Introduction to Linear Programming LP

optimization

IE

END2971

Advanced Computer Programming

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What we will and will not do…

- Linear programming (LP) is one of our (we = IE) core ability.
- You will learn how to model problems using LP and the theories of the solution techniques (simplex method, for example) in the OR lectures of Vildan hoca and Tufan Hoca.
- Here, I won’t get into details of how to build them or the theoretic backgroud. I will only give some examples in order to make sure that you understand what it is.

LP code python

- Then we will focus on how to solve LP models using some libraries that are available in Python

Pulp (library)

optimal sol'n.

- The examples 1 and 2 are from pulp's own web site:
  - https://coin-or.github.io/pulp/CaseStudies/index.html ➔ thIS ONE ☺

- You can find more information on its website.
  - https://github.com/coin-or/pulp

Example – 1 [pulp]

- A boutique brewery has two warehouses from which it distributes beer to five carefully chosen bars.
- At the start of every week, each bar sends an order to the brewery’s head office for so many crates of beer, which is then dispatched from the appropriate warehouse to the bar.
- The brewery would like to have an interactive computer program which they can run week by week to tell them which warehouse should supply which bar so as to minimize the costs of the whole operation.

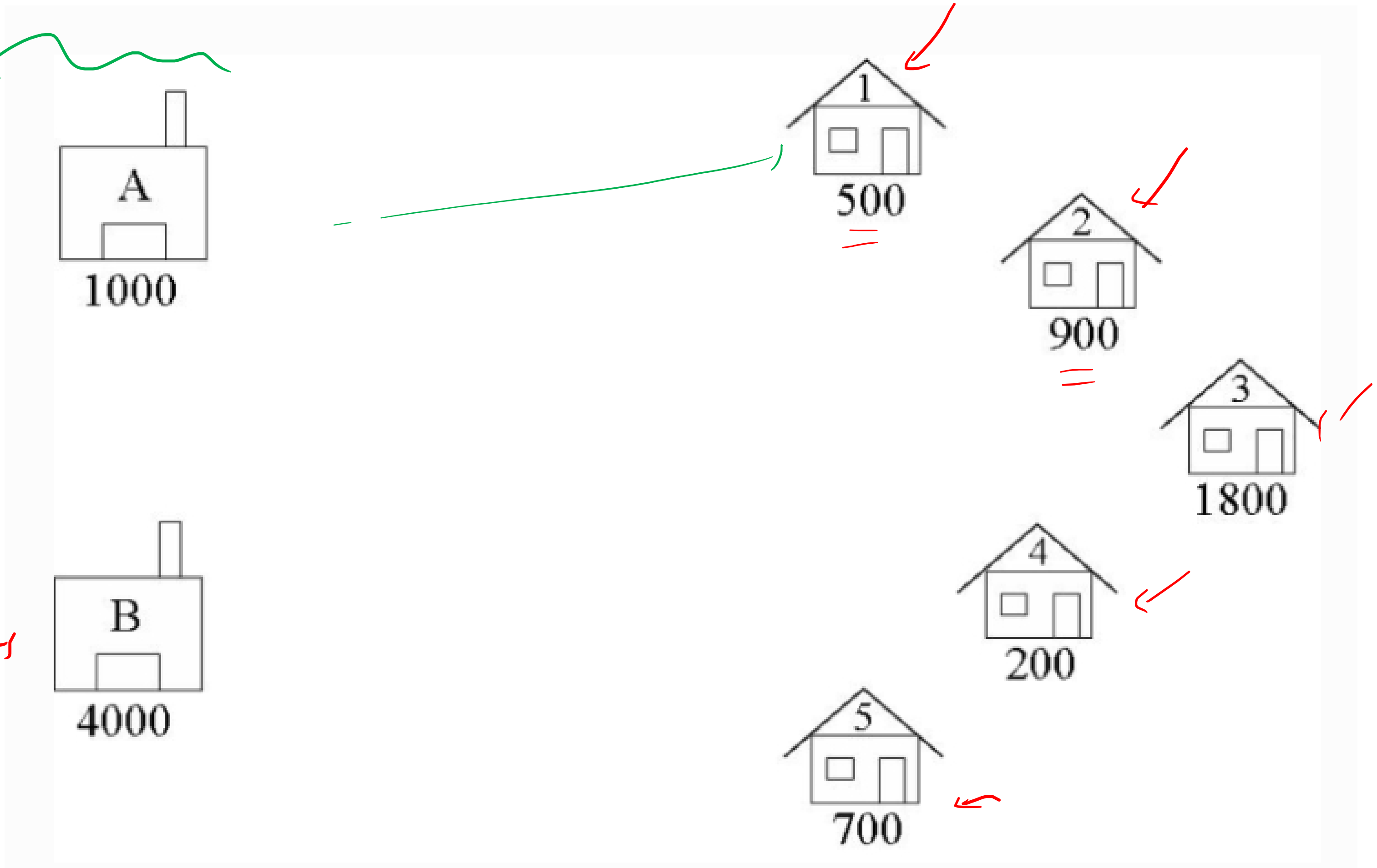
$x_{wp} = ?$

Example – 1 [pulp]

- For example, suppose that at the start of a given week the brewery has
  - boxes
  - 1000 cases at warehouse A,
  - boxes
  - 4000 cases at warehouse B,
- The bars require 500, 900, 1800, 200, and 700 cases, respectively.
- Which warehouse should supply which bar?

Example – 1 [pulp]

LP →



Example – 1 [pulp]

