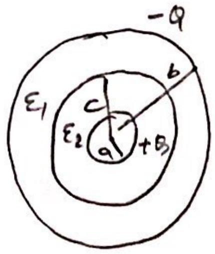
EEE 210 Electromagnetic Fields Theory

1th Examination

SOLUTIONS

1)



$$\oint_S \bar{D} \cdot \bar{ds} = Q \quad (4)$$

$$D(4\pi R^2) = Q \rightarrow \vec{D} = \frac{Q}{4\pi R^2}\hat{a}_R \quad (4)$$

$$\bar{E}_1 = \frac{\vec{D}}{\varepsilon_0 \varepsilon_1} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1 R^2}\hat{a}_R \qquad b > R > c \quad (4)$$

$$\bar{E}_2 = \frac{\bar{D}}{\varepsilon_0 \varepsilon_2} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2 R^2}\hat{a}_R \qquad c > R > a \quad (4)$$

$$V_{ba} = -\int_b^c \vec{E}_1 \cdot \vec{dl} - \int_c^a \bar{E}_2 \cdot \bar{dl} \quad (4)$$

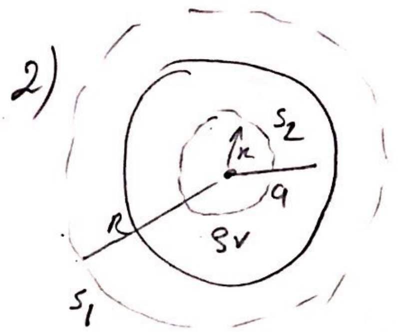
$$V_{ba} = -\frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left(-\frac{1}{R}\right)\Big|_b^c - \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left(-\frac{1}{R}\right)\Big|_c^a$$

$$V_{ba} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left(\frac{1}{c} - \frac{1}{b}\right) + \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left(\frac{1}{a} - \frac{1}{c}\right)$$

$$V_{ba} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{c} - \frac{1}{b}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{a} - \frac{1}{c}\right)\right] \quad (10)$$

$$C = \frac{Q}{V_{ba}} = \frac{4\pi\varepsilon_0\varepsilon_1\varepsilon_2}{\varepsilon_2\left(\frac{1}{c} - \frac{1}{b}\right) + \varepsilon_1\left(\frac{1}{a} - \frac{1}{c}\right)} \quad (5)$$

2)



$\underline{a > R > 0}$

$$\oint_{S_2} \vec{E}_{in} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \int_v \rho_v \, dv \quad 3$$

$$E_{in}(4\pi R^2) = \frac{1}{\varepsilon_0} \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{R}{a^2} \cdot R^2 \sin\theta \, dR \, d\theta \, d\phi \quad 3$$

$$E_{in}(4\pi R^2) = \frac{4\pi}{\varepsilon_0 a^2} \int_0^R R^3 dR$$

$$E_{in}(4\pi R^2) = \frac{4\pi}{a^2 \varepsilon_0} \frac{R^4}{4} \Rightarrow \vec{E}_{in} = \frac{R^2}{4a^2 \varepsilon_0} \hat{a_R} \quad 3$$

$\underline{R > a}$

$$\oint_{S_1} \vec{E}_{out} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \int_v \rho_v \, dv \quad 3$$

$$E_{out}(4\pi R^2) = \frac{1}{\varepsilon_0 a^2}(4\pi) \int_0^a R^3 dR \quad 3$$

$$E_{out}(4\pi R^2) = \frac{4\pi}{a^2 \varepsilon_0} \frac{R^4}{4} \Big|_0^a = \frac{4\pi a^4}{4a^2 \varepsilon_0} = \frac{4\pi a^2}{4\varepsilon_0}$$

$$\vec{E}_{out} = \frac{a^2}{4\varepsilon_0 R^2} \hat{a_R} \quad 3$$

$$V\Big|_{R=\frac{a}{2}} = -\int_\infty^a \vec{E}_{out} \cdot \vec{dl} - \int_a^{a/2} \vec{E}_{in} \cdot \vec{dl} \quad 7$$

