Reliability Engineering

Notes 3

Probability

- A quantitative measure of the likelihood of an event
- Measure of chance
- Quantitative statement about the likelihood of an event or events

Probability

- The likelihood of an occurrence on a scale from 0 (zero chance for an occurrence) to 1 (100% certainty for an occurrence) attached to a random event based on a particular mode for which the event can occur.

- It is the measure of chance which means that the chance of an event to happen

- Generally we can note that (the probability of an event to happing is the number of times that a specific event occurs relative to the sum of all possible events that can occur. This is the classical definition of the probability.
- P (success) = No. of success / No. of possible outcomes; $p = s / (s+f)$
- – q(failure) = No. of failures / No. of possible outcomes; $q = f / (s+f)$
- Where; $p + q = 1$

- Flipping a coin once can results in either a head (H) or a tail (T) ,i.e, 1 out of 2 , or 1/2.

Independent Events:

- Two events are said to be independent if the occurrence of one event does not affect the probability of occurrence of the other event.
- Example: Throwing a dice and tossing coin are independent events.

Mutually exclusive events:

- Two events are said to be mutually exclusive or disjoint if they cannot happen at the same time.
- Example: (i) When throwing a single die, the events 1, 2, 3, 4, 5 and 6 spots are all mutually exclusive because two or more cannot occur simultaneously
- (ii) Similarly success and failure of a device are mutually exclusive events since they cannot occur simultaneously.

Complementary Events:

- Two outcomes of an event are said to be complementary, if when one outcome does not occur, the other must occur.
- If the outcomes A & B have probabilities P(A) and P(B), then
- $P(A) + P(B) = 1$ $\quad$ $P(B) = P(\bar{A})$
- Example: When tossing a coin, the outcomes head and tail are complementary since
- $P(\text{head}) + P(\text{tail}) = 1$
- Therefore we can say that two events that are complementary events are mutually exclusive also. But the converse is not necessarily true .

Conditional Events;

- Conditional events are events which occur conditionally on the occurrence of another event or events.
- Consider two events A & B and also consider the probability of event A occurring under the condition that event B has occurred.
- $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Addition rule
- Event (AUB) is the union event and is defined as the event that occurs if A occurs or B occurs or both.
- Mathematically it is the union of the two events and is expressed as (AUB), (A or B)
- If the events are independent but not mutually exclusive then
- P(A U B) = P(A) + P(B) – P(A)*P(B)
- If the events are dependent then
- P(A U B) = P(A) + P(B) – P(A)*P(B I A)
- if A and B are mutually exclusive
- P(A U B) = P(A) + P(B)

- Multiplication Rule of Probability
- Event (A∩B) is the intersection event and is defined as the event that occurs if A and B occurs, i.e., the probability of A and B happening together.
- If the events are independent , then the probability of occurrence of each event is not influenced by the probability of occurrence of the other.
- $P(A \cap B) = P(A)*P(B)$

- If two events are not independent, then the probability of occurrence of one event is influenced by the probability of occurrence of the other
- $P(A \cap B) = P(B/A).P(A)$
- $= (PA/B).P(B)$

Example

- An engineer selects two components A & B. The probability that component A is good is 0.9 & the probability that component B is good is 0.95.What is the probability of both components being good.
- P(A good ∩ B good) = P(A good) (B good)
- = 0.9 x 0.95 = 0.85

- There are two lamps in a room. When turned on, one has probability of working of .90
- and the other has probability of working of .80. Only a single lamp is needed to light the room for success. What is the probability of success?

- Solution 1
- $P(A \cup B) = P(A) + P(B) - P(A)*P(B)$
- $= 0.90+0.80-0.90*0.80$
- $= 0.98$
- Solution 2
- $P= 1 - (1 - 0.90)(1 - 0.80) = 0.98.$

Example

- Now suppose 300 of the boys and 100 of the girls are interested in computer games. The school has 400 students out of 800 who like computer games. However, if a student is picked at random, what is the probability of finding a boy who is interested in computer games ?

