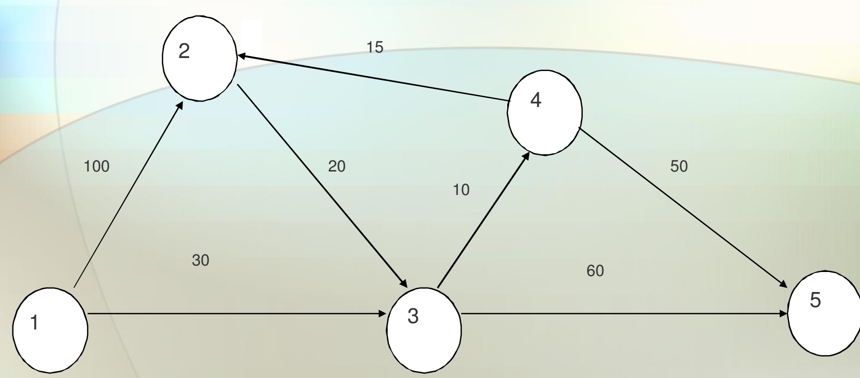
SOLUTION OF NETWORK PROBLEMS By LINGO

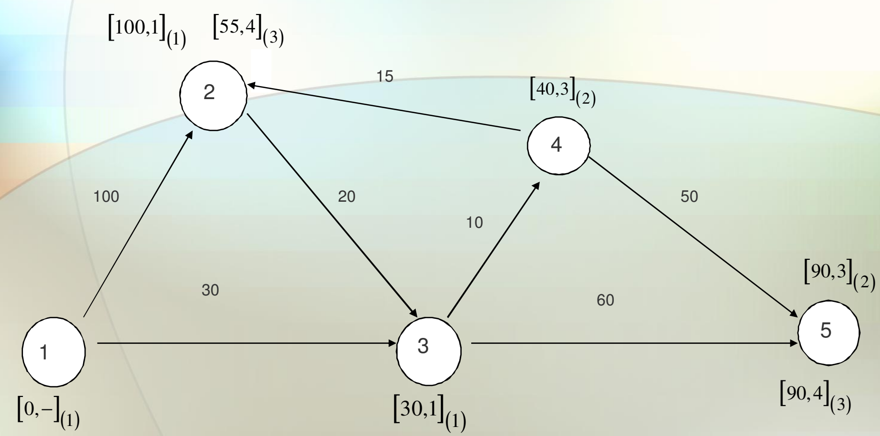
Shortest-Route Algorithms with LINGO

Network in figures gives routes and their lengths in miles between city 1 (node 1) and four other cities (nodes 2 to 5). Determine the shortest routes between city 1 and each of the remaining four cities.



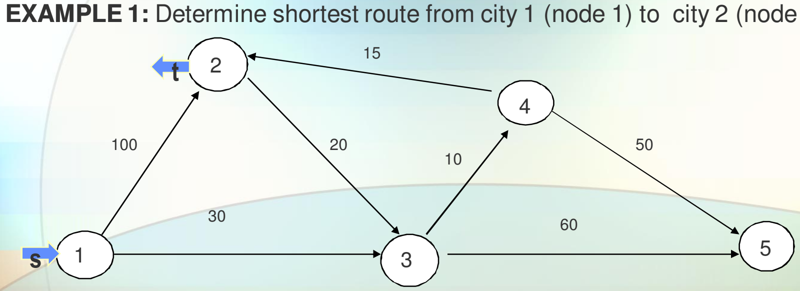
Shortest-Route Algorithms with LINGO

Dijkstra's Algorithms.



LP formulation of Shortest-Route

EXAMPLE 1: Determine shortest route from city 1 (node 1) to city 2 (node 2).



