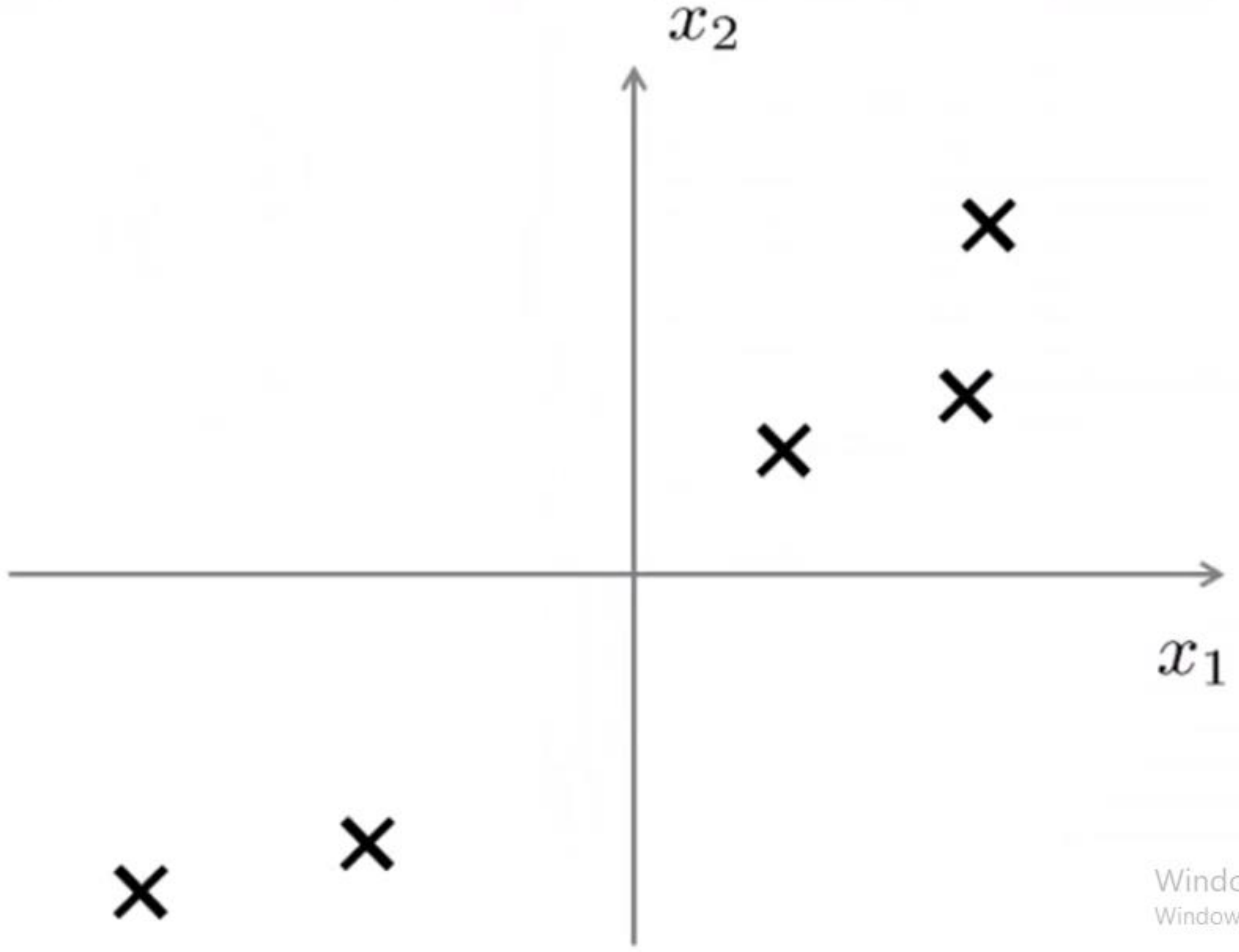
Problem Formulation

Principal Component Analysis

Unsupervised Learning

Principal Component Analysis (PCA) problem formulation

$x \in \mathbb{R}^2$



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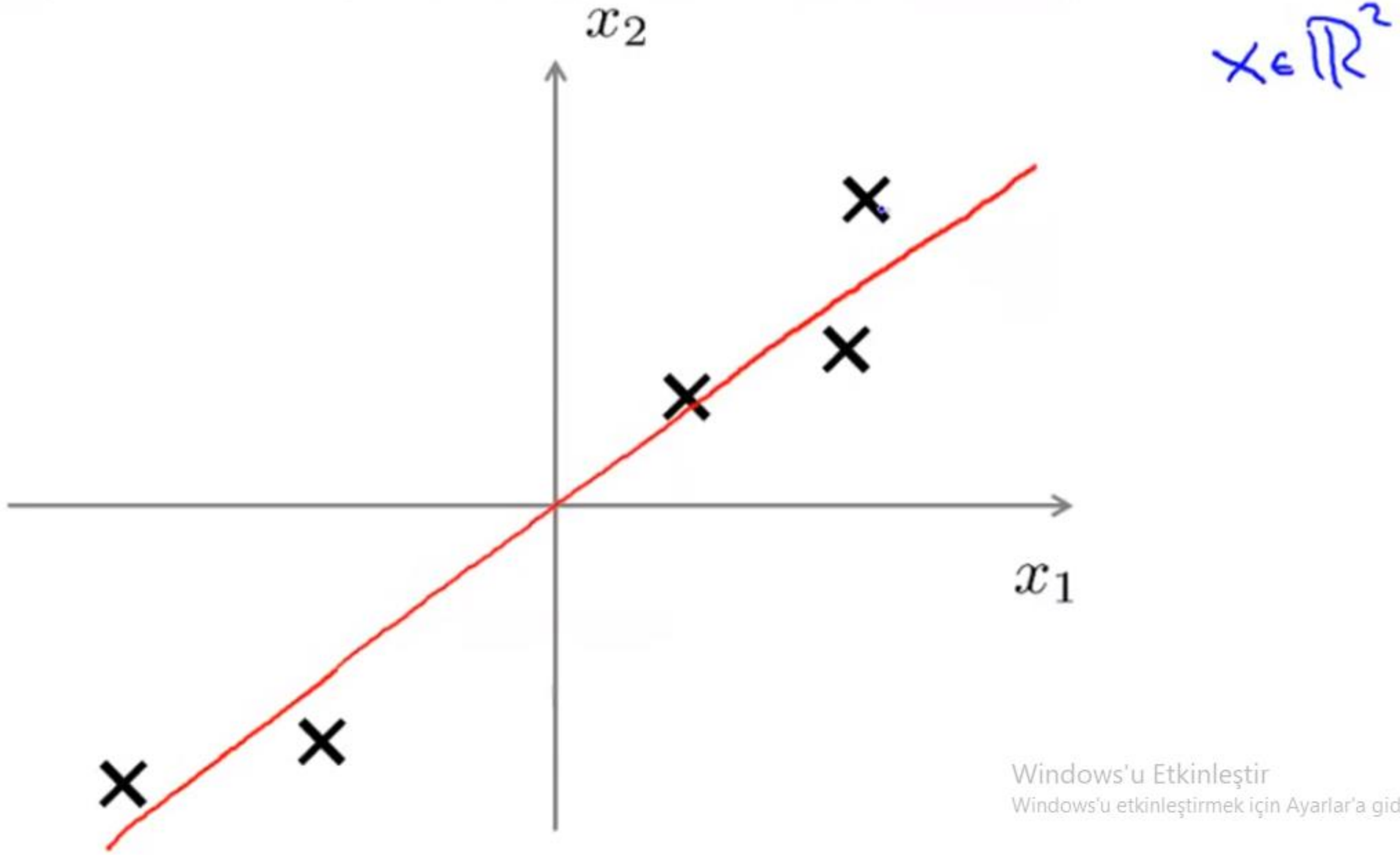

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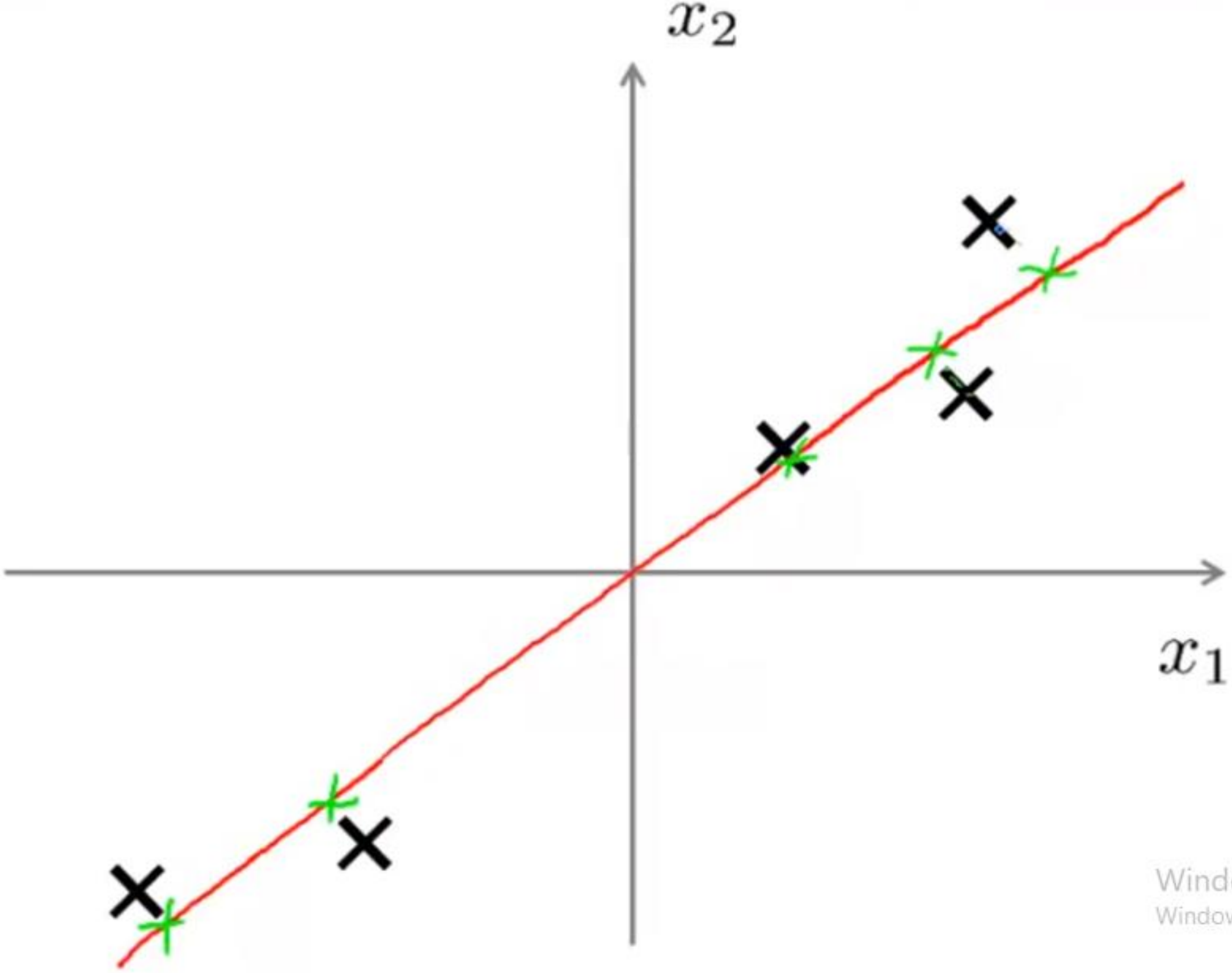

