Multiple Features

Multivariate Linear Regression

Linear Regression with Multiple Variables

Introduction

- Linear regression with multiple variables is also known as "multivariate linear regression".
- We now introduce notation for equations where we can have any number of input variables.

Multiple features (variables).

Size (feet^2) x	Price ($1000) y
2104	460
1416	232
1534	315
852	178
...	...

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Multiple features (variables).

Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
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…	…	…	…	…

$m = 47$

Notation:

- n = number of features $n = 4$
- $x^{(i)}$ = input (features) of i^{th} training example.
- $x_j^{(i)}$ = value of feature j in i^{th} training example.

$$x^{(2)} = \begin{bmatrix} 1416 \\ 3 \\ 2 \\ 40 \end{bmatrix}$$

Hypothesis:

Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

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$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

E.g. $h_\theta(x) = 80 + 0.1x_1 + 0.01x_2 + 3x_3 - 2x_4$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

$$\rightarrow h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $\boxed{x_0 = 1.}$ $\quad (x_0^{(i)} = 1)$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1} \qquad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$h_\theta(x) = \theta_0 \overset{=1}{x_0} + \theta_1 x_1 + \cdots + \theta_n x_n$$

$$= \boxed{\theta^T x.}$$

$$\underbrace{\begin{bmatrix} \theta_0 & \theta_1 & \cdots & \theta_n \end{bmatrix}}_{\theta^T} \qquad \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$(n+1) \times 1$ matrix $\qquad x$

$$\theta^T x$$

Multivariate linear regression. $\leftarrow$

Gradient Descent For MLR

$x_0 = 1$

Hypothesis: $h_\theta(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$

Parameters: $\cancel{\theta_0, \theta_1, \ldots, \theta_n}$ θ n+1 - dimensional vector

Cost function:

$$\cancel{J(\theta_0, \theta_1, \ldots, \theta_n)} = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$J(\theta)$

Gradient descent:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} \cancel{J(\theta_0, \ldots, \theta_n)} \; J(\theta)}$$

$\}$ (simultaneously update for every $j = 0, \ldots, n$)

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Feature Scaling

- We can speed up gradient descent by having each of our input values in roughly the same range.
- This is because θ will descend
  - quickly on small ranges and
  - slowly on large ranges,
- So it will oscillate inefficiently down to the optimum when the variables are very uneven.

Feature Scaling

Idea: Make sure features are on a similar scale.

E.g. x_1 = size (0-2000 feet^2)

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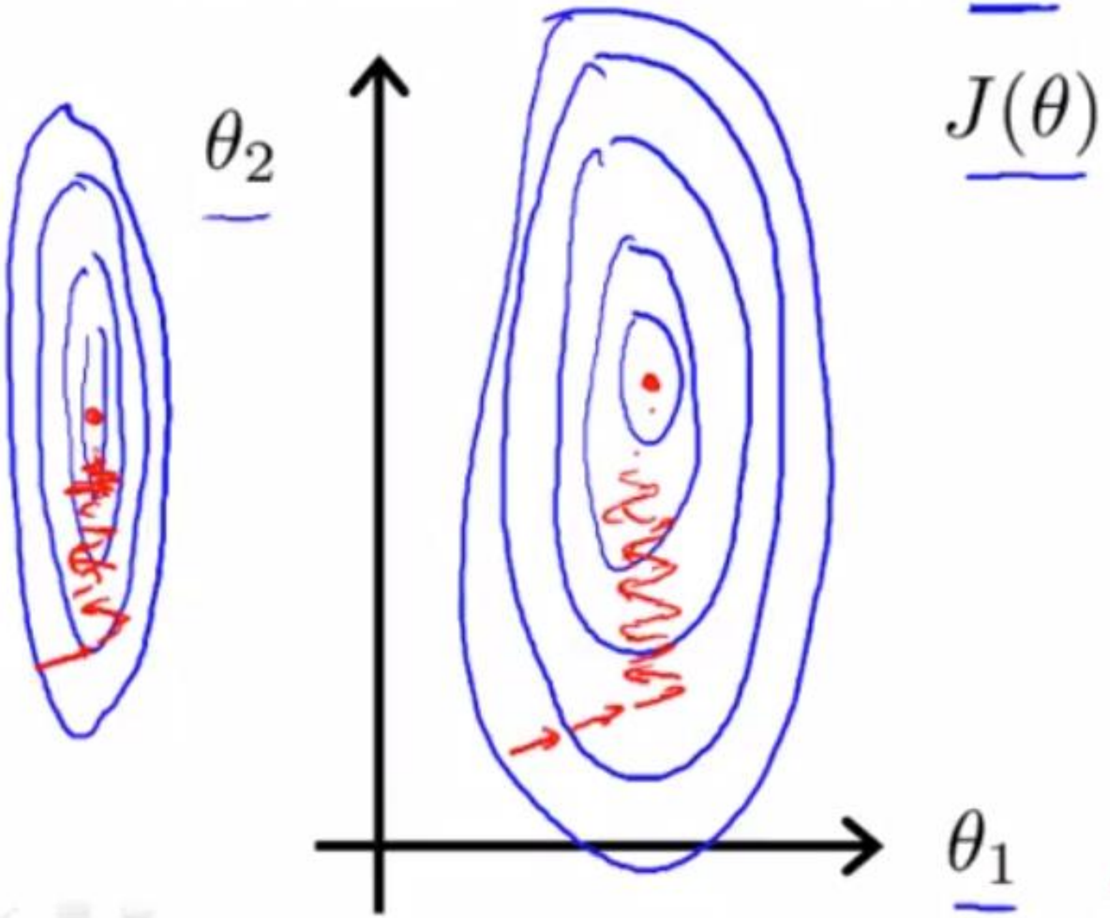
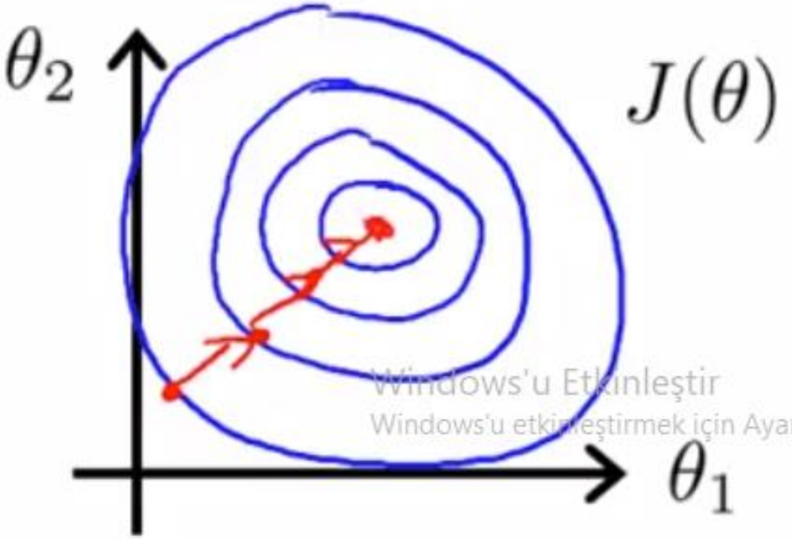
$$x_1 = \frac{\text{size (feet}^2\text{)}}{2000}$$

