Interpretation of LP Results

Why Use LP's

Finding Optimal Plan

- Solution of key decision variables

Economic Interpretations

- Reduced costs
- Shadow prices
- Solution ranging
- Sensitivity to key parameters

Sensitivity Analysis

Answering "what-if" questions

- What is the value of additional capacity
- How much would price have to change before
- How sensitive is profitability to the LPF

Interpretation of LP Results

Linear Programing Solution

Maximize $z = 6x1 + 5x2$
Subject to

$x1 + x2 \leq 5$ Eq. 2
$3x1 + 2x2 \leq 12$ Eq. 3
$x1, x2 \geq 0$

Without making any calculation. Let's find UB of solution

1. Let forget about constrains
$Z = \infty$
2. Multiply eq. 3 by 3, $9x1 + 6x2 \leq 36$

So Z=36

3. Multiply eq. 2 by 6, $6x1 + 6x2 \leq 30$

So Z=30

4. Multiply eq. 3 by 2 , and the add to eq.2 , $7x1 + 5x2 \leq 29$

So Z=29

Interpretation of LP Results

Maximize z = 6x1+ 5x2
Subject to

$x1+ x2 \leq 5$ Eq. 2
$3x1+ 2x2 \leq 12$ Eq. 3
$x1, x2 \geq 0$

So what we are doing it, that is

$a1+3\ a2 \geq 6$
$a1+2\ a2 \geq 5$

so 5 a1+12 a2 is UB so that is the Min value that UB takes.
So that is another LP problem
If we replace a1 by y1 and a2 by y2, we can get

Min 5 y1+12 y2
$y1+3y2 \geq 6$
$y1+2y2 \geq 5$
$y1, y2 \geq 0$

so this problem is dual of this problem.

Interpretation of LP Results

Lets look at here, Primal has 2 constrains, so dual has 2 variables.
RHS values of the primal will be the objective function values of the dual.
So coefficient matrix of primal will be transpose

Writing the dual to the standart form of the LPP
We generalize that if the Primal is

Max $Z=Cx$

Subject to
$A\ x \leq b$
$x \geq 0$

The Dual is
Min $W=yb$

Subject to
$A^T\ y \leq C$
$y \geq 0$

The Dual Model

Primal Problem:

Maximize

$$Z_X = C_1 X_1 + C_2 X_2 + C_3 X_3 + . \ . \ . \ . \ . \ . + C_N X_N$$

Subject to

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 + . \ . \ . \ . \ . \ . \ . + a_{1N} X_N \leq b_1$$
$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 + . \ . \ . \ . \ . \ . \ . + a_{2N} X_N \leq b_2$$
$$. \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ .$$
$$a_{M1} X_1 + a_{M2} X_2 + a_{M3} X_3 + . \ . \ . \ . \ . \ . + a_{MN} X_N \leq b_M$$
$$X_J \geq 0$$

$i = 1, 2, \ . \ . \ . \ . \ . \ ., M$
$j = 1, 2, \ . \ . \ . \ . \ . \ ., N$

The Dual Model

The associated dual problem is obtained by

- Transposing the coefficients, C_j in the objective function and the right-hand-side constants, bi, of the constraint functions.
- Transposing the rows and columns of coefficients in the constraint functions.
- Reversing the direction of the inequalities. "Greater than " inequalities appear instead of "Less than" inequalities. "Less than" in equalities replace "greater than" inequalities.
- Specifying a new set of variables or unknowns, Y_1, Y_2, Y_3, …. $Y_{M.}$. These are to be thought of as indicating the marginal value of b_1, b_2, …. b_M in the primal.
- Minimizing the value of objective function instead of maximizing its value.

The Dual Model

General form of Dual problem is as follows

Minimize

$$Z_Y = b_1 Y_1 + b_2 Y_2 + b_3 Y_3 + . \ . \ . \ . \ . \ . + b_M Y_M$$

Subject to

$$\left.\begin{array}{l} a_{11} Y_1 + a_{21} Y_2 + a_{13} Y_3 + . \ . \ . \ . \ . \ . + a_{M1} Y_M \geq C_1 \\ a_{12} Y_1 + a_{22} Y_2 + a_{23} Y_3 + . \ . \ . \ . \ . \ . + a_{M2} Y_M \geq C_2 \\ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \\ a_{1N} Y_1 + a_{2N} Y_2 + a_{M3} Y_3 + . \ . \ . \ . \ . + a_{MN} Y_M \geq C_N \\ Y_i \geq 0 \end{array}\right\} \quad \begin{array}{l} i = 1, 2, \ . \ . \ . \ . \ . \ ., M \\ j = 1, 2, \ . \ . \ . \ . \ . \ ., N \end{array}$$

MATHEMATICAL EXPLANATION TO THE DUAL

Maximize $z=6x1+ 5x2$
Subject to

$x1+ x2 \leq 5$ R1
$3x1+ 2x2 \leq 12$ R2
$x1, x2 \geq 0$

Min $5 y1+12 y2$
$y1+3y2 \geq 6$
$y1+2y2 \geq 5$
$y1, y2 \geq 0$

Optimal solution to primal x1=2, x2=3, Z=27
Optimal solution to dual y1=3, y2=1, W=27

Now Let us now add a small quantity δ - to first constrain such that the resource available now is $5+\delta$

MATHEMATICAL EXPLANATION TO THE DUAL

Assuming that x1 and x2 will remain as basic variables at the optimum and solving for x1 and x2

$$x1+ x2 \leq 5+ \delta$$
$$3x1+ 2x2 \leq 12$$
$$2\ x1+ 2x2 \leq 10+ 2\delta$$

$$x1=2-2\delta$$
$$x2=5+ \delta -x1$$
$$=5+ \delta -2+2\delta$$
$$=3+3\delta$$

So $Z= 6x1+ 5x2=6(2-2\delta)+5(3+3\delta)$

$$=27 +3\delta$$

So this 3 is same as y1=3,

So what we are telling is rate of increase is by value of the dual variable.
So the value of the dual variable at the optimum is the rate of change of the objective function for a small change in the value of the resources.
It can be viewed as the change in the objective function for a unit change of the resource at the optimum (assuming that the change is not significant enough to change the set of basic variables themselves)

ECONOMICAL INTERPRETATION OF THE DUAL

- From previous discussion we know that the objective function increases by 3 for a unit increase in the first resource.
- If we have to by the resource we will be willing to pay a maximum of 3 for the unit increase, Otherwise we will end up making a loss and it will not be profitable considering the purchase of the extra resources
- The value of the dual variable is the marginal value of the corresponding resource at the optimum.

We have defined the primal earlier as the problem as the Carpenter who makes chair and tables. Now the dual is the problem faced by the person who is assuming to be selling the resources to the carpenter.
If the person sells the extra resources for a price less than 3, the carpenter will buy and make more profit (but the seller would not want)
On the other hand if the seller charges more than 3, the carpenter will not buy the resource.
So both the carpenter and seller will agree for 3 and each will make their money and associated profit.

Performing Sensitivity Analysis

Factory Example:

- Can produce 5 types of product with 3 grinding machines, and 2 drilling machines
- Factory works 2 shifts, 8 hours per shift, 6 days / week
- Each product takes 20 hrs of assembly, with 8 workers working 48 hours / week (384 hours/week)

Formulate the LP for this problem in tableau form

	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5
Price	550	600	350	400	200
Grinding Hours	12	20		25	15
Drilling Hours	10	8	16		

Formulation Solution

	Prod 1	**Prod 2**	**Prod 3**	**Prod 4**	**Prod 5**	**RHS**
Obj	550	600	350	400	200	
Grinding	12	20		25	15	≤ 288
Drilling	10	8	16			≤ 192
Labor	20	20	20	20	20	≤ 384

Target Cell (Max)

Cell	**Name**	**Original Value**	**Final Value**
$K$11	Obj Value	10920	10920

Adjustable Cells

Cell	**Name**	**Original Value**	**Final Value**
$D$12	Prod 1	12	12
$E$12	Prod 2	7.2	7.2
$F$12	Prod 3	0	0
$G$12	Prod 4	0	0
$H$12	Prod 5	0	0

Interesting Questions

1. What is the value of an extra hour of grinding, drilling or labor?

2. How much more expensive should products 3, 4 & 5 be in order before we would start producing them?

Answering Q1

	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	
	12	7.2	0	0	0	RHS
Obj	550	600	350	400	200	
Grinding	12	20		25	15	≤288
Drilling	10	8	16			≤ 192
Labor	20	20	20	20	20	≤ 384

Obj Value 10920
10920
0

Change to 289

By Trail and Error

	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	
	11.875	7.325	0	0	0	RHS
Obj	550	600	350	400	200	
Grinding	12	20		25	15	≤ 289
Drilling	10	8	16			≤ 192
Labor	20	20	20	20	20	≤ 384

Obj Value 10926.25
10920
6.25

Value of one additional hour grinding

The Dual Model

Problem Statement:

Value each of the resources in such a way as to give a minimal overall value (cost) to the factory for producing the optimal plan.