- Being a boy and being interested in computer games, are not independent

$$
\begin{aligned}
P(\text{boy} \cap \text{likes computer games}) &= P(\text{boy}|\text{computer games}) \\
&\quad \times P(\text{likes computer games}) \\
&= 300/400 \times 400/800 \\
&= 0.375
\end{aligned}
$$

Probability Distributions in Reliability Engineering

- Most commonly used distributions are
- Discrete Distribution
- - Binomial Distribution
- Continuous Distribution
- -Weibull distribution
- -Exponential distribution
- -Normal distribution

Binomial Distribution

- To apply binomial distribution:
- -Fixed number of trials
- -Each trial must result in success or failure
- -All trials must have identcal probabilites of success.
- -All trials must be independent

- Consider a random trial having only two possible outcomes, success and failure, such a trial is referred as a “Bernoulli trialé’’
- p = probability of success,
- q= probability of failure
- p+q=1

- Binomial distribution gives the probability of exactly *k* successes in *m* attempts:

$$f(k) = \binom{m}{k} p^k q^{m-k}, \quad 0 \le p \le 1, \quad q = 1 - p, \quad k = 0, 1, 2, \ldots, m,$$

$$\binom{m}{k} \equiv C_k^m = \frac{m!}{k!(m-k)!},$$

where *p* is the probability of the defined success, *q* (or 1 – *p*) is the probability of failure, *m* is the number of independent trials, *k* is the number of successes in *m* trials, and the combinational formula is defined by

- $F(k)$, gives the probability of k or fewer successes in m trials.

$$F(k) = \sum_{i=0}^{k} \binom{m}{i} p^i q^{(m-i)}.$$

- An engineer wants to select four capacitors from a large lot of capacitors in which 10 percent are defective. What is the probability of selecting four capacitors with:
- (a) Zero defective capacitors?
- (b) Exactly one defective capacitor?
- (c) Exactly two defective capacitors?
- (d) Two or fewer defective capacitors?

- a) $f(4)=\begin{pmatrix}4\\4\end{pmatrix}(0.9)^4(0.1)^0=0.6561.$ or

$$f(0)=\begin{pmatrix}4\\0\end{pmatrix}(0.1)^0(0.9)^4=0.6561$$

(b) $f(1)=\begin{pmatrix}4\\1\end{pmatrix}(0.1)^1(0.9)^3=0.2916$

(c) $f(2)=\begin{pmatrix}4\\2\end{pmatrix}(0.1)^2(0.9)^2=0.0486$

(d) $F(2)=f(0)+f(1)+f(2)=0.9963.$

Continuous Distributions

- **Weibull Distribution**
- Weibull distribution first introduced by W. Weibull in 1937.
- In probability theory and statistics, Weibull distribution is one of most important continuous probability distributions.
- Weibull analysis is leading method for fitting life data.

- The Weibull distribution is very popular among engineers in reliability applications.
- It is valuable in reliability application because it enables to model different failure modes.
- If you want to model all the three phases of the lifecycle curve then one single distribution the Weibull distribution you can use.

- It can be used in events where the Probability of occurrence follows a “Bathtub Curve”
- Describes well the failure rate of real world components.

A random variable T is said to have a **Weibull distribution** with parameter $\beta > 0$ and $\eta > 0$ if its pdf is given by

