

SOLUTION OF NETWORK

Shortest-Path PROBLEMS

Model Description

- The shortest-path problem is a particular network model that has received a great deal of attention for both practical and theoretical reasons.
- The essence of the problem can be stated as follows: Given a network with distance c_{ij} (or travel time, or cost, etc.) associated with each arc, find a path through the network from a particular origin (source) to a particular destination (sink) that has the shortest total distance.

General Model Notation

In general, the formulation of the shortest-path problem is as follows:

Minimize $z = \sum_i \sum_j c_{ij} x_{ij} ,$

subject to:

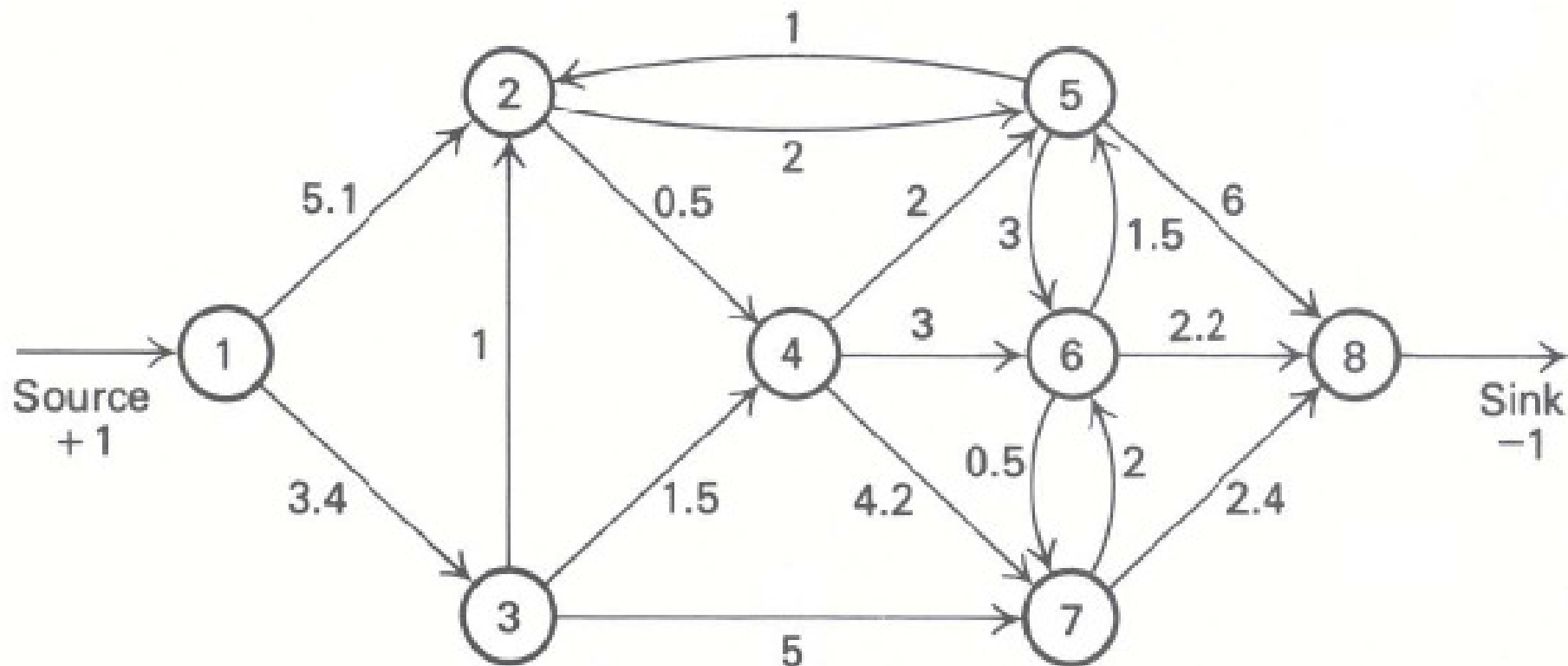
$$\sum_j x_{ij} - \sum_k x_{ki} \begin{cases} 1 & \text{if } i = s(\text{source}) \\ 0 & \text{otherwise} \\ -1 & \text{if } i = t(\text{sink}) \end{cases}$$

$x_{ij} \geq 0$ for all arcs $i-j$ in the network

Formulating Shortest-Path Problems

Example :

Find the shortest distance from node 1 to node 8 given in Figure. The numbers next to the arcs are the distance over, or cost of using, that arc.



