## **Statistical Process Control**



## **Quality Control Process**

There are three steps to follow in quality control:

- To determine the true quality characteristics,
- To determine the methods of measuring and testing true quality characteristics,
- To reveal substitute quality characteristics and to establish the relationship between substitute quality characteristics and true quality characteristics.

# **Statistical Concepts**

- Mean (μ)
- Median
- Mode
- Range (R)
- Variance  $(\sigma^2)$
- Standard Deviation (σ)



## Uncertainty

- All processes vary due to machine, tool, method, material, operator, maintenance and environmental conditions. At no time can any two products or any feature of the product be the same.
- There is little difference between the measurements / features of the processed parts, even if they are small. This explains why the specifications have tolerances.



# The cause of uncertainty – common reasons

- These changes, which are made up of many small sources and that are found at different times in each process, can be predicted in advance. However, it is difficult to identify and correct these changes.
- However, since the specific reasons for the procedure have been eliminated and over time the overall causes have been stable, they should be reduced.

**Example:** vibration, temperature, humidity, voltage fluctuation, etc.

# The cause of uncertainty – special reasons

 The specific causes of change consist of an indefinite source, unpredictable and unordered. They repeat unless you take precautions. If it is known that the specific causes have occurred, they can be easily detected and corrected.

For example; tool breakage, tool wear, poor connections, material types, inexperienced operator, etc.

## Statistical Quality Control

- It is the application of methods which ensure that the production process is carried out under normal conditions and that it plays a crucial role in ensuring that the production is taken out of control by a defect or for a special reason, and that the necessary measures are taken in time.
- Statistical principles and techniques are used in all stages of production to ensure that a product is produced in the most economical and useful way.

## Statistical Quality Control

- It is producing with minimum cost, on time and right data.
   Appropriate use of quality control techniques in statistical terminology is to help control the magnitude of non-sampling errors.
- Statistical quality control methods are divided into two:
  - Process Control
  - Product Control

### Statistical Process Control

- Statistical process control (SPC) is a tool used to reduce production of inappropriate product, meeting quality standards and ensuring compliance with predefined quality specifications.
- Thus, it provides to make decisions based on the data so that corrective and preventive activities can be initiated.

Differences between correction, corrective action and preventive action



Correction
Put fire out
(at the time)



Corrective Action
What caused fire
and how to prevent
recurrence
(after event)



Preventive Action
Stop fire from
happening
(before event)

## What is Statistical Process Control?

- ✓ Statistical process control is an industry-standard methodology for measuring and controlling quality during the manufacturing process.
- ✓ SPC performed during the manufacturing/assembly process not only eliminates the need for final inspection, but equally important significantly reduces and amount of material scrap along with direct & indirect labor waste. The result is a direct increase in bottom line profitability.
- ✓ For those who feel they cannot afford to implement SPC, the reality is they cannot afford not to.

## Statistical Process Control Objectives

- To keep the process under control by removing the special causes of uncertainty,
- Preventing defects made in the first time on the basis of prevention,
- Matching process output to desired design quality, reducing variability.

### Benefits of SPC

- It allows to determine the status of the process in advance,
- The variability in the product is reduced,
- Product quality develops,
- The scrap rate is reduced,
- Effective capacity utilization increases,
- The unit cost drops,
- Control activities are reduced,
- Poor quality costs are reduced,
- It enables the machine/process capability to be monitored,
- Determines corrective and preventive action requirements.



### The Basic Functions of SPC

- Process variability is measured.
- Process variability is controlled.
- The process is made sufficient.
- Reduction of process variability is maintained.

## The Solution Questions of SPC

You can start to quantify the value of an SPC solution by asking the following questions:

- Are your quality costs really known?
- Can current data be used to improve your processes, or is it just data for the sake of data?
- Are the right kinds of data being collected in the right areas?
- Are decisions being made based on true data?
- Can you easily determine the cause of quality issues?
- Do you know when to perform preventative maintenance on machines?
- Can you accurately predict yields and output results?

## **SPC Steps**

#### Step 1:

- It is necessary to determine what to control when establishing the quality system.
- The characteristics or factors that affect the quality of the product need to be determined.

## **SPC Steps**

#### Step 2:

- The data must be collected to evaluate the process to be controlled.
- It is impossible to collect data on all of the universe when information about the operation of a certain period is needed.
   For this reason, a certain mass is taken as an example for examination within the main mass.

## **SPC Steps**

#### Step 3:

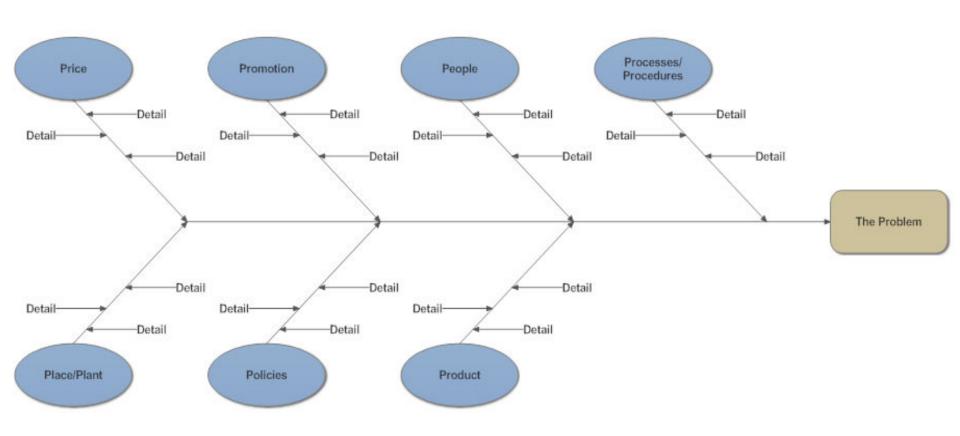
- Measurement and testing of specified quality characteristics
- Some of the techniques used to test the obtained data and quality characteristics;
  - Cause and Effect Diagram
  - Check Sheet
  - Control Charts
  - Histogram
  - Pareto Analysis

## Cause and Effect Diagram

- A cause and effect diagram examines why something happened or might happen by organizing potential causes into smaller categories. It can also be useful for showing relationships between contributing factors.
- Most cause and effect diagrams examine a similar set of possible causes for any issue analyzed.

