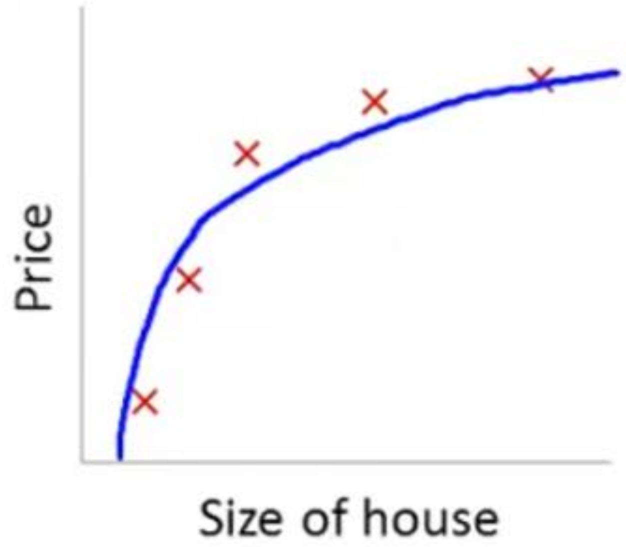
Cost Function

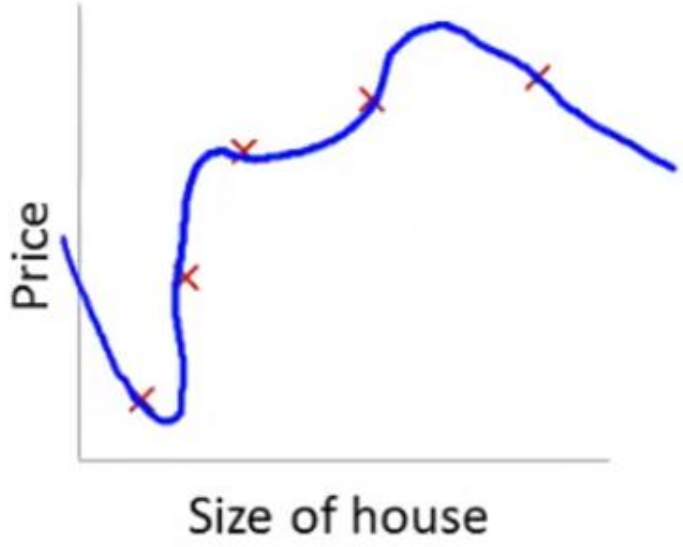
Solving the Problem of Overfitting

Regularization

Intuition



$$\theta_0 + \theta_1 x + \theta_2 x^2$$



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

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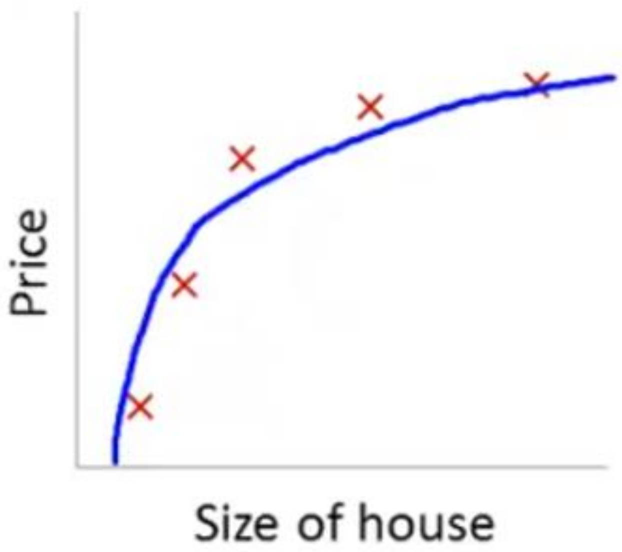

