Reliability Engineering

Notes 4

Reliability Distributions
Exponential Distribution

Exponential distribution is a special case of Weibull distributions.

If $\beta = 1$ the failure rate is constant over time, and Weibull is identical to the Exponential distribution.

The time we need to wait before an event occurs has an exponential distribution.

How long will a generating unit works without breaking down?

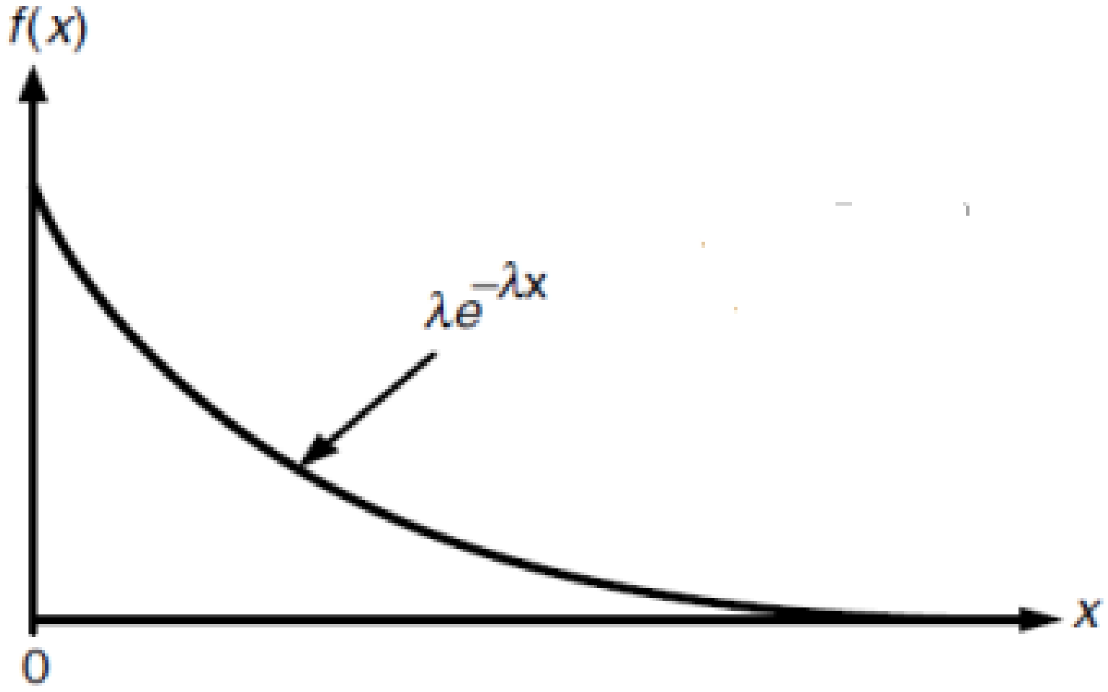
How long will a transformer works without breaking down?

