Types of Problems

Identify problems in manufacturing that can be solved with optimization techniques and identify what technique can be used.

Historical Development

- The existence of optimization methods can be traced to the days of Newton, Lagrange, and Cauchy
- The method of optimization for constrained problems, which involve the addition of unknown multipliers, became known by the name of its inventor, Lagrange
- By the middle of the twentieth century, the high-speed digital computers made implementation of the complex optimization procedures possible and stimulated further research on newer methods

Some of the major developments

- Development of the simplex method by Dantzig in 1947 for linear programming problems
- The enunciation of the principle of optimality in 1957 by Bellman for dynamic programming problems,
- Work by Kuhn and Tucker in 1951 on the necessary and sufficient conditions for the optimal solution of programming problems laid the foundation for later research in non-linear programming.
- The contributions of Zoutendijk and Rosen to nonlinear programming during the early 1960s have been very significant
- Work of Carroll and Fiacco and McCormick facilitated many difficult problems to be solved by using the well-known techniques of unconstrained optimization

Some of the major developments

- Gomory did pioneering work in integer programming, one of the most exciting and rapidly developing areas of optimization. The reason for this is that most real world applications fall under this category of problems.
- Dantzig and Charnes and Cooper developed stochastic programming techniques and solved problems by assuming design parameters to be independent and normally distributed.

Problem Formulation

- No systematic or methodical work can be done towards the solution of a problem until its existence has been detected.
- To detect a problem is to become aware of it s existence and to recognize that it is a problem. In order to be skillful at detecting problems we need to consider what forms problem
- But the first stage of problem solving is not finished until we have defined the problem clearly and made quite sure that our definition is both correct and complete.

The word problem is defined as "a question proposed for solution or consideration"

Problem Definition

Detecting and clearing the symptoms of a problem leads to the phase of problem definition.

There are three concepts to be defined:

- Subject: The subject is the topic or central theme of the problem study. When the subject is clearly defined, you automatically have a title for the problem definition.
- Scope: The scope is the area or range that the study will encompass. The scope will tell the constraints, such as time, financing, or organizational. It is always limited by the subject.
- Objectives: The objectives are the goals or credos you will be trying to accomplish. They must express exactly what you intend to accomplish in the problem definition.

Objective Function

The objective function is the mathematical function one wants to maximize or minimize, subject to certain constraints. Many optimization problems have a single.

- No objective function. In some cases (for example, design of integrated circuit layouts), the goal is to find a set of variables that satisfies the constraints of the model. The user does not particularly want to optimize anything and so there is no reason to define an objective function. This type of problems is usually called a feasibility problem.
- Multiple objective functions. In some cases, the user may like to optimize a number of different objectives concurrently. For instance, in the optimal design of panel of a door or window, it would be good to minimize weight and maximize strength simultaneously. Usually, the different objectives are not compatible; the variables that optimize one objective may be far from optimal for the others. In practice, problems with multiple objectives are reformulated as single-objective problems by either forming a weighted combination of the different objectives or by treating some of the objectives as constraints.

Definition of the subject, scope, and objectives

- Hold preliminary discussions with whoever raised the problem and get their interpretations of what the problem is.
- Read any pertinent written procedures or claims that relate to the subject of the problem definition.
- Go out and observe the experiment in action as claimed.
- Talk with the one who involved with the problem.
- Gather any other data such as quantities, times, amounts, number of failures, and so on, that may be significant in the model building process.

After all the preliminary data are gathered, analyze the findings in order to arrive at a precise definition of the problem to be modeled next.

Classification of Models

Function Models

- Descriptive: Maps, organization charts, pictures, templates, and financial statements fail into this category
- Predictive: They predict the outcome of an experiment or decisions .Decision trees , probabilistic models, forecasting models , games are examples in this category
- Normative models: Normative models indicate what should be done to achieve an established objective. Mathematical programming models (linear, integer, quadratic, convex, etc.), inventory models belong to this group.

