# SOLUTION OF NETWORK Transportation PROBLEMS

#### **Model Description**

A transportation problem basically deals with the problem of finding the best way to fulfill the demand of n demand points using the capacities of m supply points. The optimal solution will include a variable cost constraint for shipping the product from one supply point to a demand point.

### **General Description of a Transportation Problem**

- A set of m supply points from which a good is shipped. Supply point i can supply at most s<sub>i</sub> units.
- A set of **n** demand points to which the good is shipped. Demand point **j** must receive at least **d**<sub>i</sub> units of the shipped good.
- 3. Each unit produced at supply point i and shipped to demand point j incurs a variable cost of  $\mathbf{c}_{ii}$ .

#### **General Model Notation**

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} X_{ij}$$

$$s.t.\sum_{j=1}^{j=n} X_{ij} \leq s_i (i = 1, 2, ..., m)$$

$$\sum_{i=1}^{i=m} X_{ij} \ge d_j (j = 1, 2, ..., n)$$

$$X_{ij} \ge 0 (i = 1, 2, ..., m; j = 1, 2, ..., n)$$

 $X_{ij}$  = number of units shipped from *supply point i* to *demand point j* 

#### Formulating Transportation Problems

#### **Example:**

PowerCo has three (3) electric power plants that supply the electric needs of four cities (4).

- The associated supply of each plant and demand of each city is given in the table 1.
- The cost of sending 1 million kwh of electricity from a plant to a city depends on the distance the electricity must travel.

#### **Transportation Tableau**

A transportation problem is specified by

- the supply points or nodes,
- the demand points or nodes, and
- the shipping or transport costs (generally taken as linear).

So the relevant data can be summarized in a transportation tableau.

# TABLE Shipping costs, Supply, and Demand for PowerCo Example

From	То						
·	City 1	City 2	City 3	City 4	Supply (Million kwh)		
Plant 1	\$8	\$6	\$10	<b>\$9</b>	35		
Plant 2	\$9	\$12	\$13	<b>\$7</b>	50		
Plant 3	\$14	\$9	\$16	<b>\$5</b>	40		
Demand (Million kwh)	45	20	30	30			

#### **Transportation Tableau**

#### **Objective Function**

Since we want to <u>minimize</u> the total cost of shipping from plants to cities;

Minimize Z 
$$\sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} X_{ij}$$
Minimize Z =  $\{8X_{11} + 6X_{12} + 10X_{13} + 9X_{14}\} + \{9X_{21} + 12X_{22} + 13X_{23} + 7X_{24}\} + \{14X_{31} + 9X_{32} + 16X_{33} + 5X_{34}\}$ 

#### **Supply Constraints**

Each supply point has a upper bound on production capacity:

Plant 1: 
$$X_{11} + X_{12} + X_{13} + X_{14} <= 35$$

Plant 2: 
$$X_{21} + X_{22} + X_{23} + X_{24} <= 50$$

Plant 3: 
$$X_{31} + X_{32} + X_{33} + X_{34} <= 40$$

#### **Demand Constraints**

Each demand point has a needed level of electricity in kwh:

City 1: 
$$X_{11} + X_{21} + X_{31} >= 45$$

City 2: 
$$X_{12} + X_{22} + X_{32} >= 20$$

City 3: 
$$X_{13} + X_{23} + X_{33} >= 30$$

City 4: 
$$X_{14} + X_{24} + X_{34} >= 30$$

#### **Positivity Constraints**

Since a **negative** amount of electricity can not be shipped to a city all Xij's must be non negative;

$$Xij >= 0$$
 (i= 1,2,3; j= 1,2,3,4)

