

Decision Boundary

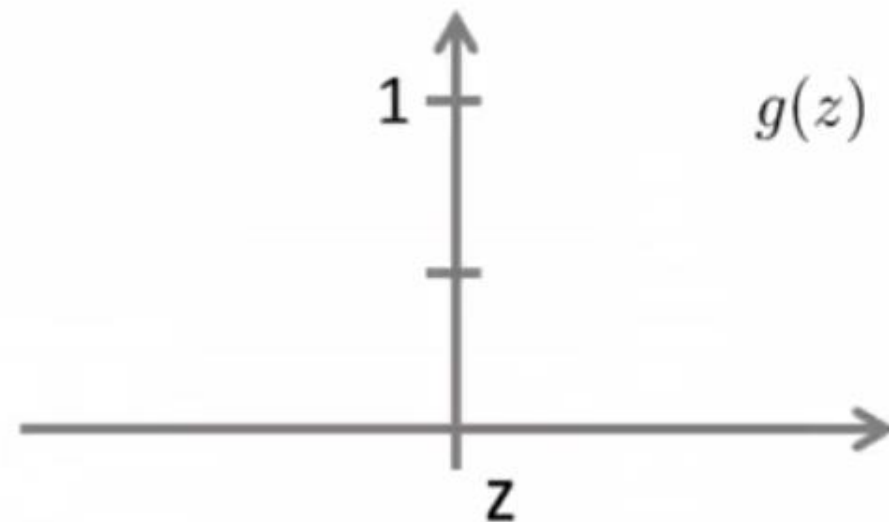
Classification and Representation

Logistic Regression

Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

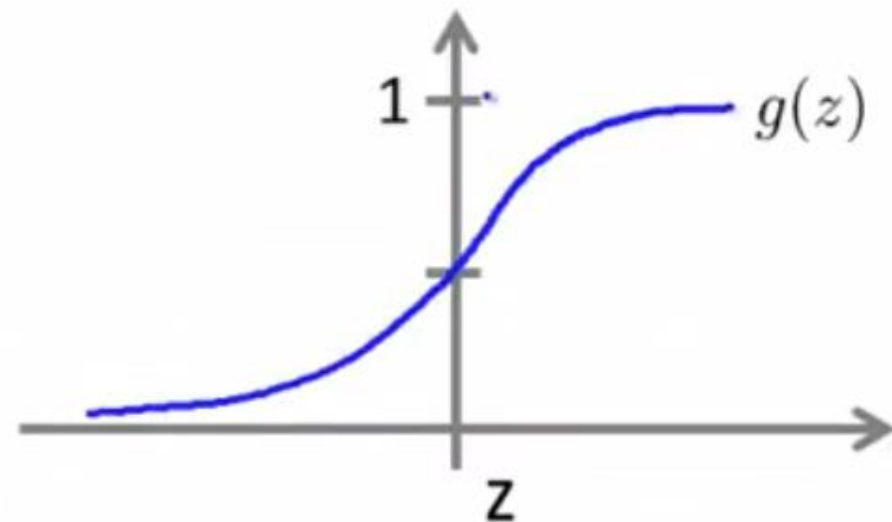


Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x)$$

$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$

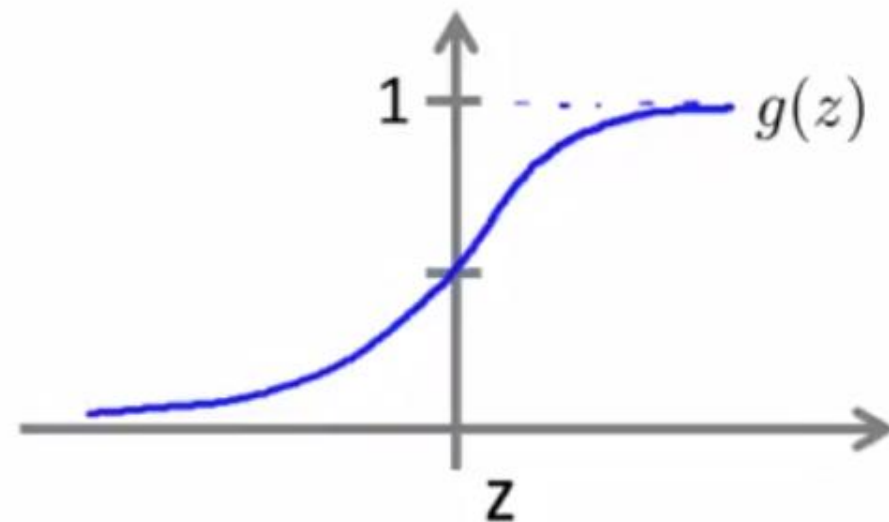


Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x) = p(y=1|x;\theta)$$

$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$

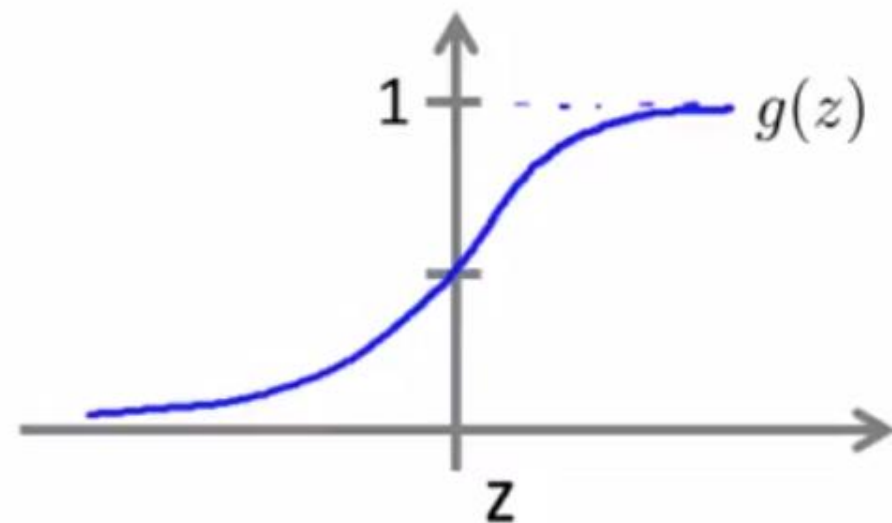


Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x) = p(y=1|x;\theta)$$

$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$



Suppose predict “ $y = 1$ ” if $h_{\theta}(x) \geq 0.5$

predict “ $y = 0$ ” if $h_{\theta}(x) < 0.5$

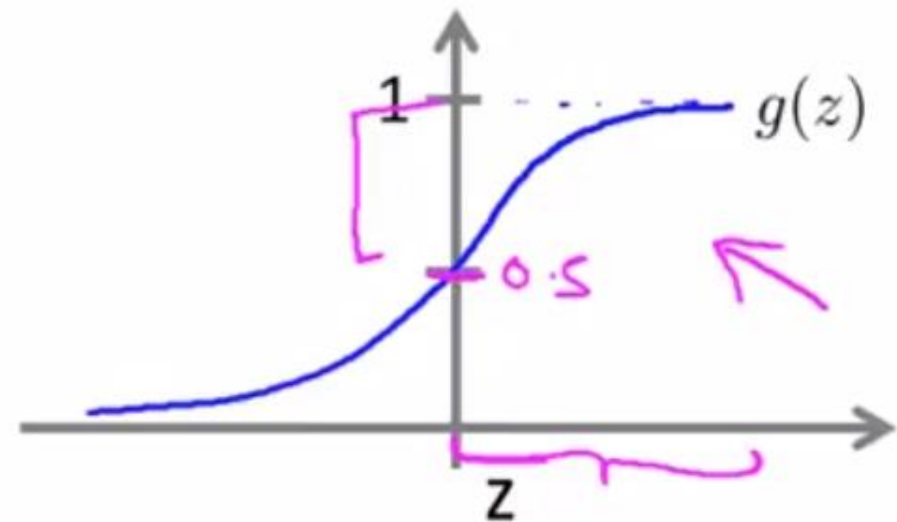
Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x) = p(y=1|x;\theta)$$

$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$

Suppose predict " $y = 1$ " if $h_{\theta}(x) \geq 0.5$

