Regularization and Bias/Variance

Bias and Variance

Advice for Applying Machine Learning

Introduction

- You've seen how regularization can help prevent over-fitting in previous lectures.
- In the lectures that we talked about today, there was no regularization…
- How does regularization affect the **bias** and **variances** of a learning algorithm?

Linear regression with regularization

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \boxed{\frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2}$$

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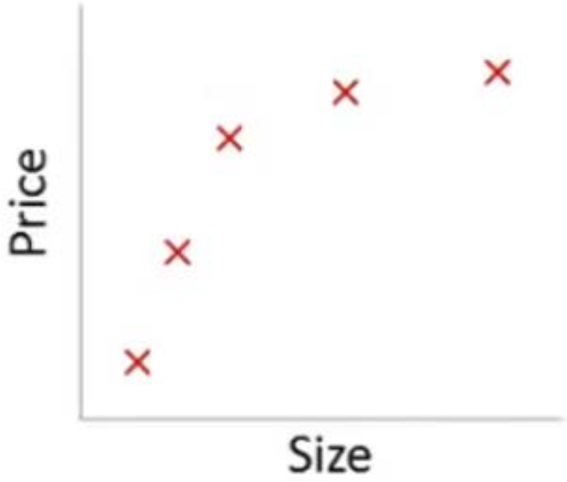
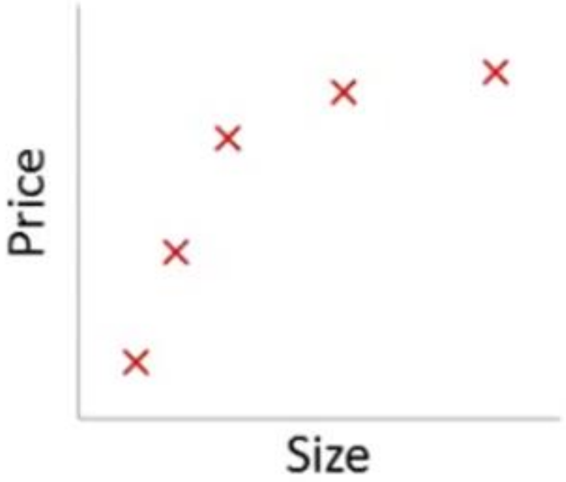


Large λ

High bias (underfit)

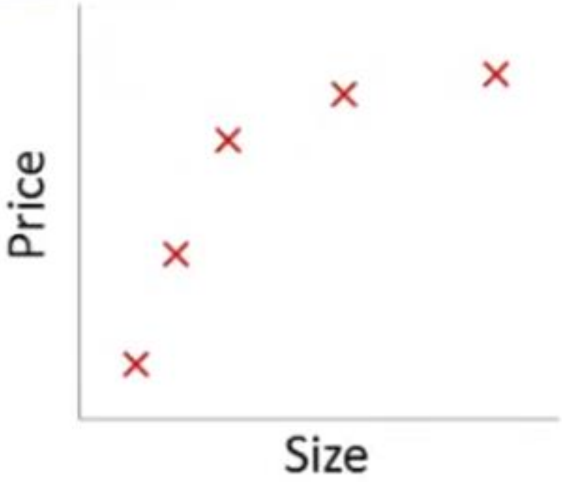
$\lambda = 10000.\ \theta_1 \approx 0, \theta_2 \approx 0, \ldots$

$h_\theta(x) \approx \theta_0$



Intermediate λ

"Just right"



Small λ

High variance (overfit)

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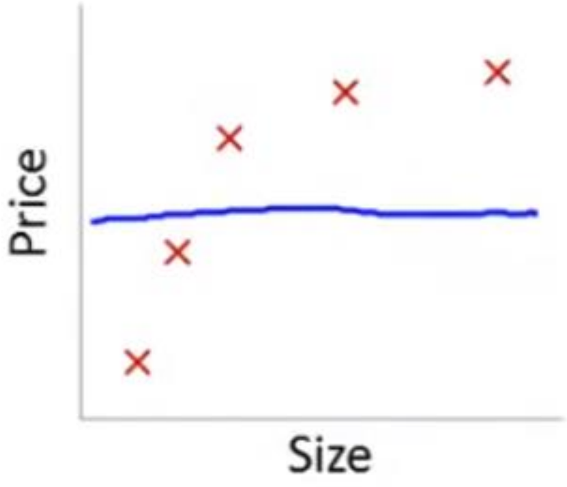

