MIP Models for MHD

END4650 - Material Handling Systems

Mehmet Güray Güler.

Yıldız Technical University

MIP models for Material Handling Design

- In this section we discuss deterministic optimization models (mixed-integer programming model) that helps designers select the required material-handling equipment.
- It minimizes a cost function subject to some specified system constraints.

MHD Selection and Assignment Model

- **MHD Selection and Assignment Model**

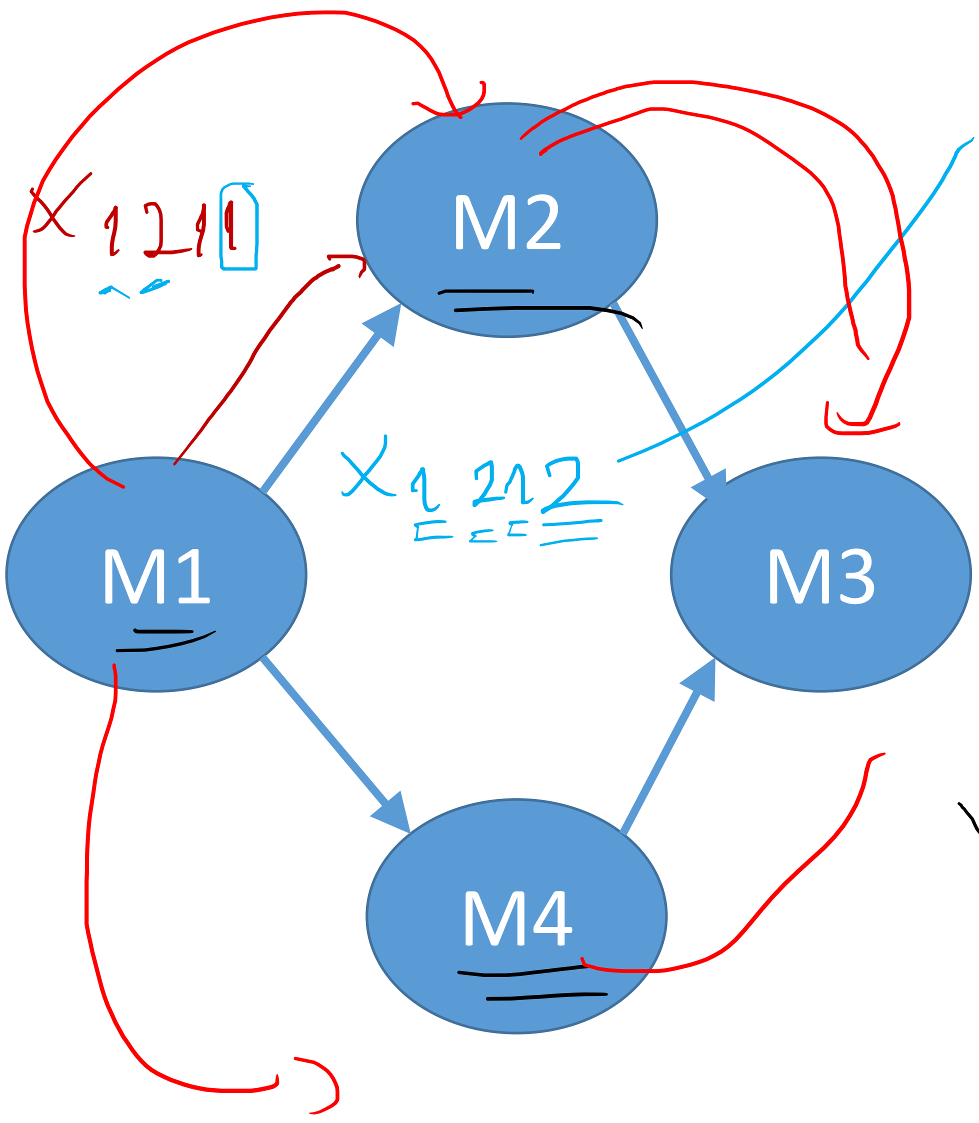
- The objective of the model for simultaneously selecting the required number and type of MHDs and assigning them to material-handling moves is to minimize the operating and annualized investment costs of the MHD.
- A material-handling move, or simply a move, is the physical move that a MHD has to execute to transport a load between a pair of machines.
- The number of moves depends not only on the volume and transfer batch size of each part type manufactured, but also on the number of machines it visits.

