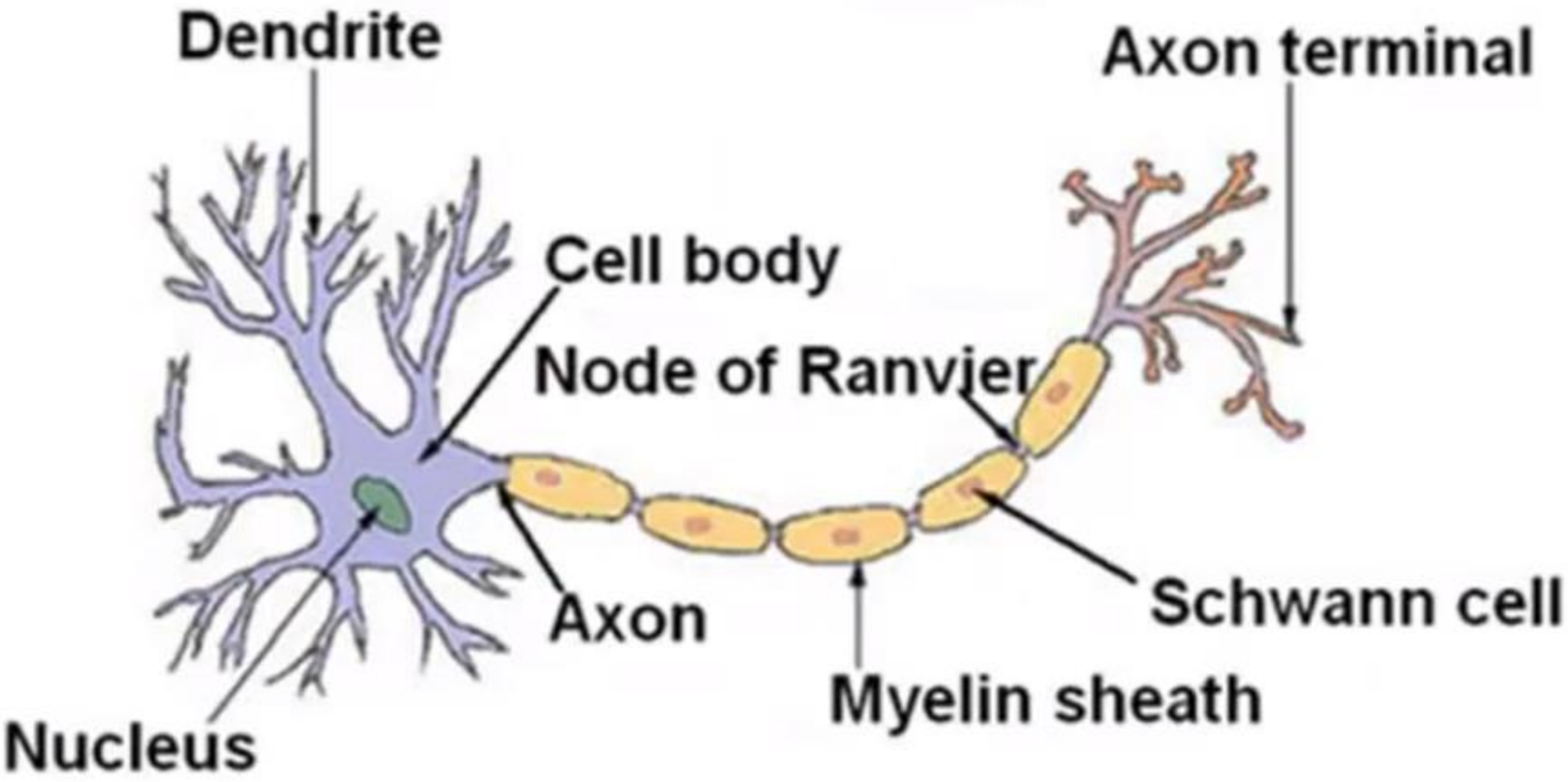
Model Representation 1

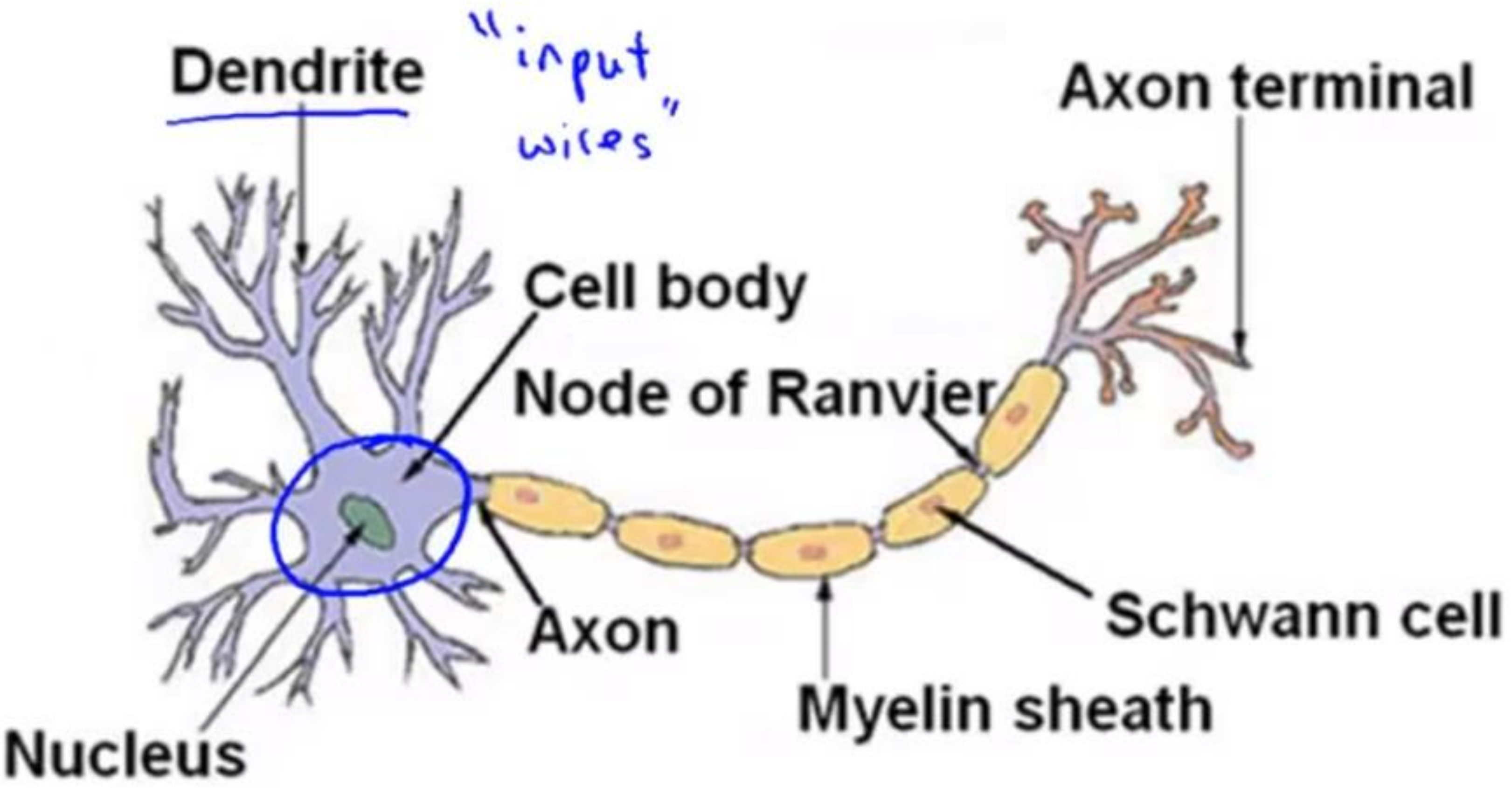
Neural Networks

Neural Networks: Representation

Neuron in the brain