Let:

y_1 = value of each hour of grinding
y_2 = value of each hour of drilling
y_3 = value of each hour of labor

The Dual Model

Profit Allocation:

Each unit of Prod 1 produces $550.
How is this profit "allocated" between the three processes?

We know that Prod 1 use 12 hours of grinding, 10 hours of drilling, and 20 hours of assembly labor.

The profit can be allocated among the three processes, but must be greater than $550, as follows:

$$\mathbf{12\ y_1 + 10\ y_2 + 20y_3 \geq \$550}$$

Now formulate the Dual

The Dual Model

Formulation of the Dual:

Minimize $288\ y_1 + 192\ y_2 + 384\ y_3$

Subject to:

$$12\ y_1 + 10\ y_2 + 20\ y_3 \geq \$550$$

$$20\ y_1 + 8\ y_2 + 20\ y_3 \geq \$600$$

$$16\ y_2 + 20\ y_3 \geq \$350$$

$$25\ y_1 + 20\ y_3 \geq \$400$$

$$15\ y_1 + 20\ y_3 \geq \$200$$

Solution to the Dual

	Grinding	Drilling	Labor	RHS
	6.25	0	23.75	
Obj	288	192	384	
Prod 1	12	10	20	≥ 550
Prod 2	20	8	20	≥ 600
Prod 3		16	20	≥ 350
Prod 4	25		20	≥ 400
Prod 5	15		20	≥ 200

Objective 10920

Shadow Prices

Valuing the Resources

	Grinding	Drilling	Labor	RHS	USED
	6.25	0	23.75		
Obj	288	192	384		
Prod 1	12	10	20	≥ 550	550
Prod 2	20	8	20	≥ 600	600
Prod 3		16	20	≥ 350	475
Prod 4	25		20	≥ 400	631.25
Prod 5	15		20	≥ 200	568.75

Objective 10920

1. Cost of producing equal to profit contribution.
2. Cost of producing higher than profit contribution.

Reduced Costs

	Grinding	Drilling	Labor	RHS	USED	Reduced Cost
	6.25	0	23.75			
Obj	288	192	384			
Prod 1	12	10	20	≥ 550	550	0
Prod 2	20	8	20	≥ 600	600	0
Prod 3		16	20	≥ 350	475	-125
Prod 4	25		20	≥ 400	631.25	-231.25
Prod 5	15		20	≥ 200	568.75	-368.75

Objective 10920

Prod 3's price would have to increase by $125 before it would be produced.

Sensitivity Analysis Report

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Primal Problem.xls]Sheet1

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
$D$12	Prod 1	12	0
$E$12	Prod 2	7.2	0
$F$12	Prod 3	0	-125
$G$12	Prod 4	0	-231.25
$H$12	Prod 5	0	-368.75

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$L$14	Grinding	288	6.25
$L$15	Drilling	177.6	0
$L$16	Labor	384	23.75

Interpretation of Shadow Prices

- Productive Capacity Constraints
  - Marginal value of additional capacity
- Raw Material Availability
  - Value of acquiring more raw material
  - Cost of cutting back on raw material

Interpretation of Shadow Prices

- Marketing Demands and Limitations
  - Cost of forced production
  - Value of increased sales
  - Should be set up as a constraint rather than a simple upper or lower bound (or use reduced cost to determine shadow price)
- Material Balance Constraints
  - May have no valid interpretation
  - Have to look at each case separately
- Quality Stipulations
  - Value or cost of the quality stipulation

Sensitivity Analysis & Stability of the Model

Stability of the Model

Sensitivity Analysis results is valid only within a particular range of solutions.

- RHS ranging
  - Within what range is the shadow price valid?
- Objective Function Ranging
  - How much of a change in an objective function coeff. is necessary before a new solution is found
- Interior Coefficient Ranging

This information is provided by commercial LP solution software.

RHS Ranges

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Primal Problem.xls]Sheet1

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
$D$12	Prod 1	12	0
$E$12	Prod 2	7.2	0
$F$12	Prod 3	0	-125
$G$12	Prod 4	0	-231.25
$H$12	Prod 5	0	-368.75

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$L$14	Grinding	288	6.25
$L$15	Drilling	177.6	0
$L$16	Labor	384	23.75

Shadow price is $6.25. Changing the hours by Δ would change profit by 6.25Δ. Within what limits is Δ valid?

Calculating RHS Ranges

Not trivial to calculate
Reported by commercial LP packages

- Grinding
  - Lower range: 230.4
  - Upper range 384
- Drilling
  - Lower range 177.6
  - Upper range
- Assembly Labor
  - Lower range 288
  - Upper range 406.1

Objective Ranges

Microsoft Excel 9.0 Answer Report
Worksheet: [Primal Problem.xls]Sheet1

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$18	Obj Value Prod 1	10920	10920

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$12	Prod 1	12	12
$E$12	Prod 2	7.2	7.2
$F$12	Prod 3	0	0
$G$12	Prod 4	0	0
$H$12	Prod 5	0	0

Over what price range is this solution

stable?

Calculating Objective Ranges

For product 3, 4 & 5

- Lower Range
  - Not produced now
  - Lower price won't change solution
  - Lower Range: - ∞
- Upper Range
  - Reduced Cost Gives Answer
  - Upper Range: Current Price – Reduce Cost

Example

Cell	Name	Final Value	Reduced Gradient	Original Price	Upper Range
$D$12	Prod 1	12	0		
$E$12	Prod 2	7.2	0		
$F$12	Prod 3	0	-125	350	475.00
$G$12	Prod 4	0	-231.25	400	631.25
$H$12	Prod 5	0	-368.75	200	568.75

Calculating Objective Ranges

For product 1 & 2

- PROD 1
  - Lower range: $500
  - Upper range $600
- PROD 2
  - Lower range: $550
  - Upper range $683.3

Example II

Red Toasters needs to produce 1000 of their new "Talking Toaster". There are three ways this toaster can be produced: manually, semi-automatically, and robotically. Manual assembly requires 1 minute of skilled labor, 40 minutes of unskilled labor, and 3 minutes of assembly room time. The corresponding values for semiautomatic assembly are 4, 30, and 2; while those for robotic assembly are 8, 20, and 4. There are 4500 minutes of skilled labor, 36,000 minutes of unskilled labor, and 2700 minutes of assembly room time available for this product. The total cost for producing manually is $7/toaster; semi automatically is $8/toaster; and robotically is $8.50/toaster.

(a) Formulate the problem of producing 1000 toasters at minimum cost meeting the resource requirements. Clearly define your variables, objective and constraints.

(b) Our union contract states that the amount of skilled labor time used is at least 10% of the total labor (unskilled plus skilled) time used. Update your formulation in (a) to handle this requirement.

(c) Any unused assembly floor time can be rented out at a profit of $0.50/minute. Update your formulation to include this possibility.

