Statistical Process Control

Example 5:

With respect to a production process, it is known that the average of the main mass is 10.1 cm and the main mass standard deviation is 0.04 cm. Under the assumption that samples of 5 units are to be selected during the manufacturing inspection, calculate the upper and lower values of the middle line of the X and R control charts using these data.

Example 5:

Control standards are known. In this case;

Xbar Chart Formulas:

$$LCL = \mu - A\sigma$$

$$CL = \mu$$

$$UCL = \mu + A\sigma$$

$$A = 3/\sqrt{n}$$

R Chart Formulas:

$$LCL = D_1\sigma$$

$$CL = d_2\sigma$$

$$ULC = D_2\sigma$$

$$D_1 = d_2 - 3d_3$$

$$D_2 = d_2 + 3d_3$$

Example 6:

The pull strength of a wire bonded lead for an integrated circuit is monitored. The table on the right provides data for 20 samples each of size three.

a. Use all the data to determine trial control limits. Construct the control limits and plot the data.

b. If necessary, revise your control limits assuming that any samples that plot outside of the control limits can be eliminated.

Sample	X1	X2	X3
1	15,40	15,60	15,30
2	15,40	17,10	15,20
3	16,10	16,10	13,50
4	13,50	12,50	10,20
5	18,30	16,10	17,00
6	19,20	17,20	19,40
7	14,10	12,40	11,70
8	15,60	13,30	13,60
9	13,90	14,90	15,50
10	18,70	21,20	20,10
11	15,30	13,10	13,70
12	16,60	18,00	18,00
13	17,00	15,20	18,10
14	16,30	16,50	17,70
15	8,40	7,70	8,40
16	11,10	13,80	11,90
17	16,50	17,10	18,50
18	18,00	14,10	15,90
19	17,80	17,30	12,00
20	11,50	10,80	11,20

Example 6:

Sample	X1	X2	X3	X	R
1	15,40	15,60	15,30	15,43	0,30
2	15,40	17,10	15,20	15,90	1,90
3	16,10	16,10	13,50	15,23	2,60
4	13,50	12,50	10,20	12,07	3,30
5	18,30	16,10	17,00	17,13	2,20
6	19,20	17,20	19,40	18,60	2,20
7	14,10	12,40	11,70	12,73	2,40
8	15,60	13,30	13,60	14,17	2,30
9	13,90	14,90	15,50	14,77	1,60
10	18,70	21,20	20,10	20,00	2,50
11	15,30	13,10	13,70	14,03	2,20
12	16,60	18,00	18,00	17,53	1,40
13	17,00	15,20	18,10	16,77	2,90
14	16,30	16,50	17,70	16,83	1,40
15	8,40	7,70	8,40	8,17	0,70
16	11,10	13,80	11,90	12,27	2,70
17	16,50	17,10	18,50	17,37	2,00
18	18,00	14,10	15,90	16,00	3,90
19	17,80	17,30	12,00	15,70	5,80
20	11,50	10,80	11,20	11,17	0,70
				15,09	**2,25**

Constants Used in Control Chart Drawing

Subgroup Size	Coefficients for Limits			Factor for center line	Coefficients for Limits			
	A	A1	A2	d2	D1	D2	D3	D4
2	2,121	3,761	1,88	1,128	0	3,686	0	3,267
3	1,732	2,394	1,023	1,693	0	4,358	0	2,575
4	1,5	1,88	0,729	2,059	0	4,698	0	2,282
5	1,342	1,596	0,577	2,326	0	4,918	0	2,115
6	1,225	1,41	0,483	2,534	0	5,078	0	2,004
7	1,134	1,277	0,419	2,704	0,205	5,203	0,076	1,924
8	1,061	1,175	0,373	2,847	0,387	5,307	0,136	1,864
9	1	1,094	0,337	2,97	0,546	5,394	0,184	1,816
10	0,949	1,028	0,308	3,078	0,687	5,469	0,223	1,777
11	0,905	0,973	0,285	3,173	0,812	5,534	0,256	1,744
12	0,866	0,925	0,266	3,258	0,924	5,592	0,284	1,716
13	0,832	0,884	0,249	3,336	1,026	5,646	0,308	1,692
14	0,8026	0,848	0,235	3,407	1,121	5,693	0,329	1,671
15	0,775	0,816	0,223	3,472	1,207	5,737	0,348	1,652