Node–Arc Incidence Tableau for this Shortest-Path Problem

Node	x12	x13	x24	x25	x32	x34	x37	x45	x46	x47	x52	x56	x58	x65	x67	x68	x76	x78	Relti	RHS
<i>Node 1</i>	1	1																	=	1
<i>Node 2</i>	-1		1	1	-1						-1								=	0
<i>Node 3</i>		-1			1	1	1												=	0
<i>Node 4</i>			-1			-1		1	1	1									=	0
<i>Node 5</i>				-1				-1			1	1	1	-1					=	0
<i>Node 6</i>									-1			-1		1	1	1	-1		=	0
<i>Node 7</i>							-1			-1					-1		1	1	=	0
<i>Node 8</i>													-1			-1		-1	=	-1
<i>Distance</i>	5,1	3,4	0,5	2,0	1,0	1,5	5,0	2,0	3,0	4,2	1,0	3,0	6,0	1,5	0,5	2,2	2,0	2,4	=	z(min)

LP Formulation of Shortest Path Problem

$$\text{Min } Z = 5.1 x_{12} + 3.4 x_{13} + 0.5 x_{24} + 2.0 x_{25} + 1.0 x_{32} + 1.5 x_{34} + 5.0 x_{37} + 2.0 x_{45} + 3.0 x_{46} + 4.2 x_{47} + 1.0 x_{52} + 3.0 x_{56} + 6.0 x_{58} + 1.5 x_{65} + 0.5 x_{67} + 2.2 x_{68} + 2.0 x_{76} + 2.4 x_{78}$$

S.T.:

$$x_{12} + x_{13} = 1$$

$$-x_{12} + x_{24} + x_{25} - x_{32} - x_{52} = 0$$

$$-x_{13} + x_{32} + x_{34} + x_{37} = 0$$

$$-x_{24} - x_{34} + x_{45} + x_{46} + x_{47} = 0$$

$$-x_{25} - x_{45} + x_{52} + x_{56} + x_{58} - x_{65} = 0$$

$$-x_{46} - x_{56} + x_{65} + x_{67} + x_{68} - x_{76} = 0$$

$$-x_{37} - x_{47} - x_{67} + x_{76} + x_{78} = 0$$

$$-x_{58} - x_{68} - x_{78} = -1$$

$$x_{ij} \geq 0$$

Shortest Path Problem

Shortest Path Problem

Shortest Path																																																							
Variables:																																																							
Ark	<table><tr><th>x₁₂</th><th>x₁₃</th><th>x₂₄</th><th>x₂₅</th><th>x₃₂</th><th>x₃₄</th><th>x₃₇</th><th>x₄₅</th><th>x₄₆</th><th>x₄₇</th><th>x₅₂</th><th>x₅₆</th><th>x₅₈</th><th>x₆₅</th><th>x₆₇</th><th>x₆₈</th><th>x₇₆</th><th>x₇₈</th></tr><tr><td>Value</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	x ₁₂	x ₁₃	x ₂₄	x ₂₅	x ₃₂	x ₃₄	x ₃₇	x ₄₅	x ₄₆	x ₄₇	x ₅₂	x ₅₆	x ₅₈	x ₆₅	x ₆₇	x ₆₈	x ₇₆	x ₇₈	Value	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		
x ₁₂	x ₁₃	x ₂₄	x ₂₅	x ₃₂	x ₃₄	x ₃₇	x ₄₅	x ₄₆	x ₄₇	x ₅₂	x ₅₆	x ₅₈	x ₆₅	x ₆₇	x ₆₈	x ₇₆	x ₇₈																																						
Value	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																																						
Distance	<table><tr><td>5,1</td><td>3,4</td><td>0,5</td><td>2</td><td>1</td><td>1,5</td><td>5</td><td>2</td><td>3</td><td>4,2</td><td>1</td><td>3</td><td>6</td><td>1,5</td><td>0,5</td><td>2,2</td><td>2</td><td>2,4</td></tr></table>	5,1	3,4	0,5	2	1	1,5	5	2	3	4,2	1	3	6	1,5	0,5	2,2	2	2,4																																				
5,1	3,4	0,5	2	1	1,5	5	2	3	4,2	1	3	6	1,5	0,5	2,2	2	2,4																																						
Objective:																																																							
5.1 x ₁₂ +3.4 x ₁₃ +0.5 x ₂₄ +2.0 x ₂₅ +1.0 x ₃₂ + 1.5 x ₃₄ +5.0 x ₃₇ +2.0 x ₄₅ +3.0 x ₄₆ +4.2 x ₄₇ + 1.0 x ₅₂ + 3.0 x ₅₆ +6.0 x ₅₈ +1.5 x ₆₅ +0.5 x ₆₇ + 2.2 x ₆₈ +2.0 x ₇₆ + 2.4 x ₇₈	<table><tr><th>Value</th></tr><tr><td>0</td></tr></table>	Value	0																																																				
Value																																																							
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Constrain																																																							
x ₁₂ +x ₁₃ =1	<table><tr><th>Formul</th><th>Value</th></tr><tr><td>0</td><td>1</td></tr><tr><td>-X₁₂+ X₂₄+ X₂₅ -X₃₂ - X₅₂ =0</td><td>0</td></tr><tr><td>-X₁₃ + X₃₂ +X₃₄ +X₃₇ =0</td><td>0</td></tr><tr><td>- x₂₄ - x₃₄ +x₄₅+ x₄₆+ x₄₇ =0</td><td>0</td></tr><tr><td>- x₂₅ - x₄₅ + x₅₂+ x₅₆ +x₅₈- x₆₅ =0</td><td>0</td></tr><tr><td>- x₄₆ - x₅₆ + x₆₅+ x₆₇+ x₆₈ -x₇₆ =0</td><td>0</td></tr><tr><td>- x₃₇ - x₄₇ - x₆₇ + x₇₆ x₇₈=0</td><td>0</td></tr><tr><td>- x₅₈ - x₆₈ - x₇₈=-1</td><td>0</td></tr><tr><td>x₁₂</td><td>0</td></tr><tr><td>x₁₃</td><td>0</td></tr><tr><td>x₂₄</td><td>0</td></tr><tr><td>x₂₅</td><td>0</td></tr><tr><td>x₃₂</td><td>0</td></tr><tr><td>x₃₄</td><td>0</td></tr><tr><td>x₃₇</td><td>0</td></tr><tr><td>x₄₅</td><td>0</td></tr><tr><td>x₄₆</td><td>0</td></tr><tr><td>x₄₇</td><td>0</td></tr><tr><td>x₅₂</td><td>0</td></tr><tr><td>x₅₆</td><td>0</td></tr><tr><td>x₅₈</td><td>0</td></tr><tr><td>x₆₅</td><td>0</td></tr><tr><td>x₆₇</td><td>0</td></tr><tr><td>x₆₈</td><td>0</td></tr><tr><td>x₇₆</td><td>0</td></tr><tr><td>x₇₈</td><td>0</td></tr></table>	Formul	Value	0	1	-X ₁₂ + X ₂₄ + X ₂₅ -X ₃₂ - X ₅₂ =0	0	-X ₁₃ + X ₃₂ +X ₃₄ +X ₃₇ =0	0	- x ₂₄ - x ₃₄ +x ₄₅ + x ₄₆ + x ₄₇ =0	0	- x ₂₅ - x ₄₅ + x ₅₂ + x ₅₆ +x ₅₈ - x ₆₅ =0	0	- x ₄₆ - x ₅₆ + x ₆₅ + x ₆₇ + x ₆₈ -x ₇₆ =0	0	- x ₃₇ - x ₄₇ - x ₆₇ + x ₇₆ x ₇₈ =0	0	- x ₅₈ - x ₆₈ - x ₇₈ =-1	0	x ₁₂	0	x ₁₃	0	x ₂₄	0	x ₂₅	0	x ₃₂	0	x ₃₄	0	x ₃₇	0	x ₄₅	0	x ₄₆	0	x ₄₇	0	x ₅₂	0	x ₅₆	0	x ₅₈	0	x ₆₅	0	x ₆₇	0	x ₆₈	0	x ₇₆	0	x ₇₈	0
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x ₇₆	0																																																						
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Shortest Path Problem

