SOLUTION OF NETWORK

THE CRITICAL-PATH METHOD

PROBLEMS

Model Description

CPM is helpful in identifying which tasks are critical for the execution of the overall project, and in scheduling all the tasks in accordance with their prescribed *precedence relationships* so that the total project completion date is minimized, or a target date is met at minimum cost.

Consider the scheduling of tasks involved in building a house on a foundation that already exists. Determine in what sequence the tasks should be performed in order to minimize the total time required to execute the project. The tasks that need to be performed in building this particular house, their immediate predecessors, and an estimate of their duration are give in following Table

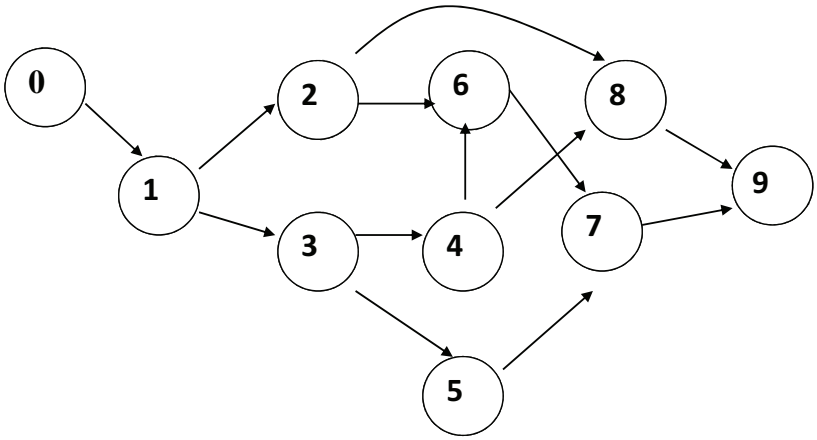
Model Description

No.	Task	Immediate predecessors	Duration	Earliest starting times
0	Start	-	0	-
1	Framing	0	2	t_1
2	Roofing	1	3	t_2
3	Siding	1	1	t_2
4	Windows	3	2.5	t_3
5	Plumbing	3	1.5	t_3
6	Electricity	2, 4	2	t_4
7	Inside Finishing	5, 6	4	t_5
8	Outside Painting	2, 4	3	t_4
9	Finish	7, 8	0	t_6

Model Description

It is useful to represent the interrelations of tasks of a given project by means of a network diagram. In this diagram, nodes represent the corresponding tasks of the project, and arcs represent the precedence relationships among tasks. The network diagram for our example is shown in following Figure.

No.	Task	Immediate predecessors
0	Start	-
1	Framing	0
2	Roofing	1
3	Siding	1
4	Windows	3
5	Plumbing	3
6	Electricity	2, 4
7	Inside Finishing	5, 6
8	Outside Painting	2, 4
9	Finish	7, 8



Task-oriented network

Model Description

As we can see, there are nine nodes in the network, each representing a given task. For this reason, this network representation is called a task- (or activity-) oriented network.

If our objective is to minimize the elapsed time of the project, we can formulate a linear programming Problem. First, we define the decision variables t_i for i = 1, 2, . . . , 6, as the earliest starting times for each of the tasks. Table gives the earliest starting times where the same earliest starting time is assigned to tasks with the same immediate predecessors. For instance, tasks 4 and 5 have task 3 as their immediate predecessor.

