

DECISION MAKING TECHNIQUES IN MANAGEMENT INFORMATION SYSTEMS (MIS)

LECTURE -9-(PROMETHEE)

MULTI CRITERIA DECISION MAKING MODELS: PROMETHEE

- One of the most efficient and easiest MCDM methodologies.
- Developed by Jean-Pierre Brans and Bertrand Mareschal at the ULB and VUB universities since 1982.
- Considers a set of criteria and alternatives. Criteria weights are determined that indicate the relative importance.
- Utilizes a function reflecting the degree of advantage of one alternative over the other, along with the degree of disadvantage that the same alternative has with respect to the other alternative.
- In scaling, there are six options allowing the user to express meaningful differences by minimum gaps between observations. When type I is used, only relative advantage matters; type 6 is based on standardization with respect to normal distribution.
- PROMETHEE I yields a partial preorder.
- PROMETHEE II yields a unique complete preorder.

MAIN STEPS

- 1. Building the outranking relation
 - DM chooses a generalized criterion and fixes the necessary parameters related to the selected criterion: a preference function is defined for each attribute
 - Multi-criteria preference index is defined as the weighted average of the preference functions
 - This preference index determines a valued outranking relation on the set of alternatives.

MAIN STEPS

- 2. Exploitating the outranking relation with regard to the chosen statement of the problem
 - For each alternative, a leaving and an entering flow are defined. A net flow is also considered.
 - A partial preorder (PROMETHEE I) or a complete preorder (PROMETHEE II) can be proposed to the DM.

RECOMMENDED GENERALIZED CRITERIA

Usual

$$P_k(a_i, a_j) = \begin{cases} 0 & \forall d \le 0 \\ 1 & \forall d > 0 \end{cases}$$

• U Shape

$$P_k(a_i, a_j) = \begin{cases} 0 & \forall d \le q_k \\ 1 & \forall d > q_k \end{cases}$$

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V Shape

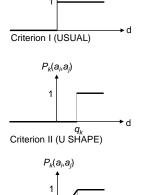
$$P_k(a_i, a_j) = \begin{cases} 0 & d \le 0 \\ d / p_k & 0 \le d \le p \\ 1 & d \ge p_k \end{cases}$$

$$P_k(a_i, a_j) = \begin{cases} 0 & \text{if } a = 0 \\ 1 - \exp(-d^2/2\sigma_k^2) & d \ge 0 \end{cases}$$

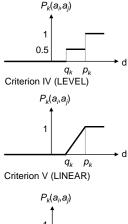
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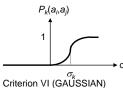
GENERALIZED CRITERIA

 $P_k(a_i,a_i)$



 p_k Criterion III (V SHAPE)





NECESSARY CALCULATIONS

o Multiattribute Preference Index

$$\pi(a_i, a_j) = \sum_k w_k P_k(a_i, a_j)$$

Leaving Flow

$$\Phi^+(a_i) = \sum_{a_j \in A} \pi(a_i, a_j)$$

• Entering Flow

$$\Phi^{-}(a_i) = \sum_{a_j \in A} \pi(a_j, a_i)$$

• Net Flow $\Phi(a_i) = \Phi^+(a_i) - \Phi^-(a_i)$

PROMETHEE I

- Two complete preorders are built:
 - Ranking the alternatives following the decreasing order of leaving flows
 - Ranking the alternatives following the increasing order of entering flows
- The intersection of the preorders yields the partial preorder.

PROMETHEE II

- A unique **complete preorder** is built:
 - Ranking the alternatives following the decreasing order of net flows.

EXAMPLE FOR PROMETHEE

a. A location problem

Let us consider the following multicriteria problem: Six criteria are considered as relevant by the decision-maker to rank six hydroelectric powerstation projects (x_1, \ldots, x_b) .

These criteria are:

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f<sub>1</sub>: manpower,
f<sub>2</sub>: power (MW),
f<sub>3</sub>: construction cost (10<sup>9</sup> $),
f<sub>4</sub>: maintenance cost (10<sup>6</sup> $),
f<sub>5</sub>: number of villages to evacuate,
f<sub>6</sub>: security level.
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The second and the last criterion have to be maximized, the others to be minimized.

EXAMPLE FOR PROMETHEE

Table 2 gives, for each criterion, the evaluations of the six actions, the type of generalized criterion specified by the decision maker, and the corresponding parameters. The six criteria are considered as having the same importance for the decision maker, so all the weights are equal.

Crit.	Min or Max	Actions						Type	Param-	
		<i>x</i> ₁	x 2	x3	x4	x ₅	x ₆	of crit.	eters	
f_1	Min	80	65	83	40	52	94	II	q = 10	
f_2	Max	90	58	60	80	72	96	III		p = 30
fi	Min	6	2	4	10	6	7	V	q = 0.5	p = 5
14	Min	5.4	9.7	7.2	7.5	2.0	3.6	IV	q = 1	p = 6
f_5	Min	8	1	4	7	3	5	I	_	
16	Max	5	1	7	10	8	6	VI	$\sigma = 5$	

EXAMPLE FOR PROMETHEE (THE PREFERENCE INDEX TABLE)

The preference index is represented in Table 3. The leaving flows are computed directly by adding the figures of each row of the table and the entering flows by adding the figures of each column.

П	x_1	x2	x3	x4	X5	x ₆	$\phi^*(x)$
x,		0.296	0.250	0.268	0.100	0.185	1.099
x2	0.462		0.389	0.333	0.296	0.500	1.980
x3	0.236	0.180		0.333	0.056	0.429	1.234
X4	0.399	0.505	0.305		0.223	0.212	1.644
X5	0.444	0.515	0.487	0.380		0.448	2.274
x ₆	0.286	0.399	0.250	0.432	0.133		1.500
$\phi^-(x)$	1.827	1.895	1.681	1.746	0.808	1.774	

EXAMPLE FOR PROMETHEE (NET FLOWS)

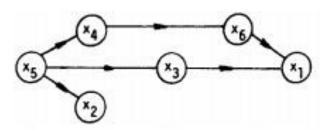
The net flows are then easily obtained by taking the difference between the leaving flows and entering flows.

Table 4

	<i>x</i> ₁	x2	x3	x4	xs	x ₆
φ(x)	-0.728	0.085	-0.447	-0.102	1.466	-0.274

EXAMPLE FOR PROMETHEE (PROMETHEE I-PARTIAL PREORDER)

	X1	X2	X 3	X4	X5	X6
Ф+	6	2	5	3	1	4
Φ-	5	6	2	3	1	4



EXAMPLE FOR PROMETHEE (PROMETHEE COMPLETE PREORDER)

	X1	X2	X 3	X4	X5	X6
Φ net	-0.728	0.085	-0.447	-0.102	1.466	-0.274
Rank	6	2	5	3	1	4

REFERENCES

- Lecture notes of "Prof. Dr. Y. İlker Topçu", http://web.itu.edu.tr/topcuil/
- o www.cwu.edu/~bayazito/AHP1.ppt
- o www.mis.boun.edu.tr/sencer/mis463/.../ahp.ppt
- o yunus.hacettepe.edu.tr/.../ppt/ahpnotes.ppt