Shortest Path Problem

Variables:

Ark	x12	x13	x24	x25	x32	x34	x37	x45	x46	x47	x52	x56	x58	x65	x67	x68	x76	x78
Value	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Distance

	5,1	3,4	0,5	2	1	1,5	5	2	3	4,2	1	3	6	1,5	0,5	2,2	2	2,4
--	-----	-----	-----	---	---	-----	---	---	---	-----	---	---	---	-----	-----	-----	---	-----

Objective:

$$5.1 x_{12} + 3.4 x_{13} + 0.5 x_{24} + 2.0 x_{25} + 1.0 x_{32} + 1.5 x_{34} + 5.0 x_{37} + 2.0 x_{45} + 3.0 x_{46} + 4.2 x_{47} + 1.0 x_{52} + 3.0 x_{56} + 6.0 x_{58} + 1.5 x_{65} + 0.5 x_{67} + 2.2 x_{68} + 2.0 x_{76} + 2.4 x_{78}$$

Constrain

Formula	Value
$x_{12} + x_{13} = 1$	1
$-x_{12} + x_{24} + x_{25} - x_{32} - x_{52} = 0$	0
$-x_{13} + x_{32} + x_{34} + x_{37} = 0$	0
$-x_{24} - x_{34} + x_{45} + x_{46} + x_{47} = 0$	0
$-x_{25} - x_{45} + x_{52} + x_{56} + x_{58} - x_{65} = 0$	0
$-x_{46} - x_{56} + x_{65} + x_{67} + x_{68} - x_{76} = 0$	0
$-x_{37} - x_{47} - x_{67} + x_{76} + x_{78} = 0$	0
$-x_{58} - x_{68} - x_{78} = -1$	-1
x_{12}	0
x_{13}	0
x_{24}	0
x_{25}	0
x_{32}	0
x_{34}	0
x_{37}	0
x_{45}	0
x_{46}	0
x_{47}	0
x_{52}	0
x_{56}	0
x_{58}	0
x_{65}	0
x_{67}	0
x_{68}	0
x_{76}	0

Çözücü Parametreleri

Hedef Hücre: $\$C\9

Eşittir: ☐ En Büyük ☒ En Küçük ☐ Değer: 0

Değişen Hücreler: arc

Kısıtlamalar:

- $\$B\$12 = \$C\12
- $\$B\$13 = \$C\13
- $\$B\$14 = \$C\14
- $\$B\$15 = \$C\15
- $\$B\$16 = \$C\16
- $\$B\$17 = \$C\17

Buttons: Çöz, Kapat, Tahmin, Seçenekler, Ekle, Değiştir, Tümünü Sıfır, Sil, Yardım

Shortest Path Problem

Shortest Path

Variables:

Ark

Value

Distance

	x12	x13	x24	x25	x32	x34	x37	x45	x46	x47	x52	x56	x58	x65	x67	x68	x76	x78
	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
	5,1	3,4	0,5	2	1	1,5	5	2	3	4,2	1	3	6	1,5	0,5	2,2	2	2,4

Objective:

5.1 x12+3.4 x13+0.5 x24+2.0 x25 +1.0 x32+
1.5 x34+5.0 x37 +2.0 x45 +3.0 x46 +4.2 x47 +
1.0 x52+ 3.0 x56 +6.0 x58 +1.5 x65 +0.5 x67 +
2.2 x68 +2.0 x76+ 2.4 x78

