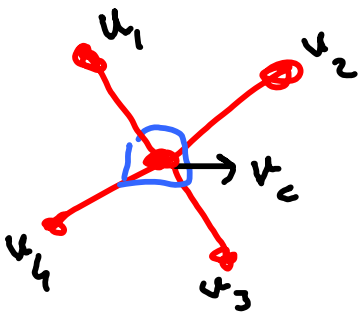Wiener index

$$W(G) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} d_G(u_i, u_j)$$

$K_{1,n}$ grafı için?

$d_G(u_c, u_i) = 1$, $i \in \{1, \dots, n\}$

$v_i \in V(G) - \{u_c\}$ olsun.

$d_G(u_i, u_j) = 2$ $i, j \in \{1, \dots, n\}$

$K_{1,5} = 25$

$$W(G) = \frac{1}{2} \left(n \cdot 1 + n \cdot \overbrace{((n-1) \cdot 2 + 1)}^{2n-1} \right)$$

$$W(G) = \frac{1}{2} \left(\cancel{n} + 2n^2 - \cancel{n} \right)$$
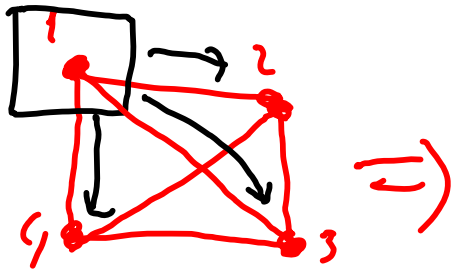
$$W(G) = n^2 \implies \boxed{W(K_{1,n}) = n^2}$$

```
top=0
for { i=0 ; i<n ; i++}
   {
      for (j=0 ; j<n ; j++}
         {
            top=top+ d[i][j]
         }
   }

printf ("G grafının wiener index
            degeri =% d , top/2 };
```
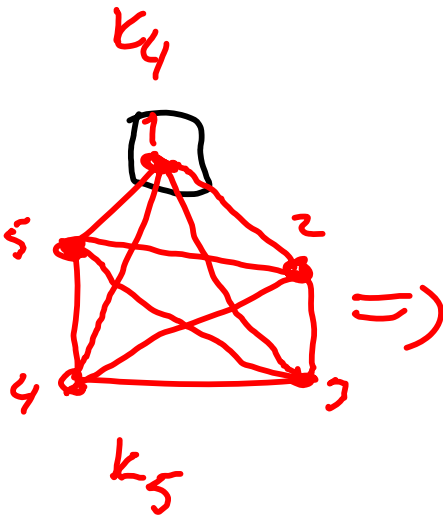
n tepeli tam grafın Wiener indeksi nedir?

$W(k_n) = ?$ (n cinsinden)

$$\underbrace{\frac{1 \text{ için}}{1+1+1}}_{} + \underbrace{\frac{2 \text{ için}}{1+1+1}}_{} + \underbrace{\frac{3 \text{ için}}{1+1+1}}_{}$$

$$\underbrace{\frac{4 \text{ için}}{1+1+1}}_{} = \underline{12}$$

$$\underline{W(k_4) = \frac{12}{2} \neq 6}$$ n cinsinden ?

$$\frac{4 + 4 + 4 + 4 + 4}{2} = 10 ✓$$

$$\boxed{W(k_5) = 10}$$

$$W(k_n) = ? \quad \frac{\left[(n-1) \cdot 1\right] \cdot n}{2}$$

$W(k_5) =$
$\frac{25-5}{2} = 10$

$$\boxed{W(k_n) = \frac{n^2 - n}{2}}$$

$$W(k_4) = \frac{16-6}{2} = 6$$

$W(P_n) = ?$ $W(C_n) = ?$ $W(W_{1,n}) = ?$

yol   Gevre   Tekerlek
  graf   graf   graf

Wiener index değerleri.