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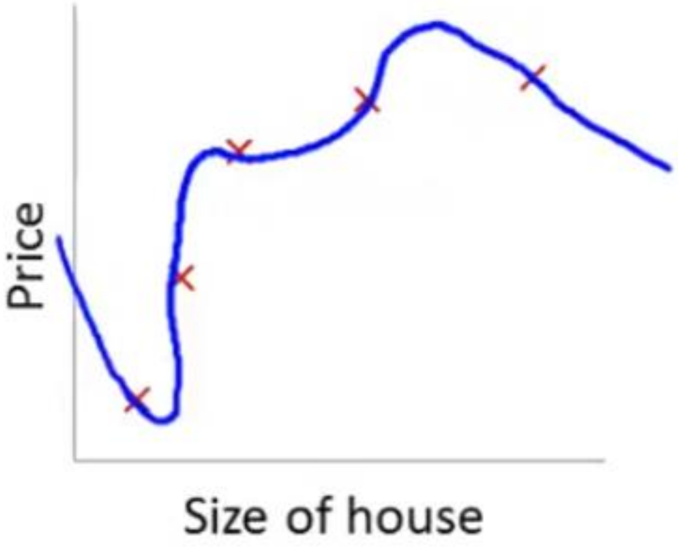
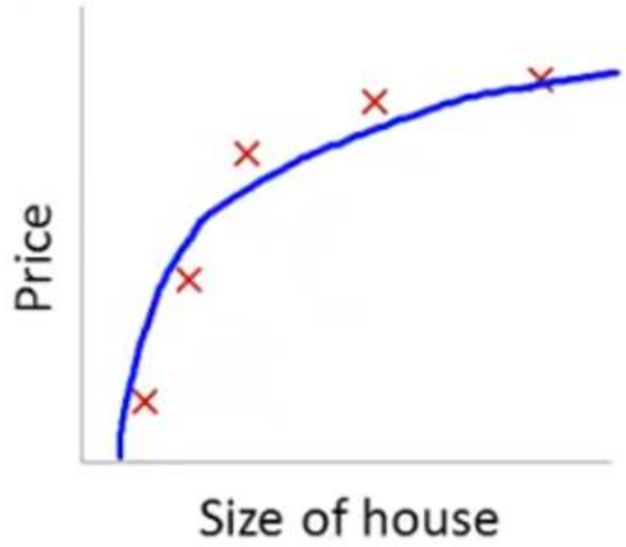


$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

Suppose we penalize and make θ_3, θ_4 really small.

$$\rightarrow \underline{\min_\theta \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2} + 1000\,\theta_3^2 + 1000\,\theta_4^2$$

Intuition



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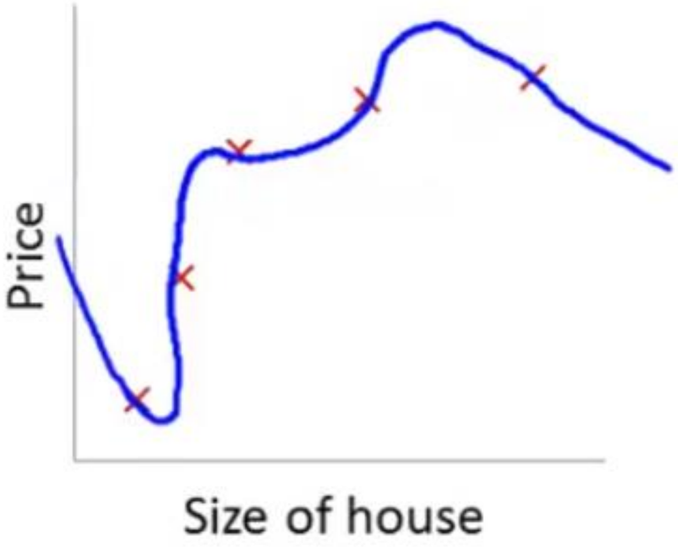

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$$\theta_3 \approx 0 \qquad \theta_4 \approx 0$$

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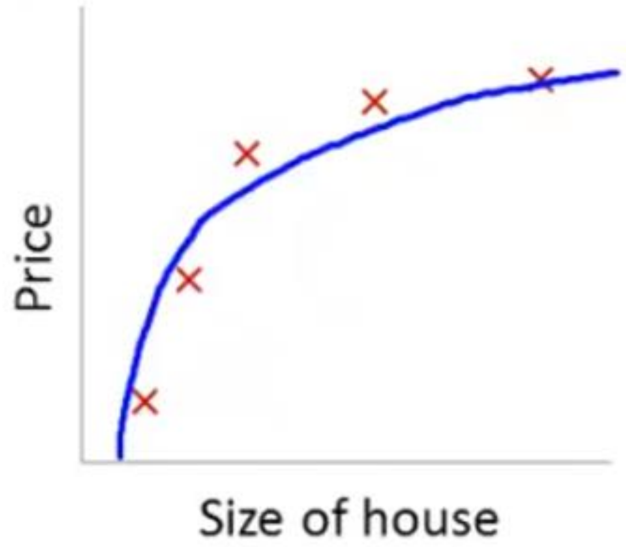

