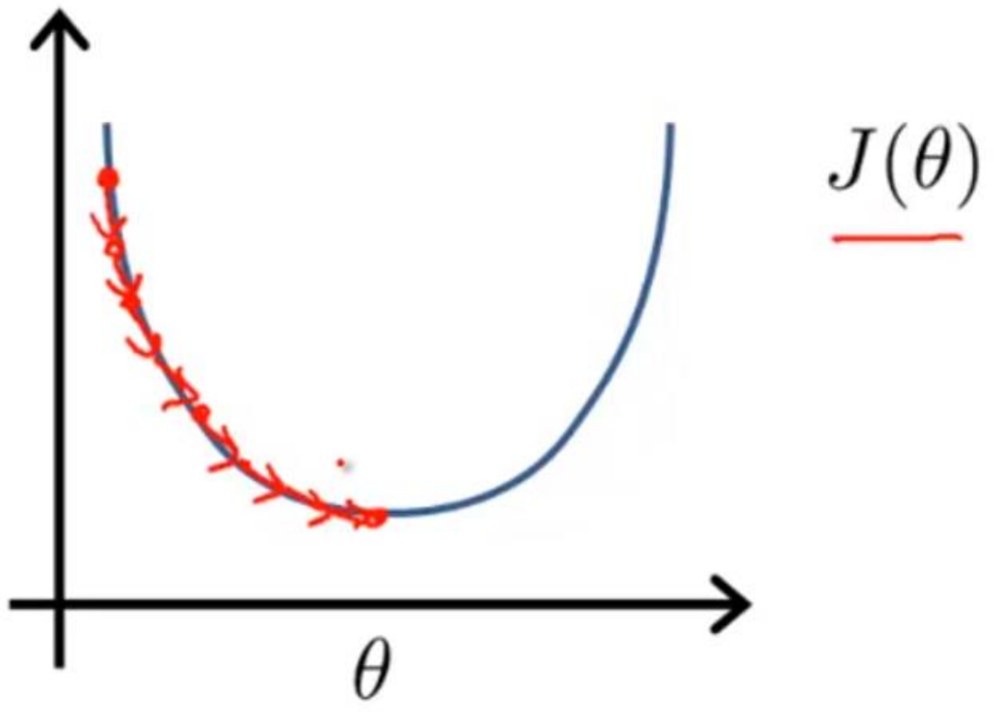
Normal Equation

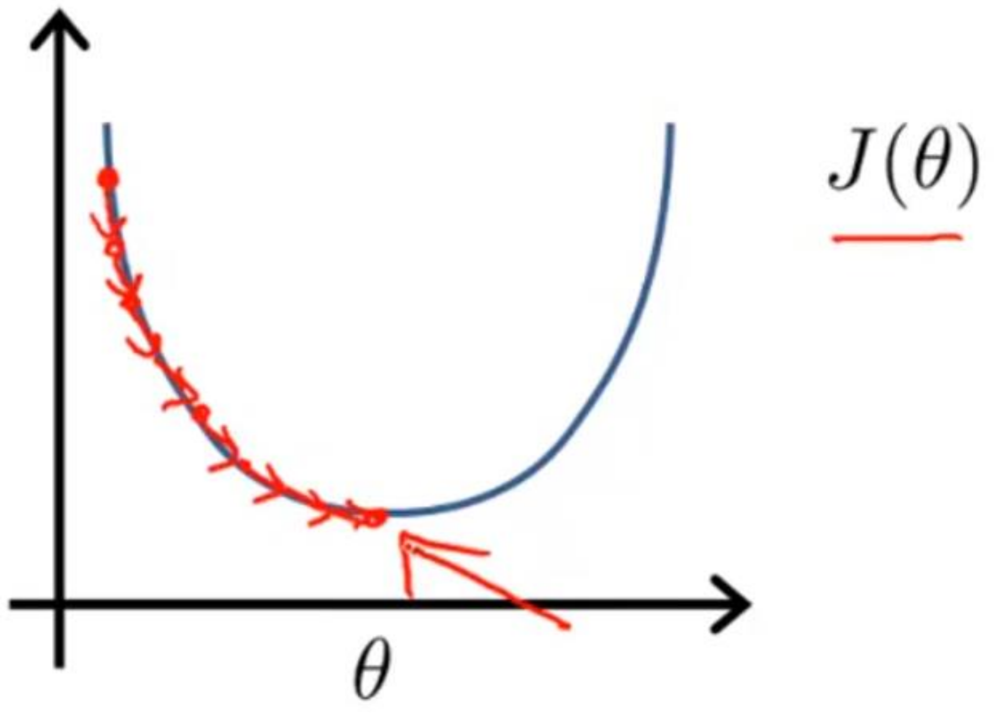
Computer Parameters Analytically

Linear Regression with Multiple Variables

Gradient Descent



Gradient Descent

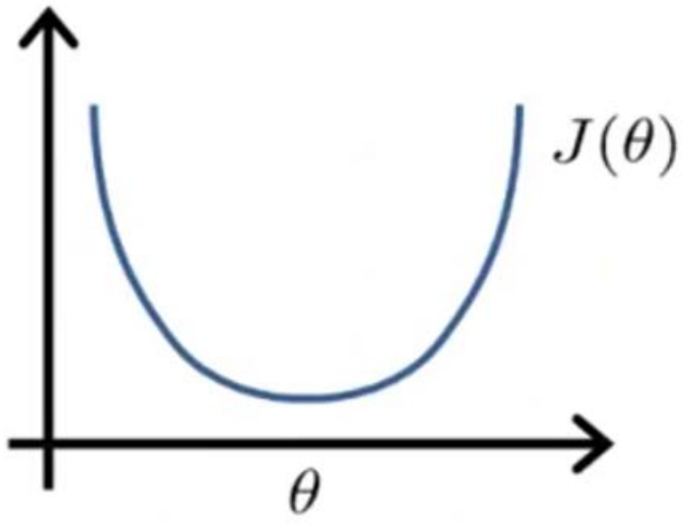


