Gradient Descent for Multiple Variables

LR with Multiple Variables

- Linear regression with multiple variables is also known as "multivariate linear regression".
- We now introduce notation for equations where we can have any number of input variables.

Size (feet ²)	Price (\$1000)
x	y
2104	460
1416	232
1534	315
852	178

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

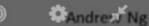






Size (feet ²)	Number of bedrooms	Number of floors	Age of home (years)	Price (\$1000)
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178





Size (feet²)		Number of bedrooms	Number of floors	Age of home (years)	Price (\$1000)
	×I	Xz	×3	*4	3,
	2104	5	1	45	460
	1416	3	2	40	232
	1534	3	2	30	315
	852	2	1	36	178
				•••	•••





	Size (feet²)	Number of bedrooms	Number of floors	Age of home (years)	Price (\$1000)
	×I	Xz	×3	*4	9
	2104	5	1	45	460
	1416	3	2	40	232
	1534	3	2	30	315
	852	2	1	36	178
NI.	1	*	1	1	

Notation:

$$\rightarrow$$
 n = number of features

 $x^{(i)}$ = input (features) of i^{th} training example.

$$x_j^{(i)}$$
 = value of feature j in i^{th} training example.

 Size (feet²)	Number of bedrooms	Number of floors	Age of home (years)	Price (\$1000)	
×1	×2	×3	** **	7	_
2104	5	1	45	460 7	_
1416	3	2	40	232	M= 47
1534	3	2	30	315	
852	2	1	36	178	
 A	*	1	1	,	

Notation:

$$\rightarrow$$
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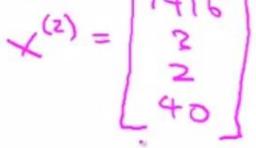
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2104	5	1	45	460 7
-> 1416	3	2	40	232 - M= 47
1534	3	2	30	315
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				l
Notation:	*	7	1	(z) = 1416
			n-4	

$$\rightarrow n$$
 = number of features

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	2104	5	1	45	460 7	_
->	1416	3	2	40	232	M= 47
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						,
Nota	tion:	人	1	1	~	(2) = 14
				n-4	~	

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For convenience of notation, define $x_0 = 1$.

$$h_{\theta}(x) = \underline{\theta_0} + \underline{\theta_1}x_1 + \underline{\theta_2}x_2 + \dots + \underline{\theta_n}x_n$$
For convenience of notation, define $x_0 = 1$. $(x_0^{(i)} = 1)$

$$X = \begin{bmatrix} X_0 \\ X_1 \\ X_N \end{bmatrix} \in \mathbb{T}^{M+1}$$

$$Q = \begin{bmatrix} Q_0 \\ Q_1 \\ Q_1 \\ \vdots \\ Q_n \end{bmatrix}$$

$$\rightarrow h_{\theta}(x) = \underline{\theta_0} + \underline{\theta_1}x_1 + \underline{\theta_2}x_2 + \dots + \underline{\theta_n}x_n$$

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$$(\times'') = i$$

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$$\begin{aligned}
\chi &= \begin{bmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_N \end{bmatrix} \in \mathbb{R}^{n+1} & O &= \begin{bmatrix} O_0 \\ O_2 \\ \vdots \\ O_N \end{bmatrix} \in \mathbb{R}^{n+1} \\
&= O \cdot \chi_1 + \dots + O \cdot \chi_N \\
&= O \cdot \chi_1 + \dots + O \cdot \chi_N
\end{aligned}$$

$$\rightarrow h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$. $(x_0) = 0$

Multivariate linear regression.

Windows'u Etkinleştir Windows'u etkinleştirmek için Ayarlar'a gidin.

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 This is a vectorization of our hypothesis function for one training example

$$h_{ heta}(x) = \left[egin{array}{cccc} heta_0 & & heta_1 & & \dots & & heta_n
ight] egin{bmatrix} x_0 \ x_1 \ dots \ x_n \end{bmatrix} = heta^T x$$

Gradient Descent for MLR

Hypothesis:
$$h_{\theta}(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

Parameters: $\theta_0, \theta_1, \dots, \theta_n$

Cost function:

Tunction:
$$J(\theta_0, \theta_1, \dots, \theta_n) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

Hypothesis: $h_{\theta}(x)=\theta^Tx=\theta_0x_0+\theta_1x_1+\theta_2x_2+\cdots+\theta_nx_n$

Parameters: $\theta_0, \theta_1, \dots, \theta_r$



n+1 - direction vector.

Cost function:

$$J(\theta_0, \theta_1, \dots, \theta_n) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

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Parameters: $\theta_0, \theta_1, \dots, \theta_n$



htl - direction vector

Cost function:

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Parameters: $\theta_0, \theta_1, \dots, \theta_n$

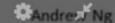
n+1 - direction vector

Cost function:

$$\frac{J(\theta_0, \theta_1, \dots, \theta_n)}{\preceq(\bullet)} = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

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Hypothesis: $h_{\theta}(x)=\theta^Tx=\theta_0x_0+\theta_1x_1+\theta_2x_2+\cdots+\theta_nx_n$

Parameters: $\theta_0, \theta_1, \dots, \theta_n$



n+1 - direction vector

Cost function:

$$\frac{J(\theta_0, \theta_1, \dots, \theta_n)}{J(\Theta)} = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Gradient descent:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \dots, \theta_n)$$

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(simultaneously update for every 3 street in A gidin.



When there are n features, we define the cost function as

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}.$$

For linear regression, which of the following are also equivalent and correct definitions of $J(\theta)$?

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} \left(\left(\sum_{j=0}^{n} \theta_{j} x_{j}^{(i)} \right) - \left(\sum_{j=0}^{n} y_{j}^{(i)} \right) \right)^{2}$$

Hypothesis:
$$h_{\theta}(x)=\theta^Tx=\theta_0x_0+\theta_1x_1+\theta_2x_2+\cdots+\theta_nx_n$$

Parameters:
$$\theta_0, \theta_1, \dots, \theta_n$$



n+1 - diversion vector

Cost function:

$$J(\theta_0, \theta_1, \dots, \theta_n) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

Gradient descent:

Repeat { $\Rightarrow \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \dots, \theta_n) - \beta \text{ (simultaneously update for every 3 sure Kinn Quinnek için, Ayana) a gidin.}$

Previously (n=1):

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})$$

$$\frac{\partial}{\partial \theta_0} J(\theta)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

Previously (n=1):

Repeat
$$\left\{ \theta_0 := \theta_0 - o \left[\frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \right] \right\}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) \underline{x^{(i)}}$$

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(simultaneously update $heta_0, heta_1$)

}

New algorithm
$$(n \ge 1)$$
:

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \dots, n$)

$$h_{ heta}(x) = \left[egin{array}{cccc} heta_0 & & heta_1 & & \dots & & heta_n
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$$heta_2 := heta_2 - lpha rac{1}{m} \sum_{i=1}^{N} (h_{ heta}(x^{(i)}))$$
 leştiy $y^{(i)}$ $x_2^{(i)}$

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$$\theta_0 := \theta_0 - o \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})$$

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$$o$$
 $heta_2:= heta_2-lpharac{1}{m}\sum_{i=1}^{N}(h_{ heta}(x^{(i)}))$ leştiy $y^{(i)}(x^{(i)})$ $x^{(i)}_2$ indows'u etkinleştirmek için Ayarlar'a girdin.

Previously (n=1):

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Previously (n=1):

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Previously (n=1):

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$$\left[\frac{\partial}{\partial \theta_0} J(\theta)\right]$$

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(simultaneously update $\hat{\theta}_0, \hat{\theta}_1$)

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