«Defect» and «Defective»

Defective:

- Non-conforming products (rejection)
- Even if there are more than one defects in a day, it does not matter; it is an important acceptance/rejection.
  - Appropriate/ Inappropriate
  - Acceptance / Rejection
  - Good/bad

Defects:

- Number of errors on the product: for example; bubble / scratch on car window
- There may be multiple errors in a product.
- There is usually a time factor. (number of errors per week, etc.)

Control Charts for Attributes

p-chart or np-chart:

- Use p chart or np chart if you can qualify a product or service only as appropriate/inappropriate, satisfactory/not satisfactory, good/defective.

u-chart or c-chart:

- For each selected product (or service), the number of errors/defects observed is determined, and in each subset, u-chart for the number of errors per unit, and c-chart for each subset of errors.

p chart (Defective rate)

- It is a chart to use when we investigate defects of products.
- Determines the average of controlled attributes.
- Evaluation is made with defective product rate.
- The points on the chart will show the fraction of defective products (p_i) for each subgroup.
- It is suitable to use for the cases where the sample volume is different.

- Examples of usage areas;
  - *Party-specific waste or efficiency*
  - *Follow-up of credit rate rejected*

p chart (Defective rate)

- p_i: defective product rate (for *i*th sample)

- For all samples;

$$\bar{p} = \text{(Total defective rate)/(Toplam product number)}$$

$$CL = \bar{p}$$

$$UCL = \bar{p} + 3\sigma_{pi}$$

$$LCL = \bar{p} - 3\sigma_{pi}$$

$$\sigma_{pi} = \sqrt{\bar{p}(1-\bar{p})/n_i}$$

$$p_i = \text{(ith sample defective product rate)}$$

np Chart (Defective Numbers)

- The figure is the same as the p chart.
- It deals with the defective numbers, not the defective proportions.
- It should be preferred when the sample volume is constant (it can also be used when the sample volume is different, but p is preferred).
- If the main mass defect ratio is "p" and the sample size is "n", the expected number of defects expected to appear in the sample will be "np".

- Examples of usage areas;
  - Incorrect sending of invoices to the customer
  - Incorrect TC ID entries

np Chart (Defective Numbers)

$$CL = n\overline{p}$$

$$UCL = n\overline{p} + 3\sigma_p$$

$$LCL = n\overline{p} - 3\sigma_p$$

$$\sigma_p = \sqrt{n\overline{p}(1-\overline{p})}$$

u Chart

- u charts are used when the number of defects found in a unit in the quality control of the produced units is taken as basis.
- It is used if the sample volume is composed of multiple product types and sample size is variable.
- Since there is no preset standard for the number of defects, the standards can not be known here.
- The points on the diagram will show the number of average defects per unit (per unit) in each subgroup "u_i"; different control limits are calculated for each subgroup.

- Examples of usage areas;
  - Following the defects for raw material with different lot sizes
  - Hospital infection cases are monitored on a daily basis (number of hospitalized patients varies)
  - Tracking of car accidents in the logistics company (the number of vehicles coming out of the road changes)

u Chart

$$CL = \overline{u}$$

$$UCL_i = \overline{u} + 3\sigma_{ui}$$

$$LCL_i = \overline{u} - 3\sigma_{ui}$$

$$\overline{u} = \frac{\sum c}{n} \qquad \text{c = Defect number}$$

$$\sigma_{ui} = \sqrt{\overline{u} / n_i}$$

c Chart

- It shows the number of defective products per lot.
- The sample volume is used if it consists of a single product variety.
- It is not the case if the standards are known.
- The sample size must be constant.
- It is similar to p and np charts.