Eccentricity(Dış Merkezlik) Temelli Topolojiksel İndeksler

The connective eccentricity index: $\xi^{ce}(G) = \sum\limits_{u \in V(G)} \left(\deg_G(u)/\varepsilon_G(u) \right).$

The eccentric connectivity index: $\xi^c(G) = \sum\limits_{u \in V(G)} \left(\deg_G(u).\varepsilon_G(u) \right).$

The total eccentricity index: $\xi(G) = \sum\limits_{u \in V(G)} \varepsilon_G(u).$

The first Zagreb eccentricity index : $M_1^*(G) = \sum\limits_{uv \in E(G)} \left(\varepsilon_G(u) + \varepsilon_G(v) \right).$

The second Zagreb eccentricity index: $M_1^{**}(G) = \sum\limits_{u \in V(G)} \left(\varepsilon_G(u) \right)^2.$

The third Zagreb eccentricity index: $M_2^*(G) = \sum\limits_{uv \in E(G)} \left(\varepsilon_G(u).\varepsilon_G(v) \right).$
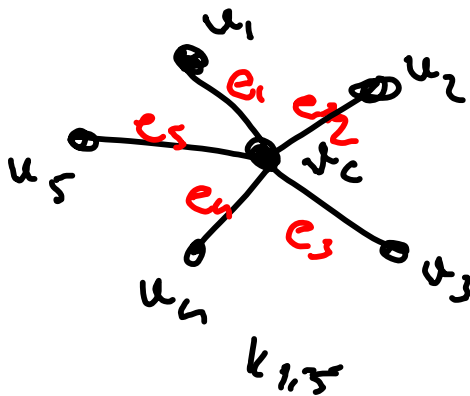
The eccentricity based geometric-arithmetic index:. $GA_4(G) = \sum\limits_{uv \in E(G)} \left(\dfrac{2\sqrt{\varepsilon_G(u).\varepsilon_G(v)}}{\varepsilon_G(u) + \varepsilon_G(v)} \right).$

New version of the ABC index namely $ABC_5(G)$: $ABC_5(G) = \sum\limits_{uv \in E(G)} \left(\sqrt{\dfrac{\varepsilon_G(u) + \varepsilon_G(v) - 2}{\varepsilon_G(u).\varepsilon_G(v)}} \right).$

Örnek)

$K_{1,5}$ grafı için yukarıdaki parametre değerlerini hesaplayınız.

$\varepsilon(v_c) = 1$

$\varepsilon(u_1) = \varepsilon(u_2) = \varepsilon(u_3) = \varepsilon(u_4)$
$= \varepsilon(u_5) = 2$

$$\xi^{ce}(K_{1,5}) = \sum_{u_i} \left(\frac{deg(u_i)}{\mathcal{E}(u_i)} \right)$$

$$= \frac{5}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 5 + \frac{5}{2} = \boxed{\frac{15}{2}}$$

→ Bu formül için kod, verilen C programına eklenebilir.

2) Eccentric Connectivity index

$$\xi^{c}(K_{1,5}) = \sum_{u_i} \left(deg(u_i) \cdot \mathcal{E}(u_i) \right)$$

$$= 5 \cdot 1 + 1 \cdot 2 + 1 \cdot 2 + 1 \cdot 2 + 1 \cdot 2 + 1 \cdot 2$$
$$\boxed{= 15}$$

3) Total Eccentricity index

$$\xi(K_{1,5}) = 1 + 2 + 2 + 2 + 2 + 2 = \underline{11}$$

4) 1. Zagreb Eccentricity index

$$M_1^*(K_{1,5}) = \sum_{uv \in E(K_{1,5})} \left(\mathcal{E}(u) + \mathcal{E}(v) \right)$$

$$= \overbrace{\mathcal{E}(u_c) + \mathcal{E}(u_1)}^{e_1} + \overbrace{\mathcal{E}(u_c) + \mathcal{E}(u_2)}^{e_2}$$
$$ \quad 1 \qquad\qquad 2 \qquad\qquad 1 \qquad\qquad 2$$

$$\overbrace{\mathcal{E}(u_c) + \mathcal{E}(u_3)}^{e_3} + \overbrace{\mathcal{E}(u_c) + \mathcal{E}(u_4)}^{e_4}$$
$$\quad 1 \qquad\qquad 2 \qquad\qquad 1 \qquad\qquad 2$$

$$\overbrace{\mathcal{E}(u_c) + \mathcal{E}(u_5)}^{e_5} \qquad = \underline{115}$$
$$\quad 1 \qquad\qquad 2$$