MHD Selection and Assignment Model

- All candidate MHD types that can perform the moves are evaluated, and the model determines an **optimal** selection and assignment.
- If necessary, we can modify the objective function of the model to incorporate equipment idle time in conjunction with capital and operating costs.
- Before presenting the model, we first give an example. We will give the definition of the model after the example.

Example

- A small manufacturing system processes two high-volume parts: P1 and P2.
- Each of 50 units of part P1 is first processed on machine M_1, and can then be sent to machine M_2 or M_4, where it is further processed and then sent to machine M3 for final processing (see Figure below).
- In addition to part P_1, the manufacturing facility processes 60 units of part P2, of which 30 are processed first on machine M1 and then on machine M2.
- Because machine M3 can do the same processing as machine M1 for part P2, the remaining 30 units of part P_2 are processed first on machine M3 and then sent to machine M2 for final processing.



X_{1122}

P1 M1 M2 D2

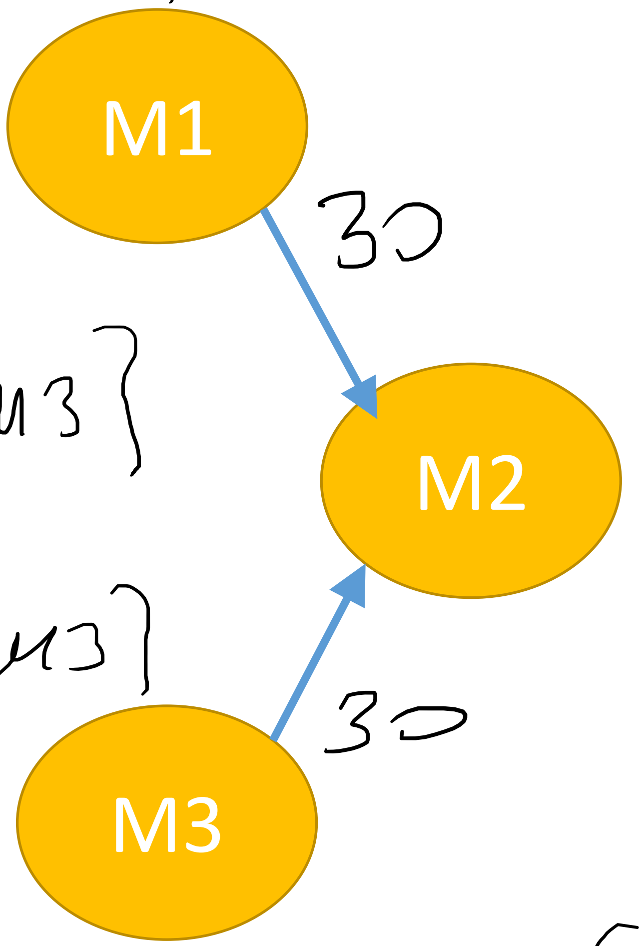
J_{pi}

$J_{11} = \{M2, M4\}$

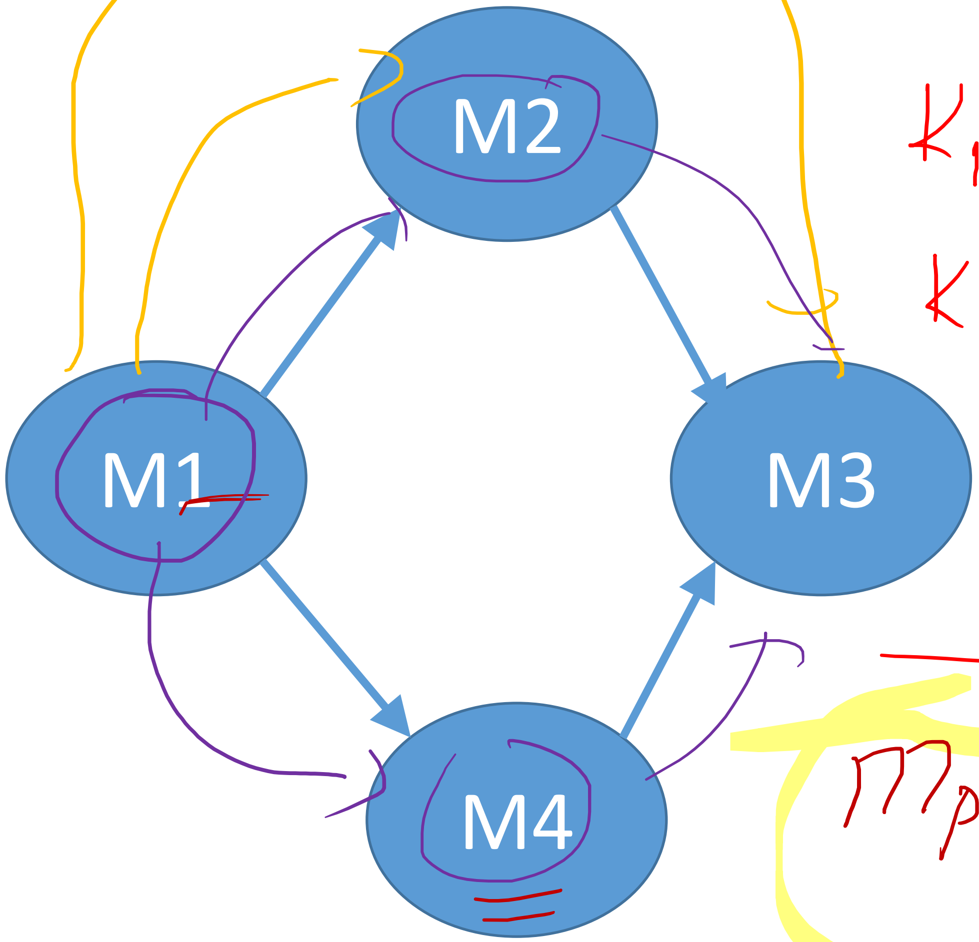
$J_{12} = \{M3\}$

$J_{14} = \{M3\}$

$J_{21} = \{M2\}$



$J_{23} = \{M2\}$



K_{pi}

$K_{11} = \{M2, M4\}$

$K_{12} = \{M3\}$

$K_{14} = \{M3\}$

M_{pi}

$M_{12} = \{M1\}$

$M_{11} = \{\ \}$

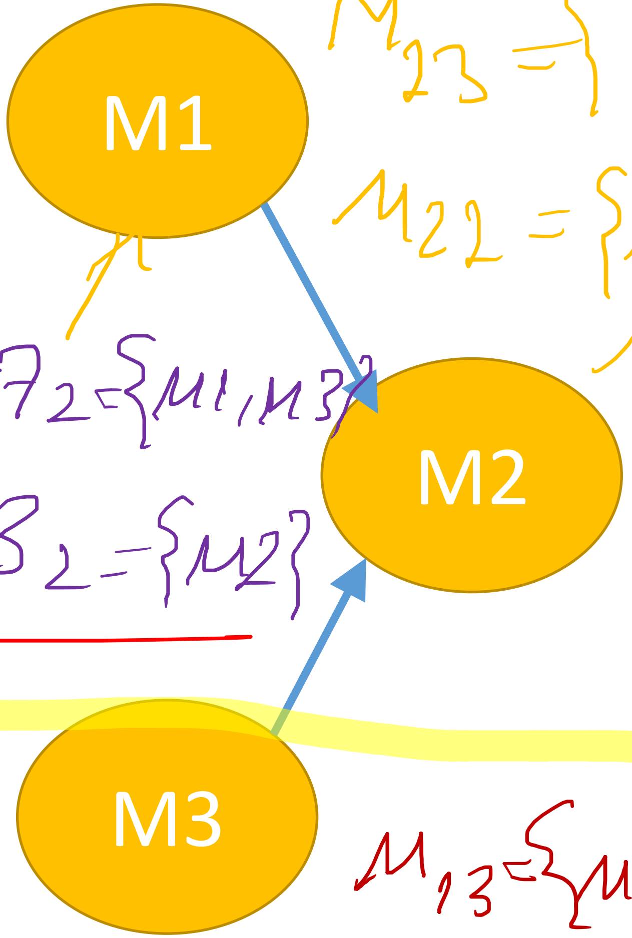
$M_{14} = \{M2\}$

$M_{13} = \{M2, M4\}$

$M_{21} = \{\ \}$

$M_{23} = \{\ \}$

$M_{22} = \{M_1, M_3\}$



$P_2 = \{M1, M3\}$

$B_2 = \{M2\}$

Example

7150 * (70%)
= 5005 minutes

P1: 50
P2: 60

- Two candidate MHDS, H_1 and H_2 , with purchase costs of $100,000 and $140,000, respectively, are being considered.
- The unit costs for transporting P_1 and P_2 on each of the MHDs between the machines are given in the following table.
- The corresponding transportation times per unit (in minutes) are provided in parentheses in the table.
- Assume that there are 7150 minutes in the planning period and each handling device is expected to make empty trips 30% of the time.
