A THEORY OF CONVEYORS*

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The common loop overhead conveyor linking two manufacturing process areas is studied as a mechanism for transforming the output flow of one area into the input flow of the other. The variables relevant to a smooth transformation of flows and their inter-relationships are explored. Findings are formulated into three principles. On the conveyor operation problem, distinguished from the design problem, two methods of simulation are developed and practical suggestions advanced for use in feasibility and optimality considerations. A fundamental observation is that a conveyor is part of a dynamic system, hence information on the connected areas is essential to its smooth operation.

1. Introduction

Conveyors are widely used in modern manufacturing plants. Sometimes they are used for delivering items such as materials, parts, or finished goods from one area to another. Sometimes they are also used for storing these items. Perhaps more frequently than these two purposes, they are used for the combined purpose of both delivering and storing these items.

Conveyors come in many different forms. A quick reference to a standard production handbook indicates that at least thirty types of material handling equipment may be regarded as fixed-path conveyor systems, under the three broad categories of gravity equipment, power-driven equipment, and pneumatic and hydraulic equipment. Thus, there are gravity roller conveyors, flat-belt conveyors, bucket elevators, pneumatic conveyors, etc. One of the most commonly used conveyors in manufacturing shops with fabrication and assembly operations is the overhead, monorail, chain conveyor with suspended carriers, moving in a continuous loop. It is simple and rugged in construction, and inexpensive in cost. It is capable of handling one or more items, with one or more loading points (stations) and unloading points. Moreover, the usual practice is to provide a large enough loop of conveyor so that the conveyor can be used for storing as well as delivering the items it is handling.

Unfortunately, experience with this type of monorail chain conveyors indicates that too often they are the sources of trouble. At the loading point, often no empty carriers will be coming by for a period of time to accommodate the items to be loaded onto the conveyor. At the unloading point, similarly, no loaded carriers will be coming by for some time to supply the consuming area with items. Such phenomena, when they occur, inevitably cause interruptions of normal manufacturing operations, revisions of preplanned schedule, and all the ensuing confusion and frustration.

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As a consequence of this, the usual practice to combat such an unhappy situation takes one or a combination of the following four forms:

(a) Run the conveyor faster in the hope that the duration of shortage period, if it should arise, will be reduced.
(b) Provide a reserve floor storage space at the loading point and a reserve stock at the unloading point for use when such shortages on the conveyor arise.
(c) Convert a long conveyor loop into two or more shorter loops and use materials handlers to transfer the items from one loop to another, in the hope that carriers on the loop with the loading point will be mostly empty, and that carriers on the loop with the unloading point will be mostly full.
(d) Replace the loop conveyor by some other type of conveyor or equipment, such as the more costly Power-and-Free type of conveyor.

While some of these alternatives have a plausible reasoning behind them, others are only illusionary in their ability to avoid shortages on the conveyor. Some of these alternatives really do not eliminate the cause of these shortages. In fact, they succeed only in shifting the occurrence of the shortages elsewhere, or in creating new difficulties.

It is the purpose of this paper to demonstrate that simple, loop conveyors *can* be operated smoothly without any difficulty provided only a few simple principles, applicable to practically all types of conveyors, are observed. *The fundamental message is that a conveyor is part of a system. Therefore, it should not be considered as an isolated object independent of the areas which it is linking together.* In the presentation a simulation method for the analysis of conveyor operations, and expressions for the design of a conveyor are also developed and explained. The scheme this paper will follow is as follows: We shall start the presentation with a general discussion of the types of mechanism linking two manufacturing areas and a scheme of classification for these mechanisms. We shall then focus our attention to a more common type of conveyor, relate the assumptions we shall be using, and define the specific problem we are tackling in this paper (Section 2). Next, we shall proceed to develop the three fundamental principles necessary for the understanding of conveyor operations (Section 3) and explain the two simulation methods which may be used to analyze the condition of a conveyor in operation (Section 4). To make use of the understanding we have thus gained we shall then show how the problem of conveyor operation, as against the problem of conveyor design, can be solved (Section 5). Finally, we shall summarize our results and indicate some of the work that remains to be done in the future in this area (Section 6).

2. Types of Conveyors, the Conveyor Problem Discussed, and the Assumptions Made

As we have indicated previously, conveyors come in many types. In practice conveyors are distinguished by their mechanical constructions. For the purpose of this paper such an identifying scheme will not be very helpful, since the basic operating characteristics are what we are interested in. Of the many factors

TABLE 1
Classification of Conveyors

	Discrete (Receptacle type)	Continuous (Belt type)
Equal Rates (Delivery type)	Type DD	Type DC
Unequal Rates (Storage-and-Delivery type)	Type SD	Type SC

that influence the operation of a conveyor, two are useful also as criteria for classifying conveyors. These are the rates of loading and unloading, and the discreteness or continuity *in time* of the loading and the unloading. When the loading rate of a conveyor is equal to its unloading rate, the quantity of items put on the conveyor at any instant is equal to the quantity taken off the conveyor at that instant, only at a different geographical location. Hence, the total quantity of the items on the conveyor at any instant remains unchanged. The conveyor performs strictly a delivery function. When the two rates are not equal, the quantity of items put on the conveyor at any instant is not equal to the quantity taken off at that instant. The total quantity of items on the conveyor changes from instant to instant. Sometimes there will be an accumulation. Sometimes there will be a depletion. Hence, the conveyor performs a storage as well as a delivery function.

If loading or unloading is done continuously in time, the conveyor must be able to accommodate such loading and unloading continuously. The conveyor must then take the form of a moving belt (thus the belt-type). If the loading or unloading is done in some small quantities at a time, the conveyor must also be able to accommodate such sudden lumped demands. The conveyor will then usually not be the belt type. It will have carriers or receptacles to accommodate the discrete loading and unloading (thus the receptacle-type). The problems of both operating and designing a conveyor become more complicated here because of the problems associated with these receptacles, problems such as the capacity of the carriers, and the spacing of the carriers. Table 1 illustrates the classifying scheme described here clearly.