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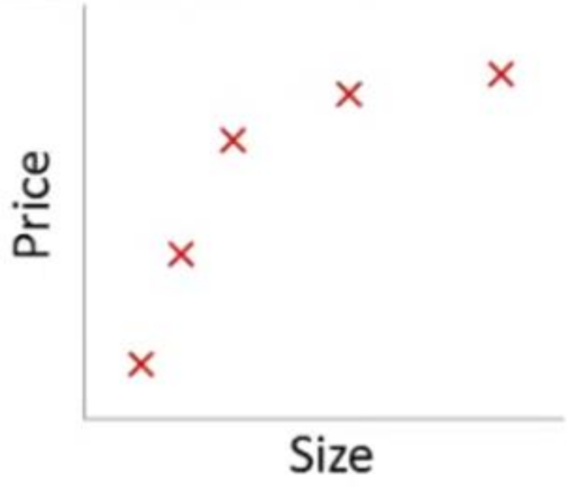

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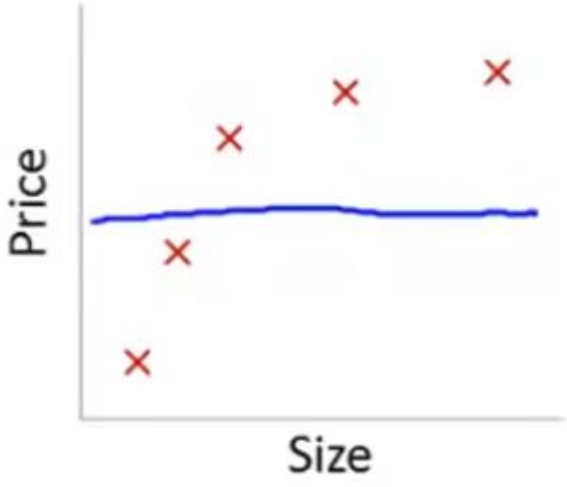

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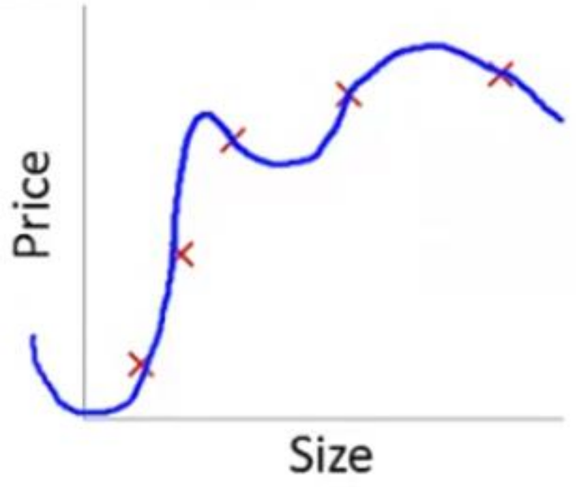

Small λ

High variance (overfit)

$\lambda = 0$

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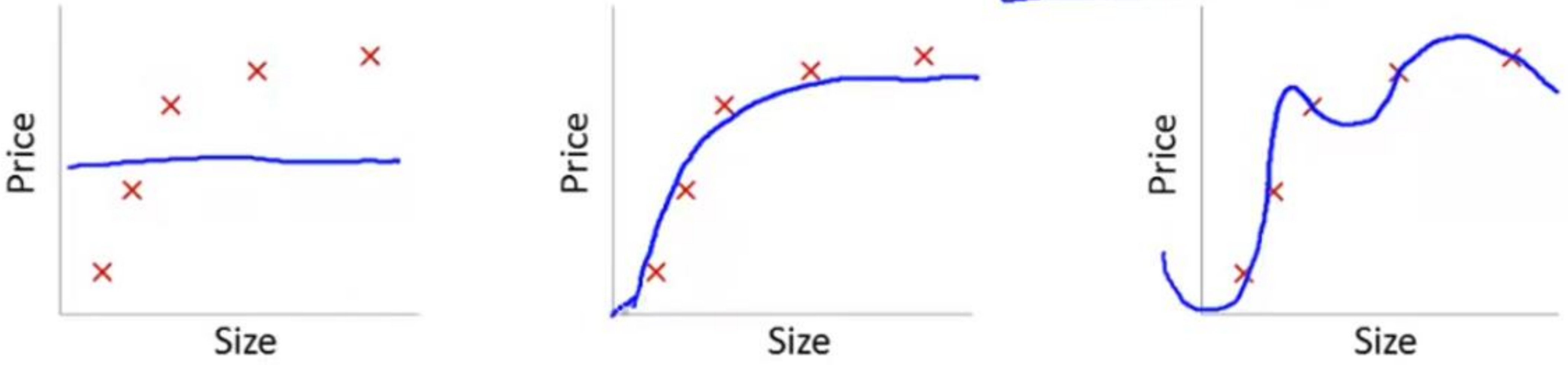