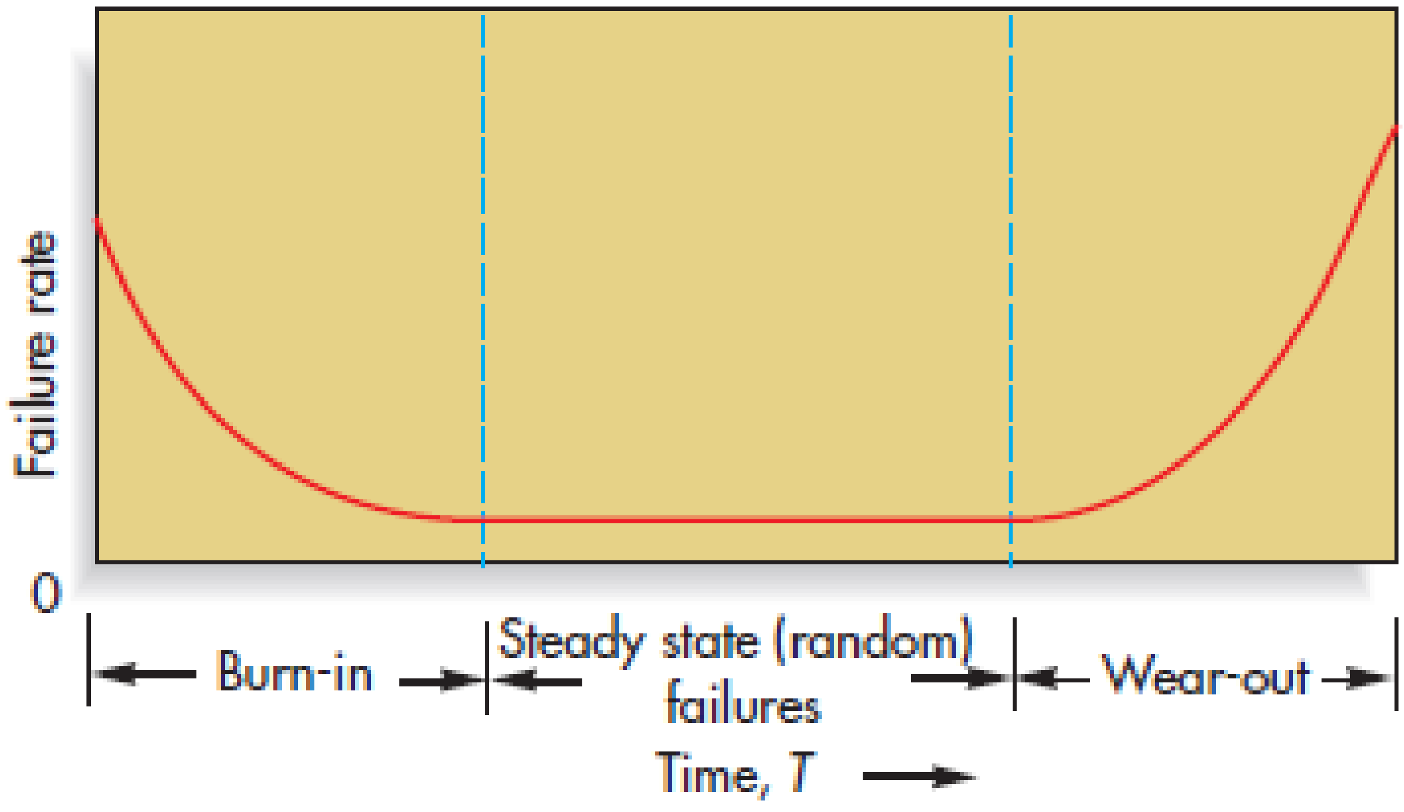
$$f_T(t) = \begin{cases} \dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1} e^{-(t/\eta)^{\beta}}, & t > 0 \\ 0, & \text{otherwise.} \end{cases}$$

- β = shape parameter
- η = scale parameter

- The cdf function
- Failure distribution function

$$F_T(t) = P(T \le t) = \begin{cases} 1 - e^{-(t/\eta)^\beta}, & t > 0, \\ 0, & t \le 0. \end{cases}$$