# Cause and Effect Diagram

Here is an example for 'Marketing Industry';



#### **Check Sheet**

- The examination and recording of test data in any case.
- Various check sheets are commonly used in businesses to collect data.
- The benefit of preparing the check sheets is that once they have been verified that the data will represent the facts, there is a way to simplify the enrollment, summarization and analysis of the data.



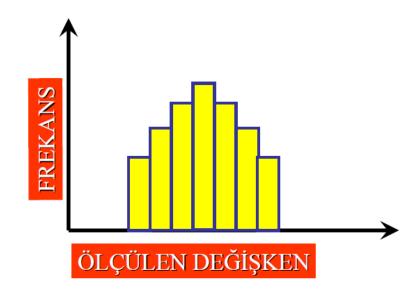
# Histogram



- The values of the horizontal axis variable are the density graphs in which the vertical axis frequencies are found and the rectangles of the length are proportional to the frequency of each interval.
- Histograms are usually used to show the frequency of occurrence of an event and to calculate whether the problem identified in the specified time interval occurs more frequently and to compare the shape of the resulting distribution with a known distribution.

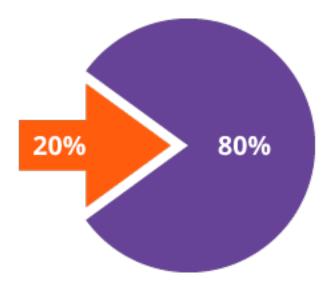
## Histogram

- Each histogram measures only one characteristic.
- Events can be tracked by making multiple histograms over time for the same feature.
- In order to be able to reflect the facts of the interpretations to be made in terms of the shape to be taken out, data of 50-100 units should be studied.
- Histograms reflect the size and distribution of symmetry and asymmetry. By following them, important clues about the structure of existing and probable problems can be obtained.



## Pareto Analysis

- The Pareto principle states that a large part of the problems are usually caused by a small number, but dominant, in connection with one another.
- This principle, also called the 80:20 rule, argues that about 80% of the results are due to about 20% of the causes of a problem.



## Pareto Analysis

Pareto analysis is a simple method for error and cost analysis. It is a tool used to determine the roles played by various factors when a result is obtained.

Sorting out the causes that constitute a problem by order of importance, distinguishing the important from the unimportant, and paying attention to the important causes.

Pareto analysis should be used to identify the next step at various stages of a quality improvement program. In addition; Pareto analysis is used to answer questions such as 'which section should the next statistical process control set' or 'what type of defect should we focus on first?'.

#### **Control Charts**

- Control charts are schematics used to monitor the inputs or outputs of processes.
- Control charts are charts that graphically show the change over time of any feature desired to be kept under control.
- Continuous control of processes can be achieved with information obtained from past experience.

## Why Control Chart?

- To ensure that the output of the process is 'NORMAL'.
- It helps in finding;
  - Is there any change in location of process mean in real time?
  - Is there any change in the spread of the process in real time?
- To keep the cost of production minimum

# **Objectives of Control Charts**

- To determine the real opportunities of the production process,
- Making adjustments to change the process output quality,
- To control the output.



#### **Control Charts**

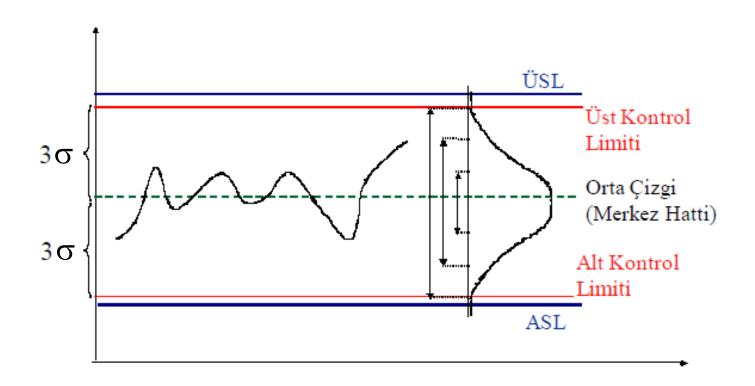
- By statistical control it is understood that the characteristics of the collected data should be determined.
- Knowing the distribution of a data set is appropriate for a particular model facilitates the analysis to be made.
- The control charts give warning messages and allow the action to be taken if the control limits or specification limits are too close or out of line.
- Control charts are often used to observe unstable processes or to provide early warning when a procedure changes.

#### **Control Charts**

#### Basic assumptions;

- The normality assumption (n > 30),
- 99.74% of the data are within  $\pm 3\sigma$  of normal distribution characteristics.
- In normal distribution;
  - The 68.20% of total data is in  $\pm 1\sigma$
  - The 95.45% of total data is in  $\pm 2\sigma$
  - The 99.74% of total data is in  $\pm 3\sigma$ .

