Storage Systems Analysis – 2

Basic Inventory Models

END4650 – Material Handling Systems

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Economic Order Quantity – EOQ

- Economic order quantity (EOQ) is the ideal quantity of units a company should purchase to meet demand while minimizing inventory costs such as
  - holding costs,
  - shortage costs, and
  - order costs.
- This production-scheduling model was developed in 1913 by Ford W. Harris and has been refined over time. It is one of the oldest classical production scheduling models.
- The EOQ formula assumes that the following remain constant
  - demand,
  - Ordering cost, and
  - holding costs

Economic Order Quantity – EOQ

- T = total annual inventory cost
- P = purchase unit price, unit production cost
- Q = order quantity
- Q* = optimal order quantity
- D = annual demand quantity
- K = fixed cost per order, setup cost (not per unit, typically cost of ordering and shipping and handling. This is not the cost of goods)
- h = annual holding cost per unit, also known as carrying cost or storage cost (capital cost, warehouse space, refrigeration, insurance, opportunity cost (price x interest))

Economic Order Quantity – EOQ

- The single-item EOQ formula finds the minimum point of the following **annual** cost function:
- Total Cost = purchase cost or production cost + ordering cost + holding cost
- **Purchase cost:**
- This is the variable cost of goods: purchase unit price × annual demand quantity. This is $P \times D$
- **Ordering cost:**
- This is the cost of placing orders: each order has a fixed cost K, and we need to order D/Q times per year. This is $K \times D/Q$
- **Holding cost:**
- the average quantity in stock (between fully replenished and empty) is $Q/2$, so this cost is $h \times Q/2$

Economic Order Quantity – EOQ

$$0 = -\frac{DK}{Q^2} + \frac{h}{2}$$

Economic Order Quantity

$$Q^* = \sqrt{\frac{2DK}{h}}$$

Economic Order Quantity – EOQ

- **Example**
- annual requirement quantity (D) = 10000 units
- Cost per order (K) = 40
- Cost per unit (P)= 50
- Yearly carrying cost per unit = 4
- Market interest = 2%

$$\sqrt{\frac{2D\cdot K}{h}} = \sqrt{\frac{2\cdot 10000\cdot 40}{4+50\cdot 2\%}} = \sqrt{\frac{2\cdot 10000\cdot 40}{5}} = 400 \text{ units}$$

