Scheduling

Notes 1

Scheduling and Sequencing Definitions Sequencing

- Sequencing means for each machine one has to establish the order in which the jobs waiting in the queue in front of that particular machine have to be processed.
- Sequencing is defined as the order in which the jobs (tasks) are processed through the machines (resources).
- Sequence refers to the order of carrying out activities.

Scheduling

- Scheduling is defined as assigning each operation of each job a start time and a completion time on a time scale of machine within the precedence relations.
- Scheduling is the timing (or timetable) to carry out the activities .

Scheduling

...Scheduling is the process of organizing, choosing, and timing resource usage to carry out all the activities necessary to produce the desired outputs at the desired times, while satisfying a large number of time and relationship constraints among the activities and the resources.

Scheduling

Production sequencing and scheduling is one of the most important activities in production planning and control. Morton and Pentico discussed how important the sequencing and scheduling role is,

...Sequencing and scheduling are forms of decision-making which play a crucial role in manufacturing as well as in service industries. In the current competitive environment, effective sequencing and scheduling has become a necessity for survival in the marketplace. Companies have to meet shipping dates committed to the customers, as failure to do so may result in a significant loss of good will. They also have to schedule activities in such a way as to use the resources available in an efficient manner.

Scheduling

Sequencing and scheduling problems occur in different industries and circumstances. The following are some examples of different situations which need sequencing or scheduling:

parts waiting for processing in a manufacturing plant;

aircraft waiting for landing clearance at an airport;

class scheduling in a school,

patients waiting in a Doctor's office;

Scheduling

- Scheduling is a decision-making process that is used on a regular basis in many manufacturing and services industries. It deals with the allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives.
- The objectives can also take many different forms. One objective may be the minimization of the completion time of the last task and another may be the minimization of the number of tasks completed after their respective due dates.

Assumptions

A variety of assumptions is made in sequencing and scheduling problems The nature of these assumptions depends on the sequencing environment. The following list contains typical assumptions generally applied to scheduling problem with variations depending on the situation.

- Each job is an entity and each job consists of several operations.
- There is no interruption. Each operation must be completed before any other operation can begin on that machine.
- Each job has m operations and there is 1 for each machines. Each machine processes all the jobs assigned to it. Two operations of a job can not be processed on the same machine.
- There is no postponement. Each job ,once started, on a machine should be performed up to the completion on that machine.
- Processing times are independent of the order in which jobs are processed.
- The time required to transfer the jobs between machines is negligible.
- Buffer stocks are allowed. It means that the jobs can wait until the next machine will be available.

Assumptions

- There is only one of each type of machine.
- Machines can be idle.
- Only one job can be processed on a given machine at a time.
- Machines never break down and they are ready during sequencing period.
- Technological constraints are known and fixed.
- There is no randomness:
    - *The number of jobs are known and fixed.*
    - *The number of machines are known and fixed.*
    - *Processing times are known and fixed.*
    - *Set-up times are known and fixed.*
    - *All quantitative values to define a problem are known and fixed.*

Scheduling Environment

- In scheduling problems **m means the number of machines** and **n means the number of jobs.**

