

LECTURE -5-

(Multi-criteria Decision Making, Elementary Methods)

MULTICRITERIA DECISION MAKING

- A single DM is to choose among a countable (usually finite) or uncountable set of alternatives that s/he evaluates on the basis of two or more (multiple) criteria (Korhonen *et al.*, 1992; Dyer *et al.*, 1992)
- MCDM consists of constructing a global preference relation for a set of alternatives evaluated using several criteria (Vansnick, 1986)
- The aim of any MCDM technique is to provide help and guidance to the DM in discovering his or her most desired solution to the problem (Stewart, 1992)

MADM - MODM

- A differentiation can be made w.r.t. number of alternatives:
- Multi Attribute Decision Making MADM Cases in which the set of decision alternatives is defined explicitly by a finite list of alternative actions – Discrete alternatives
- Multi Objective Decision Making MODM
 Those in which a is defined implicitly by a mathematical programming structure –
 Continuous alternatives

MULTI ATTRIBUTE DECISION MAKING

• MADM is making preference decisions (selecting, ranking, classifying, screening, prioritizing) over the available alternatives (finite number) that are characterized by attributes (multiple, conflicting, weighted, and incommensurable) (Yoon & Hwang, 1995)

DECISION MAKING PROCESS

- 1. Structuring the Problem
 Exploring the issue and determining whether or not MADM is an appropriate tool: If so, then alternatives for evaluation and relevant criteria can be expected to emerge
- Constructing the Decision Model
 Elicitation of preferences, performance values,
 and (if necessary) importance
- 3. Analyzing (Solving) the Problem
 Using a solution method to synthesize and
 explore results (through sensitivity and
 robustness analyses)

ELEMENTARY METHODS

- Dominance Relation (choice)
- Even-Swap (choice, trade-off, practical dominance)
- Lexicographic (ranking, noncompensatory)
- Elimination by Aspects (choice, noncompensatory)
- Maximax (choice, noncompensatory)
- Maximin (choice, noncompensatory)
- Conjunctive (classification, noncompensatory)
- Disjunctive (classification, noncompensatory)
- o Median Ranking (ranking, aggregation)

DOMINANCE RELATION

- Dominance of a over b translates a sort of agreement for all points of view in favor of a: $v_j(a) \ge v_j(b)$ where at least one of the inequalities is strict
- One alternative dominates another if the first alternative is at least as good as the second w.r.t. every attribute and **strictly** better w.r.t. at least one of them.
- Alternatives that are not dominated by any other alternatives are called as non-dominated, dominant, or efficient alternatives (Pareto optimal or efficient frontier)

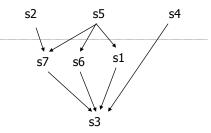
EXAMPLE FOR DOMINANCE

A textile manufacturing company wants to open a new plant. The plant requires abundant water and manpower.

7 sites are judged by the following attributes:

- Community attitude toward new plant (a1)
- Water availability (a2)
- Probability of a union within the next two years (a3)

		- 0	
	a1	a2	a3
s1	P	G	0.5
s2	Е	A	1
s3	P	P	1
s4	A	A	0.1
s5	G	Е	0.2
s6	A	G	0.9
s7	G	A	1
P:Poor, A	A:Average,	G:Good, E	:Excellent



DECISION MATRIX FOR "BUYING A NEW CAR" PROBLEM

	Price	Comfort	Perf.	Design			
Weight	5	4	3	3			
a_1	300	E (3)	E(3)	S (3)			
a_2	250	E (3)	A (2)	S (3)			
a_3	250	A (2)	E(3)	S (3)			
a_4	200	A (2)	E(3)	O (2)			
a 5	200	A (2)	A (2)	S (3)			
a_6	200	W (1)	E(3)	S (3)			
a_7	100	W (1)	A (2)	O (2)			
E:Excellen	t; A:Averag	E:Excellent; A:Average, W:Weak, S:Superior, O:Ordinary					

EVEN-SWAP METHOD (HAMMOND ET AL., 1999)

- Dominated or practically dominated alternatives are eliminated
- 2. To cancel out an attribute or an alternative, the necessary change is determined. To compensate for this needed change, the change in another attribute is assessed
- Even swap is made. An even swap increases the value of an alternative in terms of one attribute while decreasing its value by an equivalent amount in terms of another attribute (trade-off)
- 4. The now-irrelevant attribute is cancelled out or (practically) dominated alternative is eliminated
- If choice can not be made, go to 2nd step

PRACTICAL DOMINANCE

- One alternative practically dominates another if
 - the performance of the first alternative is **strictly** better than that of second w.r.t. at least one of the attributes,
 - the performance of the first alternative is at least as good as that of second w.r.t. all remaining attributes but one, and
 - the performance of the second alternative is only "**slightly**" better than that of first w.r.t. this excluding attribute

EXAMPLE FOR EVEN-SWAP

20 m.u. indifference threshold

200	
<i>a</i> ₁ 300 E E	S
a ₂ 250 E A	S
a ₃ 250 A E	S
a ₄ 200 285 A E ØS	S
a ₅ 200 A A	s
a ₆ 200 W E	S
a ₇ 100 185 W A 65	\$

	Price	Comfort	Perf.
a_1	300	E	Е
a_2	250	Е	A
a_3	250	A	Е
a 5	200 225	A	ΑE
a_6	200	W	Е

	Price	Comfort	Perf.
a_1	300	Е	E
a_2	<i>25</i> 0 275	Е -	ÆΕ
a 5	225	A	E
a 6	200	W	E

	Price	Comfort
a_2	275	Е
a 5	225 280	ΑE
· a ₆	200 310	₩ E

LEXICOGRAPHIC METHOD

- Uses the most important attribute to evaluate and rank the alternatives from best (most preferred) to worst.
- If there is a tie for some of the alternatives (performance values of alternatives are equal), use the second important attribute for these alternatives...

	Price	Comfort	Perf.	Design
Weight	5	4	3	3
a_1	300	Е	Е	S
a 2	250	Е	A	S
a 3	250	A	Е	S
a 4	200	A	Е	О
a 5	200	A	A	S
a 6	200	W	Е	S
a 7	100	W	A	О
F-Evcellen	t· A·Averag	e W·Weak	S-Superior (Ordinary

Price: a7 - a4, a5, a6 - a2, a3 - a1

Comfort: a4, a5 - a6; a2 - a3

Perf.: a4 - a5 Design: a5 - a4

Result: a7 - a4, a5 - a6 - a2 - a3 - a1

ELIMINATION BY ASPECTS METHOD

• Eliminates alternatives that do not satisfy some standard, and it continues until all alternatives except one have been eliminated.

	Price	Comfort	Perf.	Design		
Weight	5	4	3	3		
<i>a</i> ₁	300	Е	Е	S		
a_2	250	E	A	S		
a 3	250	A	Е	S		
a_4	200	A	Е	О		
a 5	200	A	A	S		
a 6	200	W	Е	S		
a 7	100	W	A	О		
E:Excellent	E:Excellent; A:Average, W:Weak, S:Superior, O:Ordinary					

Less than or equal to 300 m.u., Excellent comfort, excellent performance

Price: all alternatives satisfy Comfort: a1–a2 satisfy Performance: a1 satisfies

Result: a1 is recommended

MAXIMAX METHOD

- Optimistic attitude
- Selects the maximum (across alternatives) of the maximum (across attributes) normalized performance values, which is called "the best of the best"

r(1)	Price	Comfort	Perf.	Design	
w	0,3333	0,2667	0,2	0,2	max
a_1	0,0917	0,2143	0,1667	0,1579	0,2143
a_2	0,1101	0,2143	0,1111	0,1579	0,2143
a_3	0,1101	0,1429	0,1667	0,1579	0,1667
a_4	0,1376	0,1429	0,1667	0,1053	0,1667
a_5	0,1376	0,1429	0,1111	0,1579	0,1579
a_6	0,1376	0,0714	0,1667	0,1579	0,1667
a 7	0,2752	0,0714	0,1111	0,1053	0,2752

Maximin Method

- Pessimistic attitude
- Selects the maximum (across alternatives) of the minimum (across attributes) normalized performance values, which is called "the best of the worst"

r(1)	Price	Comfort	Perf.	Design	
W	0,3333	0,2667	0,2	0,2	min
a_1	0,0917	0,2143	0,1667	0,1579	0,0917
a_2	0,1101	0,2143	0,1111	0,1579	0,1101
a 3	0,1101	0,1429	0,1667	0,1579	0,1101
a_4	0,1376	0,1429	0,1667	0,1053	0,1053
a 5	0,1376	0,1429	0,1111	0,1579	0,1111
a_6	0,1376	0,0714	0,1667	0,1579	0,0714
a 7	0,2752	0,0714	0,1111	0,1053	0,0714

CONJUNCTIVE METHOD

- An alternative is accepted if each dimension meets a set of preset standards or thresholds. If at least one dimension doesn't meet the set, it is unacceptable.
- ${\color{blue} \circ}$ "a_i is acceptable if x_{i1} and x_{i2} ... ,and x_{ik} is acceptable" for the k attributes

	Price	Comfort	Perf.	Design
Weight	5	4	3	3
a_1	300	Е	Е	S
a_2	250	Е	A	S
a 3	250	A	Е	S
a_4	200	A	Е	О
a 5	200	A	A	S
a 6	200	W	Е	S
a 7	100	W	A	О
E-Evcellen	t. A.Averag	o W·Weak	S.Superior (Ordinary

250 m.u. or cheaper, excellent comfort, excellent perf., and superior design:

All alternatives are unacceptable

250 m.u. or cheaper, average comfort, average perf., and ordinary design:

a2, a3, a4, and a5 are acceptable

DISJUNCTIVE METHOD

- An alternative is accepted if it scores sufficiently high on at least one dimension. If no dimension meets a set of preset standards, it is unacceptable.
- o " a_i is acceptable if x_{i1} or x_{i2} ... ,or x_{ik} is sufficiently high" for the k attributes

	Price	Comfort	Perf.	Design		
Weight	5	4	3	3		
a_1	300	Е	Е	S		
a_2	250	Е	A	S		
a 3	250	A	Е	S		
a_4	200	A	Е	О		
a 5	200	A	A	S		
a 6	200	W	Е	S		
a 7	100	W	A	О		
E:Excellen	E:Excellent; A:Average, W:Weak, S:Superior, O:Ordinary					

250 m.u. or cheaper, excellent comfort, excellent perf., or superior design:

All alternatives are acceptable

MEDIAN RANKING METHOD

- Adds all attributewise ranks (ranks from each attribute) and ranks them in ascending order.
- If there is a tie for some alternatives the median value of the ranks of them is used.

	Price	Comfort	Perf.	Design	Total
<i>a</i> ₁	7	1,5	2,5	3	14
a_2	5,5	1,5	6	3	16
a 3	5,5	4	2,5	3	15
a_4	3	4	2,5	6,5	16
a 5	3	4	6	3	16
a 6	3	6,5	2,5	3	15
a 7	1	6,5	6	6,5	20

REFERENCES

 Lecture notes of "Prof. Dr. Y. İlker Topçu", <u>http://web.itu.edu.tr/topcuil/</u>