EXERCISES FOR STATISTICS

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Abbreviations

HT-One Sided: One sided hypothesis test

HT-Two Sided: Two sided hypothesis test

HT-P Value: Hypothesis test with p-value

CI: Confidence interval

PI: Prediction Interval

TL: Tolerance Limit

Type 1: Type I error problem

Type 2: Type II error problem

Power: Power of the test

Preface

This booklet contains some auxiliary exercises for the statistics course that I teach in several universities (Yıldız Technical University, Özyeğin University, Bilgi University, Şehir University and Boğaziçi Universtiy). There are two strong motivations behind preparing this booklet. First, it turns out that students feel comfortable if they have a single source to follow although one can find many sources (books, web sites, examples, etc) on the web. In fact I follow the Walpole's book in my courses and I include several questions in this booklet as well. However the questions in the book are very "engineering" type of questions and cannot attract the students' attention. Therefore I prepared some questions from scratch and also try to find interesting questions from the web for my lectures. This booklet is a mixture of all. Second, I follow a different pathway to cover the topics of Statistics. The usual way (like the one in the Walpole et al) is to cover the distributions first (like t, F and χ^2), then confidence interval and hypothesis testing. I, on the contrary, first talk about normal distribution and central limit theorem, and then cover hypothesis testing and confidence interval for a single mean. I then go back and cover t-distribution and then talk about hypothesis testing and confidence interval for a single mean with unknown variance and continue like this. Hence, in a sense, I cover the topics by taking the *transpose* of the usual coverage.

I would like to thank Dilara Sayımlar who merged all questions of recitations, problem sessions and assignments to prepare the first draft of this booklet. I would like to thank Ebru Geçici for the effort that she put to make it possible to prepare the first version of this booklet. Finally I would also like to thank Prof. Erkan Türe who shared all of his lecture notes and make it possible for me to prepare my own lecture notes.

All comments and corrections are more than welcome. I would be happy if you cite our name if you use our examples in your lectures. The single star questions are from Walpole 9th edition and the double star questions are created by us.

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0. *Probability Review*

1. A life insurance salesman sells on the average 7 life insurance policies per week. Use Poisson's law to calculate the probability that in a given week she will sell

a. 2 or more policies but less than 5 policies.

b. In the first 3 days, it is known that she could not sell any policy, what is the probability that she will not sell any policy 2 more days.

2. **Given a standard normal distribution, find the value of **k** that satisfies the following. For each part please draw the standard normal distribution plot and show your calculations on the plot as we did in the lectures.

a. $P(z > k) = 0.86$

b. $P(k < z < -0.18) = 0.4197$.

c. $P(z < k) = 0.3156$

3. **Let X~$N(\mu = 10, \sigma = 3)$. Please find the value of **k** in the following questions

a. P(X = 10) =k

b. P(X<7) = k

c. P(8<X<13) = k

d. P(X <k) = 0.80

4. **Given $X \sim N(\mu = 10, \sigma^2 = 4^2)$,

a. Find the probability that X assumes a value between 6 and 14,

b. Find the value of X that has 10% of the area to the left and

c. Find the value of X that has 5% of the area to the right.

5. **Let $X \sim N(\mu, \sigma^2)$. Find the following values.

a. $P(\mu - \sigma < X < \mu + \sigma)$

b. $P(\mu - 2\sigma < X < \mu + 2\sigma)$

c. $P(\mu - 3\sigma < X < \mu + 3\sigma)$

d. $P(\mu - 2\sigma < X < \mu + 2\sigma)$

6. **Find the following values.

a. $z_{0.15}$

b. $z_{0.05} - z_{0.10}$

c. $P(z < z_{0.05})$

d. $P(-z_{0.05} < z < z_{0.15})$

7. **Let a and b two real numbers such that $0 < a < 1$ and $0 < b < 1$. Find **k** in the following

a. $z_k = 1.96$

b. $z_k = 1$

c. $z_k = 0$

d. $P(z < z_a) = k$

e. $P(z_b < z < z_a) = k$

8. **Please answer the following questions:

a. If X $X \sim N(\mu, \sigma)$ find $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$

b. If a is a scalar such that $0 < a < 1$, find $P(z \leq z_a)$

c. Find $P(-z_{0.05} \leq z \leq z_{0.15})$
d. If X~$N(\mu = 10, \sigma = 3)$ find $P(4 \leq X \leq 16)$.
e. Find $P(-t_{0.05} \leq t \leq t_{0.15})$ where t has 8 degrees of freedom.

9. **Find the value of z if the area under a standard normal curve
(a) To the right of z is 0.3632
(b) To the left of z is 0.1131

10. **Given the normally distributed variable X with mean 18 and standard deviation 2.5, find
(a) The value of k such that $P(X<k) = 0.2236$
(b) $P(17<X<21)$

11. ***A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distributed and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

12. ***A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live
(a) More than 32 months,
(b) Less than 28 months,
(c) Between 37 and 49 months.

13. ***A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,
a. What fraction of the cups will contain more than 224 milliliters?
b. What is the probability that a cup contains between 191 and 209 milliliters?
c. How many cups will probably overflow if 230 milliliter cups are used for the next 1000 drinks?
d. Below what value do we get the smallest 25% of the drinks?

14. *A company pays its employees an average wage of $15.90 an hour with a standard deviation of $1.50. If the wages are approximately normally distributed and paid to the nearest cent,
a. What percentage of the workers receive wages between $13.75 and $16.22 an hour inclusive?
b. The highest 5% of the employee hourly wages is greater than what amount?

15. ***In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$. On average, how many manufactured ball bearings will be scrapped?

16. ***Traveling between two campuses of a university in a city via shuttle bus takes on averages, 28 minutes with a standard deviation of 5 minutes. In a given week, a bus transported passengers 40 times. What is the probability that the average transport time was more than 30 minutes? Assume the mean time is measured to the measured minute.

1. *Central Limit Theorem*

1. **I am planning to purchase a 1 GB memory card to my cellular phone to keep my pictures on this card for at least a year. Assume that the pictures that I take with my phone has a size of 4MB on the average and their standard deviation is 2 MB, i.e., they are distributed with mean 4MB and standard deviation 2MB. If I take 5 pictures per week, then what is the approximate probability that my card will be enough to keep the pictures that I took through a year? Take 52 weeks in a year and 1GB = 1000MB (Hint: If X is normally distributed with mean μ and st. deviation σ, then aX is normally distributed with mean $a\mu$ and st. deviation $a\ \sigma$).

2. Pictures on your smartphone have a mean size of 400 kilobytes (KB) and a standard deviation of 50 KB. You want to store 100 pictures on your cell phone. If we assume that the size of the pictures X_1, X_2, …, X_{100} are independent, then $\bar{X}$ has mean $\mu_{\bar{X}}$ = 400 KB and standard deviation $\sigma_{\bar{X}}$= $50/\sqrt{100}$ KB. So, find the probability that the average picture size is between 394 and 406 KB.

3. Multiple choice questions

a. C.L.T. states that, the mean of a sufficiently large number of iterates of independent random variables from a normal population, each with a well-defined mean and well-defined variance, will be approximately normally distributed.

YES **NO**

b. One of the differences between normal distribution and t-distribution is the symmetry. Normal distribution is a symmetric distribution despite t-distribution.

YES **NO**

4. Answer and explain each of the following briefly:

a. Two confidence intervals will be constructed for the mean σ of a normal population, using the same data set. The first CI will have 99% confidence level, the second 90% confidence level. Is it true that the 99% CI will be 10 percent longer than the 90% CI? Explain your reason, and show your calculations.

b. Some people activate the detector at the entrance of a shopping centre even though they have no metal objects on them. The security officer who uses the detector is curious about that. He starts an experiment as follows: For every successive 100 customers passing through the detector he counts the number of people who activate the detector with no metal object on them. What would be the probability model for this experiment? (Define the random variable here and explain what probability distribution and parameters it should have).

c. Explain briefly what the Central Limit Theorem states and why it is so important in statistics.

5. The length of a certain PVC pipe has mean 36 inches and standard deviation 0.6 inch.

a. Find the probability that the average length of a sample of 100 pieces exceeds 36.15 inches.

b. Determine the value Δ for which there is a 90% probability that the interval 36±Δ contains the average length of a random sample of 100 pieces.

c. Let X denote the average length of a random sample of n pieces. For what value of n does P(35.85<X<36.15)=0.95?

d. What is the probability that the total length of 50 pipes will exceed 1812 inches?

2. Estimators

1. Suppose that $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimators of the parameter θ. We know that $E(\hat{\theta}_1) = \theta$, $E(\hat{\theta}_2) = \theta/2$, $Var(\hat{\theta}_1) = 10$ and $Var(\hat{\theta}_2) = 4$. <u>Answer the next 2 questions using this information.</u>

a. Which of the following is the complete answer for the unbiased estimators of θ?

  a. $\hat{\theta}_1$ is an unbiased estimator for θ
  b. $\hat{\theta}_2$ is an unbiased estimator for θ
  c. Both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators for θ
  d. Both $\hat{\theta}_1$ and $2\hat{\theta}_2$ are unbiased estimators for θ
  e. Both $\hat{\theta}_1/2$ and $\hat{\theta}_2$ are unbiased estimators for θ

b. Which of the following is the minimum variance unbiased estimator for θ ?

  a. $\hat{\theta}_1$
  b. $\hat{\theta}_2$
  c. $2\hat{\theta}_2$
  d. $\hat{\theta}_1/2$
  e. There is only one unbiased estimator and nothing else to compare with it to answer the question.

2. Let $X_1, X_2, \dots, X_{2n}$ be a random sample from a population with the mean μ and standard deviation σ. Which of the following is the $E(\frac{1}{2n}\sum_{i=1}^{2n} X_i)$?

  a. μ / (2n)
  b. μ / 2
  c. μ
  d. 2μ
  e. 2nμ

3. Please, for the following, explain which one is a variable or which one is a constant and also which one refers to a population or a sample?:

$$\mu, \overline{X}, \sigma, s$$

4. X is a discrete r.v. with the following PMF, and 0<k<1 is a parameter.

X	0	1	2	3
P(X)	2k/3	k/3	2(1-k)/3	(1-k)/3

3	0	2	1	3
2	1	0	2	1

Using the above 10 independent observations from such a distribution, what is the maximum likelihood estimate of k?

3. *Data Structure*

1. The time to failure in hours of an electronic component subjected to an accelerated life test is shown in following table. To accelerate the failure test, the units were tested at an elevated temperature (read down, then across).

Electronic Component Failure Time			
124	142	137	187
174	126	150	164
153	159	130	148
130	127	165	146
154	155	115	169
183	137	144	133
162	125	119	136
165	174	142	149
140	146	166	143
169	179	135	157

a. Calculate the sample average and standard deviation.
b. Construct a histogram. (Please use 8 bins) (Hint: There is a histogram tool in MS Excel, you can use it if you wish. Or you can plot it manually.)
c. Construct a stem-and-leaf plot.
d. Find the sample median and the lower and upper quartiles.

2. Suppose that the data given below is the length of time in months before the first major repair is required on samples of Brand X and Brand Y computers.

Brand X:	8	9	11	12	18	32
Brand Y:	0	0	9	11	20	32

Make the following hand calculations:
a. Compute the sample mean for each brand.
b. Compute the sample standard deviation for Brand X.
c. Using the same scale, draw box-and-whisker plots for the two data sets.
d. Based on the above information, which brand would you prefer, and why?

3. **The height of 50 people in the IE department is given in the following.