Solution to Primal Problem

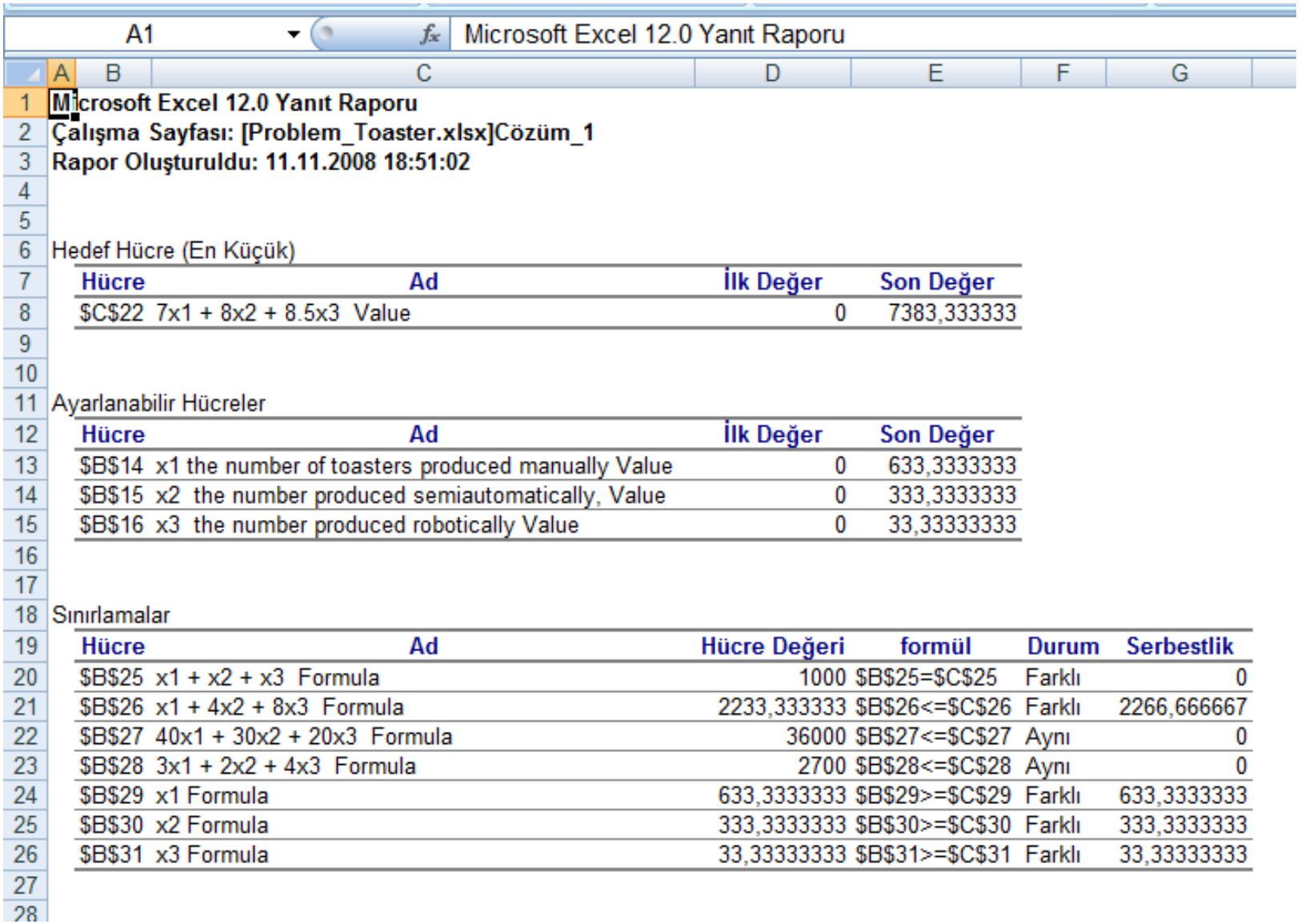
- The objective is to Minimize $7x1 + 8x2 + 8.5x3$
  Where
  x1 = the number of toasters produced manually,
  x2 =the number produced semiautomatically,
  x3= the number produced robotically
- Subject to the following constraints
  $x1 + x2 + x3 = 1000$ (produce enough toasters)
  $x1 + 4x2 + 8x3 \leq 4500$ (skilled labor used less than or equal to amount available)
  $40x1 + 30x2 + 20x3 \leq 36000$ (unskilled labor constraint)
  $3x1 + 2x2 + 4x3 \leq 2700$ (assembly time constraint)
  $x1, x2$ and $x3 \geq 0$ (positivity)

Example 2: EXCEL Solution

Dış Veri Al | Bağlantılar | Sırala ve Filtre Uygula

C22 | =+C4*B14+C5*B15+C6*B16

	A	B	C	D	E	F	G
2							
3			cost	number	skilled labor	unskilled labor	assembly
4	manual		7	1	1	40	3
5	semiautomatic		8	1	4	30	2
6	robot		8,5	1	8	20	4
7							
8							
9							
10							
11							
12	Formulation:						
13	**Variables:**	Value					
14	x1 Number of toaster produced manually	633,3333					
15	x2 Number of toaster produced semiauto	333,3333					
16	x3 Number of toaster produced robotic	33,33333					
17							
18							
19							
20							
21	**Objective:**		Value				
22	7x1 + 8x2 + 8,5x3		7383,333				
23							
24	Constrain	Formula	Value				
25	x1 + x2 + x3 produce toaster	1000	1000				
26	x1 + 4x2 + 8x3 skilled labor	2233,333	4500				
27	40x1 + 30x2 + 20x3 unskilled labor	36000	36000				
28	3x1 + 2x2 + 4x3 assembly time	2700	2700				
29	x1	633,3333	0				
30	x2	333,3333	0				
31	X3	33,33333	0				
32							



A1 | Microsoft Excel 12.0 Yanıt Raporu

Microsoft Excel 12.0 Yanıt Raporu
Çalışma Sayfası: [Problem_Toaster.xlsx]Çözüm_1
Rapor Oluşturuldu: 11.11.2008 18:51:02

Hedef Hücre (En Küçük)

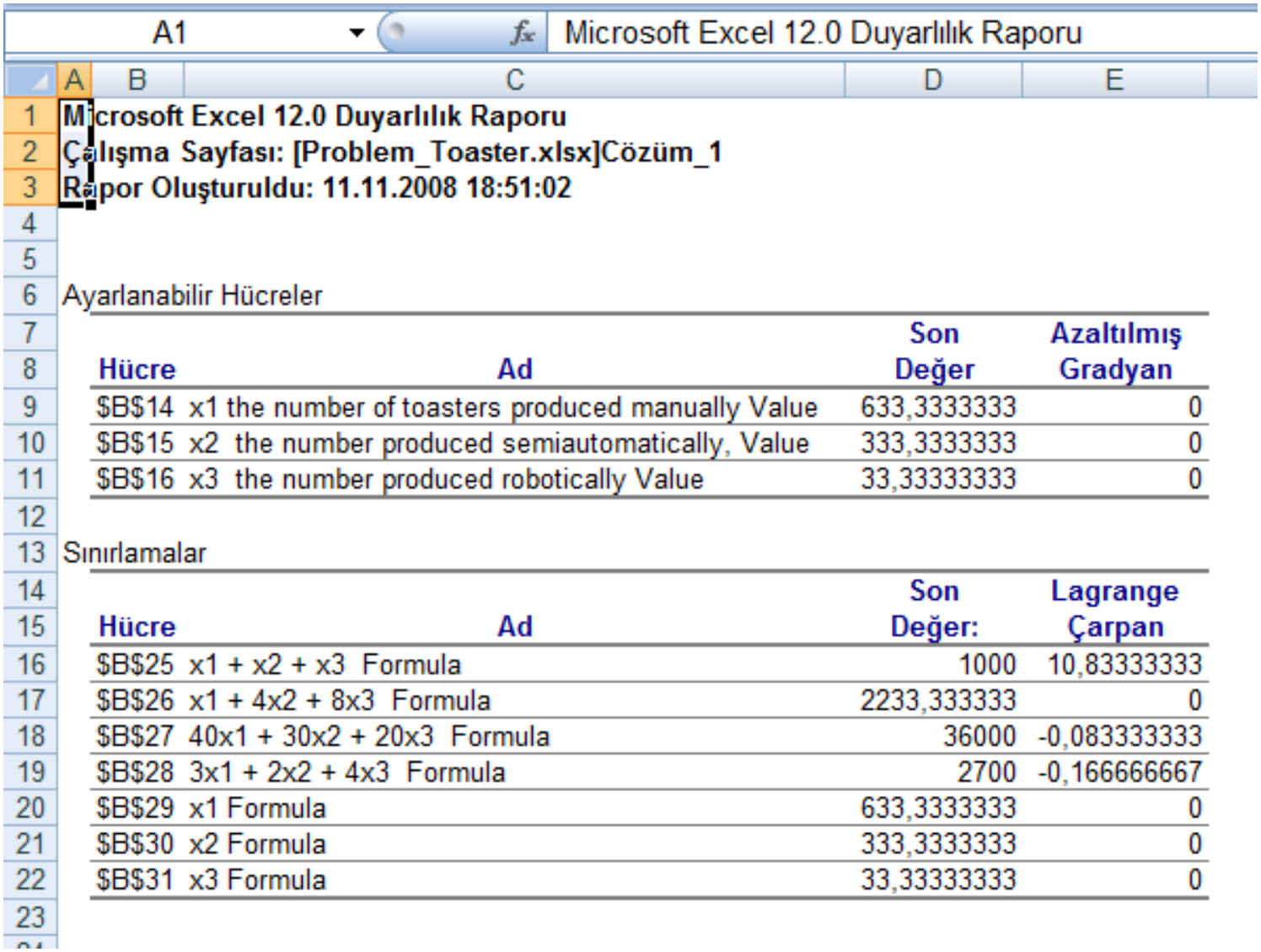
Hücre	Ad	İlk Değer	Son Değer
$C$22	7x1 + 8x2 + 8.5x3 Value	0	7383,333333