Value

10,8

Constrain

Formula

Value

x12+x13 =1
-X12+ X24+ X25 -X32 - X52 =0
-X13 + X32 +X34 +X37 =0
- x24 - x34 +x45+ x46+ x47 =0
- x25 - x45 + x52+ x56 +x58- x65 =0
- x46 - x56 + x65+ x67+ x68 -x76 =0
- x37 - x47 - x67 + x76 x78=0
- x58 - x68 - x78=-1

-1 -1

x12

1 0

x13

0 0

x24

1 0

x25

0 0

x32

0 0

x34

0 0

x37

0 0

x45

0 0

x46

1 0

x47

0 0

x52

0 0

x56

0 0

x58

0 0

x65

0 0

x67

0 0

x68

1 0

Çözücü Parametreleri

Hedef Hücre:

\$C\$9

Eşittir:

☐ En Büyük

☒ En Küçük

☐ Değer:

0

Değişen Hücreler:

arc

Kısıtlamalar:

\$B\$12 = \$C\$12
\$B\$13 = \$C\$13
\$B\$14 = \$C\$14
\$B\$15 = \$C\$15
\$B\$16 = \$C\$16
\$B\$17 = \$C\$17

Çöz

Kapat

Tahmin

Seçenekler

Ekle

Değiştir

Tümünü Sıfırla

Sil

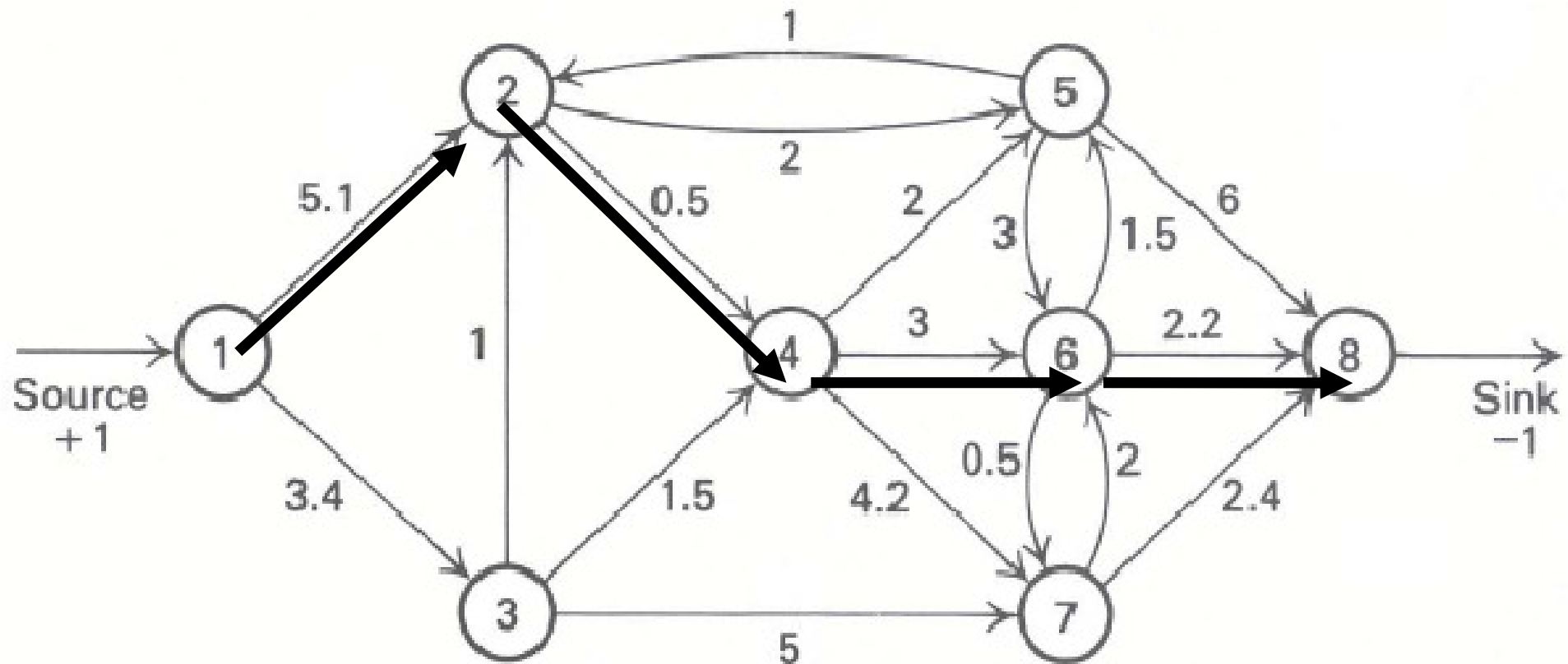
Yardım

Shortest Path Problem

1	Microsoft Excel 12.0 Yanıt Raporu			
2	Çalışma Sayfası: [Modeling_ShortestP_SP.xls]Cözüm_2			
3	Rapor Oluşturuldu: 07.12.2009 21:23:43			
4	Hedef Hücre (En Küçük)			
5	Hücre	Ad	İlk Değer	Son Değer
	$5.1 \times 12 + 3.4 \times 13 + 0.5 \times 24 + 2.0 \times 25 + 1.0 \times 32 +$ $1.5 \times 34 + 5.0 \times 37 + 2.0 \times 45 + 3.0 \times 46 + 4.2 \times 47 +$ $1.0 \times 52 + 3.0 \times 56 + 6.0 \times 58 + 1.5 \times 65 + 0.5 \times 67 +$ $2.2 \times 68 + 2.0 \times 76 + 2.4 \times 78$ Value			
6	\$C\$9		0	10,8000108
7	Ayarlanabilir Hücreler			