## **Control Limits**



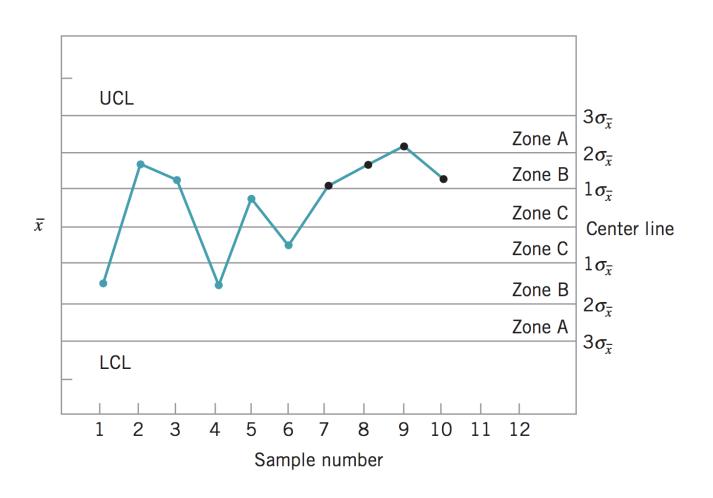
## **Specification Limits**

- ✓ The values that define certain properties of a product in an openness that will not cause any error are called 'Specifications'.
- ✓ The values that tolerances of any specification have are called 'Specification Limits'.
- ✓ Specification limits are used to determine if the product will work as planned. Specification limits of a process indicate the required state. These limits are set by management in the form of objectives.

## Control and Specification Limits

- Control limits never refer to specification limits.
- If a process fails to meet specifications, it may be under control or it may meet specifications if not under control.

## **Zones in Control Charts**



#### Western Electric Rules

The Western Electric Handbook (1956) suggests a set of decision rules for detecting nonrandom patterns on control charts. Specifically, it suggests concluding that the process is out of control if either;

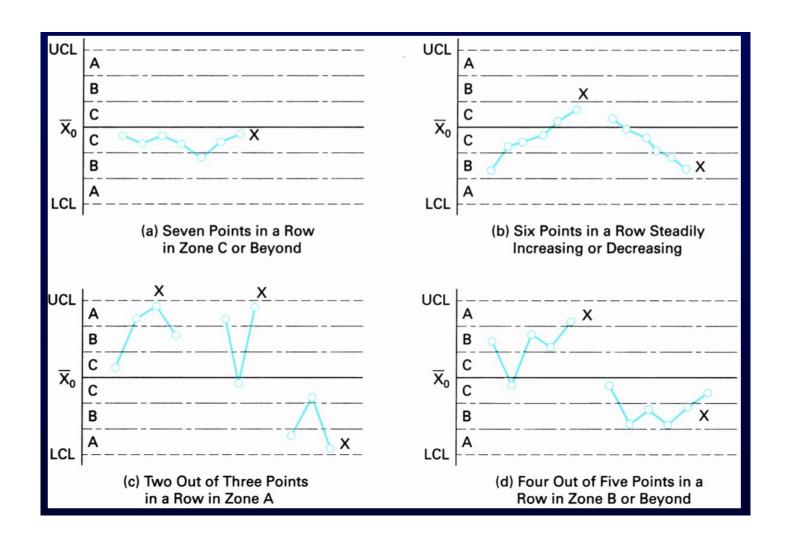
- One point plots outside the three-sigma control limits,
- Two out of three consecutive points plot beyond the twosigma warning limits,
- Four out of five consecutive points plot at a distance of onesigma or beyond from the center line, or
- Nine consecutive points plot on one side of the center line.

# Some Sensitizing Rules for Shewhart Control Charts

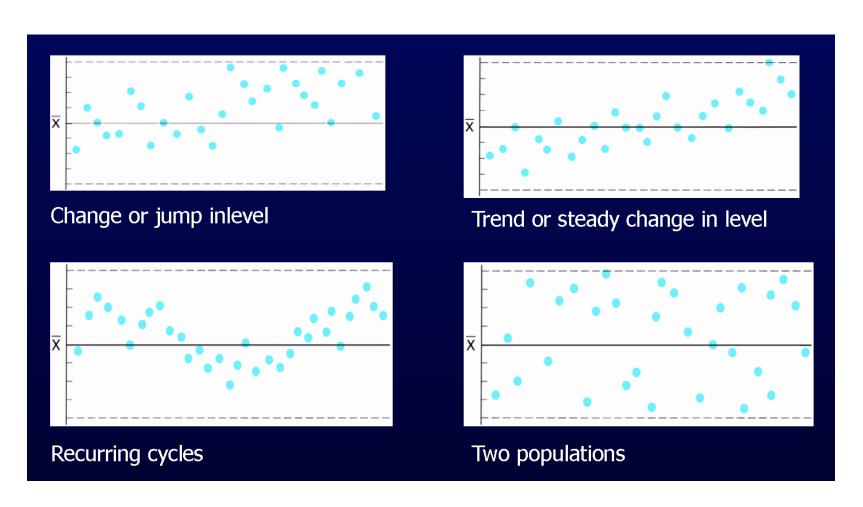
#### **Standard Action Signal:**

- 1. One or more points outside of the control limits.
- 2. Two of three consecutive points outside the two-sigma warning limits but still inside the control limits.
- 3. Four of five consecutive points beyond the one-sigma limits.
- 4. A run of nine consecutive points on one side of the center line.
- 5. Six points in a row steadily increasing or decreasing.
- 6. Fifteen points in a row in zone C (both above and below the center line).
- 7. Fourteen points in a row alternating up and down.
- 8. Eight points in a row on both sides of the center line with none in zone C.
- 9. An unusual or nonrandom pattern in the data.
- 10. One or more points near a warning or control limit.