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
$B$14	x1 the number of toasters produced manually Value	0	633,3333333
$B$15	x2 the number produced semiautomatically, Value	0	333,3333333
$B$16	x3 the number produced robotically Value	0	33,33333333

Sınırlamalar

Hücre	Ad	Hücre Değeri	formül	Durum	Serbestlik
$B$25	x1 + x2 + x3 Formula	1000	$B$25=$C$25	Farklı	0
$B$26	x1 + 4x2 + 8x3 Formula	2233,333333	$B$26<=$C$26	Farklı	2266,666667
$B$27	40x1 + 30x2 + 20x3 Formula	36000	$B$27<=$C$27	Aynı	0
$B$28	3x1 + 2x2 + 4x3 Formula	2700	$B$28<=$C$28	Aynı	0
$B$29	x1 Formula	633,3333333	$B$29>=$C$29	Farklı	633,3333333
$B$30	x2 Formula	333,3333333	$B$30>=$C$30	Farklı	333,3333333
$B$31	x3 Formula	33,33333333	$B$31>=$C$31	Farklı	33,33333333



A1 | Microsoft Excel 12.0 Duyarlılık Raporu

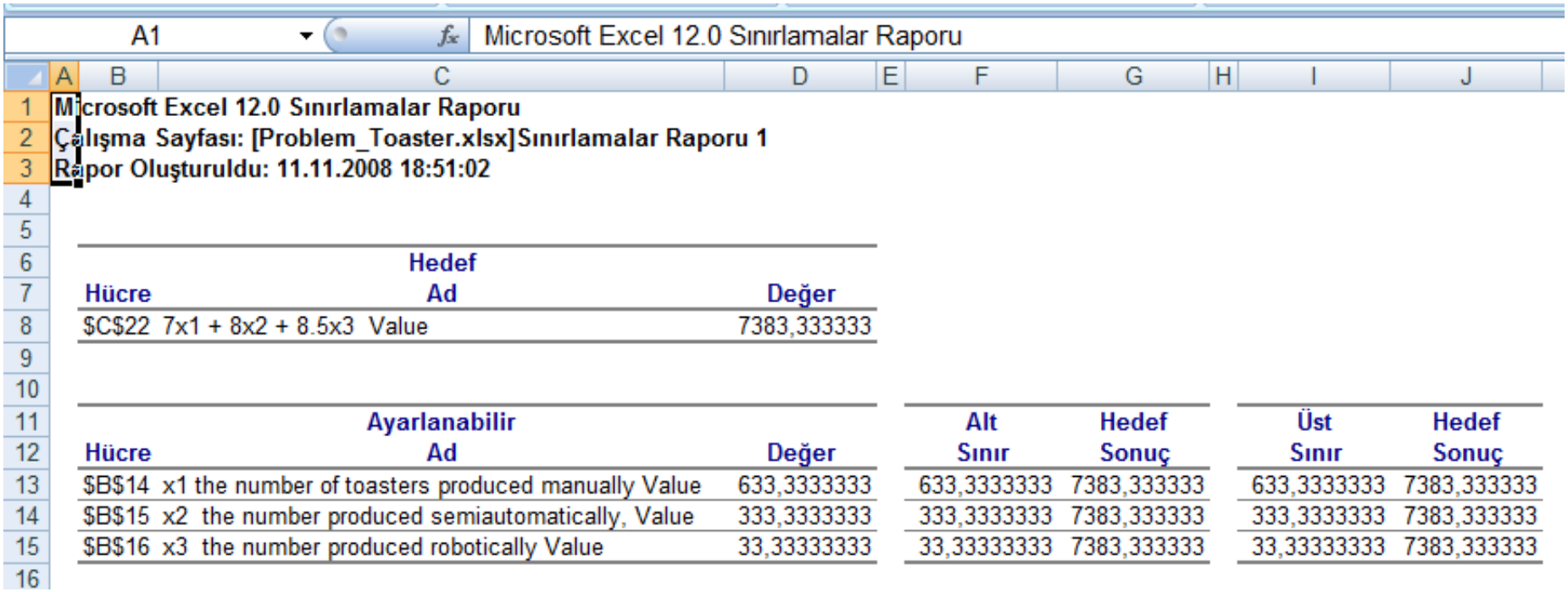
Microsoft Excel 12.0 Duyarlılık Raporu
Çalışma Sayfası: [Problem_Toaster.xlsx]Çözüm_1
Rapor Oluşturuldu: 11.11.2008 18:51:02

Ayarlanabilir Hücreler

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$14	x1 the number of toasters produced manually Value	633,3333333	0
$B$15	x2 the number produced semiautomatically, Value	333,3333333	0
$B$16	x3 the number produced robotically Value	33,33333333	0

Sınırlamalar

Hücre	Ad	Son Değer:	Lagrange Çarpan
$B$25	x1 + x2 + x3 Formula	1000	10,83333333
$B$26	x1 + 4x2 + 8x3 Formula	2233,333333	0
$B$27	40x1 + 30x2 + 20x3 Formula	36000	-0,083333333
$B$28	3x1 + 2x2 + 4x3 Formula	2700	-0,166666667
$B$29	x1 Formula	633,3333333	0
$B$30	x2 Formula	333,3333333	0
$B$31	x3 Formula	33,33333333	0



A1 — Microsoft Excel 12.0 Sınırlamalar Raporu

Microsoft Excel 12.0 Sınırlamalar Raporu
Çalışma Sayfası: [Problem_Toaster.xlsx]Sınırlamalar Raporu 1
Rapor Oluşturuldu: 11.11.2008 18:51:02