- Determine the required MHDs and assign the departmental moves to the selected MHD(s).

	From	To	M1	M2	M3	M4
P1	M1	H1		7(10)		8(5)
		H2		5(8)		2(2)
P1	M2	H1			8(6)	
		H2			4(5)	
P1	M4	H1			8(8)	
		H2			4(5)	
P2	M1	H1		2(4)		
		H2		1(2)		
P2	M3	H1		20(6)		
		H2		12(2)		

using D1 P2 to transfe Total time

$(4) \times 2131 + (6) \times 2321$

Total time to transfer P1 using D1

$(10) \times 1121 + 5 \times 1141 + 6 \times 1231 + 8 \times 1431$

500

x

y 1

	From	To	M1	M2	M3	M4
P1	M1	H1		7(10)		8(5)
		H2		5(8)		2(2)
P1	M2	H1			8(6)	
		H2			4(5)	
P1	M4	H1			8(8)	
		H2			4(5)	
P2	M1	H1		2(4)		
		H2		1(2)		
P2	M3	H1		20(6)		
		H2		12(2)		

minimize

$(100000)\, y_1 + (140000)\, y_2$

$+$

$= 7 x_{1121} + 8 x_{1141}$

$5 x_{1122} + 2 x_{1122}$

$+$

objective function

	From	To	M1	M2	M3	M4
P1	M1	H1		7(10)		8(5)
		H2		5(8)		2(2)
P1	M2	H1			8(6)	
		H2			4(5)	
P1	M4	H1			8(8)	
		H2			4(5)	
P2	M1	H1		2(4)		
		H2		1(2)		
P2	M3	H1		20(6)		
		H2		12(2)		

X_{pijd} y_d

Prod M_i M_j Design.