	X_{12}	X_{13}	X_{23}	X_{34}	X_{35}	X_{42}	X_{45}	
Minimize Z=	**100**	**30**	**20**	**10**	**60**	**15**	**50**	
Node 1	**-1**	**-1**						**=-1**
Node 2	**1**		**-1**			**1**		**= 1**
Node 3		**1**	**1**	**-1**	**-1**			**= 0**
Node 4				**1**		**-1**	**-1**	**= 0**
Node 5					**1**		**1**	**= 0**

LINGO Program Model

EXAMPLE 1:

LINGO 11.0 - LINGO Model - pr_ex6_3_6

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LINGO Model - pr_ex6_3_6

```
MIN=  100* X12 + 30 *X13 + 20* X23 +
      10*X34 + 60 *X35 + 15* X42 +
      50*X45  ;
 -X12 -  X13 = -1;
 X12 -  X23 +  X42 = 1;
 X13 +  X24 -  X34 -X35 = 0;
 X34 -  X42 -  X45 = 0;
 X35 +   X45 = 0;

END
```


Shortest-Route LINGO Solution

EXAMPLE 1:

LINGO 11.0 - Solution Report - pr_ex6_3_6

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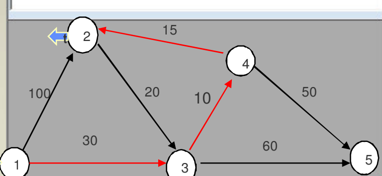
LINGO Model - pr_ex6_3_6

```
MIN=  100* X12 + 30 *X13 + 20* X23 +
      10*X34 + 60 *X35 + 15* X42 +
      50*X45  ;
 -X12 -  X13 = -1;
 X12 -  X23 +  X42 = 1;
 X13 +  X24 -  X34 -X35 = 0;
 X34 -  X42 -  X45 = 0;
 X35 +   X45 = 0;

END
```



Solution Report - pr_ex6_3_6

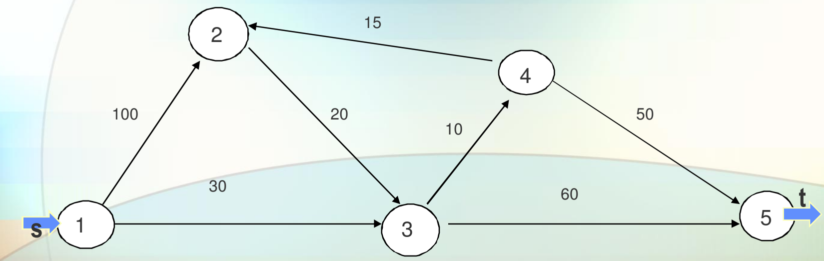
```
  Global optimal solution found.
  Objective value:                  55.00000
  Infeasibilities:                  0.000000
  Total solver iterations:             0

Variable      Value          Reduced Cost
X12         0.000000            45.00000
X13         1.000000            0.000000
X23         0.000000            0.000000
X34         1.000000            0.000000
X35         0.000000            15.00000
X42         1.000000            0.000000
X45         0.000000            15.00000
X24         0.000000            45.00000

Row      Slack or Surplus       Dual Price
1           55.00000           -1.000000
2           0.000000            75.00000
3           0.000000            20.00000
4           0.000000            45.00000
5           0.000000            35.00000
6           0.000000            0.000000
```

LP formulation of Shortest-Route

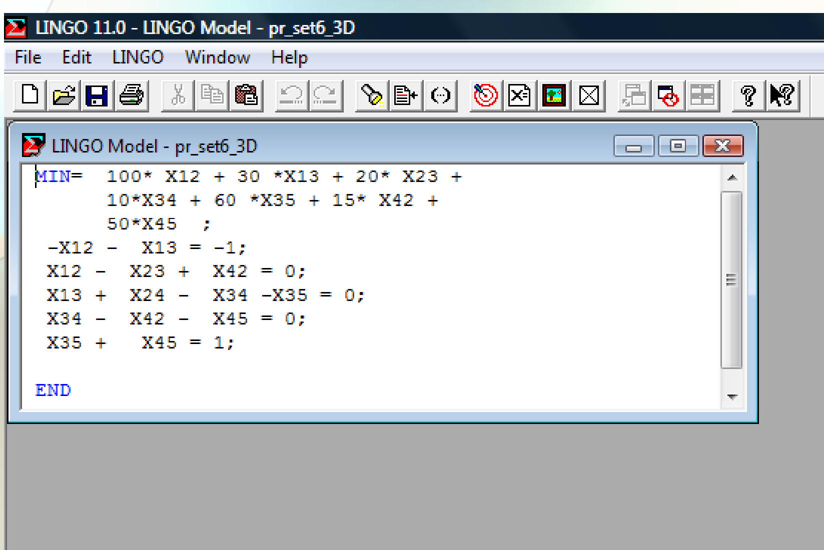
EXAMPLE 2: Determine shortest route from city 1 (node 1) to city 5 (node 5).



