

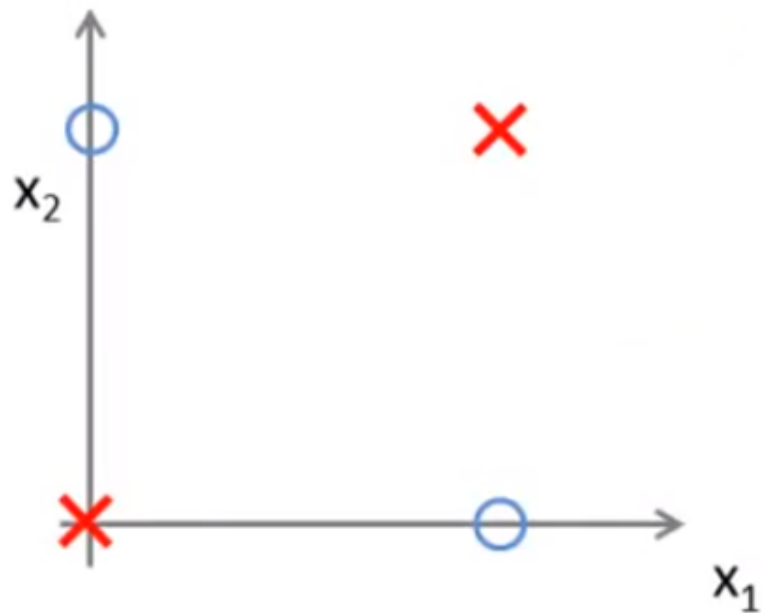
# Examples and Intuitions 1

Applications

*Neural Networks: Representation*

## Non-linear classification example: XOR/XNOR

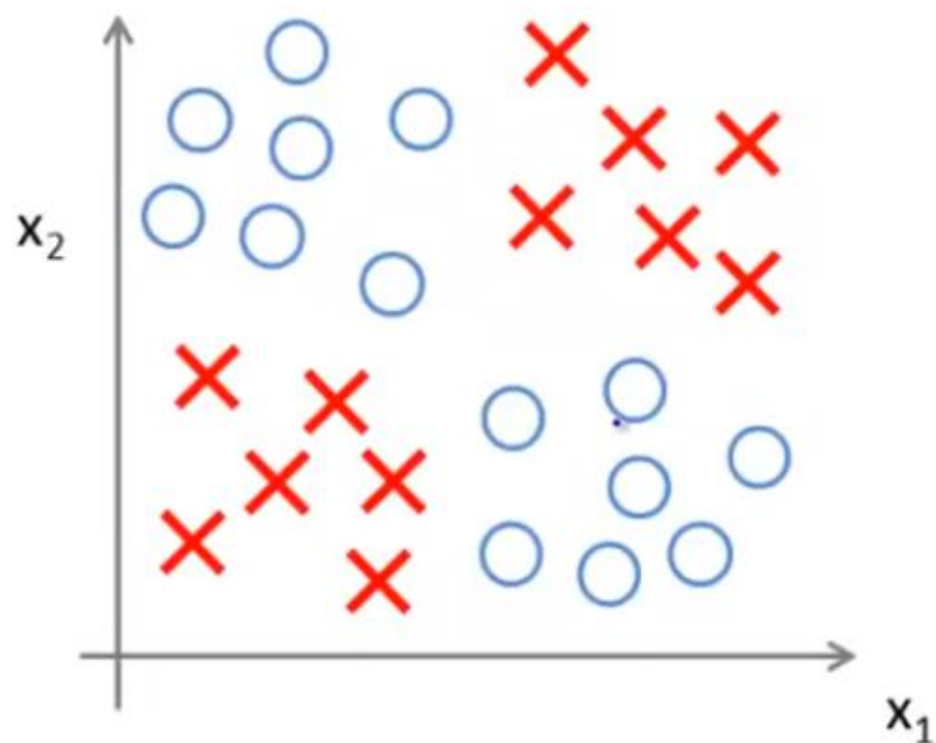
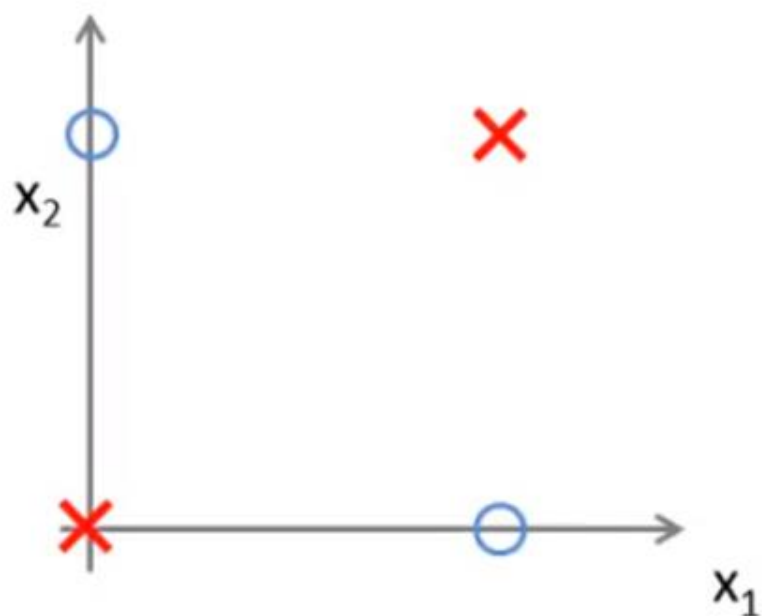
→  $x_1, x_2$  are binary (0 or 1).



A simpler version of...

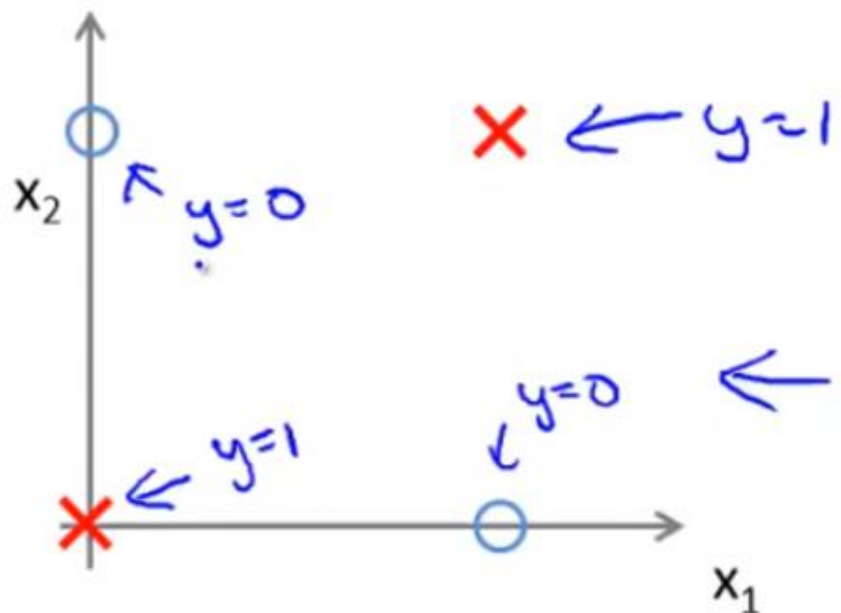
## Non-linear classification example: XOR/XNOR

→  $x_1, x_2$  are binary (0 or 1).



## Non-linear classification example: XOR/XNOR

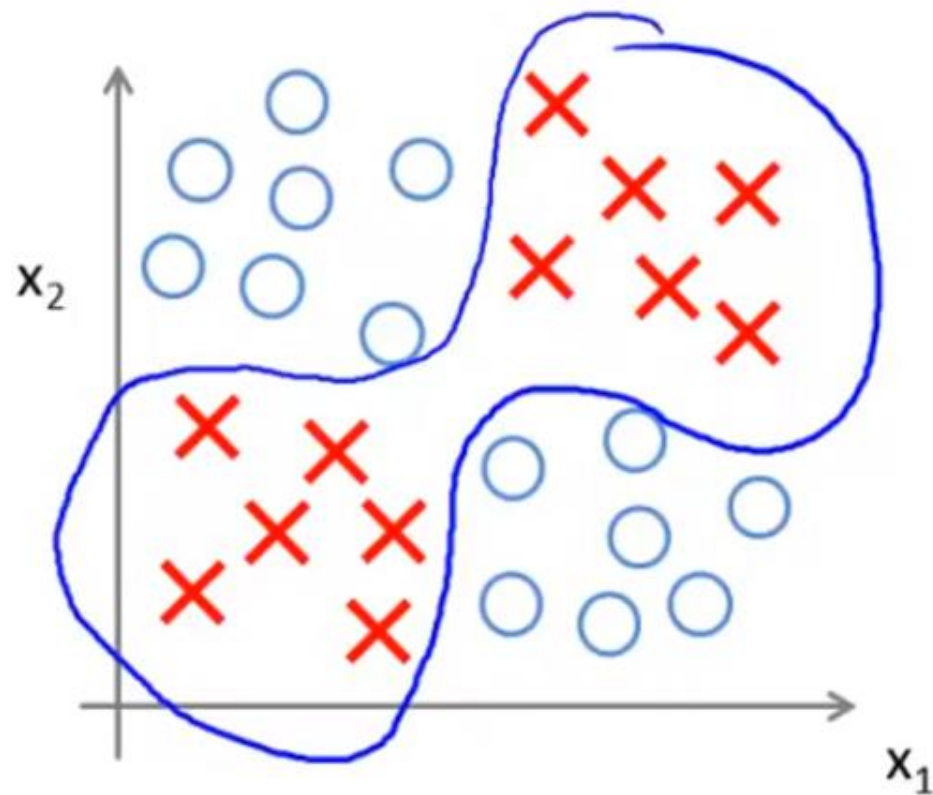
→  $x_1, x_2$  are binary (0 or 1).



$$y = \underline{x_1 \text{ XOR } x_2}$$

$$\underbrace{x_1 \text{ XNOR } x_2}$$

$$\underbrace{\text{NOT } (x_1 \text{ XOR } x_2)}$$



Exclusive OR: outputs true only when inputs differ

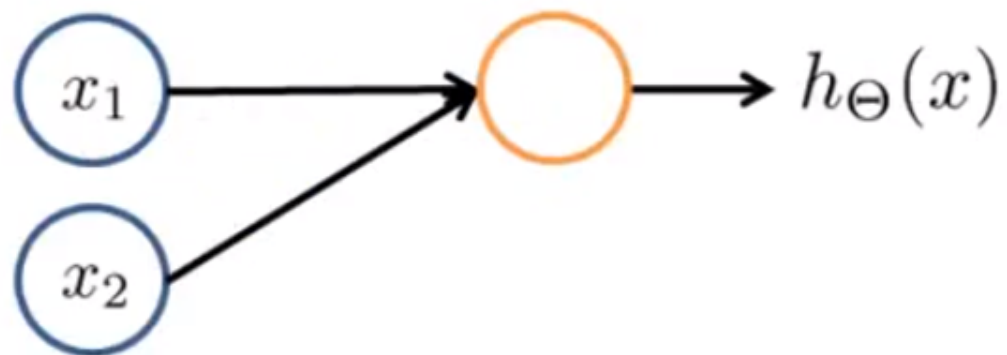
# Building Networks

- We will try to find a network that fits the XOR example.
- But before, bu we're going to start with a slightly simpler one
- We will show a network that fits the **AND** function.