#### LP Formulation of Power Co's Problem

Min Z = 
$$[8X_{11}+6X_{12}+10X_{13}+9X_{14}]+$$
  
 $[9X_{21}+12X_{22}+13X_{23}+7X_{24}]+$   
 $[14X_{31}+9X_{32}+16X_{33}+5X_{34}]$ 

S.T.: 
$$X_{11}+X_{12}+X_{13}+X_{14} <= 35$$
 (Supply Constraints)  $X_{21}+X_{22}+X_{23}+X_{24} <= 50$   $X_{31}+X_{32}+X_{33}+X_{34} <= 40$   $X_{11}+X_{21}+X_{31} >= 45$  (Demand Constraints)  $X_{12}+X_{22}+X_{32} >= 20$   $X_{13}+X_{23}+X_{33} >= 30$   $X_{14}+X_{24}+X_{34} >= 30$   $X_{15} >= 0$  (i= 1,2,3; j= 1,2,3,4)

**Transportation Problem** 

Trans	Jortan	0111	100	10111	
Transportation problem			_		
Cost of Sh	City 1	City 2	City 3	City 4	Supply
Plant-1	8	6	10	9	35
Plant-2	9	12	13	7	50
Plant-3	14	9	16	5	40
Demand		20	30	30	
Variables	x11	x12	x13	x14	
x1j : amount sent to city j	0	0	0	0	
,	x21	x22	x23	x24	
x2j : amount sent to city j	0	0	0	0	
ALI I UMOUNT SENT TO ERTY I	x31	x32	x33	x34	
x3j : amount sent to city j	0	0	0	0	
Objective Function	Value			-	
Minimize Z = {8X11+6X12+10X13+9X14} + {9X21+12X22+13X23+7X24} + {14X31+9X32+16X33+5X34}	0,000				
Constrains	Formula	Value			
X11 + X12 + X13 + X14 <= 35	0	35			
X21 + X22 + X23 + X24 <= 50	0	50			
X31 + X32 + X33 + X34 <= 40	0	40			
X11 + X21 + X31 >= 45	0	45			
X12 + X22 + X32 >= 20	0	20			
X13 + X23 + X33 >= 30	0	30			
X14 + X24 + X34 >= 30	0	30			
x11	0	0			
x12	0	0			
x13	0	0			
x14	0	0			
x21	0	0			
x22	0	0			
x23	0	0			
x24	0	0			
x31	0	0			
x32	0	0			
x33	0	0			
x34	0	_ 0			