C_{pijd} / t_{pijd} / K_d

$$\min \sum_{d=1}^{2} K_d \cdot y_d +$$

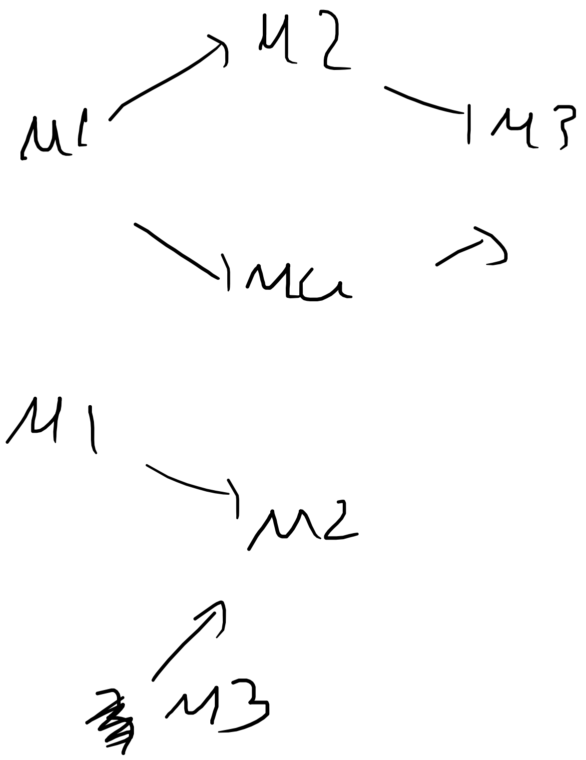
$$\sum_{p=1}^{2} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{d=1}^{2} C_{pijd} X_{pijd}$$

Time Constraint

$$\sum_{p}\sum_{i}\sum_{j} (X_{pijd}) \cdot t_{pijd} \leq (5005)\, y_d \qquad d = 1,2$$

$d = 1, \ldots, 20$

Flow Constraints:



$$\Rightarrow \sum_{d} \left(X_{112d} + X_{114d} \right) = 50$$

Variable

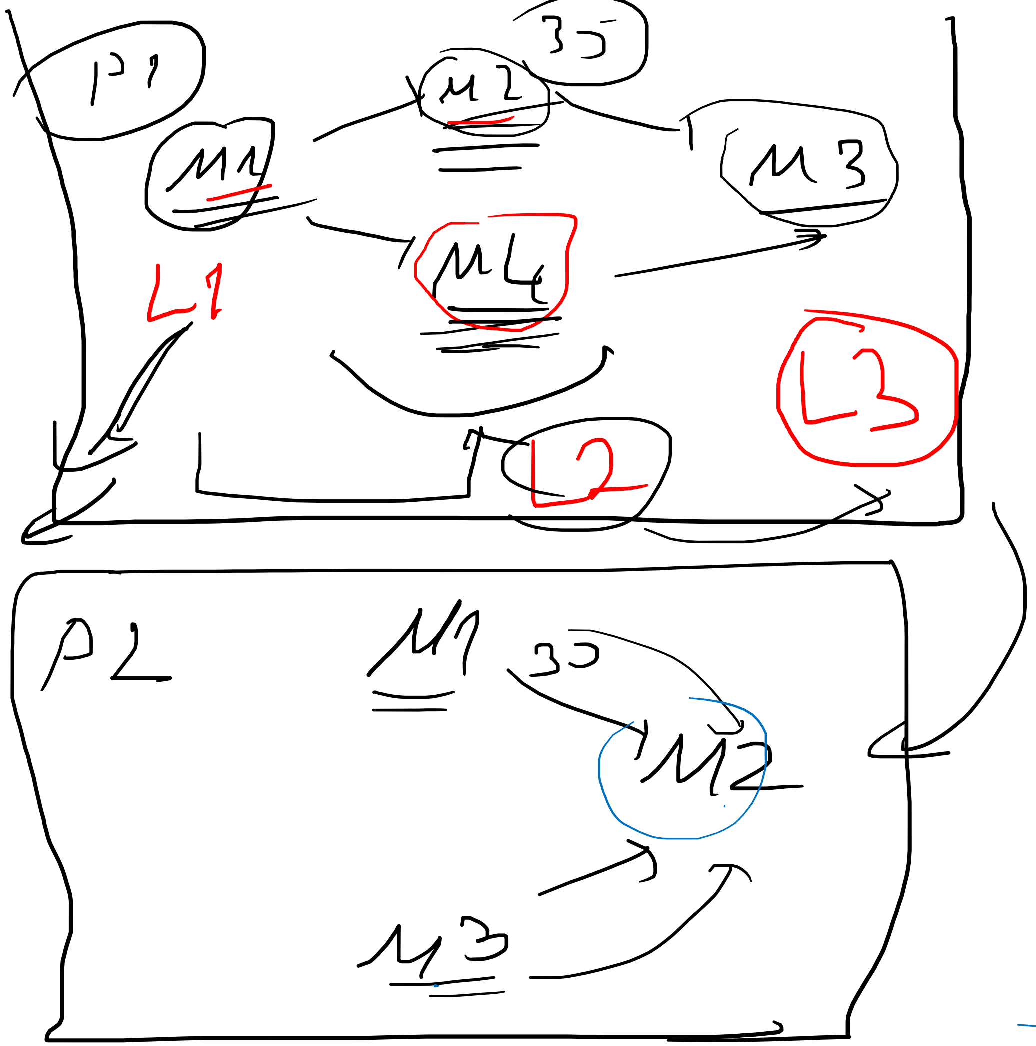
- X_{ijpd} : { the amount of prod of type p, transferred to M_j from M_i using design d.