Large λ

High bias (underfit)

$\lambda = 10000.\ \theta_1 \approx 0, \theta_2 \approx 0, \dots$

$h_\theta(x) \approx \theta_0$

Intermediate λ

"Just right"

How to choose lambda??

Small λ

High variance (overfit)

$\lambda = 0$

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Choosing the regularization parameter λ

$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4 \leftarrow$$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2 \leftarrow$$

$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

Choosing the regularization parameter λ

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$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

$J(\theta)$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}}\sum_{i=1}^{m_{cv}}(h_\theta(x^{(i)}_{cv}) - y^{(i)}_{cv})^2$$

$$J_{test}(\theta) = \frac{1}{2m_{test}}\sum_{i=1}^{m_{test}}(h_\theta(x^{(i)}_{test}) - y^{(i)}_{test})^2$$

Choosing the regularization parameter λ

$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \underline{\frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2}$$

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J_{train}

J_{cv}

J_{test}

Choosing the regularization parameter λ

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

1. Try $\lambda = 0$
2. Try $\lambda = 0.01$
3. Try $\lambda = 0.02$
4. Try $\lambda = 0.04$
5. Try $\lambda = 0.08$

$\vdots$

12. Try $\lambda = 10$

Choosing the regularization parameter λ

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

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⋮

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10.24

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Choosing the regularization parameter λ

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

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1. Try $\lambda = 0$ $\longrightarrow \min_{\theta} J(\theta) \rightarrow \theta^{(1)}$
2. Try $\underline{\lambda = 0.01}$ $\longrightarrow \min_{\theta} J(\theta) \rightarrow \theta^{(2)}$
3. Try $\lambda = \underline{0.02}$ $\longrightarrow \theta^{(3)}$
4. Try $\lambda = \underline{0.04}$ $\quad\vdots$
5. Try $\lambda = 0.08$

$\vdots$

12. Try $\underline{\lambda = 10}$ $\longrightarrow \theta^{(12)}$

$\underline{10.24}$

Choosing the regularization parameter λ

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$\vdots$

12. Try $\lambda = 10$ $\longrightarrow \Theta^{(12)} \rightarrow J_{cv}(\Theta^{(12)})$

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4. Try $\lambda = 0.04$
5. Try $\lambda = 0.08$ $\longrightarrow \Theta^{(5)}$ $J_{cv}(\Theta^{(5)})$

$\vdots$

12. Try $\lambda = 10$ $\longrightarrow \Theta^{(12)} \rightarrow J_{cv}(\Theta^{(12)})$

10.24

Pick (say) $\theta^{(5)}$. Test error:

Choosing the regularization parameter λ

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$\vdots$

12. Try $\lambda = 10$ $\longrightarrow \Theta^{(12)} \rightarrow J_{cv}(\Theta^{(12)})$

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Pick (say) $\theta^{(5)}$. Test error: $J_{test}(\Theta^{(5)})$

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Bias/variance as a function of the regularization parameter λ