# Interpretation of Control Charts



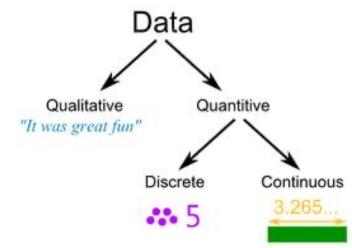
# Interpretation of Control Charts



### Data Collection in SPC

- Collected data is plotted on a graph with predetermined control limits. Control limits are determined by the capability of the process, whereas specification limits are determined by the customer's needs.
- Data that falls within the control limits indicates that everything is operating as expected. Any variation within the control limits is likely due to a common cause, the natural variation that is expected as part of the process.
- If data falls outside of the control limits, this indicates that an assignable cause is likely the source of the product variation, and something within the process should be changed to fix the issue before defects occur.

# Data Types





Discontinuous/Discrete **Continuous Data** Data

(Qualitative data types that can only take values on a certain scale)

(Quantitative data types that can be individually measured for each sample and displayed and performed on a continuous scale)

Numbers or discontinuous Counts/Proportion categories used to specify counts/proportions

**Attributes** 

Pass / fail judgments on the type of data indicating attributes such as color classifications 39

# **Quality Measures**

#### **Attribute**

- a product characteristic that can be evaluated with a discrete response
- good bad; yes no

#### **Variable**

- a product characteristic that is continuous and can be measured
- weight length

# **Types of Control Charts**

Data about continuous variables		Data about attributes					
Observations in groups	Observations as individual units	Defective	products	Defects on products			
⊼ bar− R Chart	X-R Chart	Variable Fixed subgroup size		Variable subgroup size	Fixed subgroup size		
		p Chart	np Chart	u Chart	c Chart		

## **Xbar Chart**

- Xbar chart is used to evaluate consistency of process averages by plotting the mean of each subgroup.
- It shows how individual measures or sample means are compared to the desired means or general means.
- It is used to keep track of changes that may occur over time in the process average.
- It is efficient at detecting relatively large shifts in the process average.

### R Control Chart

- R chart, on the other hand, plot ranges of each subgroup. The R chart is used to evaluate the consistency of process variation.
- It records the variation of individual observations in the sample.
- It is used to track the evolution of process variability over time.

## **Xbar-R Charts**

- In Xbar-R charts, control limits are determined according to whether or not control standards are known.
- These two charts are complementary to each other. Because a sample must have both an mean and a reasonable range of measurements, you can be assured that the process is under control.

# Xbar-R Charts-μ, σ are known;

#### **Xbar Chart Formulas**;

$$UCL = \mu + A\sigma$$

$$CL = \mu$$

$$A=3/\sqrt{n}$$

$$LCL = \mu - A\sigma$$

# Xbar-R Charts-μ, σ are known;

#### R Chart Formulas;

$$LCL = D_1 \sigma$$

$$CL = d_2 \sigma$$

$$UCL = D_2 \sigma$$

# Xbar-R Charts-μ, σ are not known;

By taking the average of the mean of the samples of n units, the process average  $\mu$  is obtained by the following formula and used as the center line in the general average graph.

$$\hat{\mu}_{\overline{x}} \pm 3\hat{\sigma}_{\overline{x}} = \overline{\overline{x}} \pm 3\frac{\hat{\sigma}}{\sqrt{n}} = \overline{\overline{x}} \pm 3\frac{\overline{R}/d_2}{\sqrt{n}} = \overline{x} \pm A_2\overline{R}$$

$$\overline{\sigma} = \frac{\overline{R}}{d_2}$$

# Xbar-R Charts-μ, σ are not known;

#### **Xbar Chart Limits:**

$$\begin{array}{ccc}
 & = & - \\
LCL = x - A_2R \\
 & = \\
CL = x \\
 & = & - \\
UCL = x + A_2R
\end{array}$$

$$A_2 = \frac{3}{d_2 \sqrt{n}}$$

# Xbar-R Charts-μ, σ are not known;

#### R Chart Limits:

$$LCL = D_{3}\overline{R}$$

$$CL = \overline{R}$$

$$D_{3} = 1 - \frac{d_{3}}{d_{2}}$$

$$UCL = \overline{R}$$

$$D_{4} = 1 + \frac{d_{3}}{d_{2}}$$

## **Xbar-S Control Charts**

- When the samples from the production have 10 or more sizes (n> = 10), the S graph is used instead of the R chart.
- When n> = 10, the standard deviation is preferred as the scattering measure, since the efficiency of R, hence the reliability, is reduced.
- The S chart is also used in the case of n <10. However, the R chart is recommended for ease of calculation.
- On the other hand, if the sample size is variable, ie if the samples taken from the production are of different sizes, the S chart is used instead of the R chart.
- The control limits on the  $\overline{X}$  and S control charts are determined separately according to whether the standards are known or not.

## Xbar-S Control Charts - μ, σ are known

The Xbar chart is created using the same calculations as the Xbar-R chart.

#### **S Chart Formulas:**

$$LCL = B_5\sigma$$

$$CL = C_4 \sigma$$

$$UCL = B_6\sigma$$

$$B_5 = c_4 - 3\sqrt{1 - c_4^2}$$

$$B_6 = c_4 + 3\sqrt{1 - c_4^2}$$

# Xbar-S Control Charts - μ, σ are not known

#### **Xbar Chart:**

$$\hat{\mu} \pm 3\hat{\sigma}_{\bar{x}} \Rightarrow \bar{\bar{x}} \pm 3\frac{\hat{\sigma}}{\sqrt{n}} \Rightarrow \bar{\bar{x}} \pm \frac{3}{\sqrt{n}} \frac{\bar{S}}{c_4} \Rightarrow \bar{\bar{x}} \pm A_3 \bar{S}$$

$$LCL = \overline{\overline{x}} - A_3 \overline{S}$$

$$CL = \overline{\overline{x}}$$

$$ULC = \overline{\overline{x}} + A_3 \overline{S}$$

$$A_{3} = \frac{3}{c_{4}\sqrt{n}}$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