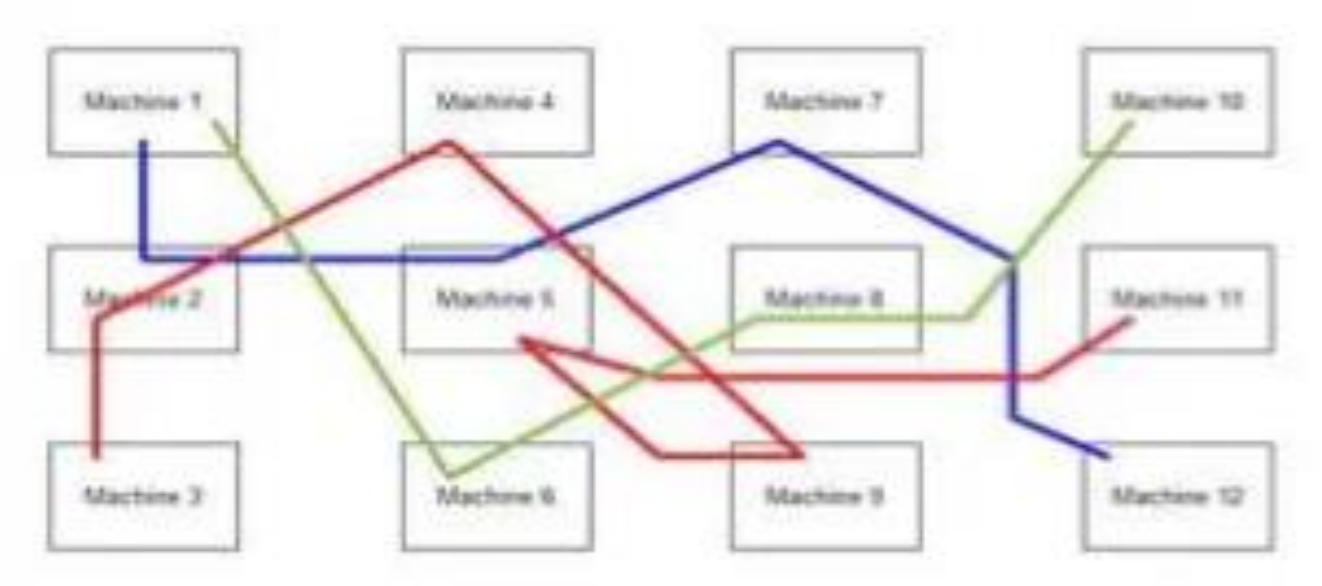
Single machine shop: one machine and n jobs to be processed.

Job shop: each job has its flow pattern and a subset of these jobs can visit each machine twice or more often. Multiple entries and exits.

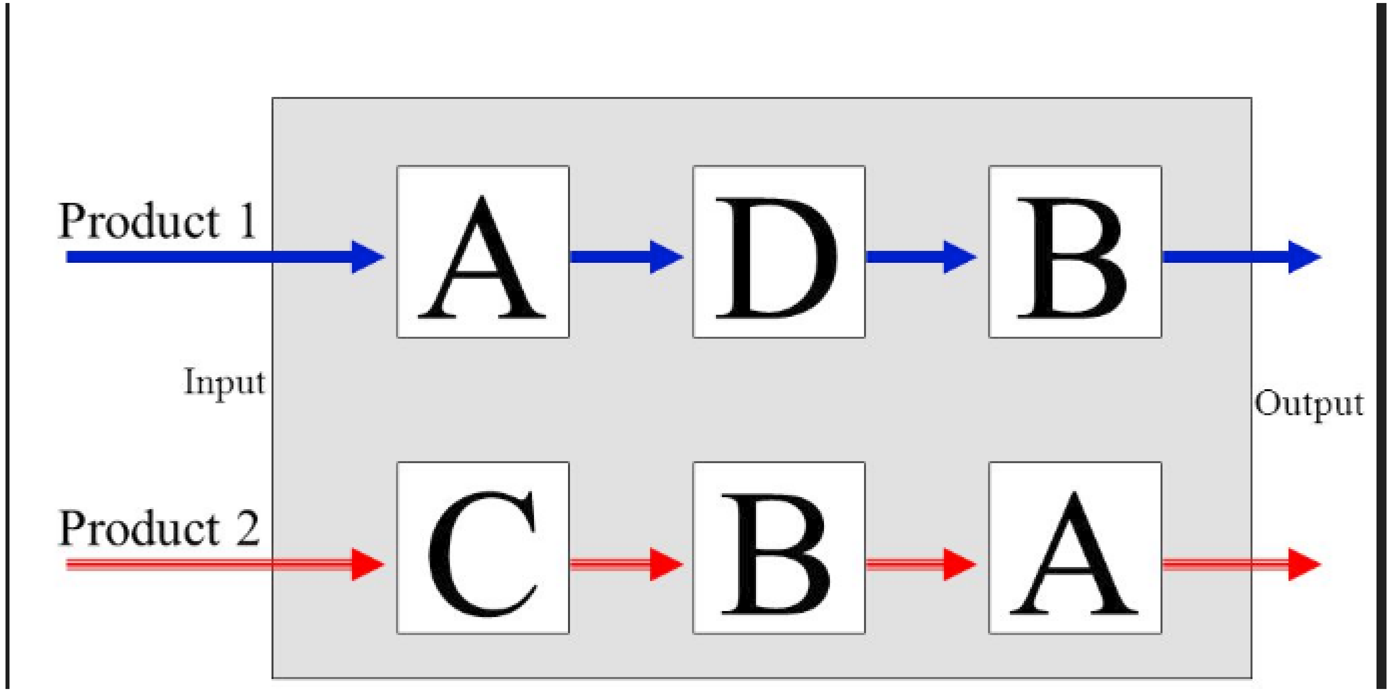
- Flow shop: A flow shop problem exists when all the jobs share the same processing order on all the machines.