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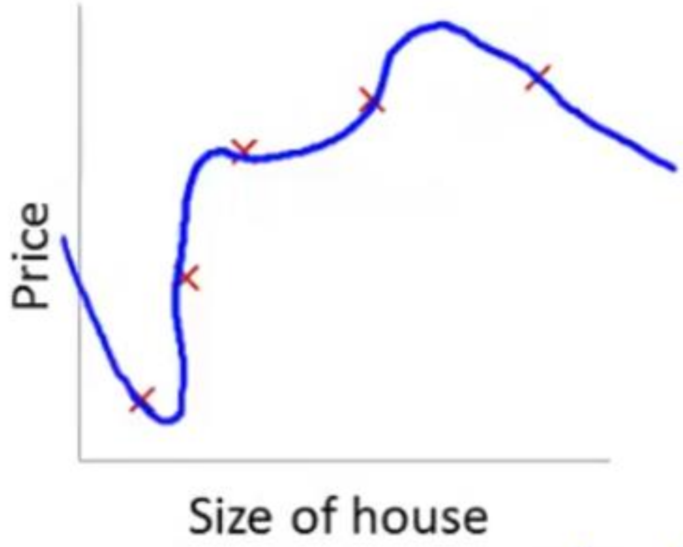

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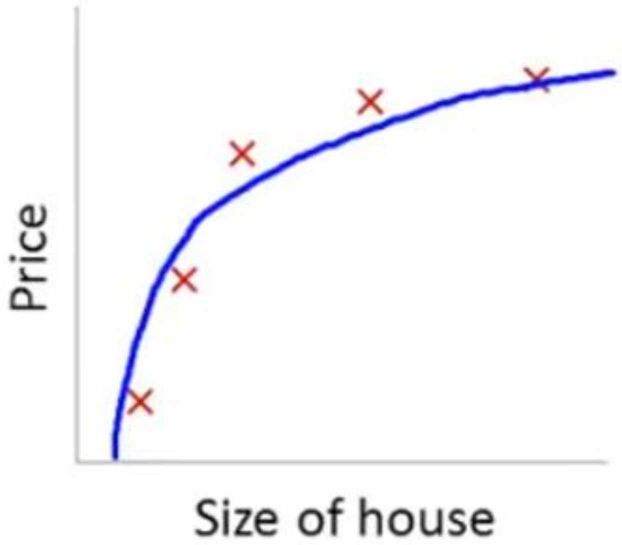

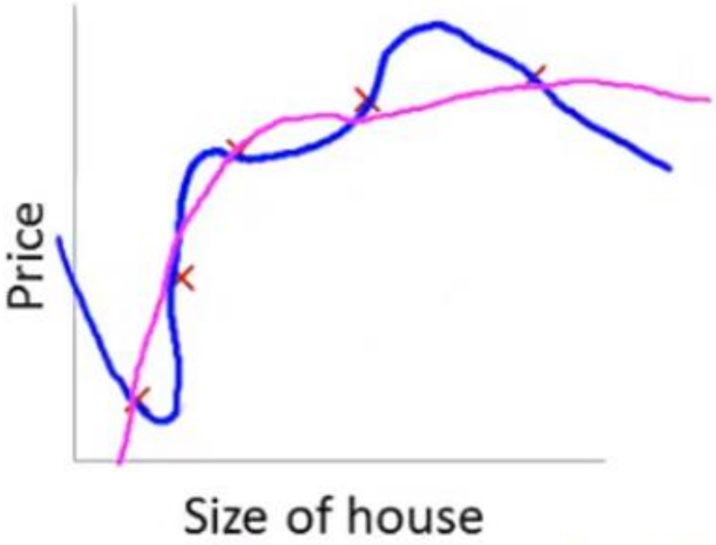

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$$\rightarrow \min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 + 1000\, \theta_3^2 + 1000\, \theta_4^2$$

$$\theta_3 \approx 0 \qquad \theta_4 \approx 0$$

Regularization.

Small values for parameters $\theta_0, \theta_1, \ldots, \theta_n$

- "Simpler" hypothesis
- Less prone to overfitting

θ_3, θ_4

≈ 0

Regularization.