## Simple example: AND

$$x_1, x_2 \in \{0, 1\}$$

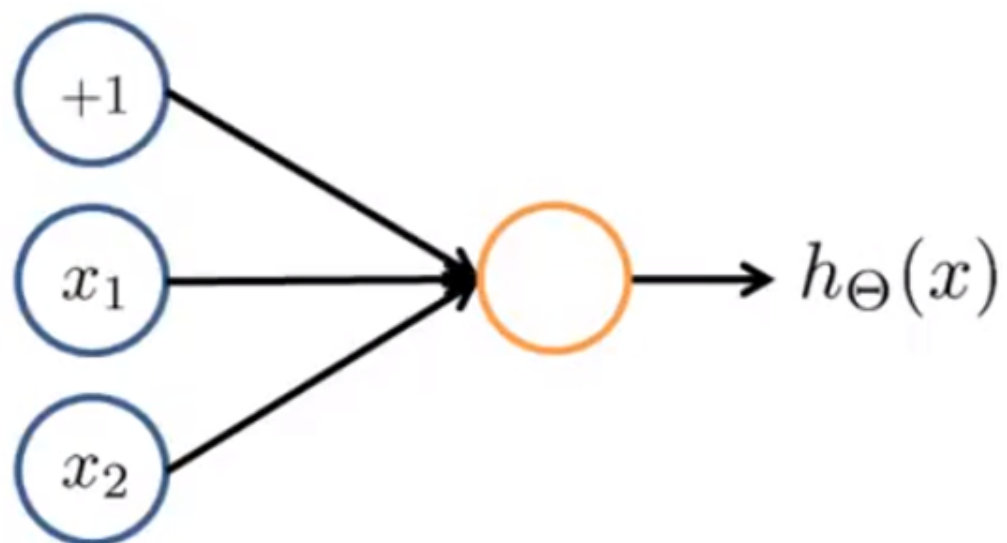
$$y = x_1 \text{ AND } x_2$$



## Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

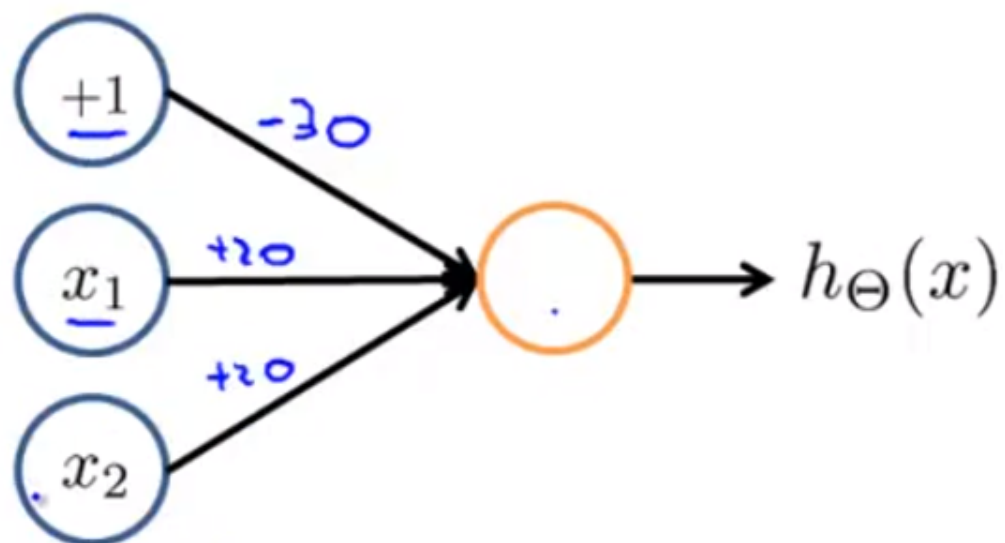
→  $y = x_1 \text{ AND } x_2$



## Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$

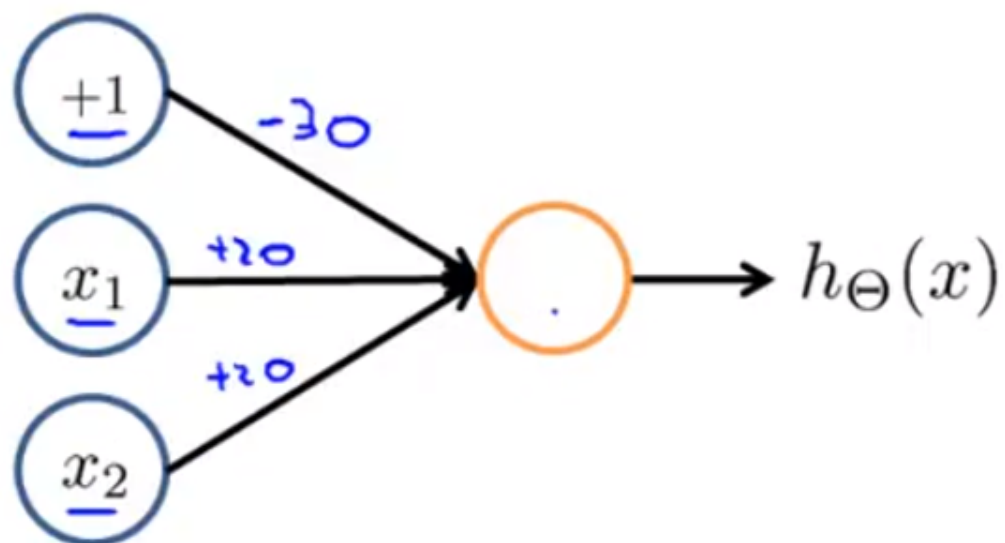




## Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$

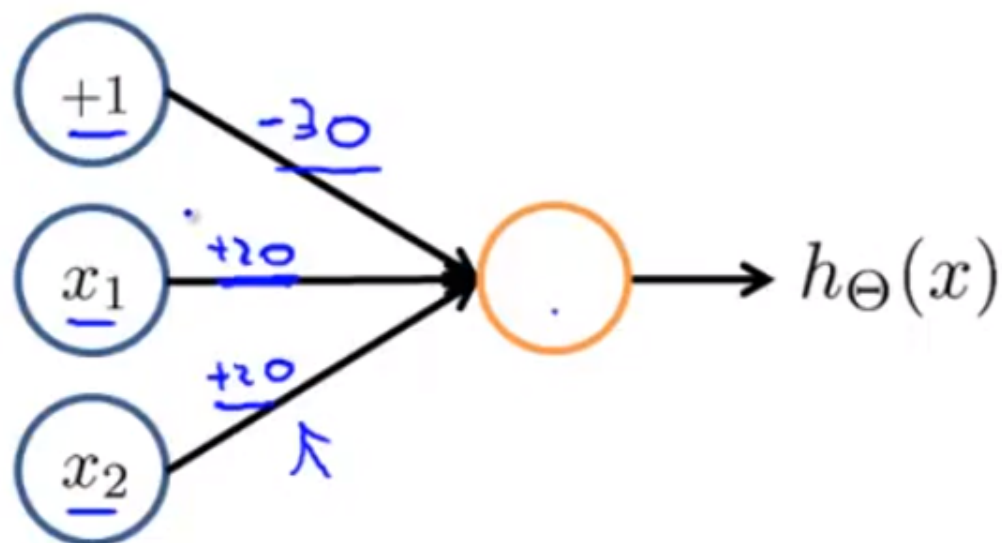


$$h_{\Theta}(x) = g(-30 + 20x_1 + 20x_2)$$

## Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$



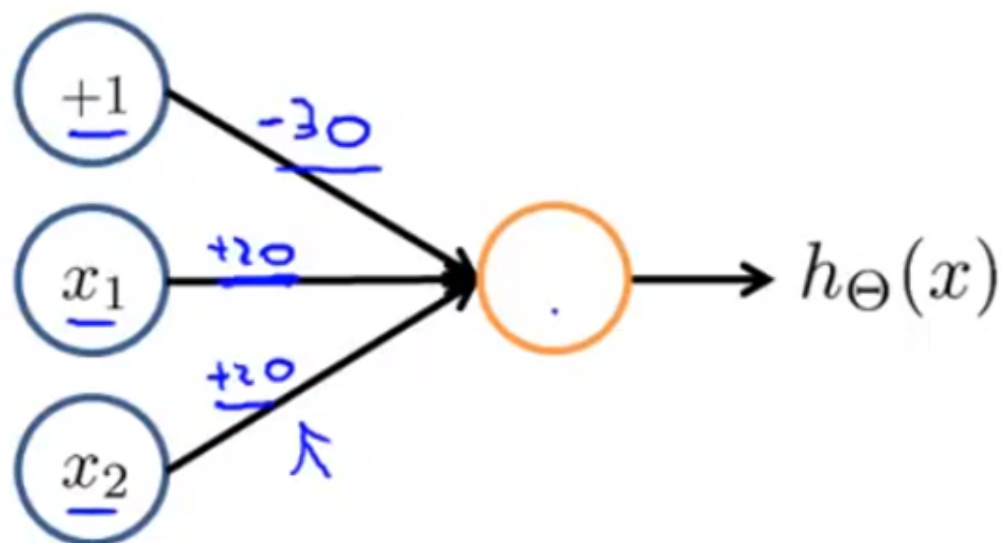
$$h_{\Theta}(x) = g(-30 + 20x_1 + 20x_2)$$

Below the equation, three blue arrows point upwards to the terms  $-30$ ,  $20x_1$ , and  $20x_2$ . Under each arrow is a handwritten label:  $\Theta_{1,0}^{(1)}$  under  $-30$ ,  $\Theta_{1,1}^{(1)}$  under  $20x_1$ , and  $\Theta_{1,2}^{(1)}$  under  $20x_2$ .

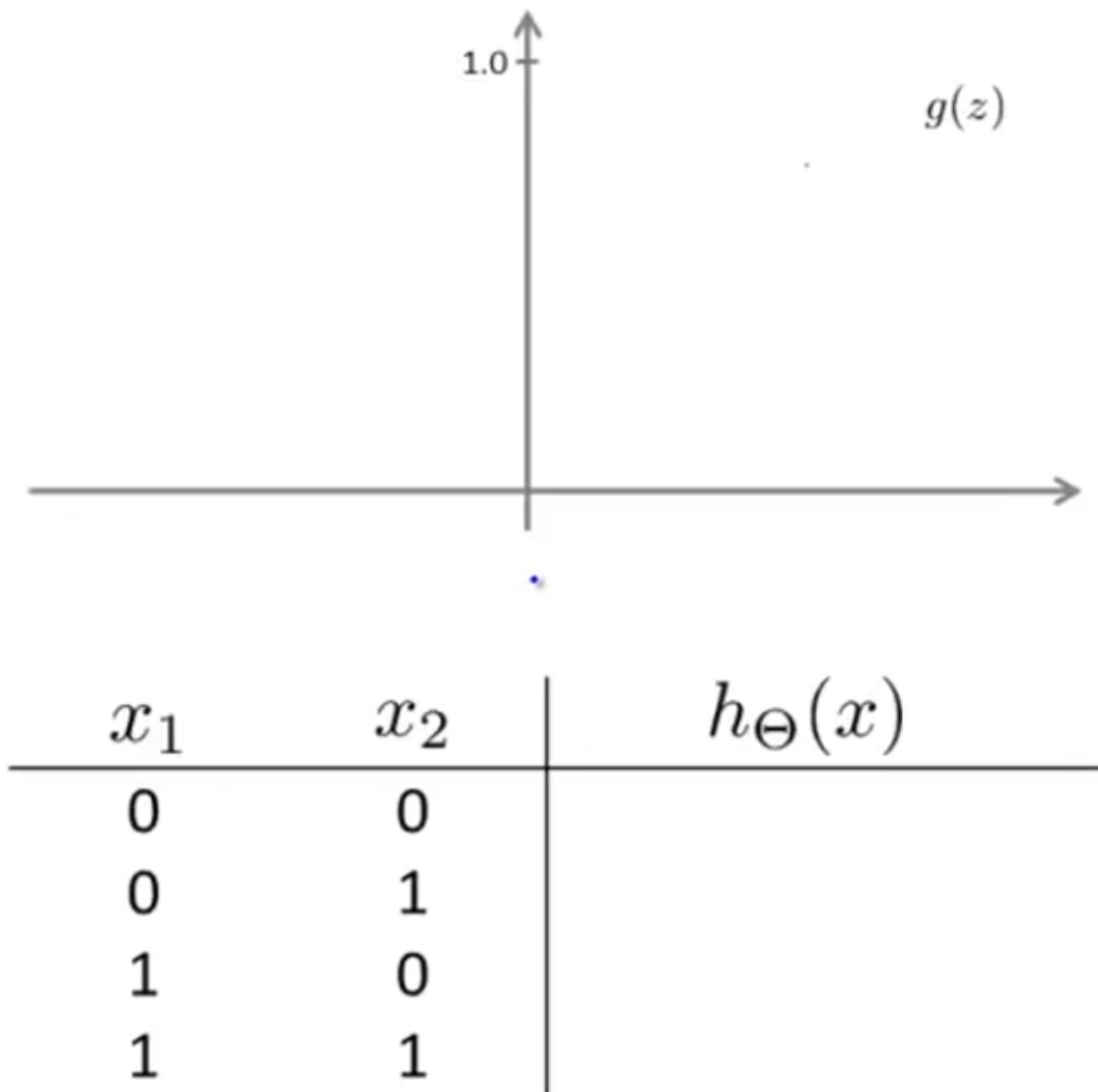
## Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$



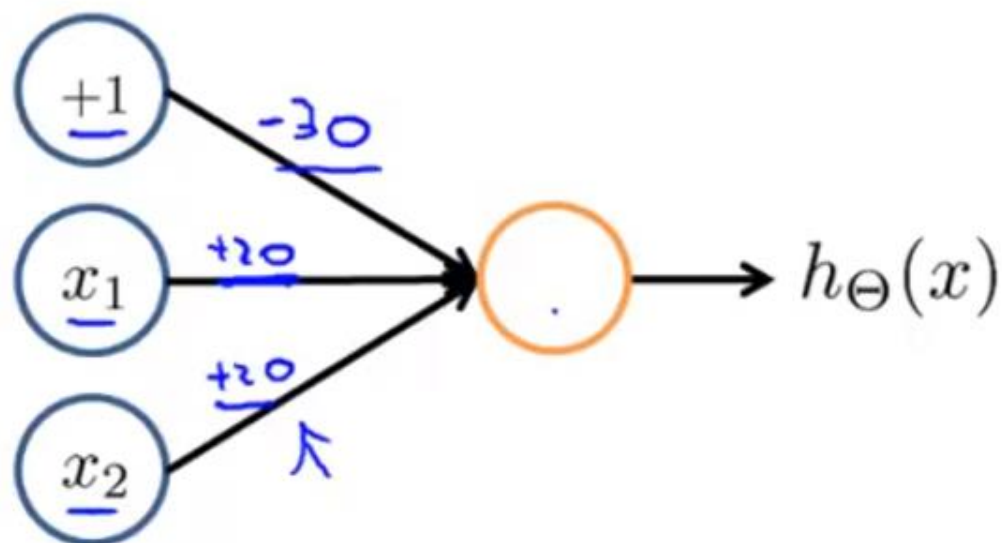
$$h_{\Theta}(x) = g(\underbrace{-30}_{w_{10}^{(1)}} + \underbrace{20}_{w_{11}^{(1)}}x_1 + \underbrace{20}_{w_{12}^{(1)}}x_2)$$