- Examples of usage areas;
  - Following the quality problems in the vehicles coming out of the production line (the number of vehicles does not change much from day to day)

c Chart

$\overline{c} = $(Total defect number)/(Total subgroup number)

$\mathrm{CL} = \overline{\mathrm{c}}$

$\mathrm{UCL} = \overline{\mathrm{c}} + 3\sqrt{\overline{\mathrm{c}}}$

$\mathrm{LCL} = \overline{\mathrm{c}} - 3\sqrt{\overline{\mathrm{c}}}$

SUMMARY

		Chart Type	Explanation
DEFECT	If the defect number per unit is significant	***c chart***	It is used to track the total "defect" of each subgroup. *The group/sample size must be constant for the defect number per group to be observed.*
		u chart	It is used to track the number of ”defects" per product/item in each of the sub-groups. *If the sample size is variable, u should be used instead of c.*
DEFECTIVE	If the defective unit number is important	***np chart***	It is used to track of the total number of ”defective" product/parts of each group of samples taken. The group size must be constant since it measures the number of defective products observed per subgroup.
		p chart	It is used to track the ”defective" product/part (waste ratio) (%) in each of subgroup taken. If the number of samples is variable, then p chart should be used instead of the np chart.

Question 1:

50 accounts per week are sent to an intermediary institution for debt collection. the error rate is monitored every week to see if there is a significant change in the process performance (error definition: accounts that can not be agreed within 2 weeks following notification).

Chart:

Reasons:

Question 2

Sub-group	Defective documents	Examined documents
1	12	300
2	9	300
3	13	300
4	7	300
5	6	300
6	10	300
7	14	300
8	7	300
9	5	300
10	6	300
11	4	250
12	9	250
13	11	250
14	18	250
15	8	250
16	6	250
17	4	400
18	6	400
19	5	400
20	8	400
21	10	400
22	7	400
23	4	200
24	5	200
25	3	200