Hedef

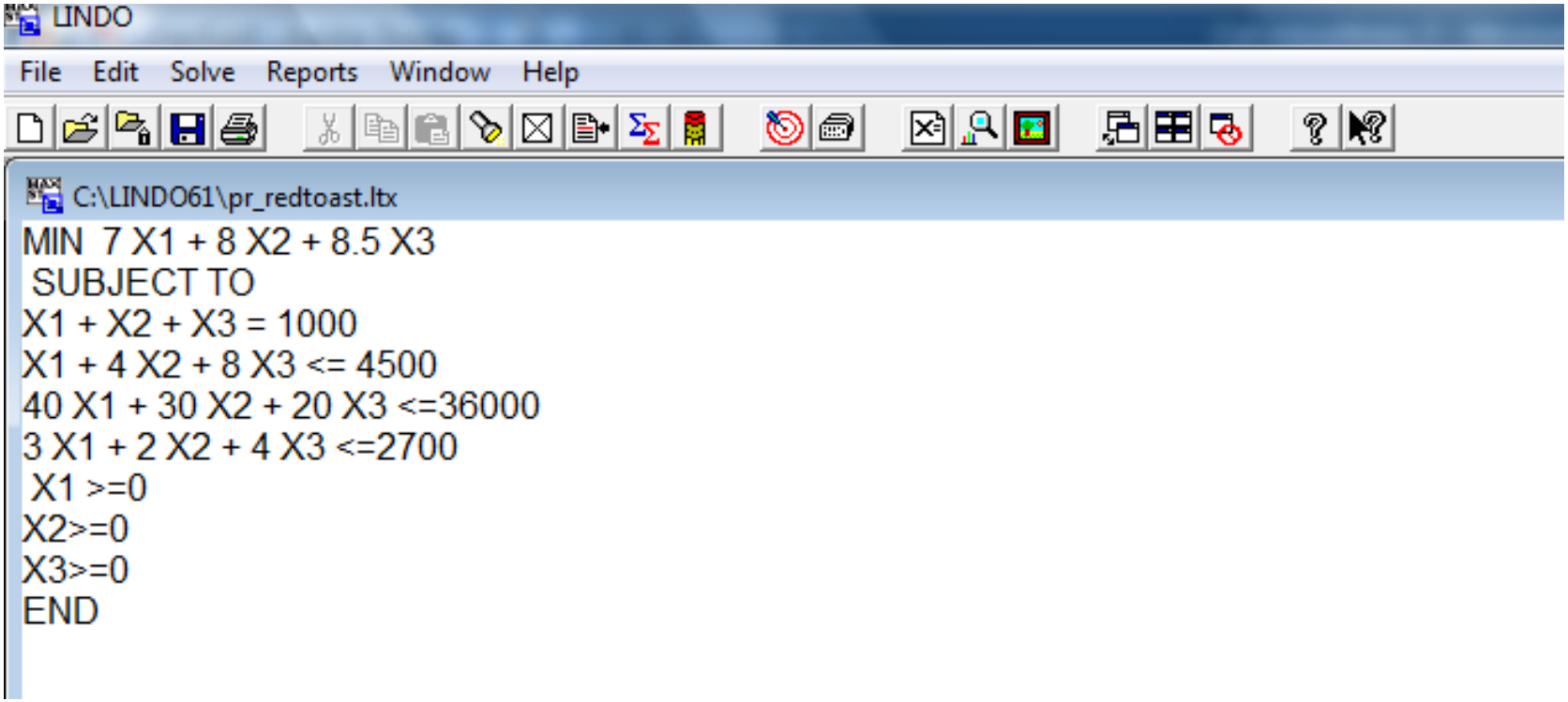
Hücre	Ad	Değer
$C$22	7x1 + 8x2 + 8.5x3 Value	7383,333333

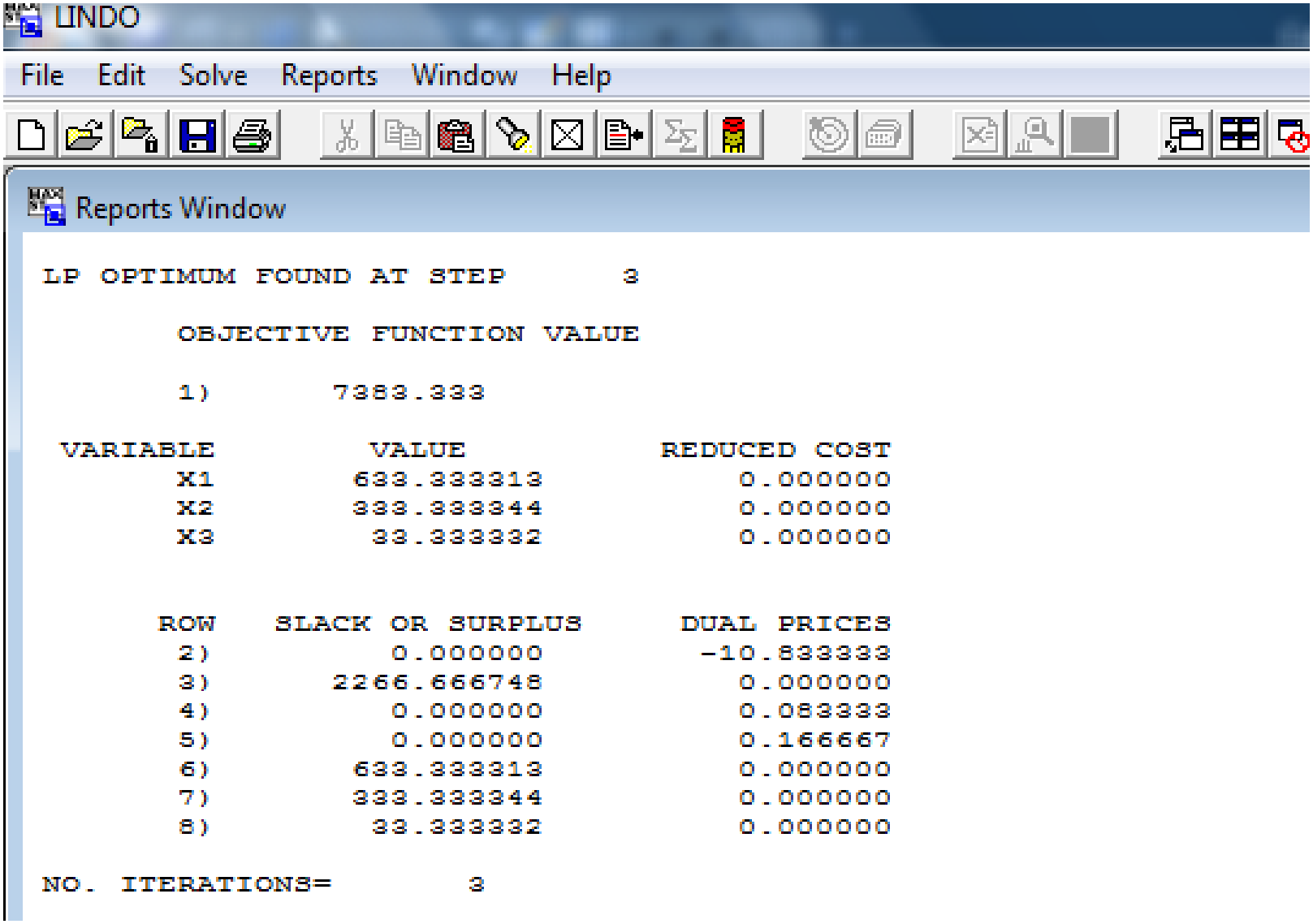
Ayarlanabilir

Hücre	Ad	Değer	Alt Sınır	Hedef Sonuç	Üst Sınır	Hedef Sonuç
$B$14	x1 the number of toasters produced manually Value	633,3333333	633,3333333	7383,333333	633,3333333	7383,333333
$B$15	x2 the number produced semiautomatically, Value	333,3333333	333,3333333	7383,333333	333,3333333	7383,333333
$B$16	x3 the number produced robotically Value	33,33333333	33,33333333	7383,333333	33,33333333	7383,333333

LINDO Program Model

Example 2: LINDO Solution




LINDO
File Edit Solve Reports Window Help
Reports Window
LP OPTIMUM FOUND AT STEP 3
OBJECTIVE FUNCTION VALUE
1) 7383.333
VARIABLE VALUE REDUCED COST
X1 633.333313 0.000000
X2 333.333344 0.000000
X3 33.333332 0.000000
ROW SLACK OR SURPLUS DUAL PRICES
2) 0.000000 -10.833333
3) 2266.666748 0.000000
4) 0.000000 0.083333
5) 0.000000 0.166667
6) 633.333313 0.000000
7) 333.333344 0.000000
8) 33.333332 0.000000
NO. ITERATIONS= 3

```
NO. ITERATIONS=       3


RANGES IN WHICH THE BASIS IS UNCHANGED:

                              OBJ COEFFICIENT RANGES
VARIABLE           CURRENT         ALLOWABLE         ALLOWABLE
                    COEF           INCREASE          DECREASE
       X1        7.000000         0.500000          INFINITY
       X2        8.000000         INFINITY          0.250000
       X3        8.500000         0.500000          2.500000

                              RIGHTHAND SIDE RANGES
      ROW           CURRENT         ALLOWABLE         ALLOWABLE
                      RHS           INCREASE          DECREASE
        2     1000.000000       170.000015         99.999992
        3     4500.000000         INFINITY       2266.666748
        4    36000.000000       999.999939       6800.000000
        5     2700.000000       500.000000         99.999992
        6        0.000000       633.333313          INFINITY
        7        0.000000       333.333344          INFINITY
        8        0.000000        33.333332          INFINITY
```

(b) Our union contract states that the amount of skilled labor time used is at least 10% of the total labor (unskilled plus skilled) time used. Update your formulation in (a) to handle this requirement.