Open shop: there are m machines and there is no restriction in the routing of each job through the machines. In other words, there is no specified flow pattern for any job.

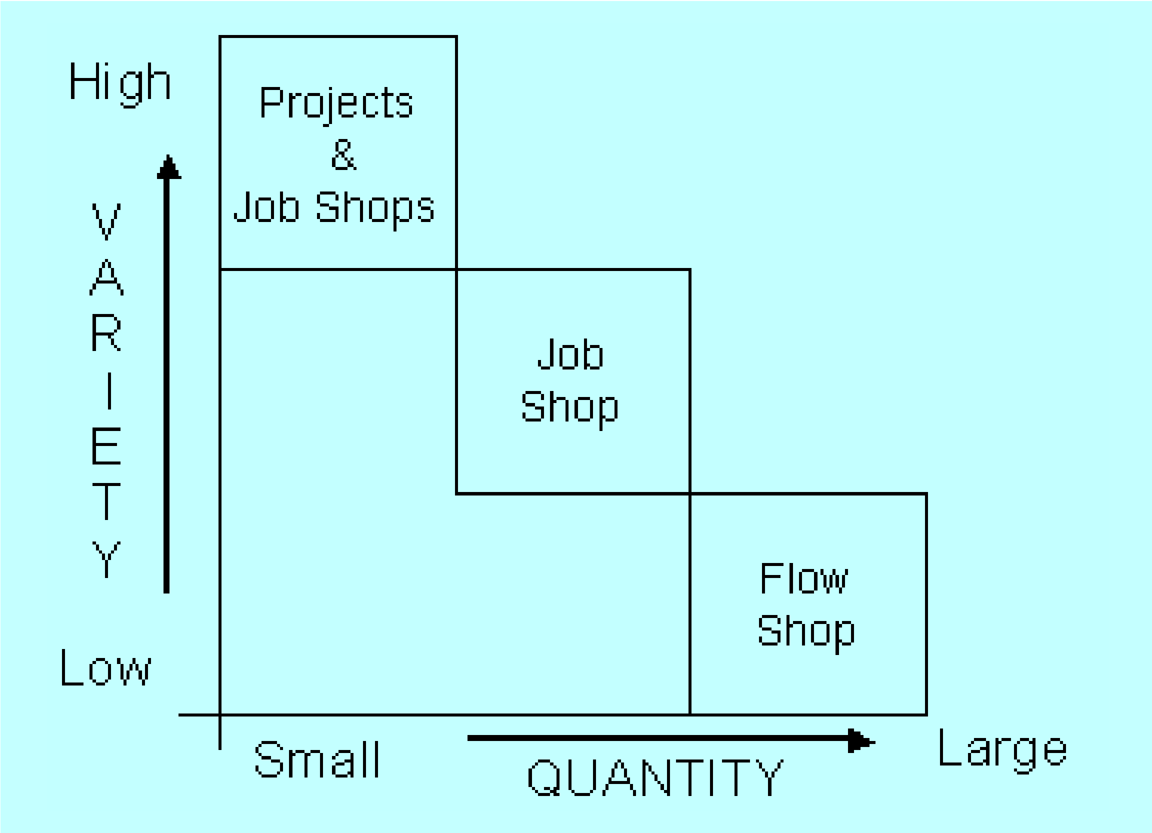
Job Shop


Machine 1
Machine 4
Machine 7
Machine 10
Machine 2
Machine 5
Machine 8
Machine 11
Machine 3
Machine 6
Machine 9
Machine 12

Flow Shop



Job shop vs. Flow shop



Notations

- P_{ij} = Processing time of job i on machine j
- d_i = Due date of job i. It is the date the job is promised to the customer. Completion of a job after its due date can be allowed, but then a penalty is incurred.
- r_i = Release date (Ready date). It is the time the job arrives at the system, i.e., the earliest time at which job i can start its processing.
- w_i = The weight of job i is basically a priority factor, denoting the importance of job i relative to the other jobs.
- W_i = Waiting time of job i.
- W_{ik} = Waiting time of job i before the k th operation.

Notations

- C_i = Completion time of job i. The time job i exits the system.
- C_{ij} = The completion time of the operation of job i on machine j.
- F_i = Flow time of job i. It is time spent by job i in the shop.

$$F_i = C_i – r_i$$

- L_i = Lateness of job i.

$$L_i = C_i - d_i$$

- L_i is positive when job j is completed late and **negative** when it is completed early.

Notations

- T_i = Tardiness of job i.
- $T_i = \max \{L_i, 0\}$
- The difference between the tardiness and the lateness lies in the fact that the tardiness never is negative.
- E_i = Earliness of job i.
- $E_i = \max \{-L_i, 0\}$

Notations

- C_{max} = Makespan. Maximum completion time. The makespan, defined as max(C1,...,Cn), is equivalent to the completion time of the last job to leave the system. A minimum makespan usually implies a good utilization of the machine(s). It is the total amount of time required to completely process all the jobs.
- F_{max} = Maximum flow time.