predict " $y = 0$ " if $h_{\theta}(x) < 0.5$



$g(z) \geq 0.5$
when $z \geq 0$

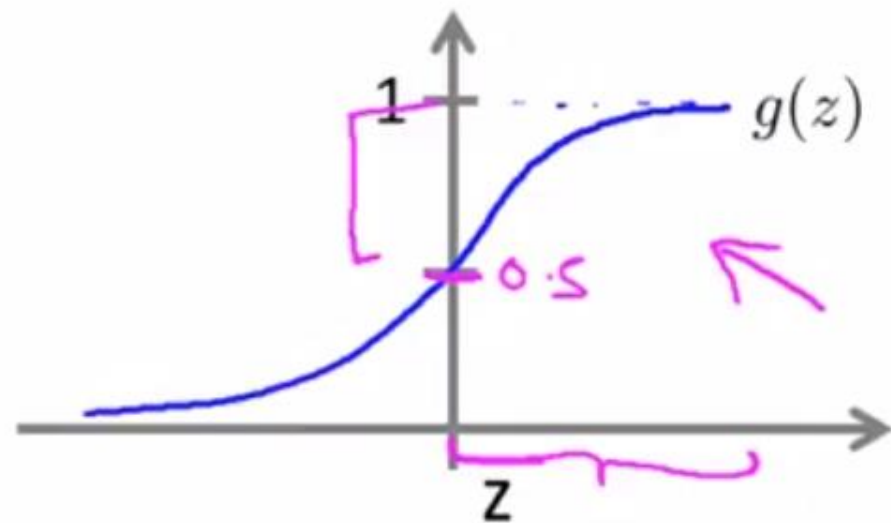
Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x) = p(y=1|x;\theta)$$

$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$

Suppose predict " $y = 1$ " if $h_{\theta}(x) \geq 0.5$

predict " $y = 0$ " if $h_{\theta}(x) < 0.5$



$$g(z) \geq 0.5$$

when $z \geq 0$

$$h_{\theta}(x) = g(\theta^T x) \geq 0.5$$

whenever $\theta^T x \geq 0$

↑
 z

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x) = p(y=1|x;\theta)$$

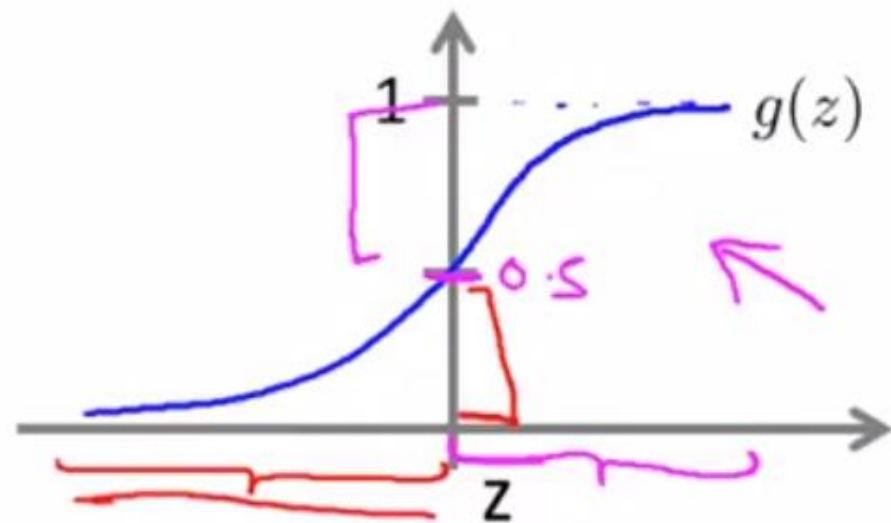
$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$

Suppose predict "y = 1" if $h_{\theta}(x) \geq 0.5$

$$\theta^T x \geq 0$$

predict "y = 0" if $h_{\theta}(x) < 0.5$

$$g(z) < 0.5$$



$$g(z) \geq 0.5$$

when $z \geq 0$

$$h_{\theta}(x) = g(\theta^T x) \geq 0.5$$

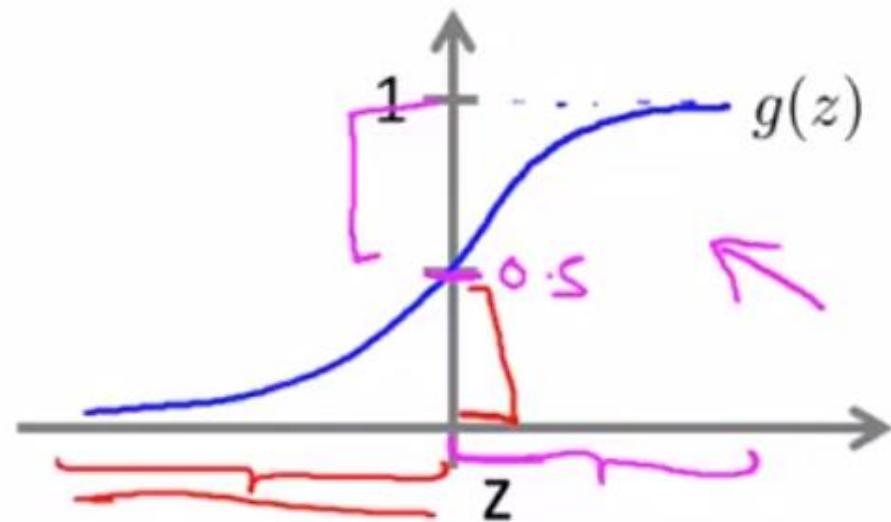
whenever $\theta^T x \geq 0$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Logistic regression