Question 2

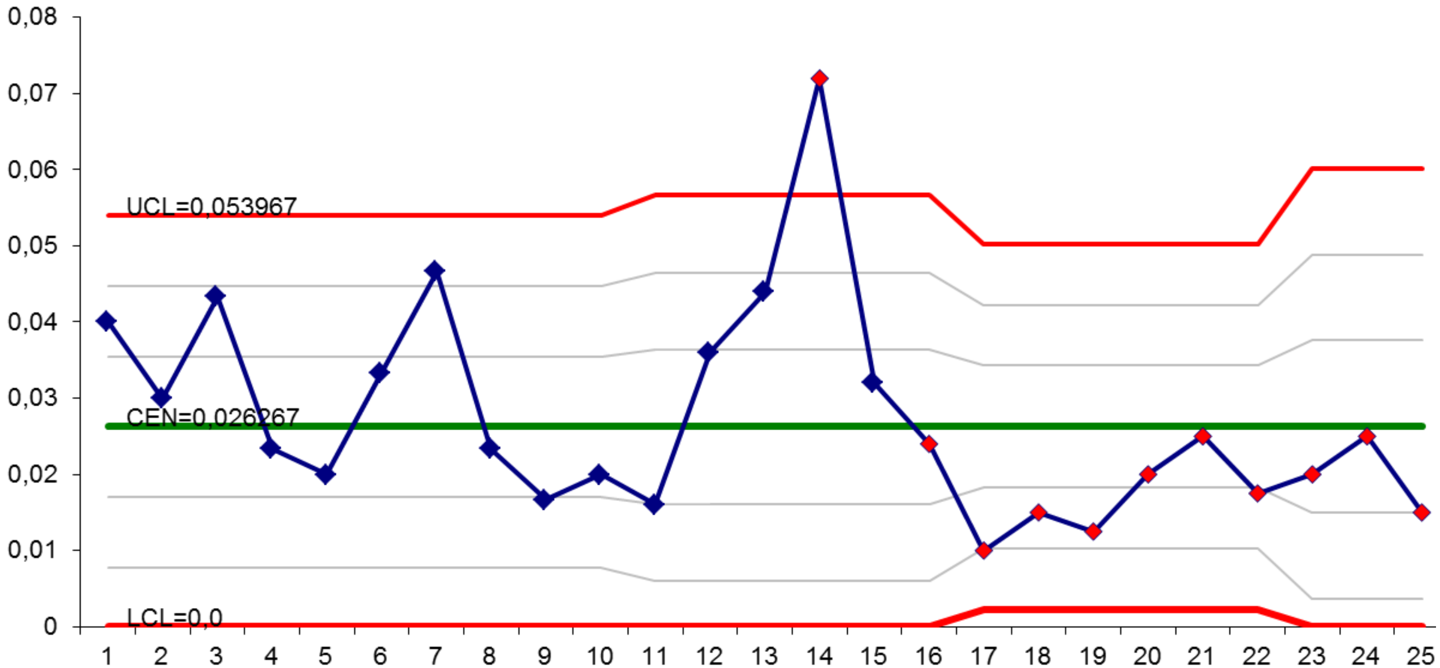
$$CL = \overline{p}$$

$$UCL = \overline{p} + 3\sigma_{pi}$$

$$LCL = \overline{p} - 3\sigma_{pi}$$

$$\sigma_{pi} = \sqrt{\overline{p}(1-\overline{p})/n_i}$$

$p_i =$ (ith sample defective product rate)



Question 3:

Parts manufactured by an injection molding process are subjected to a compressive strength test. Twenty samples of five parts each are collected, and compressive strengths are analyzed.

Chart:

Reasons:

Question 4

A textile dyeing factory paints knitting fabric batches. Recently it has been observed that customers complain that there are a lot of stains on the fabric of blue dyed fabric. In order to investigate the situation, 25 batches fabric painted in blue are examined carefully and the number of stains are determined and the following tabulated data are obtained. Fabrics are different in size and each fabric batches is thought of as a different subgroup.

Question 4

Fabric Batch Number	Fabric Amount (kg)	Stain Number on Batch (c_i)	Fabric Batch Number	Fabric Amount (kg)	Stain Number on Batch (c_i)
1	180	28	14	280	56
2	190	38	15	300	30
3	170	22	16	290	71
4	370	42	17	380	41
5	470	52	18	400	47
6	230	25	19	250	28
7	280	37	20	260	35
8	260	45	21	290	72
9	270	24	22	310	44
10	380	42	23	190	23
11	250	26	24	500	34
12	240	35	25	420	52
13	290	32	**TOPLAM**	**7450**	**981**

