DECISION MAKING TECHNIQUES IN MANAGEMENT INFORMATION SYSTEMS (MIS)

LECTURE -7-

(Constructing The Decision Model, SAW, WP, TOPSIS)

CONSTRUCTİNG DECİSİON MODEL

- Assessing performance values of alternatives w.r.t. attributes
- (If necessary) Determining relative importance of attributes
- Modeling the preference of DMs

Assessing Performance Values

- Determine how well each alternative achieves each attribute
- Performance value = score = rating = attribute level

Assessing Performance Values

- Objective evaluation
  - Natural attribute is used
  - Quantitative
  - Independent of DM
  - Attributes are readily measured in terms of some natural physical unit
    - e.g. dollars or number of people
- Subjective evaluation
  - Constructed attribute is used
  - Qualitative
  - Dependent on DM
  - No natural measuring scales exist
    - e.g. beauty or convenience

Decision Matrix

- Alternative evaluations w.r.t. attributes are presented in a decision matrix
  - Entries are performance values
  - Rows represent alternatives
  - Columns represent attributes

Attributes

- Benefit attributes

  Offer increasing monotonic utility. Greater the attribute value the more its preference
- Cost attributes

  Offer decreasing monotonic utility. Greater the attribute value the less its preference
- Nonmonotonic attributes

  Offer nonmonotonic utility. The maximum utility is located somewhere in the middle of an attribute range

Global Performance Value

- If solution method that will be utilized is performance aggregation oriented, performance values should be aggregated.
- In this case
  - Performance values are normalized to eliminate computational problems caused by differing measurement units in a decision matrix
  - Relative importance of attributes are determined

Normalization

- Aims at obtaining comparable scales, which allow interattribute as well as intra-attribute comparisons
- Normalized performance values have dimensionless units
- The larger the normalized value becomes, the more preference it has

Normalization Methods

1. Distance-Based Normalization Methods
2. Proportion Based Normalization Methods (Standardization)

Distance-Based Normalization Methods

If we define the normalized rating as the ratio between individual and combined <u>distance</u> from the origin (0,0,…,0) then the comparable rating of x_{ij} is given as (Yoon and Kim, 1989):

$$r_{ij}(p) = (x_{ij} - 0) \Big/ \left\{ \sum_{k=1}^{m} \left| x_{kj} - 0 \right|^{p} \right\}^{1/p}$$

This equation is arranged for <u>benefit attributes</u>.

<u>Cost attributes</u> become benefit attributes by taking the inverse rating ($1/ x_{ij}$)

DİSTANCE-BASED NORMALİZATİON METHODS

- Normalization (*p*=1: Manhattan distance)
- Vector Normalization (*p*=2: Euclidean distance)
- Linear Normalization (*p*=∞ : Tchebycheff dist.)

$$r_{ij}(1) = x_{ij} \,/\, \sum_{k=1}^{m} |x_{kj}|$$

$$r_{ij}(2) = x_{ij} \,/\, \sqrt{\sum_{k=1}^{m} |x_{kj}|^2}$$

$$r_{ij}(\infty) = x_{ij} \,/\, \max\{|x_{kj}|, k = 1,2,\dots,m\}$$ (BENEFIT ATTRIBUTE)

$$r_{ij}(\infty) = \min\{|x_{kj}|, k = 1,2,\dots,m\} \,/\, x_{ij}$$ (COST ATTRIBUTE)

EXAMPLE

Variable	Value
x1	35
x2	15
x3	25
x4	30
x5	20

Obtain the normalized values by using:

a. Manhattan distance
b. Euclidian distance
c. Tchebycheff distance

Example (normalization-Manhattan distance)

Variable	Value	Manhattan
x1	35	**0,28**
x2	15	**0,12**
x3	25	**0,2**
x4	30	**0,24**
x5	20	**0,16**
Total	**125**	

Example (vector normalization-euclİdian distance)

Variable	Value	x2	Euclidian
x1	35	1225	0,60
x2	15	225	0,26
x3	25	625	0,43
x4	30	900	0,52
x5	20	400	0,34
Total	**125**	**3375**	

Example (linear normalization-Tchebycheff distance)

Variable	Value	Linear
x1	35	1,00
x2	15	0,43
x3	25	0,71
x4	30	0,86
x5	20	0,57
Total	**125**	

Proportion-Based Normalization Methods

The proportion of difference between performance value of the alternative and the worst performance value to difference between the best and the worst performance values
(Bana E Costa, 1988; Kirkwood, 1997)

$r_{ij} = (x_{ij} - x_j^-) / (x_j^* - x_j^-)$ benefit attribute

$r_{ij} = (x_j^- - x_{ij}) / (x_j^- - x_j^*)$ cost attribute

*where * represents the best and – represents the worst*

(best: max. perf. value for benefit; min. perf. value for cost or ideal value determined by DM for that attribute)

Example

Variable	Value
x1	35
x2	15
x3	25
x4	30
x5	20

Obtain the normalized values by using proportion based normalization.