Letting t6 be the earliest completion time of the entire project, out objective is to minimize the project duration given by

$$\text{Minimize } t_6 - t_1,$$

subject to the precedence constraints among tasks

Model Description

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$$\text{Minimize } t_6 - t_1,$$

subject to the precedence constraints among tasks

LP Formulation of Task-Oriented Network Problem

No.	predecessors	Duration	starting times
0	-	0	-
1	0	2	t_1
2	1	3	t_2
3	1	1	t_2
4	3	2.5	t_3
5	3	1.5	t_3
6	2, 4	2	t_4
7	5, 6	4	t_5
8	2, 4	3	t_4
9	7, 8	0	t_6

these precedence relationships define the linear program given in Tableau

Minimize $t_6 - t_1$,

$t_2-t_1 \geq 2$
$t_3-t_2 \geq 3$
$t_4-t_2 \geq 1$
$t_4-t_3 \geq 2,5$
$t_5-t_3 \geq 1,5$
$t_5-t_4 \geq 2$
$t_6-t_4 \geq 3$
$t_6-t_5 \geq 4$

When this LP is solved, the minimum value gives the duration for the entire project (once you add the duration of the final task) and the start times that appear in the optimal solution will identify the activities that constrain the duration of the project.

t1	t2	t3	t4	t5	t6	Relation	RHS
-1	1					≥	2
	-1	1				≥	3
	-1		1			≥	1
		-1	1			≥	2.5
		-1		1		≥	1.5
			-1	1		≥	2
			-1		1	≥	3
				-1	1	≥	4
-1					1	=	T(min)

Task-Oriented Network Problem

CPM							
Variables:							
Ark	t1	t2	t3	t4	t5	t6	
Value	0	0	0	0	0	0	
Objective:		Value					
Minimize t6 − t1		0					
Constrain	Formula	Value					
t1	0	0					
t2-t1≥2	0	2					
t3-t2≥1	0	3					
t4-t2≥1	0	1					
t4-t3≥2,5	0	2,5					
t5-t3≥1,5	0	1,5					
t5-t4≥3	0	2					
t6-t4≥3	0	3					
t6-t5≥4	0	4					

Task-Oriented Network Problem

CPM

Variables:

Ark	t1	t2	t3	t4	t5	t6
Value	0	0	0	0	0	0

Objective:		Value
Minimize t6 – t1		0
Constrain	Formula	Value
t1	0	0
t2-t1≥2	0	2
t3-t2≥1	0	3
t4-t2≥1	0	1
t4-t3≥2,5	0	2,5
t5-t3≥1,5	0	1,5
t5-t4≥3	0	2
t6-t4≥3	0	3
t6-t5≥4	0	4

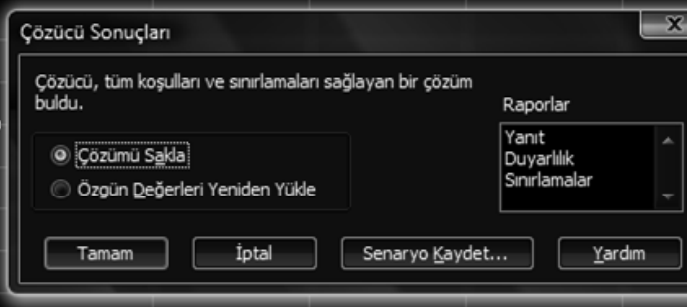
Task-Oriented Network Problem

CPM

Variables:

Ark	t1	t2	t3	t4	t5	t6
Value	0	2	5	7,5	9,5	13,5

Objective:		Value
Minimize t6 – t1		13,5
Constrain	Formula	Value
t1	0	0
t2-t1≥2	2	2
t3-t2≥1	3	3
t4-t2≥1	5,5	1
t4-t3≥2,5	2,5	2,5
t5-t3≥1,5	4,5	1,5
t5-t4≥3	2	2
t6-t4≥3	6	3
t6-t5≥4	4	4



Task-Oriented Network Problem

Microsoft Excel 12.0 Yanıt Raporu
Çalışma Sayfası: [Modeling_CPM.xls]Task-oriented
Rapor Oluşturuldu: 12.12.2009 23:10:45

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
$C$8	Minimize t6 – t1 Value	0	13,5

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
$B$4	Value t1	0	0
$C$4	Value t2	0	2
$D$4	Value t3	0	5
$E$4	Value t4	0	7,5
$F$4	Value t5	0	9,5
$G$4	Value t6	0	13,5