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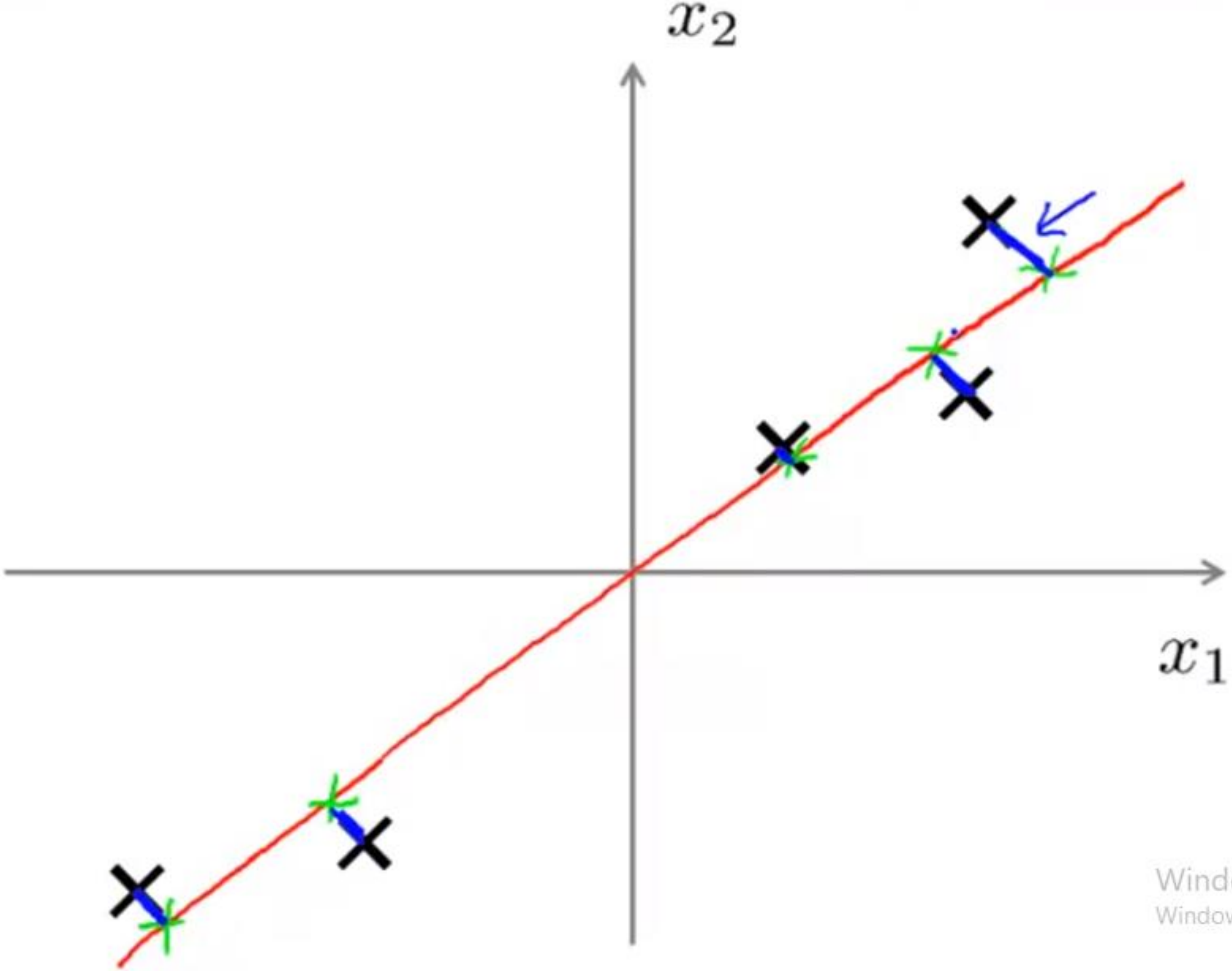

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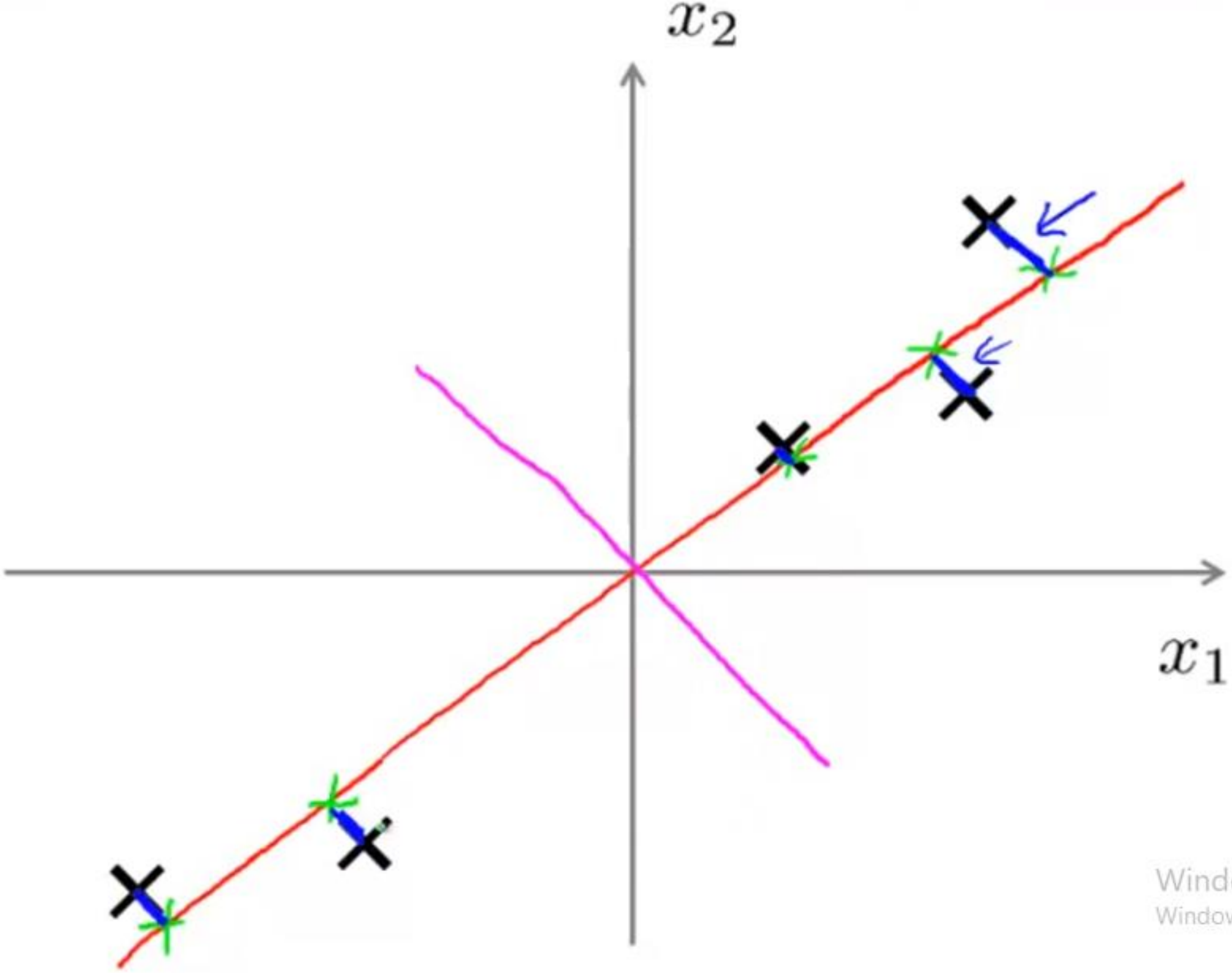

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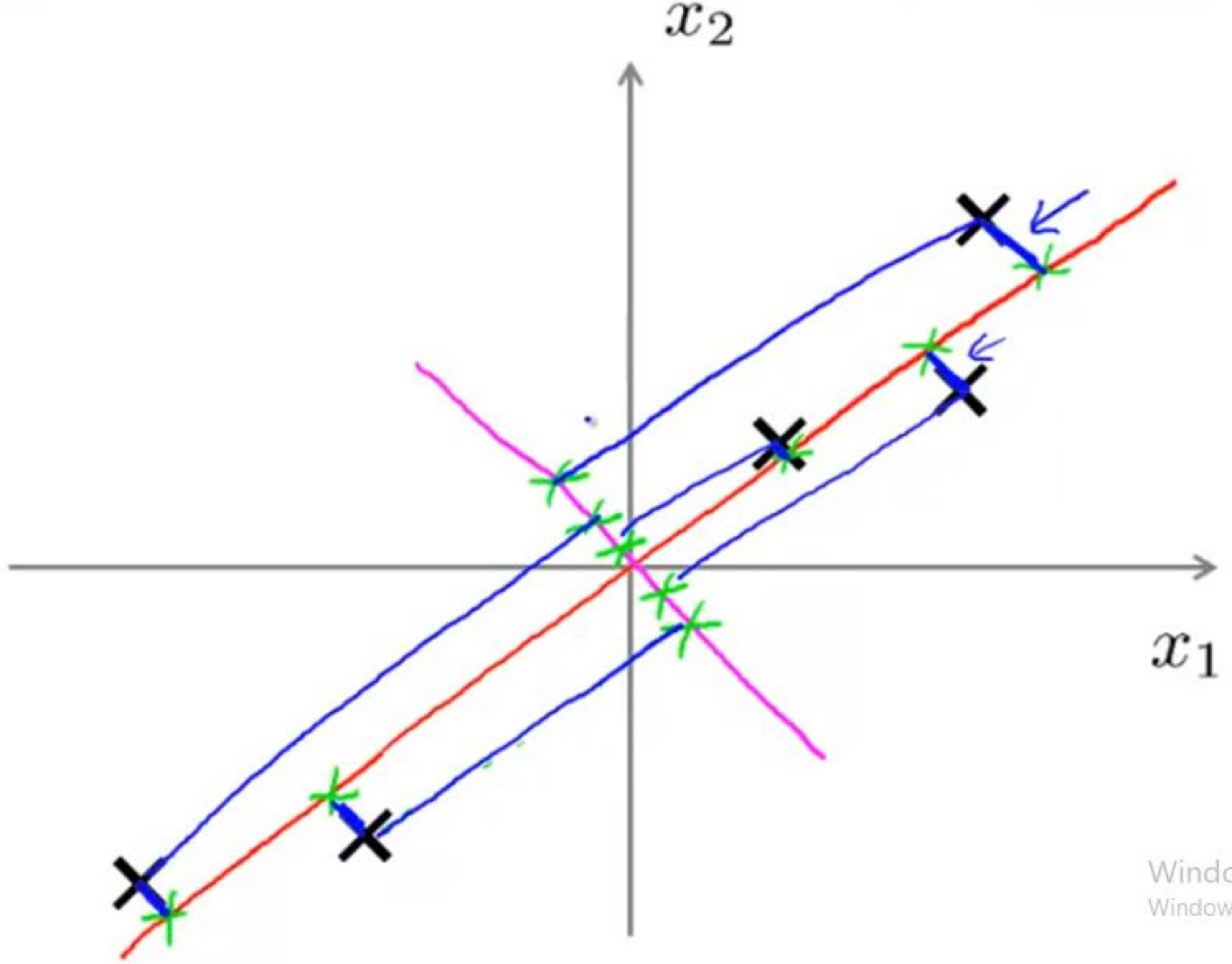