$$x_2 = \frac{\text{number of bedrooms}}{5}$$





Feature Scaling

Get every feature into approximately a $\boxed{-1 \le x_i \le 1}$ range.

$x_0 = 1$

$0 \le x_1 \le 3$ ✓

$-2 \le x_2 \le 0.5$ ✓

$-100 \le x_3 \ \boxed{100}$ ✗

$-0.0001 \le x_4 \le \boxed{0.0001}$ ✗

-3 to 3 ✓

$-\frac{1}{3}$ to $\frac{1}{3}$ ✓

Feature Scaling

E.g. $\boxed{x_1 = \frac{size-1000}{2000}}$ Average size = 100

$\boxed{x_2 = \frac{\#bedrooms-2}{5}}$ 1-5 bedrooms

$\boxed{-0.5 \leq x_1 \leq 0.5}$ $\boxed{-0.5 \leq x_2 \leq 0.5}$

$$x_1 \leftarrow \frac{x_1 - \mu_1}{s_1}$$

avg value of x_1 in training set

range (max - min)
(or standard deviation)

$$x_2 \leftarrow \frac{x_2 - \mu_1}{s_2}$$

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Exercise

- Suppose you are using a learning algorithm to estimate the price of houses in a city. You want one of your features x_i to capture the age of the house.
- In your training set, all of your houses have an age between 30 and 50 years, with an average age of 38 years.
- How do you normalize your data using mean normalization?

Normal Equation

Computing Parameters Analytically

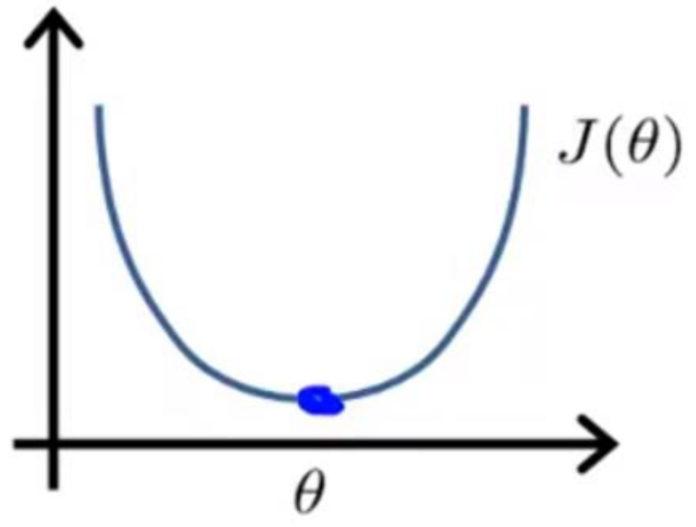
Linear Regression with Multiple Variables

Intuition: If 1D ($\theta \in \mathbb{R}$)

$$J(\theta) = a\theta^2 + b\theta + c$$

$$\frac{d}{d\theta} J(\theta) = \cdots \stackrel{\text{set}}{=} 0$$

Solve for θ



$\theta \in \mathbb{R}^{n+1}$

$$J(\theta_0, \theta_1, \ldots, \theta_m) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta) = \cdots = 0 \quad \text{(for every } j\text{)}$$

Solve for $\theta_0, \theta_1, \ldots, \theta_n$

Examples: $m = 4$.

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$$X = \underbrace{\begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}}_{m \times (n+1)} \qquad y = \underbrace{\begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}}_{m\text{-dimensional vector}}$$

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$m \times (n+1)$

m-dimensional vector

$$\boxed{\theta = (X^T X)^{-1} X^T y}$$

Exercise

- Suppose you have the training in the table below:

age (x_1)	height in cm (x_2)	weight in kg (y)
4	89	16
9	124	28
5	103	20

- You would like to predict a child's weight as a function of his age and height with the model
- $weight = \theta_0 + \theta_1 x_1 + \theta_2 x_2$
- What are X and y?

How to choose them?

Gradient Descent	**Normal Equation**
Need to choose alpha	No need to choose alpha
Needs many iterations	No need to iterate
O (kn^2)	O (n^3), need to calculate inverse of X^TX
Works well when n is large	Slow if n is very large