180	167	159	153	162
168	161	165	156	174
181	166	171	167	168
168	166	183	177	177
172	158	160	187	179
176	170	184	169	159
150	178	171	157	181
172	161	158	176	174
167	175	174	160	174
164	179	143	188	164

a. Calculate the sample average and standard deviation.
b. Construct a histogram. (Please use 8 bins) (Hint: There is a histogram tool in MS Excel, you can use it if you wish. Or you can plot it manually.)
c. Construct a stem-and-leaf plot.
d. Find the sample median and the lower and upper quartiles

4. **Consider the following data:

12	15	17	14	16	14	13	11	14

a. Find $\bar{X}$
b. Assuming that $\bar{X}$ you found in a) is 15, find s.
c. Find the mode and the median.
d. Find the lower and upper quartile.

4. *R – Simulation*

1. **In the lecture we said that in order to find the distribution of a RV, we can generate many of them. Consider the height of students in YTU. Assume that it is normally distributed with mean 170 and standard deviation of 10 cm. Consider a class of 50 people.

a. Consider the sample mean $\bar{x}$. Using simulation of 10.000 (that is, I don't want you to analytically calculate the results):
   i. What is the mean of the sample mean $\bar{x}$?
   ii. What is the standard deviation of $\bar{x}$?

b. Consider the sample standard deviation s. Using simulation of 10.000 (that is, I don't want you to analytically calculate the results)
   i. What is the mean of the sample standard deviation s?
   ii. What is the standard deviation of the sample standard deviation s?

Now, since you know R, please use R. I have attached the codes that I have used in the class for the median in our shared folder for the lecture notes. Moreover, I have a simple tutorial for R in my web site. Plus, there are thousands of websites and also stackoverflow for coding, you can use any of them.

Please, explain your steps in details, ie, how you find the expected value and the sample median of the data, how you make the simulation, etc.

2. **Let X and Y be two independent exponential random variables with β1=1 and β2=2. Define the random variable A = min(X,Y). Using simulation of 10.000 X and Y variables, please find

a. The expected value of A.
b. The variance of A.

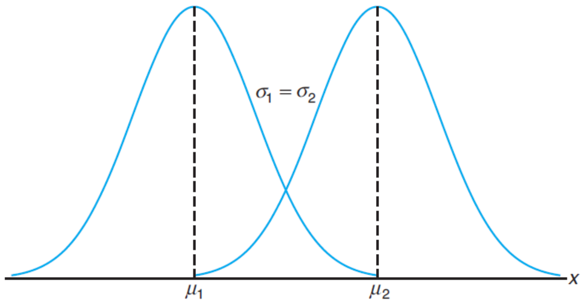
3. **For the previous question, can you *analytically* find the expected value and variance of A?
4. **Again, for the same question, can you *analytically* find the probability mass function of A?

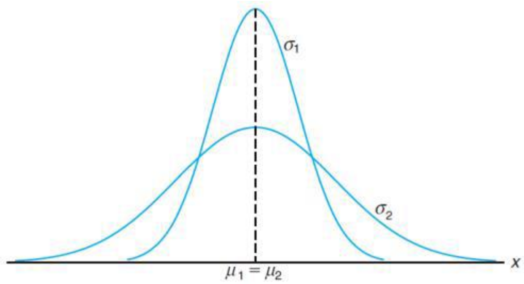
5. **We don't know the distribution of the heights of the students in our university but let's assume we know that the height of the students has a mean of 165cm and a standard deviation of 15 cm. Please explain and/or calculate the probability of the following:
a. The height of a student coming into class is between 160 and 170?
b. The average height of a class of 50 people is between 160 and 170?

6. **We don't know the ***distribution*** of the heights of the students in our university but let's assume we know that the height of the students has a mean of 165cm and a standard deviation of 15 cm. Please explain and/or calculate the probability of the following:

a. The height of a student coming into class is between 160 and 170?
b. The average height of a class of 50 people is between 160 and 170?

Consider the question above. Please write the hypothetic distribution of average height of a class of given populations of n1=20,2=50,n3=100 and plot their distributions of the





7. **In the lecture we said that in order to find the distribution of a RV, we can generate the histogram of it. Consider the height data above, i.e., the height of the students has a mean of 165 cm and a standard deviation of 15 cm. Consider a class of 50 people.

a. Find the expected value of **the sample median** of such a class
b. Find the standard deviation of **the sample median** of such a class.

Again, you can use Excel or any other software that you are familiar with. A simulation of size of 1000 will be sufficient. Larger simulations are more than welcome. Below you can find the definition of the median. Please, explain your steps in details, i.e., how you find the expected value and the sample median of the data, how you make the simulation, etc.

Given that the observations in a sample are $x_1, x_2, \ldots, x_n$ arranged in increasing order of magnitude, the sample median is

$$\bar{x} = \begin{Bmatrix} x_{(na+1)/2}, & if\ n\ is\ odd \\ \frac{1}{2}\left(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}\right), & if\ n\ is\ even \end{Bmatrix}$$

5. *Mean*

5.1 Hypothesis Testing

1. ***(HT-Two Sided)** A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that mean life span today is 70 years? Using five steps of hypothesis testing with both X and Z. Use a 0.01 level of significance.

2. ****(HT-Two Sided)** Assume from TUIK I know the heights of people in Turkey has a standard deviation of 5 cm. I assert that the height of the students in the university has a mean of 175 cm. We picked 25 people in the class and it turns out that the sample average is $\bar{X} = 177$ cm. **Using five steps** of *hypothesis testing with Z*, please make the hypothesis test with

   a. $\alpha = 0.05$
   b. $\alpha = 0.01$

   using 5 step procedure.

3. Which of the following is not a statistical hypothesis?
   a. $\mu \neq 10$
   b. $p < 0.5$
   c. $\bar{y} > 100$
   d. $\sigma^2 = 0.3$
   e. $\sigma \leq 3$

4. ****(HT-Two Sided)** Assume from TUIK I know the heights of people in Turkey has a standard deviation of 5 cm. I assert that the height of the students has a mean of 175 cm. We picked 50 people in the class and it turns out that the sample average is 170 cm. Please make the hypothesis test with $\alpha = 0.01$. Using five steps of hypothesis testing with both X and Z.

5. ***(HT-Two Sided)** A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance.
   Please **Use five steps** procedure of *hypothesis testing with X*.

6. **(HT-Two Sided)** When a robot welder is in adjustment, its mean time to perform its task is 13.250 minutes. Past experience has found the standard deviation of the cycle time to be 0.396 minutes. An incorrect mean operating time can disrupt the efficiency of other activities along the production line. For a recent random sample of 60 jobs, the mean cycle time for the welder was 13.229 minutes. Does the machine appear to be in need of adjustment? (Use $\alpha = 0.05$)

7. ***(HT-Two Sided)** An electrical firm manufactures light bulbs that have life time that is approximately *normally distributed* with mean of 800 hours and a standard deviation of 40. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a 0.05 level of significance.
   Using five steps procedure of *hypothesis testing with both X and Z.*

8. ****(HT-Two Sided)** I want to test the hypothesis $H_0: \mu = 170$ cm vs $H_1: \mu \neq 170\ cm$. From Google I know that $\sigma = 10\ cm$ for the generation between 18-22 in the world.
   a. What can you say about the distribution, mean and variance of $\bar{X}$?
   b. Please write the appropriate hypothesis and test against,
      i. $\alpha = 0.1$

ii. $\alpha = 0.05$
iii. $\alpha = 0.01$

c. Find the corresponding critical regions for
   i. $\alpha = 0.1$
   ii. $\alpha = 0.05$
   iii. $\alpha = 0.01$

d. Can you plot the hypothesis test for $\alpha = 0.10$ as we did in the lecture, i.e., please plot the distribution, show the values of the critical region, put the $\bar{x}_{obs}$ on the plot.

e. Without making any further calculations, can you say what happens to the critical region? Will it get wider or narrower?

9. ***(HT-Two Sided)** The Edison Electric Instıtute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, 46 kilowatt hours annually? *Use 5 step procedure.*

10. **(HT-Two Sided)** The life in hours of a battery is known to be approximated normally distributed, with standard deviation 1.25 hours. A random sample of 10 batteries has a mean life of 40.5 hours. Is there evidence to support the claim that battery life equals 40 hours? Use $\alpha = 0.05$ and *5 step procedure.*

11. **(HT-Two Sided)** An analyst fixes the population standard deviation at 0.00003 and uses an instrument to measure the length of lines occurring in a certain experiment that are supposed to be $0.00025m$. She computes the sample mean as $0.00029m$ for $n = 8$. Can she conclude that $\mu = 0.00025m$ at $\alpha = 0.05$?

12. ****(HT-One Sided)** I argue that the 500 ml bottled waters (like Erikli) are in fact less than 500 ml. In order to test it, we pick 10 bottles of water. Their sample mean is 480 ml and the sample standard deviation is 10ml. Is there any evidence that the mean level of water is less than 500 ml? Construct a hypothesis test and use $\alpha = 0.05$ level of significance.

13. **(HT-One Sided)** An outbreak of Salmonella-related illness was attributed to ice cream produced at a certain factory. Scientists measured the level of Salmonella in 9 randomly sampled batches of ice cream. The levels (in MPN/g) were: 0.593, 0.142, 0.329, 0.691, 0.231, 0.793, 0.519, 0.392, 0.418. Is there evidence that the mean level of Salmonella in the ice cream is greater than 0.3 MPN/g? Use $\alpha = 0.05$ level of significance.

14. Suppose that nine observations are selected at random from the normal distribution with mean 20 and unknown variance, and for these nine observations it is found that $\bar{X} = 22$ and $\sum_{i=1}^{n}(X_i - \bar{X})^2$=72.

a. **(HT-One Sided)** From the data, test of hypothesis at the level of significance 0.05

$$H_0: \mu = 20 \qquad H_1: \mu > 20$$

b. **(CI)** Construct the confidence interval for μ with α = 0.95.

15. **(HT-One Sided)** The administrator at your local hospital states that on weekends the average wait time for emergency room visits is 10 minutes. Based on discussions you have had with friends who have complained on how long they waited to be seen in the ER over a weekend, you dispute the administrator's claim. You decide to test your hypothesis. Over the course of a few weekends you record the wait time for 40 randomly selected patients. The average wait time for these 40 patients is 10.5 minutes. I want you to find whether you have enough evidence to support your hypothesis that the average ER wait time exceeds 10 minutes or not.

Assume that the standard deviation for waiting in the ER room is given as $\sigma = 2.5$ minutes and take the significance level as $\alpha = 0.05$

a. Please test the hypothesis question using the $\bar{X}$ as your test statistic. Please explicitly calculate your critical region and specify your decision based on this critical region.
b. For part (a) please plot the critical region and your observed test statistic on the plot.
c. Now answer the question using the standard normal random variable z as your test statistic.
d. For part (c) please plot the critical region and your observed test statistic on the plot.
e. Now assume that you are not given σ hence you have to calculate the sample standard deviation. It turns out that $s = 2$. Now, using the appropriate test statistic please make the hypothesis testing.
f. For part (e) please plot the critical region and your observed test statistic on the plot.

16. (HT-One Sided) If the test statistics of an upper tailed test for 20 observations is t=2.7, should the statistician agree with the claim under H_0 at the significance level of 0.05?

a. Since H_0 is rejected, the statistician agrees with the claim
b. Since H_0 is not rejected, the statistician agrees with the claim
c. Since H_0 is rejected, the statistician disagrees with the claim
d. Since H_0 is not rejected, the statistician disagrees with the claim
e. I need to know what the claim is to answer this

17. (HT-One Sided) A certain type of brick is being considered for use in a construction project. The brick will be used **unless** sample evidence strongly suggests that the true mean compressive strength ($\boldsymbol{\mu}$) is below 3200 psi. A random sample of 36 bricks has been selected, and each was subjected to a compressive strength test. The results are: $\bar{\boldsymbol{x}}$ = 3107 psi, and $\boldsymbol{s}$ = 186 psi. Assume the data follows a normal distribution.

a. **(HT-One Sided)** State the relevant hypotheses and carry out a test to reach a decision at α = 0.01. Write down your conclusion clearly. Calculate the ***P***- value.
b. Calculate the probability that the bricks will be erroneously accepted (used) when the true average strength is $\boldsymbol{\mu}$ = 3100 psi. (Use the normal distribution for the test statistic)
c. **(Sample Size)** The probability that the bricks will be erroneously accepted (used) – when actually $\boldsymbol{\mu}$ = 3100 – is desired to be at most 0.01. How many more bricks should be tested to ensure this?

