Scheduling

Notes 2

Last Lecture

- 3/5/J_m/Tmax = There is 3 jobs 5 machines job shop problem and the aim is to minimize max. Tardiness.

- 1 |ri |Cmax= There is a single machine problem. The jobs are arriving at their release dates and the goal is to minimize makespan.

Gantt Charts

- In the late 1800s, Polish engineer Karol Adamiecki developed a visual work flow chart .
- In around 1910, Henry Gantt, a management consultant and engineer, took Adamiecki's concept to the next stage, devised his own version of the chart and it was this that became widely known. Consequently, it was Henry Gantt whose name was to become associated with charts of this type.
- His chart was designed to help manufacturing supervisors see whether their work was on, ahead of, or behind schedule, and it formed the foundation of the tool we use today.

Gantt Charts

- A Gantt chart, commonly used in project management, is one of the most popular and useful ways of showing activities (tasks or events) displayed against time. On the left of the chart is a list of the activities and along the top is a suitable time scale. Each activity is represented by a bar; the position and length of the bar reflects the start date, duration and end date of the activity. This allows you to see at a glance.
- Gantt charts are most commonly used for tracking project schedules.

Gantt Charts

- A Gantt chart is „the earliest and best known type of control chart especially designed to show graphically the relationship between planned performance and actual performance”. Gantt designed his charts so that supervisors could quickly know whether production was on schedule, ahead of schedule or behind schedule. A Gantt chart, or bar chart as it is usually named, measures activities by the amount of time needed to complete them and use the space on the chart to represent the amount of the activity that should have been done in that time.

Load charts

Mechanic	8–9	9–10	10–11	11–12	12–1	1–2	2–3	3–4	4–5
Bob	JOB A			JOB G		JOB I			
Sam	JOB B			JOB H		JOB J		JOB N	
Alex	JOB C		JOB E			JOB K			JOB O
J.J.	JOB D		JOB F			JOB L	JOB M		

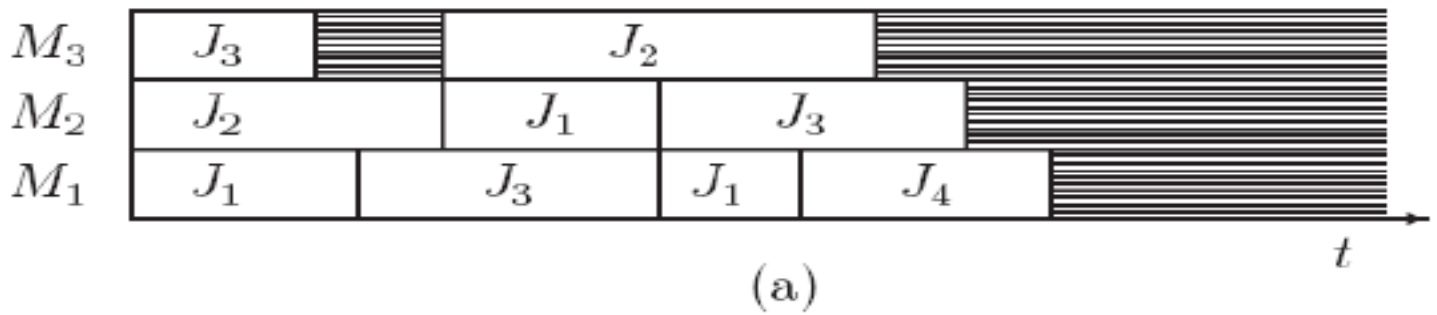
Progress charts

Gantt Charts

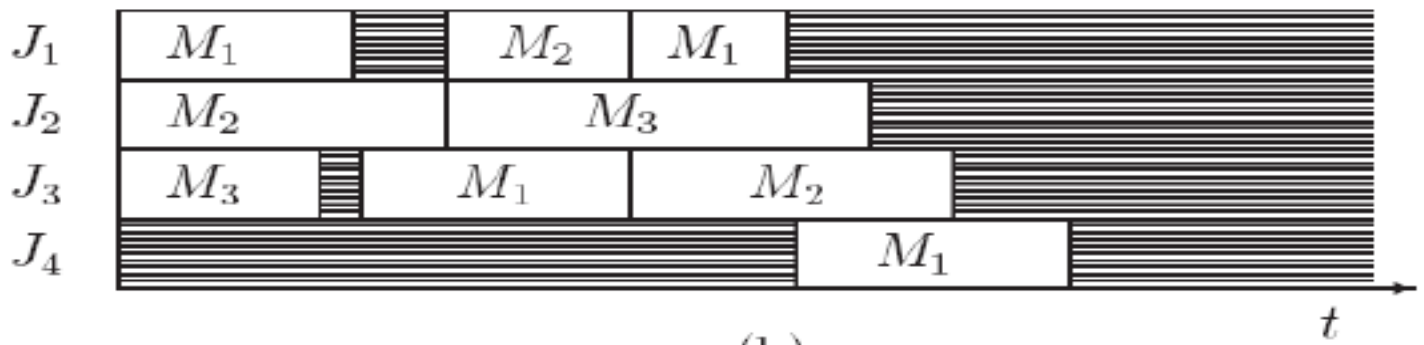
Scheduling are represented by Gantt charts