Neuron in the brain



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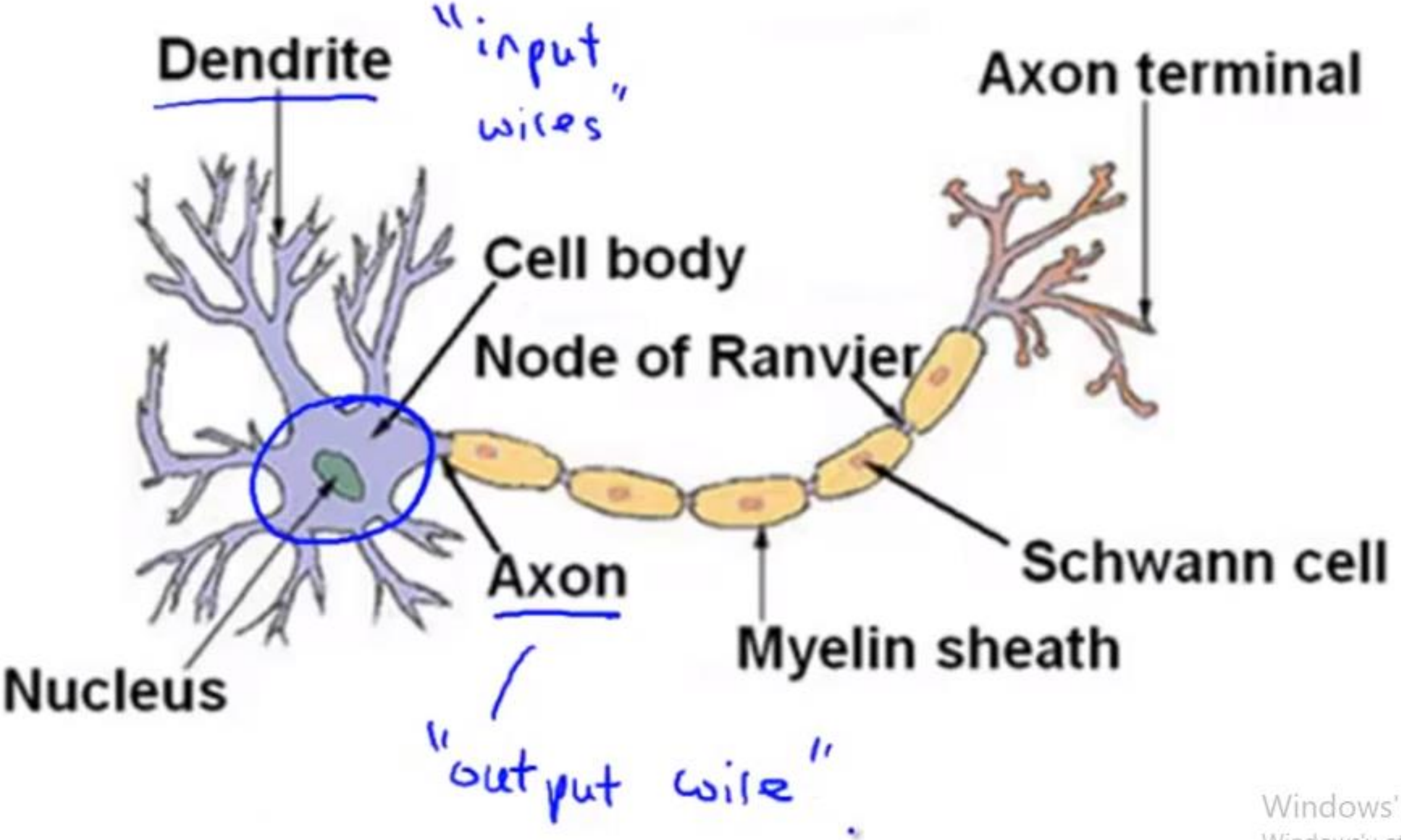
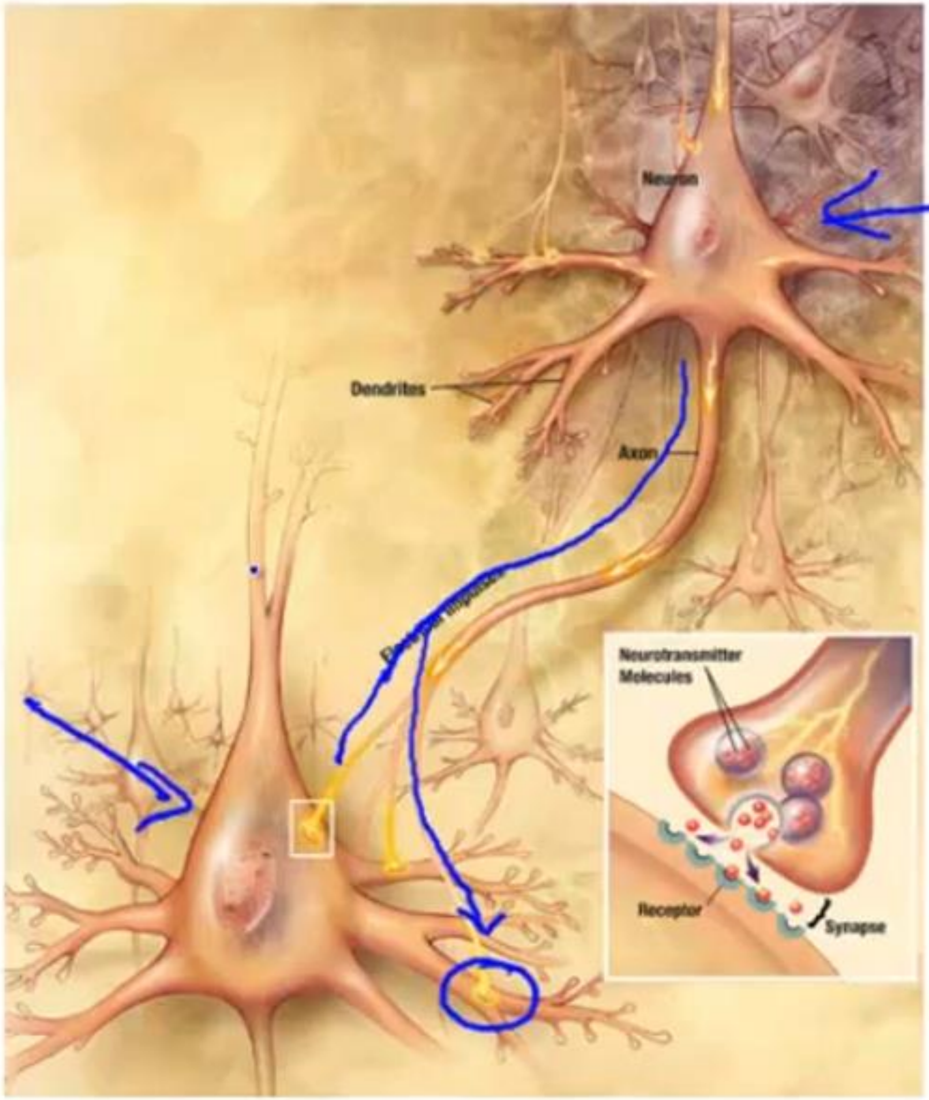