18. *(HT) A chemical engineer claims that the population mean yield of a certain batch process is 500 grams per millilitre of raw material. To check this claim he samples 25 batches each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with this claim. What conclusion should he draw from a sample that has a mean $\bar{X} = 518$ grams per milliliter and a sample standard deviation s = 40 gram? Assume the distribution of yields to be approximately normal.

5.2 Confidence Interval, Prediction Interval, Tolerance Limits

19. **(CI)** Assume you have calculated a 90% CI for the mean of a population. If you want to increase your confidence level, what happens to the CI? Does it shrink, move to left or move to right?

20. ***(CI)** The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 gram per milliliter. Assume that the population standard deviation is 0.3 gram per milliliter.

a. Find the 95% confidence interval and
b. Find the 99% confidence interval for the mean zinc concentration in the river.

21. **(CI)** What is the confidence level for the interval $\bar{x} \pm 1.23\frac{\sigma}{\sqrt{45}}$?

a. 0.1093
b. 0.2186
c. 0.6045
d. 0.7814
e. 0.8907

22. **(CI)** If I argue that the only difference between the intervals (50,105) and (25,130) for μ is the confidence level and everything else is the same, which of the following is <u>correct</u>?

a. The sample mean is 85.
b. The width of the wider interval is 55.
c. The bound of the error estimation of the wider interval is 27.5.
d. The width of the narrower interval is 105.
e. If hypothesized μ falls in the first interval, it definitely falls in the second interval.

23. ***(CI)** Gauges are used to reject all components for which a certain dimension is not within the specification $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.2. Determine the value d such that the specifications "cover" 95% of the measurements.

24. ***(CI)** The contents of seven similar containers of sulfuric acid are 9.8, 10.5, 11.4, 9.7, 10.3, 11.2, and 9.6 liters. Find a 95% *confidence interval* for the mean contents of all such containers, assuming an approximately normal distribution.

25. ***(CI)** An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed. If a sample of 9 bulbs has an average life of 780 hours, find a 95% confidence interval for the population mean of all bulbs produced by this firm if

a. the sample standard deviation is s=40?
b. the population standard deviation is known to be $\sigma = 40$?

26. If the mean life of 20 printers is 1014 hours, (where lifetimes are normally distributed with $\sigma = 25$ hours)

a. **(CI)** Construct a 95% two-sided confidence interval on the mean life.
b. **(Sample Size)** If we wanted to be 95% confident that the error in estimating the mean life is less than 5 hours, what sample size should be used?

27. **(CI)** The width of 55 glass pieces were measured. The sample mean was X= 4.05 mm, and the sample standard deviation was s=0.08 mm. Find a 90% lower confidence interval on the mean width.

28. **(CI)** A random sample of 100 recorded deaths in Turkey during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, construct a 95% confidence interval.

29. *(CI) A machine is used to fill boxes with product in an assembly line operation. The standard deviation in weight of product was known to be 0.3 kg. An improvement is implemented, after which a random sample of 16 boxes is selected and the sample variance is found to be 0.045 kg.

a. Find a 95% confidence interval on the variance in the weight of the product.

b. By looking at the result of (a) and previous value 0.3, can you say that the improvement made significant reduction in the standard deviation? Why?

30. *(PI) Due to the decrease in interest rates, the First Citizens Bank received a lot of mortgage applications. A recent sample of 50 mortgage loans resulted in an average loan amount of \$257300. Assume a population standard deviation of \$25000. For the next customer who fills out a mortgage application, find a 95% *prediction interval* for the loan amount.

**31. ***In a study conducted by the Department of Zoology at Virginia Tech, fifteen samples of water were collected from a certain station in the James River in order to gain some insight regarding the amount of orthophosphorus in the river. The concentration of the chemical is measured in milligrams per liter. Let us suppose that the mean at the station is not as important as the upper extreme of the distribution of the concentration of the chemical at the station. Concern centers around whether the concentration at the extreme is too large. Readings for the fifteen water samples gave a sample mean of 3.84 milligrams per liter and a sample standard deviation of 3.07 milligrams per liter. Assume that the readings are a random sample from a normal distribution.

a. **(PI)** Calculate a prediction interval (upper 95% prediction limit).

b. **(TL)** Calculate tolerance limit (95% upper tolerance limit that exceeds 95% of the population of values).

c. **(PI / TL)** Interpret both (part a and b); that is, tell what each communicates about the upper extreme of the distribution of orthophosphorus at the sampling station.

32. A machine produces computer memories that are square in shape. A sample of these memories is taken and the diagonals are found to be 1.12, 1.03, 0.97, 0.99, 1.02, 0.98, 1.07, 1.05, 1.01 and 1.03 centimeters.

a. **(CI)** Find a 90% *confidence interval* on the mean diagonal.

b. **(PI)** Compute a 95% *prediction interval* on a measured diagonal of a single computer memories taken from the machine.

c. **(TL)** Find the 99% *tolerance limits* that will contain 95% of the computer memories produced by this machine.

33. (CI) From past experience, the population standard deviation of rod diameters produced by a machine has been found to be $\sigma = 5.3$ inches. For a simple random sample of $n = 300$ rods, the average diameter is found to be $\bar{x} = 140$ inches. What is the 95% confidence interval for the population mean, μ?

34. (CI) If we increase the confidence level for the confidence interval keeping everything else being the same, which of the following is incorrect?

a. The confidence interval becomes wider

b. The width of the confidence interval becomes smaller

c. The bound on the error estimation becomes larger

35. (CI) Which of the following is the confidence level for $\bar{x} - 1.782\frac{s}{\sqrt{13}}, \bar{x} + 2.179\frac{s}{\sqrt{13}}$?

a. 0.05

b. 0.075

c. 0.90

d. 0.925

e. 0.95

5.3 P-value, Type-1 Error, Type-2 Error and Power

36. Indicate each of the following statements as True (T) or False (F). Give your reason briefly

Statement	True/False	Comment
In a hypothesis testing problem there is no way of reducing the probability of a Type I error (α) without simultaneously increasing the probability of a Type II error.		
In testing about μ, the probability of a Type I error α is calculated by assuming the null hypothesis H_0 is true.		
For the Z-test, if the sample size n increases, then the power of the test also increases, provided that the probability of a Type I error (α) is not changed.		
If, for testing $H_0 : \mu = \mu_0$, a test statistic is found significant and the null hypothesis is rejected, that implies there is a practically important difference between μ and μ_0.		

37. A spectrometer used for measuring CO concentration (ppm, parts per million by volume) is checked for accuracy by taking readings on a manufactured gas in which CO concentration is very precisely controlled and kept constantly at 70 ppm. If the readings suggest that the spectrometer is not working properly, it will have to be recalibrated. 10 readings are obtained as follows :

84, 76, 81, 67, 71, 68, 73, 77, 72, 75

a. **(Data)** Do the data suggest that recalibration is necessary? (Take $\alpha = 0.05$)
b. **(P-Value)** Find the *P*-value approximately.

38. **(P-Value)** If the test statistics is Z=-0.83, which of the following is the P-value for the lower tailed test?

a. 0.2033
b. 0.3051
c. 0.4066
d. 0.5934
e. 0.7967

39. **(P-Value)** If the test statistics is Z=-3.2, which of the following is the P-value for the two-tailed test?

a. 0.0007
b. 0.0014
c. 0.9972
d. 0.9986
e. 0.9993

40. **(Error Types)** If the P-value is computed as 0.38 where α=0.05 and $\mu = 99$, for the hypothesis $H_0 : \mu \geq 100$ versus $H_a : \mu < 100$, what type of error you might have made?

c. Type I error
d. Type II error
e. No error

41. ***(HT-One Sided)** A random sample of 20 students yielded a mean of $\bar{x}$ = 72 and a variance of s^2= 16 for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, Ebru hoca says that the average is 70 but I think that it is greater than 70. Please make the single sided test $H_0: \mu = 70$ vs $H_1: \mu > 70$ at using *P value approach.*

42. 8 internal consumption engines were examined for the amount of engine wear, resulting in X=3.72 (σ=1.25)

f. **(HT-One Sided)** Assuming that the distribution is normal with mean μ, test H0: $\mu \leq 3.5$ against $\mu > 3.5$ at significance level α=0.05
g. **(P-Value)** Find the p-value.

43. ***The average height of females in the freshman class of a certain college has historically been 162.5 centimeters with a standard deviation of 6.9 centimeters. A random sample of 50 females in the present freshman class has an average height of 165.2 centimeters?

h. **(CI)** Please construct a 95% confidence interval for the mean 165.2.
i. **(P-Value)** Is there reason to believe that there has been a change in the average height? Use a P-value in your conclusion. Assume the standard deviation remains the same.

44. **We want to make a test whether the average height of the students in the university is greater than 170 cm or not, i.e., we want to test the following hypothesis $H_0: \mu = 170$ vs $H_1: \mu > 170$. We will make the test using $\bar{X}$ as our test statistic and reject the null hypothesis if $\bar{X} \geq 175$.

j. **(Type 1)** Calculate the Type I error that we commit with this rejection region if $\sigma = 10$.
k. **(Power)** What is the power of the hypothesis for $\mu = 172$?
l. **(Power)** If we want to test the hypothesis for $\alpha = 0.05$, what is the power of the test for $\mu = 172$?
m. **(Power)** For $\alpha = 0.05$, what is the power of the hypothesis for $\mu = 170$?
n.

45. The mean birth weight of babies in the world is μ = 3300 grams. A survey of birth weight of babies in Nigeria will be conducted by WHO officials. The officials suspect that the average birth weight of Nigerian babies is **smaller** than the world average (due to malnutrition of mothers). WHO officials took a random sample of 150 babies from various hospitals in Nigeria and weighed them right after birth. They want to test $H_0: \mu = 3300$ vs $H_1: \mu < 3300$. The sample average was $\bar{\boldsymbol{X}}$ = 3180 grams with a standard deviation of $\boldsymbol{s}$ = 490 grams. Since the sample is large (n=150) it is safe to assume $\sigma \approx s$ and make your analysis with this assumption.

a. **(Type 1)** If the WHO officials decides to reject the null hypothesis if the sample mean is **3230** or less, that is, if$\bar{X} \leq 3230$, what is the probability that the engineer commits a Type I error?
b. **(Type 2)** What is the probability that the hypo-thesis of μ = 3300 will **not** be rejected (by mistake) if the actual mean birth weight of Nigerian babies is in fact 3150 gr?

Now assume we are given the significance level as α=0.05

c. **(Type 1)** What is the probability that the engineer commits a Type I error?
d. **(Critical Region - HT)** If we use $\bar{X}$ as our test statistic, what is the critical region for $\bar{X}$?
e. **(Type 2)** If, unknown to engineer, the true population mean were μ=3150, what is the probability that the engineer commits a Type II error?
f. **(Power)** What is power of the test for μ=3150?
g. **(P-value)** The sample you have selected has an average of $\bar{X} = 3220$. Find the p-value. What is your decision?

p-value	
Decision	

h. **(Type 2)** If, unknown to engineer, the true population mean were μ=3300, what is the probability that the engineer commits Type II error?

46. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed. The quality manager states that the average life time of the bulbs is 800 hours. If a sample of 9 bulbs has an average life of 780 hours. If the sample standard deviation is 40:

o. **(CI)** Find a 95% confidence interval for the population mean of all bulbs produced by this firm

p. **(HT-Two Sided)** Is the quality manager right? Using $\alpha = 0.01$ level of significance, please check it.

q. **(HT-Two Sided)** Please show your calculation in (b) on a plot. Show the critical region, your test statistic and tell how you decide.

r. **(HT-Two Sided)** Another engineer states that the average life time is in fact 740 hours. Using $\alpha = 0.05$ level of significance, can you say anything about this conjecture <u>without</u> making any further calculations and using your previous calculations?

s. **(PI)** As a customer, I want to buy one of those bulbs and hence I am curious about the lifetime of a single bulb. Please construct a prediction interval using $\alpha = 0.05$.

47. The Brinell hardness scale is one of several definitions used in the field of materials science to quantify the hardness of a piece of metal. The Brinell hardness measurement of a certain type of rebar used for reinforcing concrete and masonry structures was assumed to be normally distributed with a standard deviation of 10 kilograms of force per square millimeter. Using a random sample of n = 25 bars, an engineer is interested in performing the following hypothesis test:

The null hypothesis H_0: μ = 170 against the alternative hypothesis H_1: μ > 170

t. **(Type 1)** If the engineer decides to reject the null hypothesis if the sample mean is 172 or greater, that is, if $\bar{X} > 172$, what is the probability that the engineer commits a Type I error?

u. **(Type 2)** If, unknown to engineer, the true population mean were μ = 173 (or $H_1\colon \mu = 173$), what is the probability that the engineer commits a Type II error?

Now assume we are given the significance level as $\alpha = 0.05$

v. **(Type 1)** What is the probability that the engineer commits a Type I error?

w. **(Type 2)** If, unknown to engineer, the true population mean were μ = 173, what is the probability that the engineer commits a Type II error? Note that in order to find this, first you have to calculate the critical region, or the critical value of $\bar{X}$.

Critical Region	
Prob of type II error	

x. **(Power)** What is the power of the test if the true population mean were μ = 173?

y. **(P-Value)** The sample you have selected has an average of $\bar{X} = 173.5$. Find the P-value. What is your decision?

P-Value	
Decision	

z. **(Type 2)** If, unknown to engineer, the true population mean were μ = 170, what is the probability that the engineer commits a Type II error?

48. **We want to make a test whether the average height of the students in YTU is greater than 170 cm or not, i.e., we want to test the following hypothesis $\mathrm{H_0}\colon \mu = 170$ vs $\mathrm{H_1}\colon \mu > 170$.

We will make the test using $\bar{X}$ as our test statistic and reject the null hypothesis if $\bar{X} \geq 175$. We have n=16.

a. **(Type 1)** Calculate the Type I error that we commit with this rejection region if σ=10.

b. **(Power)** What is the power of the hypothesis for μ=172?

c. **(Power)** If we want to test the hypothesis for ***α=0.05***, what is the power of the test for μ=172?

d. **(Power)** For α=0.05, what is the power of the hypothesis for μ=170?

49. State whether each of the following is true (T) or false (F). If you say "false" to any part, explain briefly why you think it is false.

a. The confidence interval shrinks as the confidence level increases.
b. If a null hypothesis is rejected against an alternative at the 5 % level of significance, then using the same data and the test statistic, it must also be rejected at the 1 % level.
c. A parameter is an unknown constant, it can be estimated from sample data.
d. μ is a parameter and it is unknown.
e. s^2 is a point estimator of σ^2.
f. If X is a normal random variable with mean μ and standard deviation σ, then $\frac{X-\mu}{\sigma/\sqrt{n}}$ is a standard normal random variable.
g. If two different random samples are taken from a population we will get the same values for the sample statistics.
h. Central Limit Theorem tells the following: if we get a large sample from a population, the distribution of the sample data is approximately normal.
i. Confidence interval gives us an interval in which the true parameter lives in with a certain confidence level.
j. μ is a random variable since its value changes for each sample.
k. The significance level of a test is the probability that the null hypothesis is false.
l. A Type I error occurs when a true null hypothesis is rejected.
m. A null hypothesis is rejected at the 0.025 level of significance, but is not rejected at 0.01 level. This means the P-value of the test is between 0.01 and 0.025.
n. The power of a test is the probability of accepting a true null hypothesis.
o. If a null hypothesis is rejected against an alternative at the 5 % level of significance, then using the same data and the test statistic, it must also be rejected at the 1 % level.
p. A parameter is an unknown constant, it can be estimated from sample data.
q. If two different random samples are taken from a population we will get the same values for the sample statistics.
r. Central Limit Theorem tells the following: if we get a large sample from a population, the distribution of the sample data is approximately normal.
s. For a given confidence level, a confidence interval is used to include the true value of a population parameter, but a tolerance interval is constructed to include a certain proportion of individual population values.
t. s is a parameter and its value changes for each sample.
u. μ is an unbiased estimator and its value is constant.
v. If we decrease α (type 1 error) then β (type 2 error) increases.
w. You can decrease both errors (type 1 and type 2 error) by increasing the sample size.

50. ***A random sample of 12 shearing pins is taken in a study of the Rockwell hardness of the pin head. Measurements on the Rockwell hardness are made for each of the 12, yielding an average value of 48.50 with a sample standard deviation of 1.5. Assuming the measurements to be normally distributed.

a. **(CI)** Please construct a 90% confidence interval for the mean Rockwell hardness.
b. **(HT-Two Sided)** Test the hypothesis that μ = 48.50 against the alternative hypothesis $\mu \neq$ 48.50, at the 0.05 level of significance?

51. The following data is generated randomly from a normal distribution with $\sigma^2 = 4$:

9.285999	10.56089	8.502049	7.094592
8.950302	10.1434	11.61667	12.56335
10.91725	10.10891	10.447	9.344243
9.384795	9.238835	10.1652	8.172975
5.892067	11.52196	11.49309	9.199536

a. **(CI)** Please calculate the 90% confidence interval for the population mean.
b. **(CI)** What is the error of the CI that you have calculated.
c. **(Sample Size)** What should be the sample size if the error should not exceed 0.5?
d. **(HT-Two Sided)** My conjecture is that, the mean of the population to which the sample above belongs to is 10. Please test this using α=0.05.
e. **(HT-Two Sided)** Please make the same test for α=0.10. Do you need any further calculations?
f. **(CI)** For the question above, assume that you don't know σ^2? Can you still construct a CI? How?
g. **(HT-Two Sided)** Please repeat (d) assuming that you are not given σ^2 (i.e., you don't know σ^2).

52. **Assume that I asked to 40 female students and it turns out that average height and standard deviation of the girls in our class is 164 cm and 9 cm, respectively. For all computations, assume an approximately normal distribution.

a. **(CI)** Find a 95% confidence interval for the height of the female students.
b. **(CI)** Without making any computation, can you test whether $\mu = 163$ or not at $\alpha = 0.05$? Why? If yes, what is the result and why?
c. **(PI)** Consider *a lady* that enters from the door. I say that her height is in the interval [a, b] with 95%. What is a and b? What kind of an interval is this?
d. **(TL)** Find the 99% tolerance limits that will contain 95% of ladies in this university.
e. **(HT-Two Sided)** Please test whether $\mu = 165$ or not using $\alpha = 0.01$ level of significance. Use our 5 step procedure.
f. **(HT-Two Sided)** For the question above, please plot the test statistic's distribution function and show the observed value of the RV on the plot like we did in the lecture. You can plot it manually, no need for R.
g. **(HT-Two Sided)** Please test whether $\mu = 164$ or not using $\alpha = 0.01$ level of significance. Use our 5 step procedure.

53. **I am a bit curious about the weight of bottles of waters that are sold in the markets. Hence I just picked 10 bottles of water and measured their weights. I got the following in terms of grams:

491, 504, 498, 492, 495, 481, 497, 493, 498, 499

a. **(CI)** Please construct a confidence interval for the weight for $\alpha = 0.1$, $\alpha = 0.05$, and $\alpha = 0.01$,
b. **(HT-Two Sided)** The firms conjecture that their average weight is 500 gr. Please write the appropriate hypothesis and test is against $\alpha = 0.1$, $\alpha = 0.05$, and $\alpha = 0.01$,
c. **(HT-One Sided and CI)** What can you say about your results found in a) and b)? Is there any kind of a connection between them?
d. **(HT-One Sided and CI)** Assume that we are given $\sigma = 6$, hence we can use it in our analysis. Repeat a), b) and c) with $\sigma = 6$.

54. A special type of rope is being considered for use in the production of parachutes. The rope will be considered safe and can be used as long as the mean breaking strength μ is at least 2000 kg. Otherwise it should be rejected. A random sample of 36 segments of this type of rope has been selected, and each was subjected to a breaking test. The force that caused each rope segment to break was recorded. The data followed normal distribution and the summary statistics are: $\bar{X} = 2100$ and we know that for these ropes we have σ = 150 kg.

a. **(HT-One Sided)** State the relevant hypotheses and carry out a hypothesis test to reach a decision at $\alpha = 0.01$. Write down your conclusion clearly.

b. **(P-Value)** Calculate the *P*- value.

c. **(Type 2)** Calculate the probability that this type of rope will be erroneously accepted (used in production of parachutes) when the true mean breaking strength is $\mu = 1900$ kg. What is this probability called? (Hint: Please note that although $\mu = 1900$ makes us think that H_0 is not true, since our alternative hypothesis is $H_1\colon \mu > 2000$, taking $\mu = 1900$ favors H_0 against H_1. Hence you can read this probability as:
$P(Rejecting\ H_0|H_0 is\ true)$

d. **(Power)** What is the power of the test when $H_1\colon \mu = 2100$?

e. **(Sample Size)**The probability that the rope will be erroneously accepted (used) - when actually $\mu = 1900$ is desired to be at most 0.01. How many more segments should be tested to ensure this?

55. **Yıldızlar Elementary School has 1000 students. The manager of the school thinks that the average IQ of students at Yıldızlar is at least 110. To prove her point, she administers an IQ test to 25 randomly selected students. Among the sampled students, the average IQ is 107. Assume that the standard deviation of the IQ of the students in Turkey is 10.

a. **(HT-One Sided)** Clearly write your hypothesis

H_0:	H_1:

b. **(HT-One Sided)** Assuming a significance level of 0.05, what is your decision? Please show your calculations.

Test Statistics (z,t,Fχ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

c. **(HT-One Sided)** Show your calculation in (b) on a plot as we did in the lecture. Show the critical region and the observed test statistic.

d. **(P-value)** Find the P-value. What is your decision now? Is it the same with (b)?

P-value	
Decision	

e. **(CI)** Find a 95% confidence interval for the population mean.

f. **(PI)** What can you say about the IQ level of a single student?

56. **Assume that we want to analyze the heights of the students in the university. Following is the height of 10 students in this statistics class:

172	177	168	165	183	180	159	168	166	164

The population standard deviation given as σ=10cm

a. **(Data)** Find $\bar{X}$ and S

For the following questions assume that $\bar{X} = 170$

b. **(HT-One Sided)** Test the hypothesis H_0: μ=168 vs H_1: μ>168. Use p value approach.

Test Statistics? (t,z,χ^2,F)	
The value of observed test statistic	
P – value	
Decision	

c. **(CI)** Find a confidence interval for the mean for α=0.05.

Left CI	Right CI

d. **(Type 2)** For α=0.05, find the probability that we don't reject the null hypothesis H_0:μ=168 if the real mean is μ=170 (or $\mu_1 = 170$). What is the name of this probability?

Prob Value	
Prob Name	

e. **(Power)** Calculate the power of the test for the alternative μ_1=170.

f. **(Type 1)** What is the value of type I error for $\alpha = 0.10$?

57. **Assume that we want to analyze the weights of the students in the university. Following is the height of 10 students in this statistics class:

172	177	168	165	183	180	159	168	166	164

The population standard deviation given as σ=10cm

a. **(HT-One Sided)** Test the hypothesis H_0: μ=168 vs H_1: μ>168 using p value approach.

b. **(CI)** Find a confidence interval for the mean for α=0.05.

c. **(Type 1)** For α=0.05, find the probability that we don't reject the hypothesis that μ=170 when the real mean is μ_1=170. What is this probability called?

d. **(Power)** Calculate the power of the test for the alternative μ_1=170.