	X_{12}	X_{13}	X_{23}	X_{34}	X_{35}	X_{42}	X_{45}	
Minimize Z=	**100**	**30**	**20**	**10**	**60**	**15**	**50**	
Node 1	**-1**	**-1**						**=-1**
Node 2	**1**		**-1**			**1**		**= 0**
Node 3		**1**	**1**	**-1**	**-1**			**= 0**
Node 4				**1**		**-1**	**-1**	**= 0**
Node 5					**1**		**1**	**= 1**

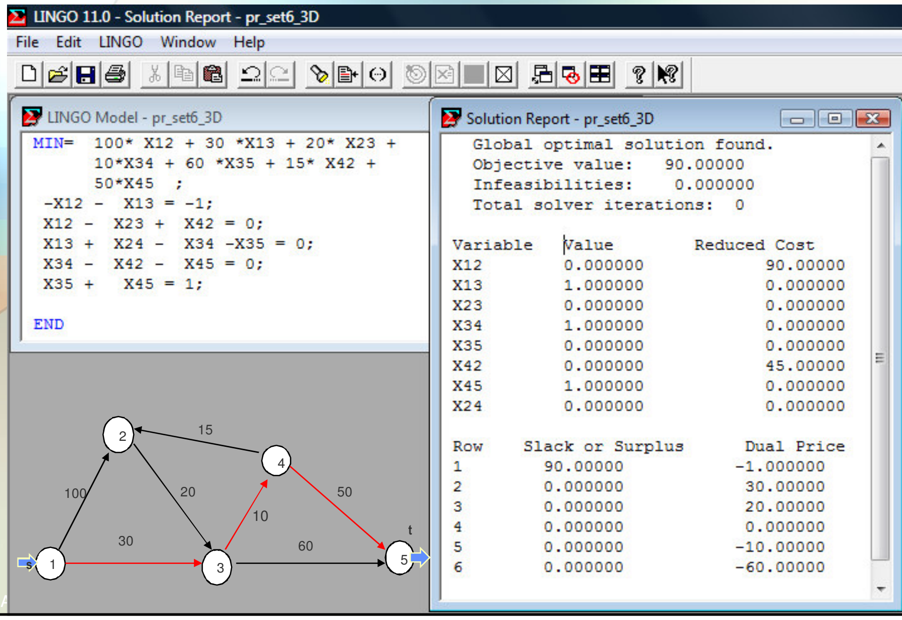
LINGO Program Model

EXAMPLE 2:



Shortest-Route LINGO Solution

EXAMPLE 2:



LINGO 11.0 - Solution Report - pr_set6_3D

LINGO Model - pr_set6_3D

```
MIN=  100* X12 + 30 *X13 + 20* X23 +
      10*X34 + 60 *X35 + 15* X42 +
      50*X45  ;
 -X12 -  X13 = -1;
 X12 -  X23 +  X42 = 0;
 X13 +  X24 -  X34 -X35 = 0;
 X34 -  X42 -  X45 = 0;
 X35 +   X45 = 1;

END
```