8	Hücre	Ad	İlk Değer	Son Değer
9	\$B\$4	Value x12	0	1,000001
10	\$C\$4	Value x13	0	0
11	\$D\$4	Value x24	0	1,000001
12	\$E\$4	Value x25	0	0
13	\$F\$4	Value x32	0	0
14	\$G\$4	Value x34	0	0
15	\$H\$4	Value x37	0	0
16	\$I\$4	Value x45	0	0
17	\$J\$4	Value x46	0	1,000001
18	\$K\$4	Value x47	0	0
19	\$L\$4	Value x52	0	0
20	\$M\$4	Value x56	0	0
21	\$N\$4	Value x58	0	0
22	\$O\$4	Value x65	0	0
23	\$P\$4	Value x67	0	0
24	\$Q\$4	Value x68	0	1,000001
25	\$R\$4	Value x76	0	0
26	\$S\$4	Value x78	0	0
27				
28				

Shortest Path Problem

	x_{12}	x_{13}	x_{24}	x_{25}	x_{32}	x_{34}	x_{37}	x_{45}	x_{46}	x_{47}	x_{52}	x_{56}	x_{58}	x_{65}	x_{67}	x_{68}	x_{76}	x_{78}
Value	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
	5,1	3,4	0,5	2	1	1,5	5	2	3	4,2	1	3	6	1,5	0,5	2,2	2	2,4



Shortest Path Problem

Equipment Replacement

Rent Car is developing a replacement plan for its car fleet for a 5-year (1996 to 2002) planning horizon. At the start of each year, a decision is made as to whether a car should be kept in operation or replaced. A car must be in service at least 2 year but must be replaced after 4 years. The following table provides the replacement cost as a function of the year a car is acquired and the number of years in operation.

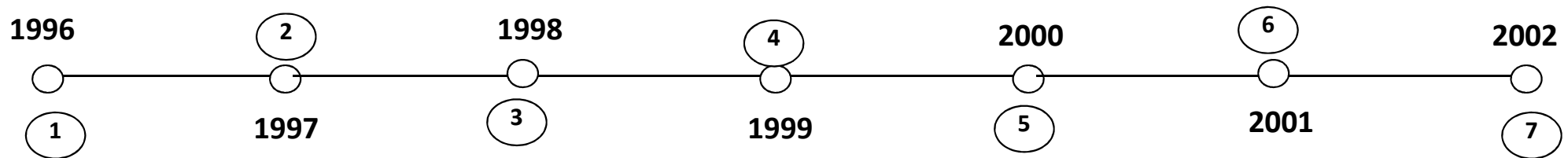
Replacement cost (\$) for given years in operation

