

#### LECTURE -2-

(Decision making under uncertainty and risk)

## COMPONENTS OF DECISION ANALYSIS

- A *state of nature* is an actual event that may occur in the future.
- A payoff matrix (decision table) is a means of organizing a decision situation, presenting the payoffs from different decisions/alternatives given the various states of nature.

#### BASIC STEPS IN DECISION ANALYSIS

- 1) Clearly define the problem at hand
- 2) List the possible alternatives
- 3) Identify the possible state of natures
- 4) List the payoff (profit/cost) of alternatives with respect to state of natures
- 5) Select one of the mathematical decision analysis methods (models)
- 6) Apply the method and make your decision

### TYPES OF DECISION-MAKING ENVIRONMENTS

- Type 1: Decision-making under certainty
  - DM *knows with certainty* the payoffs of every alternative .
- Type 2: Decision-making under uncertainty
  - DM *does not know* the probabilities of the various states of nature.
- Type 3: Decision-making under risk
  - DM *does know* the probabilities of the various states of nature.

### DECISION MAKING UNDER CERTAINTY

- Instead of state of natures, a true state is known to the decision maker before s/he has to make decision
- The optimal choice is to pick an alternative with the highest payoff

### DECISION MAKING UNDER UNCERTAINTY

- Maximax
- Maximin
- o Criterion of Realism
- o Equally likelihood
- Minimax

# DECISION TABLE / PAYOFF MATRIX

	STATES OF NATURE		
	Favorable Unfavorable		
ALTERNATIVES	market	market	
Construct large plant	\$200,000	(\$180,000)	
Construct small plant	\$100,000	(\$20,000)	
Do nothing	\$0	\$0	

# MAXIMAX (RISK SEEKING BEHAVIOR)

Choose the alternative with the maximum optimistic level

$$o_k = \max_{i=1}^m \{o_i\} = \max_{i=1}^m \{\max_{j=1}^m \{v_{ij}\}\}$$

	STATES O	F NATURE	
	Favorable	Unfavorable	Maximum in
ALTERNATIVES	market	market	$row(o_k)$
Construct large plant	\$200,000	(\$180,000)	\$200,000
Construct small plant	\$100,000	(\$20,000)	\$100,000
Do nothing	\$0	\$0	\$0

# MAXIMIN (RISK AVERSE BEHAVIOR)

Choose the alternative with the maximum security level

$$s_k = \max_{i=1}^m \{s_i\} = \max_{i=1}^m \{\min_{j=1}^m \{v_{ij}\}\}$$

	STATES OF NATURE		
	Favorable Unfavorable		
ALTERNATIVES	market	market	
Construct large plant	\$200,000	(\$180,000)	
Construct small plant	\$100,000	(\$20,000)	
Do nothing	\$0	\$0	

Minimum in row (s<sub>k</sub>) (\$180,000) (\$20,000) \$0

### **CRITERION OF REALISM**

Hurwicz suggested to use the optimism-pessimism index (a)

Choose the alternative with the maximum weighted average of optimistic and security levels

$$\max_{i=1}^{m} \{a o_i + (1-a) s_i\} \text{ where } 0 \le a \le 1$$

	STATES O	F NATURE		
	Favorable	Unfavorable	Weighted	
ALTERNATIVES	market	market	average	$\alpha=0.7$
Construct large plant	\$200.000	(\$180.000)	380α-180	86
Construct small plant	\$100.000	(\$20.000)	$120\alpha$ -20	64
Do nothing	\$0	\$0	\$0	0

#### **CRITERION OF REALISM**

$$120\alpha-20 = 0 \Rightarrow \alpha = 0.1667$$
  
 $380\alpha-180 = 120\alpha-20 \Rightarrow \alpha = 0.6154$ 

 $0 \le \alpha \le 0.1667 \Rightarrow$  "Do nothing"

 $0.1667 \le \alpha \le 0.6154 \Rightarrow$  "Construct small plant"

 $0.6154 \le \alpha \le 1 \Rightarrow$  "Construct large plant"

### **EQUALLY LIKELIHOOD**

Laplace argued that "knowing nothing at all about the true state of nature" is equivalent to "all states having equal probability"

Choose the alternative with the maximum row average (expected value)

	STATES OF NATURE		
	Favorable	Unfavorable	
ALTERNATIVES	market	market	
Construct large plant	\$200,000	(\$180,000)	
Construct small plant	\$100,000	(\$20,000)	
Do nothing	\$0	\$0	

