Reliability Engineering

Notes 5

System Reliability

- Most products are made up of a number of components. The reliability of each component and the configuration of the system consisting of these components determines the system reliability.
- System reliability is a function of the reliabilities of the (sub reliabilities of the (sub -) components and of) components and of the relationships between the components.

- The components may be in
- Series system
- Parallel system
- Combination of both
- K out of n system

- The reliability evaluation of engineering systems can be obtained by drawing Reliability Block Diagram (RBD).
- The reliability block diagram (RBD) is a pictorial way of showing the success or failure combinations for a system.
- A system reliability block diagram presents a logical relationship of the system, subsystems, and components.
- In a RBD each of the engineering system components is indicated by a block.
- Any closed path through the diagram is a success path. it is closed when the component it represents is working and is opened when the component has failed.

System Reliability

- Series System
- A system may consist of one or more of the sub-systems and each subsystem may have different sub subsystems.
- In series or chain structure, all components must function for the system to function.
- Example, lights (bulbs or LEDs) used in festivals and weddings.

Series System

- It consists of several components and all the components must run simultaneously.
- All the components must be working that it must not fail at a particular point in time, if just one component fails entire system breaks down.
- Series system component failures are statistically independent. That means, for each of these components it has its own time to failure time.

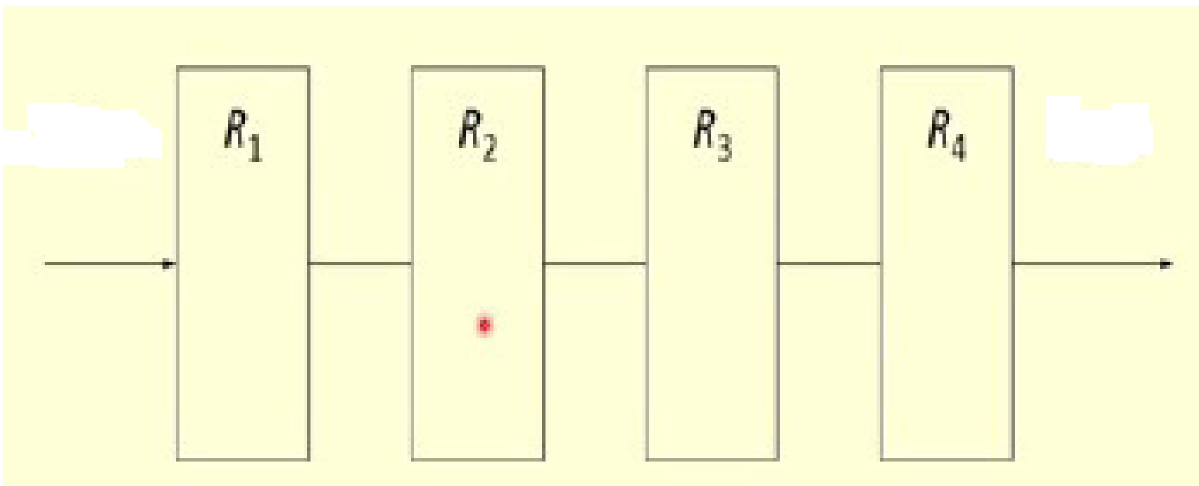
Series System

- The function of one component no way is affected by the function of the other components. So, we may assume that they are all statistically independent.
- Even though the individual components of a series system (product) might have high reliability, the series system (product) as a whole can have considerably less reliability because all its components must function (i.e., the system is dependent on each of its components). As the number of components in a series system (product) increases, the system (product) reliability decreases.

Series System

- Reliability of the series system can never be better than the reliability of the worst component in the system.
- Serial system reliability is smaller than any individual reliability of the components.
- All the reliabilites should be given for a common time period.

Series System



Series System Reliability

If R denotes the reliability of the series system, then, from the definitions of reliability and series system, we have

$$R = P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n)$$

In equation , we take the intersection of events because reliability is a probability and the definition of series system implies that for the successful operation of the system, all components must work.

Series System Reliability

The probability of success of the system is

$$P(S) = P(X_1 \cap X_2 \cap X_3 \cap \dots \dots \dots \dots \dots \dots \dots \dots \dots . \cap X_n)$$

$$or \quad P(S) = P(X_1).P(X_2).P(X_3) \dots \dots \dots \dots \dots \dots \dots \dots \dots . P(X_n)$$

Where $P(X_1)$ = Probability of success of X_1 = Reliability R_1(t)

$P(X_2)$ = Probability of success of X_2 = Reliability R_2(t)

$P(X_3)$ = Probability of success of X_3 = Reliability R_3(t)

. . . .

$P(X_n)$ = Probability of success of X_n = Reliability R_n(t)

$$R_{\text{system}}(t) = R_1(t) \times R_2(t) \times R_3(t) \times \cdots \times R_n(t)$$

$$R_{syst}(t) = \prod_{i=1}^{n} R_i\,(t)$$

Example

- A system is composed of 3 independent serially connected components
- R 1 = 0.95
- R 2 = 0.87
- R 3 = 0.82
- What is the system reliability?
- Rs = 0.95 x 0.87 x 0.82 = 0.6777

Example

- A two component series system contains identical components each having a reliability of 0.99. Evaluate the unreliability of the system.
- $Rs= (0,99)^2 = 0,9801$
- Unreliability= $Qs = 1-0,9801 = 0,0199$

Example

- A system design required 200 identical components in series. If the overall reliability must not be less than 0.99, what is the minimum reliability of each component.
- $Rs = R^{200} = 0.99$
- $R = 0.99^{1/200} = 0.99995$

Example

- A system has three components connected in series having reliabilities 0.40, 0.70, 0.80, respectively, for a mission of 400 hours. What is the percentage increase in the reliability of the system in each of the following cases?
- (i) Reliability of the first component is increased by 0.1 and that of the second and third components remains the same.
- (ii) Reliability of the second component is increased by 0.1 and that of the first and third components remains the same.
- (iii) Reliability of the third component is increased by 0.1 and that of the first and second components remains the same.

- System Reliability
- R = 0.4x0.7x0.8
- R = 0.224

	Reliability of the First Component	Reliability of the Second Component	Reliability of the Third Component	Reliability of the System	Percentage Increase in the Reliability of the System
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
	0.4	0.7	0.8	0.224	–
i)	0.5	0.7	0.8	0.280	$\frac{0.280-0.224}{0.224}\times 100=25\%$
ii)	0.4	0.8	0.8	0.256	$\frac{0.256-0.224}{0.224}\times 100 \quad 14.29\%$
iii)	0.4	0.7	0.9	0.252	$\frac{0.252-0.224}{0.224}\times 100=12.5\%$

- If a system engineer wants to improve the reliability of the series system, he/she should **concentrate** on the **improvement** of the reliability of the **poorest component**.

Parallel Systems

- Only one of N components is needed to make the system function properly.
- If all of the components fail, the system fails.
- System is composed of n independent components connected in parallel.
- Failure of all components results in the failure of the whole system.
- Example : a laptop with a power source and a battery

- If R denotes the reliability of the parallel system for the mission of t units of time, then by definition of reliability and parallel system, we have

$$R = P(E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$$

We take the union of events in equation because the definition of reliability implies that reliability is a probability and the definition of parallel system implies that for the successful operation of the parallel system only one component needs to work.

$$R = 1 - P\left(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \ldots \cap \bar{E}_n\right)$$

$$R = 1 - P\left(\bar{E}_1\right)P\left(\bar{E}_2\right)P\left(\bar{E}_3\right)\ldots P\left(\bar{E}_n\right)$$

$$= 1 - \left(1 - P\left(E_1\right)\right)\left(1 - P\left(E_2\right)\right)\left(1 - P\left(E_3\right)\right)\ldots\left(1 - P\left(E_n\right)\right)$$

$$R = 1 - \left(1 - R_1\right)\left(1 - R_2\right)\left(1 - R_3\right)\ldots\left(1 - R_n\right)$$

$$\text{or } R = 1 - \prod_{i=1}^{n}\left(1 - R_i\right)$$

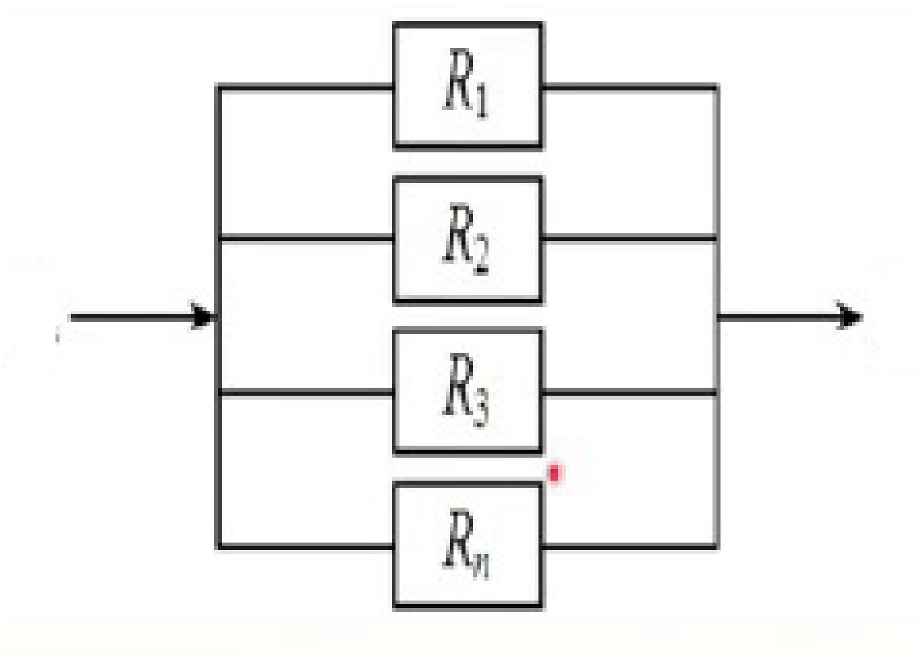
Parallel Systems

- The reliability is higher than the highest among the component reliabilities.

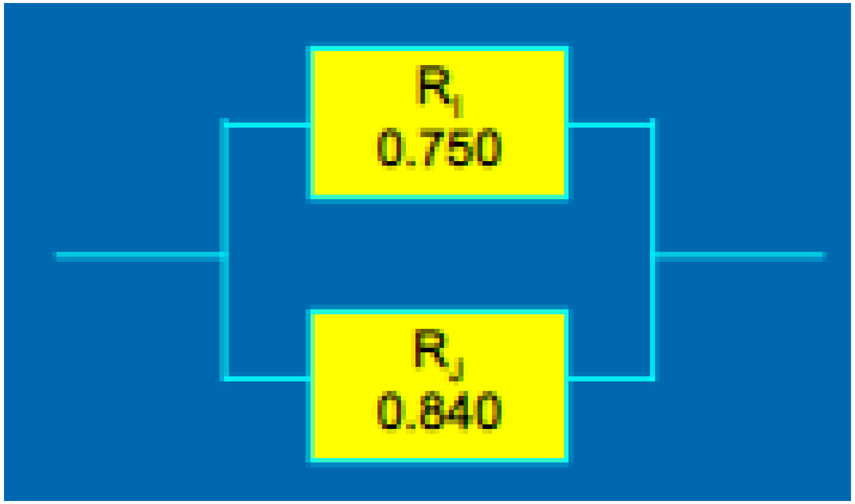
$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i)$$

- n means number of components
- R_i means the reliability of the ith component.
- $R_p = 1-(1-R_A)(1-R_B)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1-R_n)$

Parallel Systems



Example

- A system consists of 2 independent parallel connected components.
- 

- $Rp = 1 – (1-0.750)(1- 0.840)$
- $= 1- 0.04$
- $= 0.960$

Example

- A system has three components connected in parallel from a reliability point of view having reliabilities 0.20, 0.40, 0.50, respectively, for a 400 hours period of time. What is the percentage increase in the reliability of the system in the following cases?

a) Reliability of the first component is increased by 0.1 and that of the second and third components remains the same.

b) Reliability of the third component is increased by 0.1 and that of the first and second components remains the same.

- System reliability
- $R1 = 0.2 \quad R2 = 0.4 \quad R3 = 0.5$
- $Rp = 1 - (1-0.2)(1-0.4)(1-0.5)$
- $= 1 - 0.8 \times 0.6 \times 0.5$
- $= 1 - 0.240$
- $= 0.760$

- a) Reliability of the first component is increased by 0.1
- $R_p = 1- (1-0.3)(1-0.4)(1-0.5)$
- $\quad = 1- 0.7 \times 0.6 \times 0.5$
- $\quad = 1-0.21$
- $\quad = 0.79$
- System reliability increases
- [(0.790-0.760)/0.760]x100 = %3.95

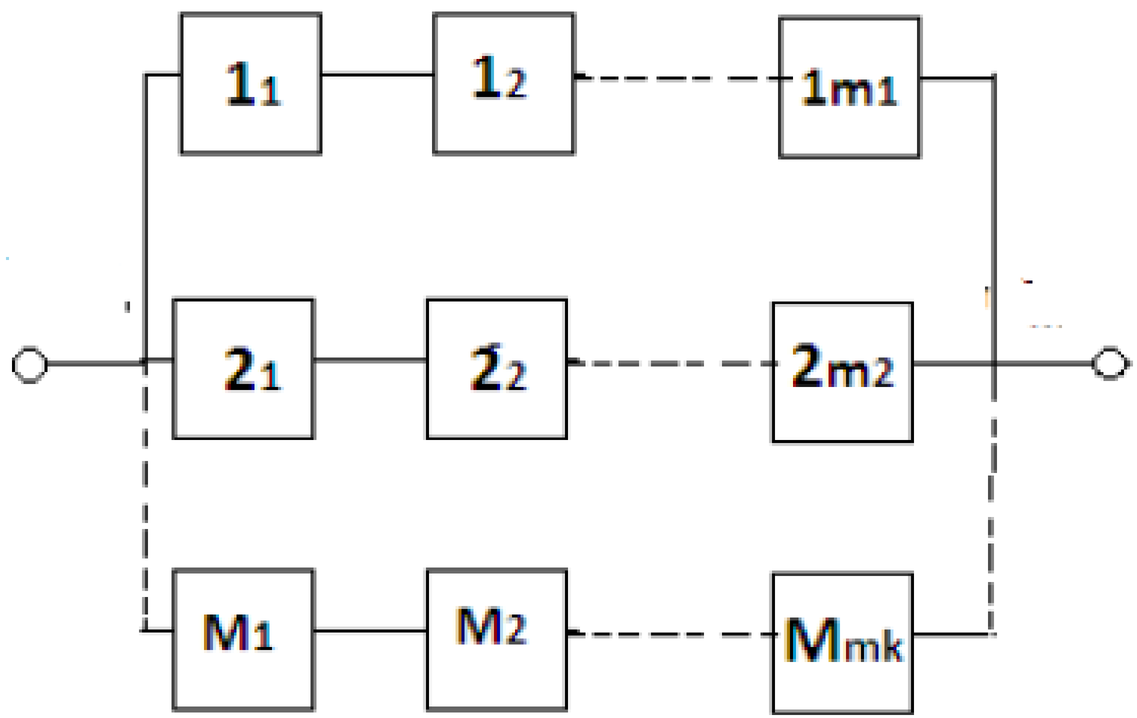
- b) Reliability of the third component is increased by 0.1
- Rp = 1- (1-0.2)(1-0.4)(1-0.6)
- = 1- 0.8x0.6x0.4
- = 1- 0.192
- = 0.808
- System reliability increases
- [(0.808-0.760)/0.760]x100 = % 6.32

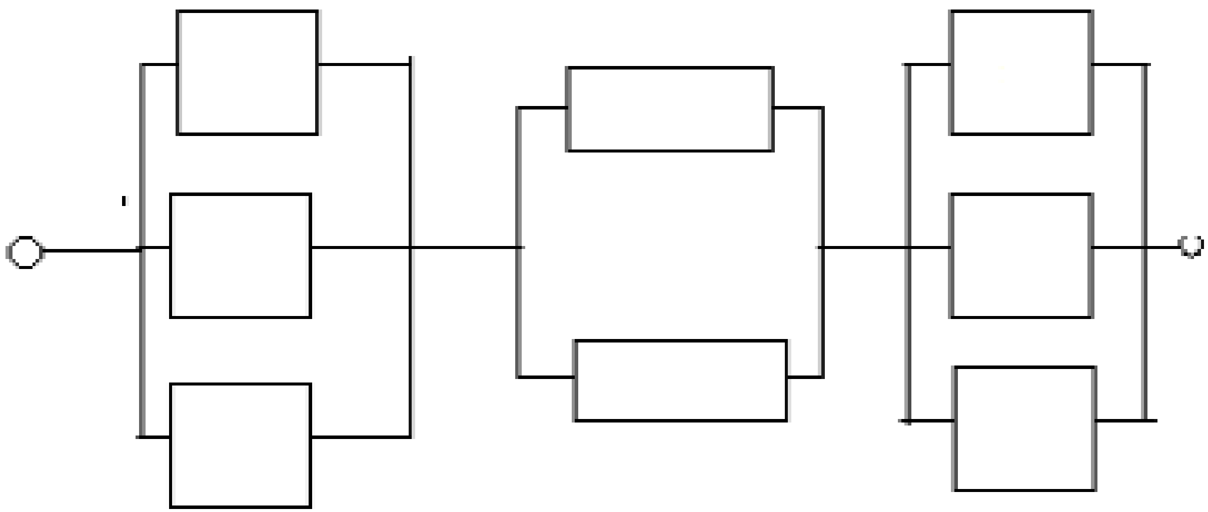
- The improvement in the reliability of the system (in percentage) is higher when reliability of the best component is increased by 0.1.
- If a system engineer wants to improve the reliability of a parallel system, he/she should concentrate on the improvement of the reliability of the best component.

Mixed Systems

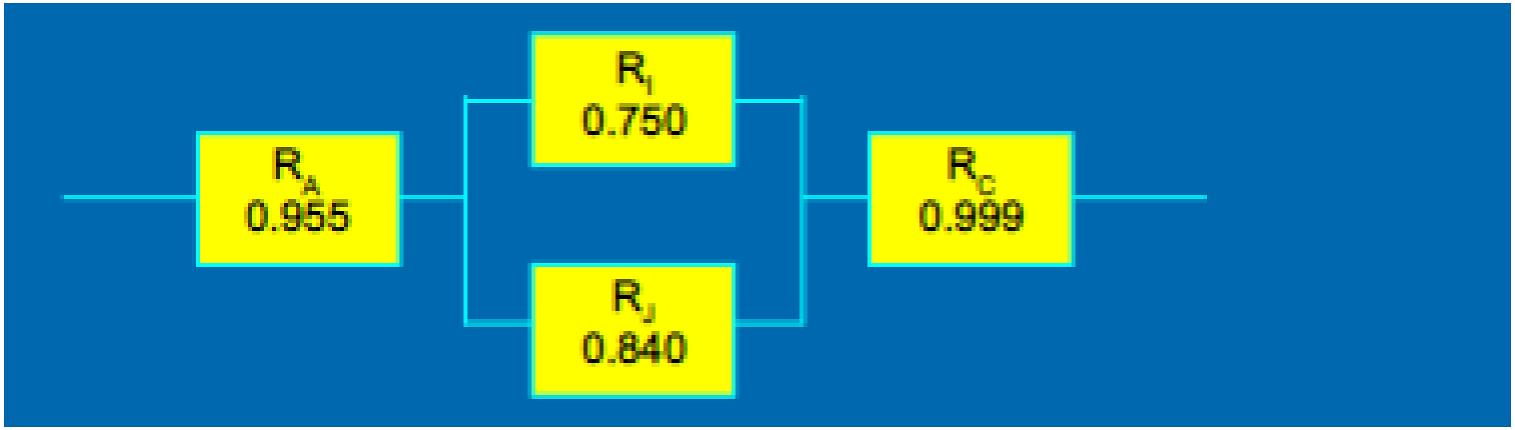
- A system is said to be a mixed system if the components of the system are connected both in series and in parallel configurations.
- To evaluate the reliability of a mixed system, we first break the reliability block diagram into series or parallel subsystems. Then we evaluate the reliability of each subsystem. Finally, we evaluate the reliability of the given mixed system by combining the reliabilities of the subsystems. It is called reduction method.

Mixed System





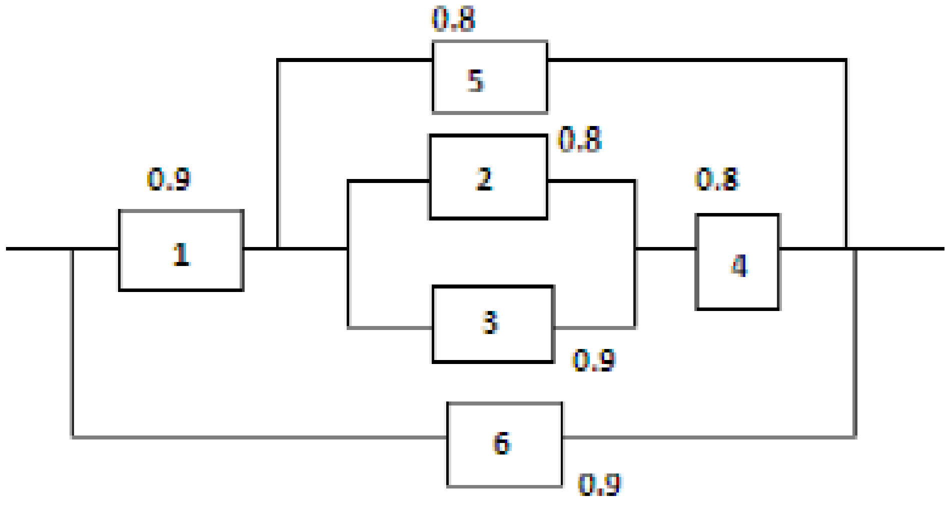
Example

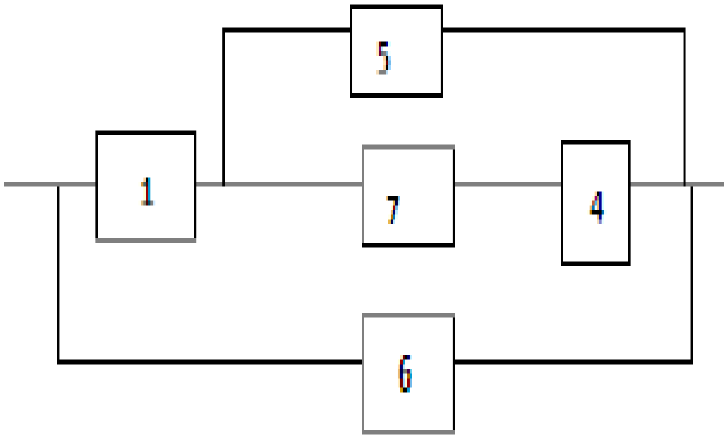


- First of all, we find the parallel system reliability.
- Rıj = 1- (1- 0.750)(1-0.840) =
- = 1- 0.04
- = 0.96
- Then, the system is reduced to series system
- Rs = (RA)(Rıj)(Rc)
- = (0.955)(0.960)(0.999)
- = 0.916

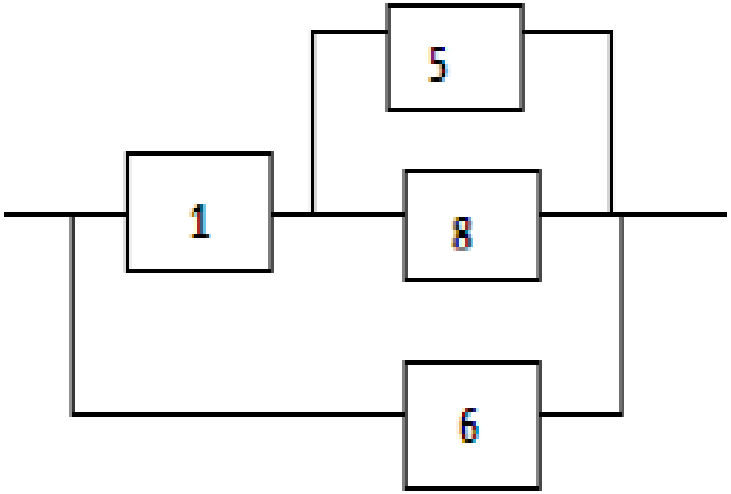
Example

Calculate the reliability of the system shown using network reduction technique?

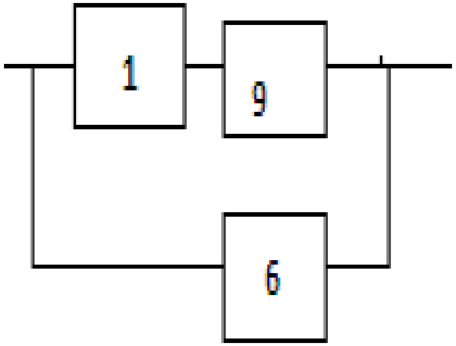




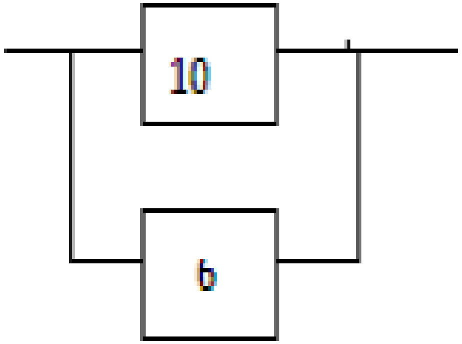
$R_7 = 1 - \{(1-0.8)(1-0.9)\} = 1 - 0.02 = 0.98$



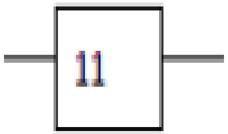
$R_8 = R_7R_4 = 0.98x0.8 = 0.784$



$R_9 = 1-\{(1-R_5)(1-R_8) = 1- (0.2x\ 0.216) = 1-0.0432 = 0.9568$



$R_{10}=R_1R_9=0.9x0.9568 = 0.86112$



$R_{11} 1- \{(1-0.86112)(1-0.9)\} = 1-(0.13888 \times 0.1) = 1-0.01388 = 0.98612$

Resources

- Quality Design and Control, Design for Reliability- I , Lecture – 44 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- Quality Design and Control, Design for Reliability- II , Lecture – 46 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- https://canmedia.mheducation.ca/college/olcsupport/stevenson/5ce/ste39590_ch04S_001-019.pdf, Supplement to Chapter 4 Reliability
- Reliability Engineering Lecture Notes, Vardhaman College of Engineering
- https://web4.uwindsor.ca/users/f/fbaki/85-222.nsf/831fc2c71873e46285256d6e006c367a/b7d85091e772a10185256f84007be5c1/$FILE/Lecture_07_ch6_222_w05_s5.pdf
- https://people.ucalgary.ca/~far/Lectures/SENG637/PDF/SENG637-03.pdf, Dependability Reliability & Dependability, Reliability & Testing of Software Systems Chapter 3: System Reliability Lecture Notes, Department of Electrical & Computer Engineering, University of Calgary, B.H. Far
- https://slideplayer.com/slide/5284571/
- https://www.egyankosh.ac.in/bitstream/123456789/35168/1/Unit-14.pdf, UNIT 14 RELIABILITY EVALUATION OF SIMPLE SYSTEMS, Lecture Notes, ignou the people's university

- **Operations Management :An Integrated Approach, R. Dan Reid, Nada R. Sanders, 4th Edition, John Wiley & Sons, Inc.**
- Power System Reliability, Lecture Notes DR. AUDIH ALFAOURY, 2017- 2018, Al-Balqa Applied University
- **Reliability Engineering,** Kailash C. Kapur , Michael Pecht, 2014 , John Wiley & Sons, Inc