Sınırlamalar

Hücre	Ad	Hücre Değeri	formül	Durum	Serbestlik
$B$18	t6-t5≥4 Formula	4	$B$18>=$C$18	Aynı	0
$B$11	t2-t1≥2 Formula	2	$B$11>=$C$11	Aynı	0
$B$12	t3-t2≥1 Formula	3	$B$12>=$C$12	Aynı	0
$B$13	t4-t2≥1 Formula	5,5	$B$13>=$C$13	Farklı	4,5
$B$14	t4-t3≥2,5 Formula	2,5	$B$14>=$C$14	Aynı	0
$B$15	t5-t3≥1,5 Formula	4,5	$B$15>=$C$15	Farklı	3
$B$16	t5-t4≥3 Formula	2	$B$16>=$C$16	Aynı	0
$B$17	t6-t4≥3 Formula	6	$B$17>=$C$17	Farklı	3
$B$10	t1 Formula	0	$B$10>=$C$10	Aynı	0

Task-Oriented Network Problem

Microsoft Excel 12.0 Duyarlılık Raporu
Çalışma Sayfası: [Modeling_CPM.xls]Task-oriented
Rapor Oluşturuldu: 12.12.2009 23:10:45

Ayarlanabilir Hücreler

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$4	Value t1	0	0
$C$4	Value t2	2	0
$D$4	Value t3	5	0
$E$4	Value t4	7,5	0
$F$4	Value t5	9,5	0
$G$4	Value t6	13,5	0

Sınırlamalar

Hücre	Ad	Son Değer:	Lagrange Çarpan
$B$18	t6-t5≥4 Formula	4	1
$B$11	t2-t1≥2 Formula	2	1
$B$12	t3-t2≥1 Formula	3	1
$B$13	t4-t2≥1 Formula	5,5	0
$B$14	t4-t3≥2,5 Formula	2,5	1
$B$15	t5-t3≥1,5 Formula	4,5	0
$B$16	t5-t4≥3 Formula	2	1
$B$17	t6-t4≥3 Formula	6	0
$B$10	t1 Formula	0	0

LP Formulation of Event-oriented network Problem

The dual problem can be interpreted as a network flow problem. Its solution identifies the critical path and the minimum duration for the project, but it also gives new information on how collaboration can reduce the time to complete a project. The transpose for this matrix defines the constraints for the dual problem. More important, this new matrix is the adjacency matrix for a new network. The columns are labeled with the new (dual) variables for a network flow problem.

X_{12}	X_{23}	X_{24}	X_{34}	X_{35}	X_{45}	X_{46}	X_{56}	Relti.	RHS
-1								=	-1
1	-1	-1						=	0
	1		-1	-1				=	0
		1	-1		-1	-1		=	0
				1	1		-1	=	0
						1	1	=	1

Maximize

$w = 2x_{12}+1\ x_{23}+1\ x_{24}+2{,}5\ x_{34}+1{,}5\ x_{35}+2\ x_{45}+3\ x_{46}+4\ x_{56}$

subject to:

$-x_{12}=-1$

$x_{12}-x_{23}-x_{24}=0$

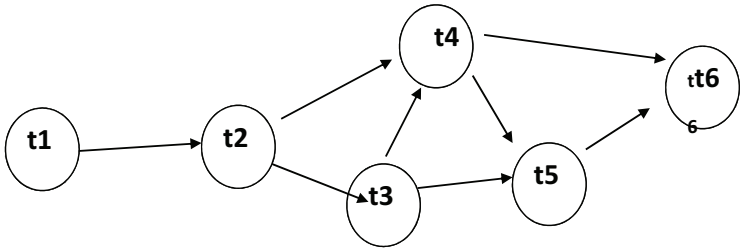
$x_{23}-x_{34}-x_{35}=0$

$x_{24}-x_{34}-x_{45}-x_{46}=0$

$x_{35}+x_{45}-x_{56}=0$

$x_{46}+x_{56}=1$

$x_{ij} \geq 0$



Event-oriented network

Event-oriented network Problem

CPM								
Variables:								
Ark	X12	X23	X24	X34	X35	X45	X46	X56
Value	0	0	0	0	0	0	0	0
Distance	2	3	1	2,5	1,5	2	3	4