 $\eta$  = The number of data points

 $ar{x}=$  The mean of the  $x_i$ 

 $x_i = {}$  Each of the values of the data

# Xbar-S Control Charts - μ, σ are not known

#### **S Chart:**

$$LCL = B_{3}^{-}S$$

$$CL = S$$

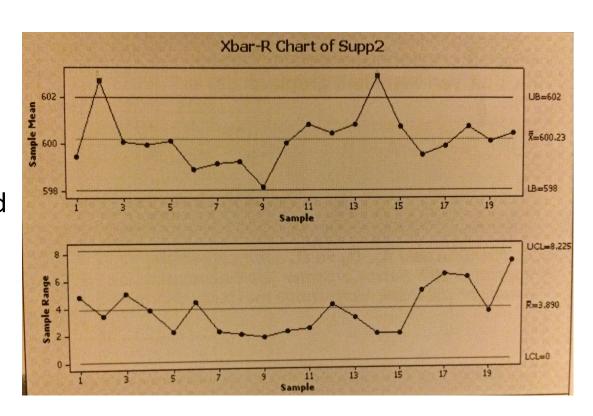
$$UCL = B_{4}^{-}S$$

$$B_3 = 1 - \frac{3}{c_4} \sqrt{1 - c_4^2}$$

$$B_4 = 1 + \frac{3}{c_4} \sqrt{1 - c_4^2}$$

## Example 1:

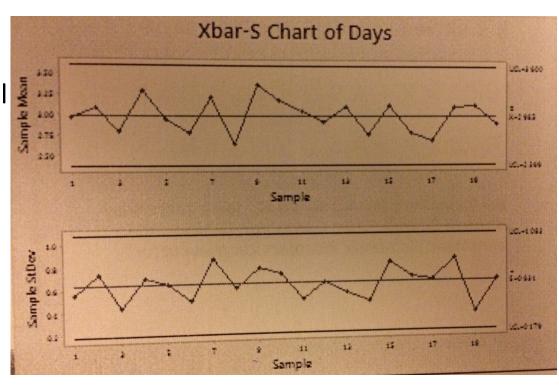
In a car production facility, the acceptance limit for shaft's length is 600 mm±2 mm. We are looking for a new supplier for this equipment. We have selected 100 samples with systematic sampling from shafts from new suppliers. After analyzing this data set of 100 data, the resulting chart is as on the right.



## Example 2:

A logistics company follows the delivery time that is critical for the customer. The time of arrival to the customer of 10 randomly selected shipments is taken as data for 20 days during the shipments are made.

After analyzing this data set of 200 data, the resulting chart is as on the right.



## Example 3:

**Xbar Chart** 

UCL = 5,64645

CL = 0.54

LCL = -4,56645

Range Chart

UCL = 18,71775

CL = 8,85

LCL = 0.0

	Samples							
Sample Number	1	2	3	4	5	Total	Х	R
1	-2	-6	4	2	0	-2	-0,4	10
2	3	-5	0	1	-2	-3	-0,6	8
3	6	7	3	-4	-1	11	2,2	11
4	-1	5	-1	6	3	12	2,4	7
5	-3	2	1	-4	-6	-10	-2	8
6	-3	3	2	8	4	14	2,8	11
7	7	9	0	5	6	27	5,4	9
8	1	8	-4	-6	4	3	0,6	14
9	-4	4	3	-6	0	-3	-0,6	10
10	-1	0	-7	-3	1	-10	-2	8
11	2	-1	3	-1	1	4	0,8	4
12	-7	-3	4	1	0	-5	-1	11
13	5	3	-1	-6	4	5	1	11
14	-2	1	-3	-1	5	0	0	8
15	-6	-2	0	3	1	-4	-0,8	9
16	-3	2	-1	5	0	3	0,6	8
17	6	5	2	-1	-2	10	2	8
18	-3	2	-4	0	2	-3	-0,6	6
19	-5	-3	0	2	4	-2	-0,4	9
20	6	-1	0	3	-1	7	1,4	7
Average							0,5 4	8,8 5

## Example 4:

The weight of a packaged foodstuff will be investigated with the help of control charts. For this purpose, the measurement values, means and range values of 25 units taken in regular intervals are given in the table. Based on this information, draw and interpret the control charts.