Normal equation: Method to solve for θ analytically.

Intuition: If 1D $(\theta \in \mathbb{R})$

$$J(\theta) = a\theta^2 + b\theta + c$$

How to minimize a function?



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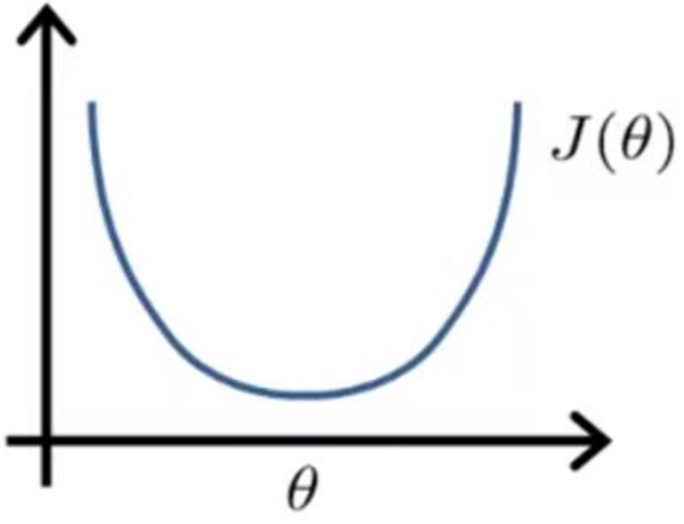
Intuition: If 1D $(\theta \in \mathbb{R})$