Question 4

$$CL = \overline{u}$$

$$UCL_i = \overline{u} + 3\sigma_{ui}$$

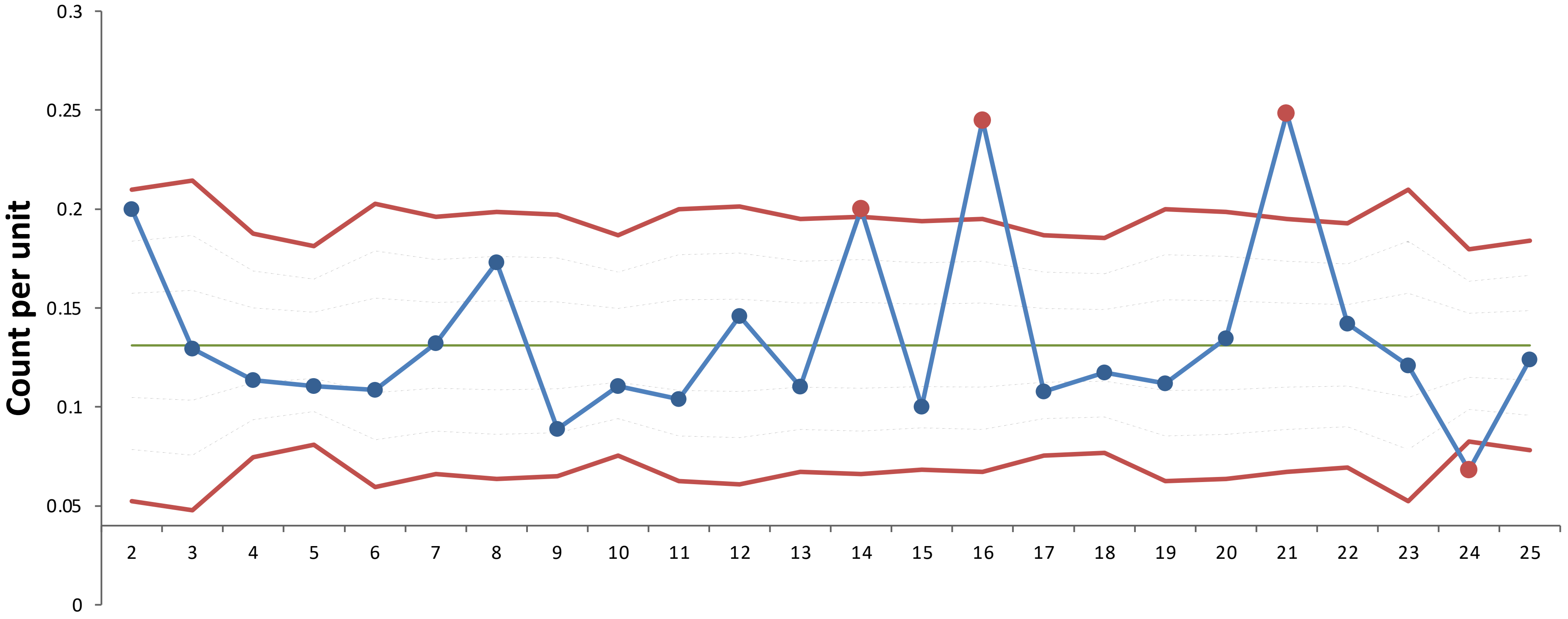
$$LCL_i = \overline{u} - 3\sigma_{ui}$$