## Simple example: AND

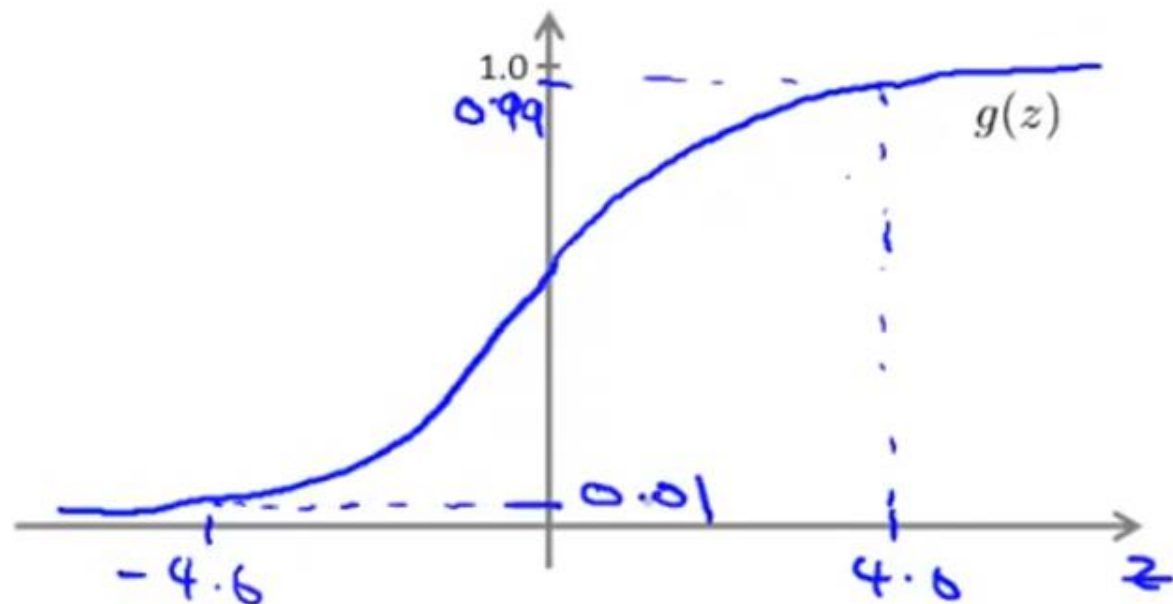
→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$



$$h_{\Theta}(x) = g(-3.0 + 2.0x_1 + 2.0x_2)$$

Handwritten labels below the equation:  $\omega_{1,0}^{(1)}$  points to  $-3.0$ ,  $\omega_{1,1}^{(1)}$  points to  $2.0x_1$ , and  $\omega_{1,2}^{(1)}$  points to  $2.0x_2$ .

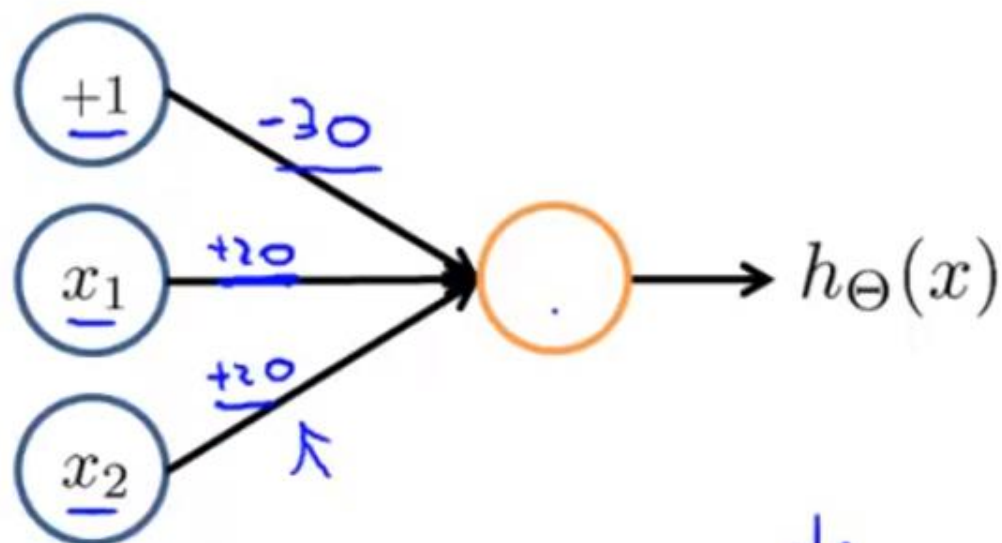


$x_1$	$x_2$	$h_{\Theta}(x)$
0	0	
0	1	
1	0	
1	1	

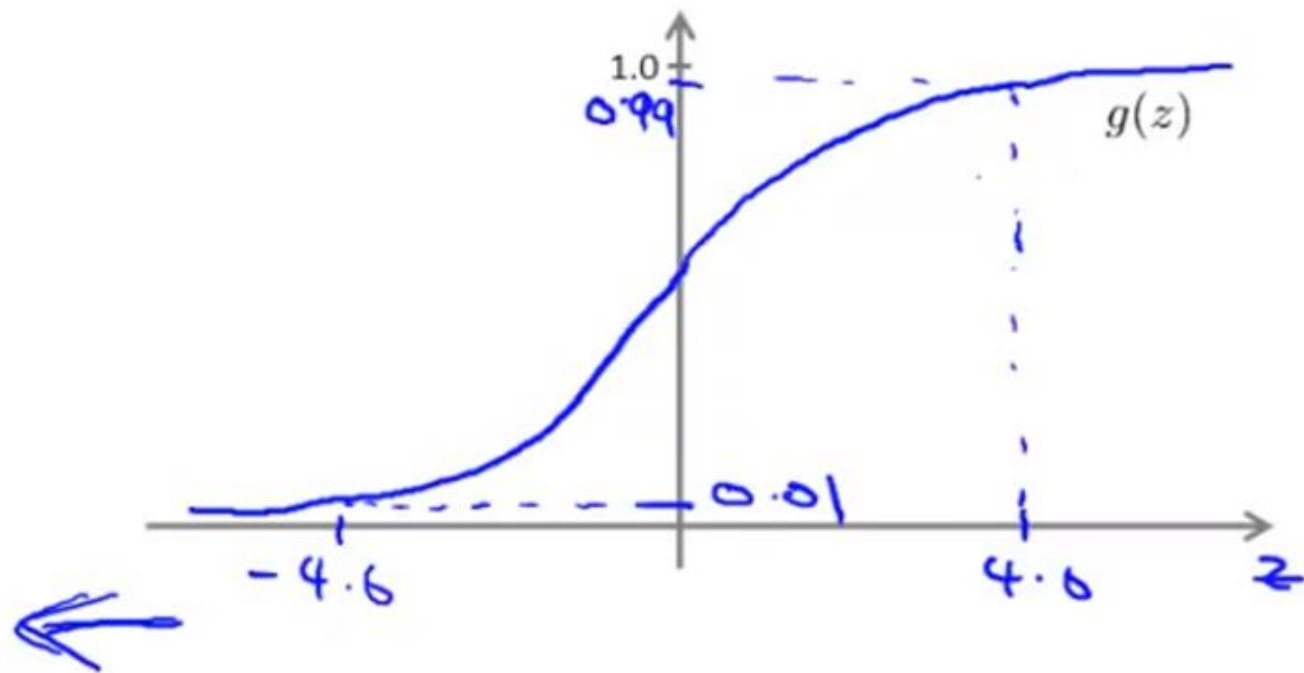
# Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$



$$\rightarrow h_{\Theta}(x) = g\left(\underbrace{-30}_{w_{(1)}^{(0)}} + \underbrace{20}_{w_{(1)}^{(1)}}x_1 + \underbrace{20}_{w_{(1)}^{(2)}}x_2\right) \leftarrow$$



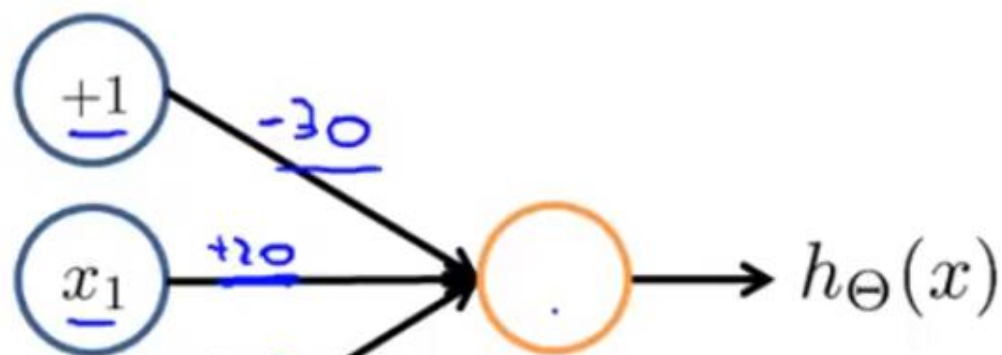
$x_1$	$x_2$	$h_{\Theta}(x)$
0	0	$g(-30) \approx 0$
→ 0	1	$g(-10) \approx 0$
1	0	$g(-10) \approx 0$
1	1	$g(10) \approx 1$



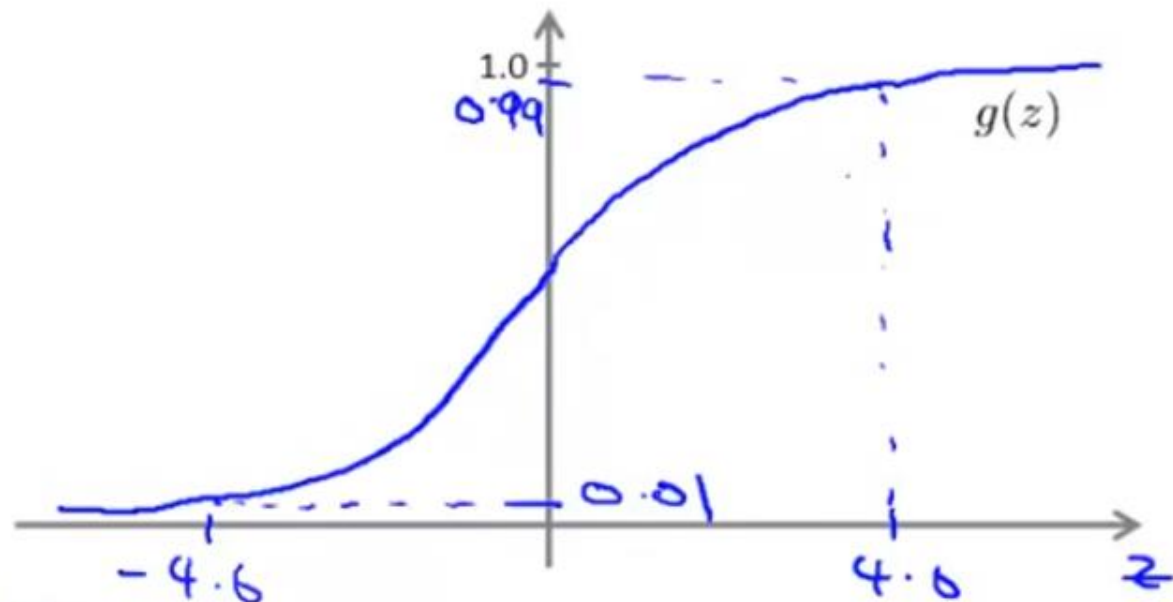
## Simple example: AND

→  $x_1, x_2 \in \{0, 1\}$

→  $y = x_1 \text{ AND } x_2$



$$\rightarrow h_{\Theta}(x) = g\left(\underbrace{-30}_{w_{10}} + \underbrace{20}_{w_{11}}x_1 + \underbrace{20}_{w_{12}}x_2\right)$$