#### **Transportation Problem**

X31   X32   X33   X34 <= 40			_		_	_						
Plant-1	nsportation problem											
Plant-2 9 12 13 7 50  Plant-3 14 9 16 5 40  Demand 45 20 30 30  Variables x11 x12 x33 x44  x1j : amount sent to city j 0 0 0 0 0  x21 x22 x28 x24  x2j : amount sent to city j 0 0 0 0 0  x31 x32 x33 x34  x3j : amount sent to city j 0 0 0 0 0  Objective Function Value  Constrains Formula Y14 x12 x13 x14 x25  X14 x12 x13 x14 x25 0 0 0  X31 x32 x33 x34 x34  Constrains Formula Value  X11 x12 x13 x14 x25 0 0 50  X31 x32 x33 x34 x4 x40  X11 x12 x13 x14 x25 0 0 50  X31 x32 x33 x34 x4 x40  X11 x21 x31 x25 x33 x34 x4 x40  X12 x22 x28 x24  X25 x26 x27  X39 x	Cost of Shi	City 1	City 2	City	/ 3	City 4	Supply					
Plant-3	Plant-1	8	6		10	9	35					
Demand 45   20   30   30   30     X11	Plant-2	9	12		13	7	50					
Variables       X11       X12       X13       X14         x1j : amount sentto city j       0       0       0       0       0         x2j : amount sentto city j       0       0       0       0       0         x3j : amount sentto city j       0       0       0       0       0         0 bjective Function       Value       Value       Value         Minimize Z = (8X11+6X12+10X13+9X14) + {5X21+12X22+1X22+7X24} + {14X31+9X32+16X33+5X34}       4	Plant-3	14	9		16	5	40					
x1j : amount sent to city j	Demand	45	20	30		30						
X21   X22   X23   X24   X25   X24   X25   X25	Variables :	x11	x12	x13		x14						
x2j : amount sent to city j	amount sent to city j	0	0	0		0						
x2j : amount sent to city j		x21	x22	x23	3	x24						
X31	amount sent to city i	0	0			0						
x3j : amount sent to city j		x31	x32	x33	5	x34						
Objective Function         Value         Cozdc@ Parametreleri           Minimize Z = {8X11+6X12+10X13+9X14} + {5X21+12X22+13X20+7X24} + {14X31+9X32+16X33+5X34}         Hedef Hücre: SS14	amount sent to city i			0								
Minimize Z = (8X11+6X12+10X13+9X14) +		Value	_									
Tahmin	imize Z = {8X11+6X12+10X13+9X14} +	1		5	ÖZÜCÜ Para	metreleri						
O,000         Eşittir: ☐ En Büyük ☐ En Küçük ☐ Değer: ☐           Constrains         Formula         Value           X11 + X12 + X13 + X14 <= 35	- I				Hedef Hücro	≘: \$B\$1	4 [55]					Çü <u>z</u>
Constrains  X11 + X12 + X13 + X14 <= 35  X21 + X22 + X23 + X24 <= 50  X31 + X32 + X33 + X34 <= 40  X11 + X21 + X31 >= 45  X12 + X22 + X32 >= 20  X13 + X23 + X33 >= 30  X14 + X24 + X34 >= 30  X15  X17  X18  X19  X19  X19  X19  X11  X11  X11	[14/31+3/32+10/33+3/34]	0.000			Esittir:	€ En	possile @	). En Minila	@ Daž	šan O		
X11 + X12 + X13 + X14 <= 35				Н	-		<u>D</u> uyuk (§	∌ ⊑⊓ <u>v</u> uçuk	⊕ nei	ger: U		Kapat
X11 + X12 + X13 + X14 <= 35  X21 + X22 + X23 + X24 <= 50  X31 + X32 + X33 + X34 <= 40  X11 + X21 + X31 >= 45  X12 + X22 + X32 >= 20  X13 + X23 + X35 >= 30  X14 + X24 + X34 >= 30  X12  X13  X14  X14  X15  X16  X17  X17  X17  X18  X18  X18  X19  X19  X19  X11  X11					x1i:x2i:x3	3i			(Fig.)	Tahmir		
X31 + X32 + X33 + X34 <= 40									()	1 20 111111		
X11 + X21 + X31 >= 45       0       45         X12 + X22 + X32 >= 20       0       20         X15 + X23 + X35 >= 30       0       50         X14 + X24 + X34 >= 30       0       30         X11       0       0         X12       0       0         X13       0       0         X14       0       0         X21       0       0         X22       0       0				- 100	-Kısıtla <u>m</u> alar	:						Segenekler
x11 + x21 + x31 >= 45       0       45 <td< td=""><td></td><td></td><td></td><td>- 18</td><td>\$B\$17 &lt;=</td><td>\$C\$17</td><td></td><td></td><td></td><td>Fkle</td><td>1</td><td></td></td<>				- 18	\$B\$17 <=	\$C\$17				Fkle	1	
X13 + X23 + X35 >= 30     0     50       X14 + X24 + X34 >= 30     0     30       x11     0     0       x12     0     0       x13     0     0       x14     0     0       x21     0     0       x22     0     0		0								<u> </u>		
x13 + x23 + x33 >= 50     0     50       x14 + x24 + x34 >= 30     0     30       x11     0     0       x12     0     0       x13     0     0       x14     0     0       x21     0     0       x22     0     0		_		- 100						Değisti	r	
x11												T <u>ü</u> münü Sıfırl
x11	+ X24 + X34 >= 30	_		-0.00					+	<u>S</u> il		r
x13 0 0 0 x14 0 0 0 x21 0 0 0 x22 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0				- 100	40422 / -	- quque						<u>Y</u> ardım
x14     0     0       x21     0     0       x22     0     0		0	0									
x21 0 0 0		0	0									
x22 0 0 0				_								
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		0		_								
		0	0									