- Y_d : { the amount of design d that is purchased.

$Y_1 = 2$ $Y_2 = 3$

Parameters

- Cost
- time.



$$X_{1121} + X_{1122} + X_{1141} + X_{1142} = 50$$

P1 M1 M2

$$X_{1121} + X_{1122} = X_{1231} + X_{1232}$$

L1 L2

$$X_{1141} + X_{1142} = X_{1431} + X_{1432}$$

$$X_{1231} + X_{1232} + X_{1431} + X_{1432} =$$

$$X_{2121} + X_{2122} = 30$$

$$X_{2321} + X_{2322} = 30$$

- We fill first define the variables:

$X_{pijd} \rightarrow \boxed{J_{pi}}$

  - X_{ijkl} number of units of part type i to be transported from machine j to k using MHD l
  - Y_l number of units of MHD type l selected

MIN 100000 Y1 + 140000 Y2 +

7 X1121+ 5 X1122+ 8 X1141 + 2 X1142 + 8 X1231 + 4 X1232 + 8 X1431 + 4 X1432

+ 2 X2121 + 1 X2122 + 20 X2321+ 12 X2322

- SUBJECT TO

2) X1121 + X1122 + X1141 + X1142 = 50
3) X1121 + X1122 - X1231 - X1232 = 0
4) X1141 + X1142 - X1431 - X1432 = 0
5) X1231+ X1232+ X1431+ X1432 = 50

P1

6) X2121 + X2122 = 30
7) X2321 + X2322 = 30

P

8) 10 X1121+ 5 x1141 + 6 X1231 + 4 X2121 + 6 X2321 + 8 X1431 <= 5005 Y1
9) 8 X1122 + 2 x1142 + 5 X1232 + 2 X2122 + 2 X2322 + 5 X1432 <= 5005 Y2

$Y_1 = 3$

$Y_L = 4$

$X_{123L} =$

- i part type index, $i=1,2,...,p$
- j machine type index, $j=1,2,...,m$
- l MHD type index, $l=1,2,...,n$
- L_i set of MHDs that can transport part i
- H length of planning period
- D_i # of units of part type i to be produced
- K_{ij} set of machines *to* which part type i can be sent from machine j for next process
- M_{ij} set of machines *from* which part type i can be sent to machine j for next process
- A_i set of machine types required for the first operation on part type i
- B_i set of machine types required for last operation on part type i
- V_l purchase cost of MHD H_l
- T_{ijkl} time required to move one unit of part type i from machine type j to k using MHD l
- C_{ijkl} unit transportation cost to move part type i from machine j to k using MHD l