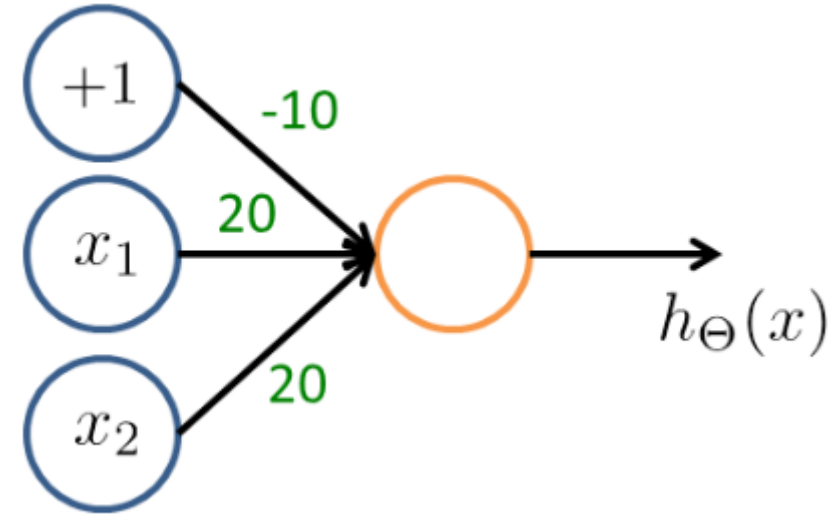


$x_1$	$x_2$	$h_{\Theta}(x)$
0	0	$g(-30) \approx 0$
0	1	$g(-10) \approx 0$
1	0	$g(-10) \approx 0$
1	1	$g(10) \approx 1$

$$h_{\Theta}(x) \approx x_1 \text{ AND } x_2$$

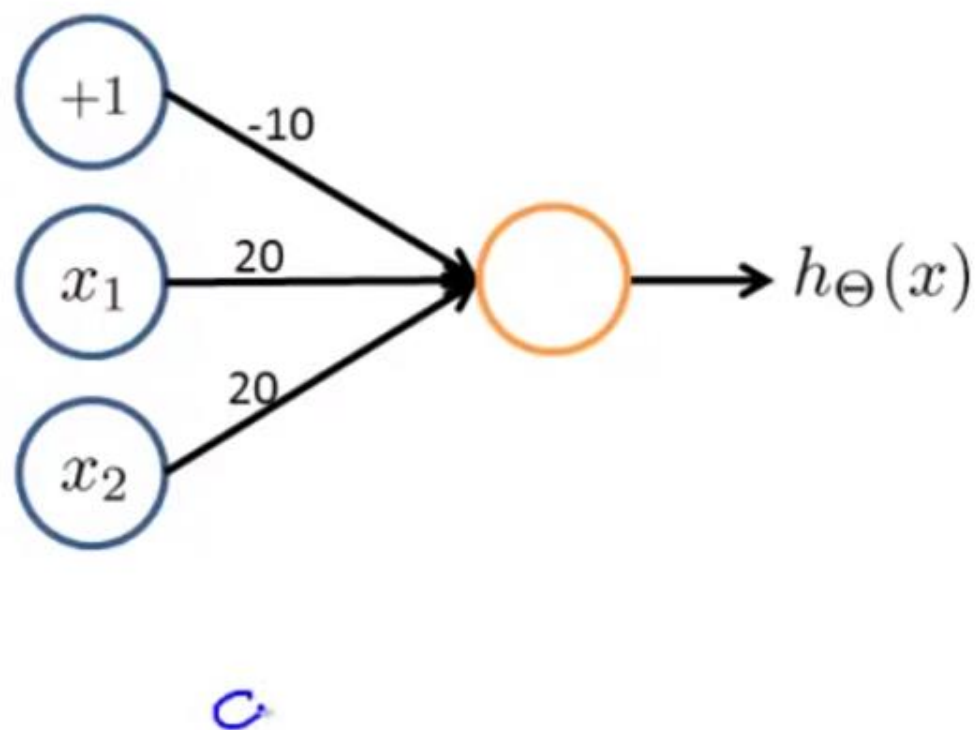
# Exercise

- Suppose  $x_1$  and  $x_2$  are binary valued (0 or 1). What boolean function does the network shown below (approximately) compute?
- (Hint: One possible way to answer this is to draw out a truth table, similar to what we did in the previous slide).



- $x_1$  AND  $x_2$
- (NOT  $x_1$ ) OR (NOT  $x_2$ )
- $x_1$  OR  $x_2$
- (NOT  $x_1$ ) AND (NOT  $x_2$ )

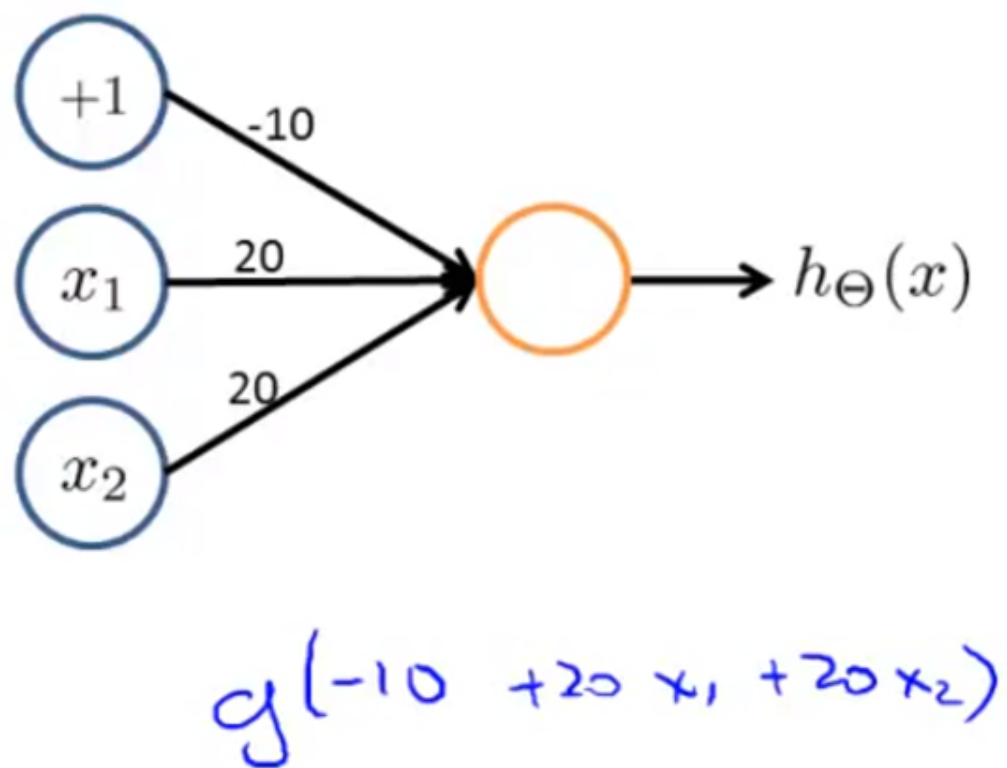
## Example: OR function



$x_1$	$x_2$	$h_{\Theta}(x)$
0	0	
0	1	
1	0	
1	1	



## Example: OR function



$x_1$	$x_2$	$h_{\Theta}(x)$
0	0	$g(-10) \approx 0$
0	1	$g(10) \approx 1$
1	0	$\approx 1$
1	1	$\approx 1$

# Examples and Intuitions 2

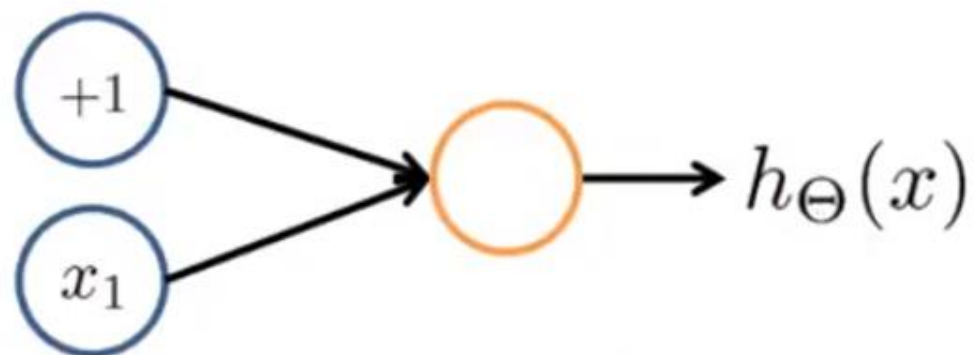
Applications

*Neural Networks: Representation*

$\rightarrow x_1 \text{ AND } x_2$

$\rightarrow x_1 \text{ OR } x_2$

**Negation:**



$x_1$	$h_{\Theta}(x)$
0	
1	

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

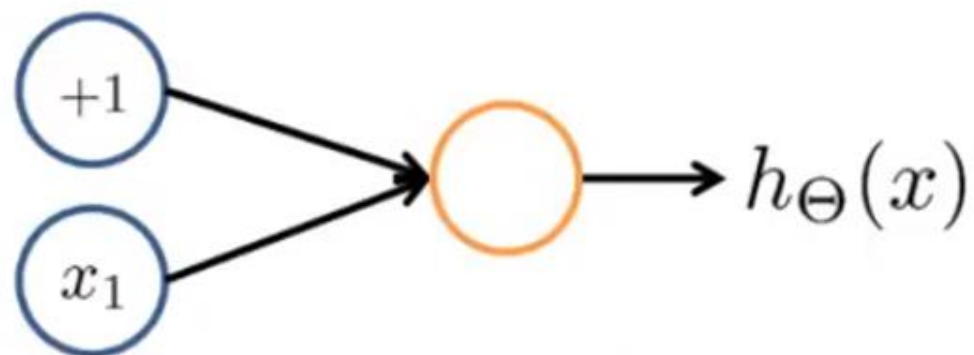
$\rightarrow x_1 \text{ AND } x_2$

$\rightarrow x_1 \text{ OR } x_2$

$\{0, 1\}$

**Negation:**

NOT  $x_1$



$x_1$	$h_{\Theta}(x)$
0	
1	

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

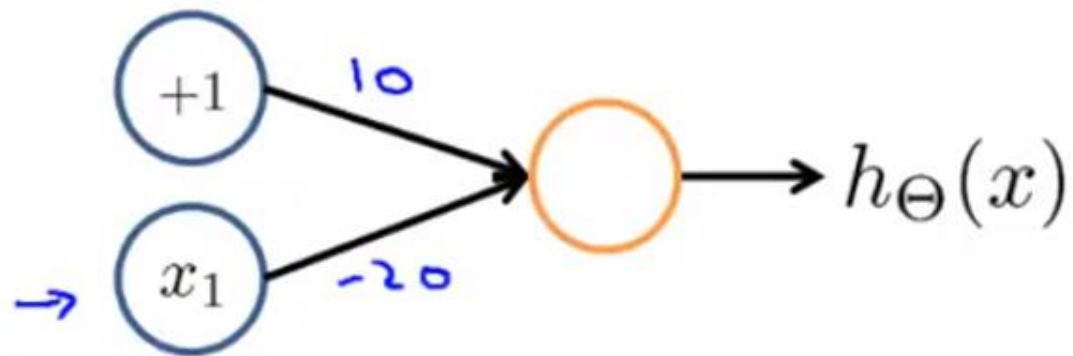
$\rightarrow x_1$  AND  $x_2$

$\rightarrow x_1$  OR  $x_2$

$\{0, 1\}$

**Negation:**

NOT  $x_1$



$x_1$	$h_{\Theta}(x)$
0	
1	

$$h_{\Theta}(x) = g(10 - 20x_1)$$

Windows'u Etkinleştir  
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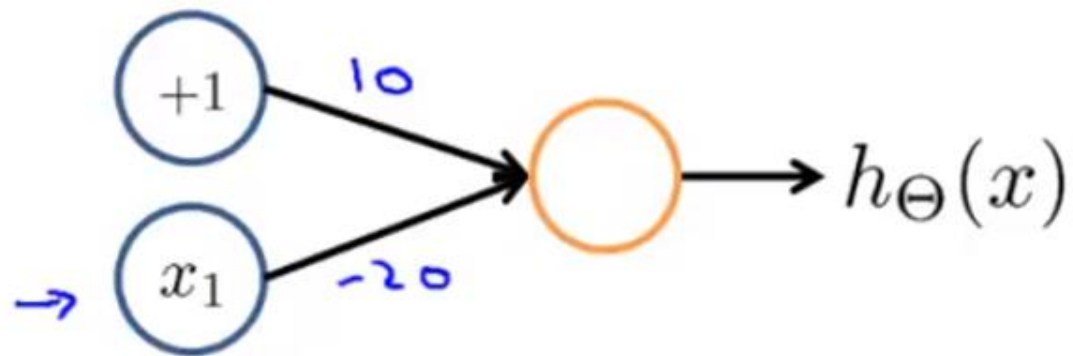
$\rightarrow x_1$  AND  $x_2$

$\rightarrow x_1$  OR  $x_2$

$\{0, 1\}$ .