Row average \$10,000 \$40,000 \$0

# MINIMAX REGRET (LOST OPPORTUNITY)

Savage defined the regret (opportunity loss) as the difference between

- the value resulting from the best action given that state of nature j is the true state and
- the value resulting from alternative i with respect to state of nature j

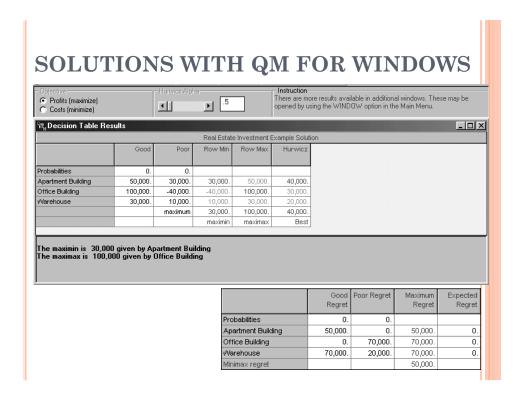
Choose the alternative with the minimum worst (maximum) regret

Regret Values	STATES OF NATURE		
	Favorable	Unfavorable	Maximum in
ALTERNATIVES	market	market	row
Construct large plant	\$0	\$180,000	\$180,000
Construct small plant	\$100,000	\$20,000	\$100,000
Do nothing	\$200,000	\$0	\$200,000

#### SUMMARY OF EXAMPLE RESULTS

METHOD	DECISION
<ul><li>Maximax</li></ul>	"Construct large plant"
<ul><li>Maximin</li></ul>	"Do nothing"
• Criterion of Realism	$depends$ on $\alpha$
• Equally likelihood	"Construct small plant"
<ul><li>Minimax</li></ul>	"Construct small plant"

The appropriate method is dependent on the personality and philosophy of the DM.



### DECISION MAKING UNDER RISK

#### Probability

- Objective
- Subjective
- Expected (Monetary) Value
   Expected Value of Perfect Information
- Expected Opportunity Loss

#### **PROBABILITY**

- A *probability* is a numerical statement about the likelihood that an event will occur
- The probability, *P*, of any event occurring is <u>greater</u> than or equal to 0 and less than or equal to 1:

$$0 \le P(event) \le 1$$

• The sum of the simple probabilities for all possible outcomes of an activity must <u>equal 1</u>

#### **OBJECTIVE PROBABILITY**

Determined by experiment or observation:

- Probability of heads on coin flip
- Probability of spades on drawing card from deck

$$P(A) = \lim_{n \to \infty} n(A) / n$$

P(A): probability of occurrence of event A n(A): number of times that event A occurs n: number of independent and identical repetitions of the experiments or observations

### SUBJECTIVE PROBABILITY

- Determined by an estimate based on the expert's
  - personal belief,
  - judgment,
  - experience,

#### and

• existing knowledge of a situation

# PAYOFF MATRIX WITH PROBABILITIES

	STATES OF NATURE Favorable Unfavorable		
ALTERNATIVES	market	market	
Probabilites	60%	40%	
Construct large plant	\$200,000	(\$180,000)	
Construct small plant	\$100,000	(\$20,000)	
Do nothing	\$0	\$0	

# EXPECTED (MONETARY) VALUE

Choose the alternative with the maximum weighted row average

$$EV(a_i) = \sum_i v_{ij} P(\theta_j)$$

	STATES OF NATURE		
	Favorable Unfavorable		
ALTERNATIVES	market	market	
Construct large plant	\$200,000	(\$180,000)	
Construct small plant	\$100,000	(\$20,000)	
Do nothing	\$0	\$0	
PROBABILITIES	0.6	0.4	