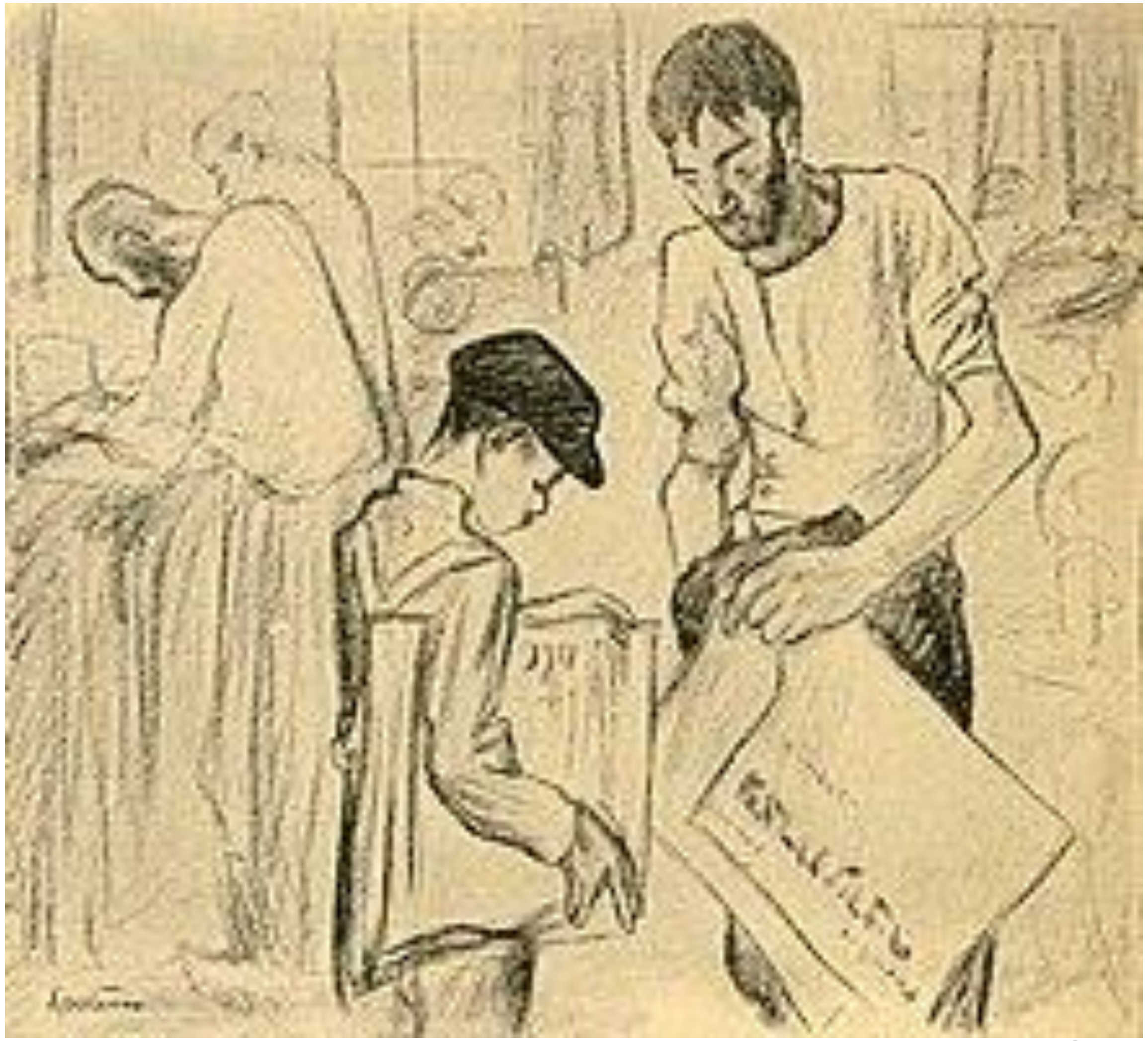
Economic Order Quantity – EOQ

- What is the total number of orders in a year?
- 10000/400 = 25

- What is the total associated cost?
- (50)(10000) + (40)(10000/500) + (5)(500/2)

Newsvendor (Newsboy) Problem

- Single period problem
- Demand is random (not deterministic like EOQ model)
- Aim is to
  - maximize the expected revenue, or
  - minimizing the expected cost
- The modern formulation relates to a paper n *Econometrica* by Kenneth Arrow, T. Harris, and Jacob Marshak



Newsvendor Problem

- r: unit selling price (Revenue)
- s: salvage
- p: penalty for lost sales
- c: unit cost of each item

- D: Random demand
- F: cumulative prob. distribution of demand
- q: order quantity
- q*: optimal order quantity

- $Profit = r \times \min(q, D) + \mathrm{s} \times (\mathrm{q} - \mathrm{D})^{+} - \mathrm{p} \times (\mathrm{D} - \mathrm{q})^{+} - \mathrm{c} \times \mathrm{q}$
- $E\ [Profit] = r \times \mathrm{E}[\min(q, D)] + \mathrm{s} \times E[(\mathrm{q} - \mathrm{D})^{+}] - \mathrm{p} \times E[(\mathrm{D} - \mathrm{q})^{+}]\ - \mathrm{c} \times \mathrm{q}$

Newsvendor Problem

- The optimal order quantity is given by the following formula

$$F(q^*) = \frac{r + p - c}{r + p - s}$$

- Or equivalently

$$q^* = F^{-1}\left(\frac{r + p - c}{r + p - s}\right)$$

- Or equivalently

$$q^* = F^{-1}\left(\frac{c_u}{c_u + c_o}\right) \; or \; q^* = F^{-1}(\alpha)$$

Where $c_u = r + p - c$ and $c_o = c - s$ are called the overage and underage costs respectively and α is called the critical ratio.

Newsvendor Problem - Example

- Uniformly Distributed Demand
- Assume we have
  - r = 10 TL
  - s = 2 TL
  - c = 5 TL
  - p = 6 TL
- And demand is uniformly distributed with $D \in [100,300]$

- Solution:
- Critical ratio is: 11/14
- $F(q^*) = \frac{11}{14}$
- $\frac{q^* - 100}{300 - 100} = \frac{11}{14}$
- q* = 257

Newsvendor Problem - Example

- Normally Distributed Demand
- Assume now the demand is normally distributed with:
- $\mu = 150 \; and \; \sigma = 20$

- Solution:
- Critical ratio is: 11/14
- $F(q^*) = \frac{11}{14}$
- $P(D < q^*) = \frac{11}{14} = 0.786$
- Converting this into standard normal we have:
- $z^* = z_{1-0.786}$
- $q^* = \mu + \sigma \times z^*$
- $q^* = 150 + 20 \times 0.792$
- $q^* = 169$