Solution Report - pr_set6_3D

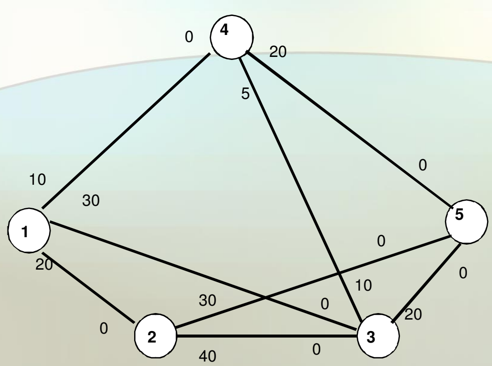
```
   Global optimal solution found.
   Objective value:     90.00000
   Infeasibilities:     0.000000
   Total solver iterations:   0
```

Variable	Value	Reduced Cost
X12	0.000000	90.00000
X13	1.000000	0.000000
X23	0.000000	0.000000
X34	1.000000	0.000000
X35	0.000000	0.000000
X42	0.000000	45.00000
X45	1.000000	0.000000
X24	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	90.00000	-1.000000
2	0.000000	30.00000
3	0.000000	20.00000
4	0.000000	0.000000
5	0.000000	-10.00000
6	0.000000	-60.00000

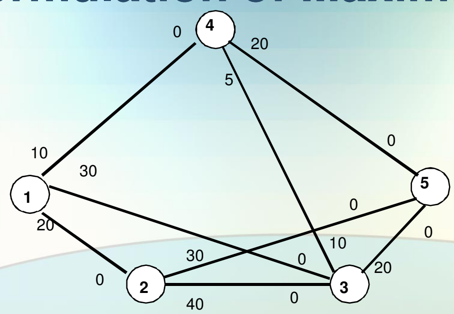
Maximal Flow Model with LINGO

Determine the Maximal Flow in the Network in figures gives bidirectional capacities. For example for arc (3,4), the flow limit is 10 units from 3 to 4, and 5 units from 4 to 3.



LP formulation of Maximal Flow

EXAMPLE 1:



	X_{12}	X_{13}	X_{14}	X_{23}	X_{25}	X_{34}	X_{35}	X_{43}	X_{45}	
Maximize Z_1=	1	1	1							
Maximize Z_2=					1		1		1	
Node 2	1		-1			1				= 0
Node 3		1	1	-1	-1					= 0
Node 4				1		-1	-1	-1	-1	= 0
Capacity	20	30	10	40	30	10	20	5	20	

LINGO Program Model

EXAMPLE 1: **Maximal Flow**

LINGO 11.0 - LINGO Model - pr_set6_4_3

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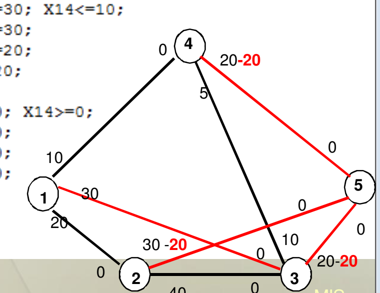
LINGO Model - pr_set6_4_3

```
MAX=   X25 + X35 + X45 ;
X12 -  X23 -  X25  = 0;
X13 +  X23 - X34 -  X35 +  X43 = 0;
X14 +  X34 - X43 -  X45 = 0;
X12<=20; X13<=30; X14<=10;
X23<=40; X25<=30;
X34<=10; X35<=20;
X43<=5; X45<=20;

X12>=0; X13>=0; X14>=0;
X23>=0; X25>=0;
X34>=0; X35>=0;
X43>=0; X45>=0;

END
```