1       500, 4       500, 2       501, 3       2003       500,75       1,1         2       500, 1       500, 500, 500, 500, 500, 3       8       2       1       2       1999,3       499,83       1         499, 499, 500, 3       500, 499, 499, 500, 3       500, 499, 500, 500, 499, 500, 500, 499, 500, 500, 498, 499, 500, 500, 500, 500, 500, 500, 500, 5	Sample number	1	2	3	4	Total	Х	R
2 500 500, 1 500, 500, 500, 1 2001,1 500,28 1  499, 499, 500, 500, 500, 499, 499, 5 1 2 9 1 1999,3 499,83 1,3  500, 499, 499, 500, 500, 499, 1 1999,3 499,83 0,9  5 1 2 9 1 1999,3 499,83 0,9  5 500, 498, 499, 500, 500, 7 9 2 1 9 2005,1 501,28 1,2  500, 501, 502, 500, 500, 7 9 2 1 9 2005,1 501,28 1,2  499, 500, 500, 500, 500, 500, 8 8 1 2 4 2000,5 500,13 0,6  498, 498, 500, 500, 500, 500, 10 6 7 3 2 2000,8 500,20 1,5  499, 500, 500, 500, 500, 1 200,6 500,15 0,8  499, 500, 500, 500, 500, 1 200,6 500,15 0,8  499, 500, 499, 500, 500, 1 200,6 500,15 0,8  499, 500, 499, 500, 500, 1 200,6 500,15 0,8  499, 500, 499, 500, 500, 1 200,6 500,15 0,8  499, 498, 499, 499, 1 200,6 500,15 0,8  499, 498, 499, 499, 1 2 1996,2 499,05 0,7  14 500, 500, 501, 501, 501, 501, 500, 500,	1		-		I -	2003	500.75	1.1
2       500       1       500       501       2001,1       500,28       1         499,       499,       500,       500,       500,       499,33       1         499,       500,       500,       499,       199,7       499,93       1,3         500,       499,       499,       500,       1999,7       499,83       0,9         500,       498,       499,       500,       1999,3       499,83       0,9         500,       498,       499,       500,       29       1       1999,3       499,83       0,9         500,       500,       498,       499,       500,       500,       2       2       2       2       2       3       1998,4       499,83       0,9       0,9       2       2       2       3       1999,3       499,83       0,9       0,9       2       2       2       3       1999,3       499,83       0,9       0,9       2       3       1,2       2       2		-						,_
3       8       2       1       2       1999,3       499,83       1         4       3       6       3       5       1999,7       499,93       1,3         5       1       2       9       1       1999,3       499,83       0,9         5       1       2       9       1       1999,3       499,83       0,9         6       2       1       8       3       1998,4       499,83       0,9         6       2       1       8       3       1998,4       499,60       2,2         500,       500,       500,       500,       500,       500,       500,       501,28       1,2         499,       500,       500,       500,       500,       500,13       0,6         498,       498,       500,       500,       500,       500,13       0,6         9       2       6       1       9       1997,8       499,45       2,7         10       6       7       3       2       2000,8       500,20       1,5         11       8       1       6       1       2000,6       500,15       0,8	2	500		500	501	2001,1	500,28	1
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5         1         2         9         1         1999,3         499,83         0,9           6         2         1         8         3         1998,4         499,60         2,2           7         9         2         1         9         2005,1         501,28         1,2           8         499,         500,         500,         500,         500,         500,13         0,6           9         2         6         1         9         1997,8         499,45         2,7           10         6         7         3         2         2000,8         500,20         1,5           499,         500,         500,         500,         499,         500,20         1,5           499,         500,         500,         500,         500,         500,         500,15         0,8           12         7         1         6         2         1999,6         499,90         0,6           499,         498,         499,         499,         2         1996,2         499,05         0,7           13         3         6         1         2         1996,2         499,05         0,7	4	-	-	-		1999,7	499,93	1,3
6       500, 498, 2 1 8 3 1998,4 499,60       2,2         500, 501, 502, 500, 9 2 1 9 2005,1 501,28 1,2         499, 500, 500, 500, 8 1 2 4 2000,5 500,13 0,6         9       2 6 1 9 1997,8 499,45 2,7         10       6 7 3 2 2000,8 500,20 1,5         499, 500, 500, 500, 6 1 2000,6 500,15 0,8       500, 500, 500, 500, 500, 500, 500, 12         11       499, 500, 499, 500, 499, 500, 7 1 6 2 1999,6 499,90 0,6         12       7 1 6 2 1999,6 499,90 0,6         499, 498, 499, 499, 13 3 6 1 2 1996,2 499,05 0,7         14       2 9 3 6 2004 501,00 1,4         500, 500, 501, 500, 500, 500, 500, 500,	_	-		-		1000.3	400.03	0.0
6         2         1         8         3         1998,4         499,60         2,2           7         9         2         1         9         2005,1         501,28         1,2           8         499, 500, 500, 500, 500, 8         500, 500, 500, 500, 500, 500, 500, 500,	5			_		1999,3	499,83	0,9
7       9       2       1       9       2005,1       501,28       1,2         499, 8       500, 500, 2       500, 500, 500, 498, 2       6       1       9       1997,8       499,45       2,7         500, 500, 500, 6       7       3       2       2000,8       500,20       1,5         499, 500, 500, 8       1       6       1       2000,6       500,15       0,8         12       7       1       6       2       1999,6       499,90       0,6         499, 499, 499, 499, 13       3       6       1       2       1996,2       499,05       0,7         14       2       9       3       6       2004       501,00       1,4         500, 500, 500, 500, 500, 500, 500, 500,	6	-			I -	1998,4	499,60	2,2