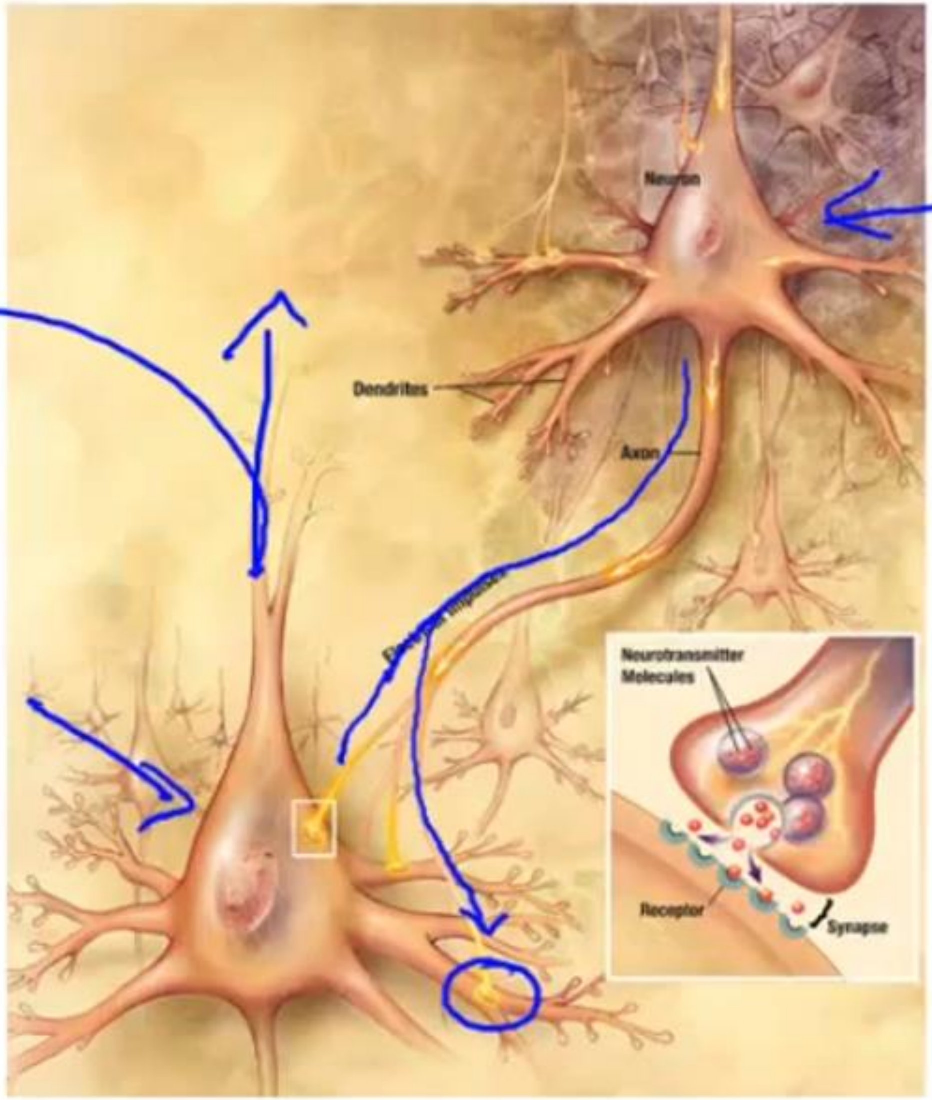
Neurons in the brain



[Credit: US National Institutes of Health, National Institute on Aging]

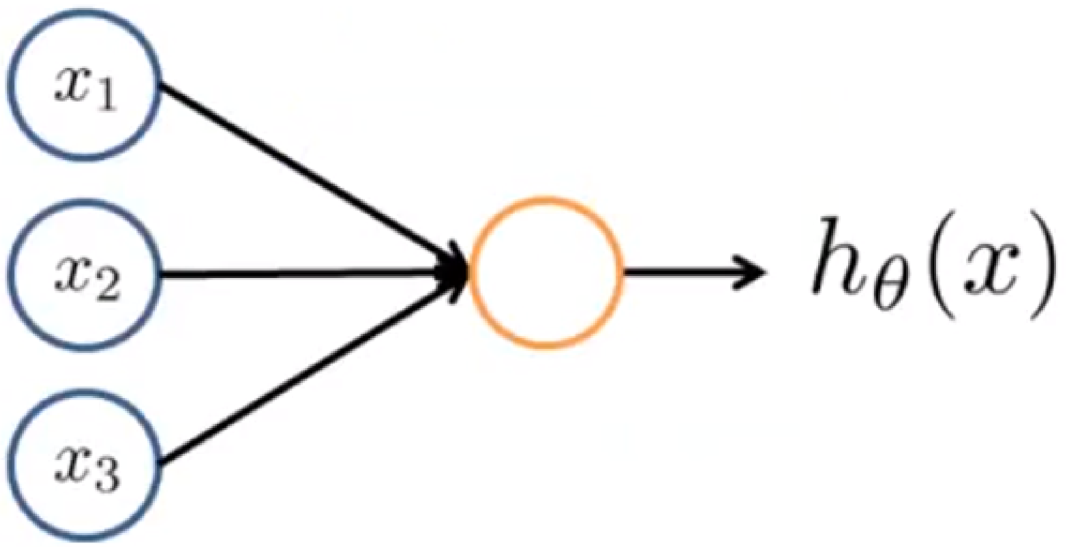
Neurons in the brain

Muscle

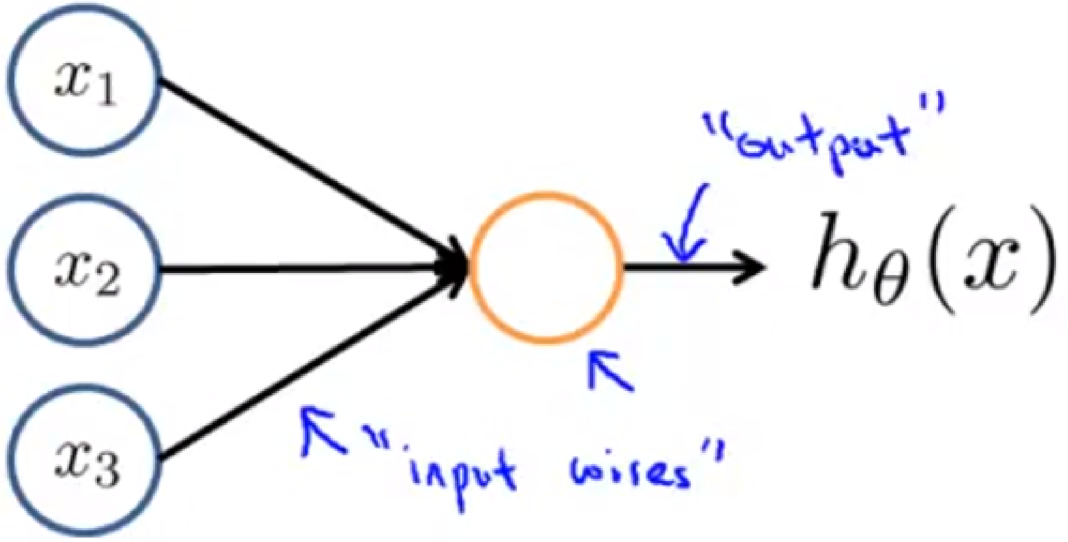


[Credit: US National Institutes of Health, National Institute on Aging]