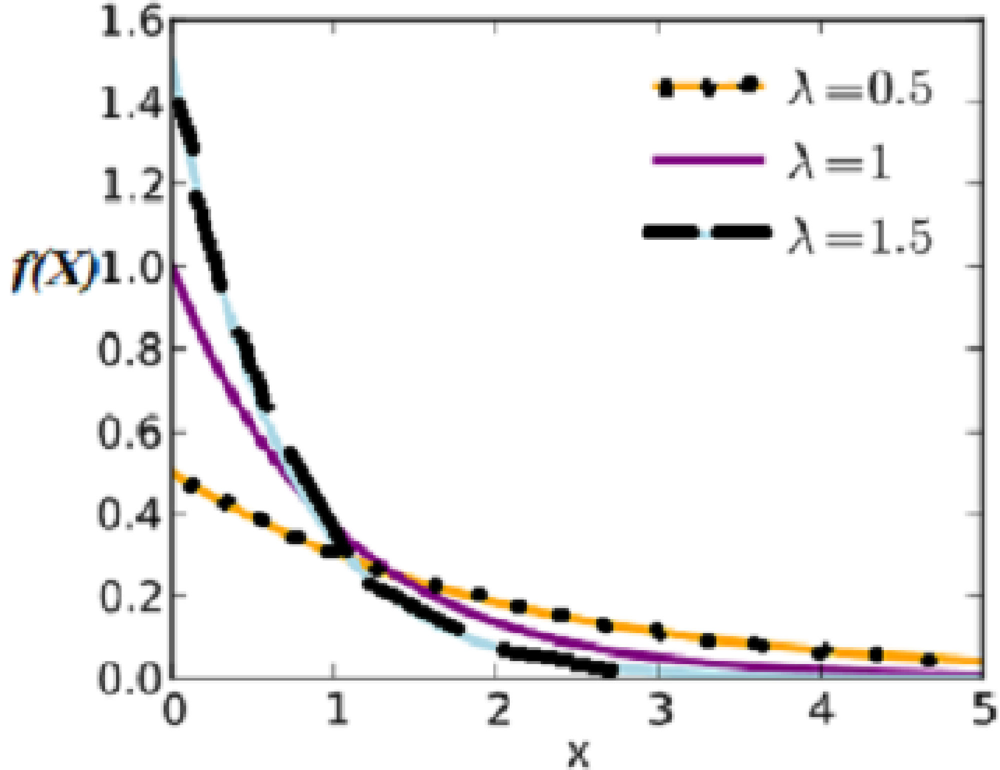
- The exponential distribution is a continuous probability density function.

A random variable X is said to have an **exponential distribution** with parameter $\lambda > 0$ if its pdf is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

λ is a parameter of the distribution. The parameter λ is a positive real number, called the constant failure rate. The exponential distribution describes a probability that decreases exponentially with increasing x.


f(x)
λe^{−λx}
x
0



1.6
1.4
1.2
f(X) 1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
x
λ=0.5
λ=1
λ=1.5

- It is one of the most commonly used distributions in reliability. The exponential distribution applies when the failure rate is constant - the graph is a straight horizontal line. It can be used to analyse the middle phase of a bath tub.

- If λ is the failure rate and t is the time, then the reliability, R, can be determined by the following equation:

$$R(x) = \int_{x}^{\infty} \lambda \, e^{-\lambda x} \, dx = e^{-\lambda x}$$

$$R(t) = e^{-\lambda t}$$

- To see that it gives sensible results, imagine that there are initially 1000 components and that λ is 10% (0.1) per hour. After one hour about 10% of the original 1000 components will have failed - leaving about 900 survivors. After two hours, about 10% of the 900 survivors will have failed leaving about 810. Similarly there will be about 729 survivors after the third hour, which means that the reliability after 3 hours is 0.729.

- Using the exponential distribution the reliability after 3 hours, with λ=0.1, is given by

$$R(t) = e^{-3\lambda} = e^{-0.3} = 0.741$$

Example

- Ten thousand new oil circuit reclosers (OCRs) are put in service. They have a constant failure rate of 0.1 per year. How many units of the original 10,000 will still be in service after 10 years? How many of the original will fail in Year 10?

In 10 years, probability of survival

$$R(10) = e^{-0.1\times 10} = e^{-1.0} = 0.3679$$

Out of 10,000 original units

10,000x0.3679=3679 should survive

Number of failures in Year 10

$$\begin{aligned}
&= (\text{number of survivors after Year 9}) - (\text{number of survivors after Year 10}) \\
&= 10,000 \times e^{-0.1\times 9} - 3679 \\
&= 1000 \times e^{-0.9} - 3679 \\
&= 4066 - 3679 \\
&= 387
\end{aligned}$$

Example

- A weather satellite has expected life of 10 years from the time it is placed into Earth's orbit.
- Determine its reliability for each of the following lengths of service (assume that Exponential
- distribution is appropriate.)
- **a.** 5 years
- **b.** 12 years
- **c.** 20 years
- **d.** 30 years