A little reflection will show that the delivery type of conveyors, be it the receptacle type or the belt type, is rather simple to understand and handle. As long as the conveyor moves at a constant speed, for whatever amount loaded on the conveyor during a time interval, there is a corresponding amount unloaded off the conveyor during the same time interval. Present loading at the loading point becomes supplies for later unloading at the unloading point, *at the same rate*. Present unloading sets up accommodations for later loading, also *at the same rate*. Since the loading point is separated from the unloading point, it is necessary to pre-store enough items on the conveyor to take care of the transit time if loading and unloading are to start at the same time. Since random fluctuations in the loading and unloading rates usually occur, it is necessary to pre-store a buffer stock to absorb such changes. Other than these, operating the delivery type of conveyor presents no difficulty.

The storage-delivery type of conveyors, on the other hand, is not so simple.

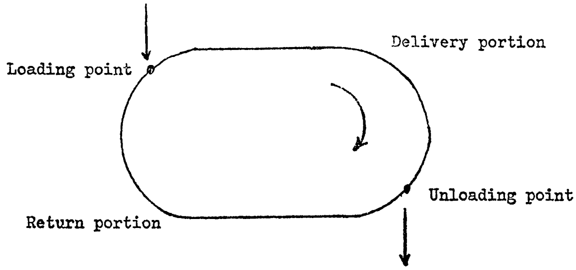

FIG. 1. Conveyor Studied

There is no such nice one-for-one relationship between what happens at the loading point and what happens at the unloading point. A section of the conveyor that has just received some loading, when it comes around to the unloading point, may be carrying too few items (or too many) for the unloading at that instant. A section that has just had some unloading when it comes around to the loading point, may be furnishing too few (or too many) empty slots to accommodate the loading at that instant. This lack of one-for-one relationship will aggravate in time, eventually causing the undesirable shortages mentioned previously to occur.

In this paper we shall attempt to examine thoroughly the more difficult case of the storage delivery type of conveyors. More specifically we shall deal with a rather primitive *discrete*, *storage-delivery* conveyor, moving in a loop with *one loading point* and *one unloading point*, and carrying only one type of item. A study of such a primitive conveyor enables us to bring out the basic principles of conveyor operation clearly. Once these basic principles are grasped, the application of them to special cases like the delivery-type of conveyors or the generalization of them to cover more involved cases such as multi-item conveyors will be straightforward. Schematically, the primitive conveyor is shown in Figure 1.

In studying a conveyor, it is important to bear in mind that the conveyor is a linkage connecting two manufacturing areas together. It serves to transport materials, parts, or finished goods from one geographical location to another.[1] What the conveyor receives at one end, the loading point, is the output of one manufacturing area, the feeding area. What the conveyor delivers at the other end, the unloading point, is the input of the other manufacturing area, the fed area. Thus the functioning of a conveyor really cannot be divorced from *the two areas* which it is connecting. In other words, the schedule of output of the feeding area and the schedule of input of the fed area must be considered in examining the conveyor. In industrial cases, it often happens that this schedule of output-input rates of the two areas, or equivalently, the schedule of loading-unloading rates for the conveyor, possesses obvious regularity. That is, the schedule often is made up from a number of basic identical patterns. Since the basic pattern repeats itself, it will be sufficient to study what happens in one

[1] Storage may be considered as delayed transport.

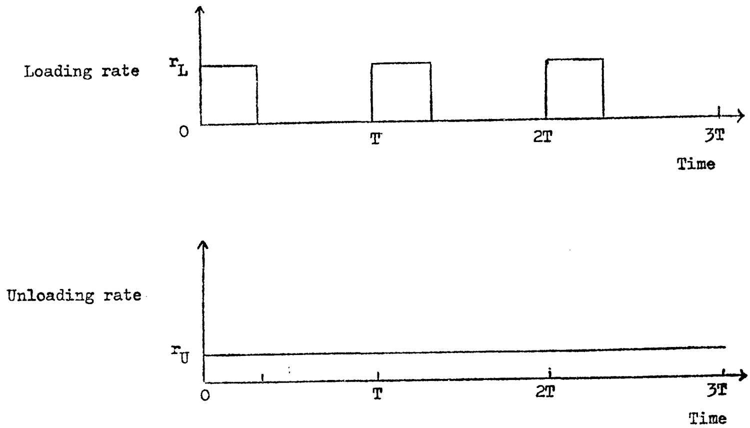

FIG. 2. Basic Production Cycles or Loading-Unloading Cycles

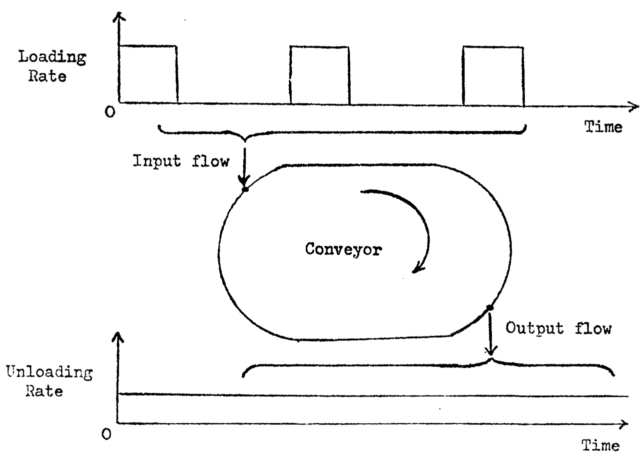

FIG. 3. Conveyor as a "Transformer" of Flows

of such patterns. We shall call such a basic pattern, the *production cycle* or *loading-unloading cycle.* Figure 2 shows a typical basic pattern.

Thus, the problem confronting us is a problem of "transformation". In its simplest form the problem arises when a conveyor such as is depicted in Figure 1 is used to transform an input "flow" into an output "flow" such as are depicted in Figure 2. Diagrammatically, the problem is shown in Figure 3. A slightly more involved situation would be a case in which the two flows are less regular but the total input is still equal to the total output, over some interval. Of course, a realistic situation would be a case in which random fluctuations are present in the two flows.

To define the task at hand more specifically, we shall address ourselves to the following problems:

(a) The determination of the variables that are relevant to the transformation process shown above, and the formulation of the relationships that must exist among these variables for the transformation process to be realizable.

(b) The verification of the understanding thus gained and the application of the derived principles to practical situations to prescribe conditions of feasibility and optimality. To facilitate our study, we shall make the following assumptions:

1. That the speed of the conveyor, once decided upon, can be held constant at that value.
2. That there are no random fluctuations in either the loading rate or the unloading rate.
3. That over some fixed period of time the total load onto the conveyor equals the total unload off the conveyor.