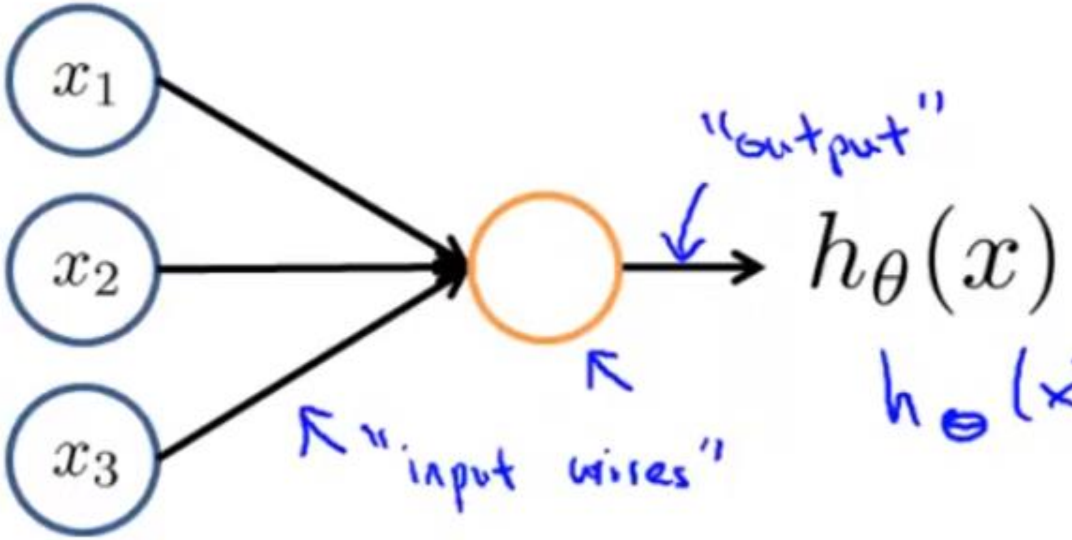
Neuron model: Logistic unit



Neuron model: Logistic unit



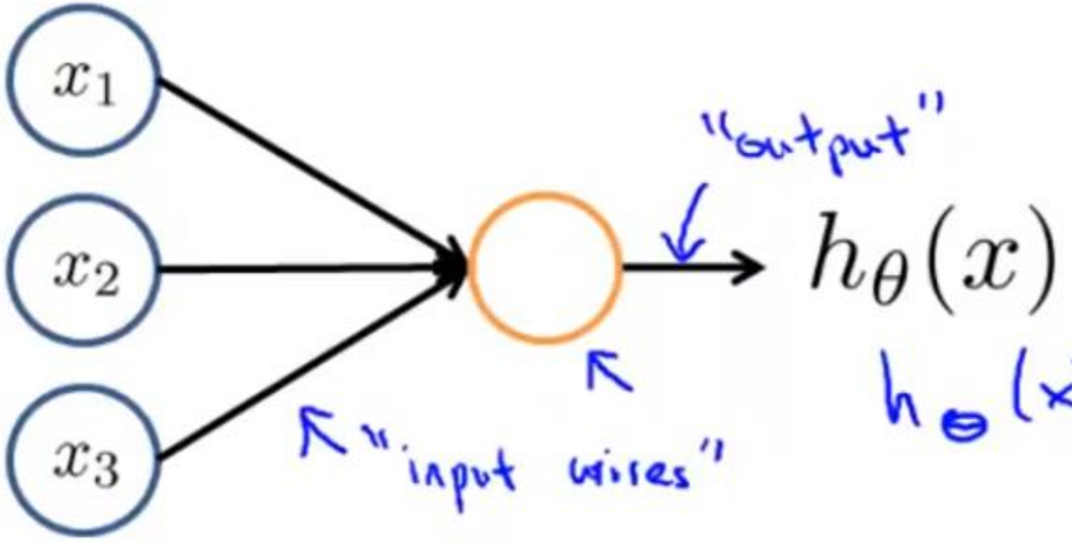
Neuron model: Logistic unit



$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

Neuron model: Logistic unit

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$



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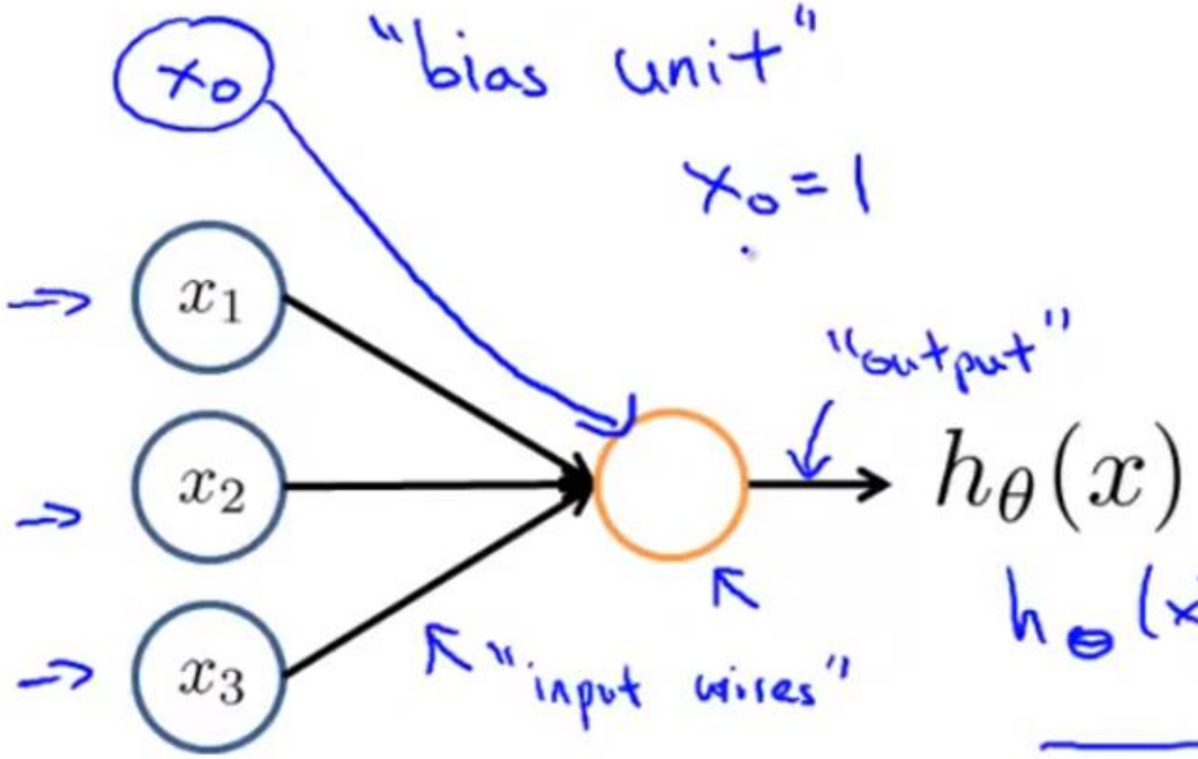
Neuron model: Logistic unit



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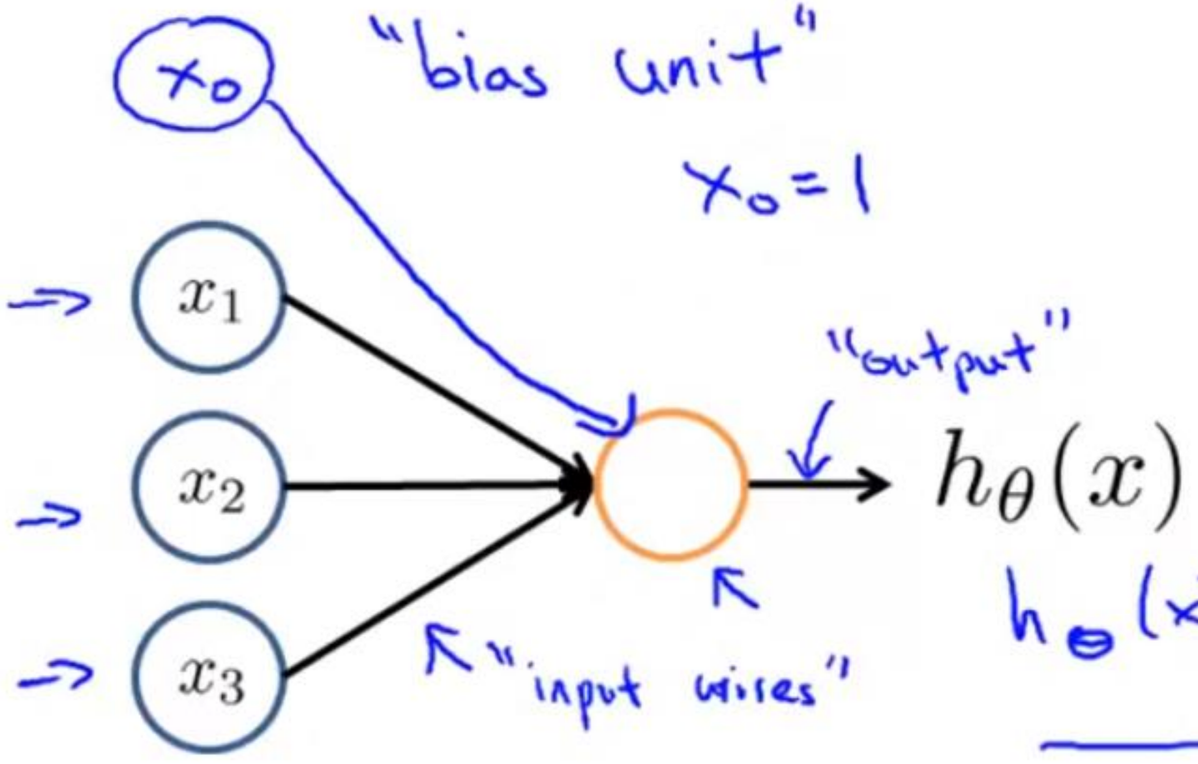