Objective:	Value
2 x12+3 x23 +1 x24 +2.5 x34 +1.5 x35+ 2 x45 +3 x46 +4 x56	0

Constrain	Formula	Value
-x12 =-1	0	-1
x12-x23-x24 =0	0	0
x23- x34 -x35 =0	0	0
x24 +x34 -x45- x46 =0	0	0
x35 + x45- x56=0	0	0
x46+ x56=1	0	1
x12	0	0
x23	0	0
x24	0	0
x34	0	0
x35	0	0
x45	0	0
x46	0	0
x56	0	0

Event-oriented network Problem

CPM

Variables:

Ark	X12	X23	X24	X34	X35	X45	X46	X56
Value	0	0	0	0	0	0	0	0
Distance	2	3	1	2,5	1,5	2	3	4

Objective: Value

2 x12+3 x23 +1 x24 +2.5 x34 +1.5 x35+
2 x45 +3 x46 +4 x56 — 0

Constrain	Formula	Value
-x12 =-1	0	-1
x12-x23-x24 =0	0	0
x23- x34 -x35 =0	0	0
x24 +x34 -x45- x46 =0	0	0
x35 + x45- x56=0	0	0
x46+ x56=1	0	1
x12	0	0
x23	0	0
x24	0	0
x34	0	0
x35	0	0
x45	0	0
x46	0	0
x56	0	0

Event-oriented network Problem

CPM

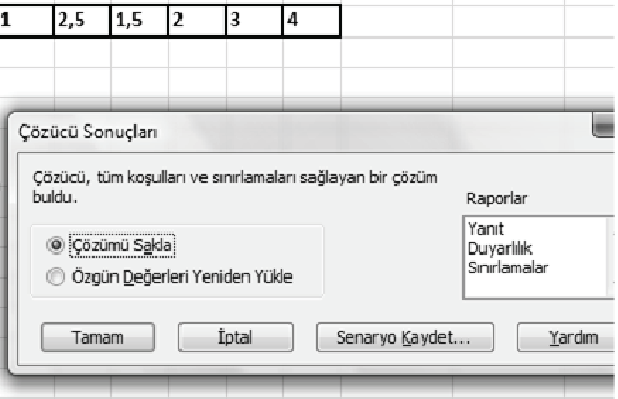
Variables:

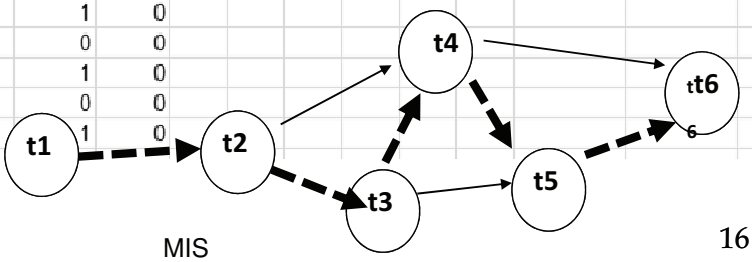
Ark	X12	X23	X24	X34	X35	X45	X46	X56
Value	1	1	0	1	0	1	0	1
Distance	2	3	1	2,5	1,5	2	3	4

Objective: Value

2 x12+3 x23 +1 x24 +2.5 x34 +1.5 x35+
2 x45 +3 x46 +4 x56 — 13,5

Constrain	Formula	Value
-x12 =-1	-1	-1
x12-x23-x24 =0	0	0
x23- x34 -x35 =0	0	0
x24 +x34 -x45- x46 =0	0	0
x35 + x45- x56=0	0	0
x46+ x56=1	1	1
x12	1	0
x23	1	0
x24	0	0
x34	1	0
x35	0	0
x45	1	0
x46	0	0
x56	1	0