$$\overline{u} = \frac{\sum c}{n}$$

c = Defect number

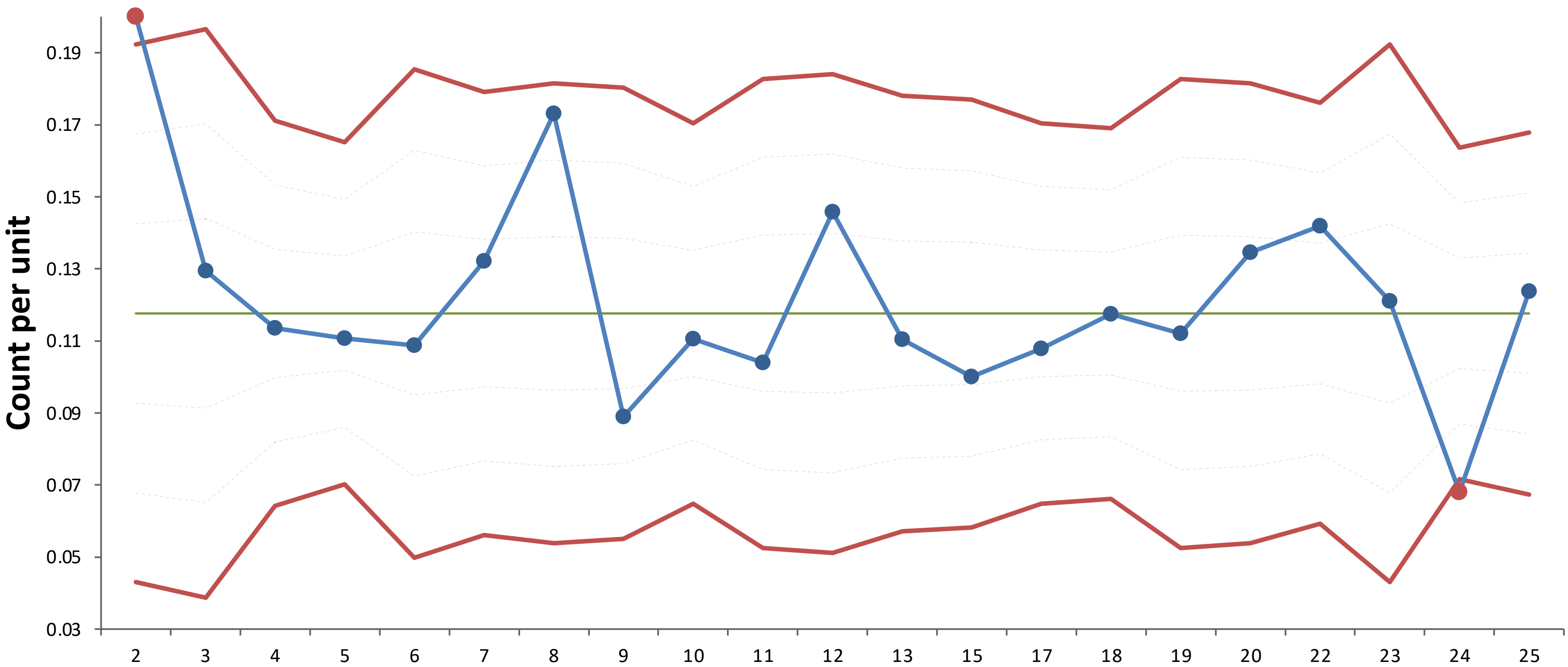
$$\sigma_{ui} = \sqrt{\overline{u} / n_i}$$