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

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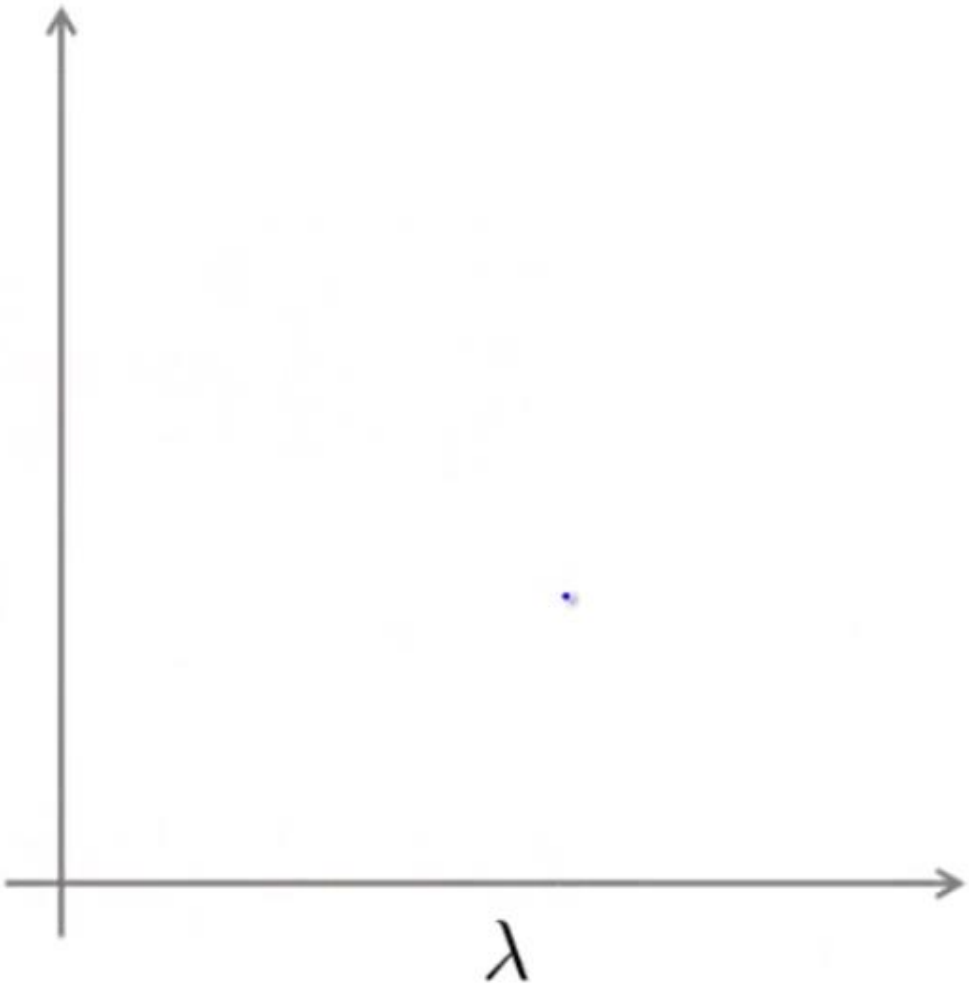

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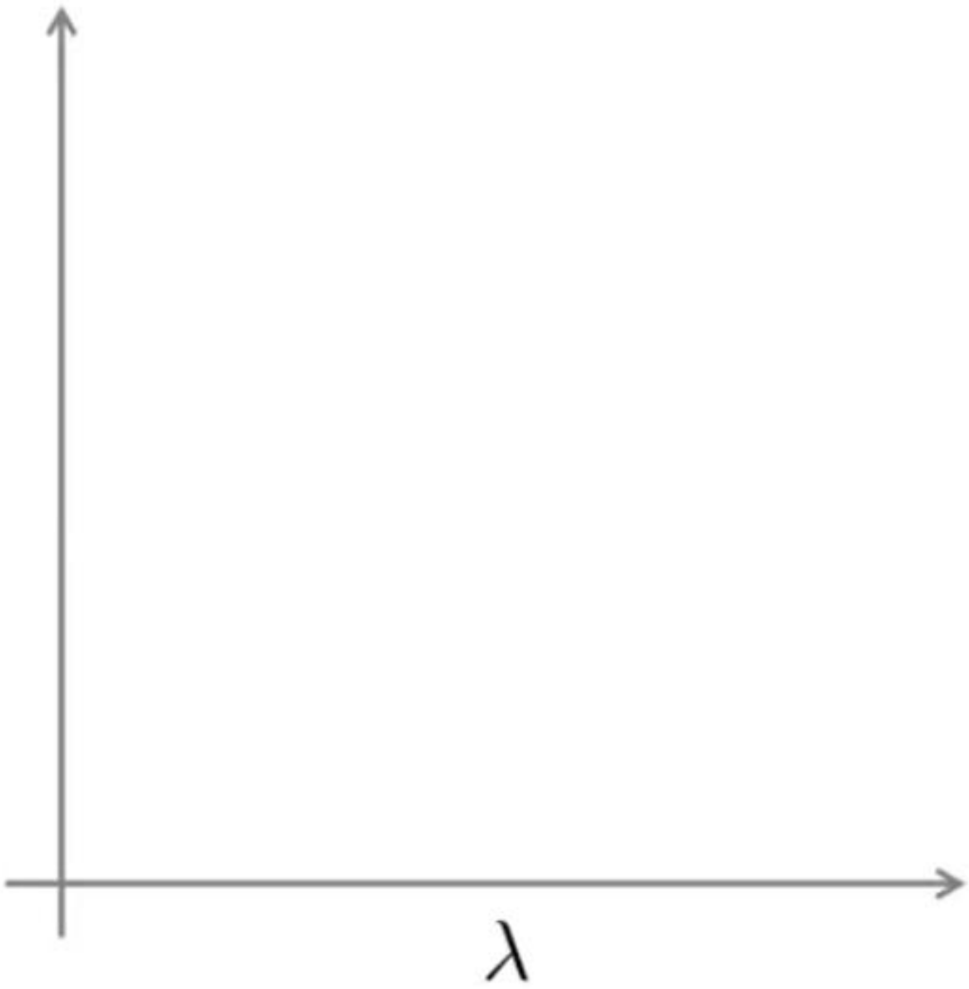