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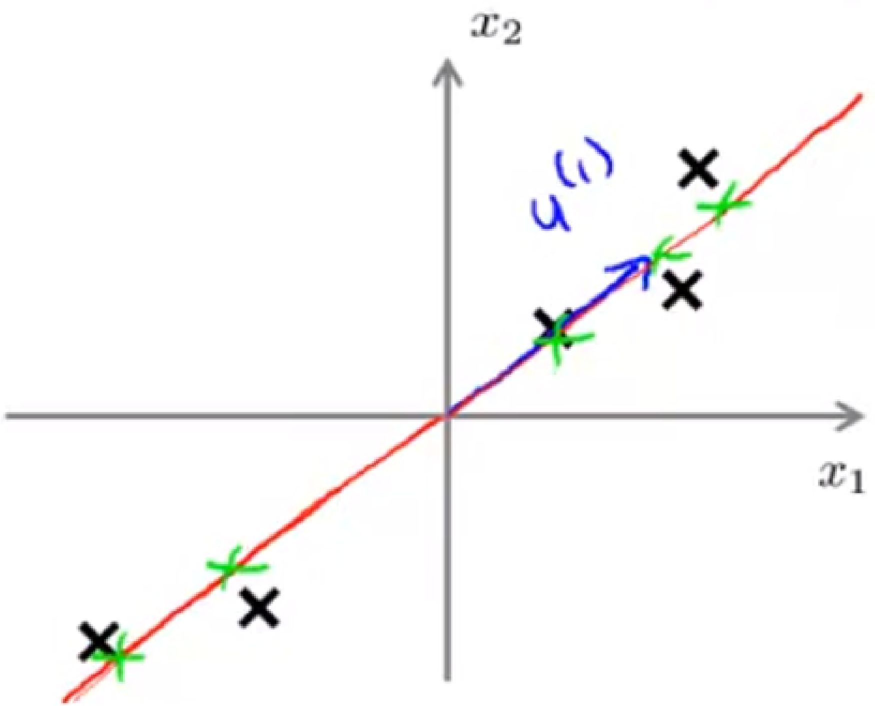

Reduce from 2-dimension to 1-dimension: Find a direction (a vector $u^{(1)} \in \mathbb{R}^n$) onto which to project the data so as to minimize the projection error.

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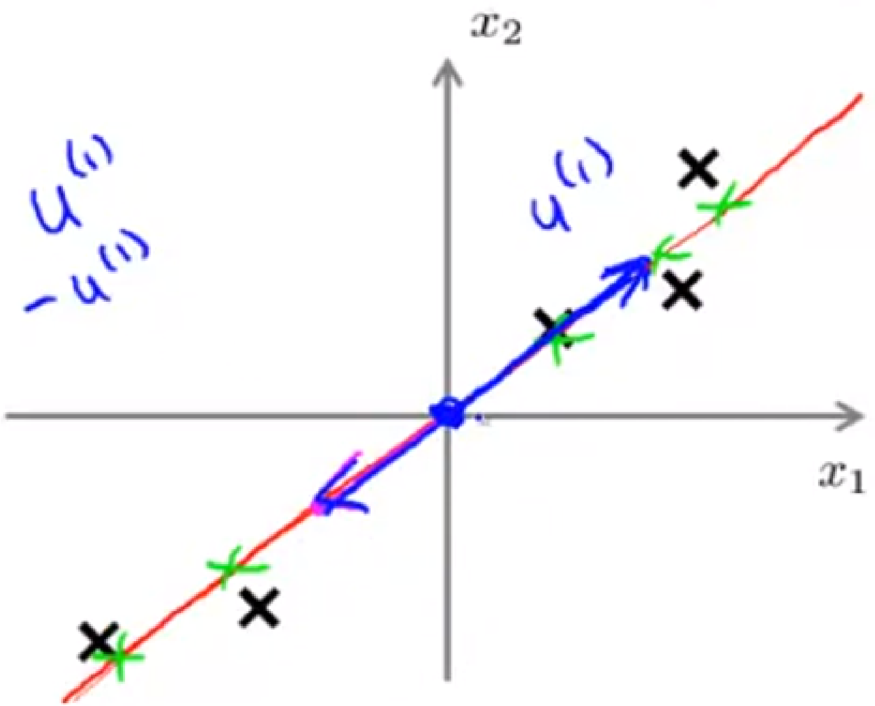

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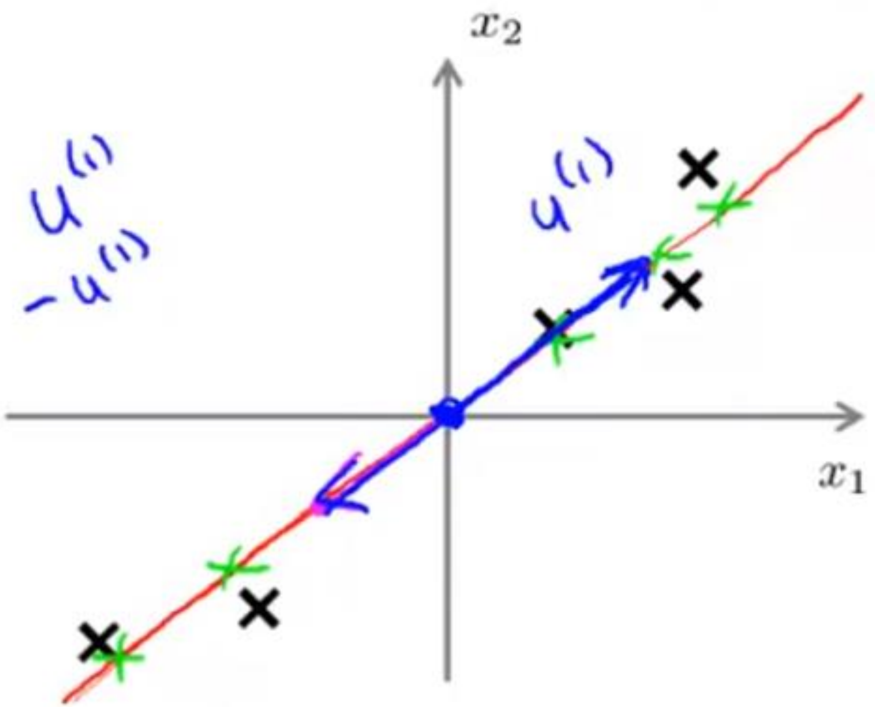
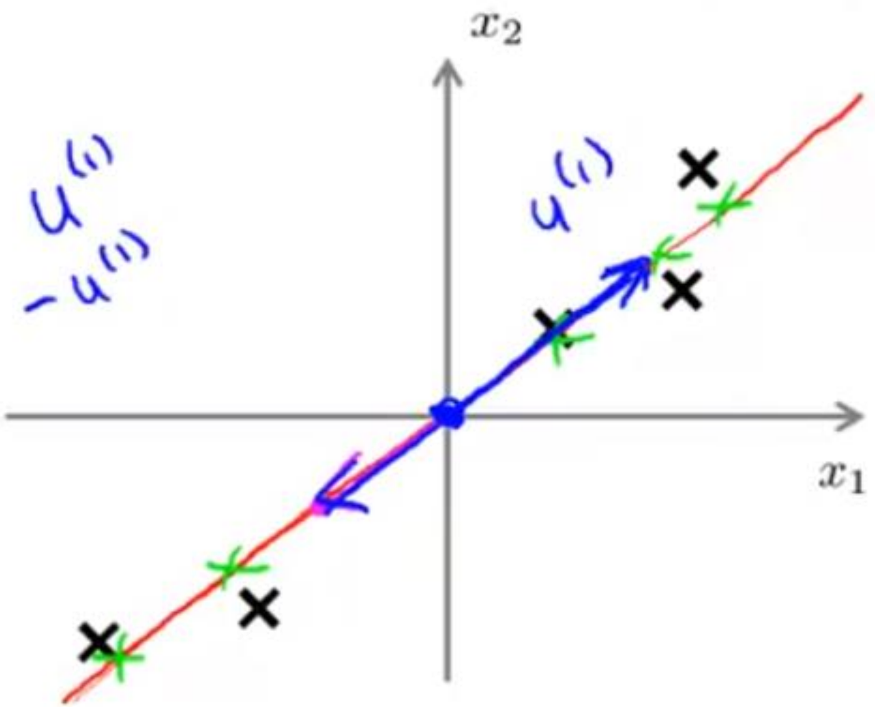


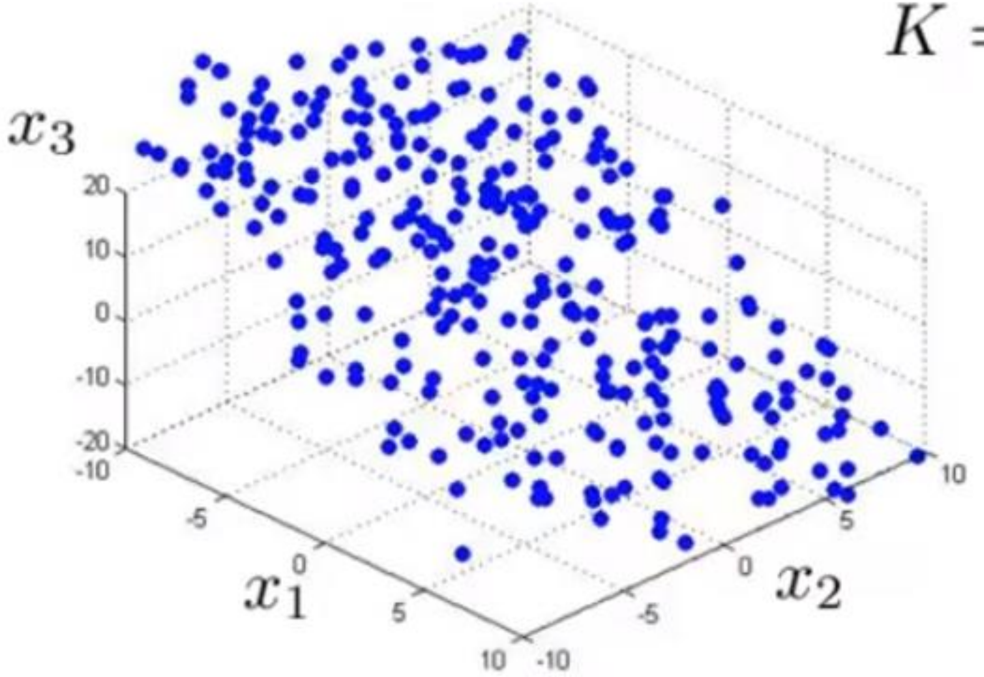
Reduce from 2-dimension to 1-dimension: Find a direction (a vector $u^{(1)} \in \mathbb{R}^n$) onto which to project the data so as to minimize the projection error.