- X_{ijkl} number of units of part type i to be transported from machine j to k using MHD l
- Y_l number of units of MHD type l selected

K_{pi}

M_{pi}

X_{pij4}

coming to = going from

$$Min \sum_{l=1}^{n} V_l Y_l + \sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{k\in K_{ij}}\sum_{l\in L_i} C_{ijkl} X_{ijkl}$$

Subject to

30

$$\sum_{j\in A_i}\sum_{k\in K_{ij}}\sum_{l\in L_i} X_{ijkl} = D_i \quad i = 1,2,\dots,p$$

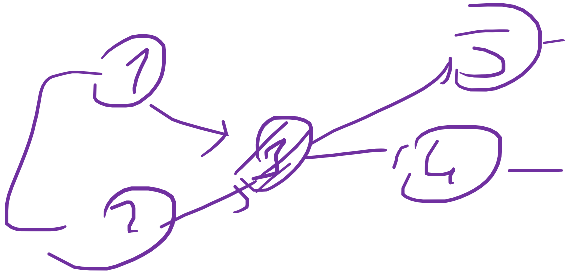
M_{12}

60

$$\sum_{k\in M_{ij}}\sum_{l\in L_i} X_{ijkl} - \sum_{k\in K_{ij}}\sum_{l\in L_i} X_{ijkl} = 0 \quad i = 1,2,\dots,p;\ j : j \notin A_i \bigcup B_i$$

$j = 1$ $\quad j = 2$

$$\sum_{j\in B_i}\sum_{k\in M_{ij}}\sum_{l\in L_i} X_{ijkl} = D_i \quad i = 1,2,\dots,p$$



$$\sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{k\in K_{ij}} X_{ijkl} T_{ijkl} \le HY_l \quad l = 1,2,\dots,n$$

$$X_{ijkl} \ge 0,\ Y_l \ge 0 \text{ and integer } an \quad i = 1,2,\dots,p;\ j = 1,2,\dots,m;\ k,l = 1,2,\dots,n$$

Assignment #~~3~~ 2

- Use python and CoinOR to model and solve the example above…

LP Equipment Selection Model

- An automobile engine cylinder manufacturing company plans to manufacture four types of cylinder: (1) a basic engine cylinder, (2) a high technology cylinder, (3) a sports car cylinder and (4) a luxury car cylinder.
- The marketing department has determined to produce 2000, 1500, 1800 and 1000 units of (1) the basic, (2) high-tech, (3) sports, and (4) luxury models, respectively according to the demand forecast.
- The facility plan department wants to design a plant and decide the number of equipment to purchase.
- Three machine types M1, M2, M3 and two material handling systems MH_1, MH_2 are available for performing the three operations and transporting products.
- The cylinder models require one or more of three operations, O_1, O_2 and O_3
- The **total** required numbers of each type of operation are 200, 200 and 150 respectively to produce the all of the demands. Well, somebody has calculated and giving it to us.
- Finished goods are transported using either of two material handling systems MH_1, MH2.

Example – 2:

- The costs of machines M1, M2 and M3 are
  - \$230,000, \$250,000 and \$310,000.
- The costs of the material handling carriers MH_1, MH_2 are
  - \$90,000 and \$130,000.
- The available budget for equipment is \$10,000,000.
- The following two matrices show the operating cost.
  - C_{ie}: The cost of performing operation i on production equipment e
    h_{pm}: The cost of handling product type p using the mat. handling system m
- The time required to perform operation i on equipment e is inversely proportional to the corresponding cost, $t_{ie} = 10/c_{ie}$.
- Similarly, the time required to transport product p on material handling system m is inversely proportional to the corresponding cost, $s_{pm} = 1/h_{pm}$.
- Each machine is available for 300 unit time and each material handling system is available for 400 unit time.