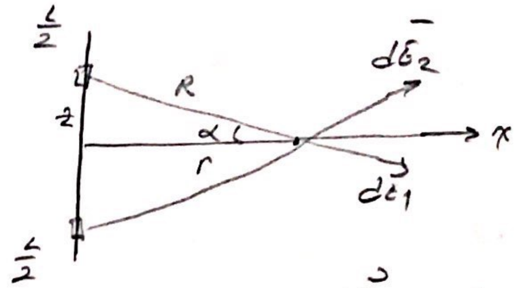
$$= -\frac{a^2}{4\varepsilon_0} \int_\infty^a \frac{dR}{R^2} - \frac{1}{4a^2 \varepsilon_0} \int_a^{a/2} R^2 dR$$

$$= \frac{a^2}{4\varepsilon R} \Big|_\infty^a - \frac{1}{12 a^2 \varepsilon_0} R^3 \Big|_a^{a/2}$$

$$= \frac{a}{4\varepsilon_0} - \frac{1}{12 a^2 \varepsilon_0}\left(\frac{a^3}{8} - a^3\right) = \frac{a}{4\varepsilon_0} + \frac{7a}{96\varepsilon_0}$$

$$= \frac{31a}{96\varepsilon_0} \text{ V.} \quad 10$$

3)



$$|d\bar{E}_1| = |d\bar{E}_2| \quad 2$$

$$|d\bar{E}_1| = \frac{\rho_l \, dz}{4\pi\varepsilon_0 R^2} \quad 2$$

$$d\vec{E}_T = 2\,\frac{\rho_l \, dz}{4\pi\varepsilon_0 R^2}\cos\alpha \; \hat{a}_x \quad 2$$

$$\tan\alpha = \frac{z}{r} \rightarrow dz = \frac{r}{\cos^2\alpha}\, d\alpha \quad 2$$

$$R = \frac{r}{\cos\alpha} \quad 2$$

$$d\vec{E}_{T_+} = \frac{\rho_l}{2\pi\varepsilon_0}\,\frac{r}{\cancel{\cos^2\alpha}}\cdot\frac{1}{\cancel{\left(\frac{r}{\cos\alpha}\right)^2}}\cdot\cos\alpha\, d\alpha\; \hat{a}_x$$

$$d\vec{E}_{T_+} = \frac{\rho_l}{2\pi\varepsilon_0 r}\cos\alpha\, d\alpha \; \hat{a}_x \quad 2$$

$$\vec{E}_{T_+} = \frac{\rho_l}{2\pi\varepsilon_0 r}\int \cos\alpha\, d\alpha \; \hat{a}_x$$

$$\vec{E}_{T_+} = \frac{\rho_l}{2\pi\varepsilon_0 r}\left(\sin\alpha\right)\Big|_0^{\frac{L}{2}/\sqrt{r^2+(\frac{L}{2})^2}} \; \hat{a}_x \quad 2$$

$$E_{T_+} = \frac{\rho_l}{4\pi\varepsilon_0}\,\frac{L}{\sqrt{r^2+L^2/4}} \quad 2$$

$$\bar{E}_{T_+} = E_{T_-} = \frac{\rho_l L}{4\pi\varepsilon_0\sqrt{r^2+(\frac{L}{2})^2}} \; \hat{a}_x \quad 10$$

$$\vec{E}_T = \frac{\rho_l L}{2\pi\varepsilon_0\sqrt{r^2+(\frac{L}{2})^2}} \; \hat{a}_x \quad 4$$

$$E_T\Big|_{r=\frac{d}{2}} = \frac{\rho_l \cdot L}{2\pi\varepsilon_0\sqrt{(\frac{d}{2})^2+(\frac{L}{2})^2}} = \frac{\rho_l \cdot L}{\pi\varepsilon_0\sqrt{d^2+L^2}}$$