5) İkinci Zegreb Eccentricity indeks

$$\mu_1^{***}(K_{1,5}) = 1^2 + 2^2 + 2^2 + 2^2 + 2^2 + 2^2 = 21$$

6) Üçüncü Zegreb Eccentricity indeks

$$\mu_2^{*}(K_{1,5}) = \sum_{uv \in E(K_{1,5})} (\varepsilon(v) \cdot \varepsilon(u))$$

$$= \left(\overbrace{\underset{1}{\varepsilon(u_c)} \cdot \underset{2}{\varepsilon(u_1)}}^{e_1} \right) + \left(\overbrace{\underset{1}{\varepsilon(u_c)} \cdot \underset{2}{\varepsilon(u_2)}}^{e_2} \right)$$

$$\left(\overbrace{\underset{1}{\varepsilon(u_c)} \cdot \underset{2}{\varepsilon(u_3)}}^{e_3} \right) + \left(\overbrace{\underset{1}{\varepsilon(u_c)} \cdot \underset{2}{\varepsilon(u_4)}}^{e_4} \right)$$

$$\left(\overbrace{\underset{1}{\varepsilon(u_c)} \cdot \underset{2}{\varepsilon(u_5)}}^{e_5} \right) = \boxed{10}$$

7) Eccentricity tensell: Geometrik - Aritmetik indeks

$$GA_q(C_{1,5}) = \sum_{uv \in E(C_{1,5})} \left(\frac{2\sqrt{\varepsilon(u) \cdot \varepsilon(u)}}{\varepsilon(u) + \varepsilon(u)} \right)$$

$$= \underbrace{\frac{2\sqrt{\varepsilon(u_c) \cdot \varepsilon(u_1)}}{\varepsilon(u_c) + \varepsilon(u_1)}}_{e_1} + \underbrace{\frac{2\sqrt{\varepsilon(u_c) \cdot \varepsilon(u_2)}}{\varepsilon(u_c) + \varepsilon(u_2)}}_{e_2}$$

$$\underbrace{\frac{2\sqrt{\varepsilon(u_c) \cdot \varepsilon(u_3)}}{\varepsilon(u_c) + \varepsilon(u_3)}}_{e_3} + \underbrace{\frac{2\sqrt{\varepsilon(u_c) \cdot \varepsilon(u_4)}}{\varepsilon(u_c) + \varepsilon(u_4)}}_{e_4}$$

$$\underbrace{\frac{2\sqrt{\varepsilon(u_c) \cdot \varepsilon(u_5)}}{\varepsilon(u_c) + \varepsilon(u_5)}}_{e_5} = ?$$

$$= \frac{2\sqrt{1 \cdot 2}}{3} + \frac{2 \cdot \sqrt{1 \cdot 2}}{3} + \cdots \cdots + \frac{2\sqrt{1 \cdot 2}}{3}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{5 \text{ tane ver.}}$$

$$= \frac{2\sqrt{2}}{3} \cdot 5 = \boxed{\frac{10\sqrt{2}}{3}}$$

8) Eccentricity Temelli Atom-Bond
Connectivity indeks:

$$ABC_5(K_{1,5}) = \sum_{uv \in E(K_{1,5})} \left(\sqrt{\frac{\mathcal{E}(u) + \mathcal{E}(v) - 2}{\mathcal{E}(u) \cdot \mathcal{E}(v)}} \right)$$

$$ABC(x_{1,5}) = 5 \cdot \left(\sqrt{\dfrac{1 + \cancel{2} - \cancel{2}}{1 \cdot 2}} \right)$$

$$= 5 \cdot \sqrt{\dfrac{1}{2}} = \dfrac{5}{\sqrt{2}} = \boxed{\dfrac{5\sqrt{2}}{2}}$$

*) Tüm indeks değerleri verilen C kodu yardımıyla hesaplanabilir.

*) Uzaklık matrisi: (A^k ile bulunan) algoritmasının verimlilik sınıfı: $\boxed{O(n^3)}$

Matris çarpımı