Small values for parameters $\boxed{\theta_0, \theta_1, \dots, \theta_n}$

— "Simpler" hypothesis

— Less prone to overfitting

$\boxed{\theta_3, \theta_4} \approx 0$

Housing:

— Features: $x_1, x_2, \dots, x_{100}$

— Parameters: $\theta_0, \theta_1, \theta_2, \dots, \theta_{100}$

Regularization.

Small values for parameters $\boxed{\theta_0, \theta_1, \ldots, \theta_n}$

- "Simpler" hypothesis
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Housing:

- Features: $x_1, x_2, \ldots, x_{100}$
- Parameters: $\theta_0, \theta_1, \theta_2, \ldots, \theta_{100}$

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \underbrace{\sum_{i=1}^{n} \theta_j^2} \right]$$

$\theta_1, \theta_2, \theta_3, \ldots, \theta_{100}$

Regularization.

Small values for parameters $\boxed{\theta_0, \theta_1, \dots, \theta_n}$

- "Simpler" hypothesis
- Less prone to overfitting

$\boxed{\theta_3, \theta_4}$ ≈ 0

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- Features: $x_1, x_2, \dots, x_{100}$
- Parameters: $\theta_0, \theta_1, \theta_2, \dots, \theta_{100}$

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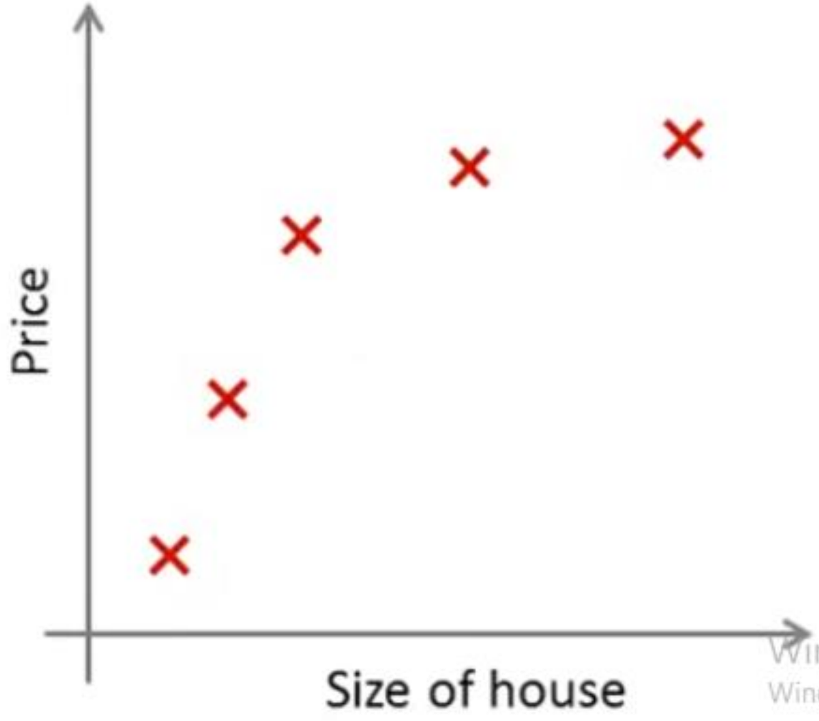
$\cancel{\theta_0}, \theta_1, \theta_2, \theta_3, \dots, \theta_{100}$ $\cancel{\theta_0}$

Regularization.