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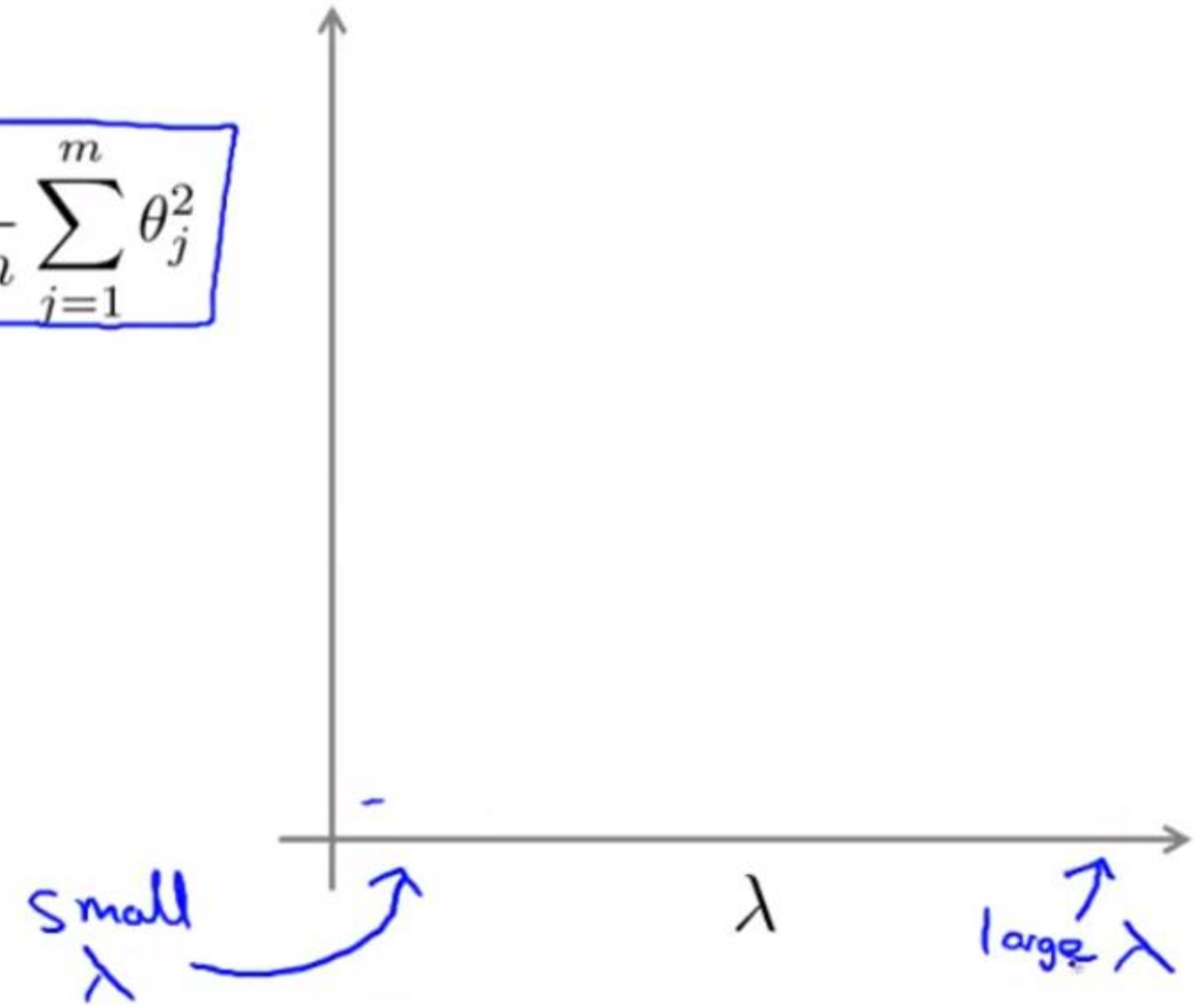

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Bias/variance as a function of the regularization parameter λ

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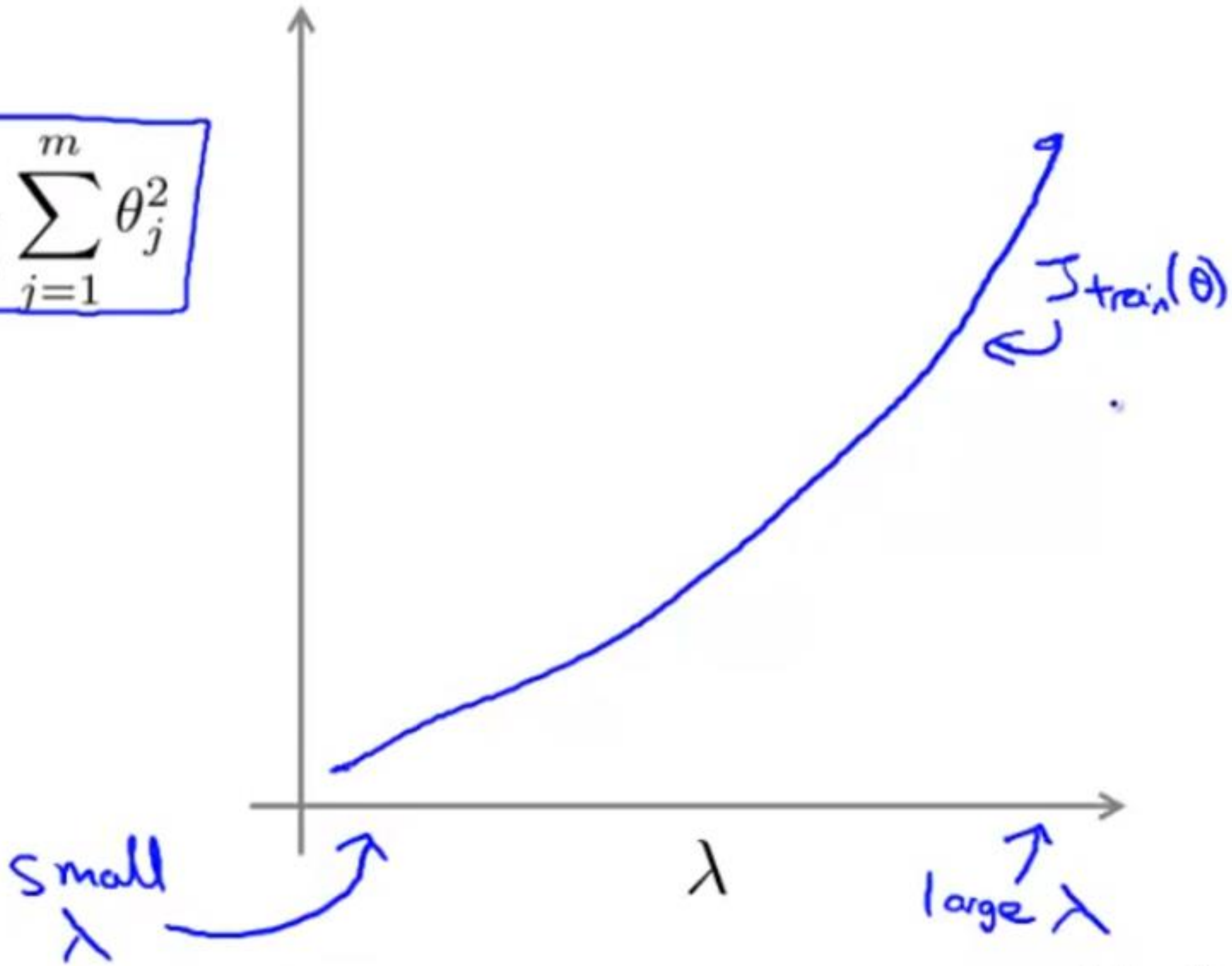