$$\rightarrow h_{\theta}(x) = g(\theta^T x) = p(y=1|x;\theta)$$

$$\rightarrow g(z) = \frac{1}{1+e^{-z}}$$



Suppose predict "y = 1" if $h_{\theta}(x) \geq 0.5$

$$\theta^T x \geq 0$$

predict "y = 0" if $h_{\theta}(x) < 0.5$

$$h_{\theta}(x) = g(\theta^T x)$$

$$\theta^T x < 0$$

$$g(z) \geq 0.5$$

when $z \geq 0$

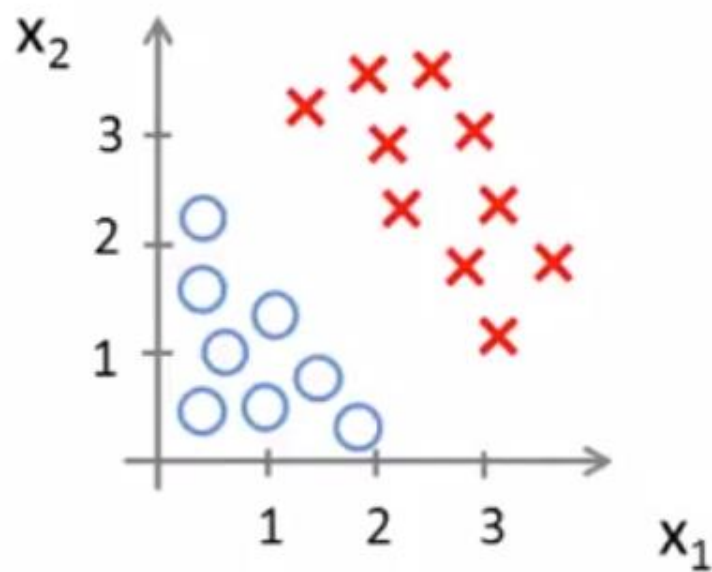
$$h_{\theta}(x) = g(\theta^T x) \geq 0.5$$

whenever $\theta^T x \geq 0$

$$g(z) < 0.5$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

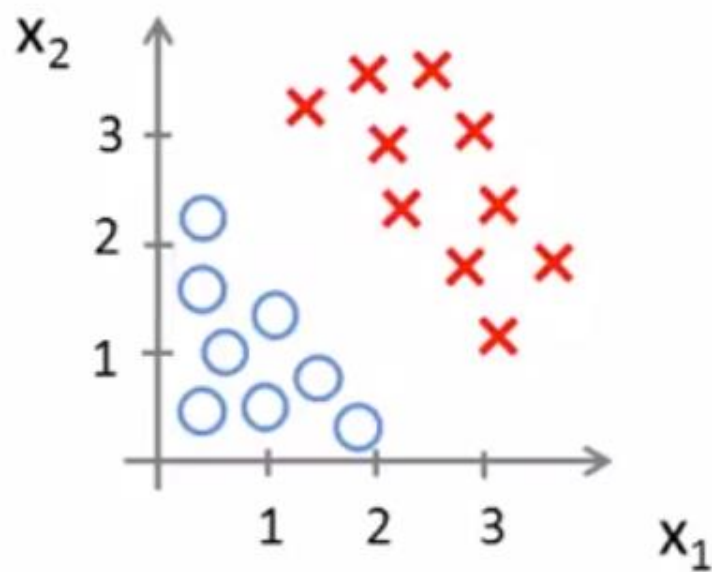
Decision Boundary



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

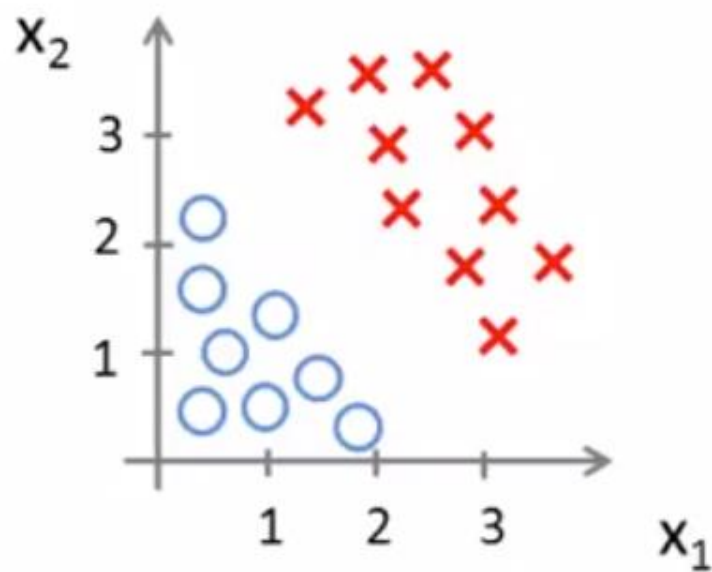
Decision Boundary



$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{//} x_1 + \underbrace{\theta_2}_{//} x_2)$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary

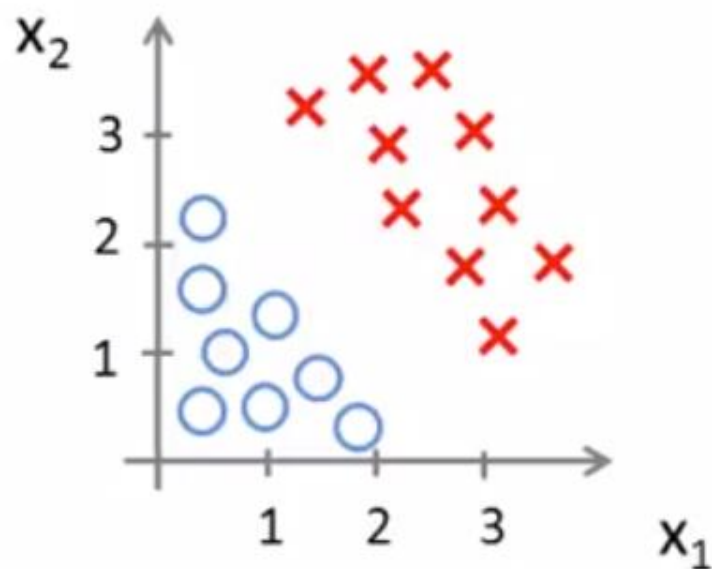


$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary

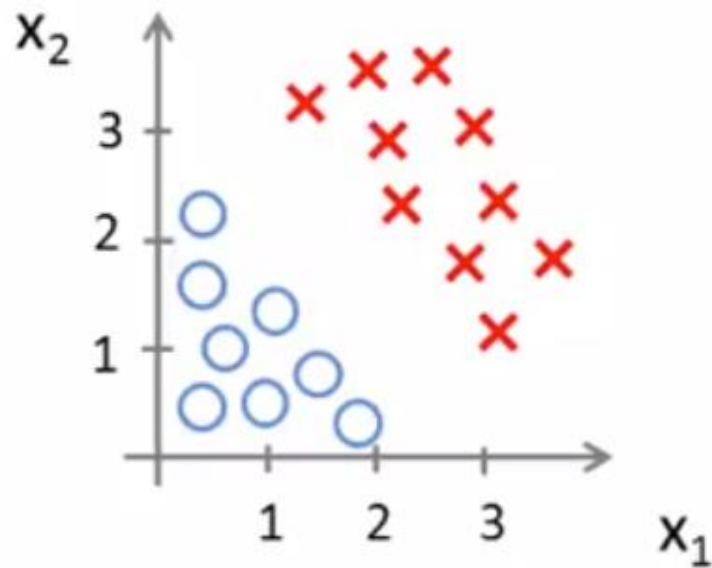


$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

Decision Boundary

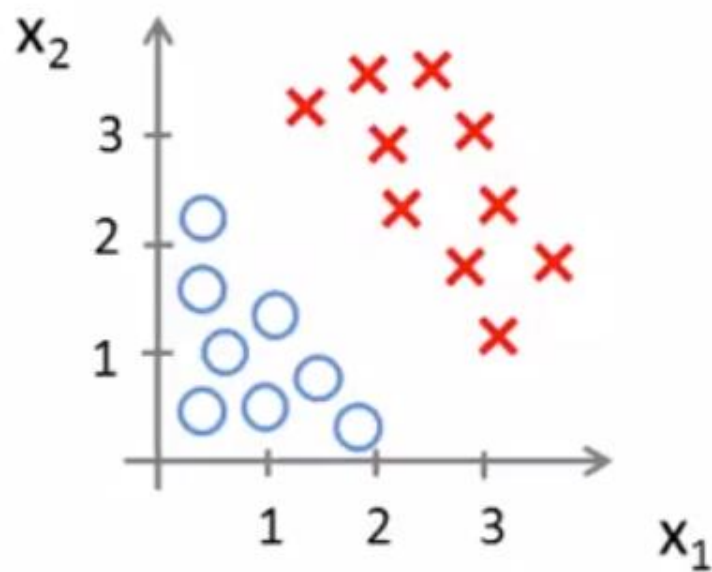


$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$
 $\theta^T x$

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

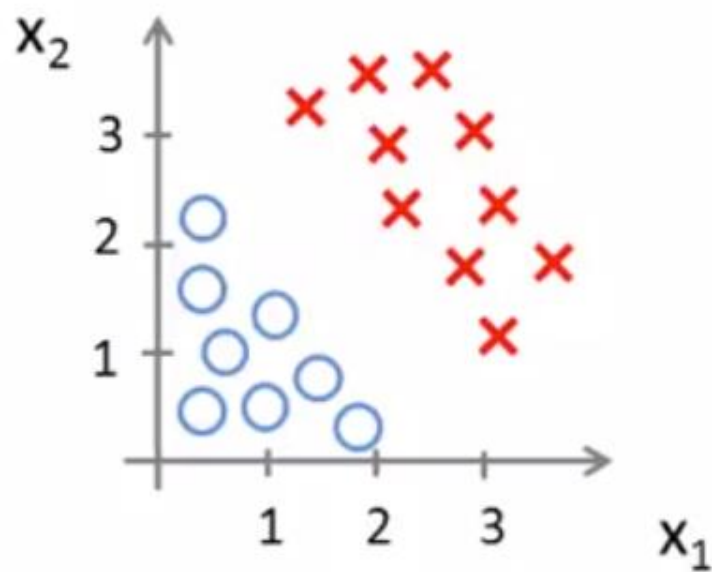
$$\theta^T x$$

$$\rightarrow x_1 + x_2 \geq 3$$

x_1, x_2

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

$\theta^T x$

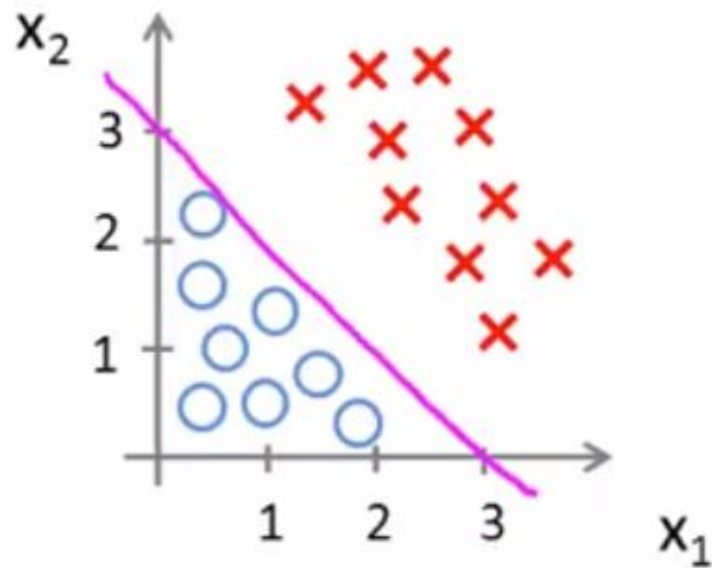
$\rightarrow x_1 + x_2 \geq 3$

x_1, x_2

$x_1 + x_2 = 3$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

$\theta^T x$

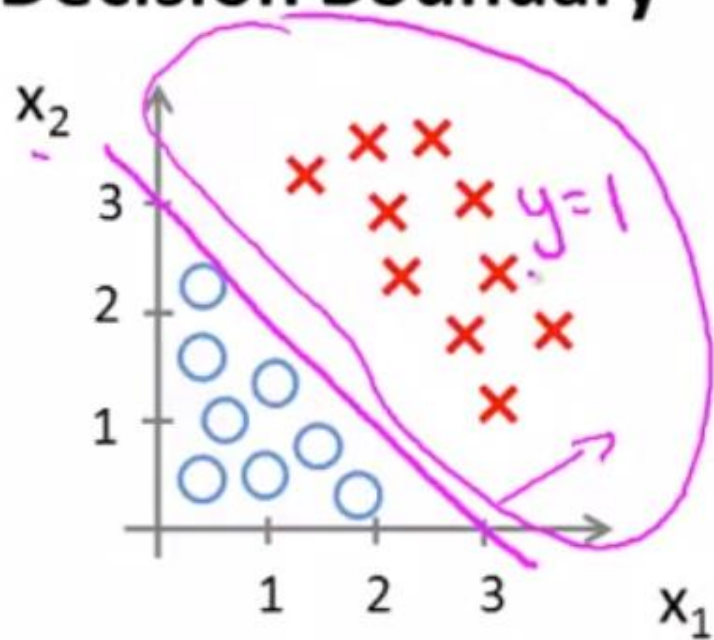
$\rightarrow x_1 + x_2 \geq 3$

x_1, x_2

$x_1 + x_2 = 3$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

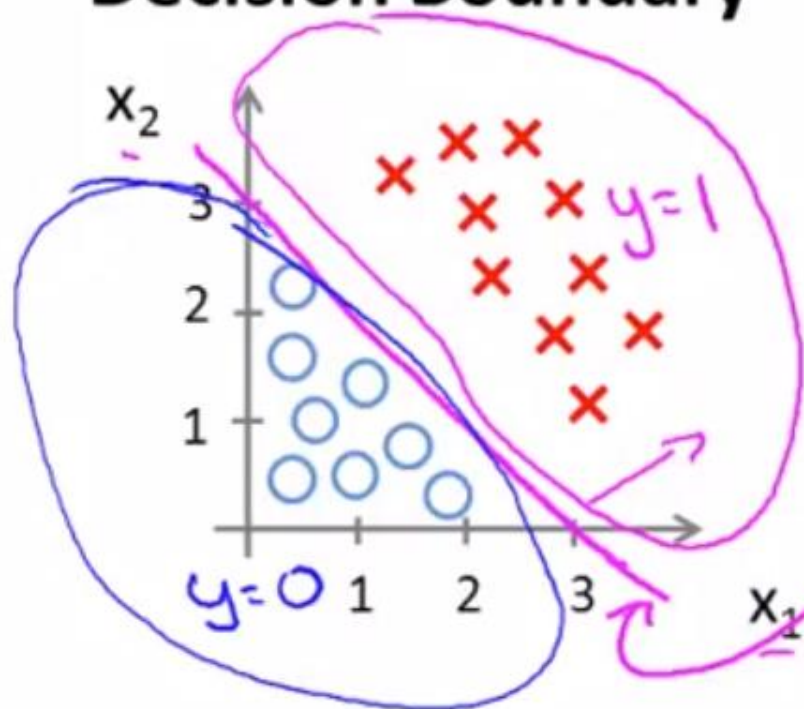
x_1, x_2

$$\underline{x_1 + x_2 = 3}$$