Reduce from n-dimension to k-dimension: Find k vectors $u^{(1)}, u^{(2)}, \ldots, u^{(k)}$ onto which to project the data, so as to minimize the projection error.

Principal Component Analysis (PCA) problem formulation

$3D \rightarrow 2D$
$K = 2$





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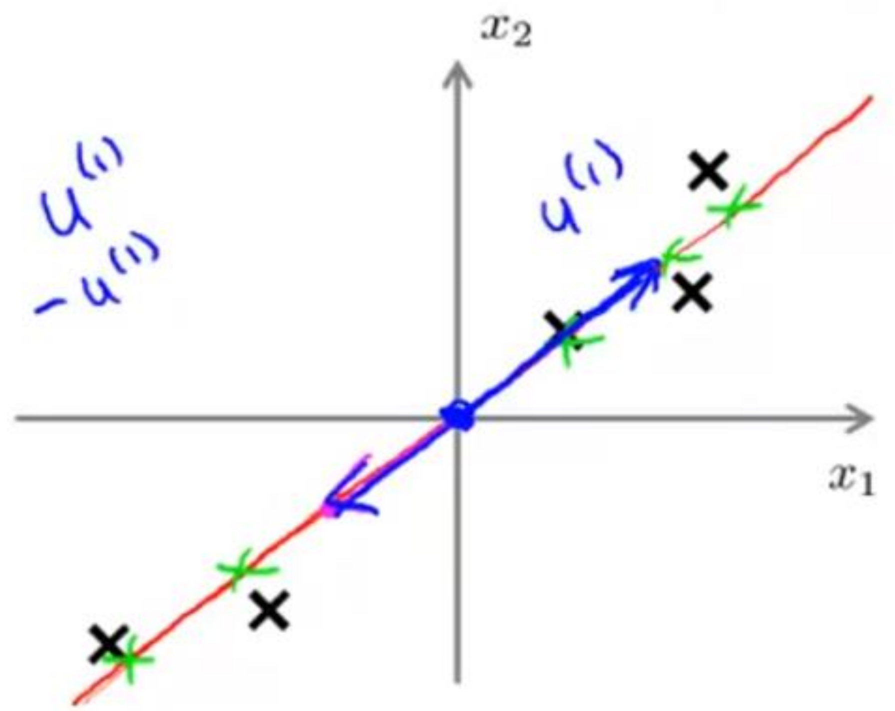

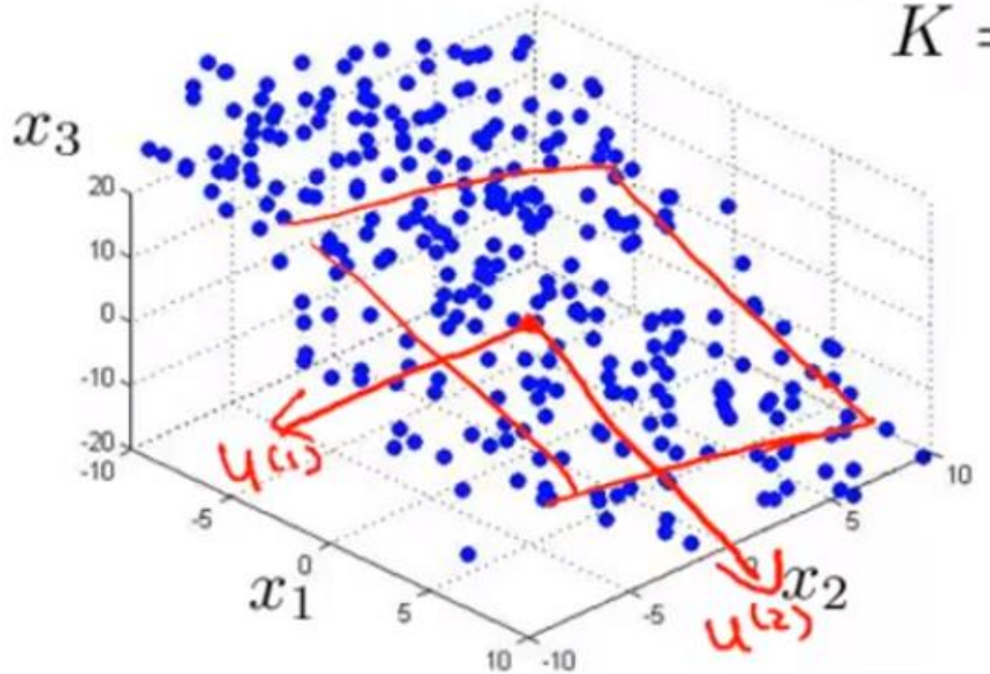

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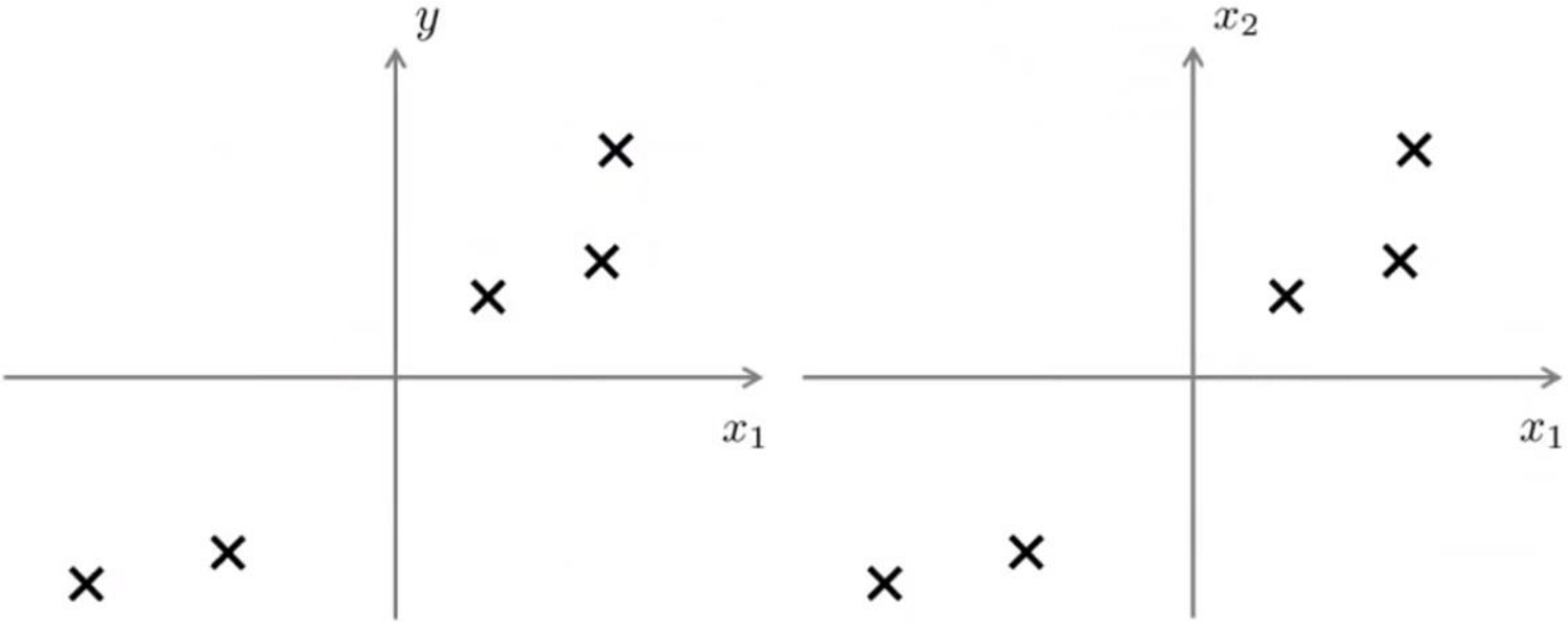
PCA is not linear regression