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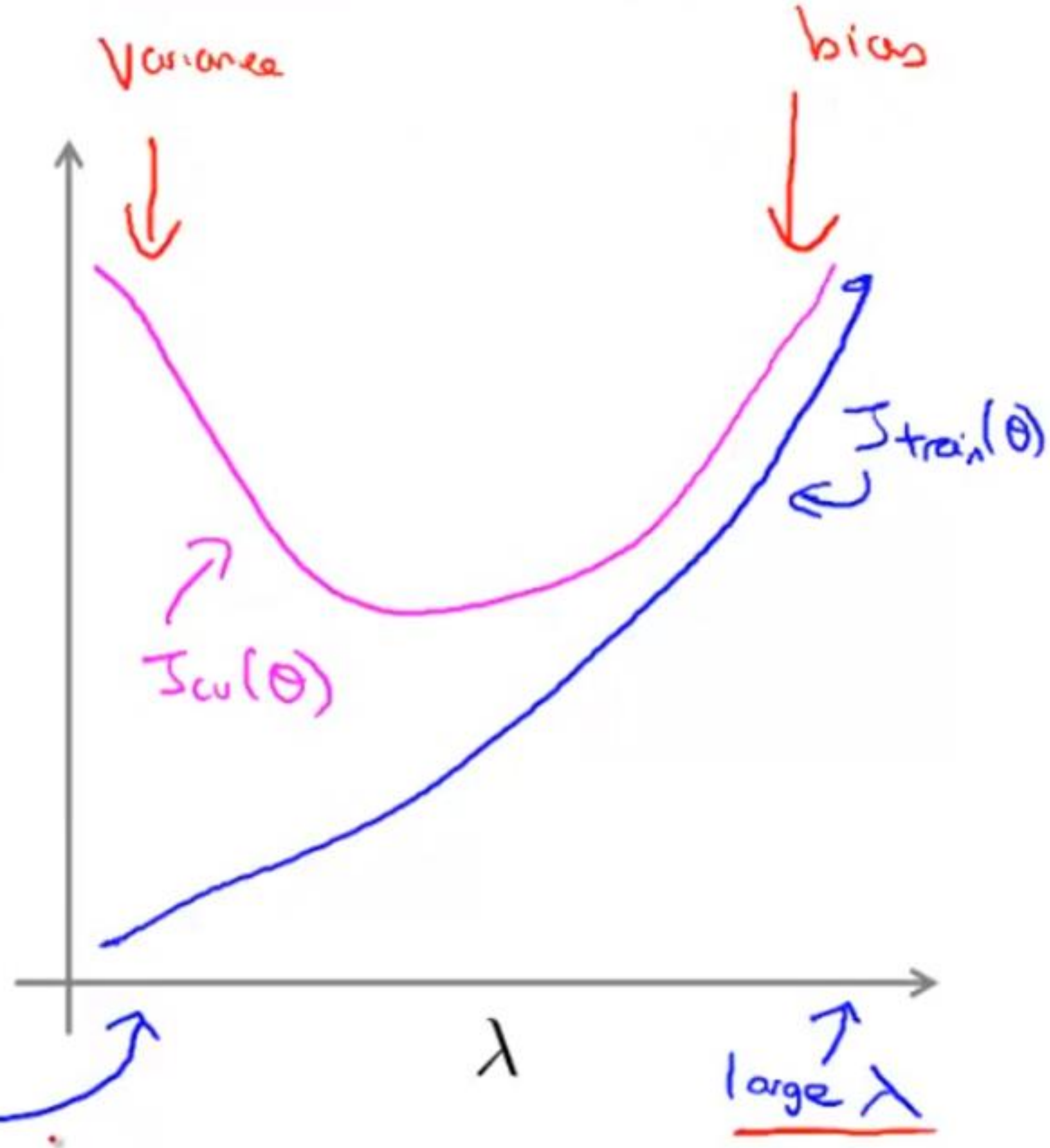