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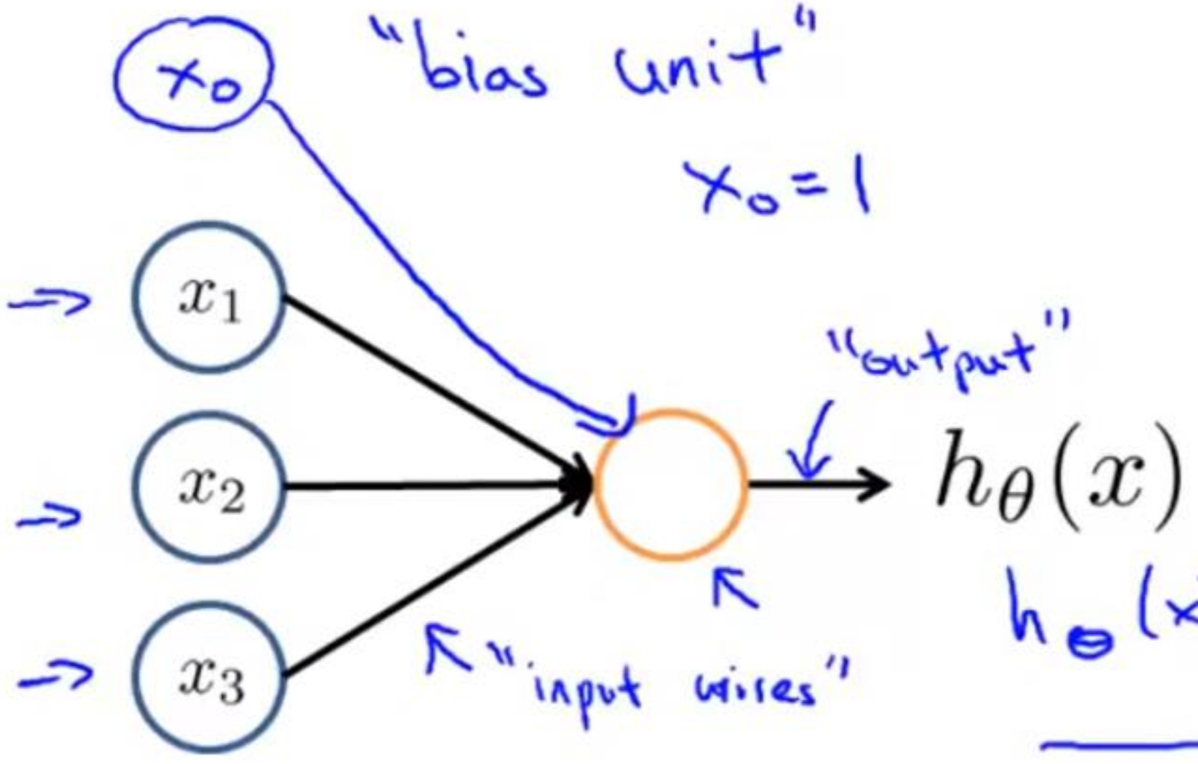
Sigmoid (logistic) activation function.

Neuron model: Logistic unit



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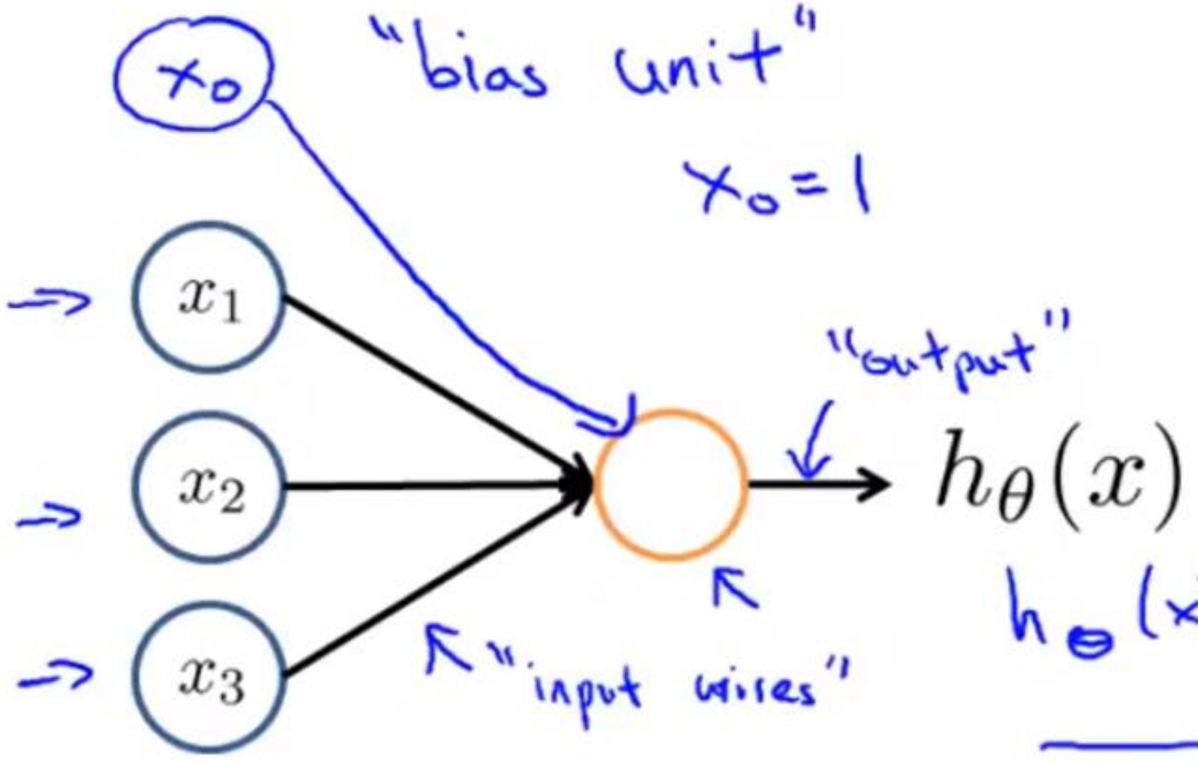
Sigmoid (logistic) activation function.

$$g(z) = \frac{1}{1+e^{-z}}$$

Neuron model: Logistic unit



$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

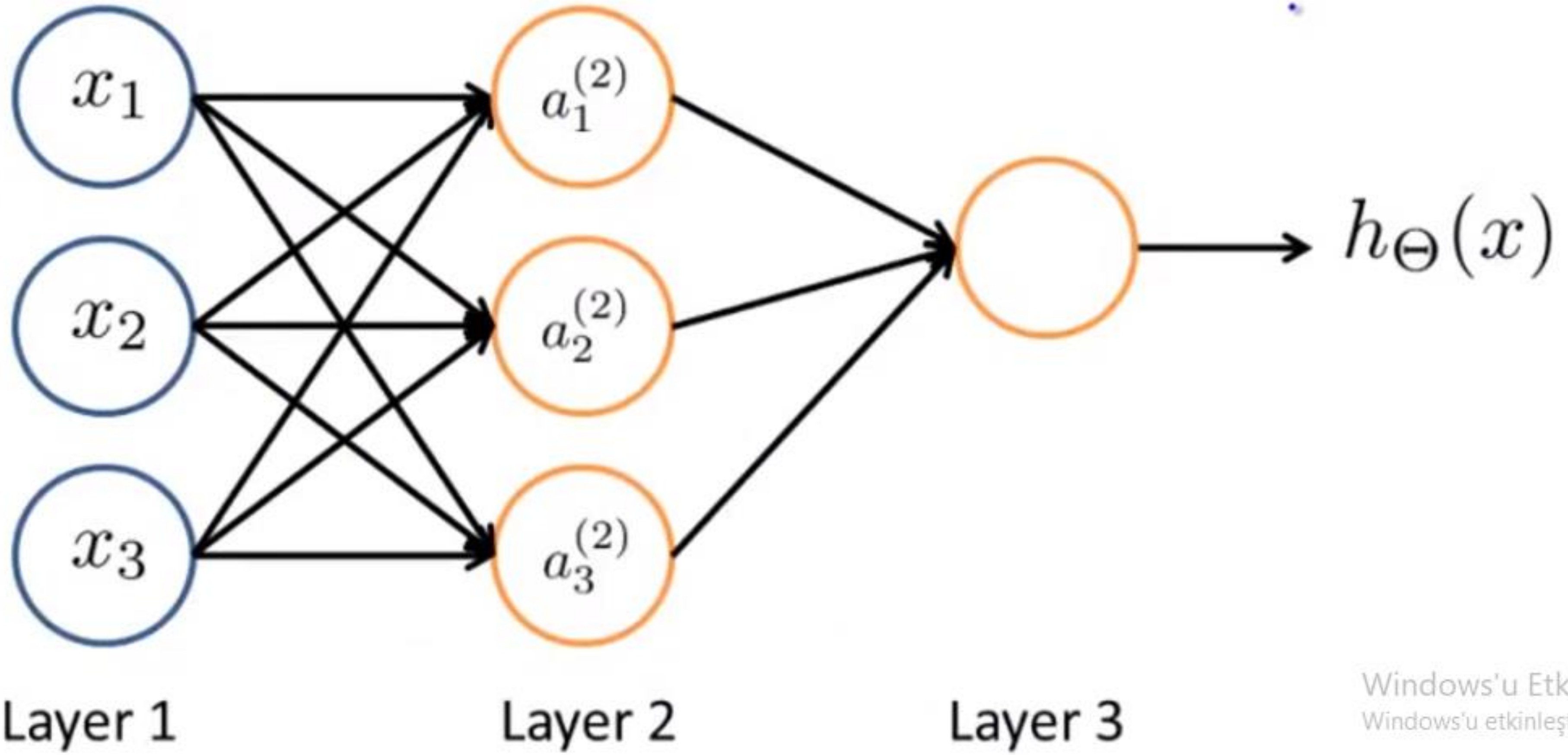
"weights"
(parameters)

