Mathematical Programming

Cutting stock problem:

Consider a big steel roll from which steel sheets of same lengths but different width have to be cut. Let us assume that the roll is 20 inch wide and the following sizes have to be cut

1. 9 inch 511 numbers
2. 8 inch 301 numbers
3. 7 inch 263 numbers
4. 6 inch 383 numbers

It is assumed that all the cut sheets have the same length (25 inches). Only one dimensional cutting allowed. The problem is to cut the sheets in such a way as to minimize wastage of the material.

Cutting stock problem:

Problem Statement:

[9 8 7 6] inches made from sheets of steel that are 20 inches wide

1. [2000] waste 2
2. [0200] waste 4
3. [0021] waste 0
4. [0003] waste 2
5. [1100] waste 3
6. [1010] waste 4
7. [1001] waste 5
8. [0110] waste 5
9. [0102] waste 0
10. [0012] waste 1

10 different pattern

Xj : numer of sheets cut using pattern j.

$2x1 + x5+x6+x7 \geq 511$ (9 inch sheets)

$2x2+x5+x8+x9 \geq 301$ (8 inch sheets)

$2x3+x6+x8+x10 \geq 263$ (7 inch sheets)

$X3+3x4+x7+2x9+2x10 \geq 383$ (6 inch sheets)

$xj \geq 0$

Cutting stock problem:

The objective function is to minimize wastage. They are all inequalities because it may not be possible to get the exact number required. This is given by

Minimize 2x1+ 4x2+0x3+ 2x4+ 3x5+ 4x6+ 5x7+ 5x8+0x9+ x10
+9(2x1 + x5+x6+x7 -511)
+8(2x2+x5+x8+x9 -301)
+7(2x3+x6+x8+x10-263)
+6(X3+3x4+x7+2x9+2x10-383)

This reduces to
Minimize 20x1+ 20x2+20x3+ 20x4+ 20x5+ 20x6+ 20x7+ 20x8+20x9+20x10-11146
So objective function Minimize $\sum_j x_j$ so it becomes mim total number of cuts, not the waste

Subject to

$\sum_j aij\ x_j \geq b_j$

Where aij is parameter of various patterns

$x_j \geq 0$

Linear Programing Solution

Maximize z=6x1+ 5x2
Subject to
x1+ x2 ≤5
3x1+ 2x2≤12
x1, x2 ≥0

Maximize z=6x1+ 5x2 + 0x3 + 0x4

With the addition of x3 and x4 are slack variables, we have 4 variable and 2 eqs.
x1+ x2+x3+x4=5
3x1+ 2x2+x3+x4=12

x3 and x4 are slack variables
x1, x2, x3, x4 ≥0

Linear Programing Solution

We have to fix any 2 variables to some arbitrary value and solve for the remaining 2 variables.
Thus there is infinite number of solution but fixing arbitrary values to zero we have only 6 solutions.

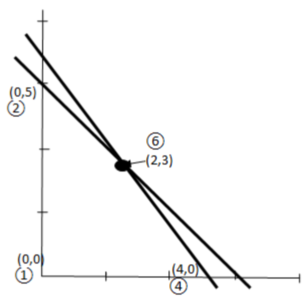
1) Solve for x3, x4 fix x1, x2
2) Solve for x2, x4 fix x1, x3
3) Solve for x2, x3 fix x1, x4
4) Solve for x1, x3 fix x2, x4
5) Solve for x1, x4 fix x2, x3
6) Solve for x1, x2 fix x3, x4

Linear Programing Solution

Solution:

1. x1=x2=0, non-basic variables, so x3=5, x4=12, z=0
2. x1=x3=0, non-basic variables, so x2=5, x4=2, z=25
3. x1=x4=0, non-basic variables, so x2=6, x3=-1, since x3=-1 that violates the positivity constrain
4. x2=x4=0, non-basic variables, so x1=4, x3=1, z=24
5. x2=x3=0, non-basic variables, so x1=5, x4=-3, violates the positivity constrain
6. x3=x4=0, non-basic variables, so x1=2, x2=3 , z=27

Six basic solution and four are feasible (satisfies all constrains). Remaining two infeasible (negative values)



We observed that 4 basic feasible solution corresponding to 4 corner points.
3 and 5 do not give feasible solution.
These shows relation with graphical methods.
So corner points evaluated by these techniques.
This solution is call basic solution.

EM Prof. Dr. Arif N. Gulluoglu

Linear Programing Solution

n variable
m equations
so we have nCm basic solutions.

- Here we have 6 solution, 4 of them is feasible. Feasible solution satisfies all constrains including the non-negativity. Feasible solutions are called basic feasible solutions.
- 2 solutions that are not satisfy constrains that are call infeasible solutions.
- And all basic feasible solutions are corner points.
- In the solution those variables we fixed to be zero is call non-basic variables, and those we solved are call basic variables.
- Graphical method, we evaluate the corner points and we do algebraic method also.
- We evaluate the corner points, the one is maximum is the optimal solution.

Linear Programing Solution

ALGEBRAIC METHOD

1. Convert inequalities into equations by adding slack variables.
2. Assuming that there are m eqs., and n variables, set n-m (non-basic) variables to zero and evaluate the solutions for remaining m basic variables. Evaluate the objective function if basic solution is feasible.
3. Perform step 2 for all nCm combinations of basic variables.
4. Identify the optimum solution as the one with the maximum (minimum) value of the objective function.

Disadvantages

1. nCm is very large number.
2. There are some infeasible solution
3. Among feasible ones, we like the improve the solution