- L_{max} = Maximum lateness.
- I_j = Idle time on M_j
- $I_j = C_{max} - \sum_{i=1}^{n} P_{ij}$

Notations for Common Machine Environment

Environment Name	Symbol / Notation	Description
Single Machine	1	One machine
Identical Machines in Parallel	P_m	P : Parallel machines “m” : number of machines
Parallel machines with different speeds	Q_m	Q : Parallel machines with different speeds “m” : number of machines
Flow Shop	F_m	F : Flow shop “m” : number of machines
Job Shop	J_m	J : Job shop “m” : number of machines
Open Shop	O_m	O: Open shop “m” : number of machines
Flexible flow shop	FFc	Instead of m machines in series there are c stages in series with at each stage a number of identical machines in parallel.

Notations for Common Processing Characteristics

Term	Notation	Description
Preemptions	*Prmp*	A job may interrupted during its processing due to arrival of a high priority job
Precedence constraints	*Prec*	When one job depends on the completion of another job, this implies precedence constraint
Breakdowns	*Brkdwn*	This implies that machines are not continuously available for processing
Recirculation	*Recrc*	When a job visits a machine more than once
Permutation	*Prmu*	The processing order of all jobs on one machine is maintained throughout the shop

Notations for Common Processing Characteristics

Term	Notation	Description
Machine eligibility restrictions	Mi	When the Mi is present, not all m machines are capable of processing job i.
Blocking	block	Blocking implies that the completed job has to remain on the upstream machine preventing (i.e., blocking) that machine from working on the next job.
No-wait	nwt	Jobs are not allowed to wait between two successive machines. This implies that the starting time of a job at the first machine has to be delayed to ensure that the job can go through the flow shop without having to wait for any machine

Notations (based on inventory and machine utilization)