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

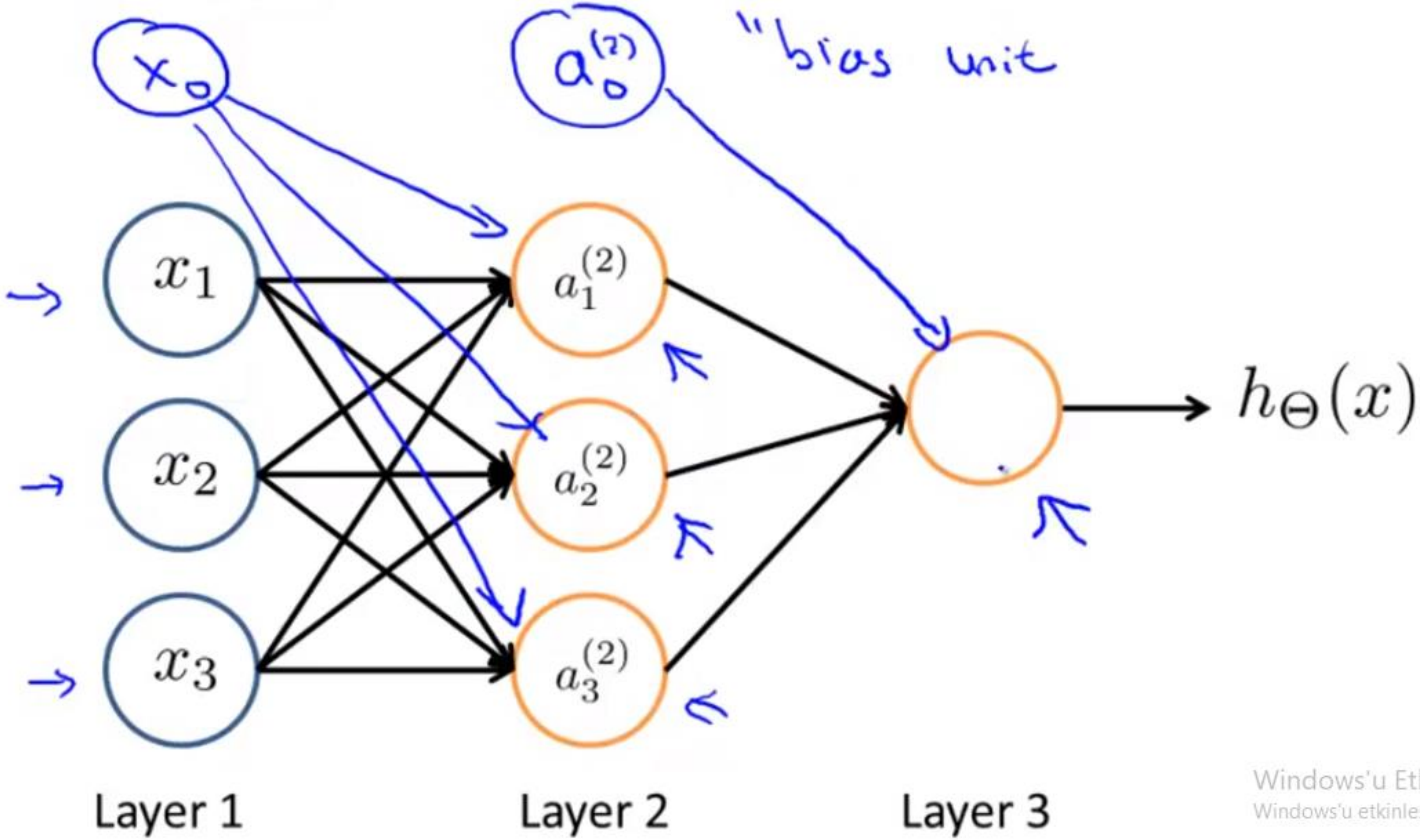
Sigmoid (logistic) activation function.

$$g(z) = \frac{1}{1+e^{-z}}$$

Neural Network



Neural Network



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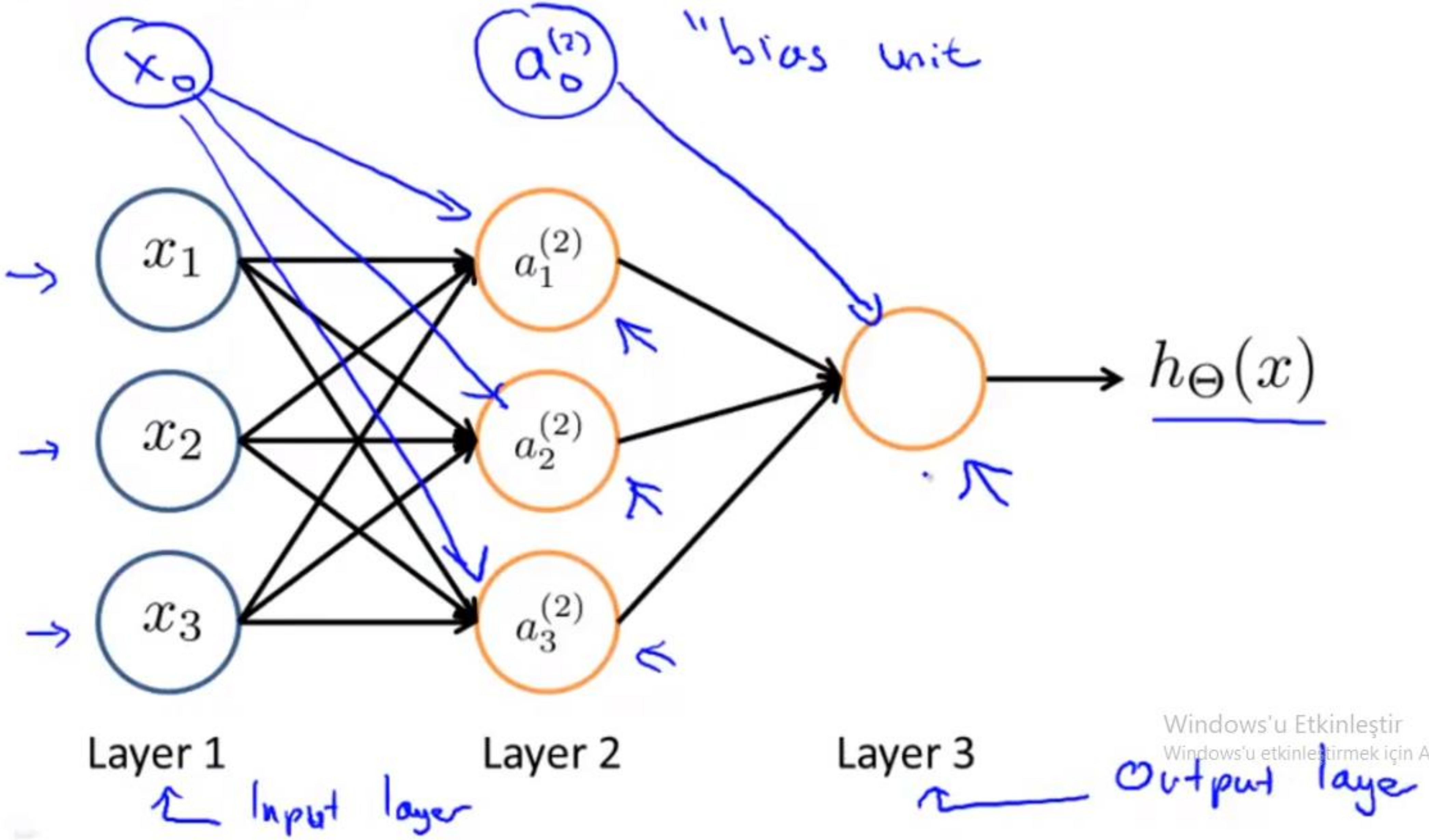