8       499, 8       500, 2       500, 4       2000,5       500,13       0,6         9       498, 498, 500, 500, 500, 500, 500, 7       500, 500, 500, 500, 500, 500, 500, 10       499, 500, 500, 500, 500, 500, 10       500, 500, 500, 500, 500, 500, 10       500, 500, 500, 500, 500, 500, 10       499, 500, 499, 500, 60, 10       499, 60, 2       1999,6       499,90       0,6         499, 498, 499, 499, 13       3       6       1       2       1996,2       499,05       0,7         14       2       9       3       6       2004       501,00       1,4         500, 500, 500, 500, 500, 500, 500, 500,		500,	501,	502,	500,			
8       8       1       2       4       2000,5       500,13       0,6         498, 498, 2       6       1       9       1997,8       499,45       2,7         500, 500, 500, 6       7       3       2       2000,8       500,20       1,5         499, 500, 8       1       6       1       2000,6       500,15       0,8         499, 500, 499, 500, 7       1       6       2       1999,6       499,90       0,6         499, 498, 499, 499, 3       6       1       2       1996,2       499,05       0,7         500, 500, 500, 501, 3       6       2004       501,00       1,4         500, 500, 500, 500, 500, 500, 500, 500,	7	9	2	1	9	2005,1	501,28	1,2
9       498, 2       6       1       9       1997,8       499,45       2,7         10       500, 500, 500, 6       500, 500, 499, 3       2       2000,8       500,20       1,5         11       8       1       6       1       2000,6       500,15       0,8         12       7       1       6       2       1999,6       499,90       0,6         13       3       6       1       2       1996,2       499,05       0,7         14       2       9       3       6       2004       501,00       1,4         500, 500, 501, 9       2       3       1       2003,5       500,88       0,9         500, 500, 500, 500, 500, 500, 500,       500, 500, 500, 500, 500, 500, 500,       500, 500, 500, 500, 500, 500, 500, 500,		499,	500,	500,	500,			
9       2       6       1       9       1997,8       499,45       2,7         10       6       7       3       2       2000,8       500,20       1,5         499,       500,       500,       500,       500,       500,       500,15       0,8         499,       500,       499,       500,       2       1999,6       499,90       0,6         499,       498,       499,       499,       499,90       0,6         499,       3       6       1       2       1996,2       499,05       0,7         500,       500,       501,       501,       501,       501,       501,00       1,4         500,       501,       3       6       2004       501,00       1,4         500,       500,       500,       500,       500,       500,88       0,9         500,       500,       500,       500,       500,       500,       500,       500,	8	8	1	2	4	2000,5	500,13	0,6
10       500, 6       7       3       2       2000,8       500,20       1,5         499, 8       500, 500, 6       500, 500, 6       1       2000,6       500,15       0,8         12       7       1       6       2       1999,6       499,90       0,6         499, 499, 499, 13       3       6       1       2       1996,2       499,05       0,7         500, 500, 500, 3       501, 501, 3       6       2004       501,00       1,4         500, 500, 501, 3       500, 3       501, 3       1       2003,5       500,88       0,9         500, 500, 500, 500, 500, 500, 500, 500,		-	-	500,				
10     6     7     3     2     2000,8     500,20     1,5       499, 500, 8     1     500, 6     1     2000,6     500,15     0,8       499, 500, 7     499, 6     2     1999,6     499,90     0,6       499, 498, 499, 499, 13     499, 499, 499, 501, 501, 501, 3 6     2     1996,2     499,05     0,7       500, 500, 500, 3     501, 501, 501, 3 6     2004     501,00     1,4       500, 500, 500, 500, 500, 500, 500, 500,	9	2	6	1	9	1997,8	499,45	2,7
11     499, 8     500, 6     500, 6     1     2000,6     500,15     0,8       12     7     1     6     2     1999,6     499,90     0,6       13     3     6     1     2     1996,2     499,05     0,7       14     2     9     3     6     2004     501,00     1,4       500, 500, 501, 9     501, 501, 3     501, 501, 3     500,88     0,9       500, 500, 500, 500, 500, 500, 500, 500,		-			I -			
11     8     1     6     1     2000,6     500,15     0,8       499, 500, 7     499, 6     2     1999,6     499,90     0,6       499, 498, 499, 499, 13     499, 499, 501, 501, 501, 3 6     2     1996,2     499,05     0,7       500, 500, 500, 3     501, 501, 501, 3 6     2004     501,00     1,4       500, 500, 500, 500, 500, 500, 500, 500,	10	6	7	3	2	2000,8	500,20	1,5
12       499, 500, 1       499, 6       2       1999,6       499,90       0,6         499, 498, 499, 3       6       1       2       1996,2       499,05       0,7         500, 500, 500, 2       501, 501, 3       6       2004       501,00       1,4         500, 500, 501, 3       500, 501, 3       1       2003,5       500,88       0,9         500, 500, 500, 500, 500, 500, 500, 500,	11	-			I -	2000 6	E00 1E	۸.
12     7     1     6     2     1999,6     499,90     0,6       499, 498, 3     499, 499, 499, 499, 500, 500, 500, 500, 3     6     1     2     1996,2     499,05     0,7       500, 500, 500, 500, 500, 500, 500, 500,						2000,0	300,13	0,0
13     3     6     1     2     1996,2     499,05     0,7       500,     500,     501,     501,     501,     501,00     1,4       500,     501,     500,     501,     501,     500,88     0,9       500,     500,     500,     500,     500,     500,     500,       500,     500,     500,     500,     500,     500,     500,	12	-	-	-		1999,6	499,90	0,6
13     3     6     1     2     1996,2     499,05     0,7       500,     500,     501,     501,     501,     501,     501,00     1,4       500,     501,     500,     501,     501,     500,88     0,9       500,     500,     500,     500,     500,     500,     500,		499,	498,	499,	499,			
14     2     9     3     6     2004     501,00     1,4       500,     501,     500,     501,     2003,5     500,88     0,9       500,     500,     500,     500,     500,     500,     500,	13		6	1	2	1996,2	499,05	0,7
15 9 2 3 1 2003,5 500,88 0,9 500, 500, 500, 500,		500,	500,	501,	501,			
15 9 2 3 1 2003,5 500,88 0,9 500, 500, 500, 500,	14	2	9	3	6	2004	501,00	1,4
500, 500, 500, 500,								
	15	9	2	3	1	2003,5	500,88	0,9
	1						57	
16 1 3 2 1 2000,7 500,18 0,2	16	1	3	2	1	2000,7	500,18	0,2