Example (proportion based normalization)

Variable	Value	Proportion
x1	35	1
x2	15	0
x3	25	0,5
x4	30	0,75
x5	20	0,25
Total	**125**	

Transformation of Nonmonotonic Attributes to Monotonic

- $\exp(-z^2/2)$ exponential function is utilized for transformation

  where $z = (x_{ij} - x_j^0) / \sigma_j$

  x_j^0 is the most favorable performance value w.r.t. attribute j

  σ_j is the standard deviation of performance values w.r.t. attribute j

Example

- The temperature values for different locations are provided below. If the best condition is 25^0 celcius. Normalize the related values.

Location	Temperature
x1	35
x2	15
x3	25
x4	30
x5	20

EXAMPLE

Variable	Value	z	exp(-z2/2)
x1	35	1,26	0,45
x2	15	-1,26	0,45
x3	25	0,00	1,00
x4	30	0,63	0,82
x5	20	-0,63	0,82
std	7,905694		

ATTRİBUTE WEİGHTİNG

- Most methods translate the relative importance of attributes into numbers which are often called as "weights" (Vincke, 1992)
- Methods utilized for assignment of weights can be classified in two groups (Huylenbroeck, 1995; Munda 1993; Al-Kloub *et al.*, 1997; Kleindorfer *et al.*, 1993; Yoon and Hwang, 1995):
  - Direct Determination
  - Indirect Determination

Weight Assignment Methods

- Direct Determination
  - Rating, Point allocation, Categorization
  - Ranking
  - Swing
  - Ratio (Eigenvector prioritization)
  - …
- Indirect Determination
  - Centrality
  - Regression – Conjoint analysis
  - Interactive

Rating

- Rating
  - Each attribute's importance is rated on a scale (e.g. 0-100)
- Point allocation
  - Allocate a specific amount of points (e.g. 100) among attributes in proportion of their importance
- Categorization
  - Assign attributes to different categories of importance, each carrying a different weight.

Ranking

- We assign 1 to most important attribute, and n to the least important. The cardinal weights can be obtained from one of the following formulas:
  - Rank reciprocal weights

    $w_j = (1/r_j) / (\sum_{k=1}^{n} 1/r_k)$
  - Rank sum weights

    $w_j = (n - r_j + 1) / (\sum_{k=1}^{n} (n - r_k + 1))$

where r_j is the rank of the jth attribute

If two attributes are tied for the nth and (n+1)th place, the number (2n+1)/2 is assigned to both of them

Swing

- DM considers a hypothetical alternative where attributes are all at their worst value.
- S/he is then asked which of the attributes would most prefer to swing from its worst value up to its best value
- S/he is then asked which attribute s/he would swing up second and so on…
- After ranking the attributes in this manner, DM is asked to give the most important attribute a weight of 100, and then assign weights to the other attributes in proportion to the importance of their ranges.

Decision Matrix

- Alternative evaluations w.r.t. attributes are presented in a decision matrix
  - Entries are performance values
  - Rows represent alternatives
  - Columns represent attributes

SAW

- Simple Additive Weighting – Weighted Average – Weighted Sum (Yoon & Hwang, 1995; Vincke, 1992...)
- A global (total) score in the SAW is obtained by adding contributions from each attribute.
- A common numerical scaling system such as normalization (instead of single dimensional value functions) is required to permit addition among attribute values.
- Value (global score) of an alternative can be expressed as:

$$V(a_i) = V_i = \sum_{j=1}^{n} w_j r_{ij}$$

Example for SAW

Normalized (Linear) Decision Matrix and Global Scores

	Price	Comfort	Perf.	Design	V_i
Norm. w	0.3333	0.2667	0.2	0.2	
a_1	0.3333	1	1	1	**.7778**
a_2	0.4	1	0.6667	1	.7334
a_3	0.4	0.6667	1	1	.7111
a_4	0.5	0.6667	1	0.6667	.6778
a_5	0.5	0.6667	0.6667	1	.6778
a_6	0.5	0.3333	1	1	.6555
a_7	1	0.3333	0.6667	0.6667	.6889

Example for SAW

0.7778 = (0.3333*0.3333)+(0.2667*1)+(0.2*1)+(0.2*1)

	Price	Comfort	Perf.	Design	V_i
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a_3	0.4	0.6667	1	1	.7111
a_4	0.5	0.6667	1	0.6667	.6778
a_5	0.5	0.6667	0.6667	1	.6778
a_6	0.5	0.3333	1	1	.6555
a_7	1	0.3333	0.6667	0.6667	.6889