$$J(\theta) = a\theta^2 + b\theta + c$$

$$\frac{\partial}{\partial \theta} J(\theta) = \dots \overset{\text{set}}{=} 0$$

S



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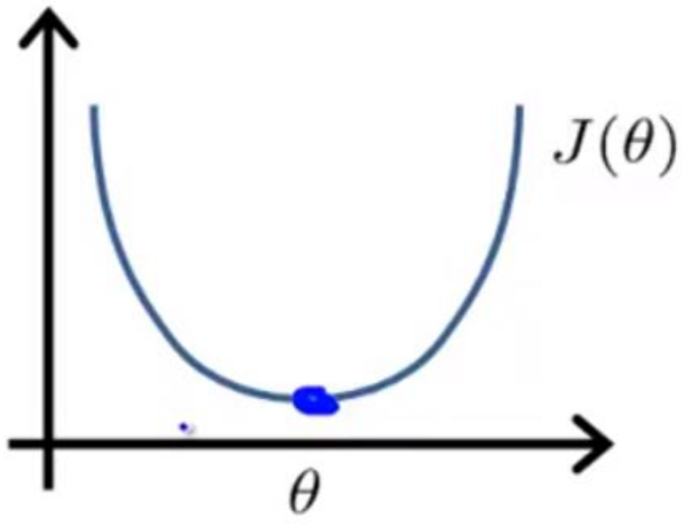
Solve for θ



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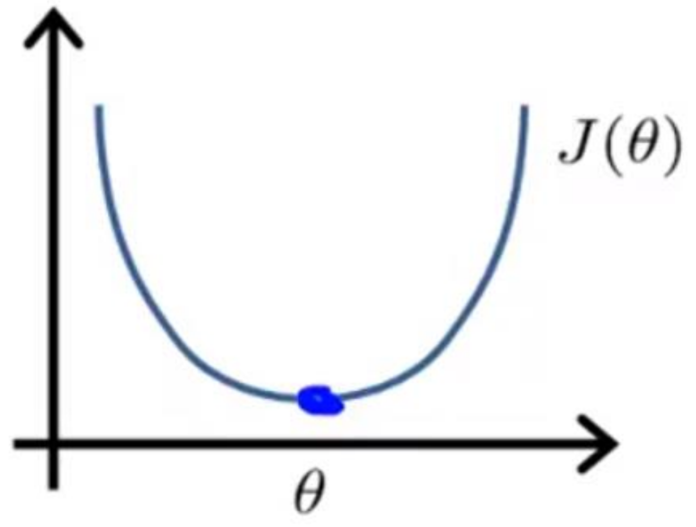

$\theta \in \mathbb{R}^{n+1}$ $\qquad J(\theta_0, \theta_1, \dots, \theta_m) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

$$\frac{\partial}{\partial \theta_j} J(\theta) = \cdots = 0 \qquad \text{(for every } j\text{)}$$

Solve for $\theta_0, \theta_1, \dots, \theta_n$

Examples: $m = 4.$

Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178

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$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}$$

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$m \times (n+1)$

m-dimensional vector

$$\boxed{\theta = (X^T X)^{-1} X^T y}$$

m **examples** $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$ **;** n **features.**

$$x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix} \in \mathbb{R}^{n+1}$$

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X

(design matrix)

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E.g. If $x^{(i)} = \begin{bmatrix} 1 \\ x_1^{(i)} \end{bmatrix}$

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$m \times (n+1)$

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$m \times (n+1)$

E.g. If $x^{(i)} = \begin{bmatrix} 1 \\ x_1^{(i)} \end{bmatrix}$

$$X = \underset{m \times 2}{\begin{bmatrix} 1 & x_1^{(1)} \\ 1 & x_2^{(1)} \\ \vdots & \vdots \\ 1 & x_m^{(1)} \end{bmatrix}} \qquad y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$$

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(design matrix)

$m \times (n+1)$

E.g. If $x^{(i)} = \begin{bmatrix} 1 \\ x_1^{(i)} \end{bmatrix}$

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$m \times 2$

$$\theta = (X^T X)^{-1} X^T y$$

Exercise

- Suppose you have the training in the table below:

age (x_1)	height in cm (x_2)	weight in kg (y)
4	89	16
9	124	28
5	103	20

- You would like to predict a child's weight as a function of his age and height with the model
- $weight = \theta_0 + \theta_1 x_1 + \theta_2 x_2$
- What are X and y?

$$\theta = \underline{(X^T X)^{-1}} X^T y$$

$$\theta = \underline{(X^T X)^{-1}} X^T y$$

$(X^T X)^{-1}$ is inverse of matrix $\underline{X^T X}$.