## **Example 4:**

#### **Xbar Chart:**

$$LCL = X - A_2R$$

$$= CL = X$$

$$= UCL = X + A_2R$$

$$A_2 = \frac{3}{d_2 \sqrt{n}}$$

#### R Chart:

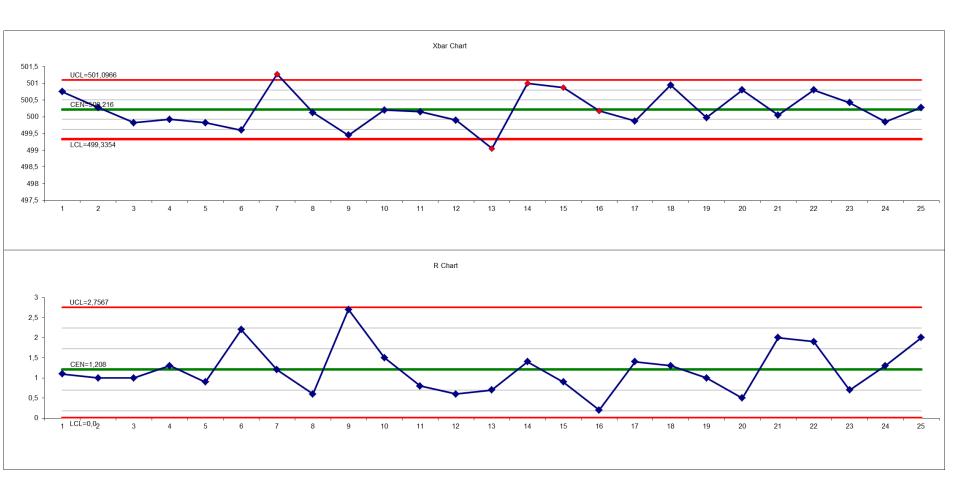
$$LCL = D_{3}\overline{R}$$

$$CL = \overline{R}$$

$$UCL = D_{4}\overline{R}$$

$$D_3 = 1 - \frac{d_3}{d_2}$$

$$D_4 = 1 + \frac{d_3}{d_2}$$



## **Example 5:**

With respect to a production process, it is known that the average of the main mass is 10.1 cm and the main mass standard deviation is 0.04 cm. Under the assumption that samples of 5 units are to be selected during the manufacturing inspection, calculate the upper and lower values of the middle line of the X and R control charts using these data.

## Example 5:

Control standards are known. In this case;

**Xbar Chart Formulas:** 

$$LCL = \mu - A\sigma$$

$$CL = \mu$$

$$UCL = \mu + A\sigma$$

R Chart Formulas:

$$LCL = D_{1}\sigma$$

$$CL = d_{2}\sigma$$

$$ULC = D_{2}\sigma$$

$$D_{1} = d_{2} - 3d_{3}$$

$$D_{2} = d_{2} + 3d_{3}$$

## Example 6:

The pull strength of a wire bonded lead for an integrated circuit is monitored. The table on the right provides data for 20 samples each of size three.

- a. Use all the data to determine trial control limits. Construct the control limits and plot the data.
- b. If necessary, revise your control limits assuming that any samples that plot outside of the control limits can be eliminated.

Sample	X1	X2	Х3
1	15,40	15,60	15,30
2	15,40	17,10	15,20
3	16,10	16,10	13,50
4	13,50	12,50	10,20
5	18,30	16,10	17,00
6	19,20	17,20	19,40
7	14,10	12,40	11,70
8	15,60	13,30	13,60
9	13,90	14,90	15,50
10	18,70	21,20	20,10
11	15,30	13,10	13,70
12	16,60	18,00	18,00
13	17,00	15,20	18,10
14	16,30	16,50	17,70
15	8,40	7,70	8,40
16	11,10	13,80	11,90
17	16,50	17,10	18,50
18	18,00	14,10	15,90
19	17,80	17,30	12,00
20	11,50	10,80	11,20

# Example 6:

Sample	X1	X2	Х3	X	R
1	15,40	15,60	15,30	15,43	0,30
2	15,40	17,10	15,20	15,90	1,90
3	16,10	16,10	13,50	15,23	2,60
4	13,50	12,50	10,20	12,07	3,30
5	18,30	16,10	17,00	17,13	2,20
6	19,20	17,20	19,40	18,60	2,20
7	14,10	12,40	11,70	12,73	2,40
8	15,60	13,30	13,60	14,17	2,30
9	13,90	14,90	15,50	14,77	1,60
10	18,70	21,20	20,10	20,00	2,50
11	15,30	13,10	13,70	14,03	2,20
12	16,60	18,00	18,00	17,53	1,40
13	17,00	15,20	18,10	16,77	2,90
14	16,30	16,50	17,70	16,83	1,40
15	8,40	7,70	8,40	8,17	0,70
16	11,10	13,80	11,90	12,27	2,70
17	16,50	17,10	18,50	17,37	2,00
18	18,00	14,10	15,90	16,00	3,90
19	17,80	17,30	12,00	15,70	5,80
20	11,50	10,80	11,20	11,17	0,70
				15,09	2,25

# **Constants Used in Control Chart Drawing**

Subgroup Size	Coefficients for Limits		Factor for center line		Coefficient	cients for Limits		
	Α	A1	A2	d2	D1	D2	D3	D4
2	2,121	3,761	1,88	1,128	0	3,686	0	3,267
3	1,732	2,394	1,023	1,693	0	4,358	0	2,575
4	1,5	1,88	0,729	2,059	0	4,698	0	2,282
5	1,342	1,596	0,577	2,326	0	4,918	0	2,115
6	1,225	1,41	0,483	2,534	0	5,078	0	2,004
7	1,134	1,277	0,419	2,704	0,205	5,203	0,076	1,924
8	1,061	1,175	0,373	2,847	0,387	5,307	0,136	1,864
9	1	1,094	0,337	2,97	0,546	5,394	0,184	1,816
10	0,949	1,028	0,308	3,078	0,687	5,469	0,223	1,777
11	0,905	0,973	0,285	3,173	0,812	5,534	0,256	1,744
12	0,866	0,925	0,266	3,258	0,924	5,592	0,284	1,716
13	0,832	0,884	0,249	3,336	1,026	5,646	0,308	1,692
14	0,80264	0,848	0,235	3,407	1,121	5,693	0,329	1,671
15	0,775	0,816	0,223	3,472	1,207	5,737	0,348	1,652