**Negation:**

NOT  $x_1$



$x_1$	$h_{\Theta}(x)$
0	$g(10) \approx 1$
1	$g(-10) \approx 0$

$$h_{\Theta}(x) = g(10 - 20x_1)$$

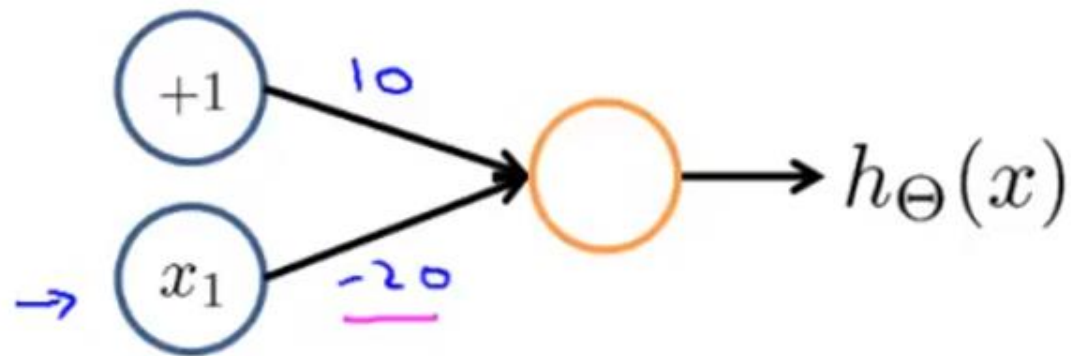
$\rightarrow x_1 \text{ AND } x_2$

$\rightarrow x_1 \text{ OR } x_2$

$\{0, 1\}$

**Negation:**

NOT  $x_1$



$x_1$	$h_{\Theta}(x)$
0	$g(10) \approx 1$
1	$g(-10) \approx 0$

$$h_{\Theta}(x) = g(10 - 20x_1)$$

$\rightarrow (\text{NOT } \underline{x_1}) \text{ AND } (\text{NOT } \underline{x_2})$

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

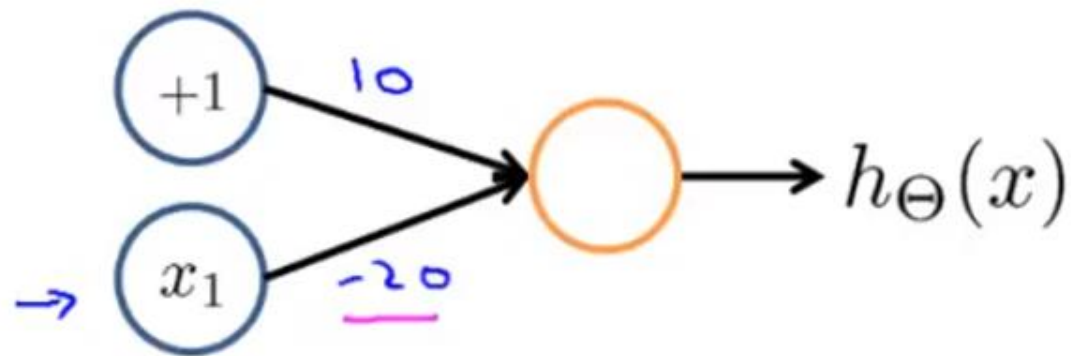
$\rightarrow x_1 \text{ AND } x_2$

$\rightarrow x_1 \text{ OR } x_2$

$\{0, 1\}$

**Negation:**

NOT  $x_1$



$x_1$	$h_{\Theta}(x)$
0	$g(10) \approx 1$
1	$g(-10) \approx 0$

$$h_{\Theta}(x) = g(10 - 20x_1)$$

$\rightarrow (\text{NOT } x_1) \text{ AND } (\text{NOT } x_2)$

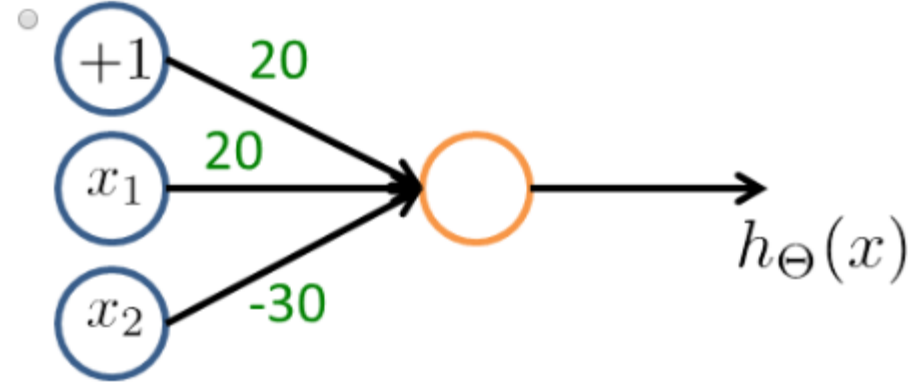
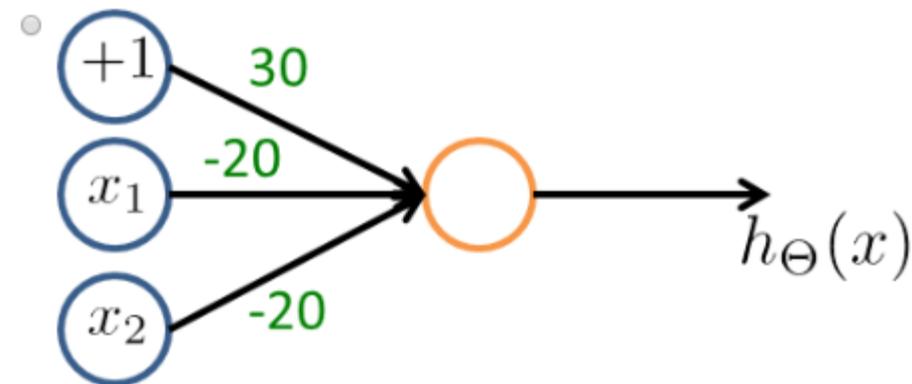
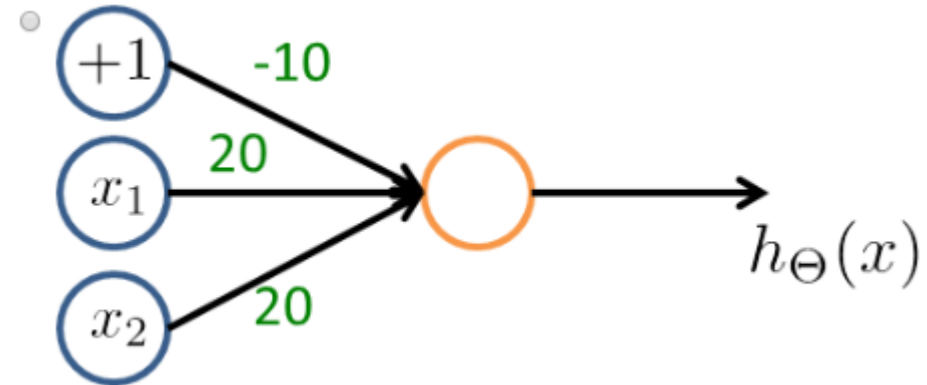
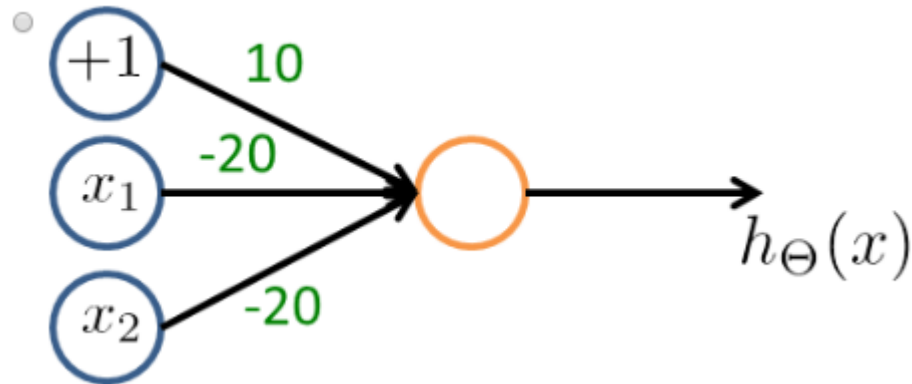
$(= 1 \text{ if and only if } x_1 = x_2 = 0)$

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

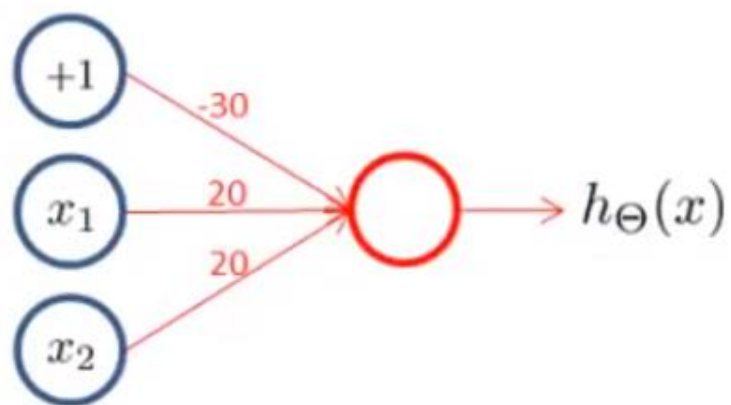


# Exercise

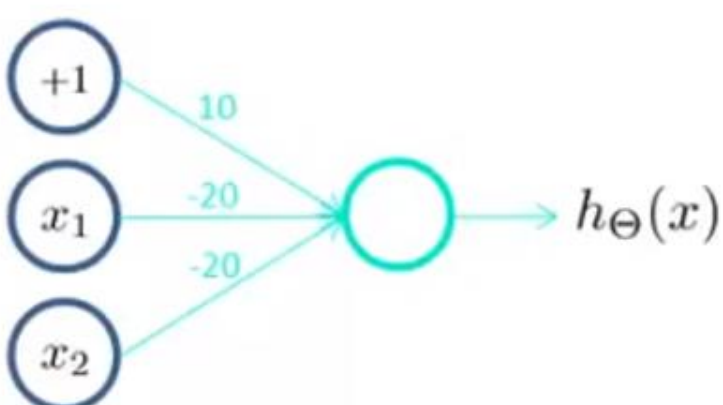
- Suppose that  $x_1$  and  $x_2$  are binary valued (0 or 1). Which of the following networks (approximately) computes the boolean function (NOT  $x_1$ ) AND (NOT  $x_2$ )?



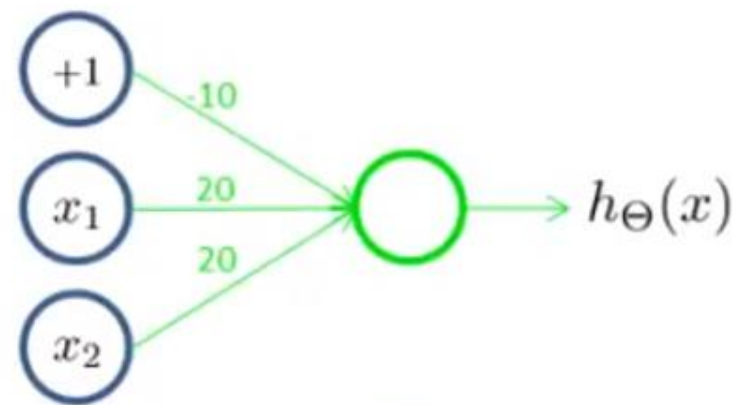
## Putting it together: $x_1$ XNOR $x_2$



\*  $x_1$  AND  $x_2$



$(\text{NOT } x_1) \text{ AND } (\text{NOT } x_2)$



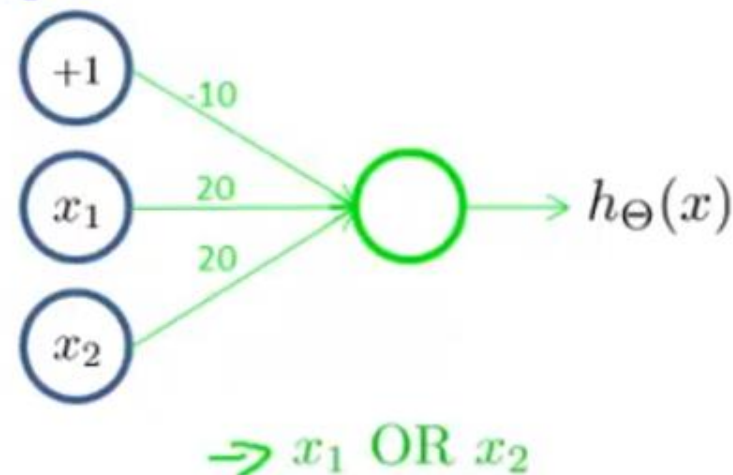
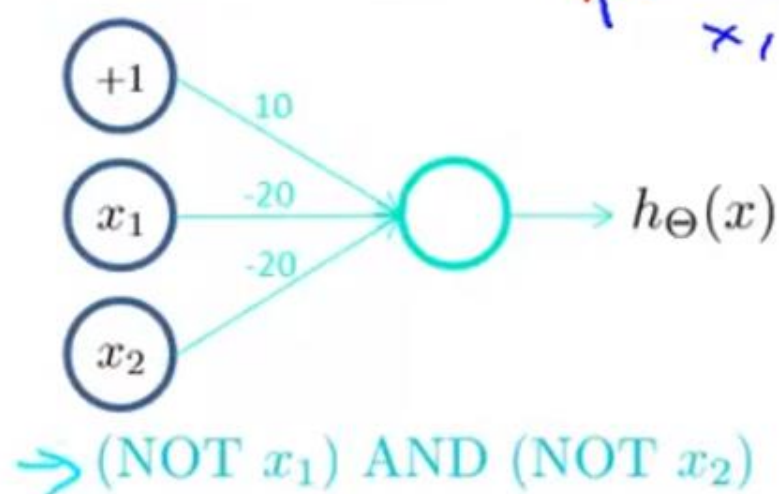
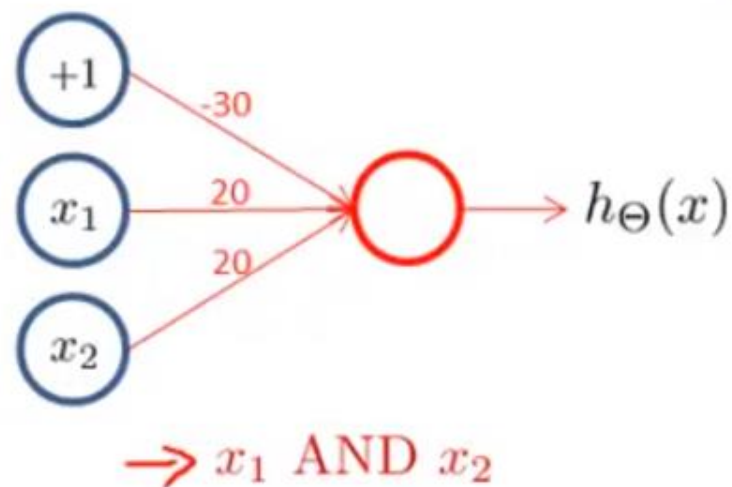
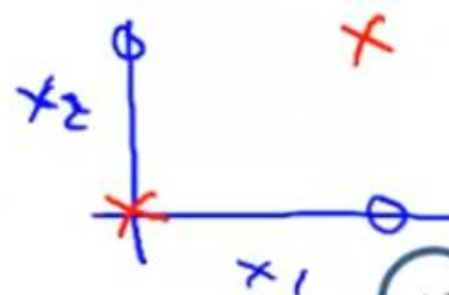
$x_1 \text{ OR } x_2$