Event-oriented network Problem

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Çalışma Sayfası: [Modeling_CPM.xls] Event-Oriented
Rapor Oluşturuldu: 12.12.2009 23:17:03

Hedef Hücre (En Büyük)

Hücre	Ad	İlk Değer	Son Değer
$C$9	2 x12+3 x23 +1 x24 +2.5 x34 +1.5 x35+ 2 x45 +3 x46 +4 x56 Value	0	13,5000135

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
$B$4	Value X12	0	1,000001
$C$4	Value X23	0	1,000001
$D$4	Value X24	0	0
$E$4	Value X34	0	1,000001
$F$4	Value X35	0	0
$G$4	Value X45	0	1,000001
$H$4	Value X46	0	0
$I$4	Value X56	0	1,000001

Sınırlamalar

Hücre	Ad	Hücre Değeri	formül	Durum	Serbestlik
$B$12	-x12 =-1 Formula	-1,000001	$B$12=$C$12	Farklı	0
$B$13	x12-x23-x24 =0 Formula	0	$B$13=$C$13	Farklı	0
$B$14	x23- x34 -x35 =0 Formula	0	$B$14=$C$14	Farklı	0
$B$15	x24 +x34 -x45- x46 =0 Formula	0	$B$15=$C$15	Farklı	0
$B$16	x35 + x45- x56=0 Formula	0	$B$16=$C$16	Farklı	0
$B$17	x46+ x56=1 Formula	1,000001	$B$17=$C$17	Farklı	0
$B$25	x56 Formula	1,000001	$B$25>=$C$25	Farklı	1,000001
$B$24	x46 Formula	0	$B$24>=$C$24	Aynı	0
$B$18	x12 Formula	1,000001	$B$18>=$C$18	Farklı	1,000001
$B$19	x23 Formula	1,000001	$B$19>=$C$19	Farklı	1,000001
$B$20	x24 Formula	0	$B$20>=$C$20	Aynı	0
$B$21	x34 Formula	1,000001	$B$21>=$C$21	Farklı	1,000001
$B$22	x35 Formula	0	$B$22>=$C$22	Aynı	0
$B$23	x45 Formula	1,000001	$B$23>=$C$23	Farklı	1,000001

Event-oriented network Problem

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Çalışma Sayfası: [Modeling_CPM.xls] Event-Oriented
Rapor Oluşturuldu: 12.12.2009 23:17:03

Ayarlanabilir Hücreler

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$4	Value X12	1,000001	0
$C$4	Value X23	1,000001	0
$D$4	Value X24	0	0
$E$4	Value X34	1,000001	0
$F$4	Value X35	0	0
$G$4	Value X45	1,000001	0
$H$4	Value X46	0	0
$I$4	Value X56	1,000001	0

Sınırlamalar

Hücre	Ad	Son Değer:	Lagrange Çarpan
$B$12	-x12 =-1 Formula	-1,000001	-3
$B$13	x12-x23-x24 =0 Formula	0	2
$B$14	x23- x34 -x35 =0 Formula	0	5
$B$15	x24 +x34 -x45- x46 =0 Formula	0	8
$B$16	x35 + x45- x56=0 Formula	0	9,5
$B$17	x46+ x56=1 Formula	1,000001	13,5
$B$25	x56 Formula	1,000001	0
$B$24	x46 Formula	0	-3
$B$18	x12 Formula	1,000001	0
$B$19	x23 Formula	1,000001	0
$B$20	x24 Formula	0	-4,5
$B$21	x34 Formula	1,000001	0
$B$22	x35 Formula	0	-3
$B$23	x45 Formula	1,000001	0