Structure

- Iconic: iconic models are physical replicas. Blueprints, toys, planetariums, and plant layouts are iconic models
- Analog: Graphs, flow charts, electrical schematic diagrams (speedometer, fuel meter), thermometers are typical analog models.
- Symbolic: Symbolic models on the other hand, have no physical resemblance to the actual system under study. Mathematical models , statistical models, quality control models belong to this category.

Classification of Models

Dimensionality Models

- This can be categorized by dimensionality with respect to the number and kinds of variables used in their construction.

Degree of Certainty Models

This category are classified as to

- Certainty: These models are deterministic in the sense that there is a certainty. Typical certainty models are basic inventory models , marginal analysis models , CPM models and linear programming models
- Conflict: Typical conflict models are game theoric, athletic competition, war, bargaining, labor negotiations and chess playing.
- Risk: In risk models, the probabilities are known in contrast to uncertainty models where future conditions and their corresponding probabilities are not known.
- Uncertainty: To solve the problems of the uncertainty model, selection is based on judgment, utility of the individual or group, or risk via subjective probabilities is required. This uncertainty model is the most common real life model.

Classification of Models

Temporal Reference

- Static models ignore time or none of the relationships vary with time .Typical static models include, break-even models, organization charts.
- Dynamic model has interactions that are time variant. Growth models, forecasting models are typical dynamic models.

Degree of Generality

- Specialized: the special purpose model is created or developed for a unique, special purpose and is only applicable to a particular problem or situation.
- General: Generality here refers to the degree to which models can be applied to different situations. Linear programming, waiting line models, financial analysis models and Markov Chains are general purpose type models.

Classification of Models

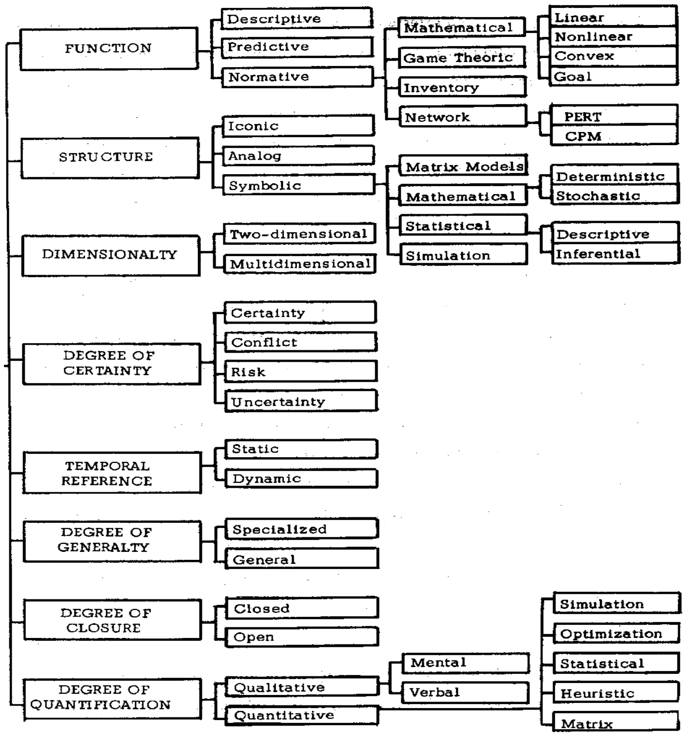
Degree of Closure

- Open: An open model, has one or more of its variables supplied from the external environment
- Closed models: In a closed model all the variables are endogenous or supplied from the controllable internal environment.

Degree of Quantification

- Qualitative: A pure qualitative model is void of mathematical description or measurement. Qualitative models are usually more flexible , robust and reflective of reality .They frequently consider intangible, human and behavioral factors that are generally ignored in quantitative models
- Quantitative: Quantitative models involve the language of mathematics, statistics, science and computers. They are extremely precise and lend themselves to testing and validation.

Classification of Models



Classification of Models

Classification based on existence of constraints.

Under this category optimizations problems can be classified into two groups as follows:

- Constrained optimization problems: which are subject to one or more constraints.
- Unconstrained optimization problems: in which no constraints exist.

Classification based on the nature of the equations involved

Based on the nature of equations for the objective function and the constraints, optimization problems can be classified as linear, nonlinear, geometric and quadratic programming problems.

- Linear programming problem: If the objective function and all the constraints are 'linear' functions of the design variables.
- Nonlinear programming problem: If any of the functions among the objectives and constraint functions is nonlinear
- Geometric programming problem: the objective function and constraints are expressed as polynomials in X.

- Quadratic programming problem: A quadratic programming problem is the best behaved nonlinear programming problem with a quadratic objective function and linear constraints.

Classification based on the permissible values of the decision variables
Under this classification, objective functions can be classified as integer and real-valued programming problems.

- Integer programming problem: If some or all of the design variables of an optimization problem are restricted to take only integer (or discrete) values
- Real-valued programming problem: A real-valued problem is that in which it is sought to minimize or maximize a real function by systematically choosing the values of real variables from within an allowed set.

Classification based on deterministic nature of the variables

- Stochastic programming problem: In this type of an optimization problem, some or all the design variables are expressed probabilistically (non-deterministic or stochastic).
- (ii) Deterministic programming problem: In this type of problems all the design variables are deterministic

Data Selection

Once the selection of the model has been made, the next step is to apply data to that model to arrive at an answer that will represent the solution to the problem.

Data collection may be time consuming but is the fundamental basis of the model-building process. The availability and accuracy of data can have considerable effect on the accuracy of the model and on the ability to evaluate the model.

The design of a procedure for measuring data depends primarily on the properties of the data itself.

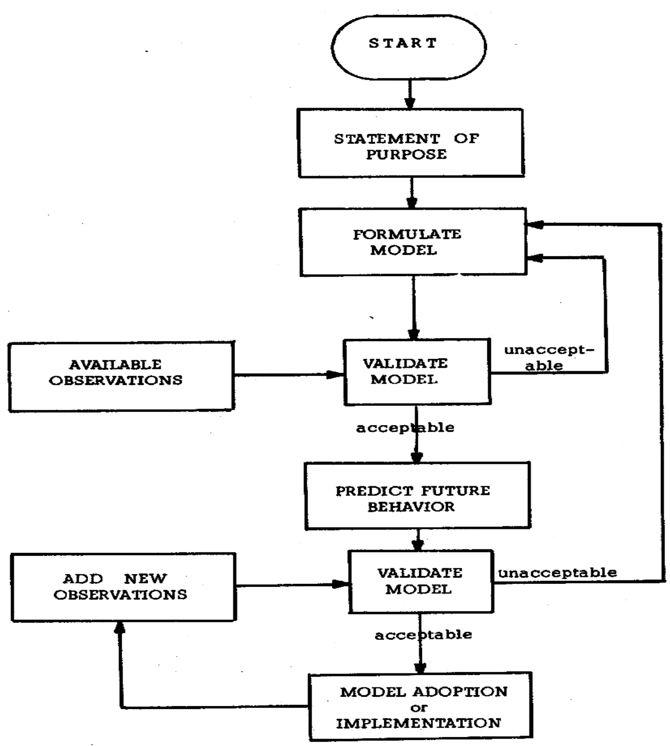
1. Identification of the experiment, or class of events or sample space to be measured.
2. Specification of the environment in which the observation should be made.
3. Specification of the changes in environment, if any, that should be made during the observation period.
4. Specification of the operations to be performed and the instruments to be used by the observer.
5. Specification of the readings to be made.
6. Specification of the analysis of the data.

Test of the Model

After obtaining this data, it must be applied to the model to give a solution. This solution must be compared against the actual problem to determine the feasibility of the solution.

