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# Computer Viewing

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# Objectives

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- Introduce the mathematics of projection
- Introduce OpenGL viewing functions
- Look at alternate viewing APIs



# Computer Viewing

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- There are three aspects of the viewing process, all of which are implemented in the pipeline,
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix
  - Clipping
    - Setting the view volume



# The OpenGL Camera

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- In OpenGL, initially the object and camera frames are the same  
Default model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- OpenGL also specifies a default view volume that is a cube with sides of length 2 centered at the origin  
Default projection matrix is an identity



Diagram illustrating the projection of a 3D object onto a 2D plane. A 3D box is shown in a coordinate system with x, y, and z axes. The box is projected onto the xy-plane ( $z=0$ ). The projection is shown as a 2D rectangle. The original 3D object is labeled "clipped out" and the projection is labeled "Projection plane  $z=0$ ".



# Moving the Camera Frame

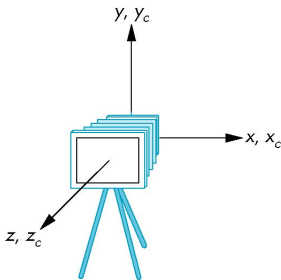
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- If we want to visualize object with both positive and negative z values we can either
  - Move the camera in the positive z direction
- Translate the camera frame
  - Move the objects in the negative z direction
- Translate the world frame
- Both of these views are equivalent and are determined by the model-view matrix
  - Want a translation (`glTranslatef(0.0,0.0,-d);`)  
 $d > 0$

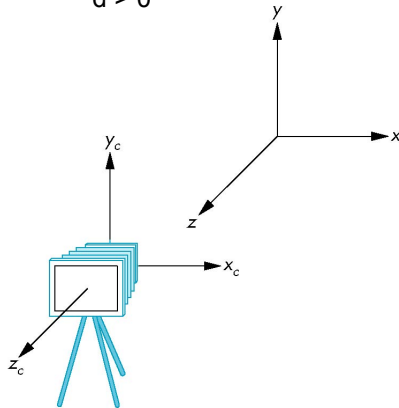
# Moving Camera back from Origin

frames after translation by  $-d$   
 $d > 0$

default frames



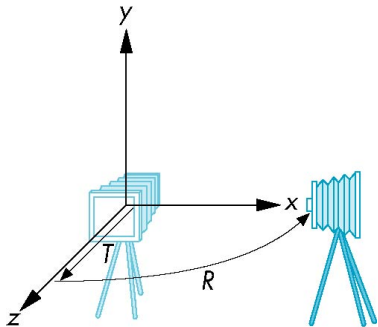
(a)



(b)

# Moving the Camera

- We can move the camera to any desired position by a sequence of rotations and translations
- Example: side view  
Rotate the camera  
Move it away from origin  
Model-view matrix  $C = TR$







# OpenGL code

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- Remember that last transformation specified is first to be applied

```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glTranslatef(0.0, 0.0, -d);
glRotatef(90.0, 0.0, 1.0,
0.0);
```



# The LookAt Function

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- The GLU library contains the function `gluLookAt` to form the required modelview matrix through a simple interface
- Note the need for setting an up direction
- Still need to initialize
  - Can concatenate with modeling transformations
- Example: isometric view of cube aligned with axes

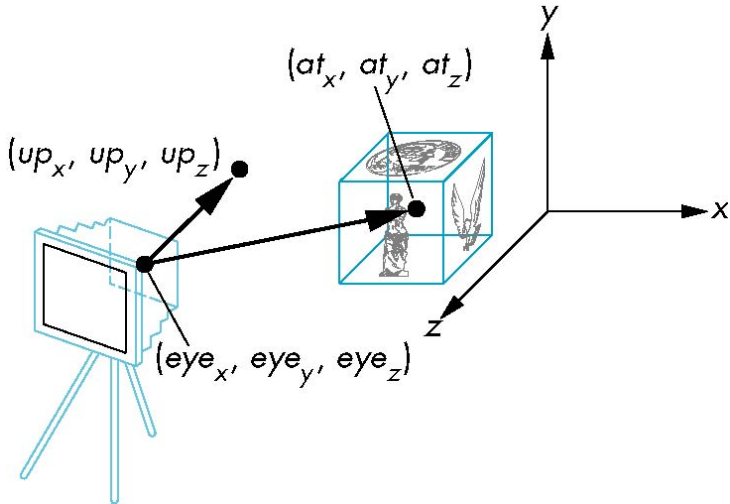
```
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();  
gluLookAt(1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0., 1.0,  
0.0);
```



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# gluLookAt

`gluLookAt(eyex, eyey, eyez, atx, aty, atz, upx, upy, upz)`





# Other Viewing APIs

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- The LookAt function is only one possible API for positioning the camera
- Others include
  - View reference point, view plane normal, view up (PHIGS, GKS-3D)
  - Yaw, pitch, roll
  - Elevation, azimuth, twist
  - Direction angles



# Projections and Normalization

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- The default projection in the eye (camera) frame is orthogonal
- For points within the default view volume

$$x_p = x$$

$$y_p = y$$

$$z_p = 0$$

- Most graphics systems use *view normalization*  
All other views are converted to the default view by transformations that determine the projection matrix  
Allows use of the same pipeline for all views



# Homogeneous Coordinate Representation

default orthographic projection

$$x_p = x$$

$$y_p = y$$

$$z_p = 0$$

$$w_p = 1$$

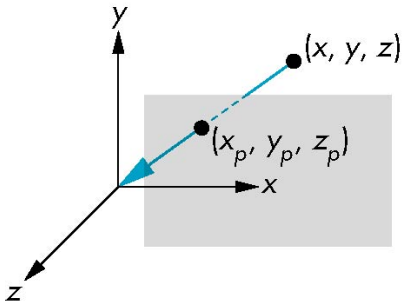
$$p_p = M p$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let  $M = I$  and set the  $z$  term to zero later

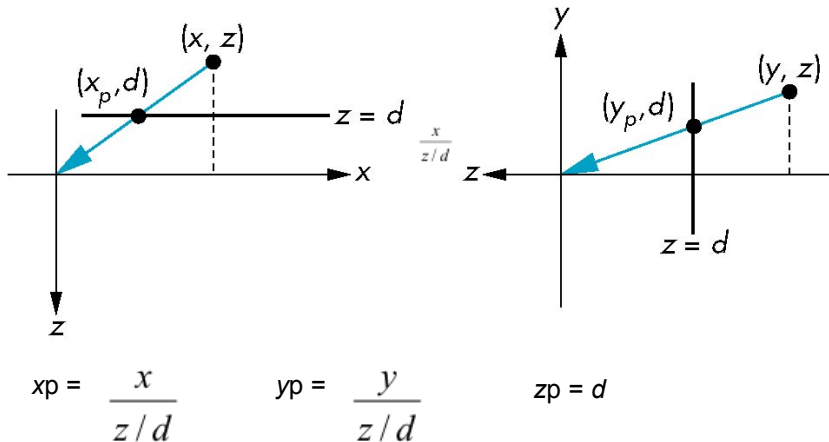
# Simple Perspective

- Center of projection at the origin
- Projection plane  $z = d, d < 0$



# Perspective Equations

Consider top and side views







# Homogeneous Coordinate Form

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consider  $\mathbf{q} = \mathbf{M}\mathbf{p}$  where  $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \square \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$



# Perspective Division

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- However  $w \neq 1$ , so we must divide by  $w$  to return from homogeneous coordinates
- This *perspective division* yields

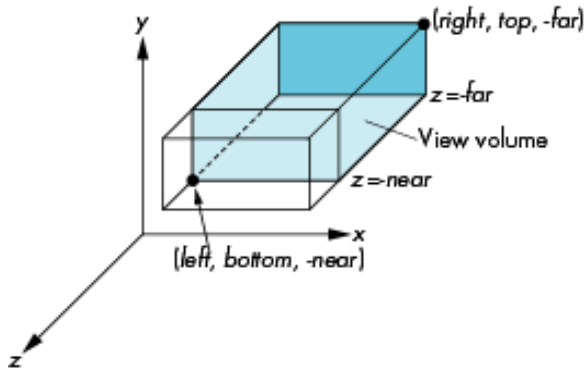
$$x_p = \frac{x}{z/d} \quad y_p = \frac{y}{z/d} \quad z_p = d$$

the desired perspective equations

- We will consider the corresponding clipping volume with the OpenGL functions

# OpenGL Orthogonal Viewing

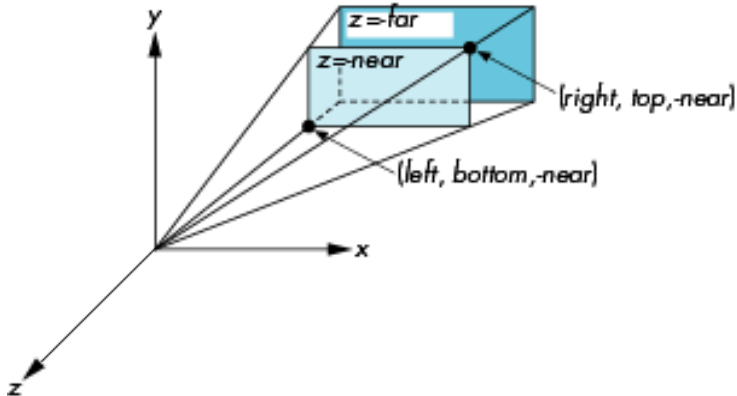
`glOrtho(left, right, bottom, top, near, far)`



**near** and **far** measured from camera

# OpenGL Perspective

`glFrustum(left, right, bottom, top, near, far)`



# Using Field of View

- With `glFrustum` it is often difficult to get the desired view
- `gluPerspective(fovy, aspect, near, far)` often provides a better interface