1)	Average number of jobs waiting for machines	$\overline{N}_w$
2)	Average number of unfinished jobs	$\overline{N}_u$
3)	Average number of jobs completed	$\overline{N}_c$
4)	Average number of jobs actually being processed	$\overline{N}_p$
5)	Average number of machines idle	$\overline{I}$
6)	Maximum machine idle time	I_{max}
7)	Average utilization	$\overline{U} = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} / m.C_{max}$

Notations

- Nw(t) = Number of jobs that are waiting between the machines at time t.
- Np(t) = Number of jobs that are being processed at time t.
- Nc(t)= Number of jobs that are completed at time t.
- Nu(t)= Number of unfinished jobs at time t.

$$Nw(t)+Np(t)+Nc(t)= n$$

$$Nw(t)+ Np(t)= Nu(t)$$

$$Nu(0) = n$$

$$Nu(Cmax) = 0$$

Notations (based on completion times)

1)	Completion time of job i	C_i
2)	The total completion time	$\sum_{i=1}^{n} C_i$
3)	The total weighted completion time	$\sum_{i=1}^{n} w_i C_i$
4)	The total weighted waiting time	$\sum_{i=1}^{n} w_i \sum_{j=1}^{m} W_{ij}$
5)	Flow time of job i	$F_i = C_i - r_i$
6)	Maximum completion time (the schedule time. or makespan)	C_{max}
7)	The total flow time	$\sum_{i=1}^{n} F_i$
8)	The total weighted flow time	$\sum_{i=1}^{n} w_i F_i$
9)	Average flow time	$\overline{F}$
10)	Maximum flow time	F_{max}

Notations (based on completion times)

11)	Waiting time of job I	$Wi = F_i - \sum_{j=1}^{m} P_{ij}$.
12)	The total waiting time	$\sum_{i=1}^{n} \sum_{j=1}^{m} W_{ij}$
13)	Average completion time	$\overline{C}$
14)	Average waiting time	$\overline{W}$

Notations (based on due-dates)

1)	Lateness of job i	$L_i = C_i - d_i$
2)	The total lateness	$\sum_{i=1}^{n} L_i$
3)	The total weighted lateness	$\sum_{i=1}^{n} w_i L_i$
4)	Average lateness	$\overline{L}$

Notations (based on due-dates)

5)	Maximum lateness	$L_{max} = \max_{1,\dots,n} \{L_i\}$
6)	Tardiness of job I	$T_i = \max \{0, L_i\}$
7)	Earliness of job I	$E_i = \max \{0, -L_i\}$
8)	Maximum Earliness	$E_{max} = \max_{1,\dots,n} \{E_i\}$
9)	The total tardiness	$\sum_{i=1}^{n} T_i$
10)	The total weighted tardiness	$\sum_{i=1}^{n} w_i T_i$
11)	Average tardiness	$\overline{T}$
12)	Maximum tardiness	$T_{max} = \max_{1,\dots,n} \{T_i\}$

Goals

According to Baker (1974), there are three common types of decision-making goals in sequencing and scheduling problems: efficient utilization of resources; rapid response to demands, and close conformance to prescribed deadlines.

- Efficient utilization of resources. Schedule activities so as to maintain high utilization of labor, equipment and space.

$$\text{Minimize} \quad C_{max} \text{ or } \bar{I}, \text{ or maximize } \bar{N}_p \text{ or } \bar{U}.$$

- Rapid response to demands. Scheduling should allow jobs to be processed at rapid rate resulting in low levels of work-in-process inventory.

$$\text{Minimize} \quad \sum_{i=1}^{n} C_i;\ \sum_{i=1}^{n} F_i;\ \sum_{i=1}^{n} L_i;\ \sum_{i=1}^{n} \sum_{j=1}^{m} W_{ij};\ \bar{C};\ \bar{F};\ \bar{L},\ \bar{N}_w, \text{ or } \bar{W}.$$

Goals

- Close conformance to meet deadlines. Scheduling should ensure that due dates are met with every time through shorter lead times.