- MTTF = 10 years

	t	R(t)
a	5	0.6065
B	12	0.3012
C	20	0.1353
d	30	0.0498

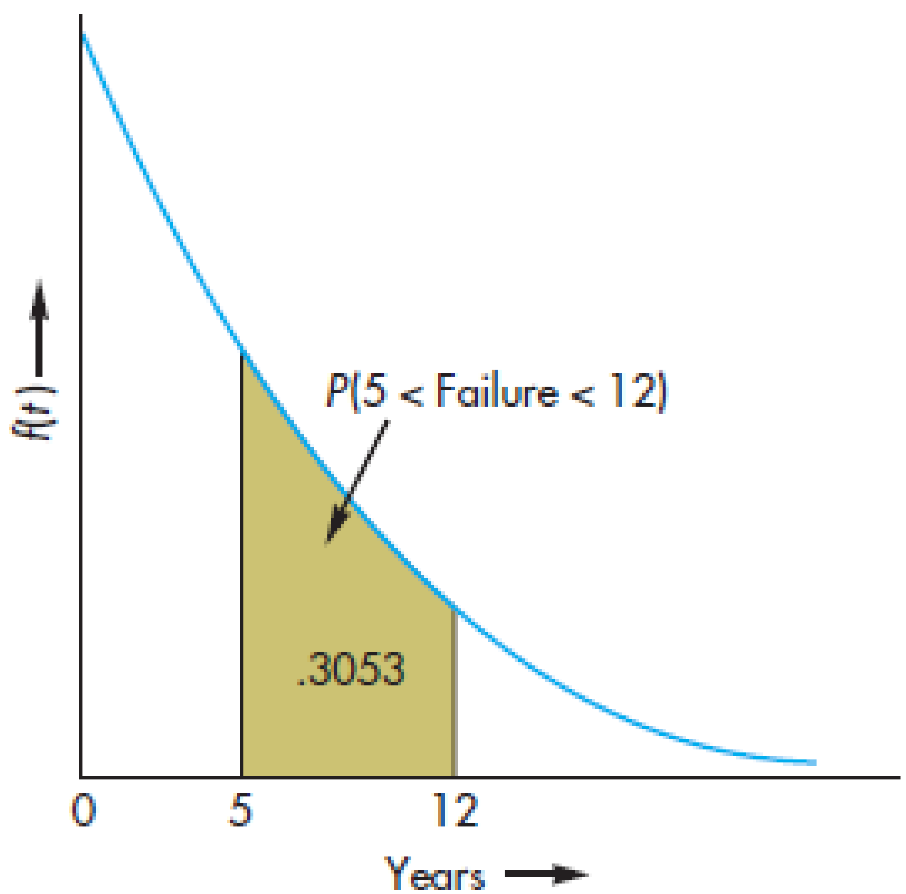
Example

- What is the probability that the satellite described in previous problem will fail between 5 and 12 years after being placed into Earth's orbit?

$$P(5 \text{ years} < \text{ failure} < 12 \text{ years}) = P(\text{failure after 5 years}) - P(\text{failure after 12 years})$$

Using the probabilities shown in the previous solution, we obtain:

$$\begin{aligned} P(\text{failure after 5 years}) &= .6065 \\ - P(\text{failure after 12 years}) &= \underline{.3012} \\ & .3053 \end{aligned}$$



f(t)
P(5 < Failure < 12)
.3053
0
5
12
Years

Example

- A particular electronic device will only function correctly if two essential components both function correctly. The lifetime of the first component is known to be exponentially distributed with a mean of 5000 hours and the lifetime of the second component (whose failures can be assumed to be independent of those of the first component) is known to be exponentially distributed with a mean of 7000 hours. Find the proportion of devices that may be expected to fail before 6000 hours use.

- First device
- λ= 1/5000

$$R(t) = e^{-\lambda t}$$

- R(t) = $e^{-\left(\frac{6}{5}\right)}$= 0,301
- Second device
- λ= 1/7000
- R(t) = = $e^{-\left(\frac{6}{7}\right)}$= 0,424
- Probability that both of the devices will work
- P(1)*P(2)= 0,301*0,424 = 0,1276
- Probability that both of the devices will fail before 6000 hours is %87

Normal Distribution

- The normal distribution occurs whenever a random variable is affected by a sum of random effects, such that no single factor dominates. This motivation is based on central limit theorem, which states that under mild conditions, the sum of a large number of random variables is approximately normally distributed. It has been used to represent dimensional variability in manufactured goods, material properties, and measurement errors. It has also been used to assess product reliability.