(a) machine-oriented

(b) job-oriented



(a)



(b)

Forward and Backward Scheduling

- In a forward scheduling procedure the assignment starts from position 1 and continues in the forward direction.
- In a backward scheduling procedure the assignment starts from the last position and continues in the backward direction.

Forward and Backward Scheduling

Static and Dynamic Scheduling

- In static scheduling, jobs are all available for processing right at time t=0 when the processing starts.
- In dynamic scheduling, jobs are not available all at time t=0 .

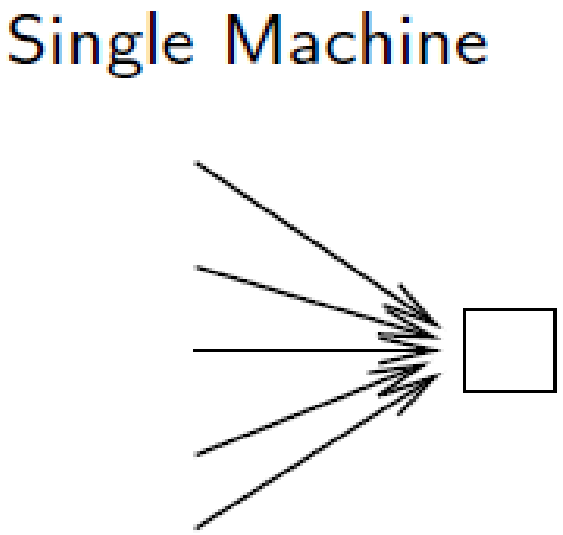
  An example is a bank where customers arrive throughout its service hours.

Feasible and Infeasible Scheduling

- A feasible schedule meets all the constraints and an infeasible schedule does not. Precedence constraints often associate with the scheduling process. A precedence constraint puts restriction on the sequence of operations. For example, a painting process may have three stages: cleaning the product, surface activation for paint adhesion, and select and application of paint. These stages must be performed in the stated sequence. A schedule is not feasible if the jobs are not processed in that sequence.

Single Machine Scheduling

- There are n jobs and 1 machine
- The jobs are all available for processing when the facility starts operation.



Common Sequencing Rules

- **First Come First Served (FCFS):** Jobs processed in the order they come to the shop.
- **Shortest Processing Time (SPT):** Jobs with the shortest processing time are scheduled first.
- **Earliest Due Date (EDD):** Jobs are sequenced according to their due dates.

$$C_i = Fi + ri = Wi + \sum_{j=1}^{m} P_{ij} + r_i = L_i + d_i$$

FCFS Example
Determine mean completion time according to FCFS

Job	1	2	3	4	5	6
Pi	3	2	5	1	8	6

Job	Fi=Ci
1	3
2	5
3	10
4	11
5	19
6	25

Mean completion time: 73/6 = 12.16

Shortest Processing Time Algorithm (SPT)

- The shortest processing time rule orders the jobs in the order of increasing processing times. Whenever a machine is free, the shortest job ready at the time will begin processing.
- SPT minimizes total flow time on a single machine with zero ready times.
- In the single machine environment with ready time at 0 for all jobs, this algorithm is also optimal in minimizing the mean flow time, total completion time, mean completion time, minimizing the mean number of jobs in the system, minimizing the mean waiting time of the jobs from the time of arrival to the start of processing, minimizing the maximum waiting time and the mean lateness.