$$\rightarrow \underline{x_1 + x_2 \geq 3}$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$\rightarrow h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Decision boundary

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

$$\theta^T x$$

$$\rightarrow \underline{x_1 + x_2 \geq 3}$$

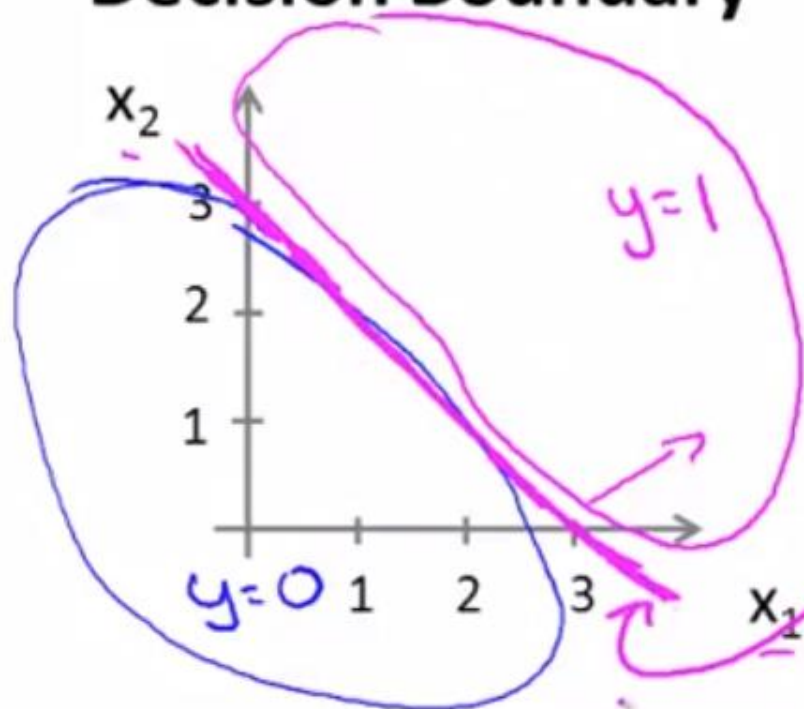
x_1, x_2

$$\underline{x_1 + x_2 = 3}$$

$$\rightarrow x_1 + x_2 < 3$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$\rightarrow h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

$$\begin{matrix} // & // & // \\ -3 & 1 & 1 \end{matrix}$$

Decision boundary

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

$$\theta^T x$$

$$\rightarrow \underline{x_1 + x_2 \geq 3}$$

x_1, x_2

$$\rightarrow h_{\theta}(x) = 0.5$$

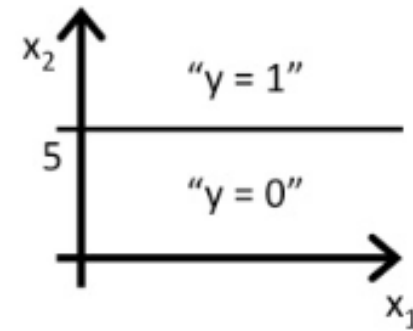
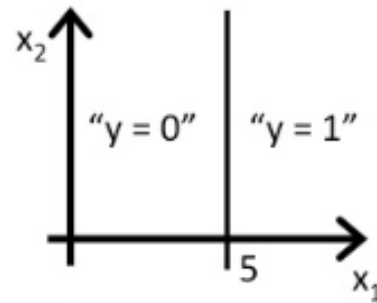
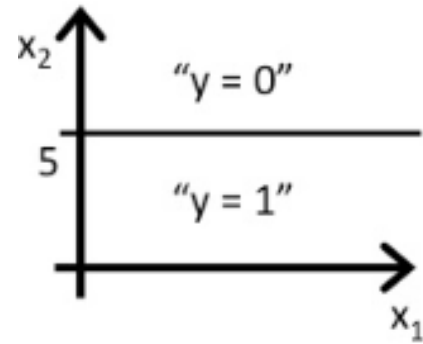
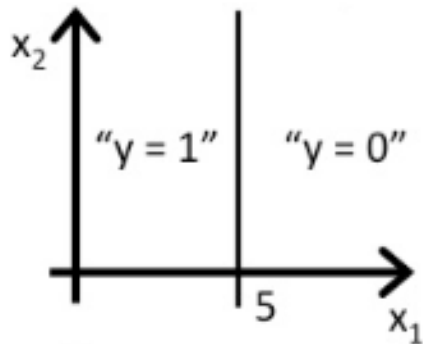
$$\boxed{x_1 + x_2 = 3}$$