$x1 + x2 + x3 = 1000$ (produce enough toasters)

$x1 + 4x2 + 8x3 \leq 4500$ (skilled labor used less than or equal to amount available)

$40x1 + 30x2 + 20x3 \leq 36000$ (unskilled labor constraint)

$3x1 + 2x2 + 4x3 \leq 2700$ (assembly time constraint)

$x1, x2$ and $x3 \geq 0$ (positivity)

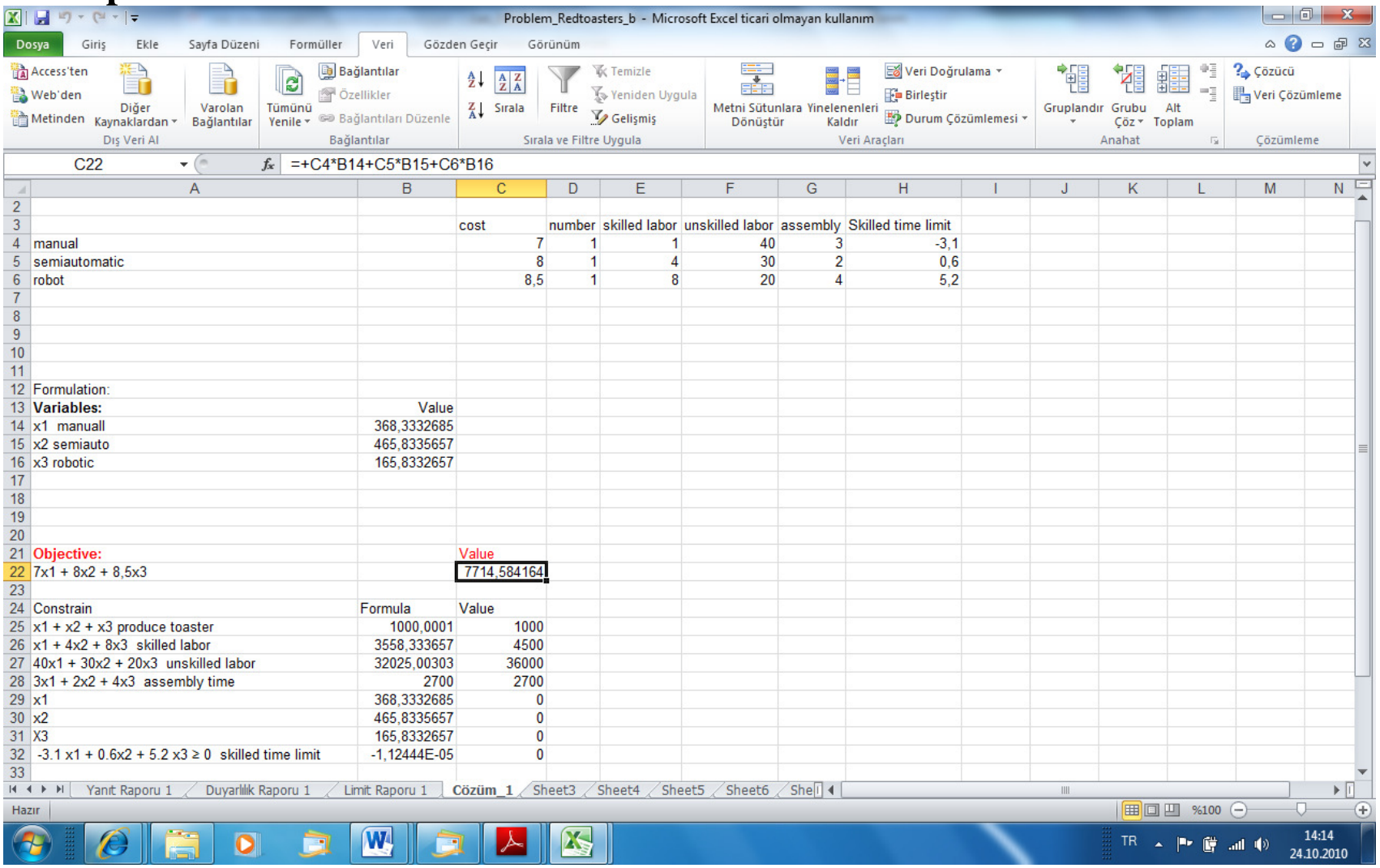
Add a constraint

$x1 + 4x2 + 8x3 \geq 0.1\ (41x1 + 34x2 + 28x3)$, Total: 40+1, 30+4, 20+8

or

$-3.1\ x1 + 0.6x2 + 5.2\ x3 \geq 0$

Example 2: EXCEL Solution



	A	B	C	D	E	F	G	H
3			cost	number	skilled labor	unskilled labor	assembly	Skilled time limit
4	manual		7	1	1	40	3	-3,1
5	semiautomatic		8	1	4	30	2	0,6
6	robot		8,5	1	8	20	4	5,2
12	Formulation:							
13	**Variables:**	Value						
14	x1 manuall	368,3332685						
15	x2 semiauto	465,8335657						
16	x3 robotic	165,8332657						
21	Objective:		Value					
22	7x1 + 8x2 + 8,5x3		7714,584164					
24	Constrain	Formula	Value					
25	x1 + x2 + x3 produce toaster	1000,0001	1000					
26	x1 + 4x2 + 8x3 skilled labor	3558,333657	4500					
27	40x1 + 30x2 + 20x3 unskilled labor	32025,00303	36000					
28	3x1 + 2x2 + 4x3 assembly time	2700	2700					
29	x1	368,3332685	0					
30	x2	465,8335657	0					
31	X3	165,8332657	0					
32	-3.1 x1 + 0.6x2 + 5.2 x3 ≥ 0 skilled time limit	-1,12444E-05	0					

Microsoft Excel 14.0 Yanıt Raporu
Çalışma Sayfası: [Problem_Redtoasters_b.xlsx]Çözüm_1
Rapor Oluşturuldu: 24.10.2010 14:01:05
Sonuç: Çözücü bir çözüm buldu. Tüm Kısıtlamalar ve uygunluk koşulları karşılandı.
Çözücü Altyapısı
Altyapı: Doğrusal Olmayan GRG
Çözüm Süresi: 0,047 Saniye.
Yinelemeler: 3 Alt problemler: 0
Çözücü Seçenekleri
Zaman Sınırı 100 saniye, Yinelemeler 1000, Precision 0,0001
Yakınsama 0,0001, Popülasyon Boyutu 100, Rastgele Kök 0, İleri Türevleri, Sınır Gerektir
En Çok Alt Problem Limitsiz, En Çok Tamsayı Çözümü Limitsiz, Tamsayı Toleransı 5%, Tamsayı Kısıtlamaları Olmadan Çöz

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
$C$22	7x1 + 8x2 + 8,5x3 Value	0	7714,584164

Değişken Hücreleri

Hücre	Ad	İlk Değer	Son Değer	Tamsayı
$B$14	x1 manuall Value	0	368,3332685	Sürekli
$B$15	x2 semiauto Value	0	465,8335657	Sürekli
$B$16	x3 robotic Value	0	165,8332657	Sürekli

Kısıtlamalar

Hücre	Ad	Hücre Değeri	Formül	Durum	Serbestlik
$B$31	X3 Formula	165,833266	$B$31>=$C$3	Farklı	165,83327
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	3558,33366	$B$26<=$C$2	Farklı	941,66634
$B$32	-3.1 x1 + 0.6x2 + 5.2 x3 ≥ 0 skilled time limit Formu	-1,1244E-05	$B$32>=$C$3	Aynı	0
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	32025,003	$B$27<=$C$2	Farklı	3974,997
$B$28	3x1 + 2x2 + 4x3 assembly time Formula	2700	$B$28<=$C$2	Aynı	0
$B$29	x1 Formula	368,333269	$B$29>=$C$2	Farklı	368,33327
$B$25	x1 + x2 + x3 produce toaster Formula	1000,0001	$B$25=$C$25	Aynı	0
$B$30	x2 Formula	465,833566	$B$30>=$C$3	Farklı	465,83357