$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0			
0	1			
1	0			
1	1			

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

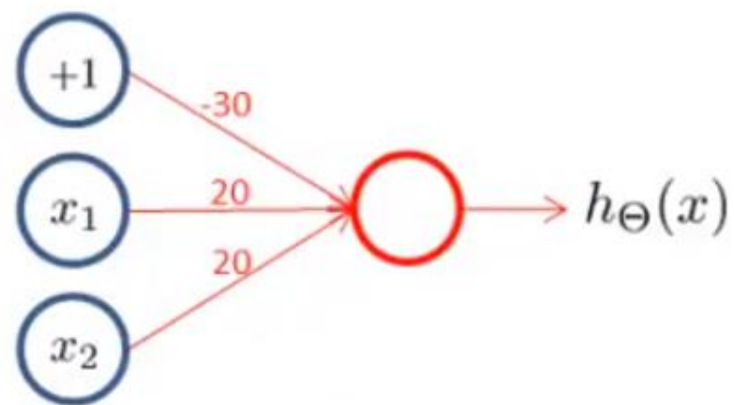
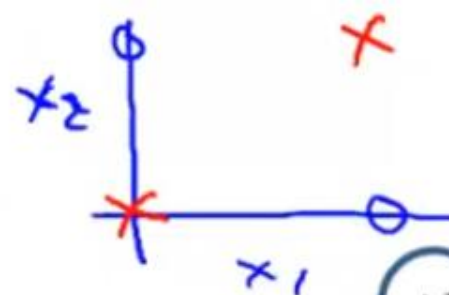
# Putting it together: $x_1$ XNOR $x_2$



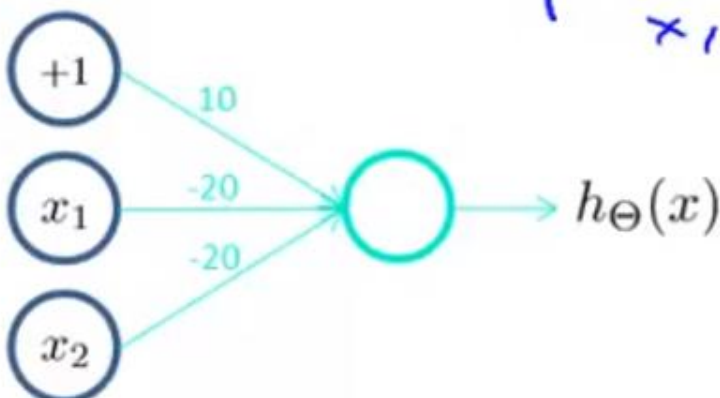
$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0			
0	1			
1	0			
1	1			

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

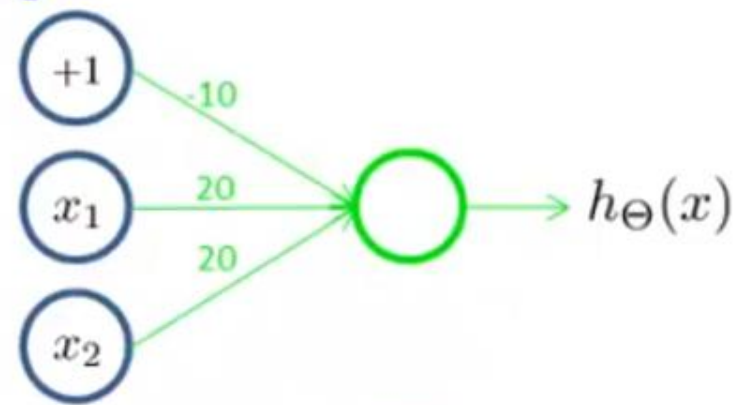
Putting it together:  $x_1$  XNOR  $x_2$



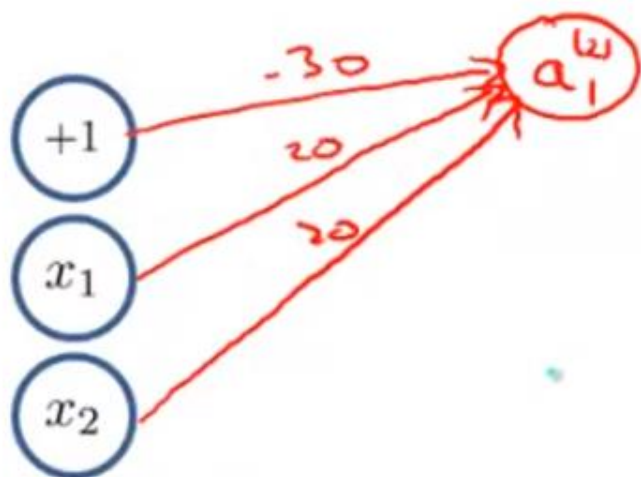
$\rightarrow x_1$  AND  $x_2$



$\rightarrow$  (NOT  $x_1$ ) AND (NOT  $x_2$ )



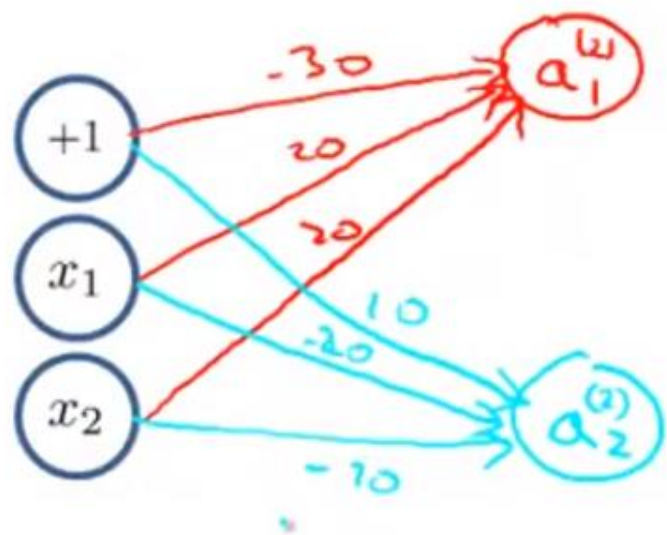
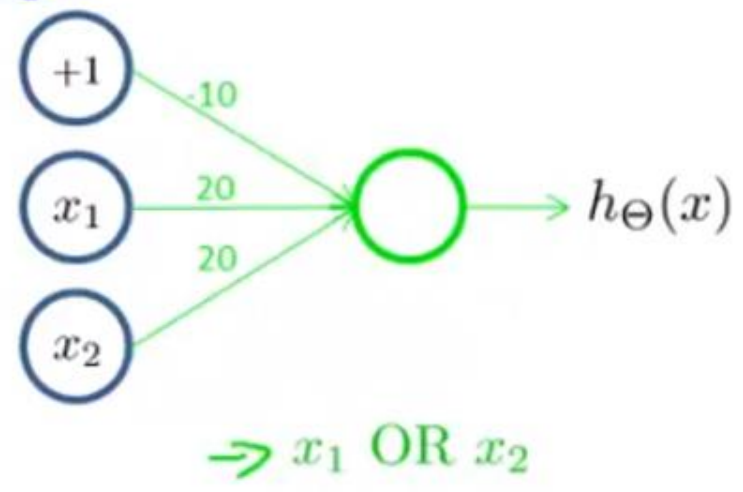
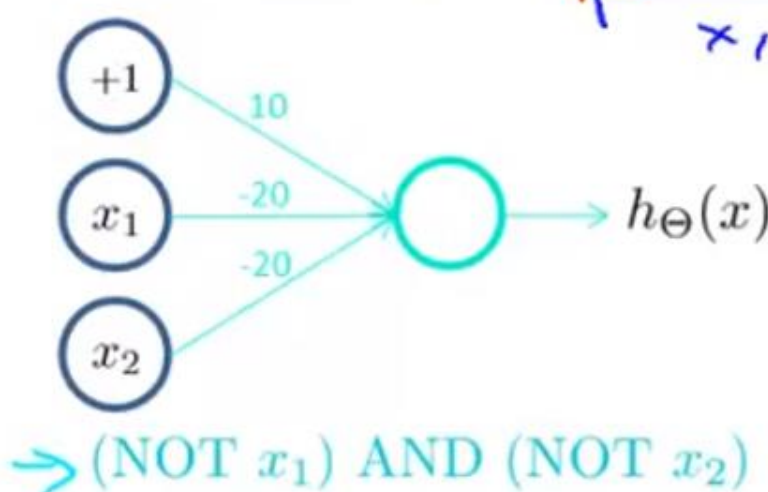
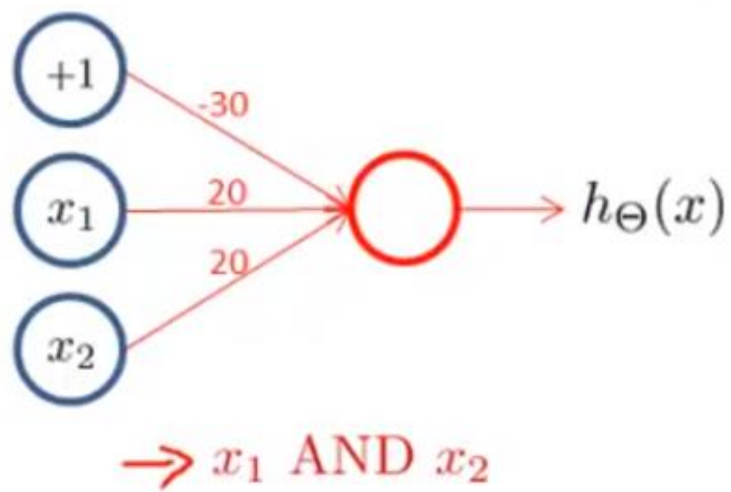
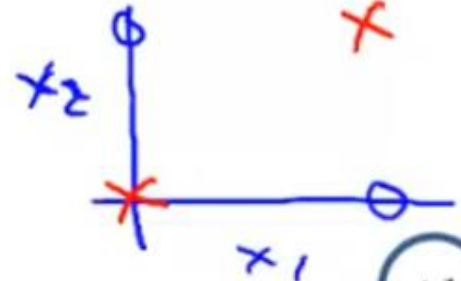
$\rightarrow x_1$  OR  $x_2$



$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0			
0	1			
1	0			
1	1			

Windows'u Etkinleştir  
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# Putting it together: $x_1$ XNOR $x_2$

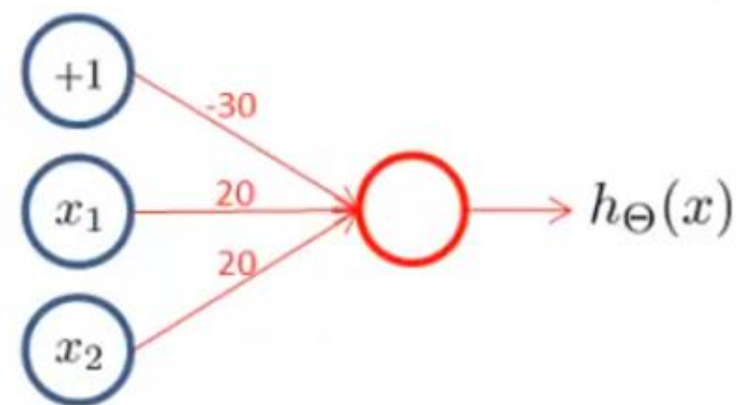
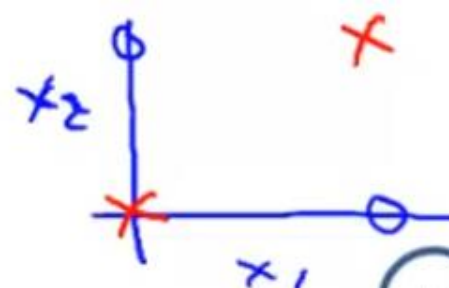