pr_set6_4_3

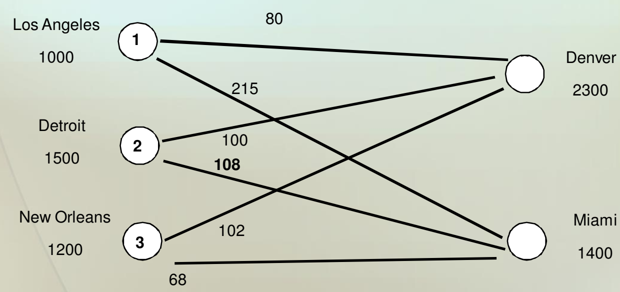
```
   Global optimal solution found.
   Objective value:60.00000
   Infeasibilities:0.000000
   Total solver iterations:  0
Variable          Value         Reduced Cost
X25            20.00000          0.000000
X35            20.00000          0.000000
X45            20.00000          0.000000
X12            20.00000          0.000000
X23            0.000000          1.000000
X13            30.00000          0.000000
X34            10.00000          0.000000
X43            0.000000          1.000000
X14            10.00000          0.000000

Row       Slack or Surplus         Dual Price
1           60.00000            1.000000
2           0.000000           -1.000000
3           0.000000            0.000000
4           0.000000           -1.000000
5           0.000000            1.000000
6           0.000000            0.000000
7           0.000000            1.000000
8           40.00000            0.000000
9           10.00000            0.000000
10           0.000000            1.000000
11           0.000000            1.000000
12           5.000000            0.000000
13           0.000000            0.000000
14           20.00000            0.000000
15           30.00000            0.000000
16           10.00000            0.000000
17           0.000000            0.000000
18           20.00000            0.000000
19           10.00000            0.000000
20           20.00000            0.000000
21           0.000000            0.000000
22           20.00000            0.000000
```

Transportation Problem

MG auto has three plants and two major distribution center. The capacities of these plants during the next quarter are 100, 1500, 1200 cars. The quarterly demands at the two distribution center are 2300 and 1400 cars. The transportation cost per car on different routes are

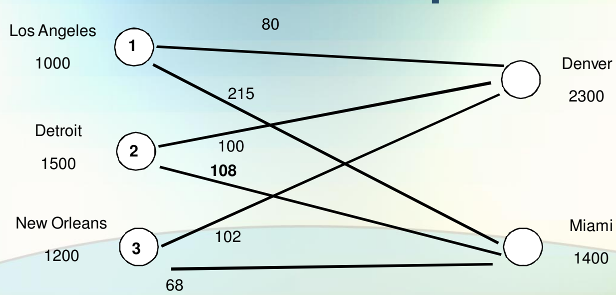
	Denver	Miami
Los Angeles	$80	$215
Detroit	$100	$108
New Orleans	$102	$68

The objective is to devise a minimum transportation cost production plan.



LP formulation of Transportation

EXAMPLE 1:



	X_{11}	X_{12}	X_{21}	X_{22}	X_{31}	X_{32}	
Min Z=	80	215	100	108	102	68	
	1	1					=1000
			1	1			= 1500
					1	1	= 1200
	1		1		1		=2300
		1		1		1	=1400

LINGO Program Model: Transportation

LINGO 11.0 - Solution Report - pr_ex5

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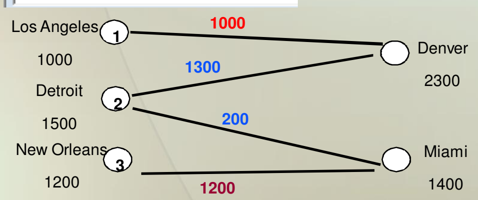
LINGO Model - pr_ex5_1_1

```
MIN=   80* X11+ 215*X12+
       100* X21+ 108*X22+
       102* X31+ 68*X32;
X11 +  X12  = 1000;
X21 +  X22  = 1500;
X31 +  X32  = 1200;
X11 + X21 + X31 = 2300;
X12 + X22 + X32 = 1400;

X11>=0; X12>=0;
X21>=0; X22>=0;
X31>=0; X32>=0;

END
```