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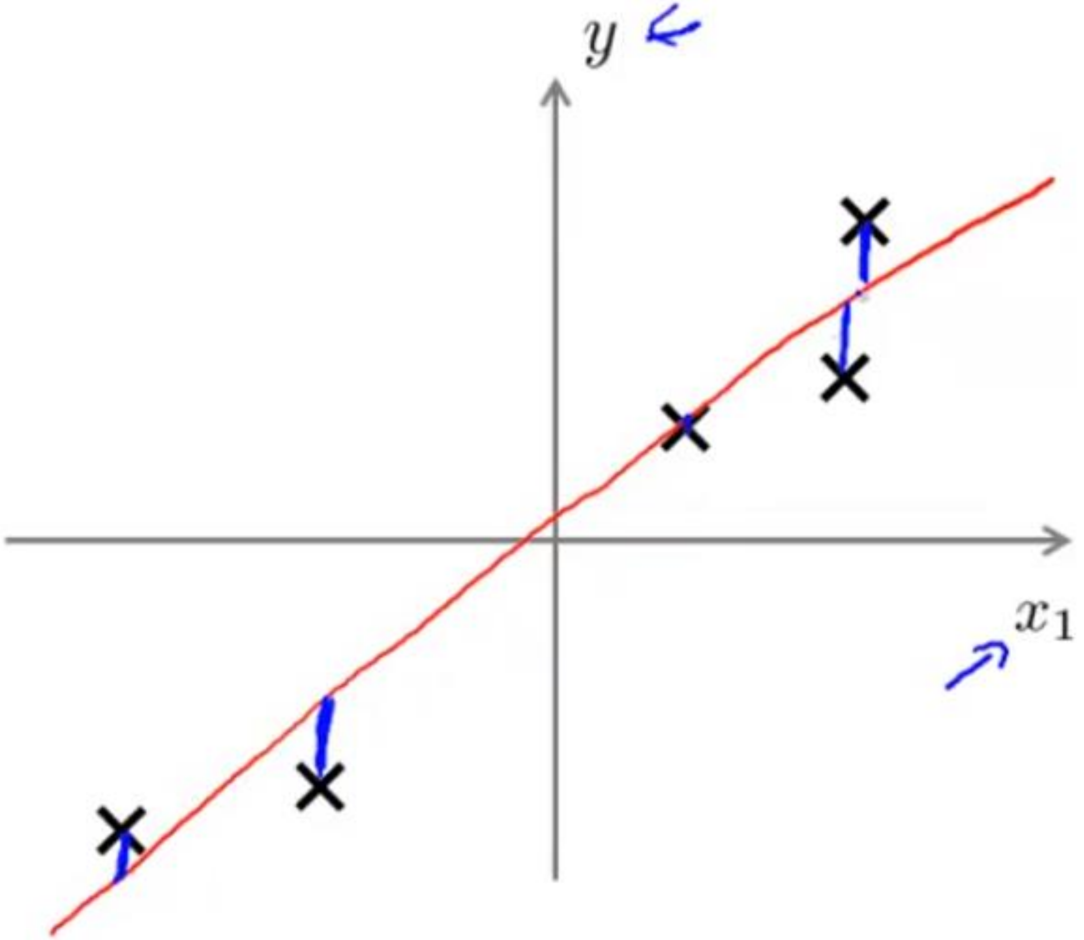

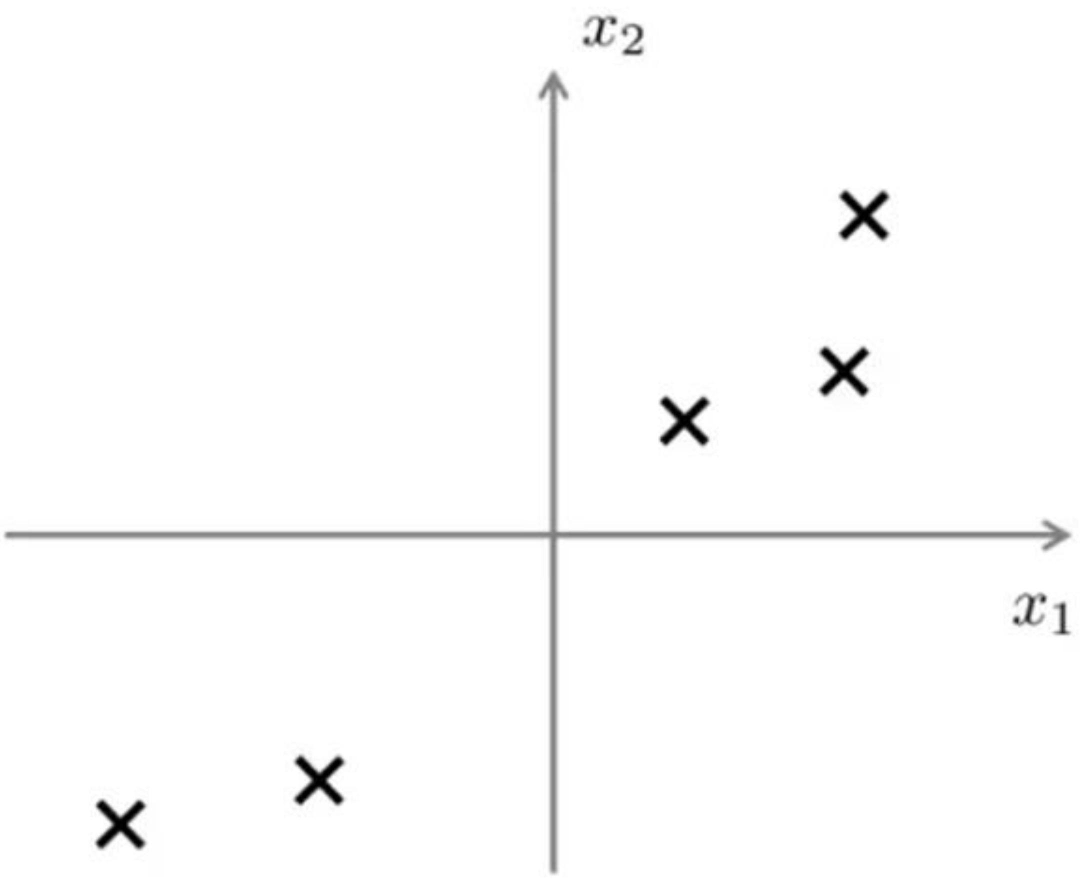

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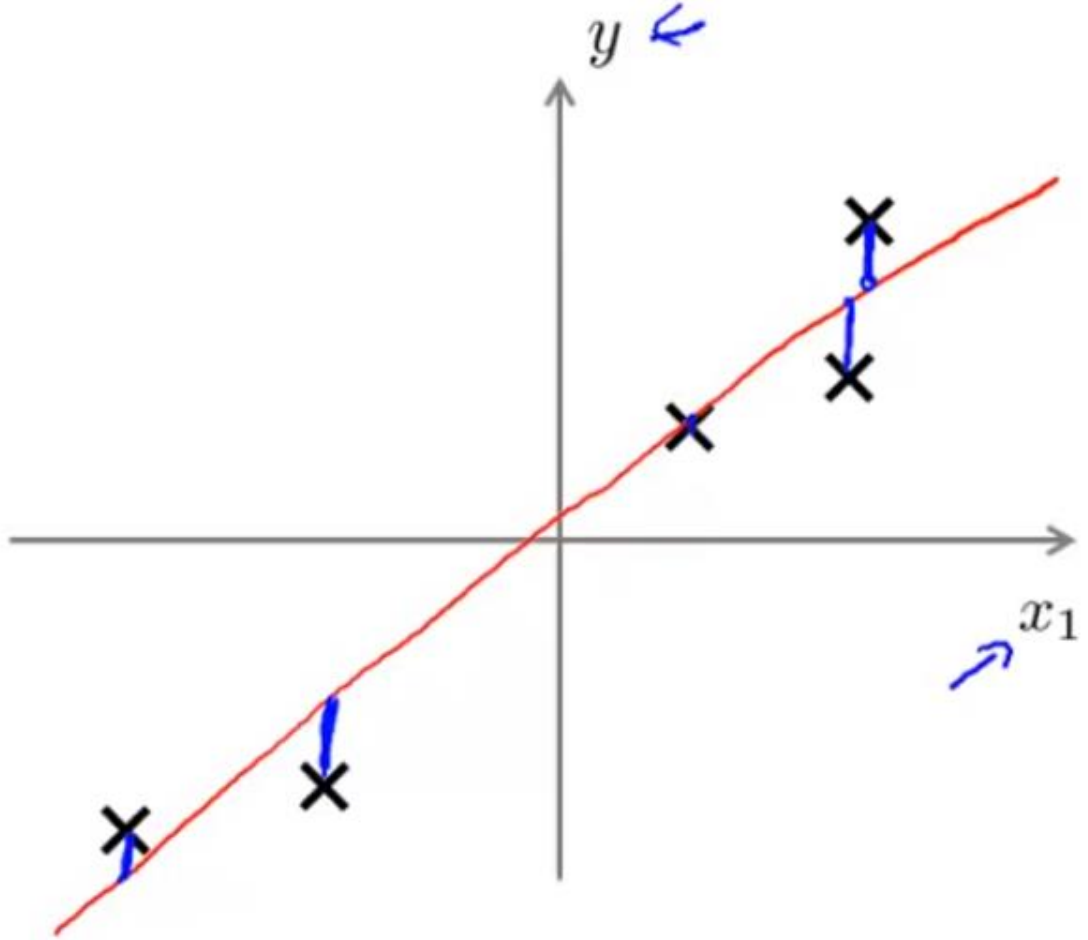
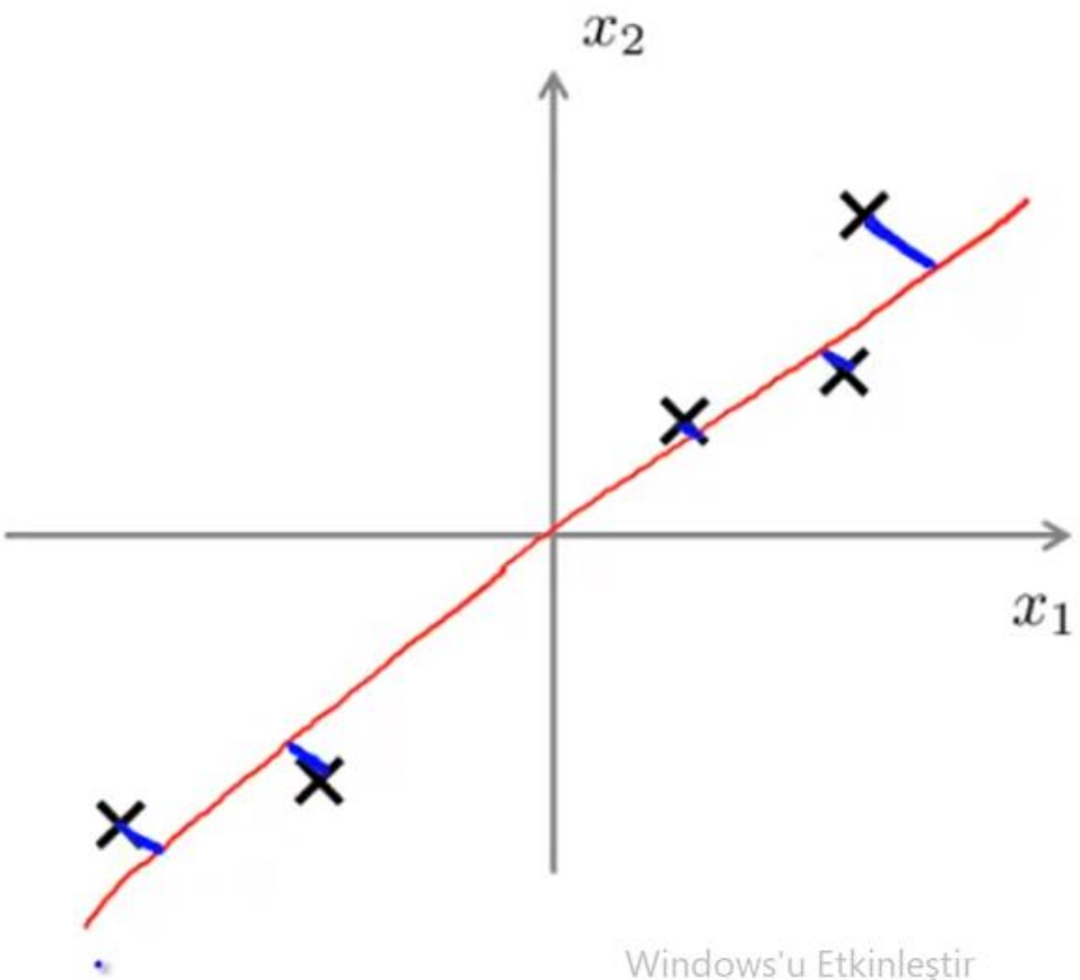