Year acquired	2	3	4
1996	3800	4100	6800
1997	4000	4800	7000
1998	4200	5100	7200
1999	4800	5700	---
2000	5300	---	---

SP-Equipment Replacement

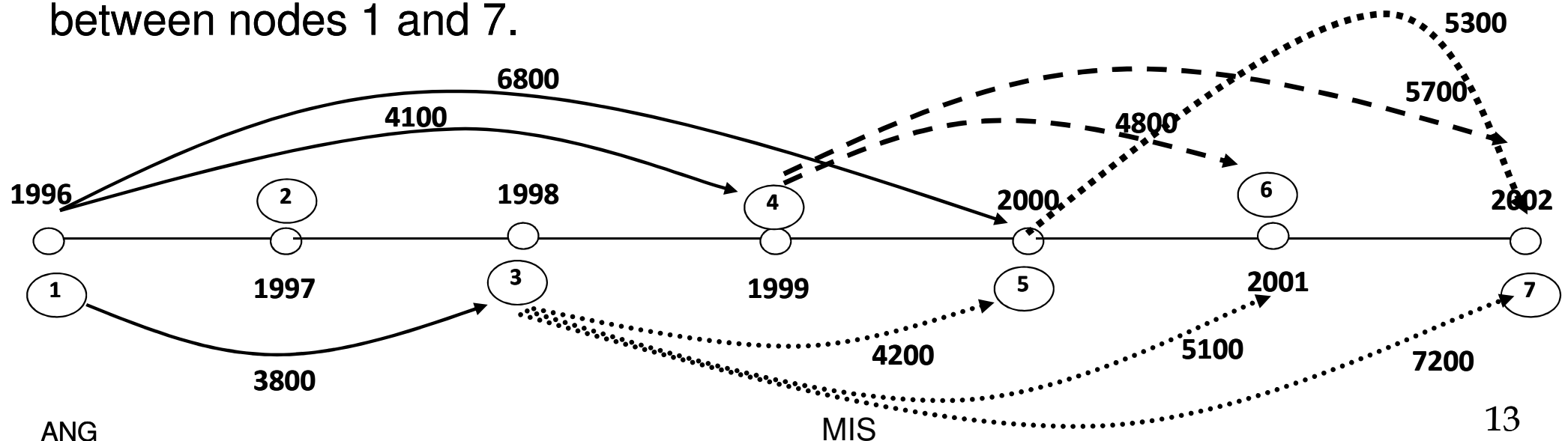
Definition of Problem:

The problem can be formulated as a network in which nodes 1 to 7 represent years 1996 to 2002.



Arcs from node 1 (year 1996) can reach only nodes 3, 4, and 5 because a car must be in operation between 2 and 4 years.

The solution of the problem is equivalent to finding the shortest route between nodes 1 and 7.



Equipment Replacement

Shortest Path-Equipment Replacement

Variables:

Arc	x_{13}	x_{14}	x_{15}	x_{35}	x_{36}	x_{37}	x_{46}	x_{47}	x_{57}
Value	0	0	0	0	0	0	0	0	0
Cost	3800	4100	6800	4200	5100	7200	4800	5700	5300

Objective:

$3800 x_{13} + 4100 x_{14} + 6800 x_{15} + 4200 x_{35} +$
 $5100 x_{36} + 7200 x_{37} + 4800 x_{46} + 5700 x_{47} +$
 $5300 x_{57}$