C_ij	6	12	8
	4	5	4
	12	5	5
H_mn	10	5	
	12	6	
	18	9	
	6	3	

Variables and Parameters.

- x_{ie} number of operations i to be performed on production equipment type e
- y_{pm} number of units of part type P_m to be transported on material handling system type m
- z_e number of units of production equipment type e selected
- k_m number of units of material handling system type m selected

- $6 X11 + 12 X12 + 8 X13 + 4 X21 + 5 X22 + 4 X23 + 12 X31 + 5 X32 + 5 X33$
- $+ 10 Y11 + 5 Y12 + 12 Y21 + 6 Y22 + 18 Y31 + 9 Y32 + 6 Y41 + 3 Y42$
- $+ 90000 NMH1 + 130000 NMH2 + 230000 NM1 + 250000 NM2 + 310000 NM3$
- SUBJECT TO
- C1) $X11 + X12 + X13 >= 200$
- C2) $X21 + X22 + X23 >= 200$
- C3) $X31 + X32 + X33 >= 150$
- C4) $0.167 X11 + 0.25 X21 + 0.083 X31 <= 300 NM1$
- C5) $0.083 X12 + 0.20 X22 + 0.200 X32 <= 300 NM2$
- C6) $0.125 X13 + 0.25 X23 + 0.200 X33 <= 300 NM3$
- C7) $Y11 + Y12 >= 2000$
- C8) $Y21 + Y22 >= 1500$
- C9) $Y31 + Y32 >= 1800$
- C10) $Y41 + Y42 >= 1000$
- C11) $0.1 Y11 + 0.0833 Y21 + 0.056 Y31 + 0.167 Y41 <= 300 NMH1$
- C12) $0.2 Y12 + 0.167 Y22 + 0.11 Y32 + 0.33 Y42 <= 300 NMH2$
- C13) $90000 NMH1 + 130000 NMH2 + 230000 NM1 + 250000 NM2 + 310000 NM3$

  $<= 1000000$

- O_i Operation type i, i=1,2,...,
- M_e Production equipment type e, i=1,2,...,
- P_i Part type p, i=1,2,...,p
- MH_m Material handling system type m, i=1,2,...,
- c_{ie} cost of performing operation i on production equipment type e
- h_{pm} cost of handling part type p using material handling system type m
- t_{ie} time required to perform operation i on production equipment type e
- s_{pm} time required to transport part type p using material handling carrier type m
- τ_e time available on production equipment type e
- σ_m time available on material handling carrier type m
- NO_i number of operations O_i to be performed
- NP_p number of units of part type P_i to be manufactured
- C_e cost of production equipment type e
- H_m cost of material handling system m
- B total budget available

$$\min \sum_{i=1}^{O}\sum_{e=1}^{E} c_{ie}x_{ie} + \sum_{p=1}^{P}\sum_{m=1}^{M} h_{pm}y_{pm} + \sum_{e=1}^{E} C_e z_e + \sum_{m=1}^{M} H_m k_m$$

$$\sum_{e=1}^{E} x_{ie} \geq NO_i \quad i = 1,2,\ldots O$$

$$\sum_{m=1}^{M} y_{pm} \geq NP_i \quad p = 1,2,\ldots P$$

$$\sum_{i=1}^{O} t_{ie}x_{ie} \leq \tau_e z_e \qquad e = 1,2,\ldots,E$$

$$\sum_{p=1}^{P} y_{pm} \leq \sigma_m k_m \quad m = 1,2,\ldots M$$

$$x, y, z, k \ are \ integer$$

$$\sum_{e=1}^{E} C_e z_e + \sum_{m=1}^{M} H_m k_m \leq B$$