Demand → D

$D_1 + D_5 = 1200 > 1000$

Total Cost

$C_{A1} \cdot X_{A1}$

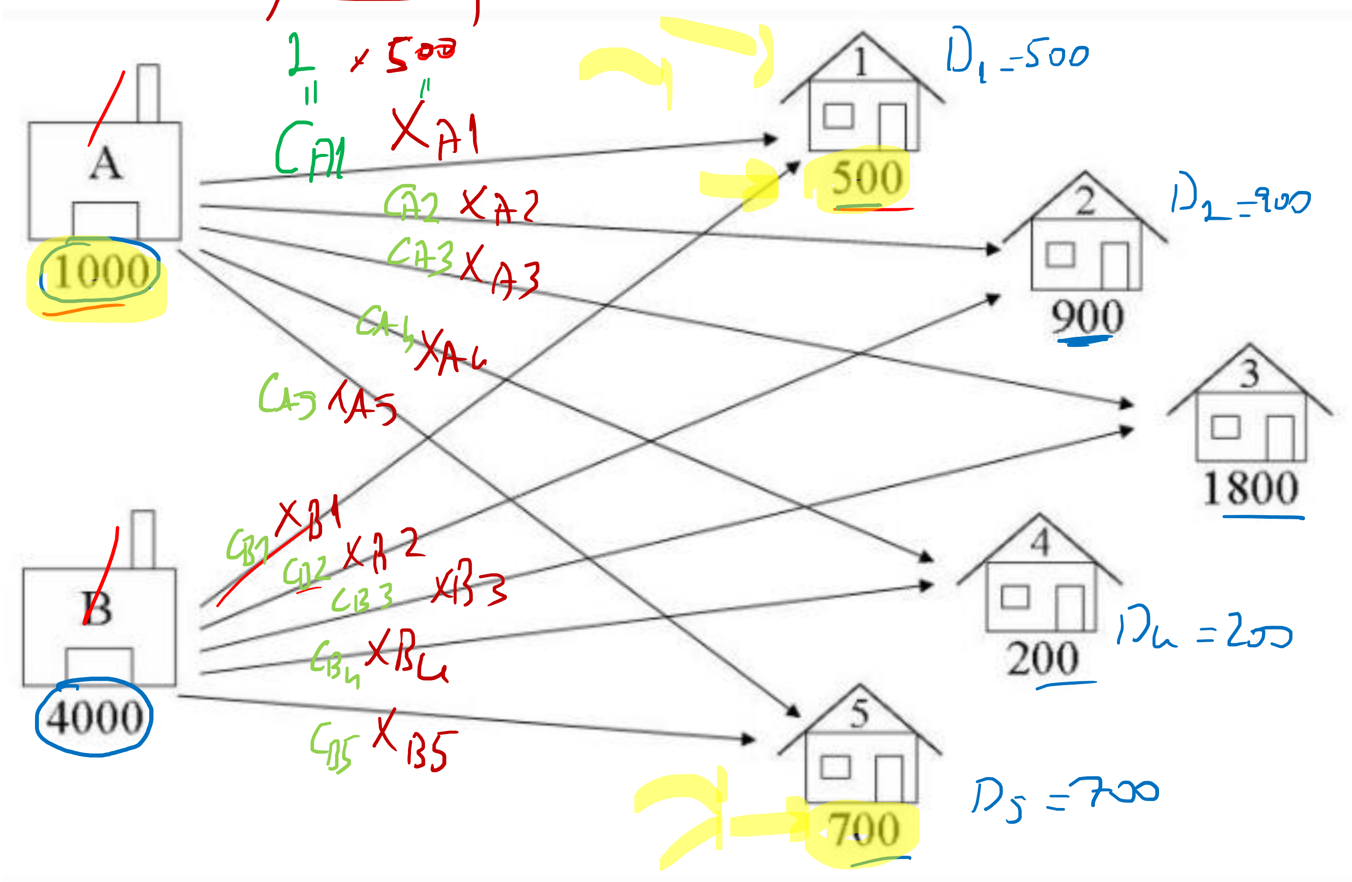
$+$

$C_{A2} \cdot X_{A2}$

$+$

$\vdots$

$C_{B5} \cdot X_{B5}$



$D_1 = 500$

$D_2 = 900$

$D_3 = 1800$

$D_4 = 200$

$D_5 = 700$

Example – 1 [pulp]

Sets/indices

W = {A, B}

Bars = {1, 2, 3, 4, 5}

- We have three definitions in LP
  - Sets/indexes
  - Parameters
  - Variables.

known in advance

to be solved (we don't know them in advance)

Parameters

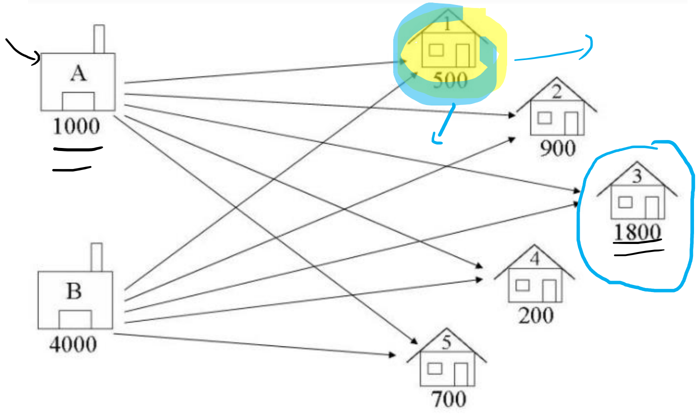
Supply = {1000, 4000}

S_A S_B

Demand = { __ __ }

- Two basic equations in LP
  - Constraints
  - Objective function.

????



Variables

X_{wp} : # of beer from WH w to bar p

Parameters

C_{wp} : cost of moving beer from WH w to bar p

S_w : supply of wh w

D_p : demand of bar p

$$\sum_{p=1}^{5} X_{Ap} = \sum_{p\in\{1,2,3,4,5\}} X_{Ap}$$

not possible if the index is a letter

Constraints:

$$X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5} \le 1000 \rightarrow \sum_{p=1}^{5} X_{Ap} \le \underbrace{1000}_{S_A}$$

$$X_{B1} + X_{B2} + X_{B3} + X_{B4} + X_{B5} \le 4000 \rightarrow \sum_{p=1}^{5} X_{Bp} \le \underbrace{4000}_{S_B}$$

$$\sum_{p=1}^{5} X_{wp} \le S_w \quad \text{---} \quad w=\{A,B\}$$

$$X_{A1} + X_{B1} \ge 500 \Rightarrow \sum_{w\in\{A,B\}} X_{w1} \ge \underbrace{500}_{D_1} \Rightarrow \sum_{w\in\{A,B\}} X_{w1} \ge D_1$$

$$X_{A2} + X_{B2} \ge 900 \rightarrow \sum_{w\in\{A,B\}} X_{w2} \ge 900 \quad \text{---} \quad \sum_{w\in\{A,B\}} X_{w2} \ge D_2$$

$$X_{A3} + X_{B3} \ge 1800$$

$$X_{A4} + X_{B4} \ge 200$$

$$X_{A5} + X_{B5} \ge 700$$

$$\sum_{w\in\{A,B\}} X_{wp} \ge D_p \quad \text{--} \quad p=\{1,2,3,4,5\}$$

Objective Function

Total Cost $= C_{A1} \cdot X_{A1} + C_{A2} X_{A2} + \text{---} + C_{B5} \cdot X_{B5}$

$$= \underline{\min} \sum_{w \in \{A,B\}} \sum_{p=1}^{5} C_{wp} \cdot X_{wp} \Rightarrow \text{total cost of the operation.}$$

Example – 1 [pulp]

The cost of transporting a unit of crates (box) of bar from WH w to bar p.

From Warehouse to Bar	A	B
1	2	3
2	4	1
3	5	3
4	2	2
5	1	3

C_{wp} : cost of transferring a unit of crate of beer from WH w to bar p.

$$\sum_{w\in W}\sum_{b\in B} c_{wb}x_{wb}$$

$\{A,B\}$ $\quad B=\{1,2,3,4,5\}$

$C_{A1}\cdot x_{A1} + C_{A2}\cdot x_{A2} + \dots + C_{B5}\cdot x_{B5}$

$\sum\limits_{w\in\{A,B\}} \sum\limits_{b\in\{1,2,3,4,5\}}$

W Warehous

B set of bars

quicksum

$$\sum_{b\in B} x_{wb} \le s_w$$

$x_{A1} + x_{A2} + \dots + x_{A5} \le s_A$

$x_{B1} + x_{B2} + \dots + x_{B5} \le s_B$

$$\sum_{w\in W} x_{wb} \ge d_b$$

$$x_{wb} \ge 0$$

quicksum

$x_{A1} + x_{B1} \ge d_1$

$x_{A2} + x_{B2} \ge d_2$

for

for loop

$w \in W = \{A,B\}$

$b \overset{in}{\in} B = \{1,2,3,4,5\}$

$w\in W, b\in B$

for b in Bars

quicksum (c[w,b] x[w,b] for w in Warehouses for b in Bars)

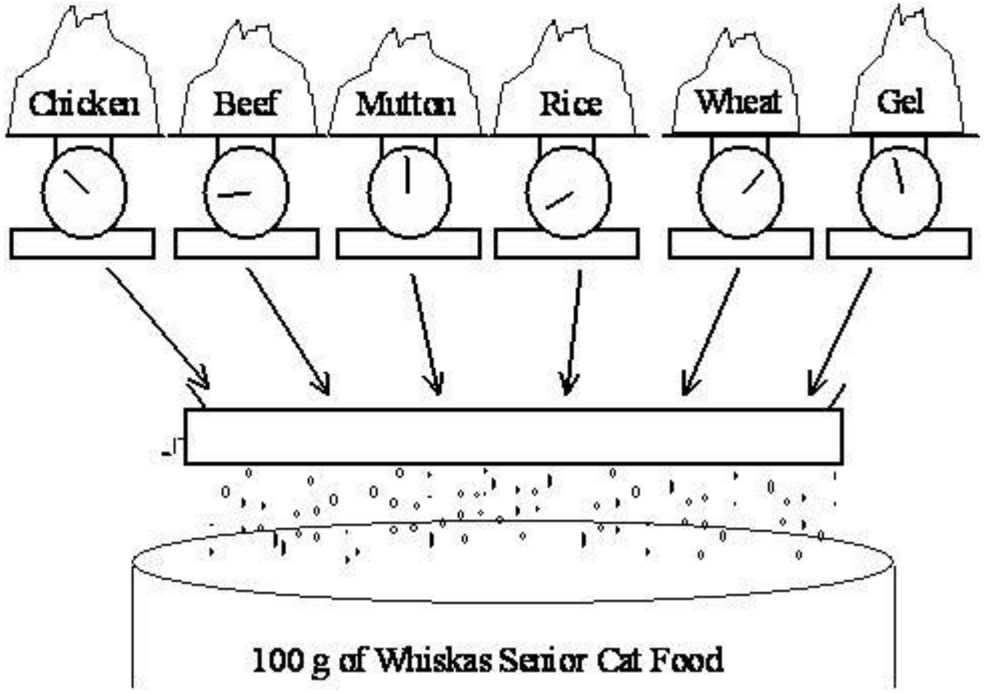
$\sum\limits_{b\in B}\sum\limits_{w\in W} c_{wb}\cdot x_{wb}$

Example – 2 [pulp]

- Whiskas cat food, shown above, is manufactured by Uncle Ben's.
- Uncle Ben's want to produce their cat food products as cheaply as possible while ensuring they meet the stated nutritional analysis requirements shown on the cans.

objective

- Thus they want to vary the quantities of each ingredient used (the main ingredients being chicken, beef, mutton, rice, wheat and gel) while still meeting their nutritional standards.



Example – 2 [pulp]

X

(i) Stuff: {Chicken, Beef, ——, Gel}

(j) Nutrition: {Prot, Fat, Fibre, Salt}

C_i

R_{ij} – Nutrition Table

Stuff2 – {1, 2, ... 6}

Nut2 : {1, 2, 3, 4}

	Stuff	Cost (Cent)	Protein	Fat	Fibre	Salt
1gr	Chicken	1.3	0.1	0.08	0.001	0.002
1gr	Beef	0.8	0.2	0.1	0.005	0.005
1gr	Mutton	1	0.15	0.11	0.003	0.007
1gr	Rice	0.2	0	0.01	0.1	0.002
	Wheat bran	0.5	0.04	0.01	0.15	0.008
	Gel	0.1	0	0	0	0
	Needed	minimize	8 (min)	6 (min)	2 (min)	0.4 (max)

X_i : the amount or percent that is used in the cat food

$R_{Beef,\ Fibre} = 0.005 \Rightarrow R_{2,3} = 0.005$

stuff   Nutrition

Stuff	Cost (Cent)	Protein	Fat	Fibre	Salt
Chicken 1	1.3	0.1	0.08	0.001	0.002
Beef 2	0.8	0.2	0.1	0.005	0.005
Mutton 3	1	0.15	0.11	0.003	0.007
Rice 4	0.2	0	0.01	0.1	0.002
Wheat bran 5	0.5	0.04	0.01	0.15	0.008
Gel 6	0.1	0	0	0	0
Needed	minimize!	8 (min)	6 (min)	2 (min)	0.4 (max)

Variable: X_i: % of the cat food of ingredient/stuff type i

$X_{chicken} = X_1 = 5 \Rightarrow$ use 5% in the cat food.

obj: $\min \sum_{i=1}^{6} c_i \cdot X_i = \sum_{i \in \{Chicken, Beef, \ldots, Gel\}} c_i \cdot X_i$

$\sum X_i = 100$

s.t.

$(0.1)(x_1) + (0.2) \cdot x_2 + (0.15) x_3 + (0)(x_4) + (0.04) x_5 + (0)(x_6) \geq 8$

$(0.08)(x_1) + (0.1) x_2 + (0.11) \cdot x_3 + (0.01)(x_4) + (0.01)(x_5) + (0)(x_6) \geq 6$

$\ldots\ldots$

$(0.002)(x_1) + \ldots + (0.008)(x_5) + (0)(x_6) \leq 0.4$

Example 3 – Giapetto's Woodcarving [Winston]

- Giapetto's Woodcarving, Inc., manufactures two types of wooden toys: soldiers and trains.
- A soldier sells for $27 and uses $10 worth of raw materials.
- Each soldier that is manufactured increases Giapetto's variable labor and overhead costs by $14.
- A train sells for $21 and uses $9 worth of raw materials.
- Each train built increases Giapetto's variable labor and overhead costs by $10.

Example 3 – Giapetto’s Woodcarving [Winston]

- The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing.
- A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor.
- A train requires 1 hour of finishing labor and 1 hour of carpentry labor.
- Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours.
- Demand for trains is unlimited, but at most 40 soldiers are bought each week.
- Giapetto wants to maximize weekly profit (revenues-costs).

Example 4 – Furnco manufacturing [Winston]

- Furnco manufactures desks and chairs.
- Each desk uses 4 units of wood, and each chair uses 3.
- A desk contributes $40 to profit, and a chair contributes $25.
- Marketing restrictions require that the number of chairs produced be at least twice the number of desks produced.
- If 20 units of wood are available, formulate an LP to maximize Furnco's profit.

Example 5 – Leary Chemical [Exercise]

- Leary Chemical manufactures three chemicals: A, B, and C.
- These chemicals are produced via two production processes: 1 and 2.
- Running process 1 for an hour costs $4 and yields
  - 3 units of A,
  - 1 units of B,
  - 1 units of C.
- Running process 2 for an hour costs $1 and produces
  - 1 unit of A
  - 1 units of B.
- To meet customer demands, at least
  - 10 units of A,
  - 5 units of B,
  - 3 units of C must be produced daily.
- Formulate the problem as a linear program

Source

- Examples 3, 4 and 5 are from
- Winston, W. L., & Goldberg, J. B. (2004). *Operations research: applications and algorithms* (Vol. 3). Belmont: Thomson Brooks/Cole.
- This book is one of the famous book of our field opeations research.

How to install PULP (CBC)

How to install Pulp

- Pulp is a modelling library than can work with CBC (an open source solver) and Gurobi (a commercial solver)
- Download the required package using the following direct link. We will install the pulp package manually from this download:
- https://github.com/coin-or/pulp-or/archive/master.zip
- Then extract the files into some folder. Then using the following command in the command prompt line (see next slides) install the package:

```
pip install -e your folder
```

- For example my command was something like this:

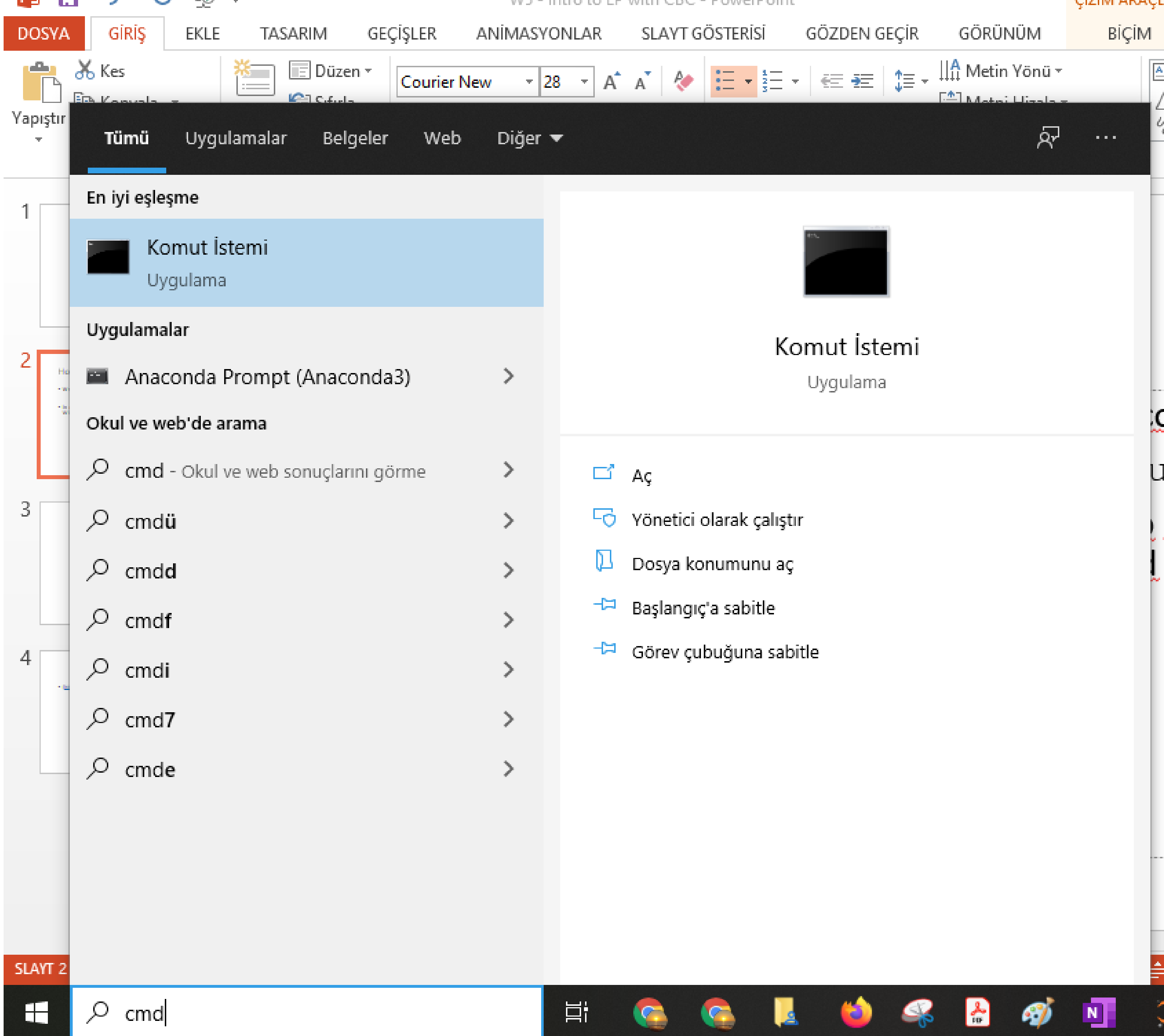
```
pip install -e C:\mgguler2\Software\PuLP\pulp-master
```

- Well, that’s all…

- If pip is not working…
- How to install pip (an easy way of installing packages)
- https://www.geeksforgeeks.org/how-to-install-pip-on-windows/

How to open your command prompt?

In order to open your *command line* go to search bar next to the Windows button at the left bottom and write `cmd` there



Some backup

Some backup

- If you encounter an error like the following:
  - *'python' is not recognized as an internal or external command, operable program or batch file.*
- Then there are two options.

Some backup

- Option 1:
  - Either go to the place where you have downloaded python and execute the same command there
  - How to find the location of your `python.exe?`
  - Write the following to your jupyter

    ```
    import os
    import sys
    os.path.dirname(sys.executable)
    ```
  - My Path is : *`C:\ProgramData\Anaconda3`*‘
  - Then you can go to there at the command prompt as follows and then execute the installation using the command given at the first slide.

Some backup

- Option 2:
- Go to one of these web sites to add python to environmental variables so that python works everywhere in the command prompts.
  - https://datatofish.com/add-python-to-windows-path/
  - https://geek-university.com/python/add-python-to-the-windows-path/
  - https://www.educative.io/edpresso/err-python-is-not-recognized-as-an-internal-or-external-command
    - *In summary: Right click your computer and find advanced computer settings and environmental variables.*
- Then you execute the installation using the command given at the «How to install Pulp» slide at any place in the command prompt (CMD)