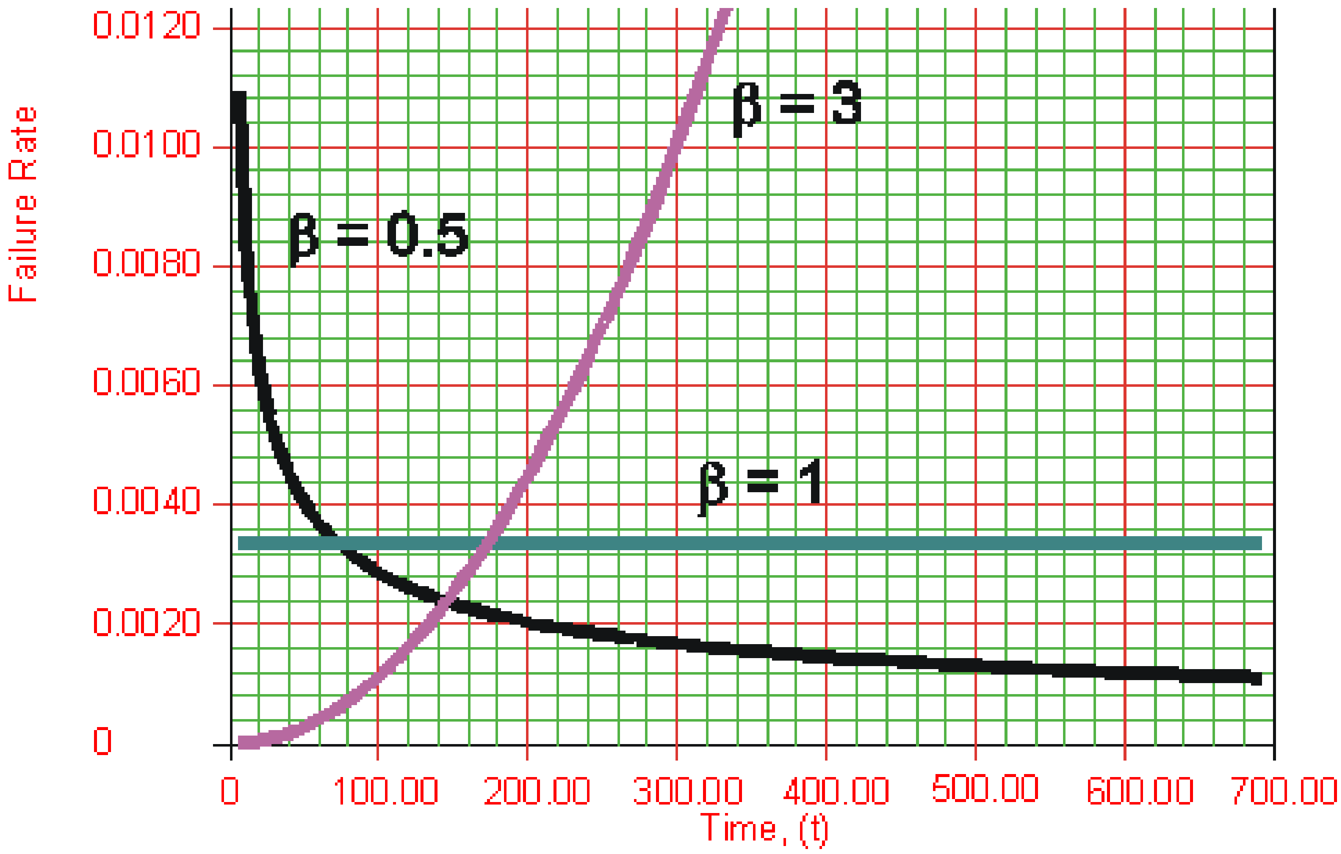
- Reliability function
- R(t) = $e^{-(t/\eta)^\beta}$,


Failure rate
0
Burn-in
Steady state (random) failures
Wear-out
Time, T

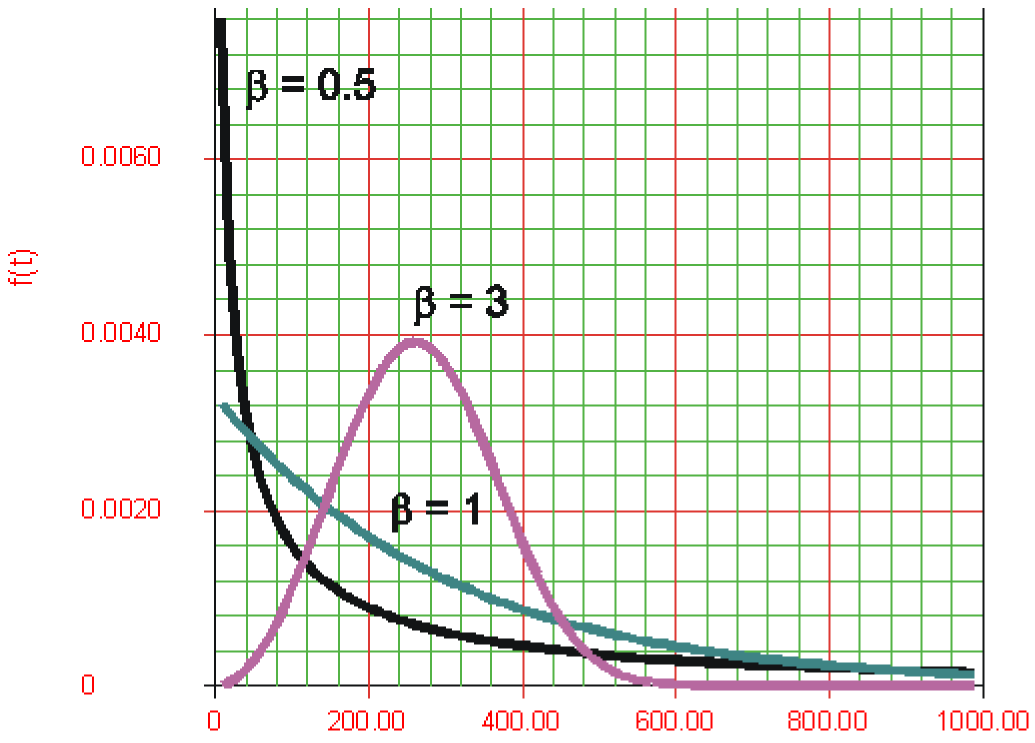
- Beta < 1 - infant mortality
- Beta = 1 - random
- 1 < Beta < 4 - early wear out
- Beta > 4 - old age wear out

- If $\beta = 1$ the failure rate is constant over time, and Weibull is identical to the Exponential distribution. If $\beta < 1$ failure rate decreases over time, and if $\beta > 1$ failure rate increases over
- time. If β takes a value between 3 and 4, then weibull distribution approaches normal distribution.
- Because of its flexibility, Weibull distribution is commonly used to model the time to failure during the burn-in ($\beta < 1$) and wear-out ($\beta > 1$) phases.