3. Three Fundamental Principles

A thorough examination of the functioning of a primitive conveyor has revealed that a conveyor is really not difficult to understand. There are only three fundamental principles governing the satisfactory operation of a conveyor. They are all very simple and are all related to its speed in some way.

The first fundamental principle may be called the SPEED RULE: *The speed of the conveyor*, in terms of the number of carriers per unit time, *must be within the permissible range*. This is a rule that specifies the range of speeds within which the conveyor may be operated. The lower limit of this range is determined by the loading rate or the unloading rate, whichever is higher. The upper limit of this range is determined by the technological capability of the conveyor or the human ability of handling the items or parts onto or off the conveyor, whichever gives the slower speed. Symbolically, this is given by:

$$\text{(1)} \qquad \operatorname{Max}(r_L, r_U) \leqq \frac{v}{s} \leqq \operatorname{Min}\left(\frac{1}{t_L}, \frac{1}{t_U}, \frac{v_c}{s}\right)$$

where r_L = loading rate
r_U = unloading rate
v = velocity or speed of conveyor
s = spacing between carriers
t_L = average time to load an item on the conveyor
t_U = average time to unload an item on the conveyor
v_c = maximum speed of the conveyor that is technologically allowable

It is assumed of course that $1/t_L$ and $1/t_U$ are greater than r_L and r_U, which is generally true in practice. The truth of this rule is immediately obvious. The conveyor can neither be moving too fast nor too slowly. On the one hand, it must be fast enough to supply items at least at the unloading rate, and at the same time, empty carriers at least at the loading rate. This consideration gives the lower limit of the range of permissible speeds. On the other hand, the conveyor must be slow enough for its speed to be below the safe technological specification, and to allow sufficient time for handling the items onto or off the conveyor. This consideration gives the upper limit of the range of permissible speeds.

The second fundamental principle may be called the CAPACITY CONSTRAINT: *The conveyor must have enough capacity,* in terms of the number of racks (not to be confused with carriers), *to accommodate the accumulated items, the intentional reserve stock, and the temporary requirements at the loading and unloading points due to the fact that these two points are geographically separated.* This is a condition on the parameters of the conveyor, parameters such as the total length of the conveyor, the spacing between carriers, and the number of racks on a carrier. Since the conveyor is a moving thing, it is not meaningful to speak of these parameters without referring to the speed of the conveyor. As every experienced hand in the factory knows, the higher the speed of a conveyor is, the more items the conveyor seems to be able to carry. And there is actually a sound reason for this. Rather than speaking of the total number of racks on a conveyor, therefore, it is more meaningful to introduce the time dimension and speak of the number of racks passing by in a unit time at a given speed.

Symbolically, the capacity constraint may be expressed as:

$$\frac{mqv}{L} = \frac{mq}{w} = \frac{qv}{s} \geqq K \tag{2}$$

where m = total number of carriers on the conveyor
q = the capacity of a carrier (number of racks per carrier), or the total number of items a carrier can accommodate
v = the speed of the conveyor
L = the length of the conveyor loop
w = L/v = revolution time of the conveyor
s = L/m = spacing between carriers
K = a constant determined by the specified amount of safety reserve stock to be carried by the conveyor, the loading-unloading schedule, and the revolution time of the conveyor

Again the truth of this condition is immediately obvious. The left hand side of this inequality is the number of racks passing by any point in a unit time when the speed of the conveyor is v, or equivalently, when the revolution time of the conveyor is w. This is also the maximum number of items the conveyor can accommodate in a unit time at that speed, or the maximum number of items the conveyor may be expected to deliver in a unit time interval. In other words the left hand side is what the conveyor can do if its revolution time is w. The right hand side, in contrast, is simply the requirements of the conveyor. The exact value of K has to be determined.

The third fundamental principle may be called the UNIFORMITY PRINCIPLE: *The conveyor must be loaded as well as unloaded uniformly throughout its entire length.* This is the most important principle of the three. It is also, in a way, related to the speed of the conveyor.

The truth of the Uniformity Principle requires a little reflection. The customary practice in dealing with conveyors has been to focus attention on the speed at which the conveyor is to be operated. However, this has been done *only* in the sense of the Speed Rule. We might say that this is a static sense.

Thus, we are given the loading rate and the unloading rate. We then furnish a long, long conveyor to give us a large enough total number of racks, and choose a speed within the permissible range, and *think* we have a workable system with ample capacity. However, almost always, we find that this is not true. On the contrary, at times we find no items coming to the unloading point and no empty carriers coming to the loading point, when we know for sure that we have these *somewhere* on the conveyor. We discover that our *timing* is off. This situation arises simply because we have not considered speed in the sense of the Capacity Constraint and the Uniformity Principle, which is a dynamic sense. Let us presently amplify this point.

In considering a loop conveyor, the most important thing to bear in mind is that there exists not only a question of "availability" of items or empty carriers, but also a question of "accessibility" of items or empty carriers. When we say that some items or empty carriers are available on the conveyor, we mean that there are some items or empty carriers *somewhere* on the conveyor. When we say that some items or empty carriers are accessible, we mean that they *will be at the proper place when they are needed*. Now, how do we obtain accessibility in addition to availability? To answer this, let us look at the adverse situation; namely the situation of inaccessibility. We ask, "When do we find that we do not have, say, empty carriers at the loading point, but instead, they are somewhere else on the conveyor, and why?" We find that this situation arises when the section of conveyor that is passing by the loading point is fully loaded whereas other sections are not. This section is fully loaded because it has either received more loading or less unloading than the other sections. We might say that this section has been favored over the others in loading or in unloading. In other words, the loading or the unloading of the conveyor has not been done *uniformly*. Putting this in a different way, we have demonstrated that accessibility can be obtained by uniformly loading and unloading the conveyor.