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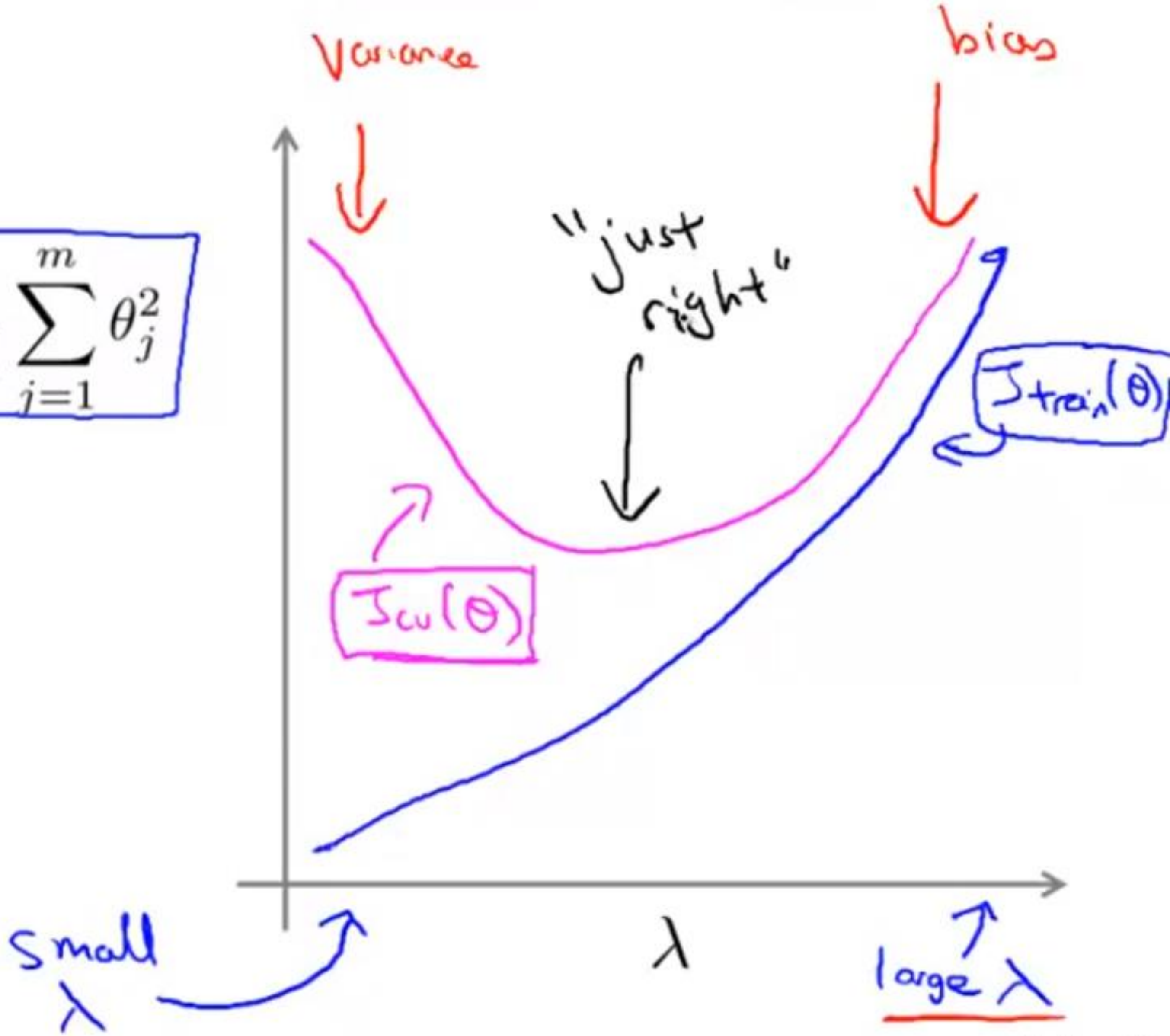

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Summary

- We understood that
  - as λ increases, our fit becomes more rigid.
  - as λ approaches 0, we tend to overfit the data.
- So how do we choose our parameter λ to get it 'just right' ?
- In order to choose the model and the regularization term λ, we need to:
  - Create a list of lambdas (i.e. $\lambda \in \{0,0.01,0.02,0.04,0.08,0.16,0.32,0.64,1.28,2.56,5.12,10.24\}$);
  - Create a set of models with different degrees or any other variants.
  - Iterate through the λs and for each λ go through all the models to learn some Θ.
  - Compute the cross validation error using the learned Θ (computed with λ) on the $J_{CV}(\Theta)$ **without** regularization or $\lambda = 0$.
  - Select the best combo that produces the lowest error on the cross validation set.
  - Using the best combo Θ and λ, apply it on $J_{test}(\Theta)$ to see if it has a *good generalization* of the problem.