STATISTICAL QUALITY CONTROL

LECTURE NOTES

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THE MEANING OF QUALITY AND QUALITY IMPROVEMENT

Dimensions of Quality (Garvin, 1987)

The quality of a product can be described and evaluated in several ways.

1. ***Performance*** (will the product do the intended job?)
   For example, you could evaluate spreadsheet software packages for a PC to determine which data manipulation operations they perform. You may discover that one outperforms another with respect to the execution speed.
2. ***Reliability*** (how often does the product fail?)
   For example, you should expect that an automobile will require occasional repair, but if the car requires frequent repair, we say that it is unreliable.
3. ***Durability*** (how long does the product last?)
   This is the effective service life of the product. Customers obviously want products that perform satisfactorily over a long period of time. The automobile and major appliance industries are examples of businesses where this dimension of quality is very important to most customers.
4. ***Serviceability*** (how easy is it to repair the product?)
   Examples include the appliance and automobile industries and many types of service industries (how long did it take a credit card company to correct an error in your bill?).
5. ***Aesthetics*** (what does the product look like?)
   This is the visual appeal of the product such as style, color, shape, packaging alternatives and other sensory features. For example, soft drink beverage manufacturers have relied on the visual appeal of their packaging to differentiate their product from other competitors.
6. ***Features*** (what does the product do?)
   Usually, customers associate high quality with products that have added features; that is, those that have features beyond the basic performance of the competition.
7. ***Perceived Quality*** (what is the reputation of the company or its product?)
   In many cases, customers rely on the past reputation of the company concerning quality of its products. For example, if you make regular business trips using a particular airline, and the flight almost always arrives on time and the airline company does not lose or damage your luggage, you will probably prefer to fly on that carrier instead of its competitors.
8. ***Conformance to Standards*** (is the product made exactly as the designer intended?)
   Manufactured parts that do not exactly meet the designer’s requirements can cause significant quality problems when they are used as the components of a more complex assembly. An automobile consists of several thousand parts. If each one is just slightly too big or too small, many of the components will not fit together properly, and the vehicle may not perform as the designer intended.

The ***traditional*** *definition of quality* is based on the viewpoint that products and services must meet the requirements of those who use them.

- *Quality* means fitness for use.

There are two general aspects of fitness for use: **quality of design** and **quality of conformance.**

Quality of design→For example, all automobiles have as their basic objective providing safe transportation for the consumer. However, automobiles differ with respect to size, appearance, and performance. These differences are the result of intentional design differences between the types of automobiles. These design differences include the types of materials used in construction, specifications on the components, reliability obtained through engineering development of engines, and other accessories or equipment.

Quality of conformance→How well the product conforms to the specifications required by the design. Quality of conformance is influenced by the choice of manufacturing processes, the training and supervision of the workforce, the type of quality assurance system used (process controls, tests, inspection activities, etc.), the extent to which these quality assurance procedures are followed, and the motivation of the workforce to achieve quality.

The ***modern*** definition of quality

- *Quality* is inversely proportional to variability.

Notice that this definition implies that if variability in the important characteristics of a product decreases, the quality of the product increases. How? Fewer repairs and warranty claims (garanti talebi) means less rework and the reduction of wasted time, effort, and money. Thus quality truly is inversely proportional to variability.

- *Quality improvement* is the reduction of variability in processes and products.

- Every product possesses a number of elements that jointly describe what the user or consumer thinks of as quality. These parameters are often called *quality characteristics*.

Since variability can only be described in statistical terms, statistical methods play a central role in quality improvement efforts. In the application of statistical methods, it is typical to classify data on quality characteristics as either **attributes** or **variables** data.

Variables data→ usually continuous measurements such as length, voltage or viscosity

Attributes data→ usually discrete data, often taking the form of counts.

- For a manufactured product, the *specifications* are the desired measurements for the quality characteristics on the components and subassemblies that make up the product, as well as the desired values for the quality characteristics in the final product.

- The largest allowable value for a quality characteristic is called the *upper specification limit (USL).*

- The smallest allowable value for a quality characteristic is called the *lower specification limit (LSL).*

- A value of a measurement that corresponds to the desired value for that quality characteristic is called the *nominal* or *target value* for that characteristic.