- The normal distribution has been used to model various physical, mechanical, electrical, or chemical properties of systems. Some examples are gas molecule velocity, wear, noise, the tensile strength of aluminum alloy steel, the capacity variation of electrical condensers, electrical power consumption in a given area, generator output voltage, and electrical resistance.

Normal Distribution

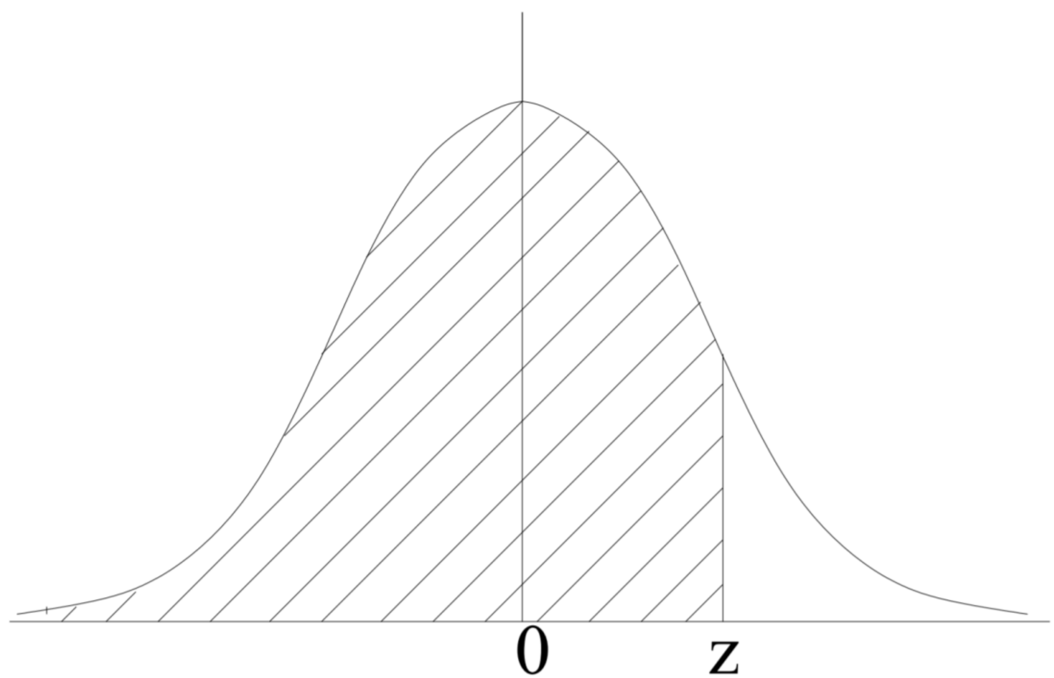
- Symmetric and bell shaped
- Two parameters, μ and σ
- If we want to calculate probabilities from different Normal distributions we convert the probability to one involving the standard Normal distribution. This process is called standardization.

Here is the formula for z-score that we have been using:

$$z = \frac{x - \mu}{\sigma}$$

- **z** is the "z-score" (Standard Score)
- **x** is the value to be standardized
- **μ** is the mean
- **σ** is the standard deviation

The tables allow us to read off probabilities of the form $P(Z < z)$.



- The normal distribution has an increasing hazard rate. The normal distribution has been used to describe the failure distribution for products that show wearout and that degrade with time. The life of tire tread and the cutting edges of machine tools fit this description. Also, mechanical items such as ball bearings, valves, and springs tend to have insignificant burn-in and steady-state phases, and start to wear out right away. In these situations, life is given by a mean value of μ, and the variability about the mean value isdefined through standard deviation.

- Obtaining Normal probabilities involves the use of the standard Normal table. The table provides areas under a Normal curve up to a specified point z , where z is a *standardized* value calculated using the formula

$$z = \frac{T - \text{Mean wear-out time}}{\text{Standard deviation of wear-out time}}$$

$$F(t) = \Phi(z) = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right)$$

Normal Distribution Table

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361

Resim şu anda görüntülenemiyor.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831

Resim şu anda görüntülenemiyor.