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{n}\theta_j^2\right]$$

regularization parameter

$$\min_\theta J(\theta)$$



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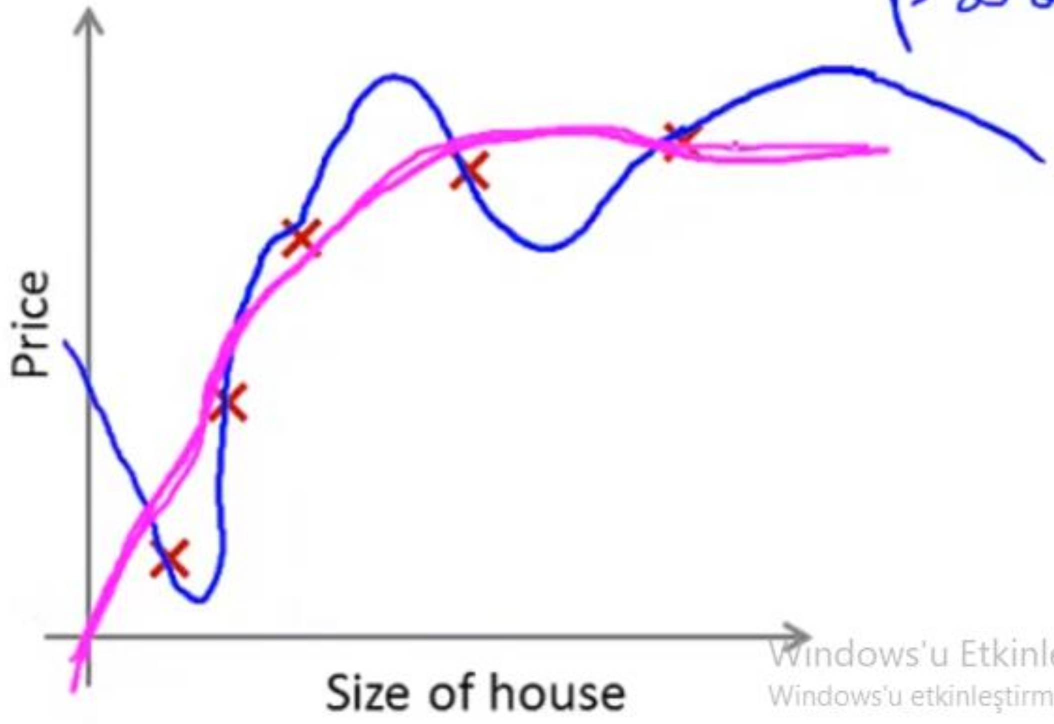


Exercise

- In regularized linear regression, we choose θ to minimize:

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

- What if λ is set to an extremely large value (perhaps too large for our problem, say λ=10^{10})?
  - Algorithm works fine; setting λ to be very large can't hurt it.
  - Algorithm fails to eliminate overfitting
  - Algorithm results in underfitting (fails to fit even the training set)
  - Gradient descent will fail to converge.

In regularized linear regression, we choose θ to minimize

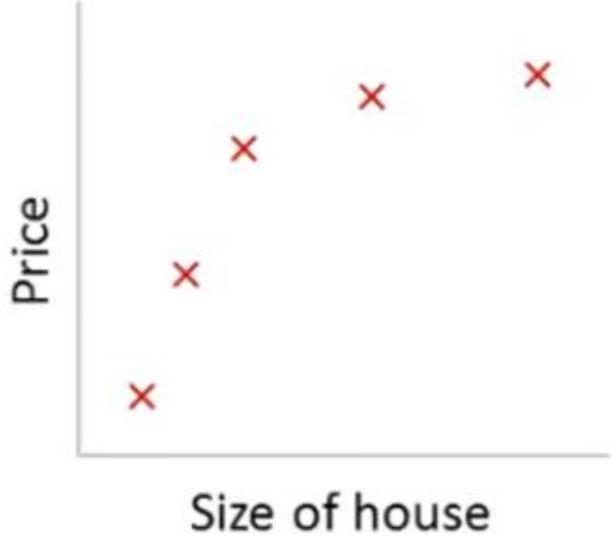
$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \underline{\lambda}\sum_{j=1}^{n}\theta_j^2\right]$$

What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?

$\underline{\theta_1}, \underline{\theta_2}, \underline{\theta_3}, \underline{\theta_4}$

$\theta_1 \approx 0, \theta_2 \approx 0$

$\theta_3 \approx 0, \theta_4 \approx 0$



$h_\theta(x)$

$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

In regularized linear regression, we choose θ to minimize

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$\underline{\theta_1}, \underline{\theta_2}, \underline{\theta_3}, \underline{\theta_4}$

$\theta_1 \approx 0, \theta_2 \approx 0$

$\theta_3 \approx 0, \theta_4 \approx 0$

$\underline{h_\theta(x) = \theta_0}$

$h_\theta(x) \to \theta_0 + \cancel{\theta_1 x} + \cancel{\theta_2 x^2} + \cancel{\theta_3 x^3} + \cancel{\theta_4 x^4}$

In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \underline{\lambda}\sum_{j=1}^{n}\theta_j^2\right]$$

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$\underline{h_\theta(x) = \theta_0}$

$h_\theta(x)$

$\theta_0 + \cancel{\theta_1 x} + \cancel{\theta_2 x^2} + \cancel{\theta_3 x^3} + \cancel{\theta_4 x^4}$

Andrew Ng