SOLUTION OF NETWORK Transportation PROBLEMS

Model Description

- A transportation problem basically deals with the problem of finding the best way to fulfill the demand of n demand points using the capacities of m supply points. The optimal solution will include a variable cost constraint for shipping the product from one supply point to a demand point.

General Description of a Transportation Problem

1. A set of **m** supply points from which a good is shipped. Supply point **i** can supply at most s_i units.

2. A set of **n** demand points to which the good is shipped. Demand point **j** must receive at least d_i units of the shipped good.

3. Each unit produced at supply point i and shipped to demand point **j** incurs a variable cost of c_{ij}.

General Model Notation

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} X_{ij}$$

$$s.t. \sum_{j=1}^{j=n} X_{ij} \leq s_i (i = 1,2,...,m)$$

$$\sum_{i=1}^{i=m} X_{ij} \geq d_j (j = 1,2,...,n)$$

$$X_{ij} \geq 0 (i = 1,2,...,m; j = 1,2,...,n)$$

X_{ij} = number of units shipped from *supply point i* to *demand point j*

Formulating Transportation Problems

Example :

PowerCo has three (3) electric power plants that supply the electric needs of four cities (4).

- The associated supply of each plant and demand of each city is given in the table 1.

- The cost of sending 1 million kwh of electricity from a plant to a city depends on the distance the electricity must travel.

Transportation Tableau

A transportation problem is specified by

- the supply points or *nodes*,

- the demand points or *nodes*, and

- the shipping or transport costs (generally taken as linear).

So the relevant data can be summarized in a <u>transportation tableau</u>.

TABLE Shipping costs, Supply, and Demand for PowerCo Example

From	To				
	City 1	City 2	City 3	City 4	Supply (Million kwh)
Plant 1	$8	$6	$10	$9	35
Plant 2	$9	$12	$13	$7	50
Plant 3	$14	$9	$16	$5	40
Demand (Million kwh)	45	20	30	30	

Transportation Tableau

Objective Function

Since we want to <u>minimize</u> the total cost of shipping from plants to cities;

Minimize Z $\displaystyle\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij} x_{ij}$

$$\text{Minimize Z} = \{8X_{11}+6X_{12}+10X_{13}+9X_{14}\} +$$
$$\{9X_{21}+12X_{22}+13X_{23}+7X_{24}\} +$$
$$\{14X_{31}+9X_{32}+16X_{33}+5X_{34}\}$$

Supply Constraints

Each supply point has a upper bound on production capacity:

Plant 1: $X_{11} + X_{12} + X_{13} + X_{14} <= 35$

Plant 2: $X_{21} + X_{22} + X_{23} + X_{24} <= 50$

Plant 3: $X_{31} + X_{32} + X_{33} + X_{34} <= 40$

Demand Constraints

Each demand point has a needed level of electricity in kwh:

City 1: $X_{11} + X_{21} + X_{31} >= 45$

City 2: $X_{12} + X_{22} + X_{32} >= 20$

City 3: $X_{13} + X_{23} + X_{33} >= 30$

City 4: $X_{14} + X_{24} + X_{34} >= 30$

Positivity Constraints

Since a **negative** amount of electricity can not be shipped to a city all Xij's must be non negative;

$X_{ij} >= 0$ (i= 1,2,3; j= 1,2,3,4)

LP Formulation of Power Co's Problem

Min Z = $[8X_{11}+6X_{12}+10X_{13}+9X_{14}]+$
$[9X_{21}+12X_{22}+13X_{23}+7X_{24}]+$
$[14X_{31}+9X_{32}+16X_{33}+5X_{34}]$

S.T.: $X_{11}+X_{12}+X_{13}+X_{14} <= 35$ (Supply Constraints)

$X_{21}+X_{22}+X_{23}+X_{24} <= 50$

$X_{31}+X_{32}+X_{33}+X_{34} <= 40$

$X_{11}+X_{21}+X_{31} >= 45$ (Demand Constraints)

$X_{12}+X_{22}+X_{32} >= 20$

$X_{13}+X_{23}+X_{33} >= 30$

$X_{14}+X_{24}+X_{34} >= 30$

Xij >= 0 (i= 1,2,3; j= 1,2,3,4)

Transportation Problem

Transportation problem

	Cost of Shi City 1	City 2	City 3	City 4	Supply
Plant-1	8	6	10	9	35
Plant-2	9	12	13	7	50
Plant-3	14	9	16	5	40
Demand	45	20	30	30	
Variables	x11	x12	x13	x14	
x1j : amount sent to city j	0	0	0	0	
	x21	x22	x23	x24	
x2j : amount sent to city j	0	0	0	0	
	x31	x32	x33	x34	
x3j : amount sent to city j	0	0	0	0	

Objective Function	Value

Minimize Z = {8X11+6X12+10X13+9X14} +
         {9X21+12X22+13X23+7X24} +
         {14X31+9X32+16X33+5X34}

0,000

Constrains	Formula	Value
X11 + X12 + X13 + X14 <= 35	0	35
X21 + X22 + X23 + X24 <= 50	0	50
X31 + X32 + X33 + X34 <= 40	0	40
X11 + X21 + X31 >= 45	0	45
X12 + X22 + X32 >= 20	0	20
X13 + X23 + X33 >= 30	0	30
X14 + X24 + X34 >= 30	0	30
x11	0	0
x12	0	0
x13	0	0
x14	0	0
x21	0	0
x22	0	0
x23	0	0
x24	0	0
x31	0	0
x32	0	0
x33	0	0
x34	0	0