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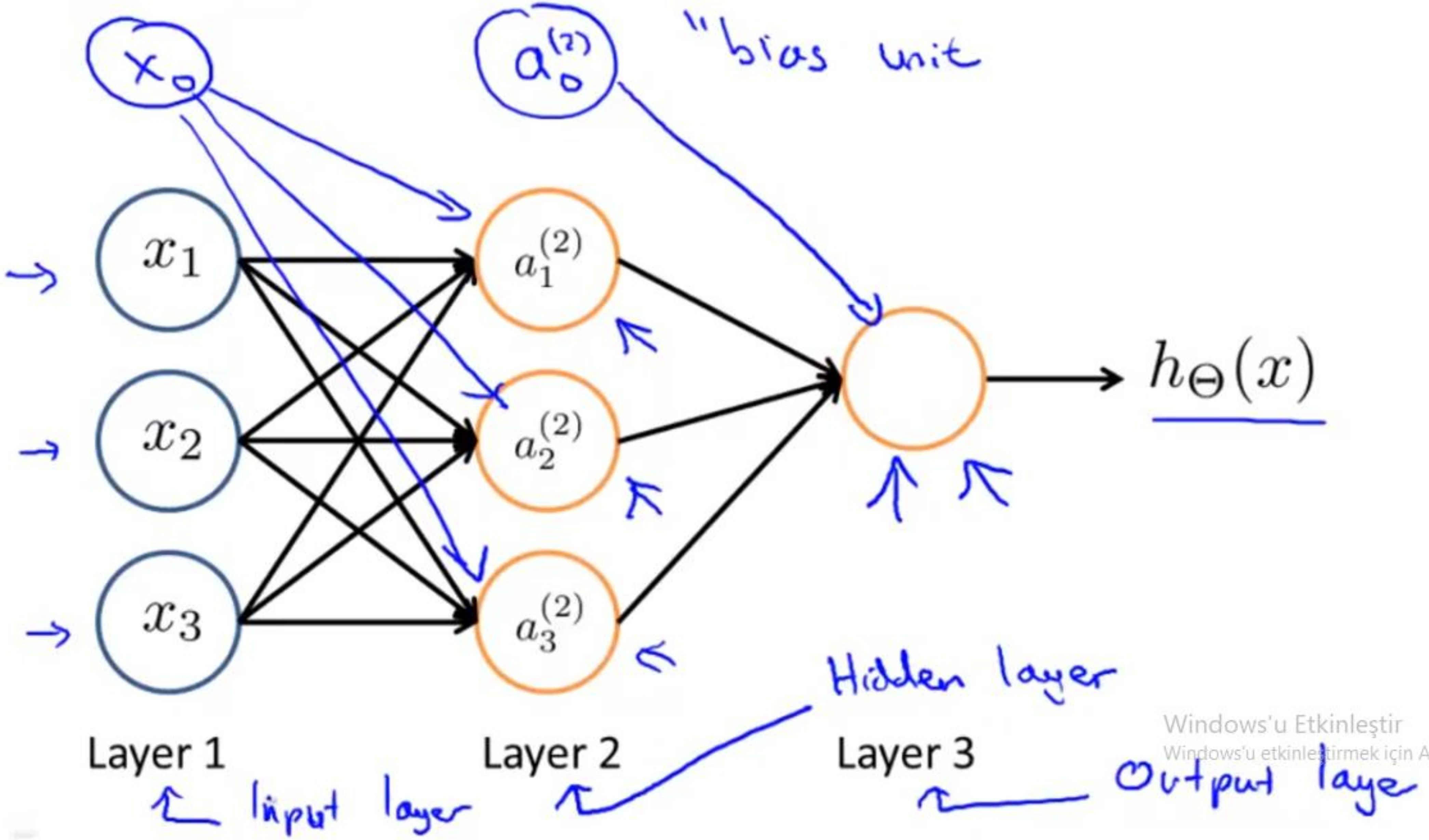

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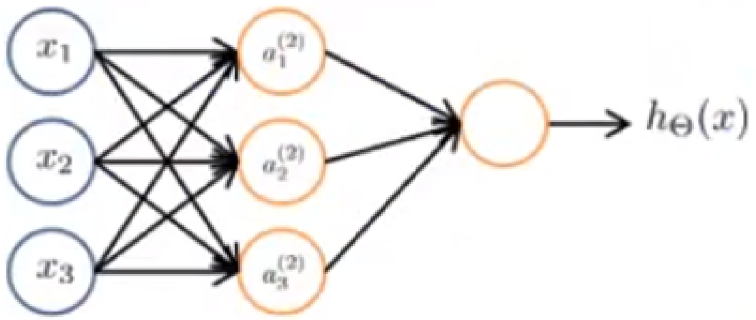
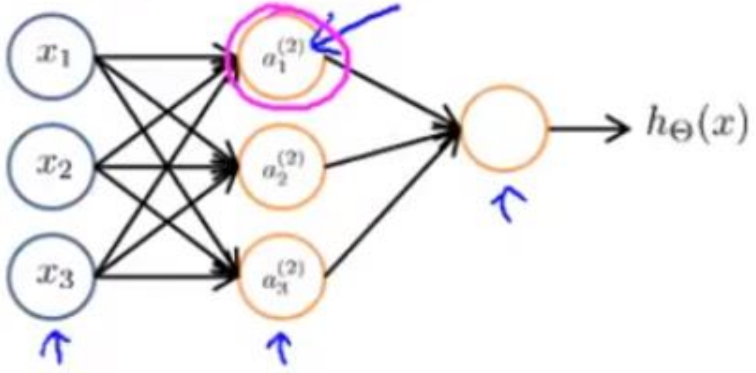
Neural Network



$a_i^{(j)} =$ "activation" of unit i in layer j

$\Theta^{(j)} =$ matrix of weights controlling function mapping from layer j to layer $j+1$

Neural Network



$\rightarrow a_i^{(j)} =$ "activation" of unit i in layer j

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$$\rightarrow a_1^{(2)} = g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3)$$
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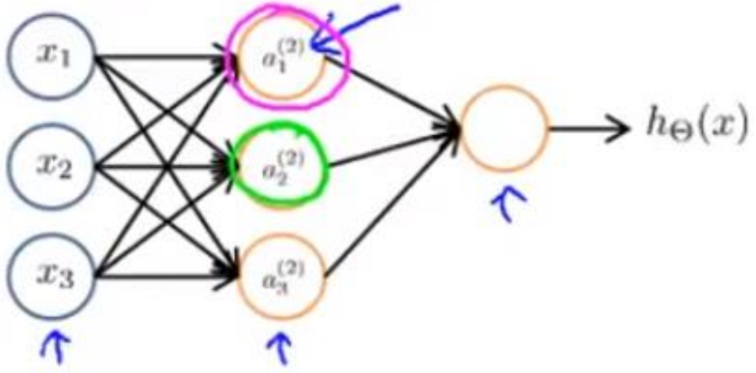
$$h_\Theta(x) = a_1^{(3)} = g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$$

Neural Network



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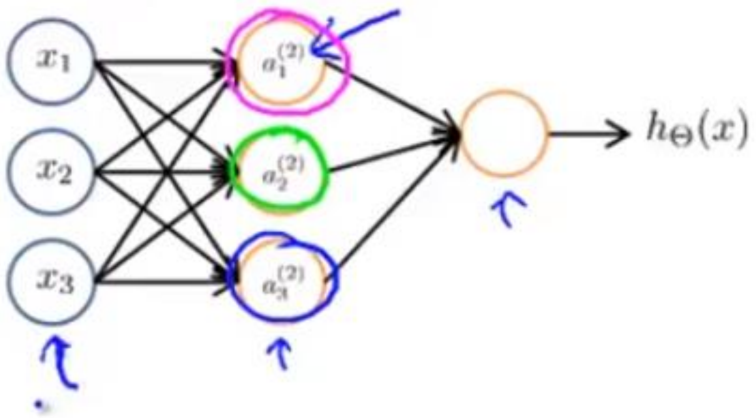
Neural Network