Having thus demonstrated the necessity of the uniformity principle, it becomes a simple matter to translate the principle into a more practical rule or rules. To load (or unload) a conveyor unformly is to load (or unload) all sections of the conveyor by the same amount of items. Now in one revolution of the conveyor all sections of the conveyor pass by the loading point once and only once. If the loading rate is constant and continuous during that time as it is under our assumptions, all sections of the conveyor will get the same amount of load in one complete revolution of the conveyor. If the conveyor makes another complete revolution, all sections of the conveyor will get another equal amount of load in addition to their previous load. If the conveyor makes a third complete revolution, all sections of the conveyor will get still another equal amount of load; and so on. And we can say the same with respect to unloading. Thus, we see that *uniformity can be obtained if the conveyor can make an integral number of revolutions during the time loading (also unloading) takes place, provided that the loading (also unloading) rate is constant and continuous without breaks.*

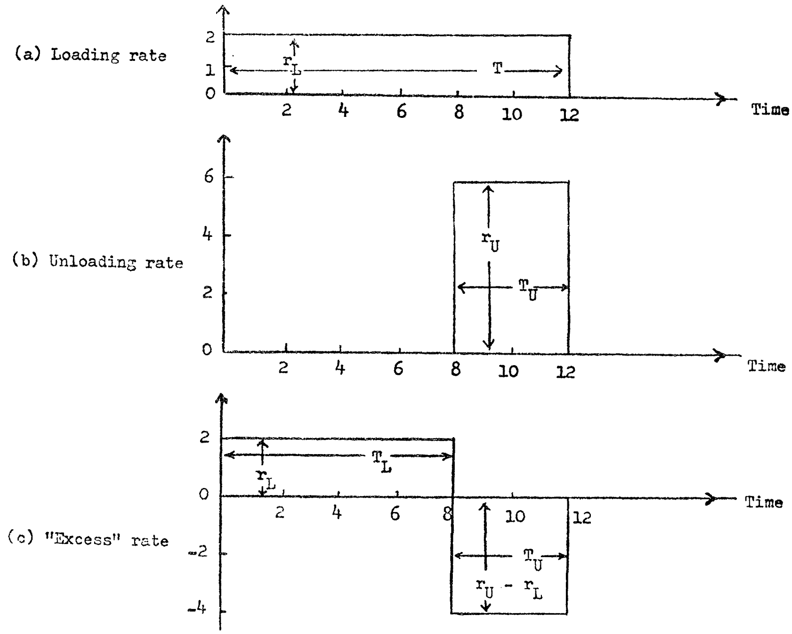

FIG. 4. Elementary Cycle

Ordinarily, in industrial cases both the loading rate and the unloading rate are constant, as they are under our assumptions stated earlier. Since total load must equal total unload by assumption, if the two rates are not equal, they will have different time durations. If both loading and unloading are done without interruptions, then the general rule for obtaining uniformity is simply to choose a conveyor speed such that the conveyor will make an integral number of revolutions during the time when loading takes place, and also an integral number of revolutions during the time when unloading takes place.

Frequently, however, the loading and unloading schedule in an industrial context can be broken up into many identical elementary patterns or cycles as is shown in Figure 2. In this case it may be feasible to consider just the basic elementary cycle, of duration T, and choose a value of the speed of the conveyor such that the conveyor will make an integral number of revolutions in the loading duration of the elementary cycle, T_L, as well as an integral number of revolutions in the unloading duration, T_U, of the elementary cycle (See Figure 4).

Sometimes it may not be possible to achieve this in one elementary cycle. The duration of the cycle, T, may be so small that it becomes impossible to find a high enough speed within the technological range dictated by the Speed Rule. Or, equivalently, the conveyor may be too long to make complete revolution in a short time. In such cases uniformity may still be achieved. It will be necessary to consider several such identical elementary cycles together. In that case, the distribution of items on the conveyor will not be quite like that in the former case. Previously, at any instant a section in the delivery portion of the conveyor has the same number of items as all sections in that portion; a section in the return portion of the conveyor also has the same number of items as all sections in that portion. Now, in contrast, when uniformity is to be achieved in several cycles, a section merely experiences the same amount of transactions of items some time during the duration of the several cycles. At any instant, it does not necessarily have the same number of items as its neighboring sections. Thus, the word "uniformity" is perhaps really not too descriptive of the situation in such cases. A "return" at the end of a number of cycles to the original condition is what will actually be happening. The accumulation of items on the conveyor when uniformity is attained in several cycles taken together is naturally higher than that resulting when uniformity is attained in one cycle. As more accommodations must be provided for a higher accumulation, attaining uniformity in several cycles must be considered less desirable. The situation will be considered again later when the method of analysis is discussed.

When the loading and unloading schedule is made up of identical cycles as shown in Figure 2, to attain uniformity in N_T cycles, the revolution time of the conveyor must satisfy the following conditions:

(3) $(N_T T)/w = N_w$, an integer not equal to one with $N_w \neq N_T$

(4) $(N_T T_L)/w =$ integer,

or $(N_T T_U)/w =$ integer with $T_L + T_U = T$, and

where N_T = the number of elementary cycles considered
T = cycle time
w = revolution time of conveyor
N_w = 2, 3, 4, $\cdots$, integer
T_L = loading duration in the elementary cycle, or more appropriately, time when conveyor has a net gain of items (See Figure 4).
T_U = unloading duration in the elementary cycle, or more appropriately, time when conveyor has a net loss of items (See Figure 4).

Expression (3) merely says that in N_T cycles the conveyor makes N_w complete revolutions. Obviously, N_w should not be made to equal to N_T, for then the revolution time of the conveyor would have to equal to the cycle time. This means that wherever the conveyor starts from at the beginning of the cycle, it would start there at the beginning of the next cycle. The same pattern of loading and unloading will fall upon the same section of the conveyor always; there is no chance of one neutralizing the other to produce uniformity on the conveyor. This obviously will not do. The condition that N_w should not be made equal to one is also obvious, for otherwise the conveyor makes one complete revolution in N_T cycles and, consequently, the same pattern of loading and unloading would fall upon the same sections of the conveyor. As mentioned previously, it is preferable to attain uniformity in $N_T = 1$ cycle.

Merely having expression (3), guaranteeing that the conveyor makes a number of complete revolutions in one or several cycles' duration, however does not necessarily guarantee uniformity. For in each cycle there is an interval when the conveyor actually gains items and there is an interval when the conveyor actually loses items. To achieve uniformity on the conveyor the net gain and the new loss must be evenly distributed throughout the length of the conveyor. That is to say, the conveyor must make an integral number of revolutions in the duration when there is a net loss of items. This is what expression (4) says. Since the two durations are related together by the fact that $T_L + T_U = T$, one of the two requirements in (4) will be sufficient if expression (3) is satisfied.