Question 4



0.3
0.25
0.2
0.15
0.1
0.05
0
Count per unit
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25

Question 4 – Revised



Question 5

The number of defective units identified in each of 12 sample groups, each consisting of n = 200 pieces, is as follows:

Sample Group	1	2	3	4	5	6	7	8	9	10	11	12
Defective product number	23	15	17	15	41	0	25	31	29	0	8	16

Question 5

$$CL = n\overline{p} = 220/12 = 18.3$$

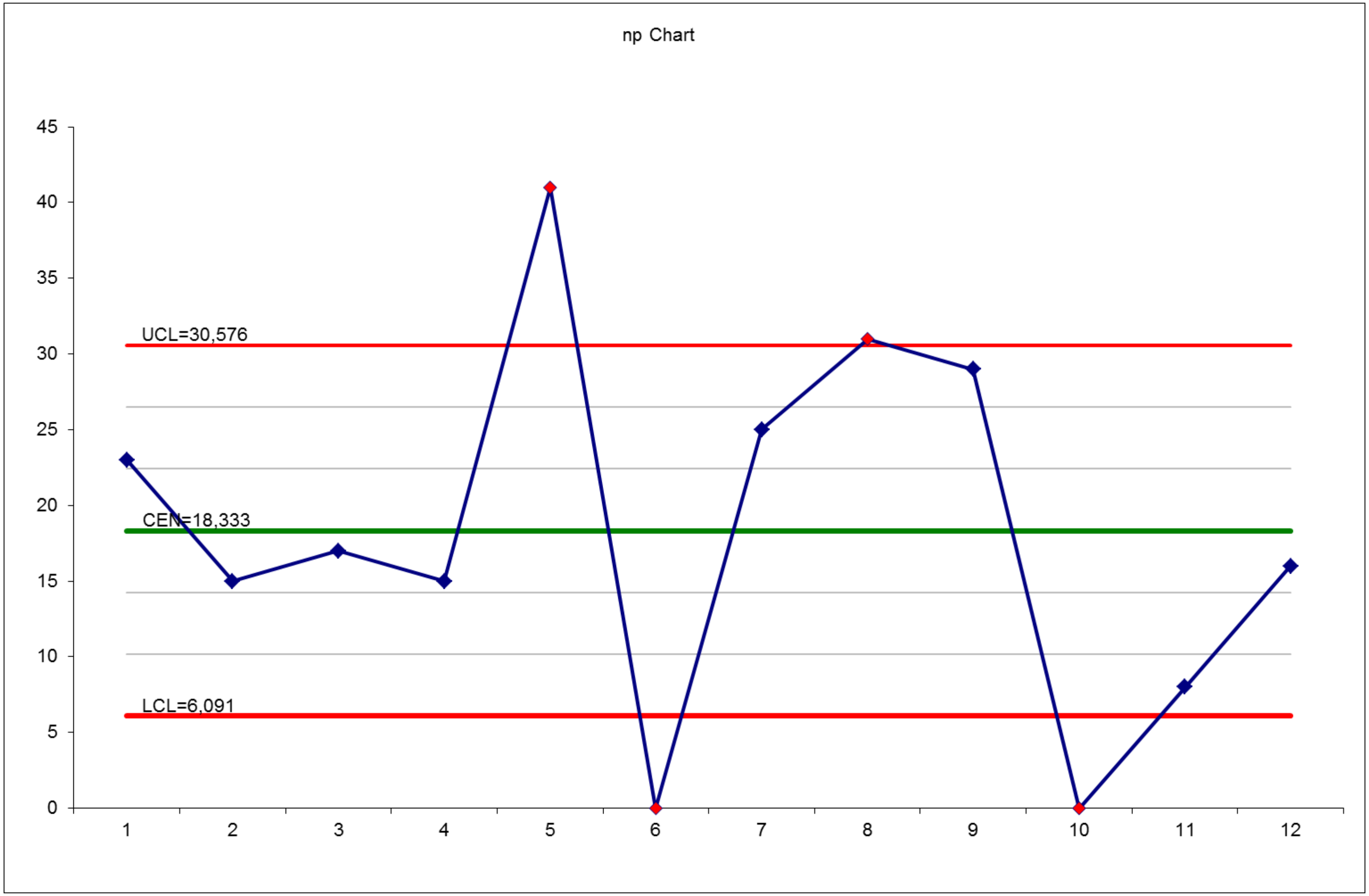
$$\overline{p} = 18.3/200 = 0.0917$$

$$UCL = n\overline{p} + 3\sigma_p = 18.3 + 3.\sqrt{18.3(1-0.0917)} = 30.575$$

$$LCL = n\overline{p} - 3\sigma_p = 18.3 - 3.\sqrt{18.3(1-0.0917)} = 6.091$$

$$\sigma_p = \sqrt{n\overline{p}(1-\overline{p})} = \sqrt{18.3(1-0.0917)} = 4.08$$

Question 5



np Chart
45
40
35
30
25
20
15
10
5
0
UCL=30,576
CEN=18,333
LCL=6,091
1
2
3
4
5
6
7
8
9
10
11
12

Question 6

The number of stains on 30 randomly selected samples of 5 square meters are given to the table. Based on this data, determine the control chart.

Sample	Defect Number	Sample	Defect Number
1	7	**16**	9
2	16	**17**	13
3	24	**18**	20
4	9	**19**	5
5	11	**20**	13
6	6	**21**	12
7	8	**22**	16
8	12	**23**	8
9	11	**24**	6
10	10	**25**	14
11	15	**26**	11
12	12	**27**	10
13	9	**28**	7
14	8	**29**	7
15	6	**30**	9