$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0			
0	1			
1	0			
1	1			

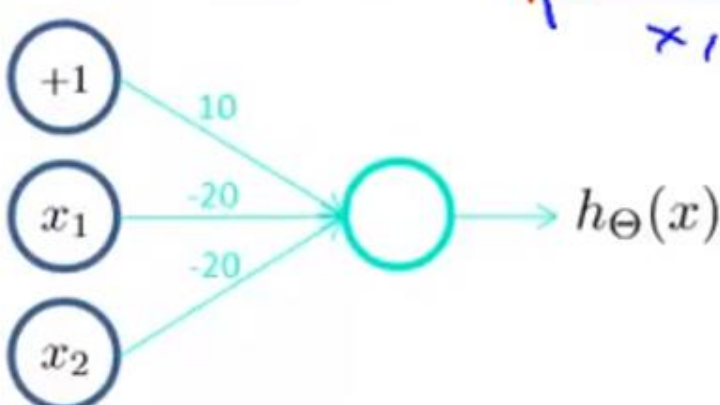
Windows'u Etkinleştir  
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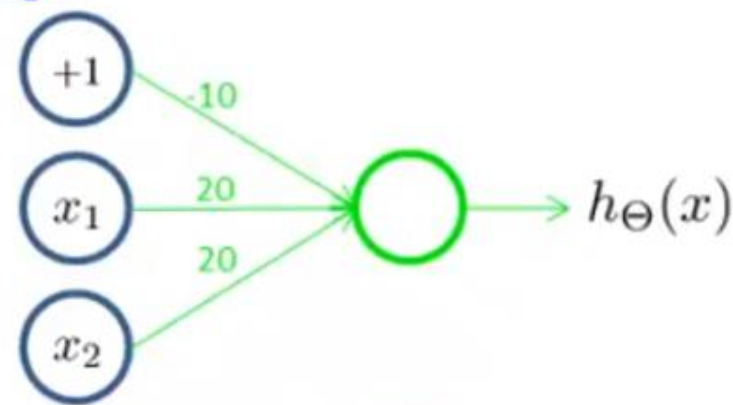
# Putting it together: $x_1$ XNOR $x_2$



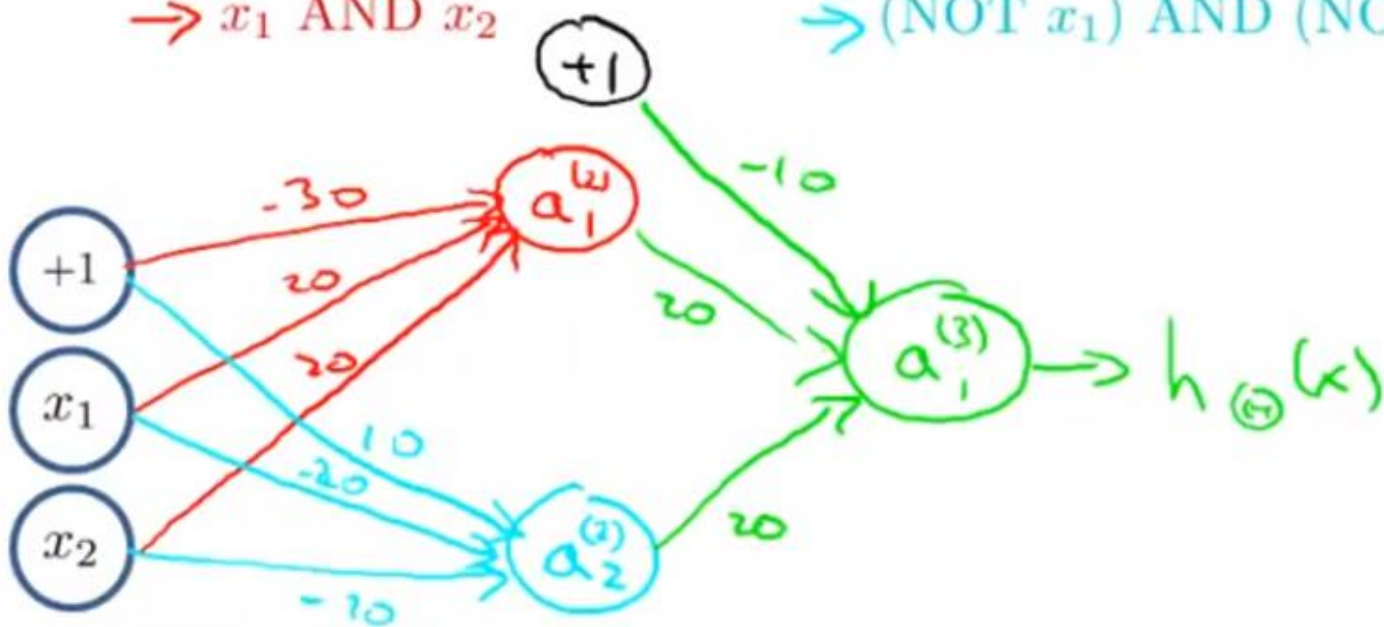
→  $x_1$  AND  $x_2$



→ (NOT  $x_1$ ) AND (NOT  $x_2$ )



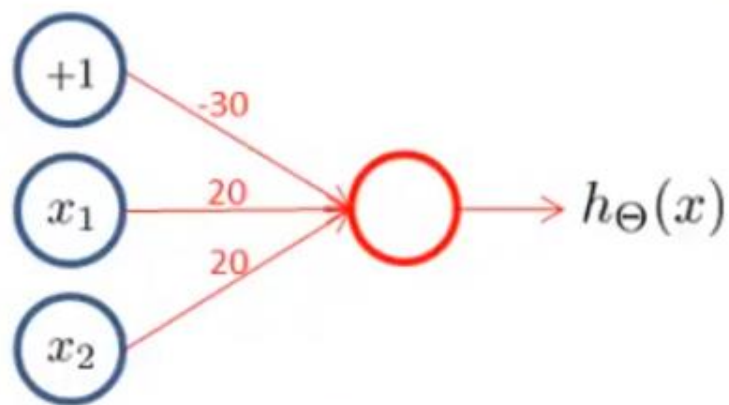
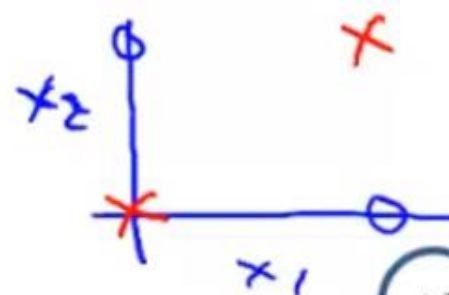
→  $x_1$  OR  $x_2$



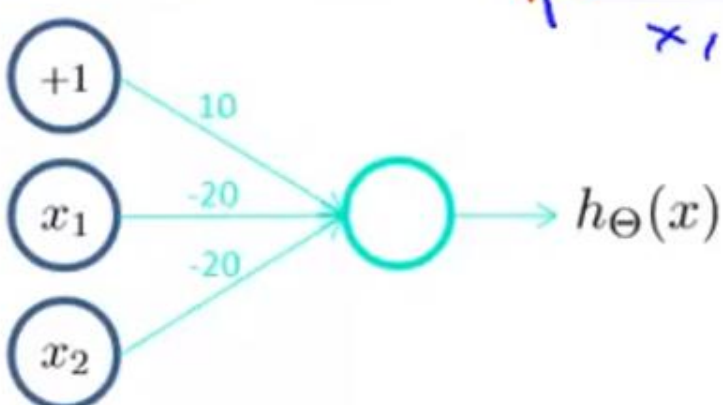
$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0	0	1	
0	1	0	0	
1	0	0	0	
1	1	1	0	

Windows'u Etkinleştir  
Windows'u etkinleştirmek için Ayarlar'a gidin.

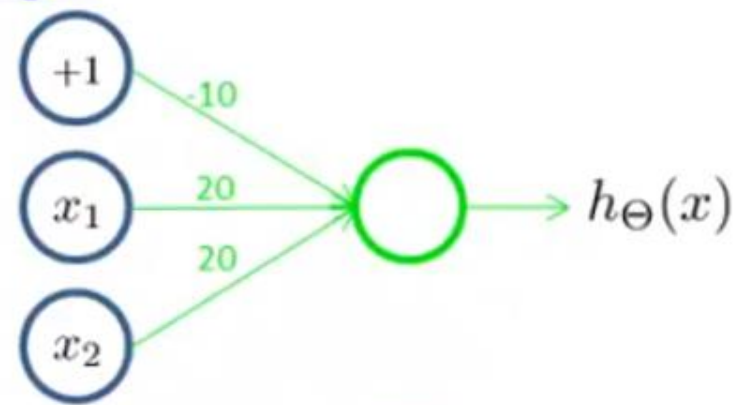
# Putting it together: $x_1$ XNOR $x_2$



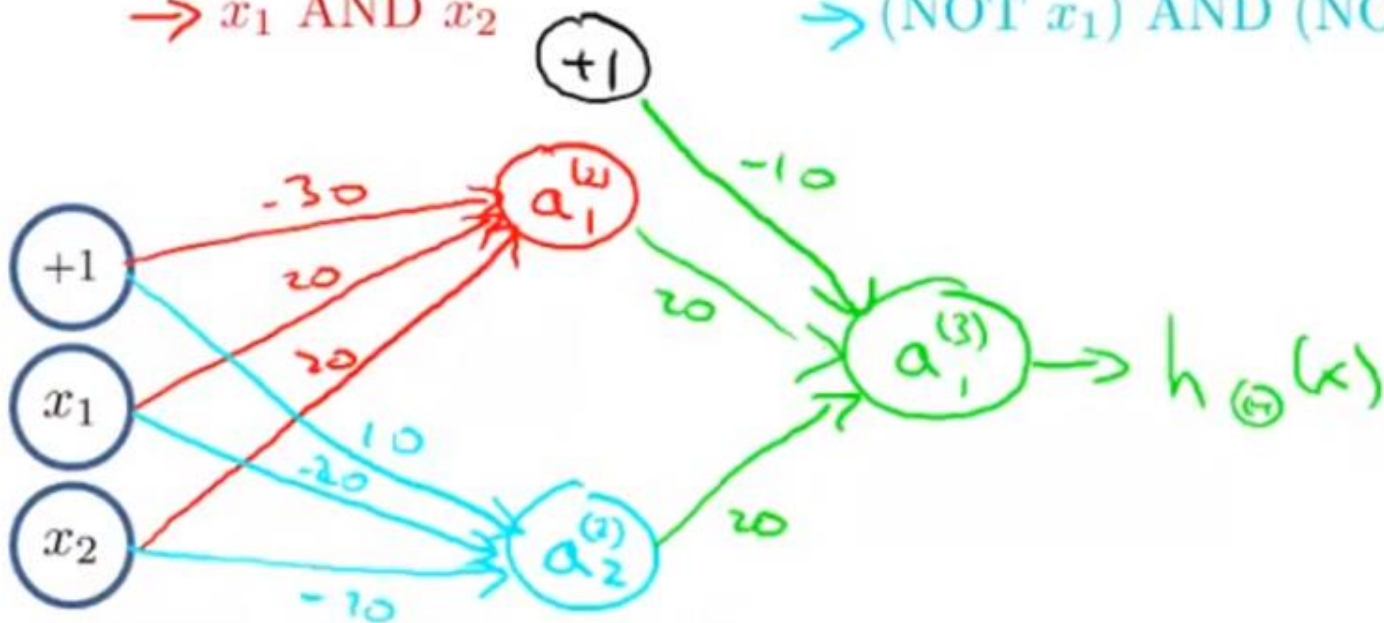
$\rightarrow x_1$  AND  $x_2$



$\rightarrow (\text{NOT } x_1) \text{ AND } (\text{NOT } x_2)$



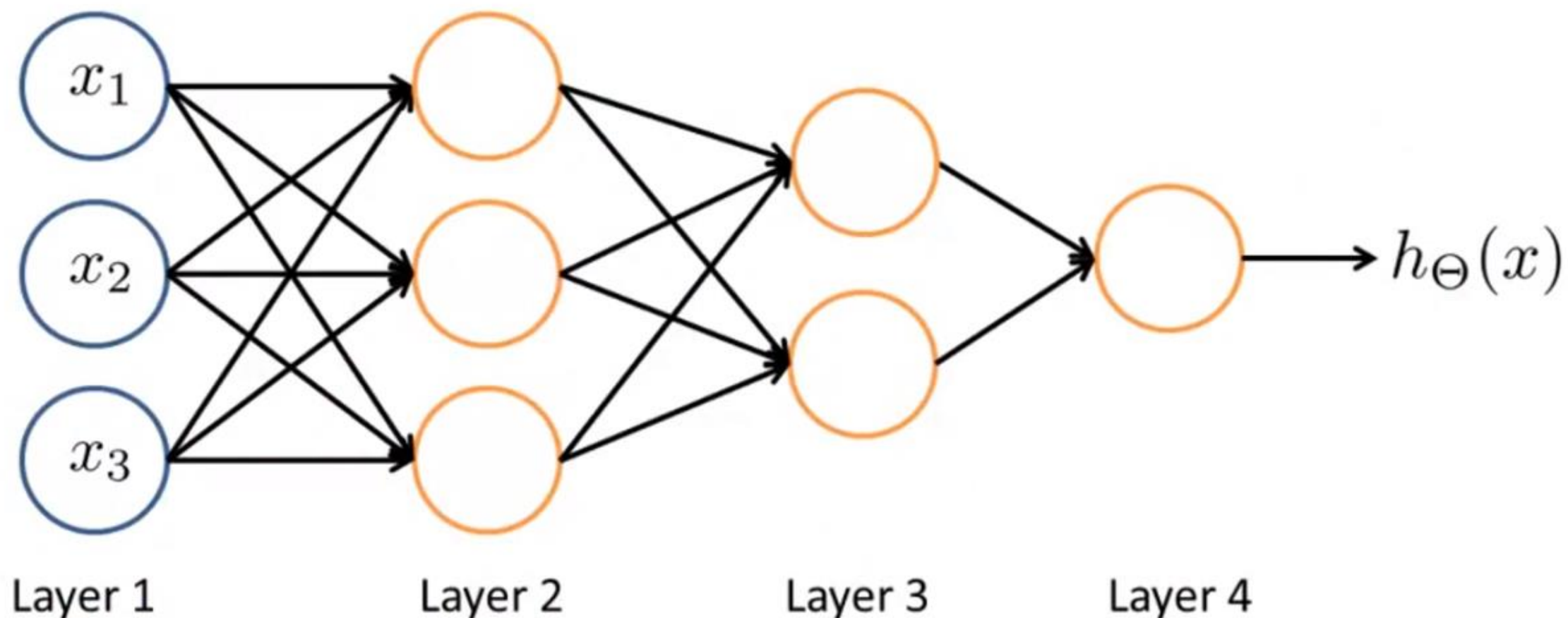
$\rightarrow x_1$  OR  $x_2$



$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1

Windows'u Etkinleştirin  
Windows'u etkinleştirmek için Ayarlar'a gidin.