While the Speed Rule deals with the relationship of speed to the loading and unloading *rates*, the Capacity Constraint and the Uniformity Principle, particularly the latter, deal with the relationship of speed to the loading-unloading *schedule*. Given the loading and unloading rates of the conveyor, the Speed Rule, expression (1), determines a range of *permissible* speeds. Given further the loading-unloading schedule, the Capacity Constraints, expression (2), and the Uniformity Principle, expressions (3) and (4), narrow the range down to a few (or just one) workable speeds, or perhaps, declare that none is workable!

Since for a given conveyor the speed of a conveyor is inversely proportional to its revolution time, we shall use these two terms interchangeably. Quite frequently it is more convenient to use one rather than the other. Also, in most practical cases the limits set on the speeds to be explored by the Speed Rule are not too restrictive. That is to say, in most cases the speeds that intuitively seem to be practical would all be within the permissible limits. It is frequently sufficient, therefore, to consider the Capacity Constraint and the Uniformity Principle only.

4. Two Methods of Stimulation

The principles discussed thus far give us some understanding of conveyor operation under certain assumptions. Granting that these assumed conditions may hold in reality, we would like to substantiate our understanding by a method that will show us what really happens on the conveyor. More importantly, the assumed conditions may not always hold true. Our imagination fails to tell us just exactly what will happen when for example, the loading and unloading schedule does not exhibit a regular pattern, or even when the conveyor cannot make an integral number of revolutions in one cycle. We then would want a method by which we can examine the condition of the conveyor from section to section as it goes from one revolution to another. Moreover, in actual practice, there are frequent random disturbances. We want a method by which the effects of random disturbances on the conveyor can be studied. We would like to see actually if a workable solution, as determined by the three fundamental principles, is still workable when random disturbances are included. We also would like to know, if these theoretically workable solutions should turn out to be not actually workable, what factors could be varied to render a solution possible and what their effects are. To meet these needs, we have developed two extremely simple methods.

They are the tool to use when it becomes impossible to study the condition analytically. These two methods are both numerical methods for *simulating* the operating conditions of the conveyor. In essence, *both involve the construction of a* *table which indicates the distribution of items on the conveyor from one instant to the next.* They are very simple to use but often they involve a tremendous amount of computation. A computer routine for these methods, therefore, would be quite helpful. With a computer routine, especially one which incorporates a sub-routine that generates random disturbances, the operation of a conveyor can be thoroughly studied.[2] We shall describe these methods here and indicate how they may be used in the next section.

A. *The First Method of Simulation*

This simple method can be easily explained through the use of an example. Consider the loading-unloading schedule as shown in Figure 4. Here the loading rate is $r_L = 2$ per minute, the unloading rate is $r_U = 6$ per minute, and the cycle time is $T = 12$ minutes. Observe also that the total load over the cycle time is equal to the total unload $(2 \times 12 = 24 = 6 \times 4)$. Consider a conveyor with a *revolution time of w minutes*, or equivalently a speed of v feet per minute. We may imagine the conveyor to be divided into w sections, each of L/w feet long with m/w carriers. Call the section under the loading point the first section, the next section that comes underneath the loading point as the conveyor moves on the second section, and so forth. Let the length of the delivery portion of the conveyor by L_d and that of the return portion be L_r, $L_d + L_r = L$. Then the section under the unloading point would be $(L_d w)/L$ sections away from the first section on the delivery portion of the conveyor, and $(L_r w)/L$ sections away on the return portion of the conveyor. In other words, in a time lapse of $(L_d w)/L$ minutes, the first section will be under the unloading point, and in a time lapse of an additional $(L_r w)/L$ minutes this section will be back at the loading point. Assuming $w = 4$, suppose we look at the condition of the conveyor once every minute. In the first minute, the first section of the conveyor is under the loading point and it receives a load of 2. This load is distributed among the m/w carriers in this section. Nothing happens in this minute to the rest of the sections. If we imagine the conveyor loop to be slit at the first section and then stretched out with the first section under the loading point, the distribution of load in the first minute is depicted by the first row of figures in Table 2.

As time elapses, the conveyor moves ahead; the section coming under the loading point will receive a load of 2 when the loading-unloading schedule so indicates, and the section coming under the unloading point will lose a load of 6, again only when the loading-unloading schedule so indicates. The distribution of load at the end of subsequent one-minute time intervals is depicted by the successive rows of figures in Table 2. Table 2 is almost self-explanatory and the method of constructing it should also be readily deducible.

B. *The Second Method of Simulation*

The first method obviously gives a detailed representation of what happens on the conveyor minute by minute. Oftentimes this is giving too much information.

[2] Such a computer routine has been developed and used successfully since June, 1956.

TABLE 2

First Method of Stimulation

Section / Time	1 (Loading Point)	2	3 (Unloading Point)	4
1st minute	2	0	0	0
2nd minute	2	2	0	0
3rd minute	2	2	2	0
4th minute	2	2	2	2
5th minute	4	2	2	2
6th minute	4	4	2	2
7th minute	4	4	4	2
8th minute	4	4	4	4
9th minute	6	4	−2	4
10th minute	6	6	−2	−2
11th minute	0	6	0	−2
12th minute	0	0	0	0

Condition of conveyor minute by minute

TABLE 3

Schedule of "Excess" Rates

Time	1	2	3	4	5	6	7	8	9	10	11	12
"Excess"	2	2	2	2	2	2	2	2	−4	−4	−4	−4

Rather than knowing precisely what happens to each section of the conveyor every minute, it is generally sufficient to have information only on the extremes in each revolution, and sometimes even on less than that. A second method of simulation was therefore developed to give a similar picture *revolution by revolution*. This requires, naturally, much less computational effort.

Again it is easier to explain the method through the use of an example. For comparison, let us use the same example and assume again $w = 4$ minutes. To simplify matters without loss of generality we may assume that the loading point and the unloading point are located at the same point. The *effect of the actual physical separation*, or the actual time lag or lead between loading and unloading, *can be later introduced, as we will demonstrate subsequently*. Thus we may take the difference between the loading rate and the unloading rate in the loading-unloading schedule and consider only the resulting "excess" rate $r_e = r_L - r_U$. When these two points are located at the same place, the unload at any time will be immediately subtracted from the load. Only the excess is loaded onto the conveyor. Figure 4c and Table 3 show this schedule of excess rates. Table 4 shows this schedule of excess rates rearranged in rows of $w = 4$ figures per row.