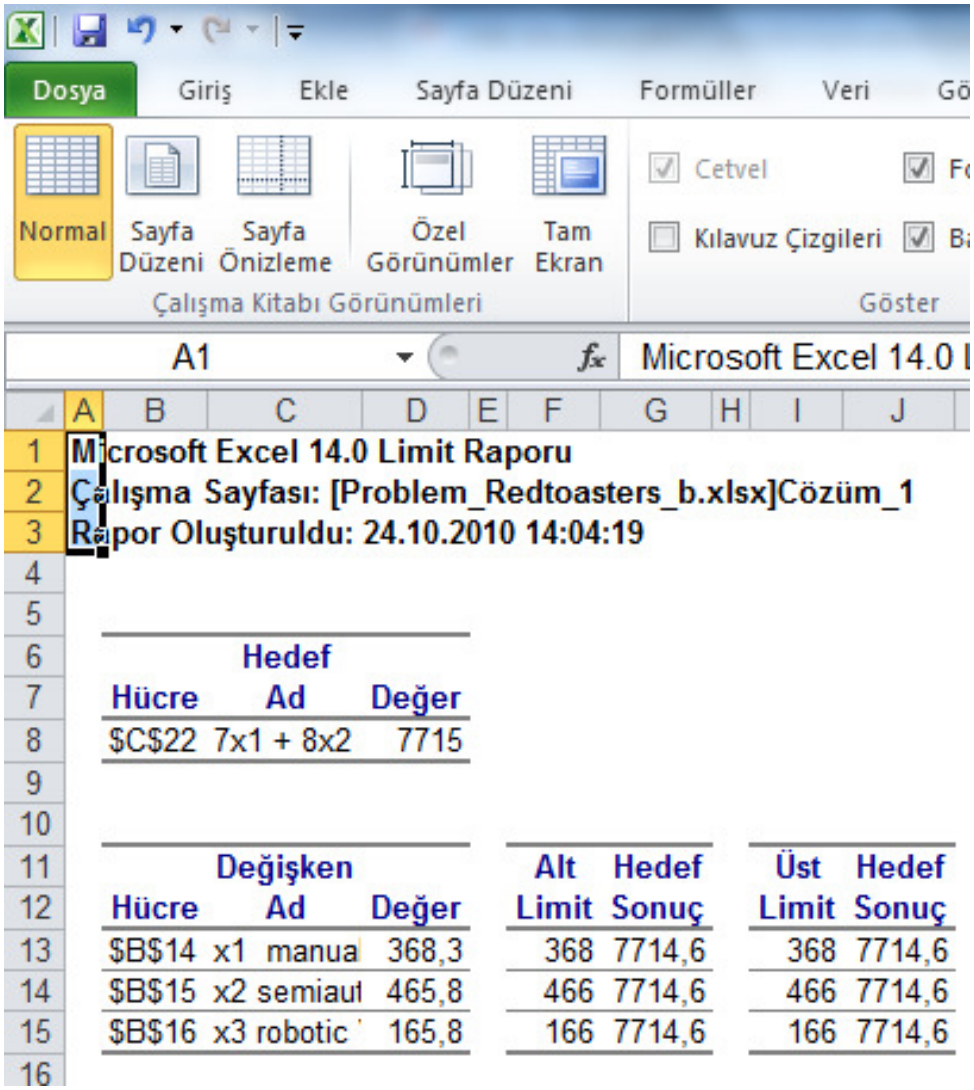
Microsoft Excel 14.0 Duyarlılık Raporu
Çalışma Sayfası: [Problem_Redtoasters_b.xlsx]Cözüm_1
Rapor Oluşturuldu: 24.10.2010 14:02:24

Değişken Hücreleri

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$14	x1 manuall Value	368,3332685	0
$B$15	x2 semiauto Value	465,8335657	0
$B$16	x3 robotic Value	165,8332657	0

Kısıtlamalar

Hücre	Ad	Son Değer	Lagrange Çarpan
$B$25	x1 + x2 + x3 produce toaster Formula	1000,0001	8,333333308
$B$31	X3 Formula	165,8332657	0
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	32025,00303	0
$B$28	3x1 + 2x2 + 4x3 assembly time Formula	2700	-0,229166659
$B$29	x1 Formula	368,3332685	0
$B$30	x2 Formula	465,8335657	0
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	3558,333657	0
$B$32	-3.1 x1 + 0.6x2 + 5.2 x3 ≥ 0 skilled time limit Formula	-1,12444E-05	0,20833334


Dosya
Giriş
Ekle
Sayfa Düzeni
Formüller
Veri
Normal
Sayfa Düzeni
Sayfa Önizleme
Özel Görünümler
Tam Ekran
Çalışma Kitabı Görünümleri
Cetvel
Kılavuz Çizgileri
Göster
A1
Microsoft Excel 14.0 Limit Raporu
Çalışma Sayfası: [Problem_Redtoasters_b.xlsx]Cözüm_1
Rapor Oluşturuldu: 24.10.2010 14:04:19
Hedef
Hücre Ad Değer
$C$22 7x1 + 8x2 7715
Değişken
Hücre Ad Değer
Alt Limit
Hedef Sonuç
Üst Limit
Hedef Sonuç
$B$14 x1 manua 368,3 368 7714,6 368 7714,6
$B$15 x2 semiaut 465,8 466 7714,6 466 7714,6
$B$16 x3 robotic 165,8 166 7714,6 166 7714,6

(c) Any unused assembly floor time can be rented out at a profit of $0.50/minute. Update your formulation to include this possibility.

$x1 + x2 + x3 = 1000$ (produce enough toasters)
$x1 + 4x2 + 8x3 \leq 4500$ (skilled labor used less than or equal to amount available)
$40x1 + 30x2 + 20x3 \leq 36000$ (unskilled labor constraint)
$3x1 + 2x2 + 4x3 \leq 2700$ (assembly time constraint)
$x1, x2$ and $x3 \geq 0$ (positivity)
Add a variable Sa to represent the assembly time slack.
Add +0,5 sa to the objective.
Change the assembly time constraint to
$3x1 + 2x2 + 4x3 + sa = 2700$ (assembly time constraint)
$Sa \geq 0$

Microsoft Excel 14.0 Yanıt Raporu
Çalışma Sayfası: [Problem_Redtoasters_c.xlsx]Çözüm_1
Rapor Oluşturuldu: 24.10.2010 16:28:52
Sonuç: Çözücü bir çözüm buldu. Tüm Kısıtlamalar ve uygunluk koşulları karşılandı.
Çözücü Altyapısı
Altyapı: Doğrusal Olmayan GRG
Çözüm Süresi: 0,078 Saniye.
Yinelemeler: 5 Alt problemler: 0
Çözücü Seçenekleri
Zaman Sınırı 100 saniye, Yinelemeler 100, Precision 0,000001
Yakınsama 0,0001, Popülasyon Boyutu 100, Rastgele Kök 0, İleri Türevleri, Sınır Gerektir
En Çok Alt Problem Limitsiz, En Çok Tamsayı Çözümü Limitsiz, Tamsayı Toleransı 5%, Tamsayı Kısıtlamaları Olmadan Çöz

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
$C$22	7x1 + 8x2 + 8,5x3+0,5 sa Value	0	7383,333333

Değişken Hücreleri

Hücre	Ad	İlk Değer	Son Değer	Tamsayı
$B$14	x1 manuall Value	0	633,3333333	Sürekli
$B$15	x2 semiauto Value	0	333,3333333	Sürekli
$B$16	x3 robotic Value	0	33,33333333	Sürekli
$B$17	sa assemly time slack Value	0	0	Sürekli