Algorithm

Principal Component Analysis

Unsupervised Learning

Data preprocessing

Training set: $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$ $\leftarrow$

Preprocessing (feature scaling/mean normalization):

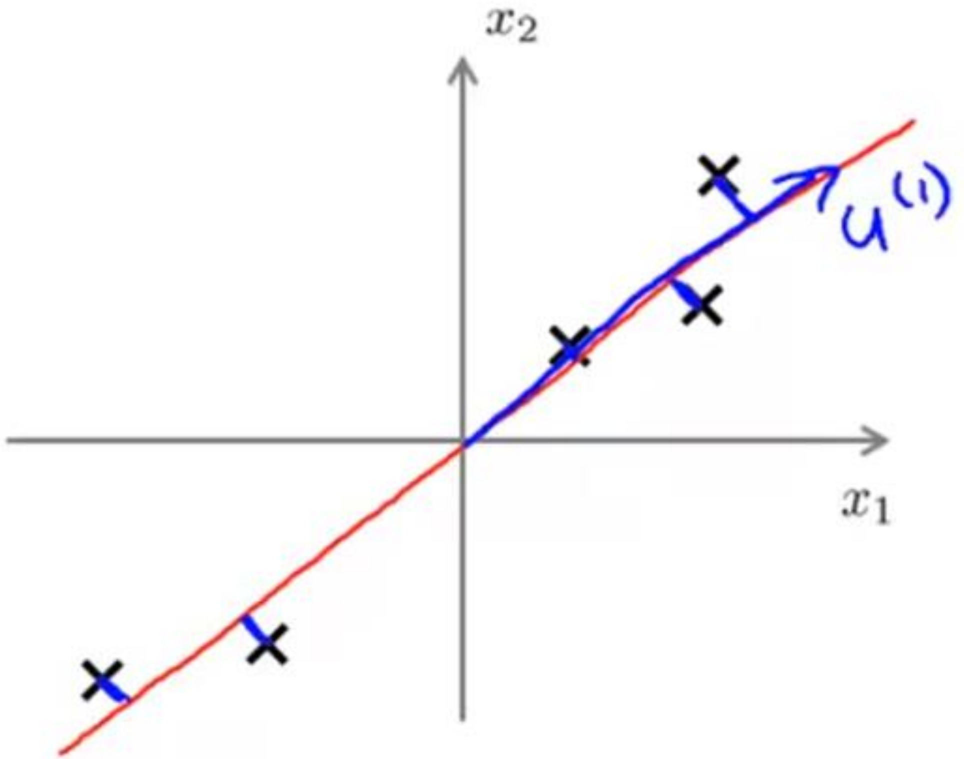
$$\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$$

Replace each $x_j^{(i)}$ with $x_j - \mu_j$.

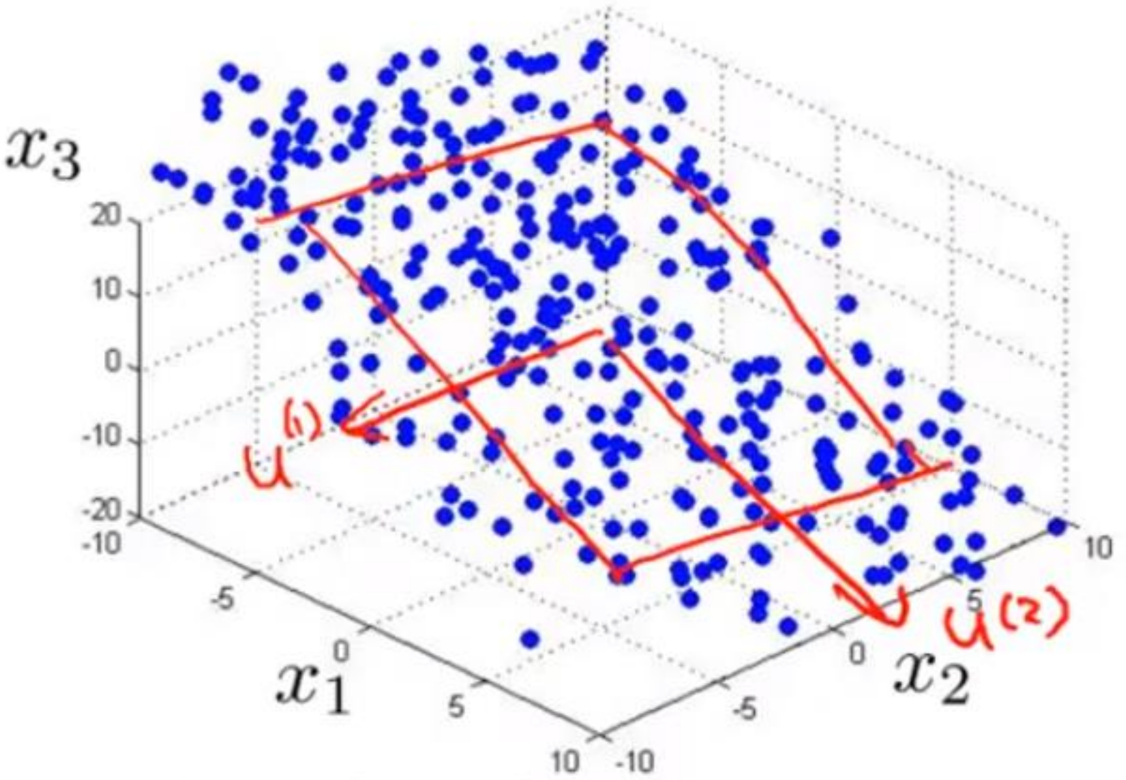
If different features on different scales (e.g., $x_1 =$ size of house, $x_2 =$ number of bedrooms), scale features to have comparable range of values.