With the passage of the first 4 minutes, the conveyor will have completed its first revolution. Also, the first 4 minutes of "excess" in the schedule will have been disposed of, with the excess in the first minute going onto the first section of the

TABLE 4

Schedule of "Excess" Rates, Rearranged

Time............ Row	1	2	3	4
1	2	2	2	2
2	2	2	2	2
3	−4	−4	−4	−4

TABLE 5

Second Method of Stimulation

Section............ Revolution	1	2	3	4
1	2	2	2	2
2	4	4	4	4
3	0	0	0	0

Condition of conveyor, at the end of each revolution

conveyor, the excess in the second minute going onto the second section of the conveyor, and so forth. Thus, the distribution of load after the first revolution is simply the first four figures in the "excess" schedule. This is also shown as the first row in Table 5.

With the passage of another 4 minutes, the conveyor will have completed another revolution, and the next 4 minutes of the "excess" schedule will have been disposed of. The "excess" in the first minute of this second 4-minute sequence falls onto the first section of the conveyor, the "excess" in the second minute falls onto the second section, and so forth. Thus, the conveyor receives a second round of "excess" on top of its first round. To get the total accumulation of load in each section we simply add algebraically the corresponding "excesses" in the two 4-minute sequences together. The second row in Table 5 shows the result of this operation.

The manner in which the load distribution on the conveyor is obtained after each complete revolution should now be clear. For any chosen revolution time w, we first break up the schedule of "excess" into sequences, each of length w, and rearrange them in the form of Table 4. To get the distribution of items shown by Table 5, we simply cumulate algebraically the corresponding figures in Table 4 row by row successively. As in the first method, a positive figure indicates an accumulation of items, and a negative figure indicates a depletion or shortage.

C. *Adjustments for the Separation of Loading Point and Unloading Point*

As indicated previously, the second method accepts the loading-unloading schedule *as it is* and works with the schedule of "excess" derived from it. Underlying this procedure is the assumption that loading and unloading, as far as a particular section of the conveyor is concerned, take effect immediately, or equivalently, that the loading point and the unloading point are located at the same point. Since actually the loading point and the unloading point are almost always at different locations, loading and unloading, as far as that particular section is concerned, do not take effect immediately. There is a time lag between the loading and the unloading of a section. Using the schedule as it is, as we have done in the example, therefore, will not give us the real peak accumulation and peak depletion which are always more acute. To avoid a distortion of this kind, we could either (a) introduce the time lag by delaying the unloading schedule (or advancing the loading schedule) appropriately, or (b) accept the schedule as it is and make an upward adjustment to the calculated peaks. The disadvantage of the former alternative is that the modification required depends upon the revolution time chosen for study. Each time a new revolution time is studied, a new modification of schedule is necessary. This is tedious. We therefore adopted the latter alternative, since the adjustments needed there are independent of the revolution time. The adjustments are always "upward" and are easily made to yield the *actual* peaks attained. The reasoning behind the method of adjustment is fairly simple to understand. There are two major cases to consider: (a) the loading rate is smaller than the unloading rate, and (b) the loading rate is greater than the unloading rate. The case in which the loading rate is equal to the unloading rate is a trivial one. This will be evident after the above two cases are studied.

When the loading rate is smaller than the unloading rate, it is economical to start loading before unloading to let the items build up on the conveyor first. In practice the loading-unloading schedule is inevitably arranged such that loading starts before unloading. Consider now the peak accumulation. This takes place at the loading point near the end of the build-up. When the unloading point is away from the loading point, the accumulated items after reaching the peak in the *last round* and thus ready to meet the demands of unloading (which is at a higher rate than the loading rate in this case), do not get to be unloaded immediately (e.g., the 9th and 10th minutes in Table 2). They remain on the conveyor temporarily while going from the loading point to the unloading point. The second method of simulation, since it assumes immediate unload, does not indicate this temporary condition. The peak accumulation given by the second method is less than the actual value attained by this temporary load added in the last round. To get the actual peak accumulation, therefore, we simply adjust the peak accumulation as given by the second method upward, by an amount equal to the loading rate which is, we note, the smaller of the two rates.

The peak depletion takes place at the unloading point when the unloading *starts*. At this time although there may be sufficient accumulation at the loading point after the new addition of load, the accumulation will not reach the unloading point until later (e.g., the 9th and 10th minutes in Table 2). The accumulation available at the unloading point is still short of the actual peak by the amount of the current addition, namely, the loading rate. Therefore, to get the actual peak depletion, we simply adjust the peak depletion as given by the second method upward, by an amount equal to the loading rate which is, we note again, the smaller of the two rates.

When the loading rate is greater than the unloading rate, it is of course economical to start unloading as soon as possible. In practice the loading-unloading schedule is inevitably arranged such that loading and unloading start at the same time. Consider then the peak accumulation. This obviously takes place at the loading point. When the unloading point is away from the loading point, the "relief" furnished by unloading cannot be achieved immediately. The actual peak accumulation of the conveyor, therefore, will be higher than that given by the second method by an amount equal to the unloading rate which is, we note, the smaller of the two rates. To get the actual peak then, we simply adjust the peak as given by the second method upward by the unloading rate.

Now consider the peak depletion. This takes place, as before, at the unloading point. When the loading point is away from the unloading point, the load is not available for use at the unloading point immediately. Therefore, the actual peak depletion will be more acute than the peak depletion given by the second method by an amount equal to the unloading rate. To get the actual peak depletion then, we simply adjust the peak depletion as given by the second method upward by the unloading rate which is, we note again, the smaller of the two rates.

As we have noted repeatedly, under the assumed conditions the adjustment of the results as given by the second method is always upward by an amount equal to the smaller of the two rates. In both of the two major cases discussed the peak accumulation is increased by this amount and the peak depletion is increased by this amount. These adjustments result in increasing the range of variation from peak depletion to peak accumulation by twice the smaller of the two rates. We may regard this as a general rule for making adjustments.

The case in which the loading rate is equal to the unloading rate can be examined in the same way. At the loading point we need something extra to accommodate the temporary new addition of load, and at the unloading point we need something so that the unloading can go ahead without having to wait for fresh supplies to come from the loading point. Since the loading rate equals the unloading rate, the adjustment to be made is simply twice either rate.