Kısıtlamalar

Hücre	Ad	Hücre Değeri	Formül	Durum	Serbestlik
$B$25	x1 + x2 + x3 produce toaster Formula	1000	$B$25=$C$25	Aynı	0
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	2233,33333	$B$26<=$C$2	Farklı	2266,6667
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	36000	$B$27<=$C$2	Aynı	0
$B$28	3x1 + 2x2 + 4x3+ sa assembly time Formula	2700	$B$28=$C$28	Aynı	0
$B$29	x1 Formula	633,333333	$B$29>=$C$2	Farklı	633,33333
$B$30	x2 Formula	333,333333	$B$30>=$C$3	Farklı	333,33333
$B$31	X3 Formula	33,3333333	$B$31>=$C$3	Farklı	33,333333
$B$32	sa Formula	0	$B$32>=$C$3	Aynı	0

Microsoft Excel 14.0 Duyarlılık Raporu
Çalışma Sayfası: [Problem_Redtoasters_c.xlsx]Cözüm_1
Rapor Oluşturuldu: 24.10.2010 16:29:14

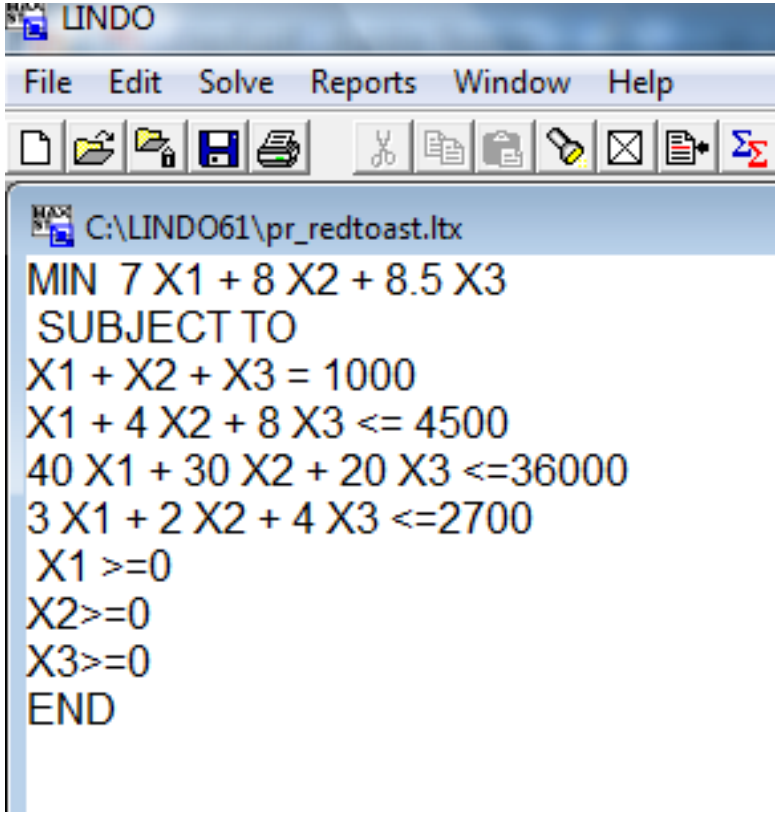
Değişken Hücreleri

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$14	x1 manuall Value	633,3333333	0
$B$15	x2 semiauto Value	333,3333333	0
$B$16	x3 robotic Value	33,33333333	0
$B$17	sa assemly time slack Value	0	0

Kısıtlamalar

Hücre	Ad	Son Değer	Lagrange Çarpan
$B$25	x1 + x2 + x3 produce toaster Formula	1000	10,83333333
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	2233,333333	0
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	36000	-0,083333333
$B$28	3x1 + 2x2 + 4x3+ sa assembly time Formula	2700	-0,166666667
$B$29	x1 Formula	633,3333333	0
$B$30	x2 Formula	333,3333333	0
$B$31	X3 Formula	33,33333333	0
$B$32	sa Formula	0	0,666666826

MORE SENSITIVITY ANALYSIS FOR LINEAR PROGRAMMING PROBLEM



(a) What is the optimal allocation of production? What is the average cost/toaster of production?
(b) How much is Red Toasters willing to pay for more assembly room time?
(c) How much will we save if we decide to produce only 950 toasters?
(d) A new production process is available that uses only 2 minutes of skilled labor, 10 minutes of unskilled labor, and an undetermined amount of assembly floor time. Its production cost is determined to be $10. What is the maximum assembly floor time that the process can take before it is deemed too expensive to use?

Solution

Microsoft Excel 12.0 Yanıt Raporu
Çalışma Sayfası: [Problem_toasters.xlsx]Çözüm_1
Rapor Oluşturuldu: 11.11.2008 20:58:58

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
$C$22	7x1 + 8x2 + 8,5x3 Value	0	7383,333333

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
$B$14	x1 manuall Value	0	633,3333333
$B$15	x2 semiauto Value	0	333,3333333
$B$16	x3 robotic Value	0	33,33333333

Sınırlamalar

Hücre	Ad	Hücre Değeri	formül	Durum	Serbestlik
$B$30	x2 Formula	333,3333333	$B$30>=$C$30	Farklı	333,3333333
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	2233,333333	$B$26<=$C$26	Farklı	2266,666667
$B$31	X3 Formula	33,33333333	$B$31>=$C$31	Farklı	33,33333333
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	36000	$B$27<=$C$27	Aynı	0
$B$28	3x1 + 2x2 + 4x3 assembly time Formula	2700	$B$28<=$C$28	Aynı	0
$B$29	x1 Formula	633,3333333	$B$29>=$C$29	Farklı	633,3333333
$B$25	x1 + x2 + x3 produce toaster Formula	1000	$B$25=$C$25	Farklı	0

Microsoft Excel 12.0 Duyarlılık Raporu
Çalışma Sayfası: [Problem_toasters.xlsx]Cözüm_1
Rapor Oluşturuldu: 11.11.2008 20:58:58

Ayarlanabilir Hücreler

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$14	x1 manuall Value	633,3333333	0
$B$15	x2 semiauto Value	333,3333333	0
$B$16	x3 robotic Value	33,33333333	0

Sınırlamalar

Hücre	Ad	Son Değer:	Lagrange Çarpan
$B$30	x2 Formula	333,3333333	0
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	2233,333333	0
$B$31	X3 Formula	33,33333333	0
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	36000	-0,083333333
$B$28	3x1 + 2x2 + 4x3 assembly time Formula	2700	-0,166666667
$B$29	x1 Formula	633,3333333	0
$B$25	x1 + x2 + x3 produce toaster Formula	1000	10,83333333

(a) What is the optimal allocation of production? What is the average cost/toaster of production?

Answer: 633.3 should be produced manually, 333.3 should be produced semiautomatically, and 33.3 produced robotically, for an average cost of (7383/1000) $7.383/toaster.

(b) How much is Red Toasters willing to pay for more assembly room time?
Answer: Value is $0.16/minute

Ayarlanabilir Hücreler

Hücre	Ad	Son Değer	Azaltılmış Gradyan
$B$14	x1 manuall Value	633,3333333	0
$B$15	x2 semiauto Value	333,3333333	0
$B$16	x3 robotic Value	33,33333333	0

Sınırlamalar

Hücre	Ad	Son Değer:	Lagrange Çarpan
$B$30	x2 Formula	333,3333333	0
$B$26	x1 + 4x2 + 8x3 skilled labor Formula	2233,333333	0
$B$31	X3 Formula	33,33333333	0
$B$27	40x1 + 30x2 + 20x3 unskilled labor Formula	36000	-0,083333333
$B$28	3x1 + 2x2 + 4x3 assembly time Formula	2700	-0,166666667
$B$29	x1 Formula	633,3333333	0
$B$25	x1 + x2 + x3 produce toaster Formula	1000	10,83333333

(c) How much will we save if we decide to produce only 950 toasters?

Answer: Objective will go down by 50(x 10.833).

(d) A new production process is available that uses only 2 minutes of skilled labor, 10 minutes of unskilled labor, and an undetermined amount of assembly floor time. Its production cost is determined to be $10. What is the maximum assembly floor time that the process can take before it is deemed too expensive to use?

Answer: Cost of $10 versus marginal cost of $10.833, leave 0.83. Unskilled labor costs $0.0833/unit.

Therefore, if the new process takes any time at all, it will be deemed too expensive.