$$\rightarrow \begin{matrix} x_1 + x_2 < 3 \\ y = 0 \end{matrix}$$

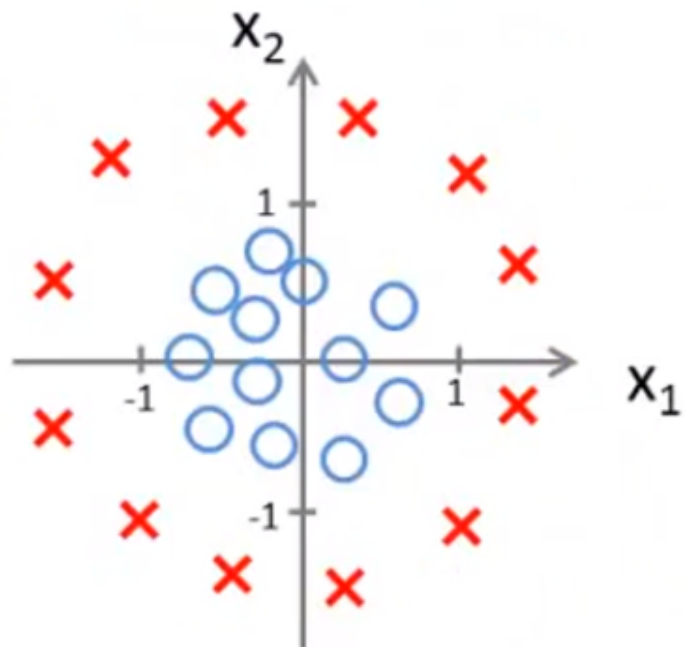
Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Exercise

- Consider logistic regression with two features x_1 and x_2 . Suppose $\theta_0 = 5$ and $\theta_1 = -1$, $\theta_2 = 0$, so that $h_\theta(x) = g(5 - x_1)$. Which of these shows the decision boundary?

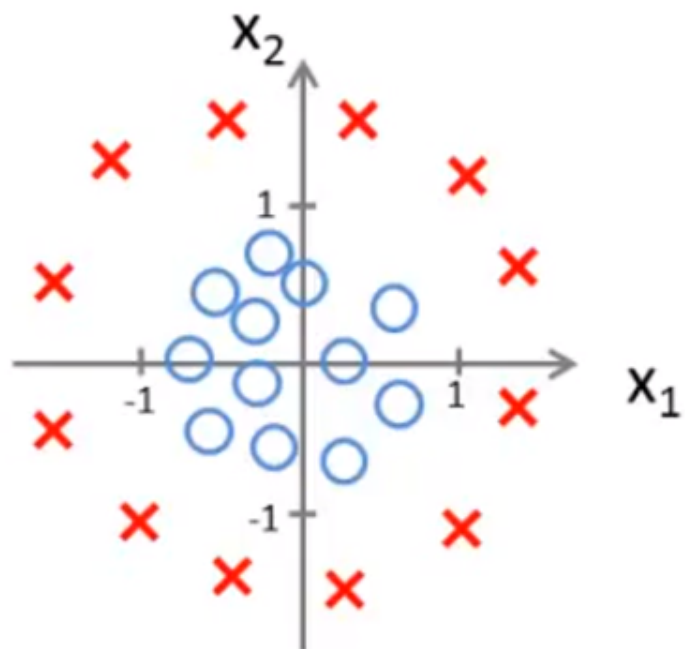


Non-linear decision boundaries



Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

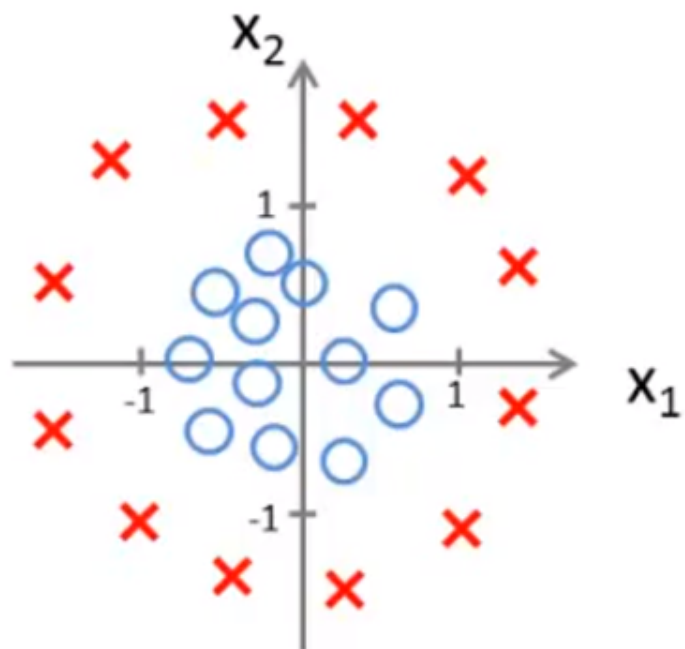
Non-linear decision boundaries



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

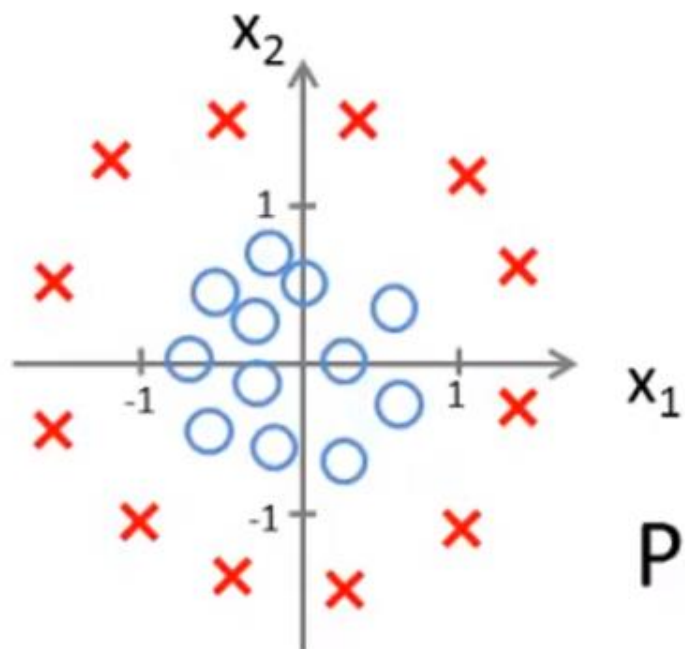
Non-linear decision boundaries



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlar'a gidin.