**Transportation Problem** Transportation problem City 2 City 1 City 3 City 4 Cost of Shipping Supply Plant-1 6 9 Plant-2 12 13 50 Plant-3 14 9 16 Demand 45 20 30 30 Variables. r13 x11 $\times 12$  $\times 14$ 25 x1j: amount sent to city j 10 x21 x22 x23 x24 0 5 x2j: amount sent to city j 45 x31 x32 x33 x34 10 30 x3j : amount sent to city j Objective Function Value Çözücü Parametreleri Minimize  $Z = \{8X11+6X12+10X13+9X14\} +$ {9X21+12X22+13X23+7X24} + Hedef Hücre: \$B\$14 Çöz {14X31+9X32+16X33+5X34} Esittir: En Büvük
En Kücük Değer: 1.020,000 Kapat Değsen Hücreler: Constrains. Formula Value Es x1j;x2j;x3j Tahmin X11 + X12 + X13 + X14 <= 35 35 35 Kısıtlamalar:  $X21 + X22 + X23 + X24 \le 50$ 50 50 Segenekler X31 + X32 + X33 + X34 <= 40 40 40 \$8\$17 <- \$C\$17 Ekle 45 X11 + X21 + X31 >= 4545 \$B\$18 <= \$C\$18 X12 + X22 + X32 >= 20 20 20 \$B\$19 <= \$C\$19 Değistir Tümünü Sıfır \$B\$20 >= \$C\$20X13 + X23 + X33 >= 30 30 30 \$B\$21 >= \$C\$21 Sil X14 + X24 + X34 >= 3030 \$B\$22 >= \$C\$22 Yardım x11 0 x1210 0 x13 25 0 x14 0 0 x21 45 0 0 x220 x23 0  $\times 24$ 0 0 x31 0 x32 10 0 x33 0 0 30 0

## Microsoft Excel Tansportation Problem Çalışma Sayfası: [Problem\_Transportation.xls]Sayfa1

Rapor Oluşturuldu: 06.12.2009 19:37:06

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
\$B\$20	{8X11+6X12+10X13+9X14}+	0,000	1.020,000
yarlanab	ilir Hücreler		
Hücre	Ad	ilk Değer	Son Değer
\$B\$12	x1j : amount sent to city j x11	0	0
\$C\$12	x1j : amount sent to city j x12	0	10
\$D\$12	x1j : amount sent to city j x13	0	25
\$E\$12	x1j : amount sent to city j x14	0	0
\$B\$14	x2j : amount sent to city j x21	0	45
\$C\$14	x2j : amount sent to city j x22	0	0
\$D\$14	x2j : amount sent to city j x23	0	5
\$E\$14	x2j : amount sent to city j x24	0	0
\$B\$16	x3j : amount sent to city j x31	0	0
\$C\$16	x3j : amount sent to city j x32	0	10
\$D\$16	x3j : amount sent to city j x33	0	0
\$E\$16	x3j : amount sent to city j x34	0	30

#### Sınırlamalar

Hücre	Ad	Hücre Değeri	formül	Durum	Serbestlik
\$B\$23	X11 + X12 + X13 + X14 <= 35 Formula	35	\$B\$23<=\$C\$23	Aynı	0
\$B\$24	X21 + X22 + X23 + X24 <= 50 Formula	50	\$B\$24<=\$C\$24	Aynı	0
\$B\$25	X31 + X32 + X33 + X34 <= 40 Formula	40	\$B\$25<=\$C\$25	Aynı	0
\$B\$26	X11 + X21 + X31 >= 45 Formula	45	\$B\$26>=\$C\$26	Aynı	0
\$B\$27	X12 + X22 + X32 >= 20 Formula	20	\$B\$27>=\$C\$27	Aynı	0
\$B\$28	X13 + X23 + X33 >= 30 Formula	30	\$B\$28>=\$C\$28	Aynı	0
\$B\$29	X14 + X24 + X34 >= 30 Formula	30	\$B\$29>=\$C\$29	Aynı	0
\$B\$30	x11 Formula	0	\$B\$30>=\$C\$30	Aynı	0
\$B\$31	x12 Formula	10	\$B\$31>=\$C\$31	Farklı	10