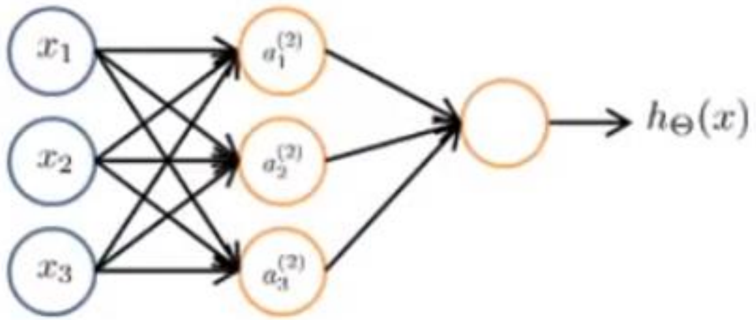
Model Representation 2

Neural Networks

Neural Networks: Representation

Forward propagation: Vectorized implementation

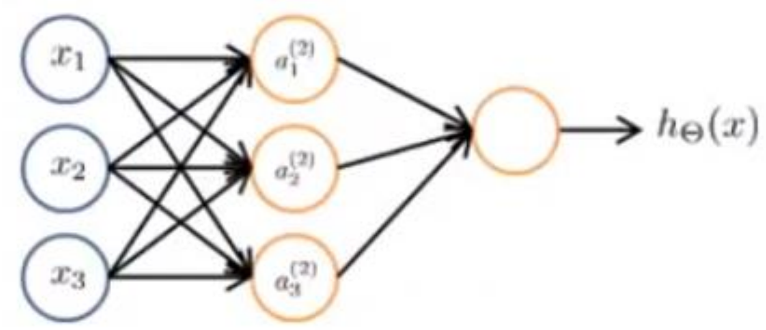


$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$
$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$
$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

$$h_\Theta(x) = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$

What are the parameters

- θ_{32}^{1} :
- it is the **first** layers parameter
- it takes the output of the **second** node in the first layer
- Provides an output for the **third** node
- $a_3^2 \leftarrow \ldots + \ \theta_{32}^{1}\, x_2 + \cdots$



$$a_1^{(2)} = g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3)$$
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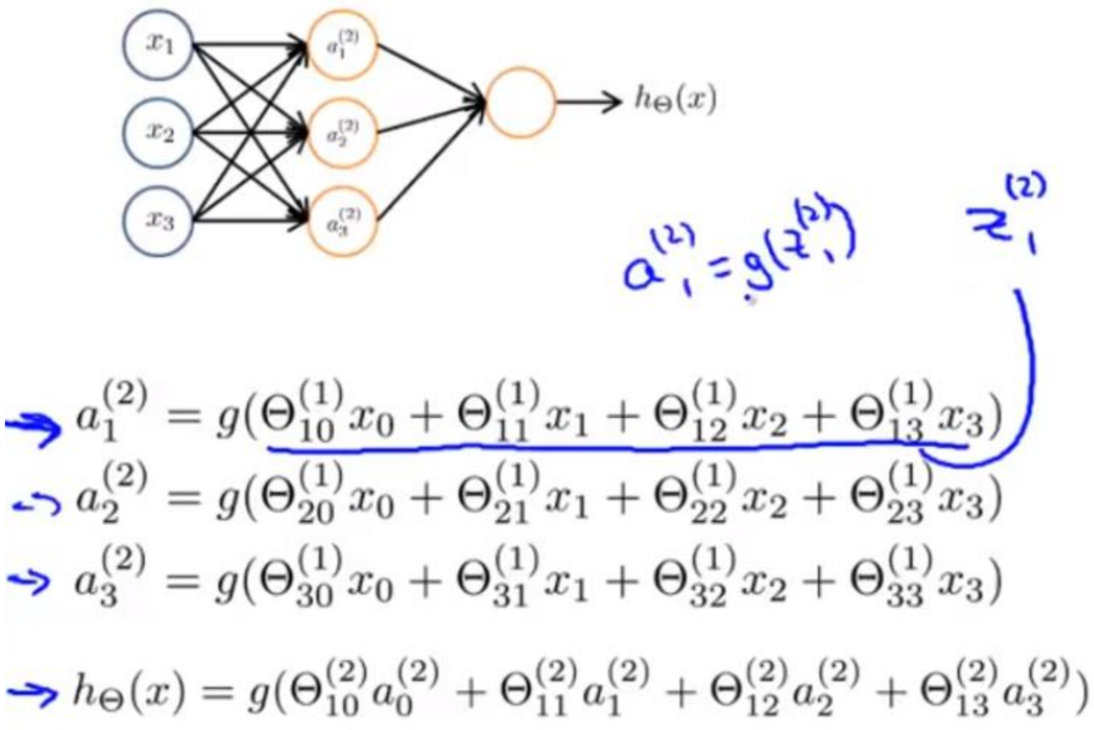
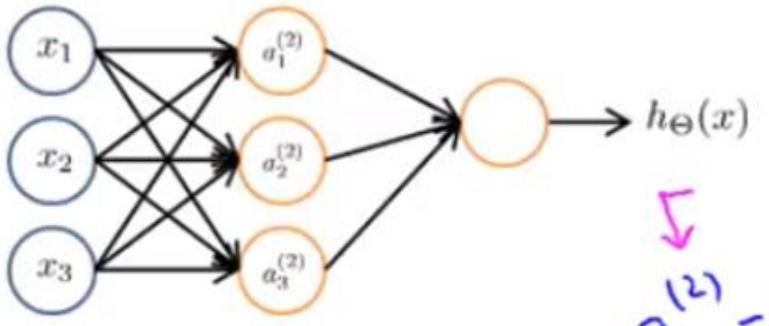
What are the parameters

- $z_1^2 = \theta_{10}^1 x_0 + \theta_{11}^1 x_1 + \theta_{12}^1 x_2 \ldots$
- Hence:
  - Find the total impact **z** using the weights $\boldsymbol{\theta}$.
  - Then squeeze this impact using the sigmoid function.
  - Assign it to the activation value **a**
- **We can Show this with following matrix notations**



Forward propagation: Vectorized implementation



$a_1^{(2)} = g(z_1^{(2)})$

$z_1^{(2)}$

$$a_1^{(2)} = g(\underbrace{\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3}_{z_1^{(2)}})$$

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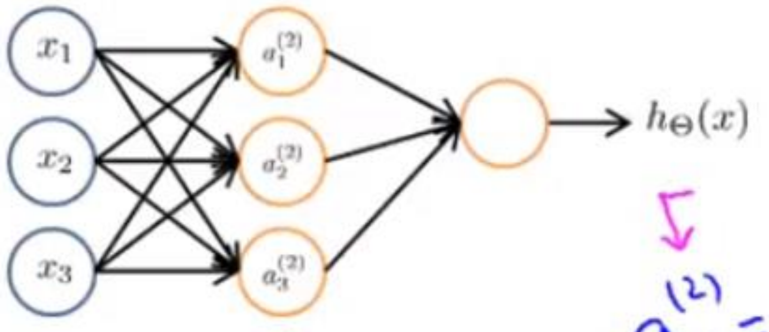
$\Theta^{(1)}$ $\times$

$a_3^{(2)} = g(z_3^{(2)})$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

Forward propagation: Vectorized implementation



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$z_1^{(2)}$

$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

$z_2^{(2)}$

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$z_3^{(2)}$

$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

$$h_\Theta(x) = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$

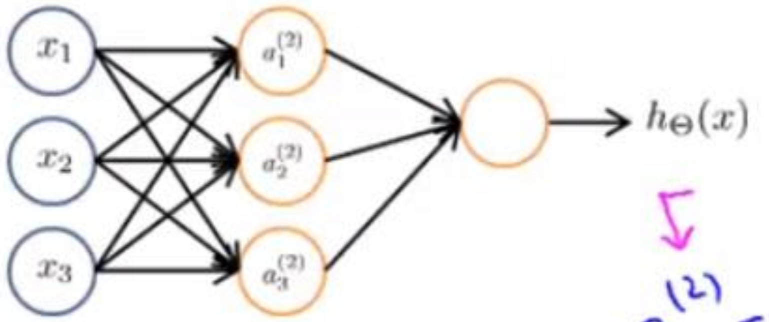
$\Theta^{(1)} x$

$a_3^{(2)} = g(z_3^{(2)})$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)}x$$
$$a^{(2)} = g(z^{(2)})$$

Forward propagation: Vectorized implementation



$a_1^{(2)} = g(z_1^{(2)})$

$$a_1^{(2)} = g(\underbrace{\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3}_{z_1^{(2)}})$$

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$\Theta^{(1)} x$

$a_3^{(2)} = g(z_3^{(2)})$

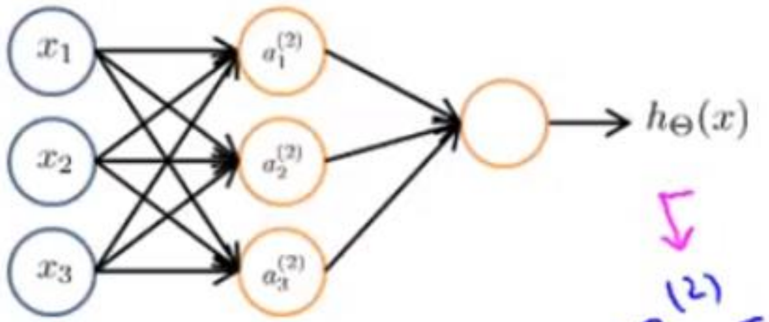
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)} x$$

$$\underbrace{a^{(2)}}_{\mathbb{R}^3} = g(\underbrace{z^{(2)}}_{\mathbb{R}^3})$$

Here we apply the sigmoid function to each of the z values.

Forward propagation: Vectorized implementation



$a^{(1)} = x$

$a_1^{(2)} = g(z_1^{(2)})$

$$a_1^{(2)} = g(\underbrace{\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3}_{z_1^{(2)}})$$

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$\Theta^{(1)} x$

$a_3^{(2)} = g(z_3^{(2)})$

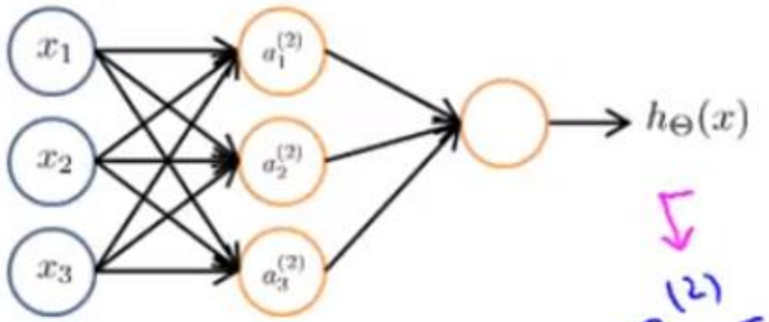
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)} \cancel{x}\, a^{(1)}$$

$$\underbrace{a^{(2)}}_{\mathbb{R}^3} = g(\underbrace{z^{(2)}}_{\mathbb{R}^3})$$

Let a(1) = x in order to have a clear representation

Forward propagation: Vectorized implementation



$a^{(1)} = x$

$a_1^{(2)} = g(z_1^{(2)})$

$z_1^{(2)}$

$$a_1^{(2)} = g(\underbrace{\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3}_{z_1^{(2)}})$$

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$\Theta^{(1)}$ x

$a_3^{(2)} = g(z_3^{(2)})$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

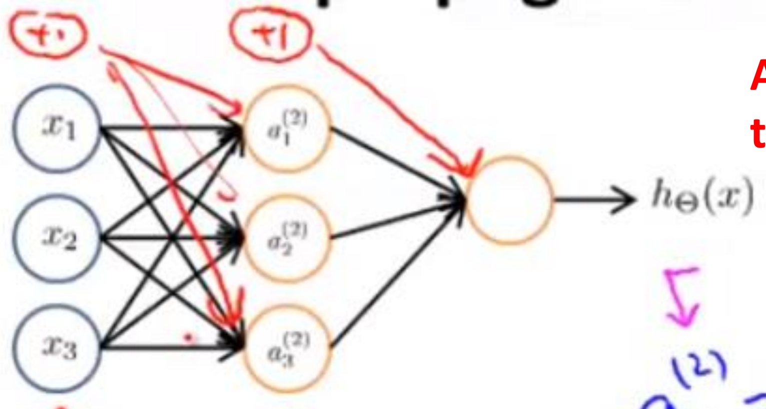
$$z^{(2)} = \Theta^{(1)} \cancel{x}\ a^{(1)}$$

$$a^{(2)} = g(z^{(2)})$$

$\mathbb{R}^3$ $\mathbb{R}^3$

$a_1^{(2)}$
$a_2^{(2)}$
$a_3^{(2)}$

Forward propagation: Vectorized implementation



Also don't forget to add the bias terms

$a^{(1)} = x$

$a_1^{(2)} = g(z_1^{(2)})$

$z_1^{(2)}$

$$a_1^{(2)} = g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3)$$

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$$a_3^{(2)} = g(\Theta_{30}^{(1)} x_0 + \Theta_{31}^{(1)} x_1 + \Theta_{32}^{(1)} x_2 + \Theta_{33}^{(1)} x_3)$$

$z_2^{(2)}$

$z_3^{(2)}$

$$h_\Theta(x) = g(\Theta_{10}^{(2)} \boxed{a_0^{(2)}} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$$

$\Theta^{(1)} x$

$a_3^{(2)} = g(z_3^{(2)})$

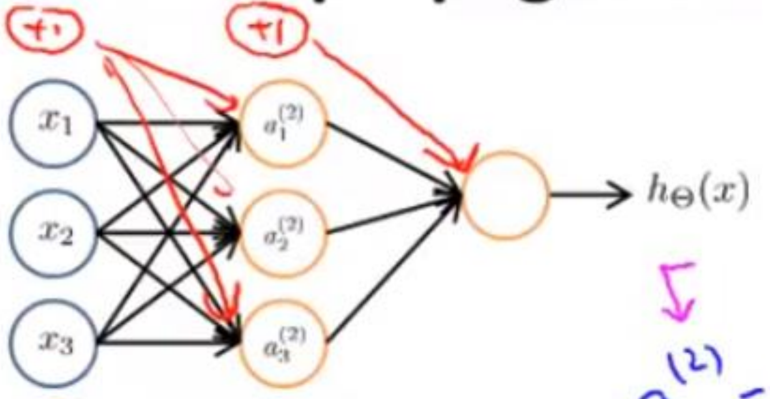
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)} \cancel{x}\, a^{(1)}$$

$$\underbrace{a^{(2)}}_{\mathbb{R}^3} = g(\underbrace{z^{(2)}}_{\mathbb{R}^3})$$

$a_1^{(2)}$
$a_2^{(2)}$
$a_3^{(2)}$

Forward propagation: Vectorized implementation



$a^{(1)} = x$

$a_1^{(2)} = g(z_1^{(2)})$

$z_1^{(2)}$

$$a_1^{(2)} = g(\underbrace{\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3}_{z_1^{(2)}})$$

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$\Theta^{(1)}$ x

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)} \cancel{x}\, a^{(1)}$$

$$\underbrace{a^{(2)}}_{\mathbb{R}^3} = g(\underbrace{z^{(2)}}_{\mathbb{R}^3})$$

$a_1^{(2)}$
$a_2^{(2)}$
$a_3^{(2)}$

Add $a_0^{(2)} = 1$. $\longrightarrow a^{(2)} \in \mathbb{R}^4$

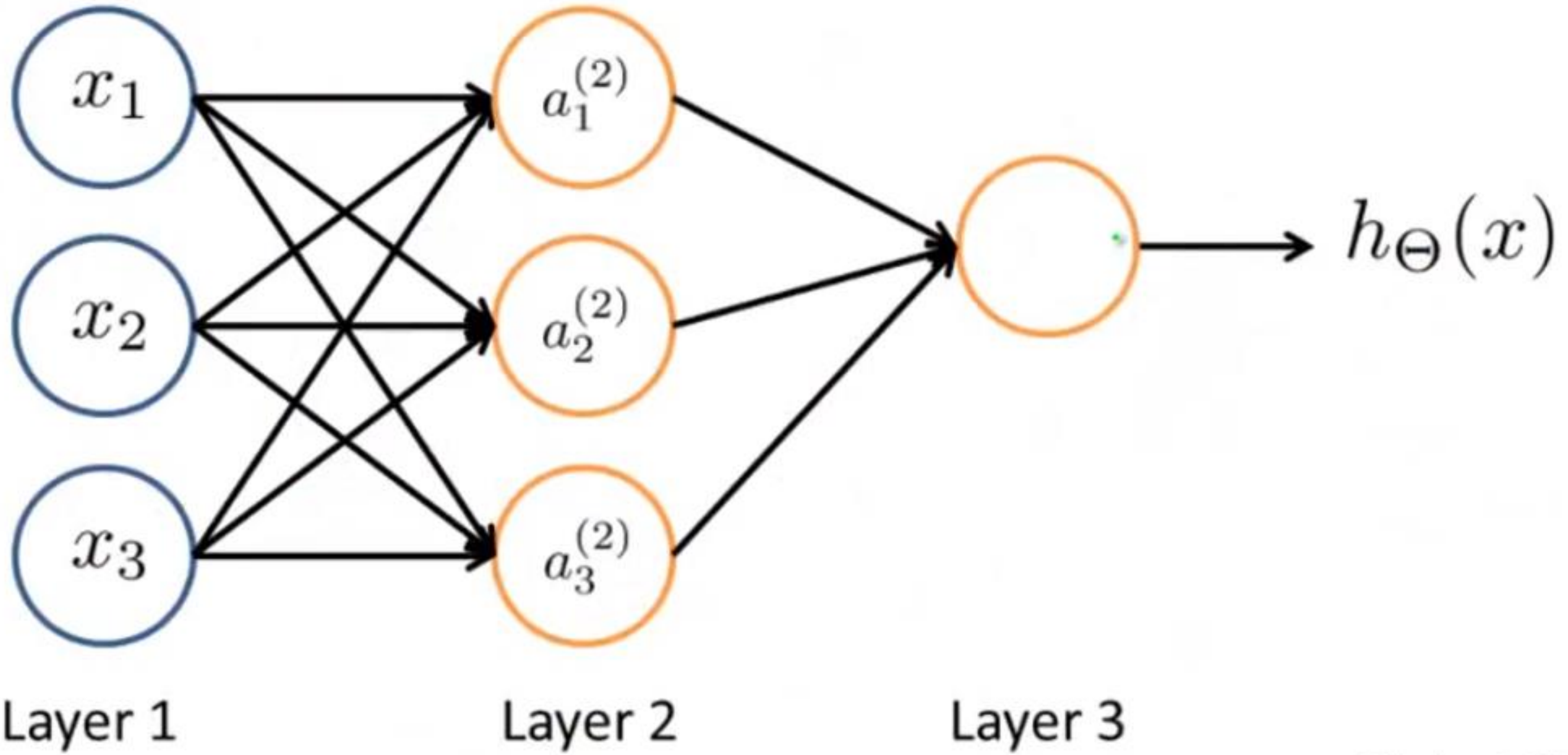
$$z^{(3)} = \Theta^{(2)} a^{(2)}$$

$$h_\Theta(x) = a^{(3)} = g(z^{(3)})$$

So what do we have?

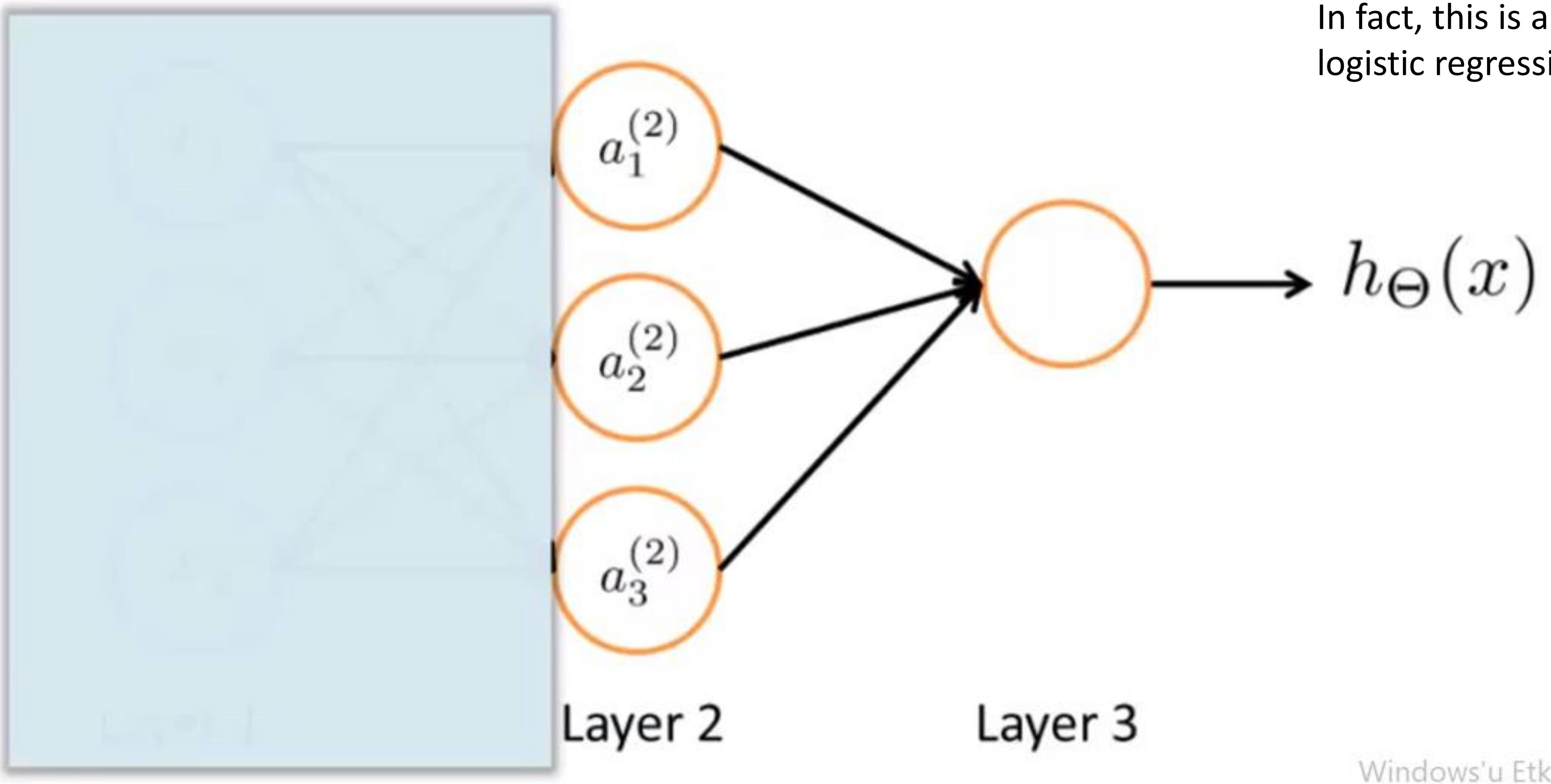
- So again, we use the weights over **a**'s to find their weighted sums **z**'s
  - $z_1^2 = \theta_{10}^1 a_0^{(1)} + \theta_{11}^1 a_1^{(1)} + \theta_{12}^1 a_2^{(2)}$ …
  - $z = \theta^{(1)} a^{(1)}$
- Then apply sigmoid function to find a's of that particular node:
  - $a^{(3)} = g\left(z^{(3)}\right)$

Neural Network learning its own features

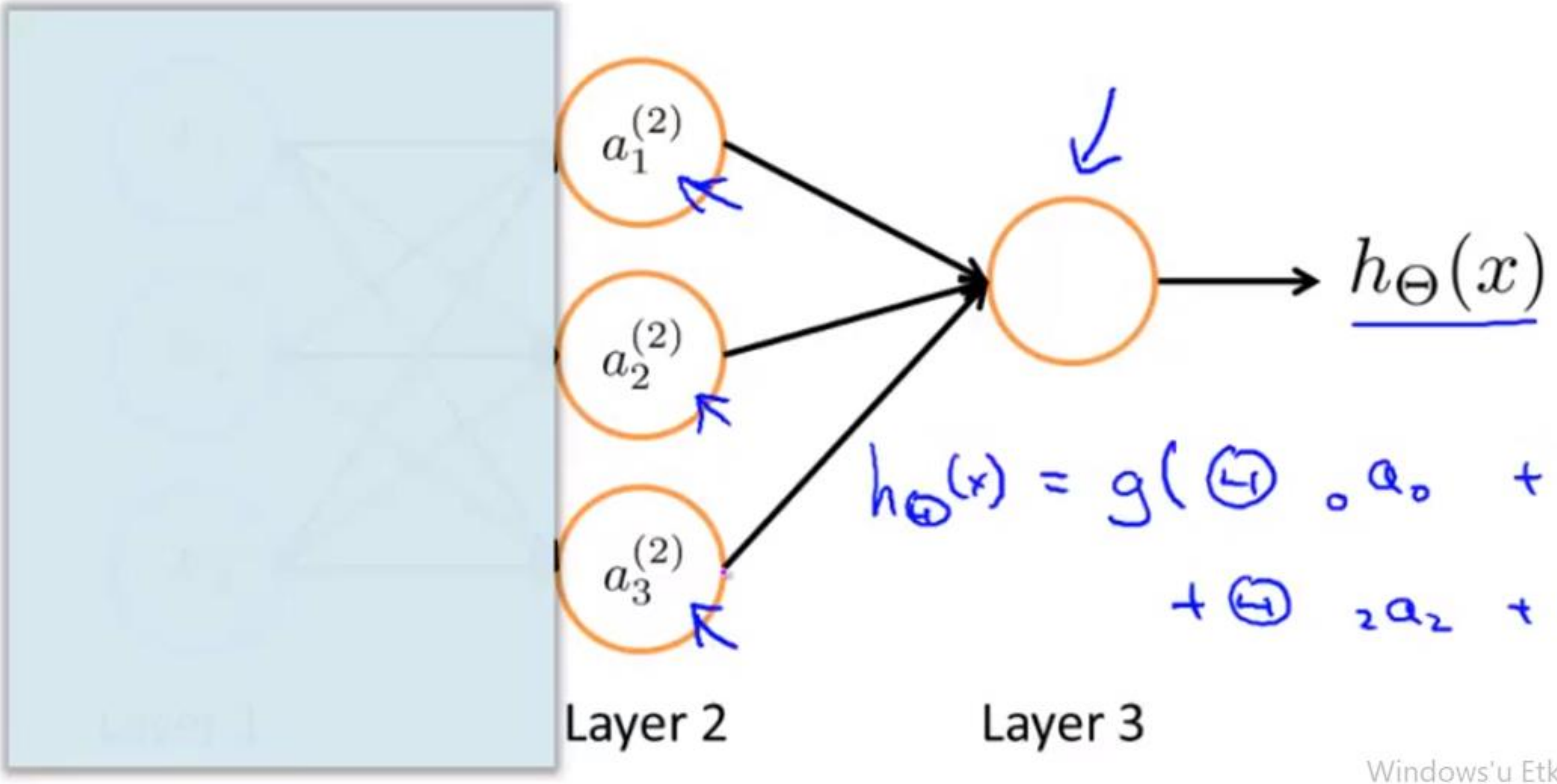


Neural Network learning its own features

In fact, this is a logistic regression!

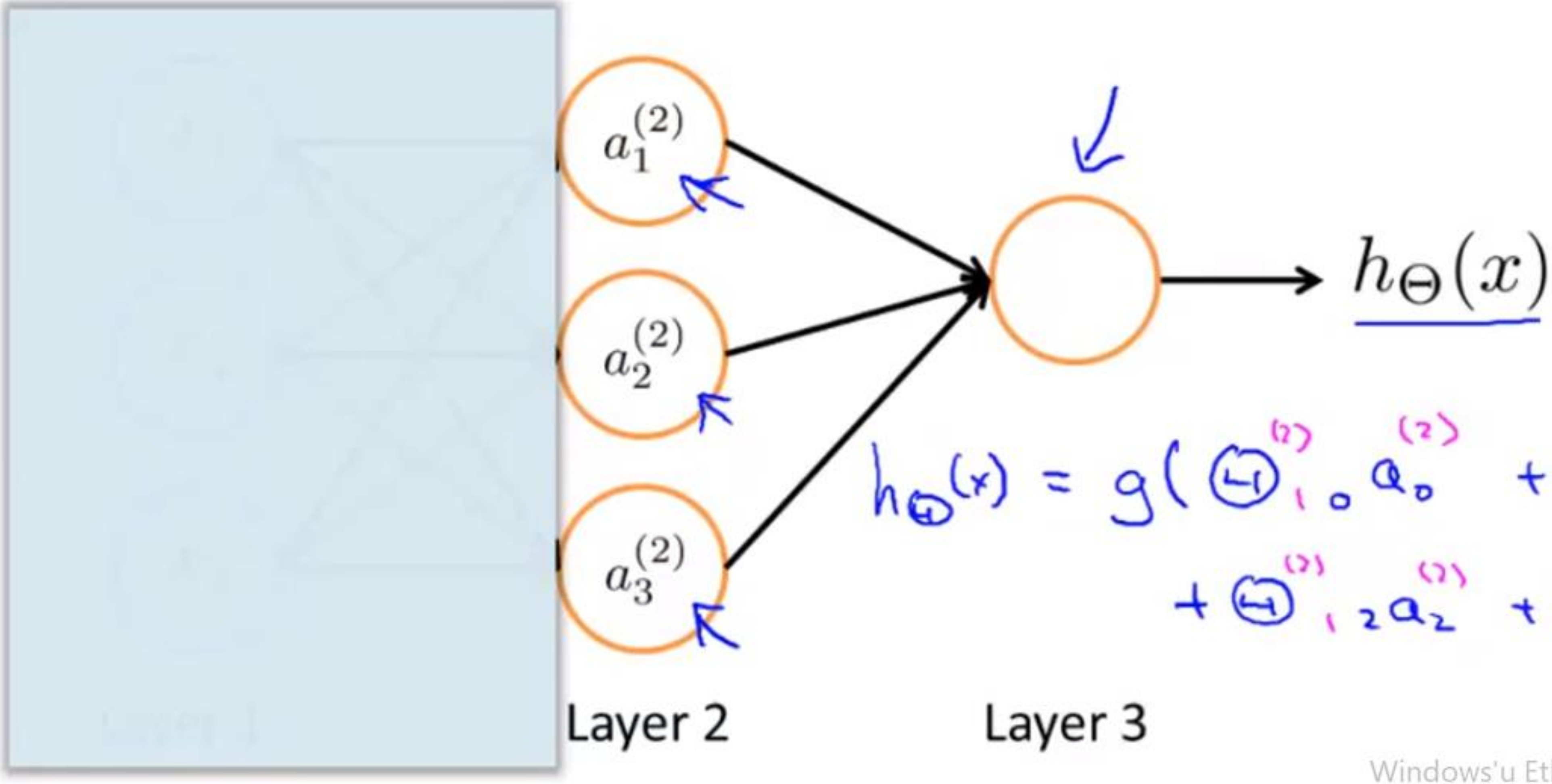


Neural Network learning its own features



$$h_\Theta(x) = g(\Theta_0 a_0 + \Theta_1 a_1 + \Theta_2 a_2 + \Theta_3 a_3)$$

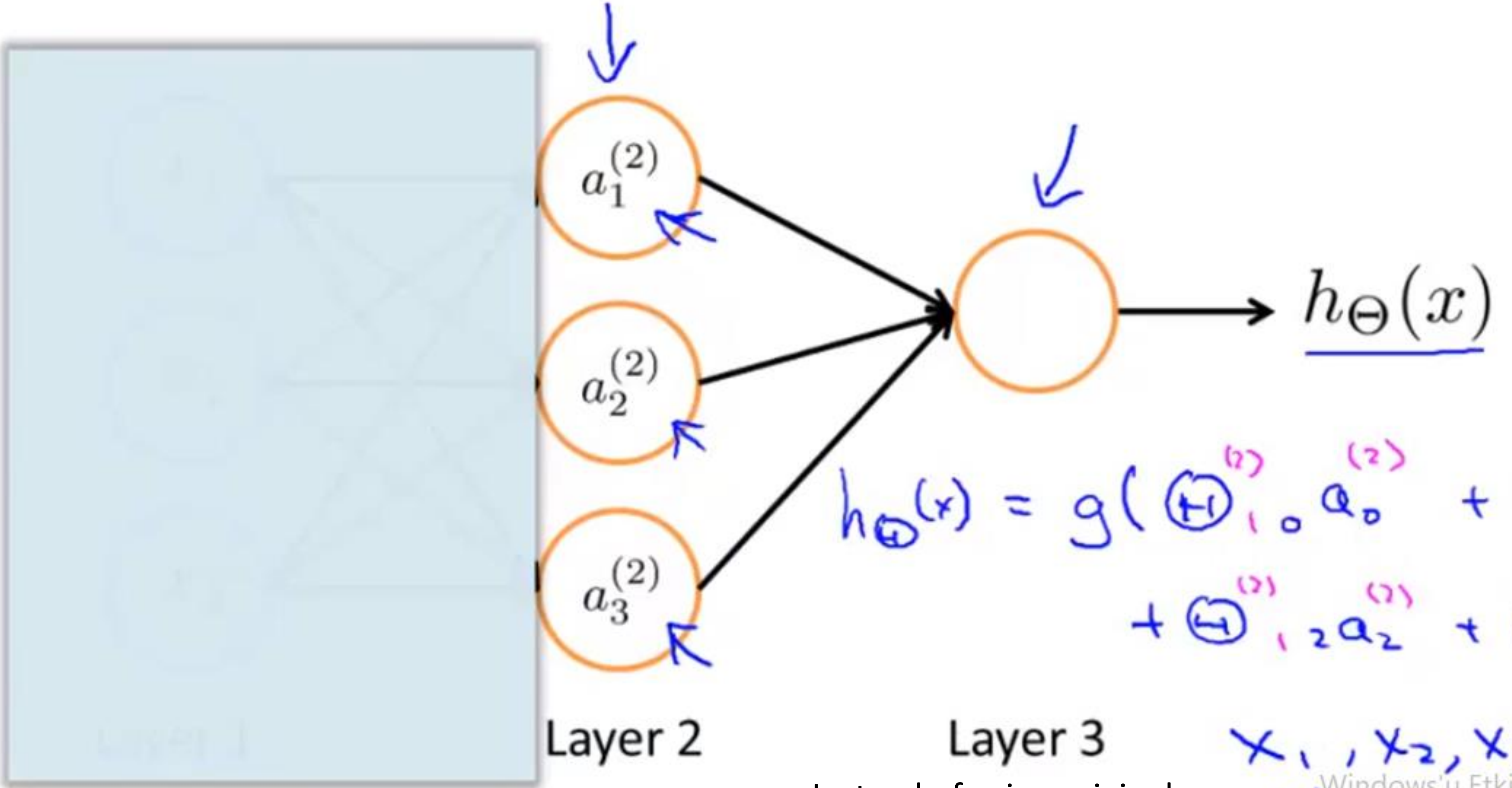
Neural Network learning its own features



$$h_\Theta(x) = g(\Theta^{(2)}_{10} a^{(2)}_0 + \Theta^{(1)}_{11} a^{(2)}_1 + \Theta^{(2)}_{12} a^{(2)}_2 + \Theta^{(2)}_{13} a^{(2)}_3)$$

Andrew Ng

Neural Network learning its own features



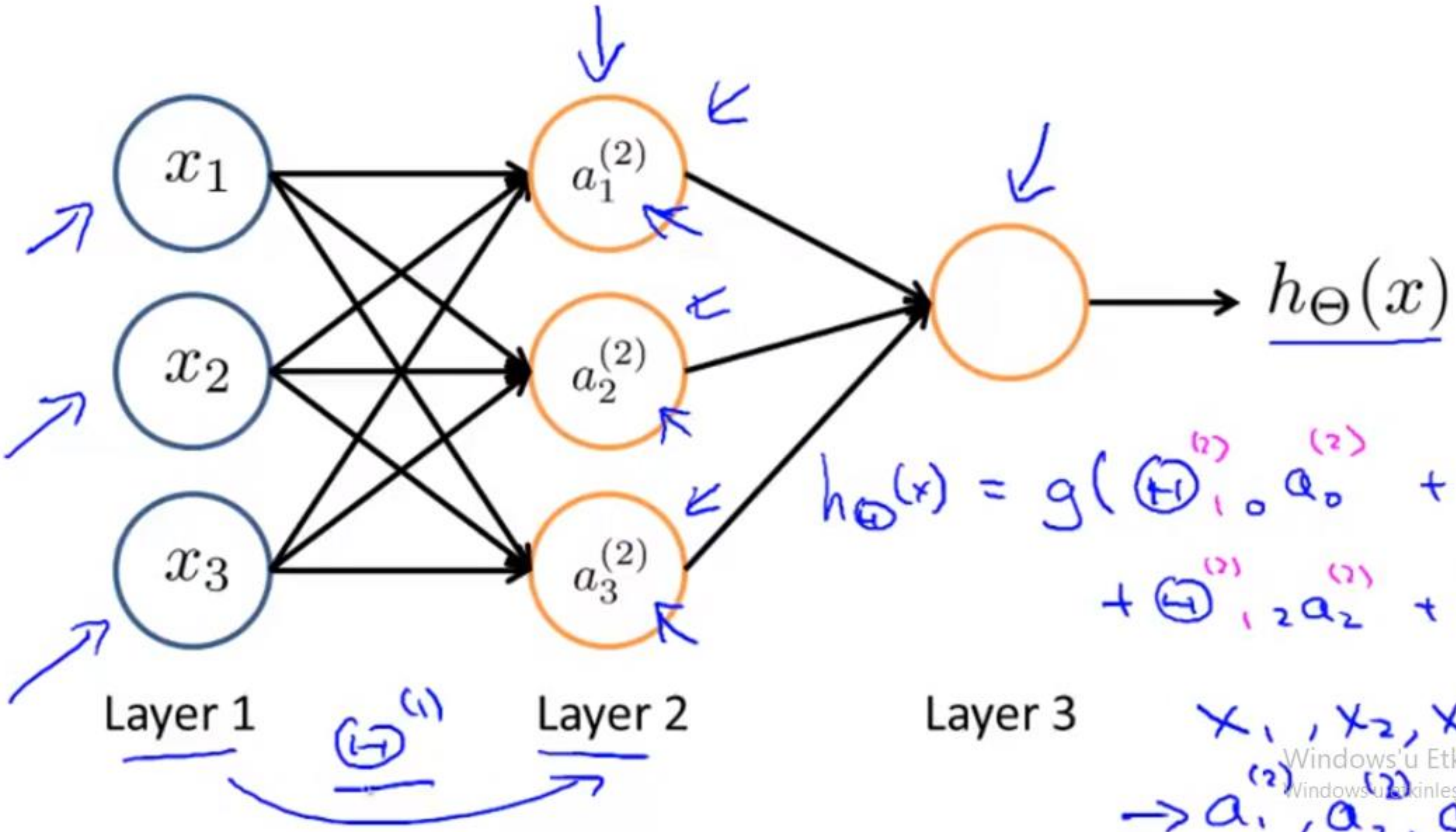
$$h_\Theta(x) = g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(1)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$$

x_1, x_2, x_3

$a_1^{(2)}, a_2^{(2)}, a_3^{(2)}$

Instead of using original features (x), we use the modifed ones (a)

Neural Network learning its own features

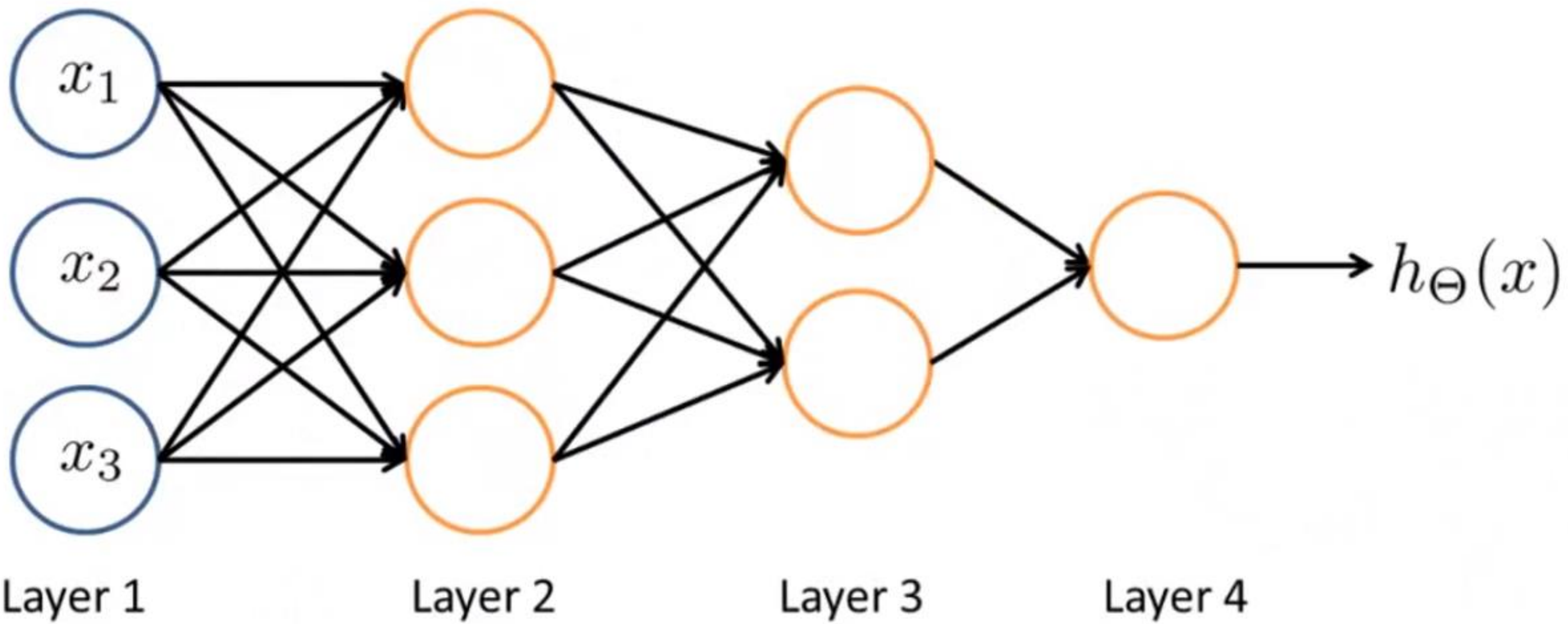


$$h_\Theta(x) = g(\Theta^{(2)}_{10} a_0^{(2)} + \Theta^{(1)}_{11} a_1^{(2)} + \Theta^{(2)}_{12} a_2^{(2)} + \Theta^{(2)}_{13} a_3^{(2)})$$

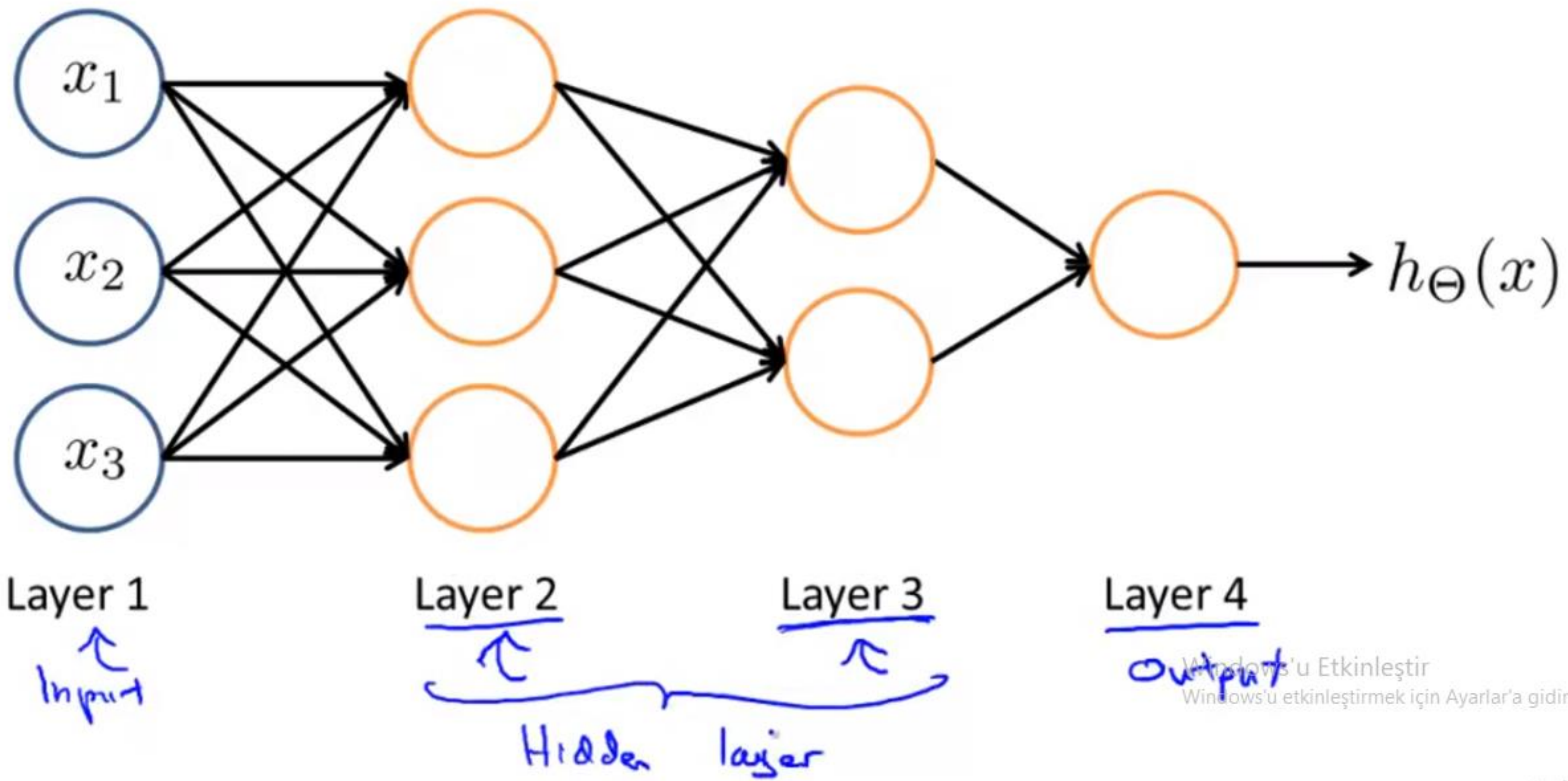
x_1, x_2, x_3

$\rightarrow a_1^{(2)}, a_2^{(2)}, a_3^{(2)}$

Other network architectures



Other network architectures



Exercise

Let $a^{(1)} = x \in \mathbb{R}^{n+1}$ denote the input (with $a_0^{(1)} = 1$).

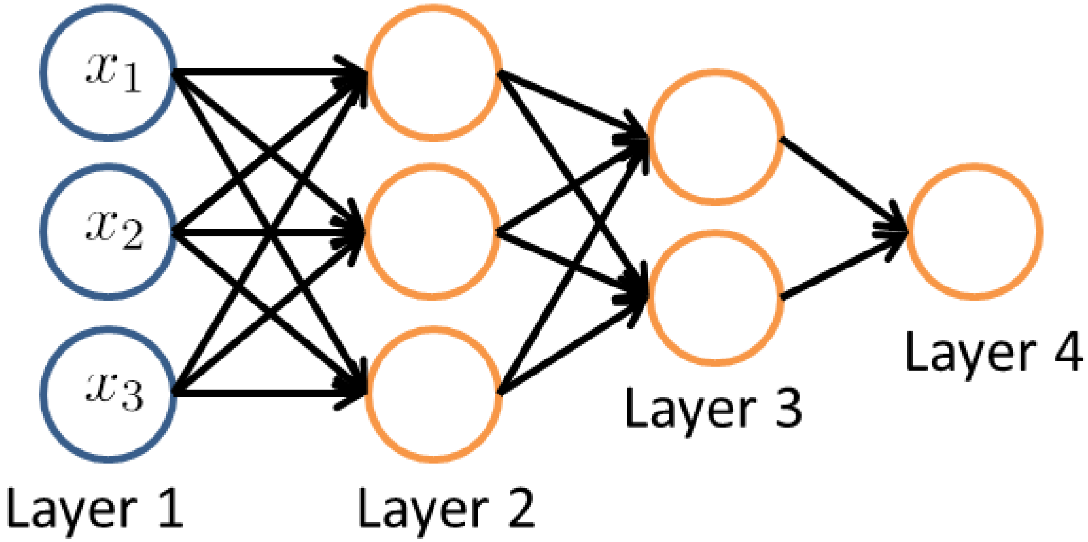
How would you compute $a^{(2)}$?

$a^{(2)} = \Theta^{(1)} a^{(1)}$

$z^{(2)} = \Theta^{(2)} a^{(1)};\ a^{(2)} = g(z^{(2)})$

$z^{(2)} = \Theta^{(1)} a^{(1)};\ a^{(2)} = g(z^{(2)})$

$z^{(2)} = \Theta^{(2)} g(a^{(1)});\ a^{(2)} = g(z^{(2)})$



Summary

- Brief summary
- Calculate the weighted sum of the outputs of the nodes

$$z^{(j)} = \Theta^{(j-1)} a^{(j-1)}$$

- This output enters to a node where the node squeezes it between 0 and 1 using the sigmoid function to generate its own output:

$$a^{(j)} = g\left(z^{(j)}\right)$$