Weibull Failure Rate



Weibull pdf


β = 0.5
β = 3
β = 1
f(t)
0.0060
0.0040
0.0020
0
0
200.00
400.00
600.00
800.00
1000.00

Example

- A mechanical system has demonstrated a Weibull failure pattern, with a shape parameter of 1.4; and the scale parameter of 500 days. Determine the reliability that the system will last for 150 days.

- $\beta = 1.4$
- $\eta = 500$
- $R(150) = e^{-(t/\eta)^{\beta}}$,
- $R(150) = e^{-(\frac{150}{500})^{1.4}}$
- $=0.8308$

Example

- Suppose that the life distribution (life in years of continuous use) of hard disk drives for a computer system follows a two-parameter Weibull distribution with the following parameters: $\beta = 3.10$ and $\eta = 5$ years.
- The manufacturer gives a warranty for 1 year. What is the probability that a disk drive will fail during the warranty period?

- F(1)= 1-R(1) = 1- $e^{-\left(\frac{1}{5}\right)^{3.10}}$ = 1-0.993212
- = 0.006788

Resources

- https://www.philadelphia.edu.jo/academics/mlazim/uploads/PSR%20Lecture%20No.6.pdf, Power System Reliability Lecture No.6 Dr. Mohammed Tawfeeq Lazim
- STATISTICS FOR ENGINEERS Fall 2015 Lecture Notes, Dewei Wang Department of Statistics University of South Carolina.
- Introduction to reliability Lecture Notes (Portsmouth Business School, April 2012)
- https://canmedia.mheducation.ca/college/olcsupport/stevenson/5ce/ste39590_ch04S_001-019.pdf, Supplement to Chapter 4 Reliability
- Quality Design and Control, Design for Reliability- I , Lecture – 43 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- Quality Design and Control, Design for Reliability- I , Lecture – 44 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur

Resources

- https://www.weibull.com/hotwire/issue14/relbasics14.htm
- https://docs.tibco.com/data-science/GUID-E94B660B-73EC-47E7-A4B2-A084AFBC09D5.html
- http://www.stats.ox.ac.uk/~marchini/teaching/L6/L6.slides.pdf
- https://www.mathsisfun.com/data/standard-normal-distribution.html
- http://math.arizona.edu/~rsims/ma464/standardnormaltable.pdf
- https://risk-engineering.org/static/PDF/slides-reliability-engineering.pdf, Overview of reliability engineering, Eric Marsden
- Introduction to Reliability Fundamentals, Donald G. Dunn, 2019 D2 Training
- Ignou The People's University, Unit 11 Reliability Lecture Notes
- Ignou The People's University, Unit 13 Introduction to Reliability Lecture Notes
- **Reliability Engineering,** Kailash C. Kapur , Michael Pecht, 2014 , John Wiley & Sons, Inc

Resources

- Reliability Engineering Lecture Notes, Vardhaman College of Engineering
- https://www.slideshare.net/CharltonInao/reliability-engineering-chapter1csi
- https://extapps.ksc.nasa.gov/Reliability/Documents/210624%20Probability%20Formulas.pdf
- https://slideplayer.com/slide/4707809/
- https://slideplayer.com/slide/5235890/
- http://slideplayer.com/slide/9536322/, Introduction to Reliability Engineering, e-Learning course, CERN
- https://slidetodoc.com/1-introduction-to-reliability-engineering-elearning-course-n
- Power System Reliability, Lecture Notes DR. AUDIH ALFAOURY, 2017- 2018, Al-Balqa Applied University
- **Probability Fundamentals and Models in Generation andBulk System Reliability Evaluation, Roy Billinton Power System Research Group University of Saskatchewan CANADA**
- **Basic Probability and Reliability Concepts, Roy Billinton Power System Research Group University of Saskatchewan CANADA**