5. The Problem of Conveyor Operation

Having gained a theoretical understanding and having developed some practical tools, we now are ready to apply them to realistic problems. There are two general conveyor problems. There is the design problem before installation of the conveyor and there is the operation problem after the installation. The design problem has a broader scope than the operation problem. With the design problem, all parameters are subject to determination. Consideration must be given to not just the characteristics of the conveyor alone but also to the relationships of the conveyor to other components of the manufacture system. The criterion to use should be economic as well as technological. We shall attempt to discuss this problem in its more restricted form in a subsequent paper.

The operation problem is actually not as involved as it may first suggest. Whereas in the design problem all of the parameters of the conveyor are unknown, here all of them are given constants. We are really not at will to vary the length of the conveyor, the capacity of the carriers, the spacing of the carriers and the like.[3] Given a certain conveyor, we may be able to control the loading-unloading schedule for the conveyor at times, but most frequently the only variable at our control is the speed of the conveyor. For a given schedule the operation problem thus becomes a problem of finding a best feasible speed and the quantity of items, if any at all, to be prestored on the conveyor to make the speed workable. We shall presently indicate how this problem may be solved by the use of the simple principles we have developed previously.

With the operation problem, basically there are two questions to be answered. One question is related to the peak depletion per section of the conveyor. The concern is whether there will be *enough items available and accessible* to the unloading point *to enable unloading as scheduled.* The other question is related to the peak accumulation per section of the conveyor. The concern is whether there will be enough empty racks available and accessible to the loading point to *accommodate the accumulation of items at their peak* or not. It is obvious from our discussion thus far that these questions cannot be answered without considering the loading-unloading schedule for the conveyor and the speed at which the conveyor is moving. That the peak accumulation per section of the conveyor and the peak depletion per section of the conveyor depend upon the manner in which loading and unloading are done, i.e., the loading-unloading schedule, is immediately obvious. The numerical examples we have produced in illustrating the simulation methods also substantiate this understanding. That for a given loading-unloading schedule the peak accumulation and peak depletion per section of the conveyor also depend upon the speed of the conveyor requires only a little reflection. A few tries with a typical loading-unloading schedule, using the simulation methods and assuming different conveyor speeds, can also bring out this point clearly.

In order that the unloading may be executed as scheduled when current supply has not yet reached the unloading point, or when the section with the peak depletion is at the unloading point, *there must obviously be some prestored items on the conveyor*. It would appear desirable if we could distribute these prestored items along the length of the conveyor as they will eventually be required, in quantity as well as in location. This, nevertheless, is hardly practical; nor is this really advisable since even if it were possible to realize this distribution, random factors and deliberate changes will make it invalid only too frequently. A simpler and more sensible way, therefore, would be to distribute the items *evenly* throughout the length of the conveyor. On a per-section basis this quantity would be equal to the actual peak depletion. Of course this is not the only quanity of items to be prestored. Generally, some stock will be prestored for meeting random disturbances in the schedule. Also, some stock may conceivably be stored on the conveyor "permanently" (relatively speaking) for some reason. These should be, for the same reason, evenly distributed on the conveyor.

In a similar manner we can reason that the way to provide accommodations on the conveyor for the peak accumulation attained should also be to provide

[3] At times these may also be altered of course, but then the problem is really a redesign problem, not an operation problem.

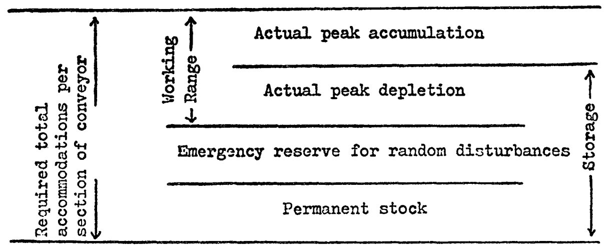

FIG. 5. Required Total Accommodations per Section of Conveyor

the same amount throughout the length of the conveyor. On a per-section basis this amount would be equal to the actual peak accumulation. Therefore, the total accommodations required, on a per-section basis, would be the sum of the peak accumulation and the quantity of accommodations required for all prestored items on a per-section basis. This is the value of K in inequality (2). (See Figure 5. No attempt has been made to draw the figure to scale.)

For a given loading-unloading schedule and a given conveyor, the operation problem then involves (a) the determination of the actual amount of accommodations available on the conveyor at a certain speed, (b) the determination of the amount of prestored items required, and the total amount of accommodations required, at that speed, (c) the determination of the feasibility of that speed by comparing what is available to what is required, and (d) when more than one speed are feasible, the selection of an optimal speed by some specified criteria. For the selection of an optimal speed, the criterion of minimizing the required prestored items is a common one. By this criterion the highest feasible speed would usually be chosen.

Now given a conveyor and a loading-unloading schedule, we can determine immediately the available accommodations on the conveyor on a per section basis, for a chosen speed (chosen possibly with the aid of the fundamental principles). This is simply mq/w. Then, depending upon how random disturbances are to be handled, there are two approaches we can follow. The first may be called the *complete simulation approach.* Assuming that we know the statistical distribution of the random disturbances, we can use the numerical methods to work out the peak accumulation and the peak depletion that can be *expected* in a section of the conveyor at that speed, for some suitably chosen time interval. We shall be working with the schedule, as modified by random disturbances generated according to the given statistical distribution, instead of the schedule as is given. Naturally, for this work an electronic computer would be helpful. The required amount of prestored items, on a per-section basis, is given by the peak depletion plus the "permanent" stock if there is to be any. The required total accommodations, on a per-section basis, is given by the sum of the peak depletion, the peak accumulation, and the "permanent" stock. Having thus obtained the available accommodations, the required total accommodations, and the required amount of prestored items, all at the chosen speed, the determination of the feasibility of that speed and the subsequent determination of its optimality by the low-stock criterion are straightforward.

The second approach may be called the *semi-simulation approach*. Here we assume that the required amount of emergency stock for meeting random disturbances has been given and is to be distributed evenly throughout the length of the conveyor. We use the second numerical method with appropriate adjustment to determine the actual peak depletion and the actual peak accumulation necessitated by the given schedule.[4] So, for any chosen speed, the required amount of prestored items, on a per-section basis, is given by the adjusted peak depletion plus the "permanent" storage per section if there is to be any. The required total accommodations, on a per-section basis, is given by the sum of the two adjusted peaks, the emergency stock per section and the "permanent" stock. With these two requirements determined the rest of the procedure is again straightforward.