58. The accuracy and precision of a new brand of digital blood pressure reader (DBPR) is under investigation. A random sample of 15 DBPRs from this brand are selected and connected to a stable source of 130 mmHg pressure that simulates a normal adult blood pressure. The DBPRs showed the following readings:

126	119	132	140	121	144	110	127	138	120	124	115	112	139	123

a. **(CI)** Calculate a 95 percent confidence interval for the mean reading μ that would result from this brand of DBPRs. What assumption(s) do you make to form this confidence interval?

b. **(CI-Variance)** Calculate a 90 percent confidence interval for the standard deviation σ of the pressure readings of this brand of DBPRs. Is this an acceptable amount of variability (precision)?

c. **(HT- Two Sided)** Does this data suggest that the mean reading μ of these DBPRs differ from 130? (α = 0.01)

d. **(P-Value)** Find the *P*-value of the hypothesis in (c).

e. **(HT- Two Sided)** Write down your suggestions about the result in (c).

f. **(Data)** Sketch a box-plot of the DBPR readings.

59. **Assume Mehmet hoca asked the weights of people in the class and have the following 10 students:

80	71	65	78	82	69	56	85	81	70

a. **(Data)** Calculate $\bar{X}$.
b. **(CI)** From TUİK we known that $\sigma = 9$ for the height of people in Turkey. Using this, calculate the confidence interval for
   i. $\alpha = 0.10$
   ii. $\alpha = 0.05$
   iii. $\alpha = 0.01$
c. **(Error)** What is the error in b1?
d. **(Sample Size)** What is the minimum sample size if the error term should be 3 kg?
e. **(HT-Two Sided)** Test the hypothesis $H_0: \mu = 68$ against $H_1: \mu \neq 68$ using the following significance levels
   i. $\alpha = 0.10$
   ii. $\alpha = 0.05$
   iii. $\alpha = 0.01$
f. **(Interpretation)** What can you say about b1 and e1, b2 and e2, b3 and e3?
g. **(Data)** Calculate s and s^2
h. **(CI - Error)** Using s, please repeat (b) and (c).
i. **(PI)** Find the prediction interval for a single student for $\alpha = 0.05$.
j. **(Data)** Calculate s and s^2
k. **(HT-Two Sided)** What can you say to the result of the hypothesis in b1 is the observed Xbar value is higher than this one without making further calculations.
l. **(HT-Two Sided)** Calculate the rejection region for b2.
m. **(HT-Two Sided)** What can you say about the rejection region in b3 when you compare it with that of b2?
n. **(HT-Two Sided)** What happens to the rejection region if σ =16 rather than 9?
o. **(HT-Two Sided)** Will the result change for the same value of $\bar{X}$, $\sigma = 4$ rather than 9? Can you answer this without further calculations?

6. *Two Means*

1. ***(CI)** Two kinds of thread are being compared for strength. Fifty pieces of each type of thread are tested under similar conditions. Brand A has an average tensile strength of 78.3 kilograms with a standard deviation of 5.6 kilograms, while brand B has an average tensile strength of 87.2 kilograms with a standard deviation of 6.3 kilograms. Construct a 95% confidence interval for the difference of the population means.

2. ***(HT)** The following data represent the running times of films produced by two motion-picture companies:

COMPANY	**TIME (Minutes)**						
1	102	86	98	109	92		
2	81	165	97	134	92	87	114

Test the hypothesis that the average running time of films produced by company 2 exceeds the average running time of films produced by company 1 by 10 minutes against the one-sided alternative that the difference is less than 10 minutes. Use a 0.1 level of significance and assume the distributions of times to be approximately normal with unequal variances.

3. **(HT)** An experiment was performed to compare the average grades of the students in the statistics class. The first group had quizzes, two midterms and a final exam whereas the second group only had single midterm and a final. 12 students in group 1 have an average grade of 75 a sample standard deviation of 3.5, while the second group's average is 72 with a sample standard deviation of 3.1 and a sample size of 15.
   a. **(HT)** Test the hypothesis that the mean grades of the groups are equal or not. Assume the populations to be approximately normal with equal variances. Use 5% level of significance.
   b. **(HT)** Show your work on the distribution plot as we did in the lecture. Show the distribution function, your critical region and the value of the test statistic.
   c. **(HT)** Does your conclusion change if you don't assume equal population variances?

Now assume that you are given the population standard deviations as 4 and 3 for group 1 and group 2, respectively.

   a. **(HT)** Please test the hypothesis that $\mu_1 = \mu_2 + 5$ groups are equal or not at $\alpha = 0.05$ significance level.
   b. **(CI)** Construct a confidence interval on the differences of two means for $\alpha = 0.05$.
   c. **(HT)** Without making further calculations, please test the hypothesis that $\mu_1 = \mu_2 + 7$.

4. A taxi company manager is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy on compact cars. Fourteen compact cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the **same cars** were then equipped with regular belted tires and driven once again over the same test course. The gasoline consumption, in kilometers per liter, was recorded as follows: (km / liter of gas)

Car / Driver	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Radial Tires	14.3	15.1	16.5	12.4	16.7	14.8	13.9	16.0	16.5	15.0	16.3	15.2	15.1	12.1
Belted Tires	14.1	14.9	16.2	12.6	16.8	14.4	13.7	15.8	16.7	14.7	16.0	14.9	15.3	11.8

   a. **(HT)** Test the hypothesis that, on the average, there is no difference in the fuel economy of the cars equipped with radial tires and those equipped with belted tires? (Take $\alpha = 0.01$)
   b. **(HT)** What assumptions for the data are needed to get reliable results?
   c. **(P-Value)** Calculate the ***P***-value.

5. An experiment was performed on the weights of the babies. The experts state that the babies that are raised in the cities are heavier than the babies raised in the villages. The data of two year old babies are given in the following. Use $\alpha = 0.05$ in your calculations.

	Number of Babies	Mean (kg)	Sample St.Dev (kg)
Babies in Cities	32	11.4	2.2
Babies in Villages	28	10.3	3.2

a. **(HT)** Assuming the standard deviation of the babies are equal, perform the hypothesis test. Please explicitly state your critical region and show where your observed test statistic value assumes its value.
b. **(HT)** One of the doctors is not comfortable with the assumption that the variances are equal. Please test the assumption with HT that the variances are equal.
c. **(CI)** Please construct a CI on the ratios of groups.
d. **(HT)** Show your results that you found in (b) on a plot as we did in the lectures. Please draw the distribution function, show the critical region on the plot, put your observed critical value and state your decision.
e. **(HT)** Using the result of (b), re-test the hypothesis in (a) if it is required.
f. **(HT - Variances)** Test the hypothesis that the st. deviation of the babies that are raised in the villages is greater than 3.
g. **(HT)** Show your results that you found in (e) on a plot as we did in the lectures. Please draw the distribution function, show the critical region on the plot, put your observed critical value and state your decision.
h. **(HT - Variances)** Please test the hypothesis that the st. deviation of the babies that are raised in the cities is equal to 3.
i. **(CI-Variances)** Please construct a CI on the standard deviation of the babies in cities.

6. In hydrolic engineering, it is important to be able to take accurate measurements on flow or discharge. Discharge data was collected using two different methods. One type of measuring device used in closed circuits is a Rotameter, a device placed inside a pipe. The second method is known as the Weigh Tank method and involves timing how long it takes to fill a container of known volume. Sample data was collected on ten pairs of flow rates (mm^3/sec):

Container	1	2	3	4	5	6	7	8	9	10
Rotameter	.292	.233	.183	.142	.117	.092	.267	.250	.217	.167
Weigh Tank	.294	.234	.184	.143	.112	.095	.259	.273	.198	.166

a. **(HT)** Is there evidence that the mean flow rate between the two measurement methods differ? ($\alpha = 0.05$).
b. **(P-Value)** Calculate the *P*-value in (a) and interpret its meaning.
c. **(HT)** What assumption(s) are needed to perform this test?

7. *****An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average wear of 86 units with a sample standard deviation of 3.5, while the samples of material 2 gave an average of 81 units and a sample standard deviation of 3.1.

a. **(HT)** Test the hypothesis that the mean abrasive wear of material 1 exceeds that of material 2 by more than 2.5 units? Assume the populations to be approximately normal with equal variances. Use 5% level of significance.

b. **(HT)** Does your conclusion change if you don't assume equal population variances?

c. **(CI)** Calculate a 95 % confidence interval for the (population) mean depth of wear of material 1.

d. **(Sample Size)** What would be the required sample size if we want the confidence interval in part (c) to have a length of at most 3 units?

8. ***(CI)** A study was conducted in which two types engines, A and B, were compared. Gas mileage, in miles per gallon, was measured. Fifty experiments were conducted using engine type A and 75 experiments were done with engine type B. The gasoline used and other conditions were held constant. The average gas mileage was 36 miles per gallon for engine A and 42 miles per gallon for engine B. Find a 95% confidence interval on $\mu_B - \mu_A$, where μ_A and μ_B are population means gas mileages for engines A and B, respectively. Assume that the population standard deviations are 6 and 8 for engines A and B, respectively.

9. ***(CI)** A study was conducted by the Department of Zoology at the Virginia Tech to estimate the difference in the amounts of the chemical orthophosphorus measured at two different stations on the James River. Orthophosphorus was measured in milligrams per liter. Fifteen samples were collected from station 1, and 12 samples were obtained from station 2. The 15 samples from station1 had an average orthophosphorus content of 3.84 milligrams per liter and a standard deviation of 3.07 milligrams per liter, while the 12 samples from station 2 had an average content of 1.49 milligrams per liter and a standard deviation of 0.80 milligram per liter. Find a 95% confidence interval for the difference in the true average orthophosphorus contents at these two stations, assuming that the observations came from normal populations with different variances.

10. ***(CI)** A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are

	Brand A	**Brand B**
$\bar{x}$	36300 kilometers	38100 kilometers
s	5000 kilometers	6100 kilometers

Compute a 95% confidence interval for $\mu_A - \mu_B$ assuming the populations to be approximately normally distributed. You may assume that the variances are equal.

11. **(CI)** A study[1] published in *Chemosphere* reported the levels of the dioxin TCDD of 20 Massachusetts Vietnam veterans who were possibly exposed to Agent Orange. The TCDD levels in plasma and in fat tissue are given in the table below. Find a 95% **confidence interval** for $\mu_1 - \mu_2$, where μ_1 and μ_2 represent the true mean TCDD levels in plasma and in fat tissue, respectively. Assume the distribution of the differences to be approximately normal.

Veteran	**TCDD Levels in Plasma**	**TCDD Levels in Fat Tissue**	d_i
1	2.5	4.9	-2.4
2	3.1	5.9	-2.8
3	2.1	4.4	-2.3
4	3.5	6.9	-3.4
5	3.1	7.0	-3.9
6	1.8	4.2	-2.4

[1] Schecter, A., Ryan, J. J., Constable, J. D., Baughman, R., Bangert, J., Fürst, P., ... & Oates, P. (1990). Partitioning of 2, 3, 7, 8-chlorinated dibenzo-p-dioxins and dibenzofurans between adipose tissue and plasma lipid of 20 Massachusetts Vietnam veterans. *Chemosphere, 20*(7-9), 951-958.

