Reliability Engineering

Notes 6

k out of n Systems

- **Redundancy**
- Redundancy in a system means that there exists an alternative parallel path for the successful operation of the system.

Redundancy

- One way to increase product reliability is to build *redundancy* into the product design in the form of backup parts.
- Redundancy is built into the system by placing components in parallel so that when one component fails the other component takes over.

Classification of redundancy

- Active Redundancy
- Standby Redundancy

Active Redundancy

- In active redundancy, all the components connected in parallel are turned on at the beginning of operation of the system, and continue to perform until they fail.
- Thus, in active redundancy, all components are simultaneously in the operating mode.
- All the components of the system are turned on at the beginning of the operation of the system.
- All components are active at the same time.
- Each component is able to meet the functional requirements of the system.
- Each component satisfies the minimum reliability condition for the system.

Active Redundancy

- **Fully Redundant System:** A k-out-of-n system is known as a **fully redundant** system if k = 1. But for k = 1, the k-out-of-n system is the same as the parallel system. Thus, the parallel system is nothing but a 1-out-of-n system and is also known as a **fully redundant** system.

Active Redundancy

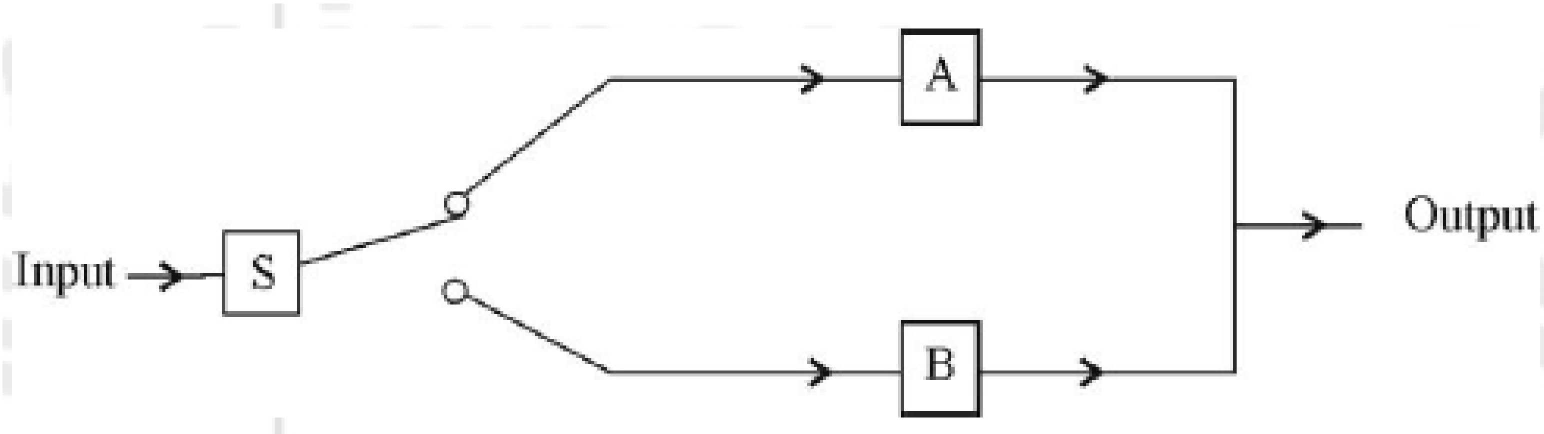
- A k-out-of-n system is said to be a **non-redundant system** if k = n. But for k = n, the k-out-of-n system is simply the series system, Thus, a series system is nothing but an n-out-of-n system and is also known as a **non-redundant** system. Now, we define a partially redundant system. A k-out-of-n system is said to be a **partially redundant system** if $1 < k < n$.

Standby Redundancy

- The components are connected in parallel but do not start operating simultaneously from the beginning of the operation of the system.
- A standby system consists of an active unit or subsystem and one or more inactive (standby) units that become active in the event of the failure of the functioning unit. The failures of active units are signaled by a sensing subsystem, and the standby unit is brought to action by a switching subsystem.