Example

- The mean life of a certain ball bearing can be modelled using a Normal distribution with a mean of six years and a standard deviation of one year. Determine:
- **a.** The probability that a ball bearing will fail *before* seven years of service.
- **b.** The probability that a ball bearing will fail *after* seven years of service (i.e., find its reliability).
- **c.** The service life that will provide a failure probability of 10 percent.

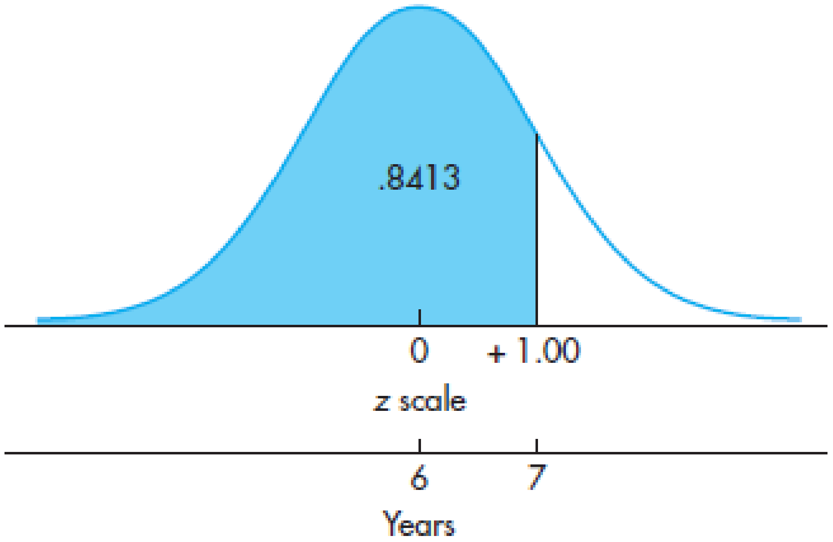
Wear-out mean = 6 years

Wear-out standard deviation = 1 year

Wear-out is Normally distributed

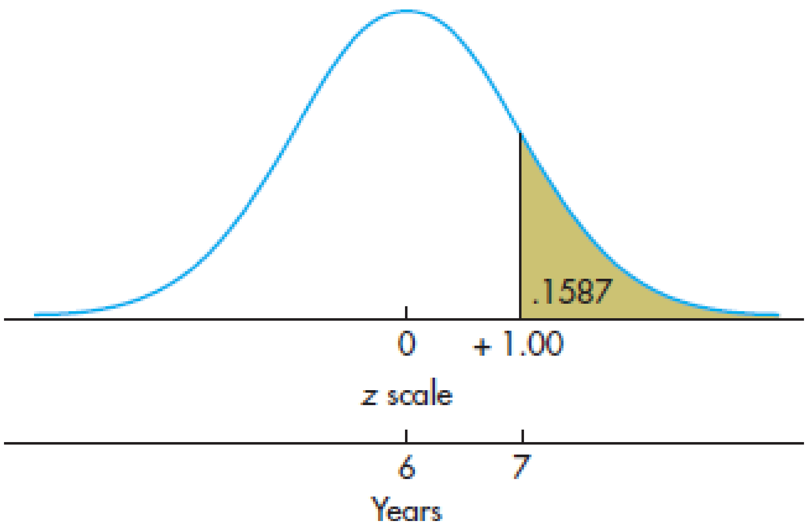
a. Calculate z using the above formula:

$$z = \frac{7 - 6}{1} = +1.00$$



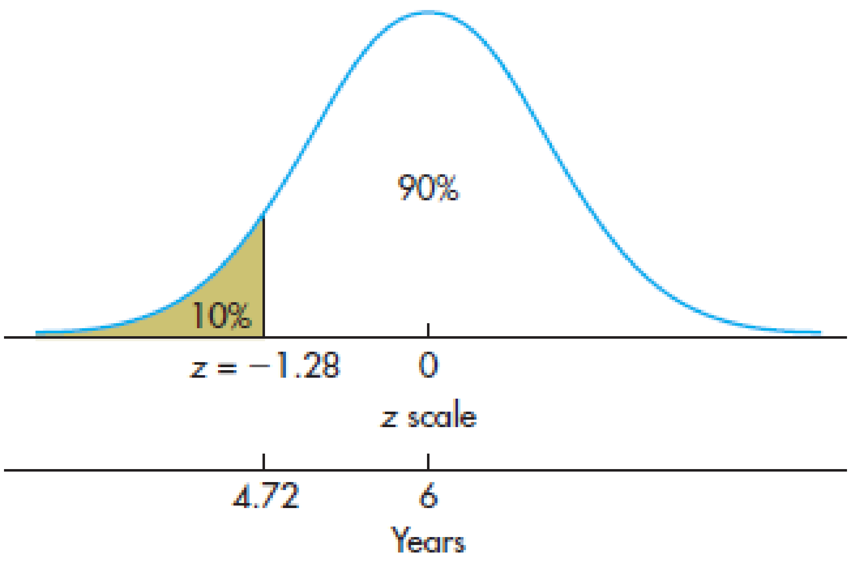
b. Subtract the probability determined in part *a* from 1.00

$1.00 - .8413 = .1587$



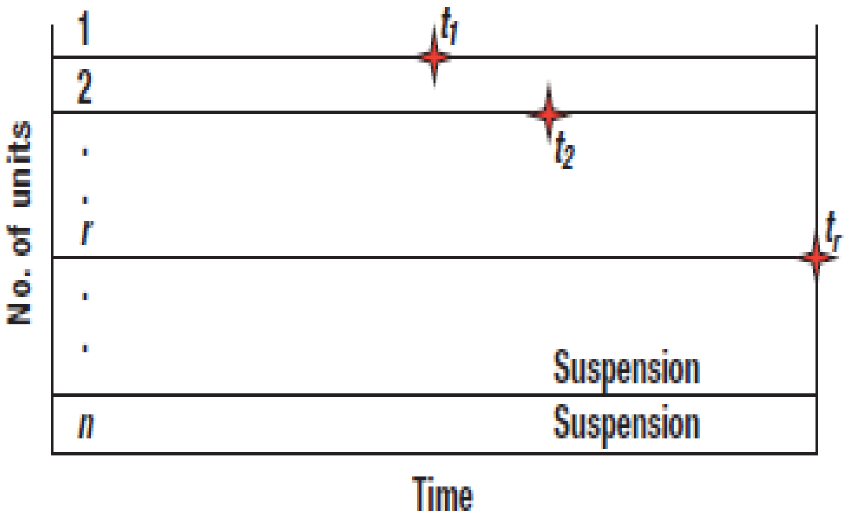
- $z = -1.28 = (t-6)/1 = 4.72$

c. Use the standard Normal table in reverse, i.e., find the value of z that corresponds to an area under the curve (starting from the left side) of .10. Thus, $z = -1.28$.
Now, insert this in the z formula above:



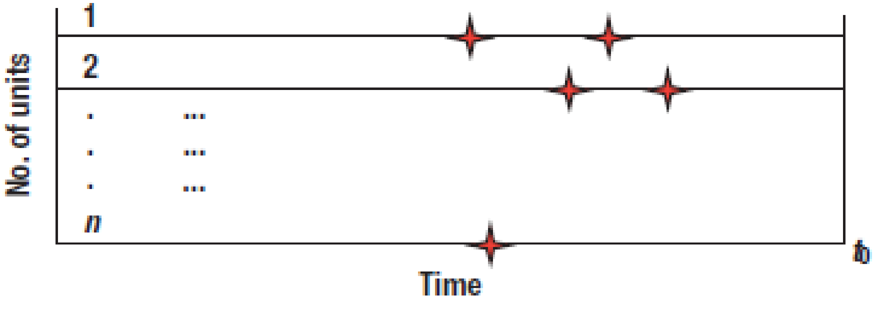
Life Testing

- For reliability tests , the time to failure can be assumed to follow an exponential distribution, the constant failure rate can be estimated by life testing. There are various ways to test the items.
- One of them is a failure-truncated test, in which n items on individual test stands are monitored to failure. The test ends as soon as there are r failures (without replacement $r \leq n$)
- The total time on test, T, considering both failed and unfailed units



Failure-truncated test. Failures are denoted by ✦.