The above two approaches for solving the conveyor operation problem are simple approaches. They are also rather conservative approaches in that they tend to give answers that are very "safe". They require a larger amount of prestored items than is necessary, and a larger amount of accommodations than is necessary. In specifying an even distribution of prestored items on the conveyor, these approaches assume that the peak shortage could occur anywhere on the conveyor. This is not necessarily so. Moreover, when there are also other stored items such as items stored to meet random disturbances, all such items are really indistinguishable. Conceivably there is a good chance of borrowing items for one purpose from those stored for other purposes, thereby reducing the storage requirements and the accommodation requirements. In requiring the total required accommodations, on a per-section basis to equal to the sum of the peak depletion and the peak accumulation and others, these approaches also exhibit conservativeness. This assumes that the two peaks will actually happen in the same section of the conveyor, which of course, is hardly likely. For these reasons these two approaches are by no means the best approaches for solving the conveyor operation problem. It is desirable to examine modifications of them as well as other alternatives. At the present time the empirical approach seems to be more promising than others. Perhaps in time as experience with conveyor operations accumulates, a more satisfactory rule can be developed.

Thus far the limited experience in solving conveyor operation problems suggests that a variation of the second approach will also produce results that are practically workable. This modified approach has the advantage that it requires less prestored items and that it can be easily explained. The modification introduced is a different way of making adjustments to the calculated peaks to get the actual peaks. Only the calculated peak accumulation is adjusted, upwards by the loading rate. The calculated peak depletion is not adjusted. This, consequently, reduces the required amount of prestored items. The intuitive reasoning for this modification is as follows: By using the simulation method under the as-

[4] When the loading-unloading schedule consists of identical cycles, it should be possible to work out the working range mathematically without resorting to simulation. However, this writer has not been successful in his attempts in this direction.

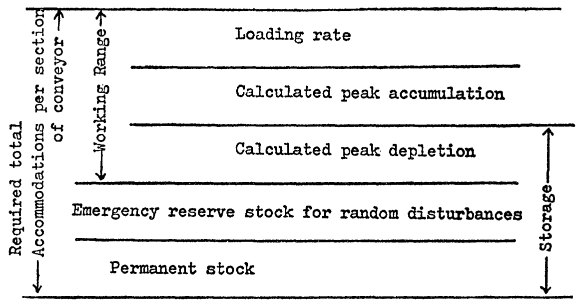

FIG. 6. Required Total Accommodations per Section of Conveyor, a Practical Modification

sumption that the loading point and the unloading point are located at the same point we are *really examining the return portion* of the conveyor. Now the difference between the delivery portion of the conveyor and the return portion of the conveyor is only in that the former is required to *accommodate temporarily an extra load in each conveyor section equal to the loading rate.* The adjustment proposed thus assures that at the loading point there will be sufficient accommodations. The question may be raised as to the sufficiency of items at the unloading point. The answer to this lies in the fact that an emergency stock for random disturbances is *always* provided. There is a good chance of borrowing from this emergency stock if the demand for items at the unloading point in the normal course of events should exceed the prestored quantity which is equal to the peak depletion on a per-section basis. Figure 6 illustrates the essence of this modified approach. Again no attempt has been made to draw the figure to scale.

6. Discussion

In this paper we have studied rather thoroughly one form of the coupling mechanism that connects two manufacturing areas. This is a subject that has been traditionally given very little attention. While the design and operation of the manufacturing areas themselves have received careful analysis, the coupling mechanism such as conveyors receives only a casual treatment. Considerations are limited to purely technological topics such as stresses in the conveyor rail structure and the sizes of the drive motors. Consequently, the result is a system that is an unbalanced integration of some otherwise well-designed areas, with poor utilization, if not frequent disruption, of its potential performance.

Our study of conveyors as one form of coupling mechanism between two manufacturing areas has indicated that the usual unhappy experiences with conveyors can be easily avoided. We have evolved out of our study some fundamental principles and numerical methods for analyzing conveyor operations. The fundamental principles are all very simple, yet they lead to a clear understanding of the operation and design problems of the conveyors in common use. The numerical methods of analysis are also simple, and are adaptable for use on electronic computers. They are thus convenient tools for analyzing involved cases which exhibit no mathematical simplicity.

The most important result of our study is perhaps the observation that a conveyor, or any linking mechanism, is not an isolated object. It is part of a large system. Consequently, attention should neither be directed only to the conveyor nor only to the areas which it is linking together. A conveyor is a moving object. As long as the output of one area is fed to a conveyor, or as long as the input of an area is fed by a conveyor, the question of timing arises. One more dimension, the temporal dimension, is added to the spacial dimensions. Information on quantities alone is not sufficient; information on timing becomes necessary. Any deviation from the predetermined plan for an area would be reflected immediately on the conveyor. This, in due time, could hamper smooth operation of the conveyor and, eventually, affect the other areas adversely if appropriate actions are not taken. It follows from this that up-to-date knowledge of the loading-unloading schedule of the conveyor (or output-input schedule of the areas) and communication channels and decision centers for processing such relevant information are really indispensable for the proper management of conveyor operations.

The conveyor problem cannot be said to have been completely solved. The conveyor we have studied is a conveyor in its simplest form and our tools can still be improved. The understanding we have gained thus far by studying the simple conveyor only arouses our curiosity in the more complex conveyors, such as multi-item, multi-loading point, and multi-unloading point conveyors. Furthermore, if we widen our horizon to cover more than just the two neighboring manufacturing areas, we have really a network of individual conveyors interlinked together in some manner. In fact, for some questions of optimization a manufacture system should perhaps be considered as a network of flows of the input and output rates of the individual areas.

These are the directions in which generalizations of our simple conveyor problem may go. In a forthcoming paper we shall discuss the design phase of the conveyor problem and in subsequent papers we hope to extend our discussions to cover an even larger area. It is difficult to foresee the obstacles and intricacies that may be encountered in tackling these generalizations. Nevertheless, we believe that armed with the understanding of conveyor operations and the numerical methods of analysis developed in this study, these will be overcome in time in arriving at some satisfactory results.