WP

- Weighted Product (Yoon & Hwang, 1995)
- Normalization is not necessary!
- When WP is used weights become exponents associated with each attribute value;
  - a positive power for benefit attributes
  - a negative power for cost attributes
- Because of the exponent property, this method requires that all ratings be greater than 1. When an attribute has fractional ratings, all ratings in that attribute are multiplied by 10^m to meet this requirement

$$V_i = \prod_j (x_{ij})^{w_j}$$

Example for WP

Quantitative Decision Matrix and Global Scores

	Price	Comfort	Perf.	Design	V_i
Norm. w	.3333	.2667	.2	.2	
a $_1$	300	3	3	3	**.3108**
a $_2$	250	3	2	3	.3045
a $_3$	250	2	3	3	.2964
a $_4$	200	2	3	2	.2944
a $_5$	200	2	2	3	.2944
a $_6$	200	1	3	3	.2654
a $_7$	100	1	2	2	.2843

Example for WP

$\mathbf{0.3108} = 300^{-0.3333} * 3^{0.2667} * 3^{0.2} * 3^{0.2}$

	Price	Comfort	Perf.	Design	V_i
Norm. w	.3333	.2667	.2	.2	
a_1	300	3	3	3	**.3108**
a_2	250	3	2	3	.3045
a_3	250	2	3	3	.2964
a_4	200	2	3	2	.2944
a_5	200	2	2	3	.2944
a_6	200	1	3	3	.2654
a_7	100	1	2	2	.2843

TOPSIS

- Technique for Order Preference by Similarity to Ideal Solution (Yoon & Hwang, 1995; Hwang & Lin, 1987)
- Concept:

  Chosen alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution
- Steps:
  - Calculate normalized ratings
  - Calculate weighted normalized ratings
  - Identify positive-ideal and negative-ideal solutions
  - Calculate separation measures
  - Calculate similarities to positive-ideal solution
  - Rank preference order

STEPS

- Calculate normalized ratings
  - Vector normalization (Euclidean) is used
  - Do not take the inverse rating for cost attributes!
- Calculate weighted normalized ratings
  - $v_{ij} = w_j * r_{ij}$
- Identify positive-ideal and negative-ideal solutions

$$a^* = \{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\} = \left\{ \left(\max_i v_{ij} \middle| j \in J_1 \right), \left(\min_i v_{ij} \middle| j \in J_2 \right) \middle| i = 1, \ldots, m \right\}$$

$$a^- = \{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\} = \left\{ \left(\min_i v_{ij} \middle| j \in J_1 \right), \left(\max_i v_{ij} \middle| j \in J_2 \right) \middle| i = 1, \ldots, m \right\}$$

where J_1 is a set of benefit attributes and J_2 is a set of cost attributes

STEPS

- Calculate separation measures
  - Euclidean distance (separation) of each alternative from the ideal solutions are measured:

$$S_i^* = \sqrt{\sum_j (v_{ij} - v_j^*)^2} \qquad S_i^- = \sqrt{\sum_j (v_{ij} - v_j^-)^2}$$

- Calculate similarities to positive-ideal solution

$$C_i^* = S_i^- / (S_i^* + S_i^-)$$

- Rank preference order
  - Rank the alternatives according to similarities in descending order.
  - Recommend the alternative with the maximum similarity

Example for TOPSIS

- Normalized (Vector) Decision Matrix

	Price	Comfort	Perf.	Design
Norm. w	0.3333	0.2667	0.2	0.2
a_1	0.5108	0.5303	0.433	0.4121
a_2	0.4256	0.5303	0.2887	0.4121
a_3	0.4256	0.3536	0.433	0.4121
a_4	0.3405	0.3536	0.433	0.2747
a_5	0.3405	0.3536	0.2887	0.4121
a_6	0.3405	0.1768	0.433	0.4121
a_7	0.1703	0.1768	0.2887	0.2747

Weighted Normalized Ratings & Positive–Negative Ideal

	Price	Comfort	Perf.	Design
a_1	0.1703	0.1414	0.0866	0.0824
a_2	0.1419	0.1414	0.0577	0.0824
a_3	0.1419	0.0943	0.0866	0.0824
a_4	0.1135	0.0943	0.0866	0.0549
a_5	0.1135	0.0943	0.0577	0.0824
a_6	0.1135	0.0471	0.0866	0.0824
a_7	0.0568	0.0471	0.0577	0.0549
a^*	.0568	.1414	.0866	.0824
a-	.1703	.0471	.0577	.0549

Separation Measures & Similarities to Positive Ideal Solution

	S^*	S^-	C^*	Rank
a_1	0.1135	0.1024	0.4742	5
$\boldsymbol{a_2}$	0.0899	0.1022	**0.5321**	1
a_3	0.0973	0.0679	0.4111	6
a_4	0.0787	0.0792	0.5016	3
a_5	0.0792	0.0787	0.4984	4
a_6	0.11	0.0693	0.3866	7
a_7	0.1024	0.1135	0.5258	2

REFERENCES

- Lecture notes of “Prof. Dr. Y. İlker Topçu”, http://web.itu.edu.tr/topcuil/