Non-linear decision boundaries

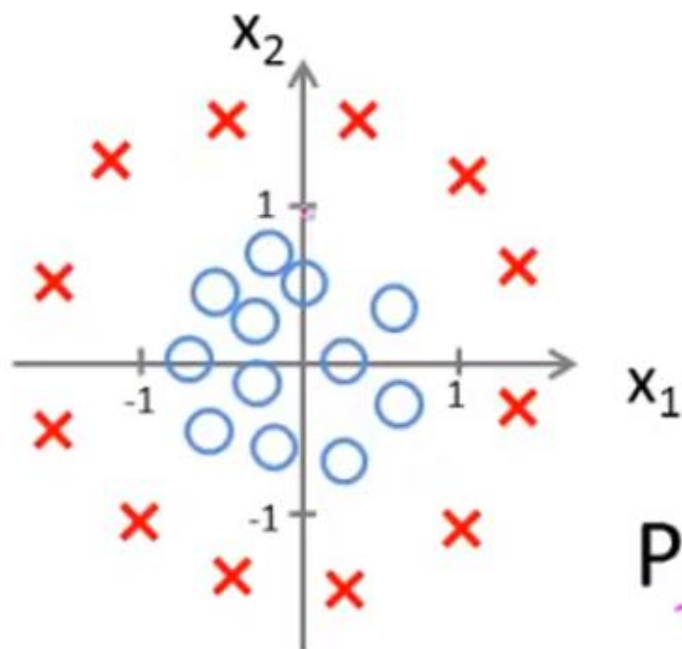


$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Handwritten blue annotations: $\theta_0 = -1$, $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 1$, $\theta_4 = 1$. The parameter vector is shown as $\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

Predict “ $y = 1$ ” if $-1 + x_1^2 + x_2^2 \geq 0$

Non-linear decision boundaries



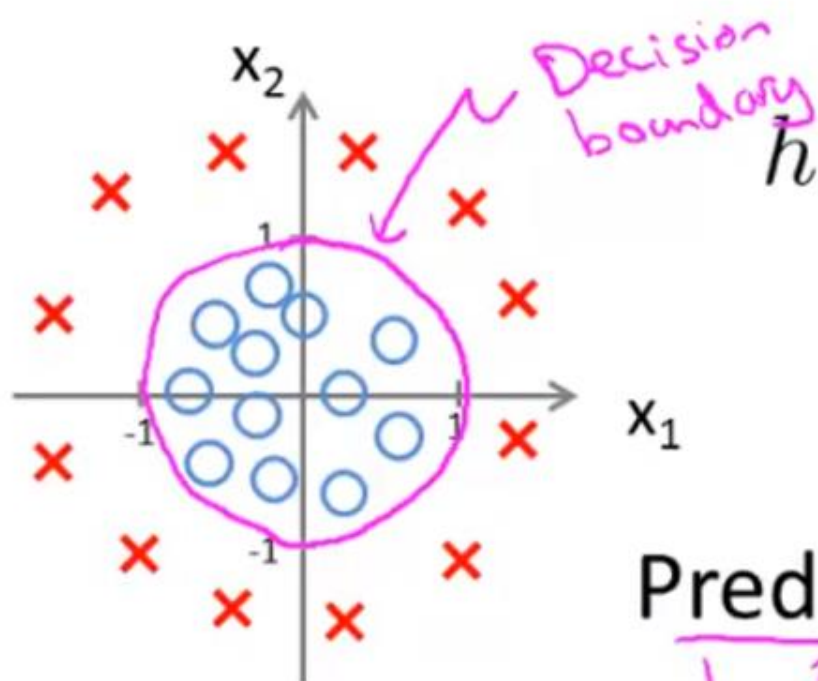
$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Handwritten annotations: $\theta_0 = -1$, $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 1$, $\theta_4 = 1$. The parameter vector is $\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

Predict " $y = 1$ " if $-1 + x_1^2 + x_2^2 \geq 0$

Handwritten annotations: $x_1^2 + x_2^2 = 1$ (in a pink box), $x_1^2 + x_2^2 \geq 1$ (under the inequality).

Non-linear decision boundaries



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

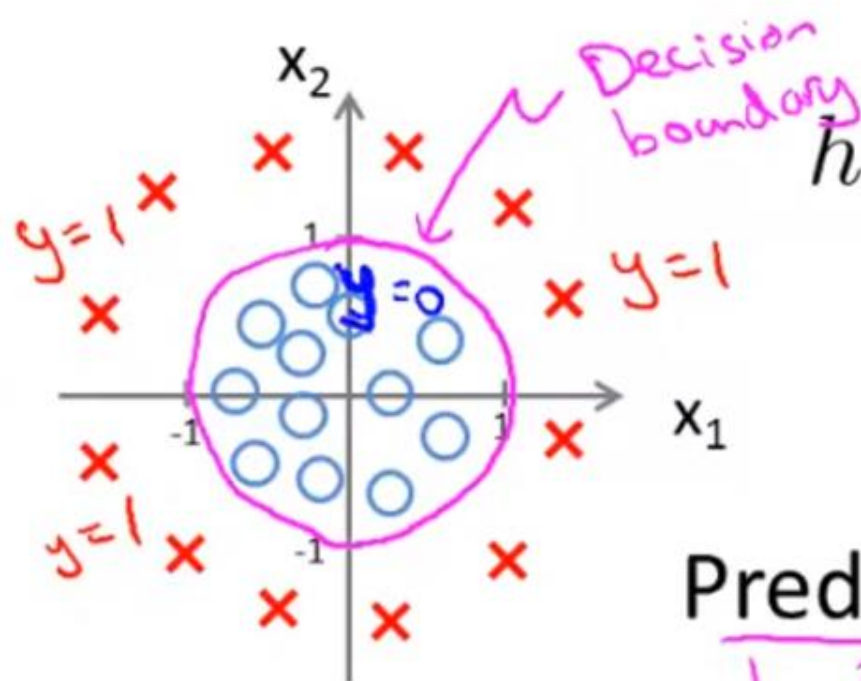
Handwritten annotations above the equation: $\theta_0 = -1$, $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 1$, $\theta_4 = 1$.

$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Predict " $y = 1$ " if $-1 + x_1^2 + x_2^2 \geq 0$

Handwritten notes below the inequality: $x_1^2 + x_2^2 = 1$ (in a magenta box) and $x_1^2 + x_2^2 \geq 1$ (with a red arrow pointing to the inequality).

