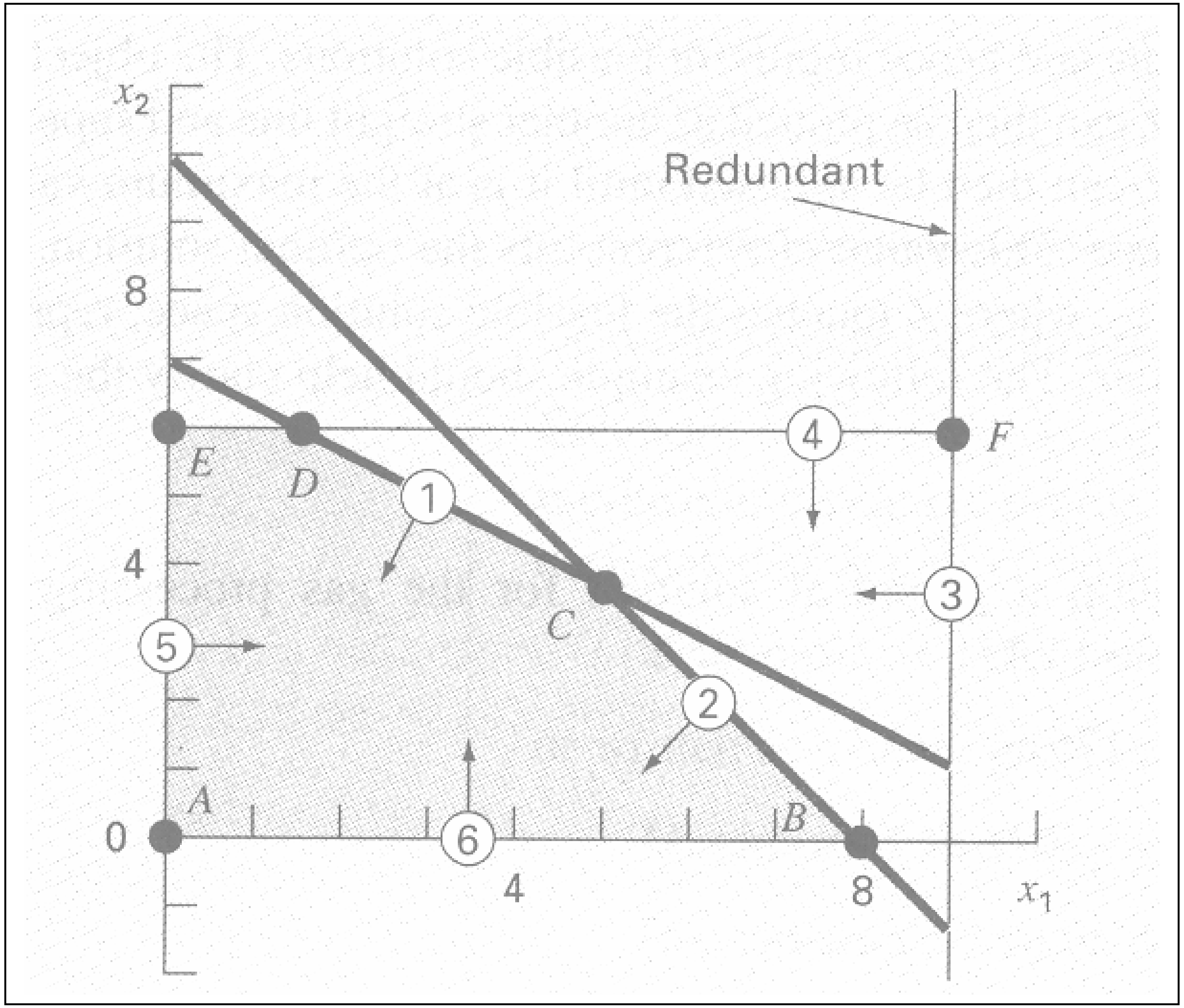
$Z = 150\ X_1 + 175\ X_2$	Maximize profit
$7\ X_1 + 11\ X_2 \leq 77$	Material constrain
$10\ X_1 + 8\ X_2 \leq 80$	Time constrain
$X_1 \leq 9$	"Regular" storage constrain
$X_2 \leq 6$	"Premium" storage constrain
$X_1, X_2 \geq 0$	Positivity constrain

Graphical Solution to the LP (Prob. I)

$Z = 150\ X_1 + 175\ X_2$

$7\ X_1 + 11\ X_2 \leq 77$	Cons(1)	This can be reformulated	$X_2 = -7/11\ X_1 + 7$
$10\ X_1 + 8\ X_2 \leq 80$	Cons(2)		$X_2 = -10/8\ X_1 + 10$
$X_1 \leq 9$	Cons(3)		
$X_2 \leq 6$	Cons(4)		
$X_1 \geq 0$	Cons(5)		
$X_2 \geq 0$	Cons(6)		

Graphical Solution to the LP (Prob. I)



Graphical Solution to the LP (Prob. I)

Next, Objective function can be added to the plot. To do this, a value of Z must be chosen. For example, for Z=0 the function becomes

$$0=150\ X_1 + 175\ X_2 \text{ and solving for } X_2 = -150/175\ X_1$$

- Since we like to maximize Z, say Z=600, and

$X_2 = -150/175\ X_1 + 600/175$

- So we can keep increase the Z in the feasible region, Make it Z=1400, at this point

$X_1 = 4.9$

$X_2 = 3.9$