$$\text{Minimize} \qquad L_{max};\ T_{max};\ NT;\ \sum_{i=1}^{n} T_i\ ;\ \overline{T}\text{, or } \sum_{i=1}^{n} w_i T_i\ .$$

$$C_i = Fi + ri = Wi + \sum_{j=1}^{m} P_{ij} + r_i = L_i + d_i$$

Presentation of a Scheduling Problem

1. Type

- n/m/A/B
- n: Number of jobs
- m: Number of machines
- A: Type of facility (Machine environment)
- B: Objective function

Presentation of a Scheduling Problem

- 2/3/Fm/Cmax : It is a 2 jobs 3 machines flow-shop problem. The objective is to minimize the makespan.

Presentation of a Scheduling Problem

2. Type

- $\alpha \mid \beta \mid \gamma$
- The α field describes the machine environment.
- The β field provides details of processing characteristics and constraints.
- The γ field describes the objective function.

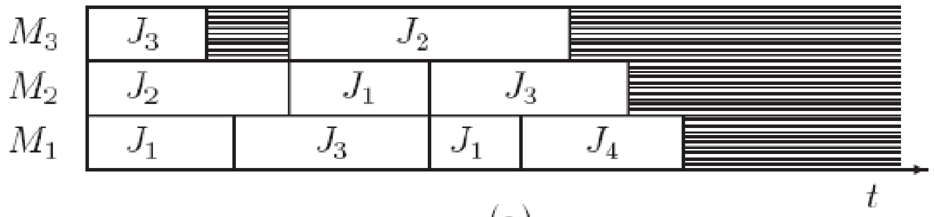
Presentation of a Scheduling Problem

- $FFc \mid r_i \mid w_iT_i$: It denotes a flexible flow shop. The jobs have release dates and the objective is the minimization of the total weighted tardiness.
- $Jm \mid\mid C_{max}$: It denotes a job shop problem with m machines. There is no recirculation, so a job visits each machine at most once. The objective is to minimize the makespan.

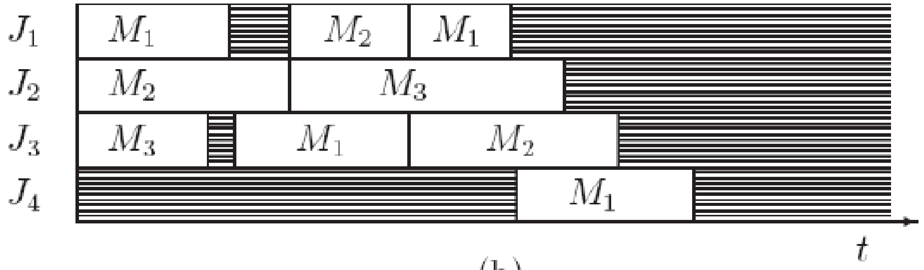
Visualization

Scheduling are represented by Gantt charts

(a) machine-oriented
(b) job-oriented



(a)



(b)

Resources

- Algorithms for Sequencing and Scheduling, Ibrahim M. Alharkan
- Scheduling, Theory, Algorithms and Systems, Michael L. Pinedo, Third Edition, Springer, 2008.
- Sıralama ve Programlama, Hüseyin Başlıgil
- Çizelgeleme Ders Notları, Prof. Dr. Hüseyin Başlıgil
- http://www.ques10.com/p/8177/assumptions-in-the-sequencing-problem-1/
- http://www.ques10.com/p/15411/what-are-assumptions-of-job-sequencing/
- Marco Chiarandini, Introduction to Scheduling: Terminology and Classification Lecture Notes, Department of Mathematics & Computer Science University of Southern Denmark, 2011.
- www.processmodel.com/blog/process-improvement-job-shop/
- http://nptel.ac.in/courses/Webcourse-contents/IIT-Delhi/project%20and%20production%20management/mod7/mod75.htm
- http://nptel.ac.in/courses/110106045/downloads/Module%207.pdf
- http://slideplayer.com/slide/5189836/