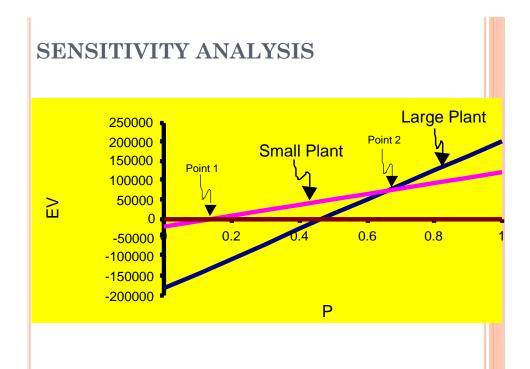
Expected Value \$48,000 \$52,000 \$0

#### SENSITIVITY ANALYSIS

EV (Large plant) = 
$$$200,000P - $180,000 (1 - P)$$

EV (Small plant) = 
$$$100,000P - $20,000(1 - P)$$

EV (Do nothing) = 
$$\$0P + \$0(1 - P)$$



# EXPECTED VALUE OF PERFECT INFORMATION

- A consultant or a further analysis can aid the decision maker by giving exact (perfect) information about the true state: the decision problem is no longer under risk; it will be under certainty.
- Is it worthwhile for obtaining perfectly reliable information: is EVPI greater than the fee of the consultant (the cost of the analysis)?
- EVPI is the maximum amount a decision maker would pay for additional information

# EXPECTED VALUE OF PERFECT INFORMATION

EVPI = EV with perfect information
- Maximum EV under risk

	STATES O	STATES OF NATURE		
	Favorable Unfavorable			
ALTERNATIVES	market	market		
Construct large plant	\$200,000	(\$180,000)		
Construct small plant	\$100,000	(\$20,000)		
Do nothing	\$0	\$0		
PROBABILITIES	0.6	0.4		

Expected Value \$48,000 \$52,000

EV with perfect information: 200\*.6+0\*.4=120

Maximum EV under risk: 52

EVPI = 120 - 52 = 68

### **EXPECTED OPPORTUNITY LOSS**

Choose the alternative with the minimum weighted row average of the regret matrix

$$EOL(a_i) = \sum_{j} r_{ij} P(q_j)$$

Regret Values	STATES C	F NATURE	Expected
	Favorable	Unfavorable	Opportunity
ALTERNATIVES	market	market	Loss
Construct large plant	\$0	\$180,000	\$72,000
Construct small plant	\$100,000	\$20,000	(\$68,000)
Do nothing	\$200,000	\$0	\$120,000
PROBABILITIES	0.6	0.4	

#### **DECISION ANALYSIS APPLICATION I**

- Suppose that a construction company is trying to make a decision on the number of condos (apartments) to build in the next year. The situation of economy is very effective during this decision making process and the related data is determined as follows. Determine the best alternative by using:
  - Maximax
  - Maximin
  - Criterion of realism (a=0.6)
  - Equally likelihood
  - Minimax

	States of Nature		
Alternatives	S1 (Good)	S2 (Fair)	S3 (Bad)
A1 (1000 units)	10.000.000	4.000.000	-5.000.000
A2 (750 units)	7.000.000	3.000.000	-3.000.000
A3 (500 units)	4.000.000	2.000.000	-2.000.000
A4 (250 units)	2.500.000	1.000.000	-1.000.000

## ANSWER TO DECISION ANALYSIS APPLICATION I

- Maximax = Alternative 1 (10.000.000)
- Maximin = Alternative 4 (- 1.000.000)
- Criterion of realism (a=0.6) = Alternative 1 (4.000.000)
- Equally likelihood = Alternative 1 (3.000.000)
- Minimax = Alternative 2 (3.000.000)

#### **DECISION ANALYSIS APPLICATION II**

- Suppose that a construction company is trying to make a decision on the number of condos (apartments) to build in the next year. The situation of economy is very effective during this decision making process and the related data is determined as follows. Moreover, the probabilities for each state of nature are given. Obtain the values of the following parameters regarding the case:
- o Expected (monetary) value
- o Epected value of perfect information
- Expected opportunity loss

	States of Nature		
Alternatives	S1 (Good)	S2 (Fair)	S3 (Bad)
Probabilities	30%	50%	20%
A1 (1000 units)	10.000.000	4.000.000	-5.000.000
A2 (750 units)	7.000.000	3.000.000	-3.000.000
A3 (500 units)	4.000.000	2.000.000	-2.000.000
A4 (250 units)	2.500.000	1.000.000	-1.000.000

# ANSWER TO DECISION ANALYSIS APPLICATION II

- Expected (monetary) value = Alternative 1 (4.000.000)
- Expected value of perfect information = 800.000
- Expected opportunity loss = Alternative 1 (800.000)

### **REFERENCES**

 Lecture notes of "Prof. Dr. Y. İlker Topçu", <u>http://web.itu.edu.tr/topcuil/</u>