$$x_j^{(i)} \leftarrow \frac{x_j^{(i)} - \mu_j}{s_j}$$

Principal Component Analysis (PCA) algorithm

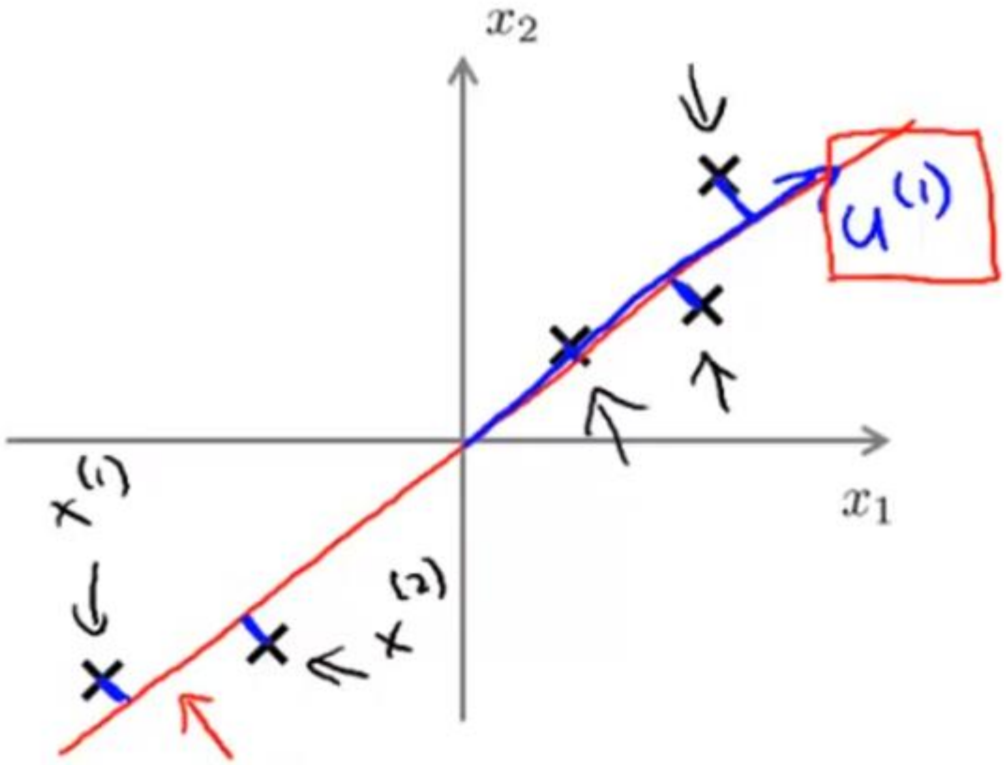


Reduce data from 2D to 1D



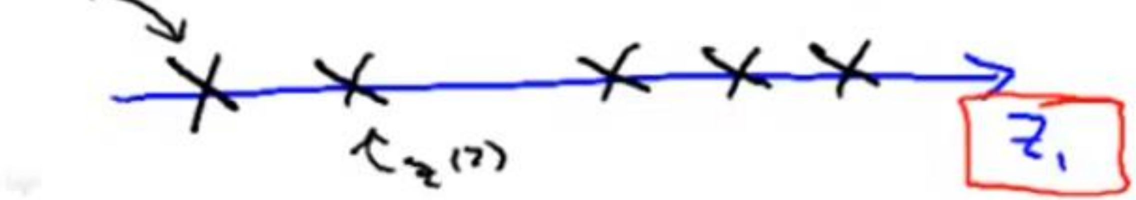
Reduce data from 3D to 2D

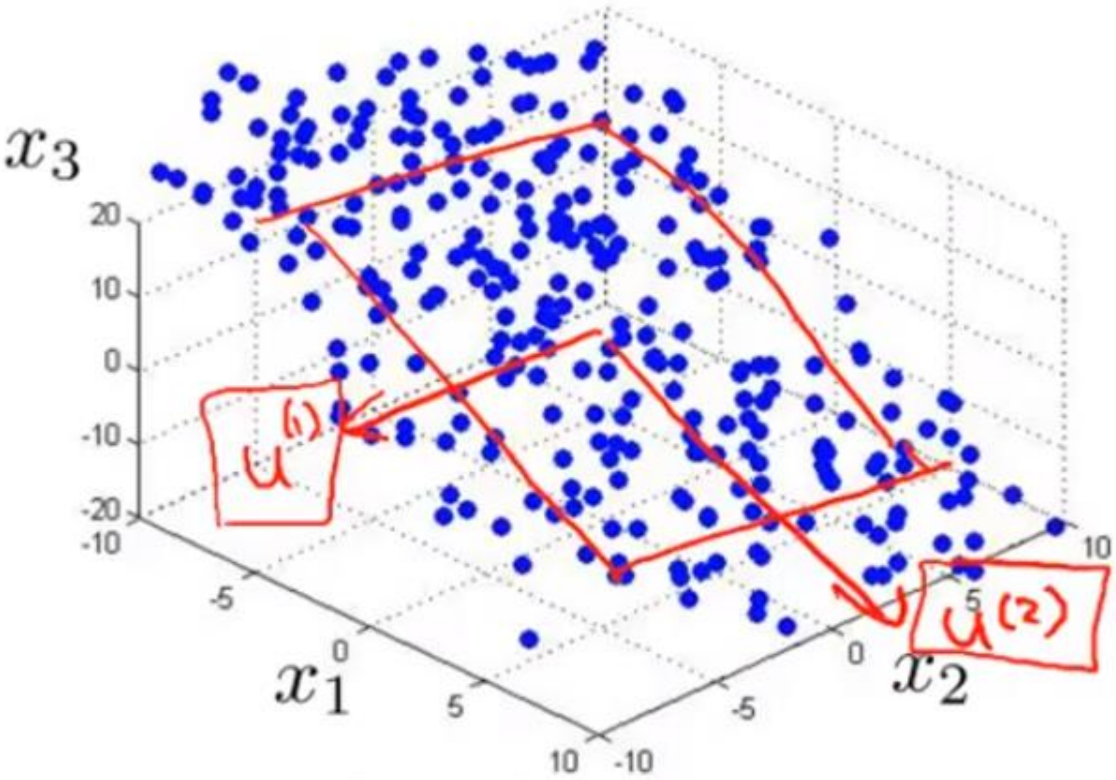
Principal Component Analysis (PCA) algorithm



Reduce data from 2D to 1D

$x^{(i)} \in \mathbb{R}^2 \rightarrow z^{(i)} \in \mathbb{R}$





Reduce data from 3D to 2D

$x^{(i)} \in \mathbb{R}^3 \rightarrow z^{(i)} \in \mathbb{R}^2$

$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$.

Principal Component Analysis (PCA) algorithm

Reduce data from n-dimensions to k-dimensions

Compute "covariance matrix":

$$\Sigma = \frac{1}{m} \sum_{i=1}^{n} (x^{(i)})(x^{(i)})^T$$

Compute "eigenvectors" of matrix Σ:

```
[U,S,V] = svd(Sigma);
```

Principal Component Analysis (PCA) algorithm

From `[U,S,V] = svd(Sigma)`, we get:

$$\rightarrow U = \begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \dots & u^{(n)} \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times n}$$

k

$$x \in \mathbb{R}^n \longrightarrow z \in \mathbb{R}^k$$

$$\begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \dots & u^{(k)} \\ | & | & & | \end{bmatrix}$$

Principal Component Analysis (PCA) algorithm

From `[U,S,V] = svd(Sigma)`, we get:

$$\rightarrow U = \begin{bmatrix} \underbrace{\begin{matrix} | & | \\ u^{(1)} & u^{(2)} \\ | & | \end{matrix}}_{k} & \dots & \begin{matrix} | \\ u^{(n)} \\ | \end{matrix} \end{bmatrix} \in \mathbb{R}^{n \times n}$$

$$x \in \mathbb{R}^n \longrightarrow z \in \mathbb{R}^k$$

$$z = \underbrace{\begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \dots & u^{(k)} \\ | & | & & | \end{bmatrix}}_{\substack{n \times k \\ U_{reduce}}}^{T} x = \underbrace{\underbrace{\begin{bmatrix} — (u^{(1)})^T — \\ \vdots \\ — (u^{(k)})^T — \end{bmatrix}}_{k \times n} \underbrace{x}_{n \times 1}}_{k \times 1}$$

$$z \in \mathbb{R}^k$$

Andrew Ng

Principal Component Analysis (PCA) algorithm summary

- After mean normalization (ensure every feature has zero mean) and optionally feature scaling:

$$\texttt{Sigma} = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)})(x^{(i)})^T$$

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$$\underline{X} = \begin{bmatrix} — & x^{(1)T} & — \\ & \vdots & \\ — & x^{(m)T} & — \end{bmatrix}$$

$\boxed{\text{Sigma} = (1/m) * X' * X;}$

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- After mean normalization (ensure every feature has zero mean) and optionally feature scaling:

```
Sigma =
```
$$\frac{1}{m}\sum_{i=1}^{m}(x^{(i)})(x^{(i)})^T$$

```
[U,S,V] = svd(Sigma);
Ureduce = U(:,1:k);
z = Ureduce'*x;
```