7	6.0	10.0	-4.0
8	3.0	5.5	-2.5
9	36.0	41.0	-5.0
10	4.7	4.4	0.3
11	6.9	7.0	-0.1
12	3.3	2.9	0.4
13	4.6	4.6	0.0
14	1.6	1.4	0.2
15	7.2	7.7	-0.5
16	1.8	1.1	0.7
17	20.0	11.0	9.0
18	2.0	2.5	-0.5
19	2.5	2.3	0.2
20	4.1	2.5	1.6

12. ***(HT-Two Sided)** A taxi company manager is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Twelve cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars were then equipped with regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, was recorded as follows:

	Radial Tires	**Belted Tires**
1	4,2	4,1
2	4,7	4,9
3	6,6	6,2
4	7	6,9
5	6,7	6,8
6	4,5	4,4
7	5,7	5,7
8	6	5,8
9	7,4	6,9
10	4,9	4,7
11	6,1	6
12	5,2	4,9

13. *****A study was conducted in which two types of engines, A and B, were compared. Gas mileage, in miles per gallon, was measured. Fifty experiments were conducted using engine type A and 75 experiments were done with engine type B. The gasoline used and other conditions were held constant. The average gas mileage was 36 miles per gallon for engine A and 42 miles per gallon for engine B., respectively. Assume that the **sample** standard deviations are 6 and 8 for engines A and B respectively.

a. **(CI)** Find a 95% confidence interval on $\mu_B - \mu_A$, where μ_A and μ_B are population mean gas mileages for engines A and B? Do we need any assumption?
b. **(HT)** Test the hypothesis that $H_0: \mu_A = \mu_B$ vs $H_1: \mu_A \neq \mu_B$
c. **(Interpretation)** What is the relation between (a) and (b)?
d. **(CI - Variances)** Find a 95% confidence interval on σ_A/σ_B where σ_A and σ_B are population standard deviation for gas mileages for engines A and B?
e. **(HT-Variances)** Test the hypothesis that $H_0: \sigma_A = \sigma_B$ vs $H_1: \sigma_A \neq \sigma_B$
f. **(CI)** Assume now that in fact we know the **population** standard deviations and they are indeed 6 and 8. How will your answer change in (a)?

7. Variance

7.1. Single Variance

1. Which of the following is the P(8.906 < χ^2 < 27.203) where χ^2 is a chi-squared random variable with degrees of freedom v=20?

a. 0.025
b. 0.10
c. 0.875
d. 0.975
e. 0.90

2. If you have a lower tailed test for testing the true variance with 0.05 significance, which of the following would be your critical value?

a. $\chi^2_{0.05;n-1}$
b. $\chi^2_{0.95;n-1}$
c. $\chi^2_{0.025;n-1}$
d. $\chi^2_{0.975;n-1}$
e. $\chi^2_{0.025;n-1}$ and $\chi^2_{0.975;n-1}$

3. A company produces metal pipes of a standard length, and claims that the standard deviation of the length is at least 1.7 cm. One of its clients decides to test this claim by taking a sample of 16 pipes and checking their lengths. They found that the standard deviation of the sample is 1.5 cm. Does this undermine the company's claim? Show all your work. Use $\alpha = 0.05$ level of significance.

4. A tensile strength test was performed in order to determine the strength of a particular adhesive for a glass-to-glass assembly. The data are: 16, 14, 19, 18, 19, 20, 15, 18, 17, 18. Assuming that the strengths are normally distributed,

a. Find the probability that the true mean exceeds 18.5.
b. Find P(2< σ < 3).

5. A sample of size 12 is selected from a normal population with μ=50 and σ=8.

a. Find the number a so that P($S^2 < a$) = 0.9.
b. Find the probability that the variance of the sample is at most twice of the variance of the population.

6. *A manufacturer of car batteries claims that the batteries will last, on average, 3 years with a variance of 1 year. If 5 of the these batteries have lifetimes 1.9, 2.4, 3.0, 3.5, and 4.2 years,

a. Test the hypothesis of manufacturers claim that $\sigma^2 = 1$ at the 0.05 level of significance.
b. Which assumption do you need?
c. Show your work on the distribution plot as we did in the lecture. Show the distribution function, your critical region and the value of the test statistic.
d. Construct a confidence interval for the sigma at 0.01 level of significance.

7. *A manufacturer of car batteries claims that the batteries will last, on average, 3 years with a variance of 1 year. 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years. We want to test whether $\sigma = 1$ or not. Assume that the life time is distributed normally. Using $\alpha = 0.05$, what is your conclusion?

Test Statistics (z,t,Fχ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

8. *A random sample of 20 students yielded a mean of $\bar{x}$ = 72 and a variance of s^2= 16 for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2.

9. A manufacturer is interested in the output voltage of a power supply used in PC. Output voltage is assumed to be normally distributed with the standard deviation 0.25 V and the manufacturer wishes to test if the true average output voltage differs from 5 V. Based on n=8 units, he computed the average output voltage as 5.15. <u>Answer the next 6 questions using this information.</u>

a. **(HT-Two Sided)** Which of the following hypothesis should be used to test this?
   - a. $H_0: \mu \leq 5\ vs. H_a: \mu > 5$
   - b. $H_0: \mu \geq 5\ vs. H_a: \mu < 5$
   - c. $H_0: \mu = 5\ vs. H_a: \mu \neq 5$

a. **(HT-Two Sided)** Which of the following test statistics should be used for testing the true average output voltage?
   - a. t
   - b. z
   - c. chi-square
   - d. both chi-square and z
   - e. both t and chi-square

b. **(HT-Two Sided)**Which of the following is the test statistics for testing the true average output voltage?
   - a. -1.697
   - b. -0.600
   - c. 0.212
   - d. 0.600
   - e. 1.697

c. **(CI)**Which of the following is the 95% confidence interval for the true average output voltage?
   - a. (4.6600 , 5.6400)
   - b. (4.7388 , 5.5613)
   - c. (4.9768 , 5.3232)
   - d. (5.0046 , 5.2954)
   - e. we do not have a large sample to compute this

d. **(HT-Variance)** Output voltage is assumed to be normally distributed with the standard deviation 0.25 V. I claim that the standard deviation should be different. Based on the 8 units, I computed the standard deviation as 0.30 V. Which of the following would be the test statistics for my claim?
   - a. 4.86
   - b. 8.40
   - c. 9.60
   - d. 10.08
   - e. 11.52

e. **(HT-Variance)** Output voltage is assumed to be normally distributed with the standard deviation 0.25 V. I claim that the standard deviation should be different. Based on the 8 units, I computed the standard deviation as 0.30 V. Which of the following hypothesis would you use to test the standard deviation?

   a. $H_0: \sigma^2 = 0.0625$
   b. $H_0: \sigma^2 = 0.09$
   c. $H_0: \sigma^2 = 0.25$
   d. $H_0: \sigma^2 = 0.30$
   e. $H_0: \mu = 0.30$

10. The accuracy and precision of a new brand of digital blood pressure reader (DBPR) is under investigation. A random sample of 15 DBPRs from this brand are selected and connected to a stable source of 130 mmHg pressure that simulates a normal adult blood pressure. The DBPRs showed the following readings:

126	119	132	140	121	144	110	127	138	120	124	115	112	139	123

a. **(Data)** Calculate the mean and the standard deviation of the sample.

From this point on, instead of the values that you found in (a), use mean as 125 and st dev as 10.5.

b. **(CI)** Calculate a 95% confidence interval for the mean reading μ that would result from this brand of DBPRs. What assumption(s) do you make to form this confidence interval?

c. **(CI-Variance)** Calculate a 90 percent confidence interval for the standard deviation of the pressure readings of this brand of DBPRs.

d. **(HT- Two Sided)** Does this data suggest that the mean reading μ of these DBPRs differ from 130? (α = 0.01)

Test Statistics (z,t,F,χ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

7.2. Two Variances

11. An industrial plant wants to determine which of the two types of fuel - gas or electric- will produce more useful energy at a lower cost. One measure of economical energy production, called the plant investment per delivered quad (shortly PIPDEQ) ratio, shows how much an industrial plant pays for energy (the smaller this ratio, the more economical is the source of energy).

A random sample of 15 plants using electrical utilities and another random sample of 15 plants using gas utilities were taken. The PIPDEQ ratios were calculated for each plant in each sample. The summary statistics are given as follows:

ELECTRIC UTITLITY PLANTS	$\sum X_i = 786$	$\sum X_i^2 = 95699.04$
GAS UTILITY PLANTS	$\sum y_i = 595.5$	$\sum y_i^2 = 60628.79$

e. Calculate the mean and variances of two populations.

Mean1	Mean2	Var1	Var2

Assume that the values that you found in (a) are $\bar{X} = 52$ and $\bar{Y} = 40$, and $s_x = 62$ and $s_y = 51$

f. Can we conclude from these data that the two types of plants have equal variances with respect to PIPDEQ ratios? (α= 0.10)

Test Statistics (z,t,Fχ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

g. Assuming equality of variances, do the data provide sufficient evidence to conclude that the mean PIPDEQ ratios are the same for the two types of plants? (α= 0.05)

Test Statistics (z,t,Fχ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

h. Calculate the *P*-value and comment.

12. In testing for the difference in the abrasive wear of two materials, random samples of size 12 and 10 of these samples are taken respectively. If $s_1 = 4$ and $s_2 = 5$, can we assume that population variances are equal? Please, use $\alpha = 0.10$

13. An experiment was performed on the weights of the babies. The experts state that the babies that are raised in the cities are heavier than the babies raised in the villages. The data of two year old babies are given in the following. Use $\alpha = 0.05$ in your calculations.

	Number of Babies	**Mean (kg)**	**Sample Std.Dev (kg)**
Babies in Cities	32	11.4	2.2
Babies in Villages	28	10.3	3.2

a) Assuming the standard deviation of the babies are equal, perform the hypothesis test. Please explicitly state your critical region and show where your observed test statistic value assumes its value.
b) One of the doctors is not comfortable with the assumption that the variances are equal. Please test the assumption that the variances are equal.
c) Show your results that you found in (b) on a plot as we did in the lectures. Please draw the distribution function, show the critical region on the plot, put your observed critical value and state your decision.
d) Using the result of (b), re-test the hypothesis in (a) if it is required.
e) Please test the hypothesis that the std. deviation of the babies that are raised in the cities is equal to 3.
f) Show your results that you found in (e) on a plot as we did in the lectures. Please draw the distribution function, show the critical region on the plot, put your observed critical value and state your decision.

8. *Proportions*

1. *A builder claims that at most 50% of all homes being constructed today in Istanbul obey the new earthquake regulations. Would you agree with this claim if a random survey of new homes in this city showed that 8 out of 15 obeyed pumps installed? Using 0.10 level of significance:
   (a) Make a hypothesis test using binomial distribution
   (b) Make a hypothesis test using normal approximation.
   (c) Construct a confidence interval on the proportion

2. A telephone company is trying to decide whether some new lines in a large community should be installed underground. Because a small surcharge will be added to telephone bills to pay for the extra installation costs, the company has decided to survey customers and proceed only if the survey strongly indicates that more than 60% of all customers favor underground installation despite the surcharge. The company took a random sample of 320 customers and found that 224 customers favor underground installation and is ready to pay the surcharge. What should the company do? Use P-value.

3. *A fabric manufacturer believes that the proportion of orders for raw material arriving late is $p = 0.6$. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that $p = 0.6$ should be rejected in favor of the alternative $p < 0.6$. Use the binomial distribution.

   (a) Find the probability of committing a type I error if the true proportion is $p = 0.6$.
   (b) Find the probability of committing a type II error for the alternatives $p = 0.3$, $p = 0.4$, and $p = 0.5$.

4. A random sample of 100 families (households) is selected from a large district. They are asked two questions:
   (a) Are you the owner of the house or the renter?
   (b) If you are the owner, will you be paying the mortgage (bank credit) for at least 10 more years?

   25% of the household said yes to both questions. Calculate a 95% confidence interval for *p*, the true proportion of mortgage paying home owners in this district.

5. A Wall Street Journal report on a survey by an advertisement agency indicates that matters of taste cannot be ignored in television advertising. Based on a mail survey of 3440 people, 40% indicated that they found TV commercials to be poor in taste.
   (a) Find a 95% confidence interval for the percentage of TV viewers who find TV commercials to be poor in taste.

6. *A random sample of 200 voters in a town is selected, and 114 are found to support an annexation suit.
   (a) Find the 96% confidence interval for the fraction of the voting population favoring the suit.
   (b) What can we assert with 96% confidence about the possible size of our error if we estimate the fraction of voters favoring the annexation suit to be 0.57?

7. A statistician is claiming that majority of all MBA's continue their formal education by taking courses within 10 years of graduation. This will be tested with $H_0: p \leq 0.5$ versus $H_a: p > 0.5$. Using a sample of 200 people, he found that 111 had taken coursework since receiving their MBA. <u>Answer the next 3 questions using this information.</u>
   a. Which of the following is the point estimate for the true proportion of MBA's continue their formal education by taking courses within 10 years of graduation?

      a. 0.445
      b. 0.50
      c. 0.555
      d. 100
      e. 111

b. Which of the following is the 98% confidence interval for the true proportion of MBA's continue their formal education by taking courses within 10 years of graduation?

    a. (0.5100 , 0.5999)
    b. (0.4972 , 0.6128)
    c. (0.4861 , 0.6239)
    d. (0.4731 , 0.6369)
    e. (0.4645 , 0.6455)

c. How many MBA's we should sample to find the width of a 95% confidence interval for the true proportion to be at most 0.05, irrespective of the sample proportion?

    a. 94
    b. 380
    c. 1519
    d. 1537
    e. 2113

8. "Do you use the same password for all your social networking sites?" A recent survey in the USA found that 32 % of social network users use the same password for all their social networking sites.

    a. A similar study will be conducted in Turkey and the same question will be asked to a random sample of people from among the social network users. It is desired to estimate this proportion with 90% confidence with a margin of error at most 0.03. How many people should be sampled to achieve this?
    b. Now suppose a random sample of 340 social network users in Turkey were asked this question and 136 of them said yes. Find a 90% confidence interval for the proportion of Turkish social network users who use the same password everywhere.
    c. Can we say that the percentage in Turkey is more than the percentage in the USA? Use 1% level of significance to test.
    d. Calculate the P-value for the test statistic in part (c).
    e. Calculate the probability of a Type II error when p = 0.40, that is, the probability that the null hypothesis in (c) will not be rejected (by mistake) if the actual proportion in Turkey is 0.40.

9. A textile marketing company manager is trying to find what percent of the university students recognize their brand name for youth sportswear. If less than 20 percent of the university students recognize their brand the company will create a new brand name for their product and mount an expensive campaign to promote the new brand. Otherwise they will keep the current brand name and try to further promote it. A survey of four university campuses was designed to reach 800 university students. The students were asked if they recognize the brand as a youth sportswear producer. 128 students in the survey said they recognize it. (Others didn't either recognize it or thought it was a brand for children.)

a. State your hypothesis.

H_0:	H_1:

b. What should the company do? (Take α= 0.01)

Test Statistics (z,t,Fχ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

c. Show your calculation over test statistic in (b) on a plot as we did in the lecture. Show the critical region and the observed test statistic.

d. Find the P-value. What is your decision now? Is it the same with (b)?

P-value	
Decision	

e. Construct a 99% confidence interval for p, the proportion who recognize the brand.

Lower CI	Upper CI

f. The manager would like to ensure that the error in estimating should be at most 0.02 with 95% confidence. Is the current sample size enough? If not how large a sample is needed for that precision?

10. A study of American medical doctors was reported in New York Times in January 27, 1988. The doctors were randomly assigned to one of the two groups. 11,037 doctors in the first group took one aspirin every other day. 11,034 doctors in the second group took no aspirin. After five years, there were 208 deaths in the first group from the heart attacks and 381 deaths in the second group. Can you conclude that the death rate from heart attacks is the same for the two groups of doctors? ($\alpha = 0.01$)

11. A vote is to be taken among the residents of a town and the surrounding county to determine whether a proposed chemical plant should be constructed. The construction site is within the town limits and, for this reason many voters in the country believe that the proposal will pass because of the large proportion of town voters who favor the construction. To determine if there is a significant difference in the proportions of town voters and county voters favoring the proposal, a poll is taken. 140 of 300 town voters favor the proposal and 390 of 620 county residents favor it.

a. Would you agree that the proportion of town voters favoring the proposal is higher than the proportion of county voters? Use a $\alpha = 0.05$ level of significance.

b. Calculate a 90% CI for the true difference between the proportions of town voters and county voters favoring the proposal.

12. YTU Pharmaceuticals (YTU-P) is working on a new drug which cures headaches. In order to test the efficiency of the drugs, the physicians worked in two groups. In the first group there are 30 patients. They received the drugs and it turns out that 21 patients has recovered from headache. The second group has 20 people. Instead of drugs, they have received "sugar pills" and 9 of them has recovered from the headache.

a. Clearly write your hypothesis

H_0:	H_1:

b. Assuming a significance level of 0.05, what is your decision? Please show your calculations.

Test Statistics (z,t,Fχ^2)	
Critical Region	
Observed Test Stat Value	
Decision	

c. Find the P-value. What is your decision now? Is it the same with (b)?

P-value	
Decision	

d. Please construct a 90% confidence interval on the proportion of the first group.

13. *A manufacturer turns out a product item that is labelled either "defective" or "not defective." In order to estimate the proportion defective, a random sample of 100 items is taken from production, and 10 are found to be defective. Following implementation of a quality improvement program, the experiment is conducted again. A new sample of 100 is taken, and this time only 6 are found to be defective. Give a 95% confidence interval on $p1 - p2$, where p1 is the population proportion defective before improvement and p2 is the proportion defective after improvement.

14. A process that manufactures metal washers is supposed to be operating with a targeted mean diameter $\mu = 12.5\ mm$ and a standard deviation $\sigma = 0.025$ mm. The process is continuously monitored by taking random samples from the production line. The last sample of 25 washers showed a normal-like distribution with the following results: $\bar{X}$ = 12.54 mm and s = 0.028 mm.

a. **(HT-Two Sided)** Does this sample suggest that the process mean has deviated (differs) from the targeted mean value of 12.5 mm.? (Take α = 0.05).

b. **(CI)** Calculate a 99% confidence interval for μ, the actual process mean.

c. **(CI)** A washer is considered defective if its diameter exceeds 12.58 mm or if its diameter is less than 12.42 mm. According to sample results what percent of the washers will be defective? (Assume σ = 0.025 is given). Comment on the answer!

d. **(Proportion-HT)** The quality engineer claims that the proportion of defective washers does not exceed 0.05. In the last 4 samples taken, out of a total of 100 washers checked, he found 4 washers with diameter above 12.580, and 2 washers with diameter below 12.420 and the rest were in between. Do these data support him? You may simply use the P- value to answer this question.

9. *Goodness of Fit*

1. *A machine is supposed to mix peanuts, hazelnuts, cashews, and pecans in the ratio 5:2:2:1. A can containing 500 of these mixed nuts was found to have 269 peanuts, 112 hazelnuts, 74 cashews, and 45 pecans. At the 0.05 level of significance, test the hypothesis that the machine is mixing the nuts in the ratio 5:2:2:1.

2. *A criminologist conducted a survey to determine whether the incidence of certain types of crime varied from one part of a large city to another. The particular crimes of interest were assault, burglary, and theft. The following table shows the numbers of crimes committed in three areas of the city during the past year.

	Type of Crime	
District	**Theft**	**Burglary**
1	162	118
2	310	196
3	258	193

Can we conclude from these data at the 0.01 level of significance that the occurrence of these types of crime is dependent on the city district?

Test Statistic (t, z, χ^2, F)	
Critical Region	
Observed Test Stat Value	
Decision	

3. The following two-way frequency table summarizes the results of a study done on a random sample of 1000 high-school students. The students were asked questions about their smoking status as well as their parents' smoking habits and classified accordingly:

		Student's Status Regarding Cigarette Smoking			
		Never smokes	Occasionally smokes	Regularly smokes	Total
Parents' Status Regarding	No parents smoke	300	120	80	500
	Only one smoke	120	100	80	300
	Both parents smoke	40	60	100	200
	Total	460	280	260	1000

Can we conclude that the parental smoking status and student smoking behavior are independent? ($\alpha = 0.01$). Show all the steps of a hypothesis testing problem. Write the expected frequency of each cell in the table above.

4. The attitudes of different age groups for preferred types of office communications is investigated. A separate random sample of 100 respondents from each age group (Young, Middle, and Senior) is selected and the preferred method of office communications for every person in each group is recorded. The data are summarized as follows. Do the data provide evidence that attitude towards type of communication differ between age groups? Clearly write the hypotheses you test. Show your work with all steps. Give a clear conclusion.

	Type of Communication Preferred				
Age group	**Group**	**Phone**	**E-**	**Other**	**TOTAL**
Young (< 35 years)	18	30	36	16	100
Middle Age (35 – 50)	24	36	24	16	100
Senior Group (> 50)	36	30	12	22	100

5. We always assumed that the heights of people are normally distributed. Now we can test it. Assume we have the following frequency table for 50 students:

Range	Count
150-155	1
155-160	5
160-165	4
165-170	10
170-175	12
175-180	7
180-185	6
185-190	5

Please test whether the data comes from a normal population with $\mu = 170cm$ and $\sigma = 10$ cm.

6. **We want to test whether the height of the people in the university is distributed normally with mean 170 and standard deviation 10. We have picked 53 people. The histogram is given below. For $\alpha = 0.10$ test the hypothesis that the distribution comes from the stated distribution.

Range	Number of People
150-160 cm	5
160-170 cm	20
170-180 cm	22
180-190 cm	4
190-200 cm	2

Test Statistics? (t,z,χ2,F)	
The value of observed test statistic	
Critical Region	
Decision	

10. Linear Regression

10.1 Simple Linear Regression

1. **For the given X and Y data below please answer the questions for the regression model $Y = \beta 0 + \beta 1 X + \epsilon$

a. Find the values of b_0 and b_1
b. Find the s_{b1} , s_{b2}
c. Please test the hypothesis β_0=50 and β_1=30 at α=0.05 level of significance.
d. Find 90% confidence interval for both β_0 and β_1.

.

X	6.49	36.32	1.63	44.43	15.53	31.68	36.11	15.31	21.80	30.38
Y	232.22	1102.69	91.56	1335.36	504.27	970.68	1092.80	494.09	692.89	930.76

2. **Using the data in **question 1(in linear regression)** with Excel:

a. Make a scatter plot of the points
b. Make a regression

Please attach your results to your assignment sheet. *(Hint: You can ask google "how to make regression in Excel")*

3. We search for a possible relationship between the amount of sales and the size of the shop of the following form:

$$y = \beta_0 + \beta_1 x$$

We have the following data

Observation	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Meter Sq(x)	1.7	1.6	2.8	5.6	1.3	2.2	1.3	1.1	3.2	1.5	5.2	4.6	5.8	3
Sales (y)	3.7	3.9	6.7	9.5	3.4	5.6	3.7	2.7	5.5	2.9	10.7	7.6	11.8	4.1

a. Find the following quantities: b_0,b_1, SSE, s, $s_{b_0} s_{b_1}$
b. Make the tests $H_0: \beta_0 = 0$ and $H_0: \beta_1 = 0$. Provide the observed test statistics and associated p-value.
c. I want to open a new shop which has a size of 5-m^2. What is the average number of sales?

4. **Following is the data for 14 A101 Stores. We want to analyze the relation between the size of the shop and its sales.