Question 6

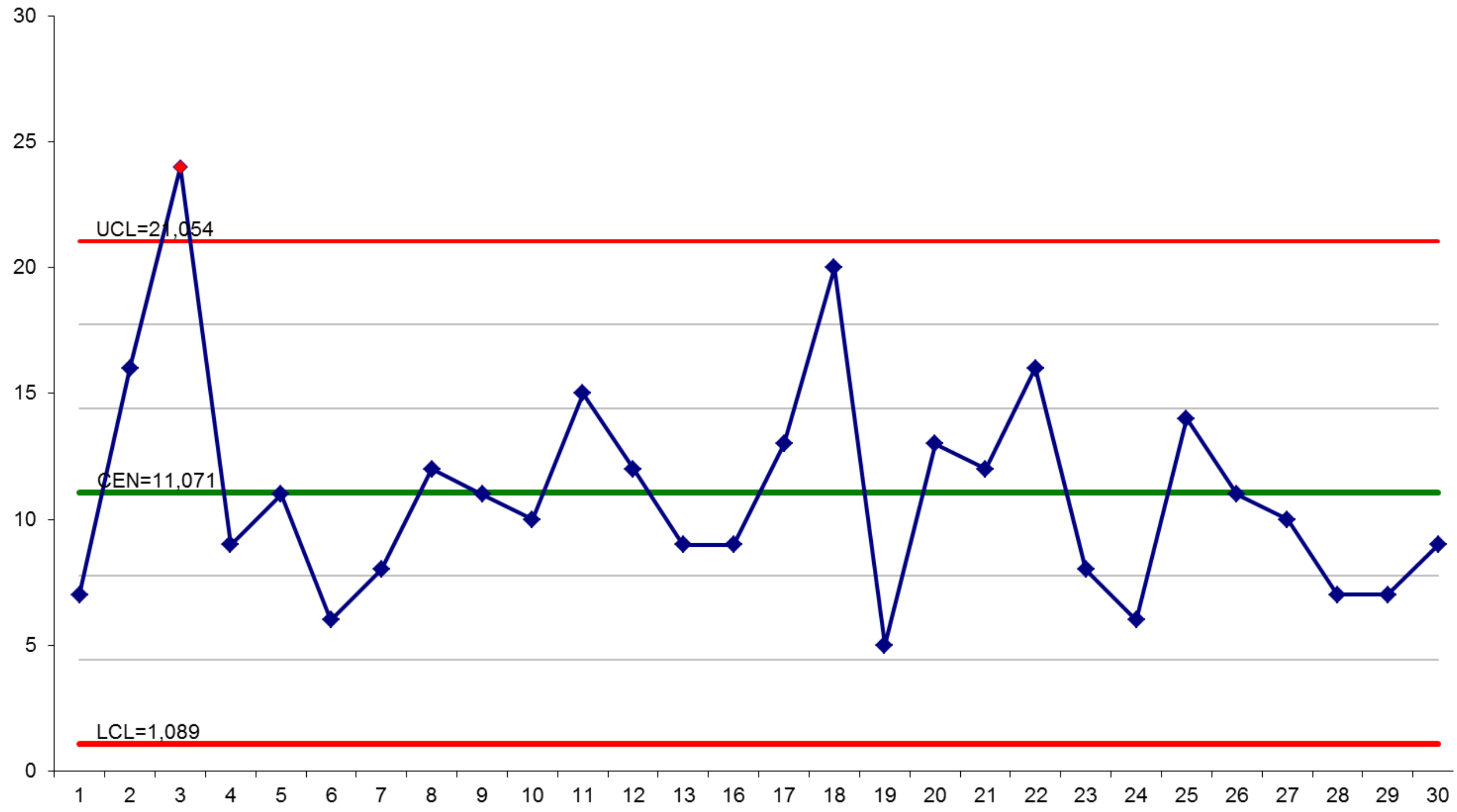
$\bar{c}$ = (Total defect number)/(Total sub-group number)

$$\mathrm{CL} = \overline{\mathrm{c}}$$

$$\mathrm{UCL} = \overline{\mathrm{c}} + 3\sqrt{\overline{\mathrm{c}}}$$

$$\mathrm{LCL} = \overline{\mathrm{c}} - 3\sqrt{\overline{\mathrm{c}}}$$

Question 6



UCL=21,054
CEN=11,071
LCL=1,089
0
5
10
15
20
25
30
1 2 3 4 5 6 7 8 9 10 11 12 13 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Question 7:

The thickness of the paint applied to the vehicles at the BMW production plant is measured every 100 cars. If the paint is too thin, there is a risk of rusting; when it is too thick, the paint has an orange peel problem over time and the cost is increasing.

Chart:

Reasons:

Question 8:

An iron and steel molding company thinks that the quality of the raw steel supplied is inconsistent and it causes variance in its manufacturing processes.

They want to follow the number of stains they observe in every 20 meters of steel coils supplied.

Chart:

Reasons: