Queueing Applications in MHD

END4650 - Material Handling Systems

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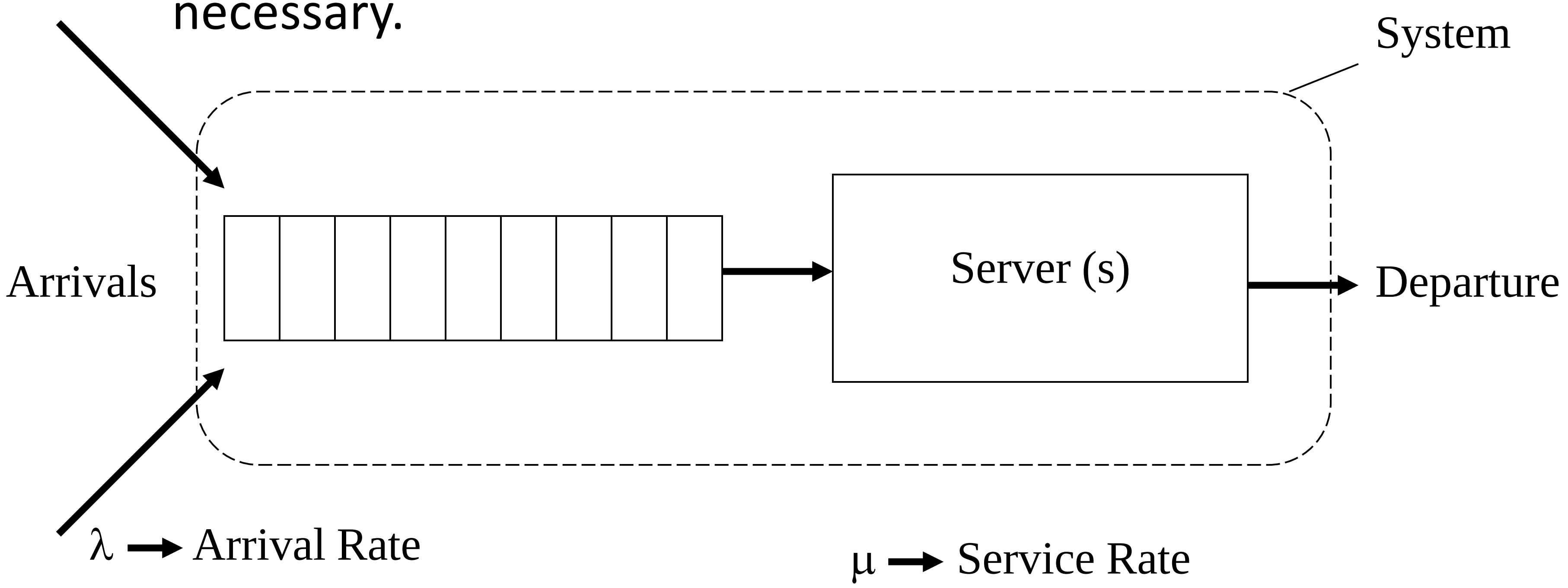
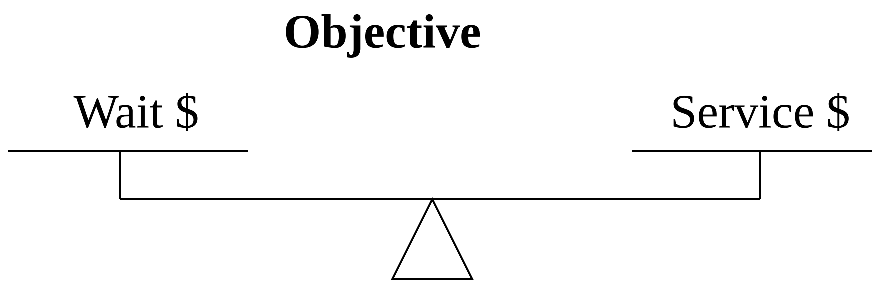
Queueing Analysis

- Mathematical analysis of queues (waiting lines which occur whenever the current demand for service exceeds the current capacity to provide that service).
- Suitable for "quick and dirty" evaluations when a high degree of detail is not necessary.



Queueing Analysis

- Simple Systems
  - FIFO (FCFS)
  - Single Server, Single Queue - M/M/1 System
  - Poisson Arrivals
  - Exponential Service Times
- **Limited:** Mathematical complexity becomes hard to deal if the above is not true
- Simulation…



Queueing Analysis

Interarrival Time	Arrival Time	Service Time	Service Start	Service End	Waiting in the queue
0		1			
2		3			
4		4			
3		2			
1		3			
3		1			

Queueing Analysis

Interarrival Time	Arrival Time	Service Time	Service Start	Service End	Waiting in the queue
0	0	1	0	1	0
2	2	3	2	5	0
4	6	4	6	10	0
3	9	2	10	12	1
1	10	3	12	15	2
3	13	1	15	16	2

λ = avg. arrival rate

μ = avg. service rate

U = utilization = what portion of the time does the machine works.

W = Avg. Time a customer spends in the system

W_q = Avg. Time a customer spends in line (queue)

L_q = Avg. number of customers waiting in line(queue)

L = Avg. number of customers in system

P_n = probability that there are n customers in the system

Queueing Analysis

For example

- $L_q = 3$
  - on the average there are 3 entities waiting in the queues
- L=8
  - On the average there are 8 customers in the system (Queue + server)
- $W_q = 0.5$ min
  - On the average, every customer spend 0.5 min in the queue.
- $W = 1.9$ min
  - On the average, every customers spends 1.9 minutes in the system (queue + server)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$L = \frac{\lambda}{\mu-\lambda}$$

<u>also</u> ($L = L_q + \lambda/\mu$)

W_q = L_q/λ = (Little's Law)

W = L/λ = (Little's Law)

U = λ/μ =

$$\mathbf{P_n} = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

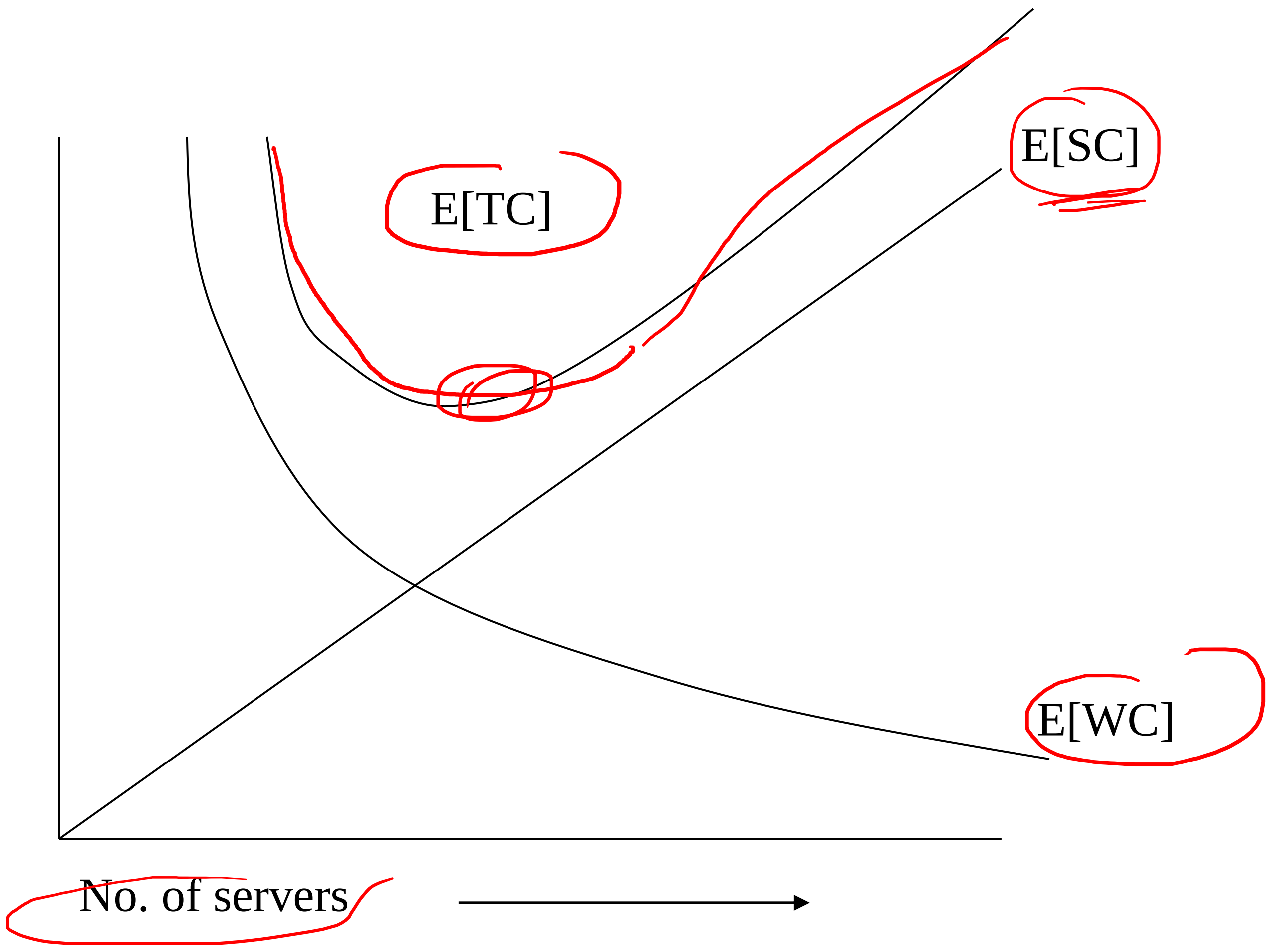
$$\mathbf{P_0} = 1 - \frac{\lambda}{\mu}$$

Type I. **Internal Customers**

Example: Forklift truck drivers
Cost: Lost productivity of drivers

Type II. **External customers**

Example: People waiting for cabs
Cost: ??



Queueing Systems: Example-1

- Manufacturing engineers at the Widget Manufacturing Company recently convinced their manager to purchase a more expensive, but flexible machine that can do multiple operations simultaneously.
- The rate at which parts arrived at the machine that was replaced by the flexible machines follows a Poisson process with a mean of 10 parts per hour.
- The service rate of the flexible machine is 15 units parts per hour compared with the 12 units per hour service rate of the machine it replaced. (All service times follow an exponential distribution.)

Queueing Systems: Example-1

- The engineers and manager were convinced that the company would have sufficient capacity to meet higher levels of demand, but just after a two months of purchasing the machines it turned out that the input queue to the flexible machine was excessively long and part flow times at this station were so long, that the flexible machine became a severe bottleneck.

Queueing Systems: Example-1

- The engineers noticed that more parts were routed through this machine, and that the parts arrival rate to the flexible machines had increased from 10 per hour to about 14 per hour, but were puzzled why the part flow time at this station doubled from 30 minutes to one hour and the work-n-process (WIP) inventory increased nearly threefold from 5 to 14 when the arrival rate only increased 40%.
- Use a queuing model to justify the results observed at Widget Manufacturing Company.

Queueing Systems: Example-1

	Old Machine	New Machine
Arrival Rate(λ)	10	14
Service Rate(μ)	12	15

7 8 5

$L = \frac{\lambda}{\mu - \lambda}$

$W = L/\lambda$

$L \quad \frac{10}{12-10} = 5 \qquad \frac{14}{15-14} = 14$

$W \quad 5/10 = 0.5 = 30\,\text{min} \qquad \frac{14}{14} = 1\ \text{hour}$

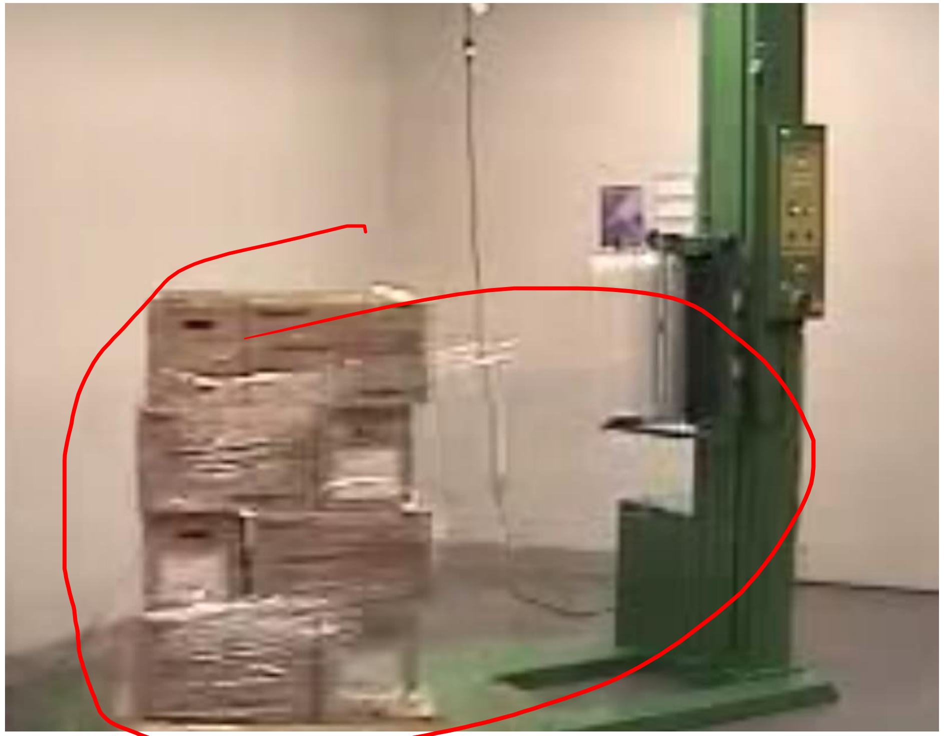
5 min

Queueing Systems: Example-1

	System 1	System 2
L	10/2 = 5	14/1 = 14
W	5/10 = 0.5	14/14 = 1

Queueing Systems: Example 2

- Yıldız Manufacturing Company uses a simple stretching operation for its products.
- The forklift drivers brings a pallet and waits for the boxes to be strecthed at this machine.
- Again, on the average 8 pallets arrive to this stretching station in an hour.
- Yıldız Manufacturing is considering two machines which have different specifications.



6 minutes

Queueing Systems: Example 2

- The stretching machine 1:
  - The machine can make 10 stretches in an hour.
  - The cost of stretching machine per hour is 50 TL.
    - Assume that it includes all kind of costs like the strectch film, the electricity it uses, etc.
  - This cost is **not** incurred when the machine is idle, i.e., not working.
- The stretching machine 2:
  - Makes %20 more jobs than the previous one (12 stretching per hour), 5 m /stretch
  - However its cost is 50% higher, i.e., 75 TL per hour.
  - Like the previous one, it does **not** incur any cost if it is idle.

Queueing Systems: Example 2

- There are also costs for the operators (or drivers):
  - The operators drive the forklift to the strecthing machine, and make the products filmed if there is no one in the queue.
  - If there is someone in the queue, then the operators wait for the other operators to finish their stretching job.
  - Every operator is paid 25 TL per hour.
  - Hence their cost is incurred during they are waiting in the queue of the stretching machine or during they are getting the stretching service.

- Which machine do you recommend and why?

	System 1	System 2
Parameters	$\lambda = 8/\text{hour}$, $\mu = 10/\text{hour}$ $C_u = 50/h$, $C_o = 25/h$	$\lambda = 8/\text{hour}$, $\mu = 12/h$ $C_u = 75$ $C_o = 25/h$
Cost of operators (per hour)	$W = L/\lambda$ $L = \frac{\lambda}{\mu - \lambda} = 8/2 = 4$ $W = 4/8 = 0.5$ hours $C_o = (4)(0.5)(25)$ $= 50$ TL/hour	$W = L/\lambda$ $L = \frac{8}{12-8} = 2$ $W = 2/8 = 0.25$ $C_o = (2)\left(\frac{1}{4}\right) \cdot \cancel{50}\ 25$ 12.5 $\cancel{= 25\text{ TL}}$ /hour

	System 1	System 2
Machine Cost	$(50) \cdot \frac{8}{\underbrace{12}_{80\%}} = 40\$/\text{hour}$	$(75) \cdot \frac{8}{12} = 50\$/\text{hour}$
Total Cost	90€/hour	62.5/hour

Queueing Systems: Exercise

- Machinists who work at a tool-and-die plant must check out tools from a tool center. An average of ten machinists per hour arrive seeking tools.
- At present, the tool center is staffed by a clerk who is paid \$6 per hour and who takes an average of 5 minutes to handle each request for tools.
- Since each machinist produces \$10 worth of goods per hour, each hour that a machinist spends at the tool center costs the company \$10.
- The company is deciding whether or not it is worthwhile to hire (at \$4 per hour) a helper for the clerk.
- If the helper is hired, the clerk will take an average of only 4 minutes to process requests for tools. Assume that service and interarrival times are exponential.
- Should the helper be hired?