$$X = \begin{bmatrix} — x^{(1)T} — \\ \vdots \\ — x^{(m)T} — \end{bmatrix}$$

$\boxed{\texttt{Sigma} = (1/m) * X' * X;}$

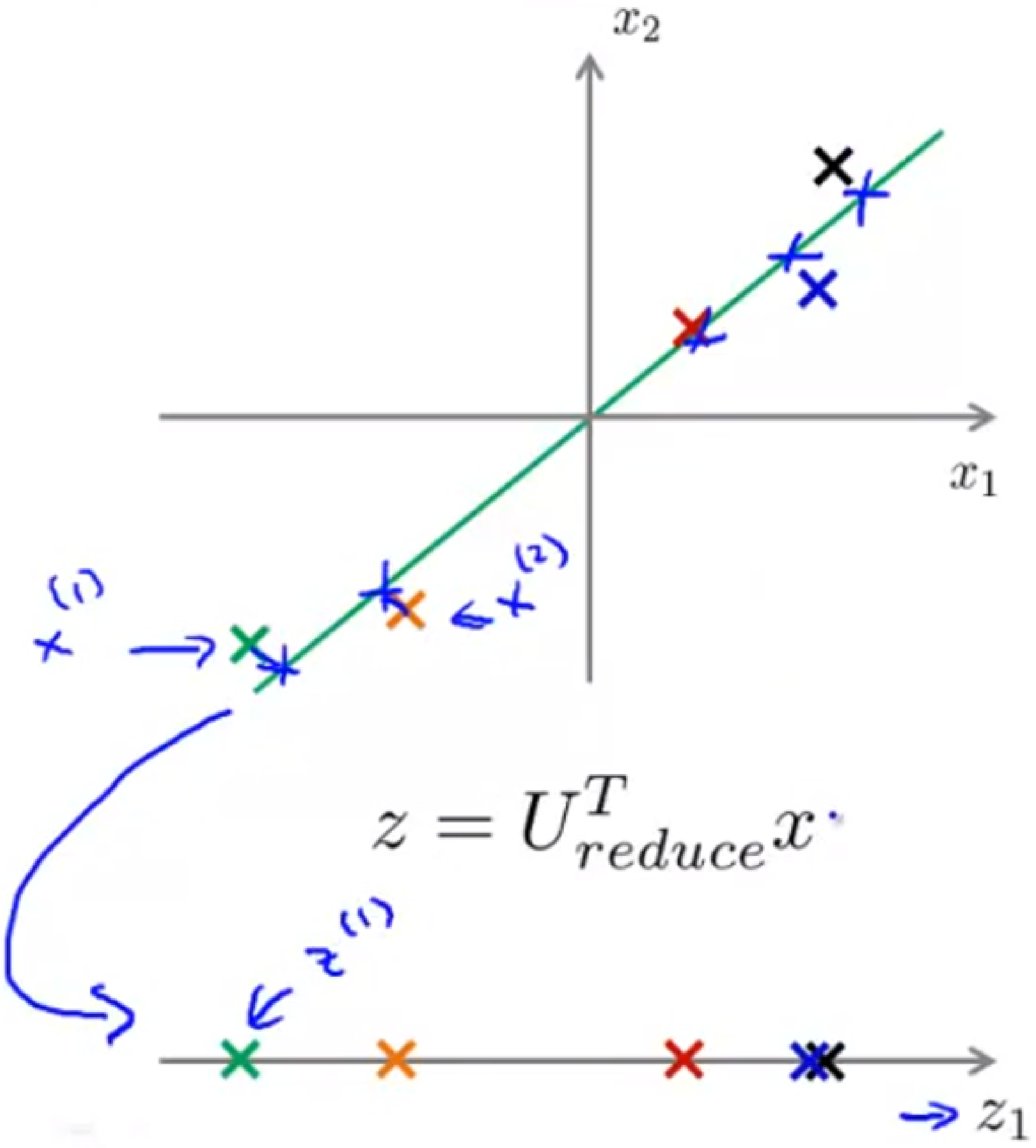
$x \in \mathbb{R}^n$ $\cancel{x_0 = 1}$

Reconstruction from compressed representation

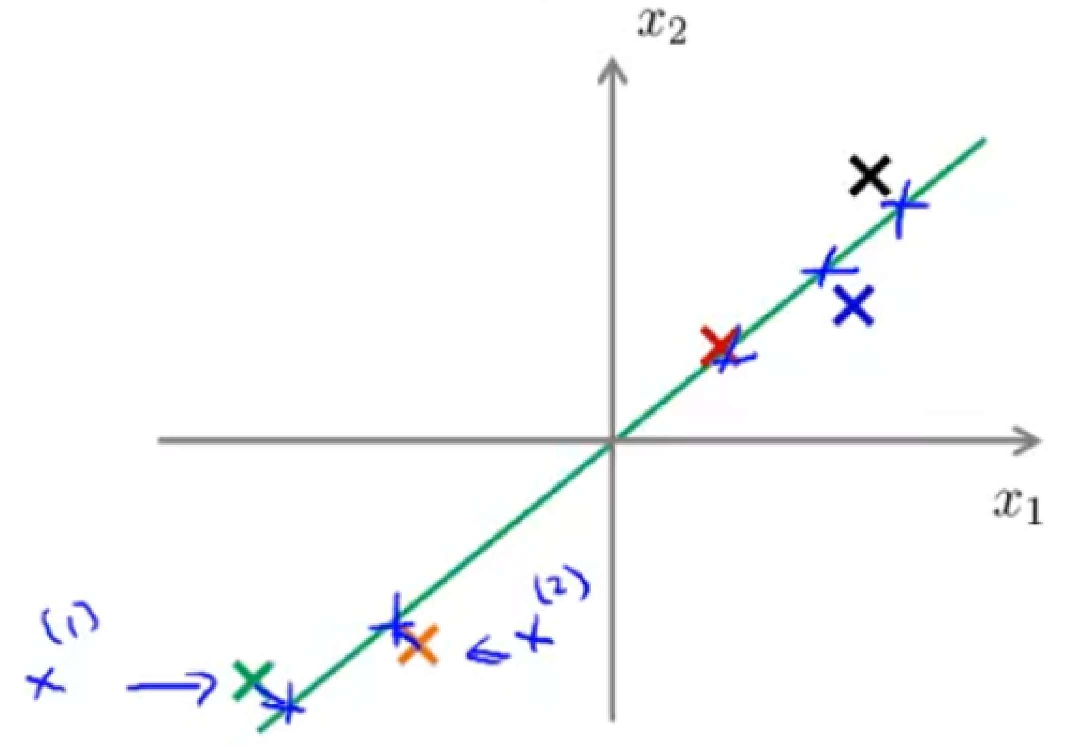
Principal Component Analysis

Unsupervised Learning

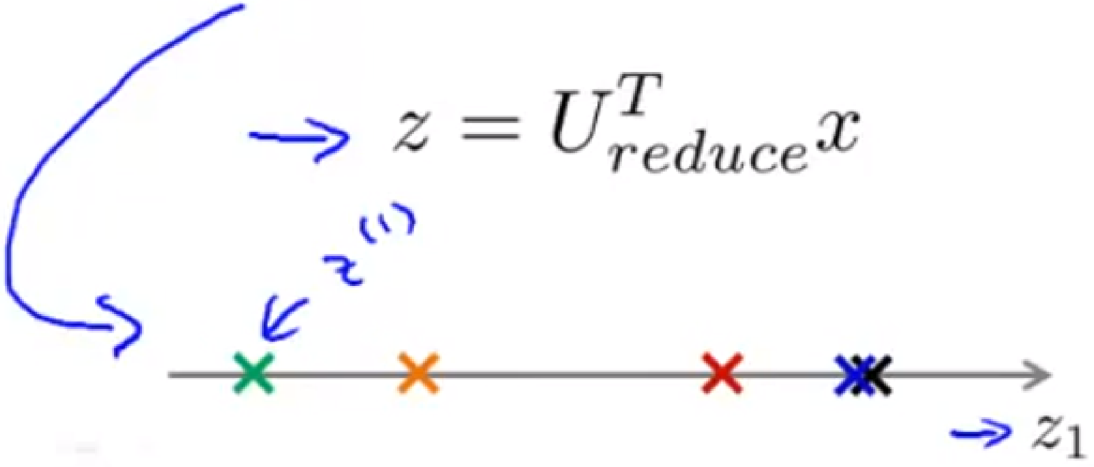
Reconstruction from compressed representation

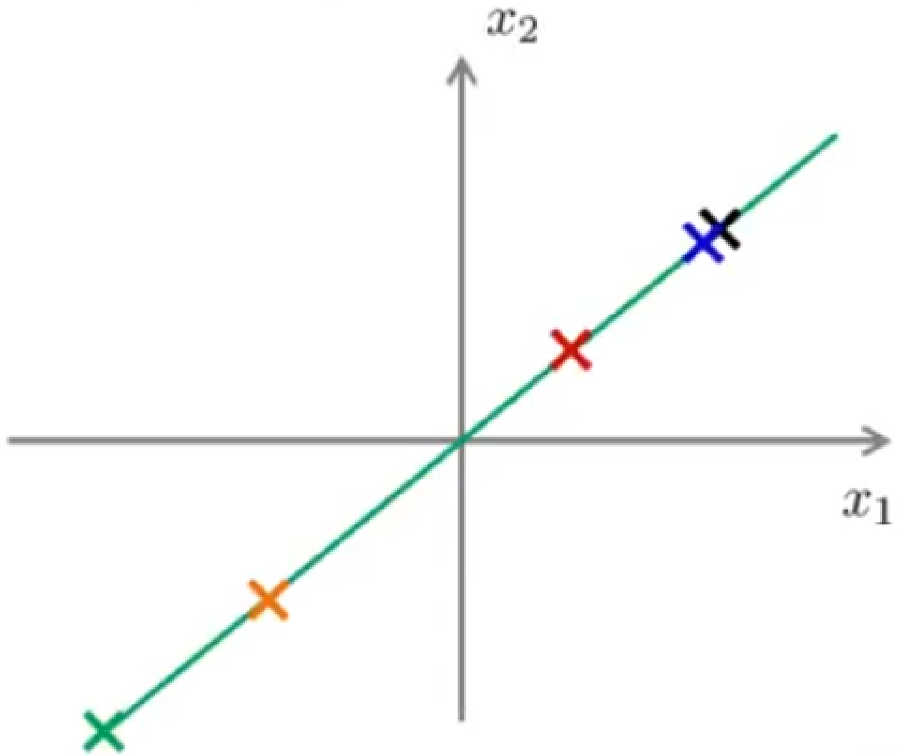


Reconstruction from compressed representation



$$\rightarrow z = U_{reduce}^T x$$



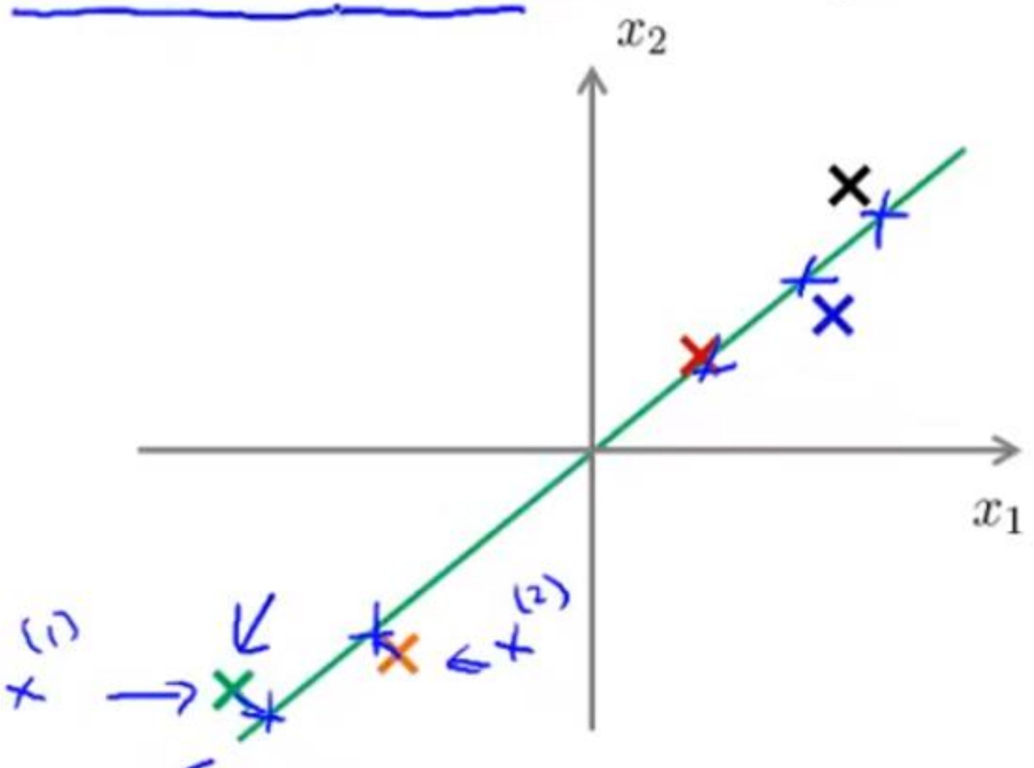


$$z \in \mathbb{R} \rightarrow x \in \mathbb{R}^2$$

$$\approx x$$

$$\underbrace{x_{approx}}_{\in \mathbb{R}^n} = \underbrace{\underbrace{U_{reduce}}_{n \times k} \cdot \underbrace{z}_{k \times 1}}_{n \times 1}$$

Reconstruction from compressed representation



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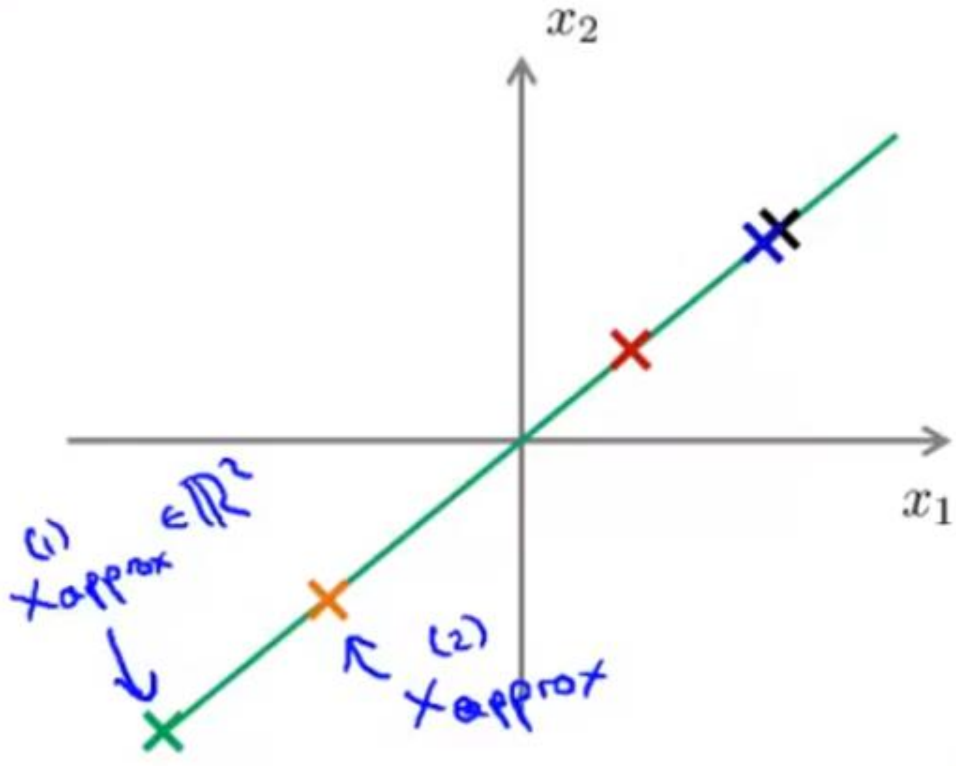

$$z \in \mathbb{R} \rightarrow x \in \mathbb{R}^2$$

$$x \approx \underbrace{x_{approx}^{(1)}}_{\mathbb{R}^n} = \underbrace{\underbrace{U_{reduce}}_{n \times k} \cdot \underbrace{z^{(1)}}_{k \times 1}}_{n \times 1}$$