Non-linear decision boundaries



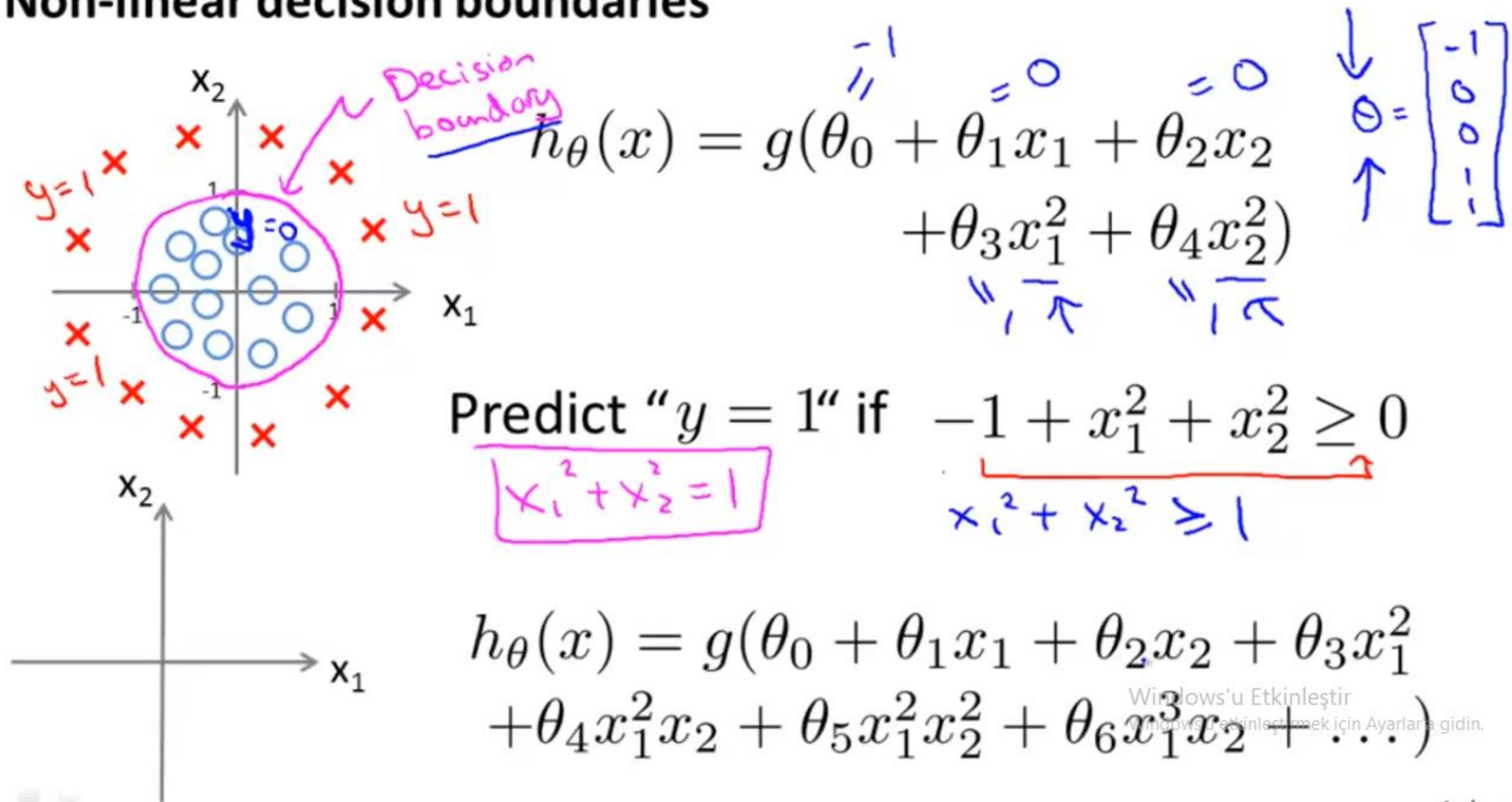
$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

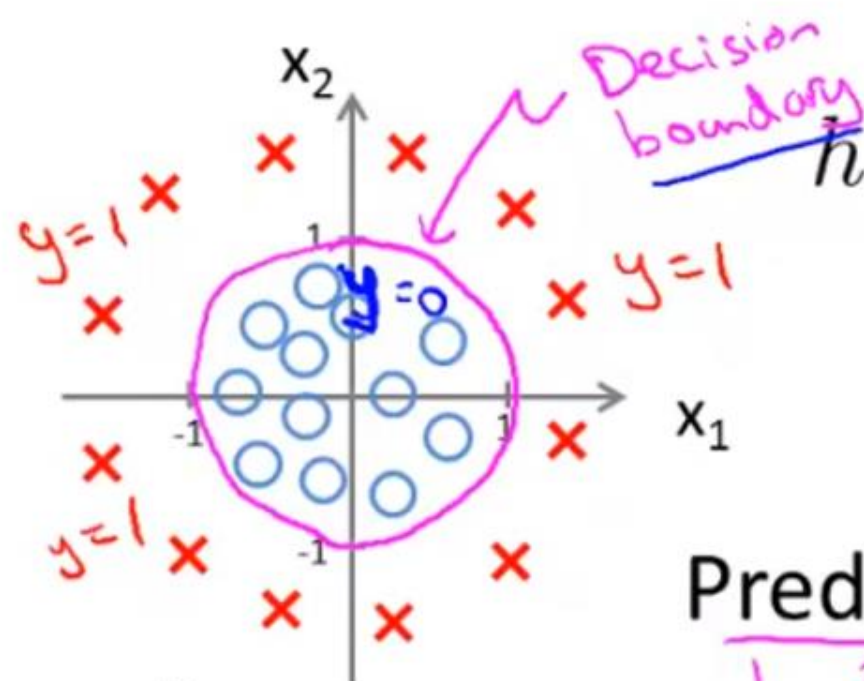
Predict " $y = 1$ " if $-1 + x_1^2 + x_2^2 \geq 0$

$x_1^2 + x_2^2 \geq 1$

Non-linear decision boundaries



Non-linear decision boundaries



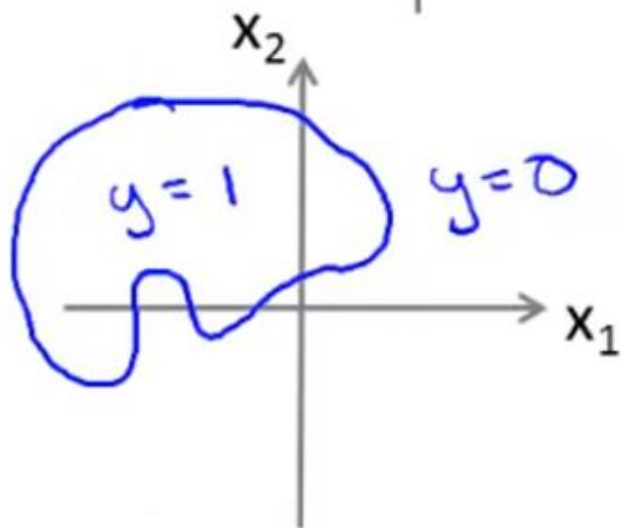
$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

$\begin{matrix} -1 & =0 & =0 \\ \uparrow & & \end{matrix}$
 $\begin{matrix} \theta_0 & \theta_1 & \theta_2 \\ \theta_3 & \theta_4 \end{matrix}$

$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Predict "y = 1" if $-1 + x_1^2 + x_2^2 \geq 0$

$\boxed{x_1^2 + x_2^2 = 1}$
 $\underbrace{-1 + x_1^2 + x_2^2}_{x_1^2 + x_2^2 \geq 1} \geq 0$



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_1^2 x_2 + \theta_5 x_1^2 x_2^2 + \theta_6 x_1^3 x_2 + \dots)$$

Windows'u Etkinleştir
Windows'u etkinleştirmek için Ayarlara gidin.