# Neural Network intuition



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Windows'u etkinleştirmek için Ayarlar'a gidin.



# Multiclass Classification

Applications

*Neural Networks: Representation*

## Multiple output units: One-vs-all.



Pedestrian



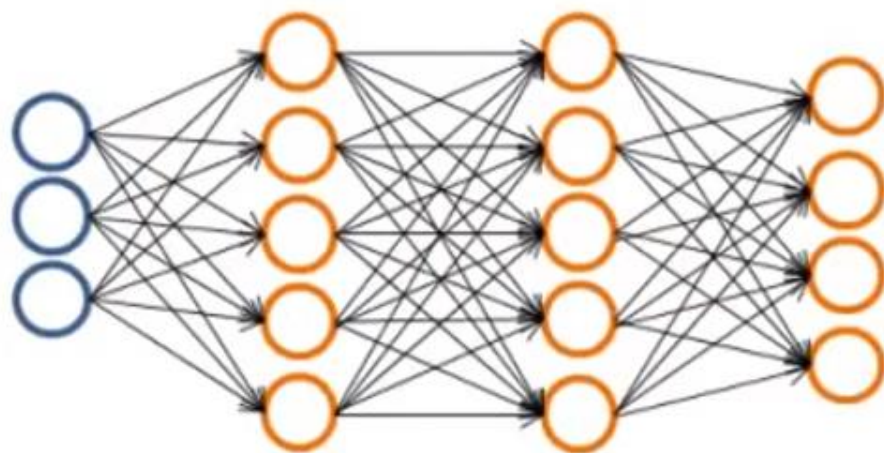
Car



Motorcycle



Truck



$$h_{\Theta}(x) \in \mathbb{R}^4$$

Want  $h_{\Theta}(x) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ,  $h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ , etc.  
when pedestrian      when car      when motorcycle

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## Multiple output units: One-vs-all.



Pedestrian



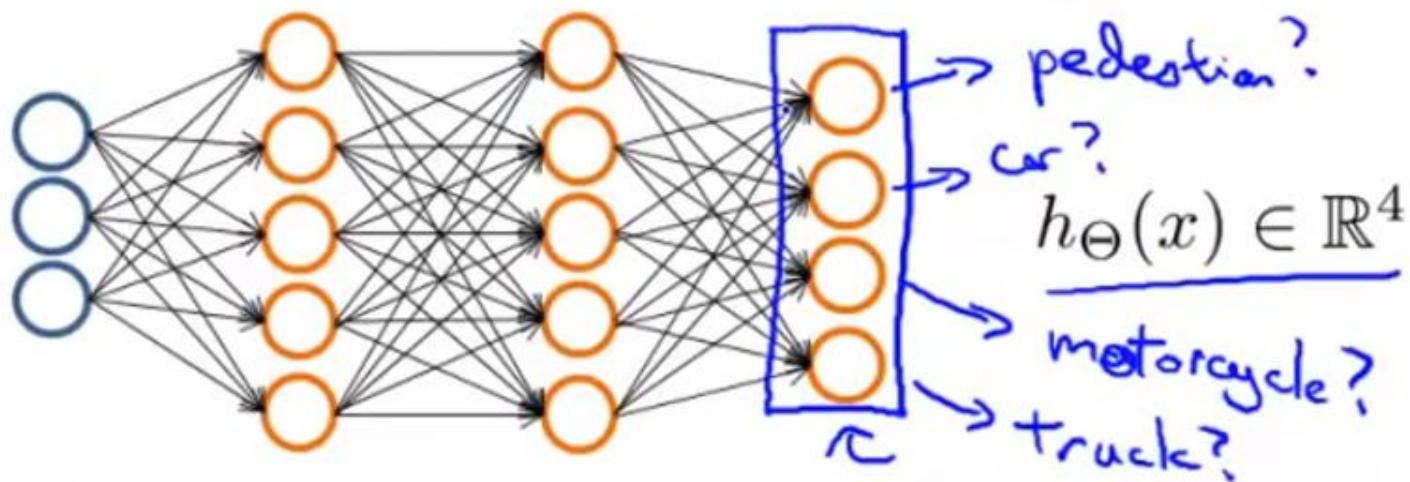
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Truck



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when pedestrian

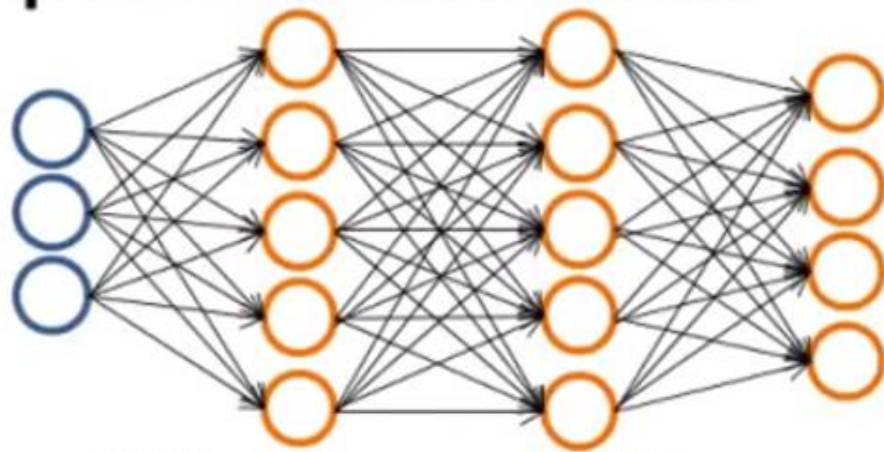
$h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ ,  
when car

$h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ ,  
when motorcycle

etc.

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## Multiple output units: One-vs-all.



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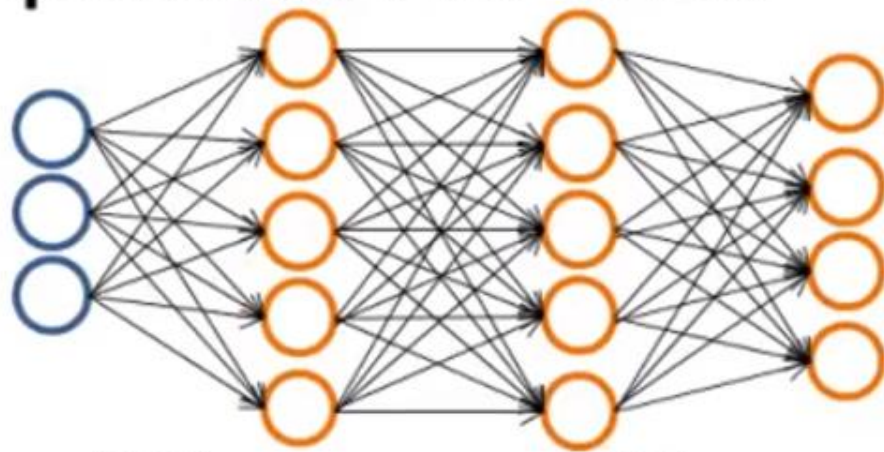
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when pedestrian      when car      when motorcycle



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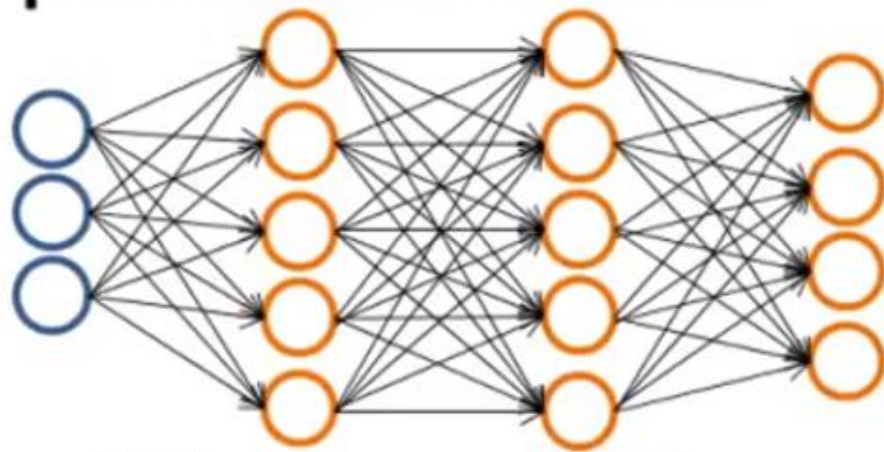
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when pedestrian      when car      when motorcycle

Training set:  $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})$

$y^{(i)}$  one of  $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$   
pedestrian   car   motorcycle   truck

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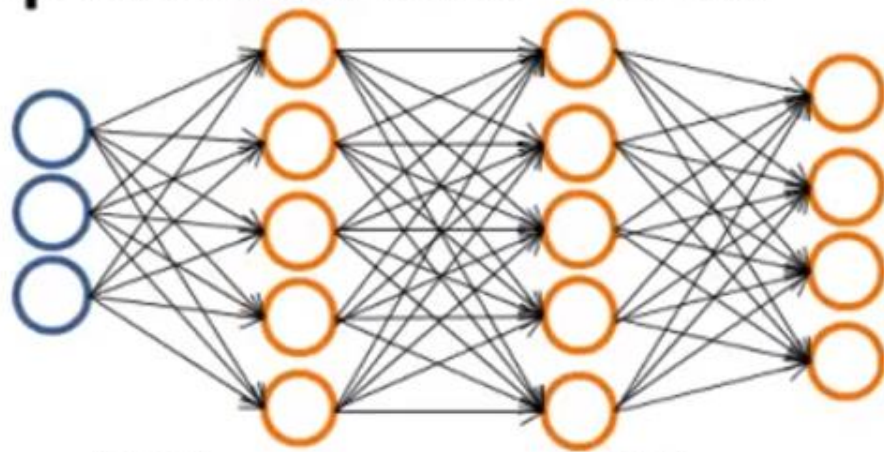
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 $\nearrow$  pedestrian    $\nearrow$  car    $\nearrow$  motorcycle    $\nearrow$  truck

~~Previously  
 $y \in \{1, 2, 3, 4\}$~~

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pedestrian   car   motorcycle   truck

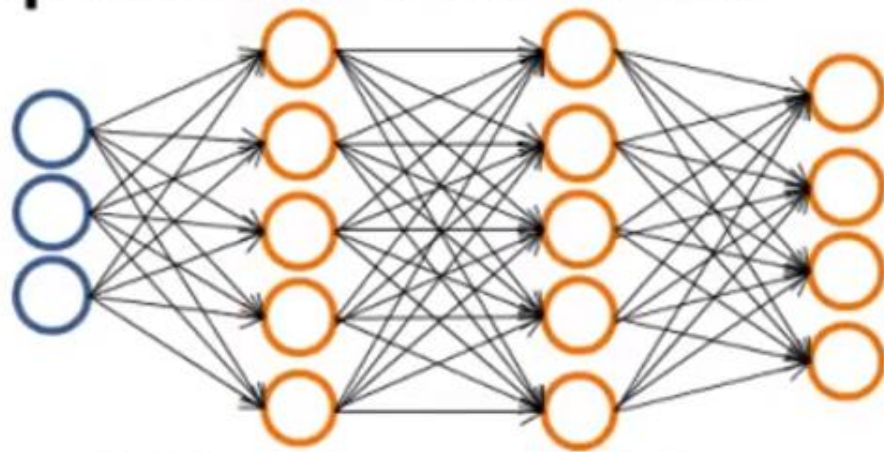
$(x^{(i)}, y^{(i)})$   
 $\uparrow$

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$\frac{h_{\Theta}(x^{(i)}) \approx y^{(i)}}{\mathbb{R}^4}$



# Exercise

- Suppose you have a multi-class classification problem with 10 classes.
- Your neural network has 3 layers, and the hidden layer (layer 2) has 5 units.
- Using the one-vs-all method described here, how many elements does  $\theta^{(2)}$  have?