Octave: `pinv(X'*X)*X'*y`

$$\theta = \underline{(X^T X)^{-1}} X^T y$$

$(X^T X)^{-1}$ is inverse of matrix $\underline{X^T X}$.

Set $\underline{A} = \underline{X^T} \underline{X}$

$$\boxed{(X^T X)^{-1}} = A^{-1}$$

Octave: `pinv(X'*X)*X'*y`

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$$\boxed{\theta = \boxed{(X^TX)^{-1}X^Ty}}$$

$(X^TX)^{-1}$ is inverse of matrix X^TX.

Set $A = X^TX$

$\boxed{(X^TX)^{-1}} = A^{-1}$

Octave: `pinv(X'*X)*X'*y`

X' X^T

$\text{pinv}(X^T*X)*X^T*y$

$(X^TX)^{-1}X^Ty$

$$\boxed{\theta = \boxed{(X^T X)^{-1} X^T y}}$$

$(X^T X)^{-1}$ is inverse of matrix $X^T X$.

Set $A = X^T X$

$\boxed{(X^T X)^{-1}} = A^{-1}$

When to choose gradient descent and when to choose normal equations???

No need for feature scaling!

Octave: `pinv(X'*X)*X'*y`

X' $\quad$ X^T

pinv(X^T*X)*X^T*y

$\theta = (X^T X)^{-1} X^T y$

$\min_{\theta} J(\theta)$

Feature Scaling

$0 \le x_1 \le 1$

$0 \le x_2 \le 1000$

$0 \le x_3 \le 10^{-5}$

m training examples, n features.

Gradient Descent

- Need to choose α.
- Needs many iterations.

Normal Equation

- No need to choose α.
- Don't need to iterate.

m training examples, n features.

Gradient Descent

- Need to choose α.
- Needs many iterations.
- Works well even when n is large.

Normal Equation

- No need to choose α.
- Don't need to iterate.

m training examples, n features.

Gradient Descent

- Need to choose α.
- Needs many iterations.
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Normal Equation

- No need to choose α.
- Don't need to iterate.
- Need to compute $\boxed{(X^TX)^{-1}}$
- Slow if n is very large.

m training examples, n features.

Gradient Descent	Normal Equation
• Need to choose α.	• No need to choose α.
• Needs many iterations.	• Don't need to iterate.
• Works well even when n is large.	• Need to compute $\boxed{(\boxed{X^TX})^{-1}}$ $\underline{n \times n}$ $O(n^3)$
	• Slow if n is very large.

m training examples, n features.

Gradient Descent	Normal Equation
• Need to choose α.	• No need to choose α.
• Needs many iterations.	• Don't need to iterate.
• Works well even when n is large.	• Need to compute $\boxed{(\boxed{X^TX})^{-1}}$ $n \times n$ $O(n^3)$
	• Slow if n is very large.
	$n=100$
	$n=1000$

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m training examples, n features.

Gradient Descent

- Need to choose α.
- Needs many iterations.
- Works well even when n is large.

Normal Equation

- No need to choose α.
- Don't need to iterate.
- Need to compute $\boxed{(\boxed{X^T X})^{-1}}$ $n \times n$ $O(n^3)$
- Slow if n is very large.

$n = 100$
$n = 1000$
$n = 10000$

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m training examples, n features.

Gradient Descent

- Need to choose α.
- Needs many iterations.
- Works well even when n is large.

$n = 10^6$

Normal Equation

- No need to choose α.
- Don't need to iterate.
- Need to compute $\boxed{(\boxed{X^T X})^{-1}}$ $n \times n$ $O(n^3)$
- Slow if n is very large.

$n = 100$

$n = 1000$

$n = 10000$

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Gradient Descent	**Normal Equation**
Need to choose alpha	No need to choose alpha
Needs many iterations	No need to iterate
O (kn^2)	O (n^3), need to calculate inverse of X^TX
Works well when n is large	Slow if n is very large