Solution Report - pr_ex5_1_1

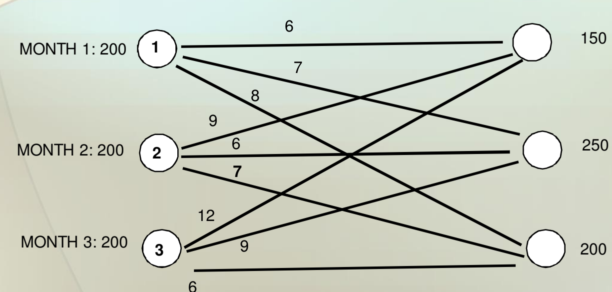
```
Global optimal solution found.
Objective value:   313200.0
Infeasibilities:   0.000000
Total solver iterations: 0
Variable         Value      Reduced Cost
X11           1000.000          0.000000
X12           0.000000          127.0000
X21           1300.000          0.000000
X22           200.0000          0.000000
X31           0.000000          42.00000
X32           1200.000          0.000000

Row      Slack or Surplus       Dual Price
1           313200.0          -1.000000
2           0.000000          -80.00000
3           0.000000          -100.0000
4           0.000000          -60.00000
5           0.000000           0.000000
6           0.000000          -8.000000
7           1000.000           0.000000
8           0.000000           0.000000
9           1300.000           0.000000
10          200.0000           0.000000
11          0.000000           0.000000
12          1200.000           0.000000
```



Transportation Problem

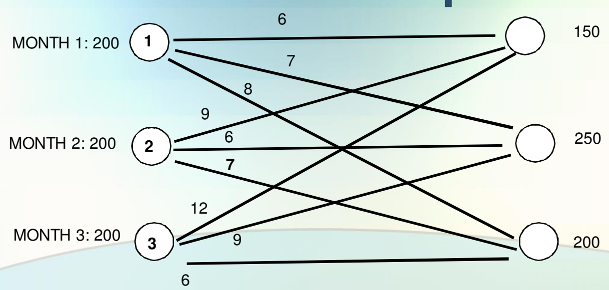
Formulate the following production problem as a transportation model. The demands for a given item are 150, 250, 200 units for the next three months. The demand may be satisfied by excess production in an earlier month held in stock for later consumption, production in the current month, excess production in a later month backordered for preceding months.

The variable production cost per unit in any month is $6.00. A unit produced for later consumption will incur a storage cost at the rate of $1 per unit per month. On the other hand, backordered items incur a penalty cost of $3.00 per unit per month. The production capacity in each of the next three months is 200 units. The objective is to devise a minimum cost production plan.



LP formulation of Transportation

EXAMPLE 1:



	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	
Min Z=	6	7	8	9	6	7	12	9	6	
	1	1	1							=200
				1	1	1				= 200
							1	1	1	= 200
	1			1			1			=150
		1			1			1		=250
			1			1			1	=200

LINGO Program Model Transportation

LINGO 11.0 - Solution Report - pr_exercise_79

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LINGO Model - pr_exercise_79

```
MIN=  6* X11+ 7*X12+8*X13 +
      9* X21+ 6*X22+7*X23 +
      12* X31+ 9*X32+6*X33;
X11 +  X12 +  X13  = 200;
X21 +  X22 +  X23  = 200;
X31 +  X32 +  X33  = 200;
X11 + X21 + X31 = 150;
X12 + X22 + X32 = 250;
X13 + X23 + X33 = 200;

X11>=0; X12>=0; X13>=0;
X21>=0; X22>=0; X23>=0;
X31>=0; X32>=0; X33>=0;

END
```

Solution Report - pr_exercise_79

```
Global optimal solution found.
Objective value: 3650.000
Infeasibilities: 0.000000
Total solver iterations: 5
Variable    Value        Reduced Cost
X11         150.0000         0.000000
X12         50.00000         0.000000
X13         0.000000         4.000000
X21         0.000000         4.000000
X22         200.0000         0.000000
X23         0.000000         4.000000
X31         0.000000         4.000000
X32         0.000000         0.000000
X33         200.0000         0.000000

Row     Slack or Surplus       Dual Price
1        3650.000            -1.000000
2        0.000000            -1.000000
3        0.000000             0.000000
4        0.000000            -3.000000
5        0.000000            -5.000000
6        0.000000            -6.000000
7        0.000000            -3.000000
8        150.0000             0.000000
9        50.00000             0.000000
10        0.000000             0.000000
11        0.000000             0.000000
12        200.0000             0.000000
13        0.000000             0.000000
14        0.000000             0.000000
15        0.000000             0.000000
16        200.0000             0.000000
```