Value

Constrain

Formula Value

$x_{13} + x_{14} + x_{15} = 1$

0 1

$-x_{13} + x_{35} + x_{36} + x_{37} = 0$

0 0

$-x_{14} + x_{46} + x_{47} = 0$

0 0

$-x_{15} - x_{35} + x_{57} = 0$

0 0

$-x_{36} - x_{46} = 0$

0 0

$-x_{37} - x_{47} - x_{57} = -1$

0 -1

x_{13}

0 0

x_{14}

0 0

x_{15}

0 0

x_{35}

0 0

x_{36}

0 0

x_{37}

0 0

x_{46}

0 0

x_{47}

0 0

x_{57}

0 0

Equipment Replacement

Shortest Path-Equipment Replacement										
Variables:										
Arc	x13	x14	x15	x35	x36	x37	x46	x47	x57	
Value	0	0	0	0	0	0	0	0	0	
Cost	3800	4100	6800	4200	5100	7200	4800	5700	5300	
Objective:										
3800 x13 + 4100 x14 + 6800 x15 + 4200 x35 + 5100 x36 + 7200 x37 + 4800 x46 + 5700 x47 + 5300 x57										
Constrain										
x13+x14+x15 =1	0	1								
-X13+ X35+ X36 + X37 =0	0	0								
-X14 + X46 +X7 =0	0	0								
- x15 - x35 +x57 =0	0	0								
- x36 - x46 =0	0	0								
- x37 - x47 - x57=-1	0	-1								
x13	0	0								
x14	0	0								
x15	0	0								
x35	0	0								
x36	0	0								
x37	0	0								
x46	0	0								
x47	0	0								
x57	0	0								

Çözücü Parametreleri

Hedef Hücre:

Eşittir: ☐ En Büyük ☒ En Küçük ☐ Değer:

Değişen Hücreler:

Kısıtlamalar:

- \$B\$12 = \$C\$12
- \$B\$13 = \$C\$13
- \$B\$14 = \$C\$14
- \$B\$15 = \$C\$15
- \$B\$16 = \$C\$16
- \$B\$17 = \$C\$17

Ekle Değiştir Sil

Çöz Kapat Seçenekler Tümünü Sıfırla Yardım

Equipment Replacement

Shortest Path-Equipment Replacement

Variables:

Arc	x13	x14	x15	x35	x36	x37	x46	x47	x57
Value	0	1	0	0	0	0	0	1	0
Cost	3800	4100	6800	4200	5100	7200	4800	5700	5300

Objective:

3800 x13 + 4100 x14 + 6800 x15 + 4200 x35 +
5100 x36 + 7200 x37 + 4800 x46 + 5700 x47 +
5300 x57

Value

9800

Constrain

Formule	Value
x13+x14+x15 =1	1 1
-X13+ X35+ X36 + X37 =0	0 0
-X14 + X46 +X7 =0	0 0
- x15 - x35 +x57 =0	0 0
- x36 - x46 =0	0 0
- x37 - x47 - x57=-1	-1 -1
x13	0 0
x14	1 0
x15	0 0
x35	0 0
x36	0 0
x37	0 0
x46	0 0
x47	1 0
x57	0 0

Çözücü Sonuçları

Çözücü, tüm koşulları ve sınırlamaları sağlayan bir çözüm buldu.

Raporlar

Yanıt
Duyarlılık
Sınırlamalar

☒ Çözümü Sakla

☐ Özgün Değerleri Yeniden Yükle

Tamam

İptal

Senaryo Kaydet...

Yardım

Equipment Replacement

A	B	C	D	E	
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Microsoft Excel 12.0 Yanıt Raporu

Çalışma Sayfası: [Problem_EquipmentReplacement.xls]Cözüm_2

Rapor Oluşturuldu: 05.12.2009 13:24:42

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
	3800 x13 +4100 x14 +6800 x15 +4200 x35+ 5100 x36 +7200 x37 +4800 x46 +5700 x47 +		
\$C\$9	5300 x57 Value	0	9800,0098

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
\$B\$4	Value x13	0	0
\$C\$4	Value x14	0	1,000001
\$D\$4	Value x15	0	0
\$E\$4	Value x35	0	0
\$F\$4	Value x36	0	0
\$G\$4	Value x37	0	0
\$H\$4	Value x46	0	0
\$I\$4	Value x47	0	1,000001
\$J\$4	Value x57	0	0

Sınırlamalar

Equipment Replacement

	x13	x14	x15	x35	x36	x37	x46	x47	x57
Value	0	1	0	0	0	0	0	1	0
	3800	4100	6800	4200	5100	7200	4800	5700	5300