- If the model does not reflect the reality of the problem, there are four primary reasons that should be investigated.
  - The model may be including variables into the problem that have no relevance to the problem.
  - The model may fail to include a variable that has a significant impact on the problem
  - The models may not accurately reflect the relationship of a variable to the solution.
  - Finally, even if the model represents the reality of the problem, the solution may not represent a feasible solution if the data fed into the model is not accurate.

The Modeling Process



Simulation

Use of computer to mimic the behavior of a system

Want to know "how some aspects of the system should be operated"

Simulation

- How do you prepare to do a simulation?
- What are the steps for a simulation?
- What are typical objectives?
- What are some pitfalls to avoid?

Simulation

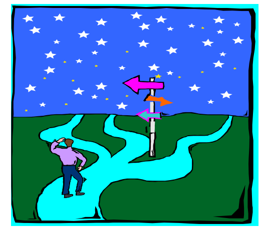
- How do you prepare to do a simulation?
  - Selecting an application
  - Software selection:
    - Simulation languages (like ………………):
    - Programming languages (Fortran, C++,) give greater flexibility, but difficulty
  - Quality, features, ease of use, cost, etc.

- What are the steps for a simulation?
  - Define objective, scope, requirements
  - Collect and analyze system data
  - Build the model
  - Validate the model
  - Conduct experiments (with the model!)
  - Present the results

Simulation

- What are typical objectives?
  - Performance analysisCapacity/constraint analysis
  - Configuration comparison
  - Optimization
  - Sensitivity analysis
  - Visualization

Simulation



- What are some pitfalls to avoid?
  - Unclear objectives
    - ♦ Unfocused simulation runs
    - ♦ Little or no information
  - Unskilled modelers
    - ♦ Quality of the results depends on quality of the model:
    - ♦ Only simulate a well defined system!
  - Unavailable data
    - ♦ "A simulation model can run with incorrect data, but it can't run with incomplete data"
  - Unmanaged expectations
    - ♦ Undertake a simulation project <u>only</u> if there is a clearly defined question to be answered or decision to be made
  - Underestimated requirements
    - ♦ Low level of detail
    - ♦ Rough answers are OK
    - ♦ Need more detail

Mathematical Programming (**MP**)

Programming is used in the sense of planning

Features of Mathematical Programing Models:

- Involve optimization
  - maximization, ie profit maximization
  - minimization, ie cost minimization

- Involve mathematical relationships
  - equations, equalities, inequalities
  - logical dependencies

Mathematical Programming

Types of MP Models

- Linear programming models
    - all equations are linear
- Non-linear programming models
    - • equations can be non-linear
- •Integer programming models
    - • can force solution in integers
    - • can make either/or decisions

Dynamic Programming 4%

- Product Selection
- Road Layout
- Optimizing product
- Transportation Problems
- Many other applications

Use of Probabilistic Models

Type of Model	Deterministic	Probabilistic
Linear Programming	👍	
Dynamic Programming	👍	👍
Simulation	👍	👍

Why Build Models?

- Better understanding of relationships
- Analyze model to determine alternatives
- Can perform experiments too costly in the real world
- Understand the implied cost of constraints
- Can repeat the experiment millions of times with different values

Examples of Possible Models

Example:

We need to build a model to optimize harvest selection and long log deliveries to 3 customers:

- saw mill
- pulp mill
- log exports

Answer the following questions:

1. What is the objective of the log delivery process
2. What "constrains" the log delivery process
3. What question would like answered from the model

What information do we have?

- Cruise information
- Stumpage prices
- Harvest & transportation costs
- Log purchase prices
- Customer demands / orders

What’s the Objective?

- Maximize profit
- Minimize costs
- Sustainability

What are the constraints?

- Can’t harvest more than is there
- Productivity constraint
- Customer demands

What questions to answer?

- Which areas to harvest?
- How much of each log to sell to each customer?

What other types of questions could you ask?

How would you solve this problem?