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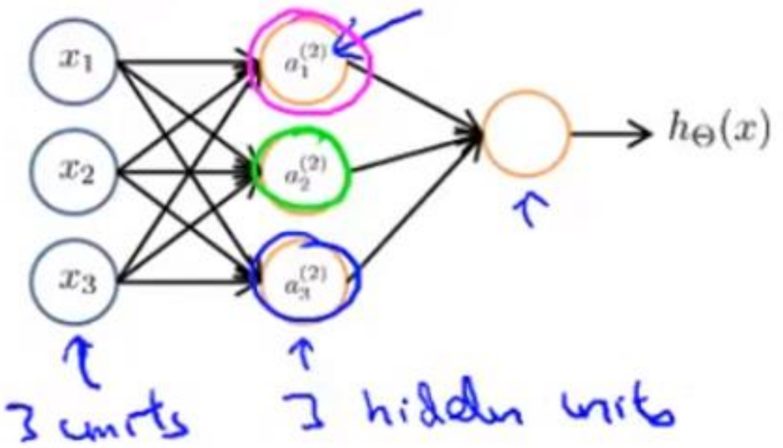
Neural Network



$\rightarrow a_i^{(j)} =$ "activation" of unit i in layer j

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$\Theta^{(1)}$

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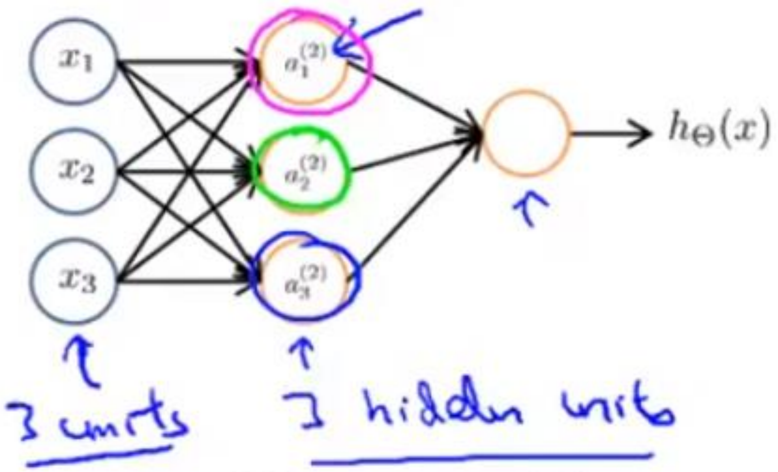
Neural Network



$\rightarrow a_i^{(j)} =$ "activation" of unit i in layer j

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$\Theta^{(1)} \in \mathbb{R}^{3\times 4}$

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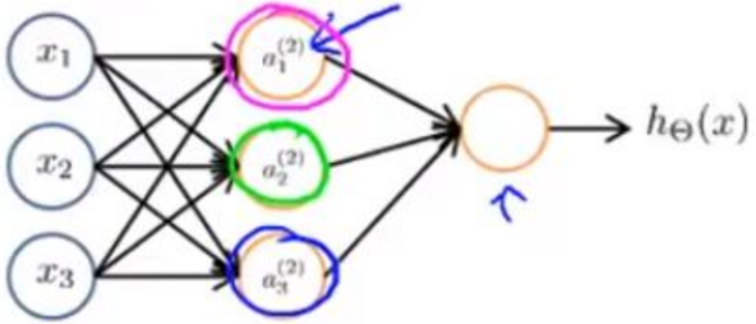
Neural Network



3 units   3 hidden units

→ $a_i^{(j)} =$ "activation" of unit i in layer j

→ $\Theta^{(j)} =$ matrix of weights controlling function mapping from layer j to layer $j+1$

$\Theta^{(1)} \in \mathbb{R}^{3\times 4}$

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→ If network has s_j units in layer j, s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ will be of dimension $s_{j+1} \times (s_j + 1)$.

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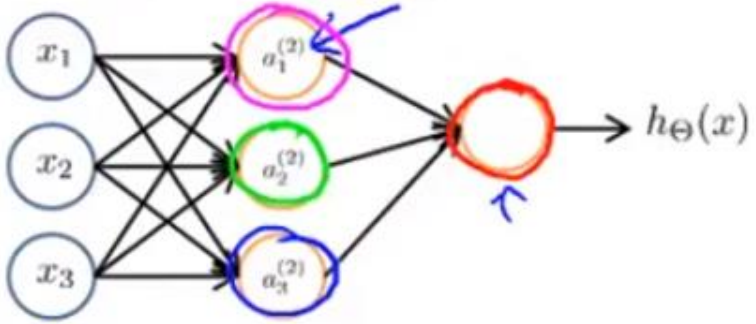
Neural Network



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$\Theta^{(2)}$

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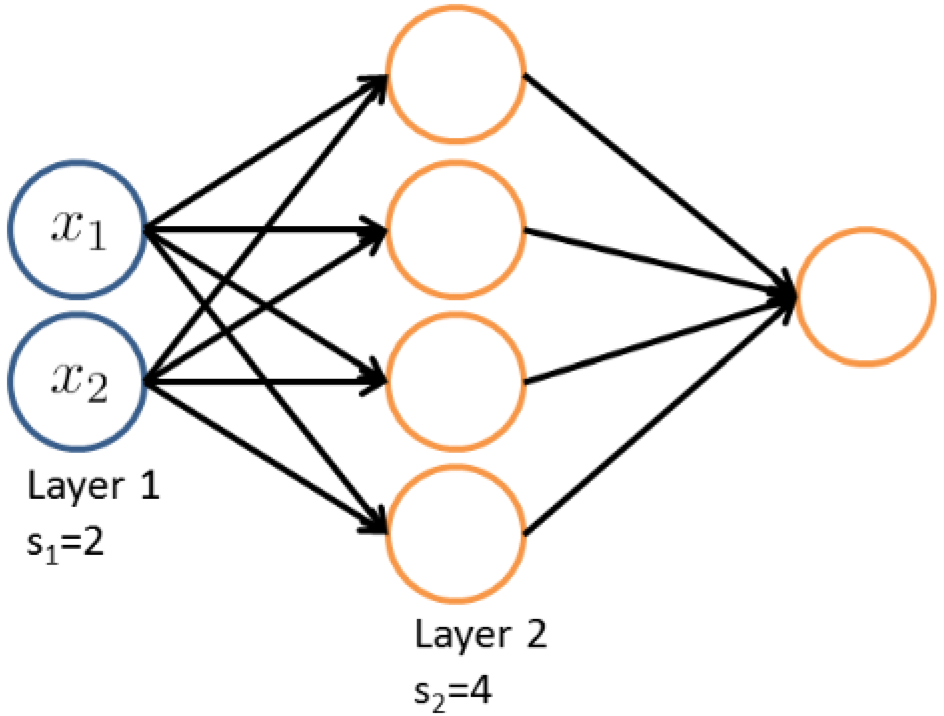
$\rightarrow$ If network has s_j units in layer j, s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ will be of dimension $\boxed{s_{j+1}} \times (s_j + 1)$.

$s_{j+1} \times (s_j + 1)$

Exercise

- What is the dimension of $\Theta^{(1)}$ (Hint: add a bias unit to the input and hidden layers)?



Summary

- At a very simple level, neurons are basically computational units that
  - take inputs (dendrites) as electrical inputs (called "spikes")
  - that are channeled to outputs (axons).
- our dendrites are like the input features x_1\cdots x_n$x_1 \cdots x_n$,
- the output is the result of our hypothesis function
- In neural networks, we use the same logistic function as in classification, yet we sometimes call it a sigmoid (logistic) activation function.

Summary

- If we have single layer, this is what we have

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \\ a_3^{(2)} \end{bmatrix} \rightarrow h_\theta(x)$$

- Where the value of each node is calculated by:

$$\begin{aligned} a_1^{(2)} &= g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3) \\ a_2^{(2)} &= g(\Theta_{20}^{(1)} x_0 + \Theta_{21}^{(1)} x_1 + \Theta_{22}^{(1)} x_2 + \Theta_{23}^{(1)} x_3) \\ a_3^{(2)} &= g(\Theta_{30}^{(1)} x_0 + \Theta_{31}^{(1)} x_1 + \Theta_{32}^{(1)} x_2 + \Theta_{33}^{(1)} x_3) \\ h_\Theta(x) = a_1^{(3)} &= g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)}) \end{aligned}$$

Summary

- We apply each row of the parameters to our inputs to obtain the value for one activation node.
- Our hypothesis output is the logistic function applied to the sum of the values of our activation nodes, which have been multiplied by yet another parameter matrix $\theta^{(2)}$ containing the weights for our second layer of nodes.