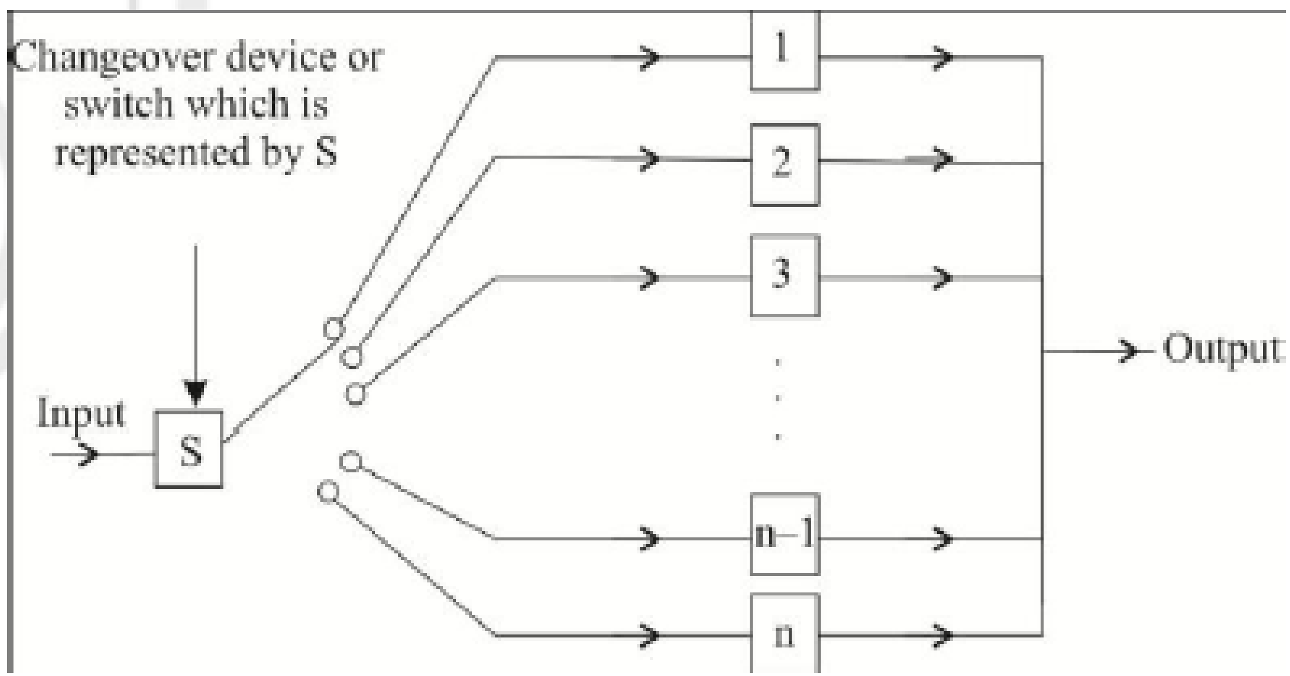
- There is a changeover device. The function of the changeover device is to sense the failure of normally operating component and in case of a failure, to bring a standby component into the normal operating mode.
- The simplest standby configuration is a two-unit system, (A and B)

- A switch in the standby systems can put any component into operation. In the 2-component standby system, initially this switch is connected to component A and turns it on, the switch is represented by S. In this setup, component B will remain in standby (reserve or inoperative) mode till such time as component A performs its function successfully. As soon as component A fails, the switch senses the failure and puts the component B into operation.


Input
S
A
B
Output

- In general, if we have a standby system having n components, namely, 1, 2, 3, …, n, then there exists a switch, say S, which can put any one of the n components into operation. The system works in the same way as the two-component standby system.
- Initially the switch is connected to component 1 and turns it on . Here we are assuming that only one component is in the normally operating mode. Until the component 1 performs its intended function successfully, the remaining (n-1)components, namely 2 to n, remain in the standby (reserve or inoperative) mode. As soon as component 1 fails, the switch senses the failure and turns component 2 on.

- Till such time as the component 2 performs its intended function successfully, the remaining (n-2) components, namely 3 to n, remain in the standby (reserve or inoperative) mode. As soon as component 2 fails, the switch senses the failure and turns the component 3 on. This process continues until the failure of the nth component. When the nth component fails, the system will go into failure mode. If there are more than one components in the normally operating mode, the process will work in the same way as it works for one normally operating component.



Changeover device or
switch which is
represented by S
Input
S
1
2
3
n–1
n
Output

- Standby system can be divided into two categories

- Standby system with perfect switching
- Standby system with imperfect switching

- **By perfect switching**, the changeover device/switch will neither fail during the operation of a component nor during the changeover of a failed component to the next available standby component. In reliability terms, we are assuming that the changeover device is 100% reliable.
- **By imperfect switching**, the changeover device/switch may fail either during the operation of a component or during the time of bringing a standby component in place of a normally operating failed component.

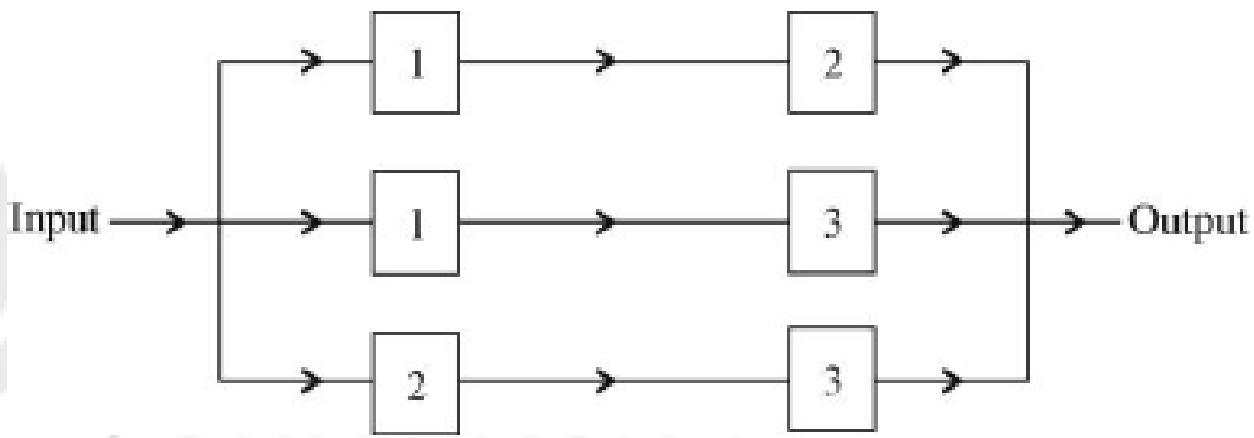
k out of n Systems

- A system having n components is said to be k-out-of-n system if and only if at least k components out of n are required to function successfully for the successful operation of the system.
- In k-out-of-n system, although all n components are in operation, the k components that are essential for the system to operate are called the **basic components**. The (n - k) components that are added with the objective to improve the reliability of the system are known as **redundant components**.

- For example, suppose a system has n components connected in parallel and at least k, $(1 \leq k \leq n)$ components are needed for the successful operation of the system. In such a system, we say that the number of **basic components** is k and the remaining (n-k) components are known as **redundant** components.
- The basic purpose of introducing redundant components in a system is to improve the reliability of the system. The reliability of a system increases as we introduce redundant components in a system .
- For example, airplane with 4 engines can fly with only 2 engines.

- for $k = 1$, the k-out-of-n system is the same as the parallel system
- for $k = n$, the k-out-of-n system is simply the series system

- The series and parallel systems are particular cases of k-out-of-n system when k = n and k = 1, respectively.
- The possible paths for a 2- out-of-3 system having components 1, 2 and 3 are shown in figure.



Input
1
2
1
3
2
3
Output

Reliability of k-out-of-n system

- We suppose all the n components of the system are identical have the same reliability and independent.
- We can apply binomial distribution for evaluating the reliability of the system.

$$f(k) = \binom{m}{k} p^k q^{m-k}, \quad 0 \le p \le 1, \quad q = 1 - p, \quad k = 0, 1, 2, \ldots, m,$$

$$\binom{m}{k} \equiv C_k^m = \frac{m!}{k!(m-k)!},$$

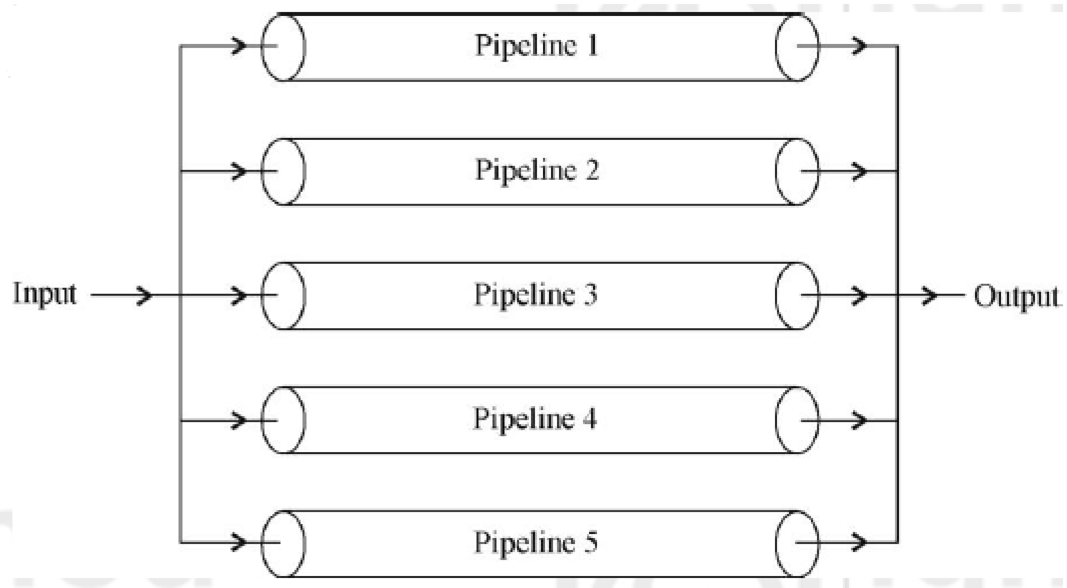
Therefore, if R_s denotes the reliability of k-out-of-n system for a given time, then

R_s = Probability of success of at least k components

$$\text{or } R_s = \sum_{x=k}^{n} {}^{n}C_x \, R^x Q^{n-x}$$

Example

- Consider a piping system having 5 pipes connected in parallel as shown in figure. Assume that all pipes are identical and independent. If reliability of smooth flow of the liquid from each pipeline is 0.80 for a mission of 1 year, evaluate the reliability of the system working successfully. The system is said to work successfully if at least 3 pipelines perform their intended function successfully.


Pipeline 1
Pipeline 2
Input
Pipeline 3
Output
Pipeline 4
Pipeline 5

$$R_s = \sum_{k=3}^{5} {}^5C_k \; R^k (1-R)^{5-k}$$

$$= {}^5C_3 R^3 (1-R)^2 + {}^5C_4 R^4 (1-R) + {}^5C_5 R^5 (1-R)^0$$

$$= 10(0.8)^3 (1-0.8)^2 + 5(0.8)^4 (1-0.8) + (0.8)^5$$

$$= 10 \times 0.512 \times 0.04 + 5 \times 0.4096 \times 0.2 + 0.32768$$

$$= 0.2048 + 0.4096 + 0.32768$$

$$= 0.94208$$

Non-identical components in k out of n system

- For independent but not identical components we can not apply binomial distribution.
- The case of non-identical components is handled by considering all possible mutually exclusive and exhaustive cases for each possible value of k.

Example

- We have 4 pipelines, which are independent but not identical. It is given that the reliabilities of 4 pipelines for a mission of 1000 hours are R1 = 0.60, R2 = 0.70, R3 = 0.80 and R4 = 0.75, respectively. Evaluate the reliability of 2-out-of-4 system.

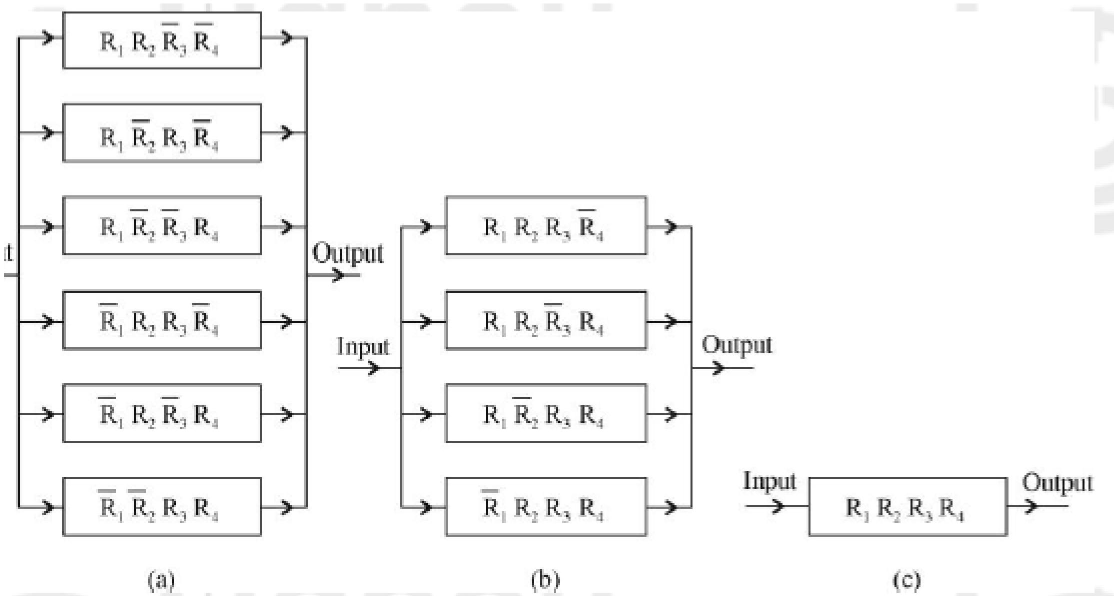
- For k = 2,
- the number of mutually exclusive cases = 6

These six mutually exclusive cases are $R_1R_2\overline{R}_3\overline{R}_4, R_1\overline{R}_2R_3\overline{R}_4,$ $R_1\overline{R}_2\overline{R}_3R_4, \overline{R}_1R_2R_3\overline{R}_4, \overline{R}_1R_2\overline{R}_3R_4, \overline{R}_1\overline{R}_2R_3R_4$, where $\overline{R}_i$ denotes the unreliability of the component i, i = 1, 2, 3, 4.

For k = 3,
the number of mutually exclusive cases = 4

These four mutually exclusive cases are $R_1R_2R_3\overline{R}_4, R_1R_2\overline{R}_3R_4, R_1\overline{R}_2R_3R_4,$ $\overline{R}_1R_2R_3R_4$

- For k = 4,
- the number of mutually exclusive cases = 1
- $R_1R_2R_3R_4$



Mutually exclusive paths for (a) k = 2; (b) k = 3 and (c) k = 4.

- To evaluate reliability for k = 2, we simply have to add the reliabilities of the six mutually exclusive cases (by addition law of probability for mutually exclusive events).

$$\begin{aligned}
R_a &= R_1R_2\bar{R}_3\bar{R}_4 + R_1\bar{R}_2R_3\bar{R}_4 + R_1\bar{R}_2\bar{R}_3R_4 + \bar{R}_1R_2R_3\bar{R}_4 \\
&\quad + \bar{R}_1R_2\bar{R}_3R_4 + \bar{R}_1\bar{R}_2R_3R_4 \\
&= 0.6\times0.7\times(1-0.8)(1-0.75) + 0.6\times(1-0.7)0.8(1-0.75) \\
&\quad + 0.6\times(1-0.7)(1-0.8)\times0.75 + (1-0.6)\times0.7\times0.8(1-0.75) \\
&\quad + (1-0.6)\times0.7(1-0.8)0.75 + (1-0.6)(1-0.7)0.8\times0.75 \\
&= 0.6\times0.7\times0.2\times0.25 + 0.6\times0.3\times0.8\times0.25 + 0.6\times0.3\times0.2\times0.75 \\
&\quad + 0.4\times0.7\times0.8\times0.25 + 0.4\times0.7\times0.2\times0.75 + 0.4\times0.3\times0.8\times0.75 \\
&= 0.021 + 0.036 + 0.027 + 0.056 + 0.042 + 0.072 \\
&= 0.254
\end{aligned}$$

$$R_b = R_1R_2R_3\bar{R}_4 + R_1R_2\bar{R}_3R_4 + R_1\bar{R}_2R_3R_4 + \bar{R}_1R_2R_3R_4$$

$$= 0.6\times0.7\times0.8(1-0.75) + 0.6\times0.7(1-0.8)0.75 + 0.6(1-0.7)0.8\times0.75$$

$$+ (1-0.6)0.7\times0.8\times0.75$$

$$= 0.084 + 0.063 + 0.108 + 0.168$$

$$= 0.423$$

$$R_c = R_1R_2R_3R_4 = 0.6\times0.7\times0.8\times0.75 = 0.252$$

The reliability of the whole system is given by

$$R = R_a + R_b + R_c = 0.254 + 0.423 + 0.252 = 0.929$$

Standby System with Perfect Switching Reliability

- To evaluate the reliability of the standby system with perfect switching, we assume that the standby component(s) will not fail in its/their standby position. Now, let Ei be the event that component i performs its intended function successfully, where i = 1, 2, 3, …, n. Let Q denote the unreliability of the component i, given that components 1to(i -1) have failed. Further, if R and Q denote the reliability and unreliability of the standby system,

$$Q = P\left(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_3 \cap \ldots \cap \overline{E}_n\right), \text{where } \overline{E}_i \text{ denotes complement of event } E_i$$

$$= P\left(\overline{E}_1\right) P\left(\overline{E}_2 \middle| \overline{E}_1\right) P\left(E_3 \middle| \overline{E}_1 \overline{E}_2\right) \ldots P\left(\overline{E}_n \middle| \overline{E}_1 \overline{E}_2 \ldots \overline{E}_{n-1}\right)$$

[using the concept of conditional probability]

$$= Q_1 Q_2 Q_3 \ldots Q_n$$

reliability (R) of the standby system is given by $R = 1 - Q$

- A standby system has three components 1, 2, 3, where component 1 is normally operating and components 2, 3 are standby components. The reliability of component 1 is 0.95. The reliability of component 2 given that component 1 has failed is 0.96 and that of component 3 given that components 1 and 2 have failed is 0.98. Evaluate the reliability of the system under the assumption that the switch is perfect.

$$Q_1 = 1 - 0.95 = 0.05, \quad Q_2 = 1 - 0.96 = 0.04, \quad Q_3 = 1 - 0.98 = 0.02$$

$$R = 1 - Q_1 Q_2 Q_3$$

$$= 1 - (0.05)\,(0.04)\,(0.02) = 1 - 0.00004 = 0.99996$$

Standby System with Imperfect Switching Reliability

- To explain the concept, let us consider a simple two-component standby system with one normally operating component (say, A) and one standby component (say, B). To evaluate the reliability of the system, let us assume that the switch fails only at the time of changeover. Therefore, the unreliability of the system is given by

$$Q = P(\text{successful changeover}) \times P(\text{system failure given successful changeover})$$

$$+ P(\text{unsuccessnu changeover}) \times P(\text{system failure given unsuccessful changeover})$$

$$\Rightarrow Q = P_s Q_A Q_B + \overline{P}_s Q_A, \quad \left[\begin{array}{l} \because \text{in case of successful changeover, system} \\ \text{failure} \Rightarrow \text{A fails and then component B fails.} \\ \text{In case of unsuccessful changeover, system} \\ \text{failure} \Rightarrow \text{failure of component A.} \end{array} \right]$$

- Consider a two-component standby system with A as normally operating component and B as standby component. The reliability of component A is 0.90 while the reliability of component B given that A has failed is 0.95. Assume that the switch can fail only at the time of changeover with a probability of failure 0.03. Evaluate the reliability of the system.

$$Q = P_s Q_A Q_B + \overline{P}_s Q_A$$

$$= (1-0.03)\times(1-0.90)(1-0.95)+0.03\times(1-0.90) \text{ [using given reliabilities]}$$

$$= 0.97\times0.10\times0.05+0.03\times0.10 = 0.00485+0.003 = 0.00785$$

- the reliability of the system
- = 1 – 0.00785 = 0.99215

Improving System Reliability

- Use redundancy
- Improve component design.
- Improve production and/or assembly techniques.
- Improve testing.
- Improve preventive maintenance procedure.
- Improve user education.
- Improve system design.

- Overall system reliability is a function of the reliability of individual components, improvements in their reliability can increase system reliability.
- Unfortunately, inadequate production or assembly procedures can negate even the best of designs, and there are often sources of failures. As you have seen, system reliability can be increased by the use of backup components. Failures in actual use can often be reduced by upgrading user education and refining maintenance recommendations or procedures. Finally, it may be possible to increase the overall reliability of the system by simplifying the system.

Resources

- **Operations Management :An Integrated Approach, R. Dan Reid, Nada R. Sanders, 4th Edition, John Wiley & Sons, Inc.**
- **Reliability Engineering,** Kailash C. Kapur , Michael Pecht, 2014 , John Wiley & Sons, Inc
- **UNIT 15 RELIABILITY EVALUATION OF k-OUT-OF-n AND STANDBY SYSTEMS,** , Lecture Notes, ignou the people's university
- **UNIT 11 RELIABILITY** , Lecture Notes, ignou the people's university
- https://people.ucalgary.ca/~far/Lectures/SENG637/PDF/SENG637-03.pdf, Dependability Reliability & Dependability, Reliability & Testing of Software Systems Chapter 3: System Reliability Lecture Notes, Department of Electrical & Computer Engineering, University of Calgary, B.H. Far