Observation	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Meter Sq(x)	1.7	1.6	2.8	5.6	1.3	2.2	1.3	1.1	3.2	1.5	5.2	4.6	5.8	3
Sales (y)	3.7	3.9	6.7	9.5	3.4	5.6	3.7	2.7	5.5	2.9	10.7	7.6	11.8	4.1

When we put the data to Excel and run the regression, we have the following data. Please answer the questions according to the following output:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.95
R Square	0.90
Adjusted R Square	0.90
Standard Error	0.97
Observations	14.00

	Coefficients	*St Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.96	0.53	1.83	0.09	-0.18	2.11
Xi	1.67	0.16	10.64	0.00	1.33	2.01

a. What is the value of intercept (b0) and the slope (b1)?
b. We said that t-stat is for a hypothesis testing, what is that hypothesis?
c. According to the P values, what can you say about b0 and b1?
d. Can you find a Confidence Interval for b0 and b1?
e. What is the confidence interval at x=3?
f. What is the prediction interval at x=3?

5. **We want to find the relation between the height and the weight of the people as follows using linear regression:

$$Weight = b_0 + b_1 Height$$

Height	170	171	172	173	175	176	177	178	180	182
Weight	71.5	61.9	78	73.3	71.5	81.9	83	77.5	85.3	82.6

a. Find the coefficients b_0 *and* b_1
b. Test at the 0.05 level of significance whether the linear model is adequate.
Alternative b) Find the estimated weight for a 190cm tall person.

6. **Following is the height and weight data of the students in our Mat216 class. Some of the calculations are given as Sxx =140.4, b1 = 1.15, Syy = 296.74.

Height	170	171	172	173	175	176	177	178	180	182
Weight	71.5	61.9	78	73.3	71.5	81.9	83	77.5	85.3	82.6

a. Please fill the ANOVA table below. Show your calculations!

Source	**Degrees of Freedom**	***SS***	***MS***		***F*-ratio**	***P*-value**
						0.01

b. For the following questions assume that $b0$=-124.

c. Please predict the weight of a student who comes to the class with a height of 180 ($\alpha = 0.01$).

d. What is the confidence interval of the students whose height is 180? ($\alpha = 0.01$)

7. The Ministry of Health is conducting a Maternal Risk Factors Survey. For this purpose, they chose several hospitals and collected data about mothers who recently delivered babies. Two variables they observed are **AGE** of the mother, and her blood pressure (**BP**). The original data set is very large, but a random sample of 17 observations is selected below:

AGE (*x*)	15	18	18	28	20	31	25	34	29	35	32	22	33	24	33	26	24
BP (*y*)	127	129	132	140	131	142	133	144	139	143	138	131	137	129	141	132	130

a. Find the equation of the least squares regression line for predicting BP from AGE. (If you are using EXCEL to answer this question, make sure to copy all necessary details from the EXCEL sheet to your answer paper.) HINT: $\Sigma\, x\, y$ = 60,922.

S_{xx} = ? S_{yy} = ? S_{xy} = ? $\bar{x} =$ $\bar{y} =$

b. Complete the ANOVA table below. Test the hypothesis that there is no linear relation between BP and AGE.

Source	Degrees of Freedom	*SS*	*MS*	*F*-ratio	*P*-value

c. Calculate the correlation coefficient between BP and AGE. Comment on the value.

d. How good does the regression line fit the data? Explain your answer.

e. The age of a mother who just delivered a baby in one of these hospitals is given as 41. How would you predict her blood pressure? Can we rely on this predicted value? Explain.

f. Calculate a 95 % confidence interval for the slope of the regression line between BP and AGE.

g. Use the confidence interval in part (e) to test the hypothesis that the slope =1 versus slope ≠ 1.

h. Calculate a 95% confidence interval for the mean blood pressure of a mother in the survey if her age is 25.

8. **You can find four columns of data: y and x. The result of the full regression model is given in the following:

SUMMARY OUTPUT						
Regression Statistics						
Multiple F	0.839045					
R Square	0.703996					
Adjusted I	0.666996					
Standard I	5.19145					
Observati	10					
ANOVA						
	df	*SS*	*MS*	*F*	*ignificance F*	
Regressio	1	512.7908	512.7908	19.02668	0.0024063	
Residual	8	215.6092	26.95115			
Total	9	728.4				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	11.75	35.95	0.33	0.75	-71.15	94.65
Y	0.43	0.10	4.36	0.00	0.20	0.66

a) What is the regression model
b) What is the value of prediction when x=182?
c) Find a 95% CI for β_0.
d) Find a **90%** CI for β_1.
e) What is your decision for the hypothesis for $\beta_0 = 0$? Why? Explain.
f) What is your decision for the hypothesis for $\beta_1 = 0$? Why? Explain.

10.2 Multiple Linear Regression

9. The infant mortality data of the 50 states in the USA is analyzed. Using EXCEL, a regression model was fitted for the infant mortality as the dependent variable (***IMR*** = rate of deaths of newborns for 1000 live births) using 3 predictor (independent) variables (***CDR*** = Child death rate = deaths per 100,000 children aged 1-14; ***HSDRP*** = High school drop-outs aged 16-19; ***LBW*** = percent of low-birth weight babies). The EXCEL output is shown below:

REGRESSION STATISTICS	
R Square	??
Adjusted R Square	0,813
Standard Error	??
No. of Observations	??

ANOVA TABLE					
	df	*SS*	*MS*	*F-ratio*	*Significance F*
Regression	??	69,39	??	??	< 0.0001
Residual	??	14,74	0,321		
Total	??	??			

	Coefficients	*Standard Error*	*t-statistic*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	1,0868	0,5461	1,99	0,0525	-0,0125	2,1861
CDR	0,0354	0,0064	??	0,0000	0,0226	0,0483
HSDRP	-0,0721	0,0331	??	0,0348	-0,1388	-0,0054
LBW	0,7770	0,0786	9,89	0,0000	0,6188	0,9351

a. Write down the regression model used to analyze these data. List the **assumptions** of this model briefly.

b. Complete the missing entries (??) in the output above. Show your calculations here.

c. Write down the **predicted** regression equation. When ***CDR*** (Child death rate) increases by 1 unit, how does the response variable (***IMR***) change?

d. What percent of the variability is explained by this model? Do you think the model fits the data well?

(Assume that residual plots show no violation of assumptions.)

e. Explain briefly **how** you would **test the assumptions** of the model.

f. Use the EXCEL output to test the hypothesis that there is no linear relation between the dependent variable ***Y*** and any of the ***X***-variables (that is, test the hypothesis that all β_i coefficients in the model are zero). Write your conclusions.

g. Use the EXCEL output to test the hypothesis that the infant mortality rate ***IMR*** in a state does not depend on high school drop-out rate (***HSDRP***) in that state. Write your conclusion. ($\alpha = 0.05$).

h. Find a 95% confidence interval for the coefficient of LBW in the regression model.

10. In an experiment to study the influence of ***temperature*** (X_1) and ***sunshine*** (X_2) on the response Y = the alpha-acid contents of hops (=şerbetçiotu), observations were made on twelve plots.

The model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ was fitted to the data and the following results were obtained (selected results from the computer output):

Predictor	Coefficient	St. Error	*t*-ratio	*P*-value
Constant	415.11	-----	5.03	0.000
Temperature	–6.593	-----	-----	0.208
Sunshine	–4.504	1.071	-----	0.002
S (standard error of the estimate) = 24.45		***R*** **–** square = 0.768		

a. Find the predicted value of Y when $x_1 = 19$, and $x_2 = 43$.

b. Prepare the ANOVA table for this experiment. (The information given is sufficient! Show your work.)

c. Test the hypothesis that both regression coefficients are (simultaneously) zero. ($\alpha = 0.05$)

d. Test the hypothesis that temperature has no linear effect on Y. ($\alpha = 0.05$)

11. The Sweethouse Estate is a real estate agency that assesses and sells houses in a large ***suburban area***. They use a regression model to predict the value of a house based on several explanatory variables.

They use the following variables to fit a multiple regression model:

(Y) : Assessed value of the house (in 1000 \$);

(X_1) : Size (in square meter);

(X_2) : Age of house (in years);

(X_3) : An indicator that shows how preferable the street of the house is (scaled from 1 to 10, 10 =the most popular);

(X_4) : Fireplace exists or not (1 means house has a fireplace, 0 means no).

They have data for 60 houses already sold in the last 12 months and they used these to fit various regression models to the data. Some of the results are summarized below:

Model Number	Variables in the	R^2, percent of variance	Standard error of the estimate (s)
1	X_1 , X_2	70.6 %	40.9
2	X_1 , X_2 , X_3	74.3 %	38.6
3	X_1, X_2 , X_4	71.8 %	40.4
4	X_1, X_3 , X_4	72.7 %	40.5
5	X_1 , X_2 , X_3 , X_4	74.5 %	38.8

a. Write down the regression model to analyze these data. What are the assumptions of this model?

b. Calculate the Mallow's C_p statistic for Model 2. Show your calculations and the formula used!

c. Based on the given information, which model would you prefer for assessment? Explain in detail.

d. For the model you selected in part (c), test the hypothesis that "there is no linear relation between the dependent variable *Y* and any of the *X*-variables (that is, test the hypothesis that all β_i coefficients in the model are zero).

e. Should we use this regression equation to assess the value of the houses in ***downtown area***? Explain.

12. Consider the example that we did in the lecture. The data that consist of measurements for nine infants on the next slide. The purpose is to relate Y, the length of an infant to the variables

x1 - Age (days), **x2 -** Length at Birth (cm),

x3 - Weight at Birth (kg), and **x4 -** Chest Size at Birth (cm).

The data is given in the following.

Infant Length, y (cm)	Age, x_1 (days)	Length at Birth, x_2 (cm)	Weight at Birth, x_3 (kg)	Cheast Size at Birth, x_4 (cm)
57.50	78.00	48.20	2.75	29.50
52.80	69.00	45.50	2.15	26.30
61.30	77.00	46.30	4.41	32.20
67.00	88.00	49.00	5.52	36.50
53.50	67.00	43.00	3.21	27.20
62.70	80.00	48.00	4.32	27.70
56.20	74.00	48.00	2.31	28.30
68.50	94.00	53.00	4.30	30.30
69.20	102.00	58.00	3.71	28.70

The result of the regression of the full model is given in the following:

SUMMARY OUTPUT

Regression Statistics	
R Square	0.991
Adj R	???
St Err	0.861
Observatioi	???

ANOVA	*df*	*SS*	*MS*	*F*	*P value*
Regression	??	318.27442	79.5686	???	0.000254
Residual	4	2.9655807	???		
Total	??	321.24			

	Coefficients	*St. Err.*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	7.148	16.460	???	0.687	-38.552	52.847
X1	0.100	???	0.295	0.783	-0.843	1.043
X2	0.726	0.786	0.924	0.408	-1.456	???
X3	3.076	1.059	2.904	0.044	???	6.017
X4	-0.030	0.166	-0.180	0.866	-0.492	0.432

a. Complete the missing entries (??) in the output above. Show your calculations here.

b. What is the predicted equation, write down the model?

c. Are all of the parameters significant? Which are significant, which are not?

d. If weight at birth increases by one unit, how much does the length of the infant increase?

e. How much of the percent in the variability is explained by this model?

f. How do we test whether there is a linear relationship between Y and any of X variables? What is the result of this test?

g. What does p value of x2 (0.408) mean here?

h. In the following we perform the regression analysis for the variables x2 and x3 only:

	Coeff	*St. Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	2.183	2.801	0.779	0.465	-4.670	9.037
X2	0.958	0.059	16.156	0.000	0.813	1.103
X3	3.325	0.233	14.260	0.000	2.755	3.896

Now the p-value of x2 changes to zero. Both of the hypothesis that is associated with these p values (0.408 in the previous table and 0.000 in this table) is $H_0: \beta_2 = 0$ vs $H_0: \beta_2 \neq 0$. Why do we observe a change here?

- **Questions containing the * mark are taken from the Walpole 9th edition:**

Walpole R.E., Myers R.H., Myers S.L., Ye K., 2011, Probability & Statistics for Engineers & Scientists: Pearson Prentice Hall.

- **Questions marked as ** was created by the authors.**