Transportation Problem

Transportation problem						
	Cost of Shi	City 1	City 2	City 3	City 4	Supply
Plant-1		8	6	10	9	35
Plant-2		9	12	13	7	50
Plant-3		14	9	16	5	40
Demand		45	20	30	30	
Variables		x11	x12	x13	x14	
x1j : amount sent to city j		0	0	0	0	
		x21	x22	x23	x24	
x2j : amount sent to city j		0	0	0	0	
		x31	x32	x33	x34	
x3j : amount sent to city j		0	0	0	0	

Objective Function — Value

Minimize Z = {8X11+6X12+10X13+9X14} +
{9X21+12X22+13X23+7X24} +
{14X31+9X32+16X33+5X34}

0,000

Constrains	Formula	Value
X11 + X12 + X13 + X14 <= 35	0	35
X21 + X22 + X23 + X24 <= 50	0	50
X31 + X32 + X33 + X34 <= 40	0	40
X11 + X21 + X31 >= 45	0	45
X12 + X22 + X32 >= 20	0	20
X13 + X23 + X33 >= 30	0	30
X14 + X24 + X34 >= 30	0	30
x11	0	0
x12	0	0
x13	0	0
x14	0	0
x21	0	0
x22	0	0
x23	0	0
x24	0	0

Çözücü Parametreleri

Hedef Hücre: $B$14

Eşittir: ○ En Büyük ● En Küçük ○ Değer: 0

Değişen Hücreler:
x1j;x2j;x3j

Kısıtlamalar:
$B$17 <= $C$17
$B$18 <= $C$18
$B$19 <= $C$19
$D$20 >= $C$20
$B$21 >= $C$21
$B$22 >= $C$22

Çöz
Kapat
Tahmin
Seçenekler
Ekle
Değiştir
Tümünü Sıfırl
Sil
Yardım

Transportation Problem

Transportation problem

Cost of Shipping	City 1	City 2	City 3	City 4	Supply
Plant-1	8	6	10	9	35
Plant-2	9	12	13	7	50
Plant-3	14	9	16	5	40
Demand	45	20	30	30	
Variables	x11	x12	x13	x14	
x1j : amount sent to city j	0	10	25	0	
	x21	x22	x23	x24	
x2j : amount sent to city j	45	0	5	0	
	x31	x32	x33	x34	
x3j : amount sent to city j	0	10	0	30	

Objective Function	Value
Minimize Z = {8X11+6X12+10X13+9X14} + {9X21+12X22+13X23+7X24} + {14X31+9X32+16X33+5X34}	1.020.000

Constrains	Formula	Value
X11 + X12 + X13 + X14 <= 35	35	35
X21 + X22 + X23 + X24 <= 50	50	50
X31 + X32 + X33 + X34 <= 40	40	40
X11 + X21 + X31 >= 45	45	45
X12 + X22 + X32 >= 20	20	20
X13 + X23 + X33 >= 30	30	30
X14 + X24 + X34 >= 30	30	30
x11	0	0
x12	10	0
x13	25	0
x14	0	0
x21	45	0
x22	0	0
x23	5	0
x24	0	0
x31	0	0
x32	10	0
x33	0	0
x34	30	0

Çözücü Parametreleri

Hedef Hücre: $B$14

Eşittir: ○ En Büyük ● En Küçük ○ Değer: 0

Değşen Hücreler:
x1j;x2j;x3j

Kısıtamalar:
$B$17 <= $C$17
$B$18 <= $C$18
$B$19 <= $C$19
$B$20 >= $C$20
$B$21 >= $C$21
$B$22 >= $C$22

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Transportation Problem

Çalışma Sayfası: [Problem_Transportation.xls]Sayfa1

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Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
$B$20	{8X11+6X12+10X13+9X14} +	0,000	1.020,000

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
$B$12	x1j : amount sent to city j x11	0	0
$C$12	x1j : amount sent to city j x12	0	10
$D$12	x1j : amount sent to city j x13	0	25
$E$12	x1j : amount sent to city j x14	0	0
$B$14	x2j : amount sent to city j x21	0	45
$C$14	x2j : amount sent to city j x22	0	0
$D$14	x2j : amount sent to city j x23	0	5
$E$14	x2j : amount sent to city j x24	0	0
$B$16	x3j : amount sent to city j x31	0	0
$C$16	x3j : amount sent to city j x32	0	10
$D$16	x3j : amount sent to city j x33	0	0
$E$16	x3j : amount sent to city j x34	0	30

Sınırlamalar

Hücre	Ad	Hücre Değeri	formül	Durum	Serbestlik
$B$23	X11 + X12 + X13 + X14 <= 35 Formula	35	$B$23<=$C$23	Aynı	0
$B$24	X21 + X22 + X23 + X24 <= 50 Formula	50	$B$24<=$C$24	Aynı	0
$B$25	X31 + X32 + X33 + X34 <= 40 Formula	40	$B$25<=$C$25	Aynı	0
$B$26	X11 + X21 + X31 >= 45 Formula	45	$B$26>=$C$26	Aynı	0
$B$27	X12 + X22 + X32 >= 20 Formula	20	$B$27>=$C$27	Aynı	0
$B$28	X13 + X23 + X33 >= 30 Formula	30	$B$28>=$C$28	Aynı	0
$B$29	X14 + X24 + X34 >= 30 Formula	30	$B$29>=$C$29	Aynı	0
$B$30	x11 Formula	0	$B$30>=$C$30	Aynı	0
$B$31	x12 Formula	10	$B$31>=$C$31	Farklı	10

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