- Another test situation is called time-truncated testing.
- Testing for these units continues until some predetermined time.



Time-truncated test. ✦, Failures.

- For life testing, two different types of tests can be conducted.
- Replacement or without replacement.
- In replacement, failed items replaced as soon as they fail.

- Estimate the MTBF for the following reliability test situations:
- (a) Failure terminated, with no replacement. Twelve items were tested until the fourth failure occurred, with failures at 200, 500, 625, and 800 hours.
- (b) Time terminated, with no replacement. Twelve items were tested up to 1000 hours, with failures at 200, 500, 625, and 800 hours.
- (c) Failure terminated, with replacement. Eight items were tested until the third failure occurred, with failures at 150, 400, and 650 hours.
- (d) Time terminated, with replacement. Eight items were tested up to 1000 hours, with failures at 150, 400, and 650 hours.
- (e) Mixed replacement/nonreplacement. Six items were tested through 1000 hours on six different test stands. The first failure on the test stand occurred at 300 hours, and its replacement failed after an additional 400 hours. On the second test stand, failure occurred at 350 hours, and its replacement failed after an additional 500 hours. On the third test stand, failure occurred at 600 hours, and its replacement did not fail up to the completion of the test. The items on the other three test stands did not fail for the duration of the test.

(a) $\text{MTBF}(e) = (200 + 500 + 625 + 800 + 8(800))/4 = 2{,}131$ hours

(b) $\text{MTBF}(e) = (200 + 500 + 625 + 800 + 8(1000))/4 = 2{,}531$ hours

(c) $\text{MTBF}(e) = (8)(650)/3 = 1{,}733$ hours

(d) $\text{MTBF}(e) = (8)(1000)/3 = 2{,}667$ hours

(e) $\text{MTBF}(e) = (700 + 850 + 1000 + (3)(1000))/5 = 1{,}110$ hours.

Resources

- https://www.philadelphia.edu.jo/academics/mlazim/uploads/PSR%20Lecture%20No.6.pdf, Power System Reliability Lecture No.6 Dr. Mohammed Tawfeeq Lazim
- STATISTICS FOR ENGINEERS Fall 2015 Lecture Notes, Dewei Wang Department of Statistics University of South Carolina.
- Introduction to reliability Lecture Notes (Portsmouth Business School, April 2012)
- https://canmedia.mheducation.ca/college/olcsupport/stevenson/5ce/ste39590_ch04S_001-019.pdf, Supplement to Chapter 4 Reliability
- Quality Design and Control, Design for Reliability- I , Lecture – 43 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- Quality Design and Control, Design for Reliability- I , Lecture – 44 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- Introduction to Reliability Fundamentals, Donald G. Dunn, 2019 D2 Training
- Ignou The People's University, Unit 11 Reliability Lecture Notes
- Ignou The People's University, Unit 13 Introduction to Reliability Lecture Notes
- **Reliability Engineering,** Kailash C. Kapur , Michael Pecht, 2014 , John Wiley & Sons, Inc

Resources

- https://www.weibull.com/hotwire/issue14/relbasics14.htm
- https://docs.tibco.com/data-science/GUID-E94B660B-73EC-47E7-A4B2-A084AFBC09D5.html
- http://www.stats.ox.ac.uk/~marchini/teaching/L6/L6.slides.pdf
- https://www.mathsisfun.com/data/standard-normal-distribution.html
- http://math.arizona.edu/~rsims/ma464/standardnormaltable.pdf
- https://risk-engineering.org/static/PDF/slides-reliability-engineering.pdf, Overview of reliability engineering, Eric Marsden
- Power System Reliability, Lecture Notes DR. AUDIH ALFAOURY, 2017- 2018, Al-Balqa Applied University
- **Probability Fundamentals and Models in Generation andBulk System Reliability Evaluation, Roy Billinton Power System Research Group University of Saskatchewan CANADA**
- **Basic Probability and Reliability Concepts, Roy Billinton Power System Research Group University of Saskatchewan CANADA**