Shortest Processing Time Algorithm

- $\bar{F} = \frac{1}{n}\sum_{i=1}^{n} F_i = \frac{1}{n}\sum_{i=1}^{n} W_i + P_i = \frac{1}{n}\sum_{i=1}^{n} W_i + \frac{1}{n}\sum_{i=1}^{n} P_i$

- Total flow time (F) = $P_1 + (P_1+P_2) + (P_1+P_2+P_3) + (P_1+P_2+....+P_n)$
- $= nP_1 + (n-1)P_2 + ……+P_n$

Example
Find the mean flow time according to SPT

Job	1	2	3	4	5	6
Pi	3	2	5	1	8	6

Optimal order for SPT: 4- 2 – 1- 3 – 6 – 5
F1 = 1
F2 = 1+2 = 3
.
.
F6= 1+2+3++5+6+8= 25
Total flow time = total completion time= 1+3+6+11+17+25 = 63
Mean flow time = 63/6 = 10,5

Earliest Due Date (EDD) Algorithm

- Due date is the time at which a task is to be completed. Any delay results in tardiness.
- Earliest due date algorithm minimizes
- **Maximal Tardiness (Tmax),**
- **Maximal Lateness (Lmax)**
- Total tardiness and mean tardiness are equivalent objectives.
- Total lateness and mean lateness are equivalent objectives.

Sort the jobs such that $d_1 \leq d_2 \leq \ldots \leq d_n$. Process the jobs on the machine according to this order.

- Lateness and Tardiness are both measures related to customer service
- Average tardiness is a more relevant customer service measurement as illustrated below

Calculating job lateness and tardiness

Job	Completion Date	Due Date	Lateness	Tardiness
A	10	15	-5	0
B	13	15	-2	0
C	17	10	7	7
D	20	20	0	0
		Average	0	1.75

Example (EDD)
Consider the following 6/1//T_{max} problem. Determine T_{max}

Job	1	2	3	4	5	6
di	7	3	8	12	9	3
Pi	1	1	2	4	1	3

- Optimal EDD order: 6-2- 1-3-5-4 or 2-6-1-3-5-4

Job	Ci	Li	Ti
6	3	0	0
2	4	1	1
1	5	-2	0
3	7	-1	0
5	8	-1	0
4	12	0	0

Example

Apply SPT and EDD rule to the following single machine problem
Find Tmax, Lmax, mean tardiness and mean flow time

Job	1	2	3	4
P_i	4	2	6	5
d_i	8	12	11	10

SPT

Job	Pi	Ci	di	Li	Ti
2	2	2	12	-10	0
1	4	6	8	-2	0
4	5	11	10	1	1
3	6	17	11	6	6

Mean Completion time = Mean Flow Time: 36/4 = 9
Mean Tardiness: 7/4 = 1.75

EDD

Job	Pi	Ci	di	Li	Ti
1	4	4	8	-4	0
4	5	9	10	-1	0
3	6	15	11	4	4
2	2	17	12	5	5

Mean Completion Time = Mean Flow Time: 45/4 = 11.25
Mean Tardiness: 9/4 = 2.25

Comparing the sequencing rules

- The FCFS rule seems to be the least effective of the rules. The primary limitation of FCFS rule is that long jobs will tend to delay their jobs. The FCFS rule also has the advantage of simplicity. However, for service systems in which customers are directly involved, the FCFS rule is by far the dominant priority rule.
- SPT rule always results in the lowest (i.e., optimal) average completion (flow) time, it can result in lower in-process inventories. And, because it often provides the lowest (optimal) average tardiness, it can result in better customer levels. Finally, since it always involves a lower average number of jobs at the work center, there tends to be less congestion in the work area. However, the managers are generally thinking of due dates, so they may not use SPT, because it doesn't incorporate due dates. The major disadvantage of the SPT rule is that it tends to make long jobs wait, perhaps for rather long times.
- The EDD rule directly addresses due dates and usually minimizes lateness. Its main limitation is that it does not take processing time into account.

Resources

- Algorithms for Sequencing and Scheduling, Ibrahim M. Alharkan
- Sıralama ve Programlama, Hüseyin Başlıgil
- Çizelgeleme Ders Notları, Prof. Dr. Hüseyin Başlıgil
- Marco Chiarandini, Introduction to Scheduling: Terminology and Classification Lecture Notes, Department of Mathematics & Computer Science University of Southern Denmark, 2011.
- http://www.gantt.com/
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- http://riot.ieor.berkeley.edu/Applications/Scheduling/algorithms.html

Resources

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- https://courses.cs.washington.edu/courses/csep521/03sp/lectures/scheduling.pdf
- http://wps.prenhall.com/wps/media/objects/7117/7288732/krm9e_SuppJ.pdf
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