STATISTICAL METHODS FOR QUALITY IMPROVEMENT

There are three major areas useful in quality improvement.

- **Statistical Process Control** (A control chart is one of the primary techniques of Statistical Process Control or SPC)
- **Design of Experiments** (extremely helpful in discovering the key variables influencing the quality characteristics of interest in the process. A designed experiment is an approach to systematically varying the controllable input factors in the process and determining the effect these factors have on the output product parameters. Statistically designed experiments are invaluable in reducing the variability in the quality characteristics and in determining the levels of the controllable variables that optimize process performance)
- **Acceptance Sampling** (closely connected with inspection and testing of product, which is one of the earliest aspects of quality control. Inspection can occur at many points in a process. Acceptance sampling, defined as the inspection and classification of a sample of units selected at random from a larger batch or lot and the ultimate decision about disposition of the lot, usually occurs at two points: incoming row materials or components, or final production)

STATISTICAL PROCESS CONTROL

The manufacturing processing must be stable and that all individuals involved with the process (including operators, engineers, quality-assurance personnel, and management) must continuously seek to improve process performance and reduce variability on key parameters. On-line statistical process control (SPC) is a primary tool for achieving this objective.

If a product is to meet customer requirements, generally it should be produced by a process that is stable or repeatable. More precisely, the process must be capable of operating with little variability around the target or nominal dimensions of the product's quality characteristics. SPC is a powerful collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability

The basic SPC problem solving tools are called '*magnificent seven*' and these seven tools are:

1. Histogram or stem-and-leaf display
    - The shape shows the nature of the distribution of the data
    - The central tendency (average) and variability are easily seen
    - Specification limits can be used to display the capability of the process

2. Check sheet

    In the early stages of an SPC implementation, it will often become necessary to collect either historical or current operating data about the process under investigation. A check sheet can be very useful in this data collection activity.
    - Simplifies data collection and analysis
    - Spots problem areas by frequency of location, type, or cause

3. Pareto chart
    - Identifies most significant problems to be worked first

4. Cause and effect (Fishbone) diagram
    - All contributing factors and their relationship are displayed
    - Identifies problem areas where data can be collected and analyzed

5. Defect concentration diagram

    A picture of the unit showing all relevant views. The various types of defects are drawn on the picture, and the diagram is analyzed to determine whether the location of the defects on the unit conveys any useful information about the potential causes of the defects.

6. Scatter diagram
    - Identifies the relationship between two variables
    - A positive, negative, or no relationship can be easily detected

7. Control chart
    - Helps reduce variability
    - Monitors performance over time
    - Allows process corrections to prevent rejections
    - Trends and out of control conditions are immediately detected

Chance and Assignable Causes of Quality Variation

In many production processes, regardless of how well designed or carefully maintained it is, a certain amount of inherent or natural variability will always exist. This natural variability is often called a "stable system of chance causes." A process that is operating with only **chance causes of variation** present is said to be **in statistical control**. In other words, the chance causes are an inherent part of the process.

Other kinds of variability may occasionally be present in the output of a process. This variability in key quality characteristics usually arises from three sources:

Improperly adjusted machines

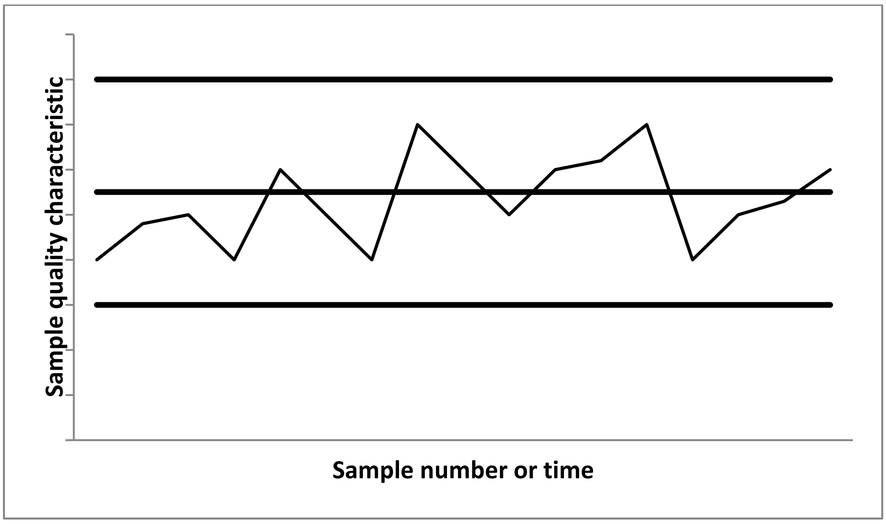
Operator errors

Defective raw material

Such variability is generally large when compared to the natural variability, and it usually represents an unacceptable level of process performance. We refer to these sources of variability that are not part of the chance cause pattern as "assignable causes." A process that is operating in the presence of **assignable causes** is said to be **out of control**.

A major objective of SPC is to quickly detect the occurrence of assignable causes of process shifts so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured. The *control chart* is an on-line process control technique widely used for this purpose.

STATISTICAL BASIS OF THE CONTROL CHART



The chart contains a **center line (CL)** that presents the average value of the quality characteristic corresponding to the in-control state (that is only chance causes are present)

Two other horizontal lines, called upper control limit (UCL) and the lower control limit (LCL), are also shown on the chart. These control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them.

As long as the points plot within the control limits, the process is assumed to be in control and no action is necessary. However, a point that plots outside of the control limits is interpreted as evidence that the process is out of control, and investigation and corrective action are required to find and eliminate the assignable cause or causes responsible for this behavior.

Even if all the points plot inside the control limits, if they behave in a systematic or nonrandom manner, then this could be an indication that the process is out of control. For example, if 18 of the last 20 points plotted above the center line but below the UCL and only two of these points plotted below the CL but above the LCL, we could be very suspicious that something was wrong. If the process is in control, all the plotted points should have an essentially random pattern. (see pages 195-197 in the text book)

There is a close connection between control charts and hypothesis testing.

Suppose that the vertical axis is the sample average, $\bar{x}$. If the current value of $\bar{x}$ plots between the control limits, we conclude that the process mean is in control; that is, it is equal to some value μ_0. If $\bar{x}$ exceeds either control limit, we conclude that the process mean is out of control; that is, it is equal to some value $\mu_1 \neq \mu_0$.

H_0: $\mu = \mu_0$ (process mean is in control) If $LCL \leq \bar{x} \leq UCL$, conclude H_0 ($\mu = \mu_0$).

H_1: $\mu \neq \mu_0$ (process mean is out of control) If $\bar{x} < LCL$ or $\bar{x} > UCL$, conclude H_1 ($\mu = \mu_1 \neq \mu_0$).

$UCL = \mu_0 + L\sigma_0$

$CL = \mu_0$ (target or nominal value)

$LCL = \mu_0 - L\sigma_0$

$\boldsymbol{\mu_0}$ and $\boldsymbol{\sigma_0}$ are the in control process parameters

L is the distance of the control limits from the centerline, expressed in standard deviation units.

The general theory of control charts was first proposed by Dr. Walter S. Shewhart, and control charts developed according to these principles are often called Shewhart Control Charts

Control Charts for central tendency and variability are collectively called ***variables control charts***.

Many quality characteristics are not measured on a continuous scale or even a quantitative scale. In these cases, we may judge each unit of product as either conforming or nonconforming on the basis of whether or not it possesses certain attributes, or we may count the number of nonconformities (defects) appearing on a unit of product. Control charts for such quality characteristics are called ***attributes control charts.***

How we analyze the performance of a control chart?

Type I error: Concluding the process is out of control when it is really in control

Type II error: Concluding the process is in control when it is really out of control

Following will be discussed later in detail.

Operating Characteristic Curve (OC Curve): Displays probability of type II error

Average Run Length (ARL)

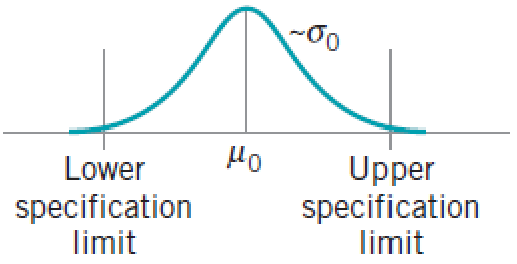
Average Time to Signal (ATS)

CONTROL CHARTS FOR VARIABLES

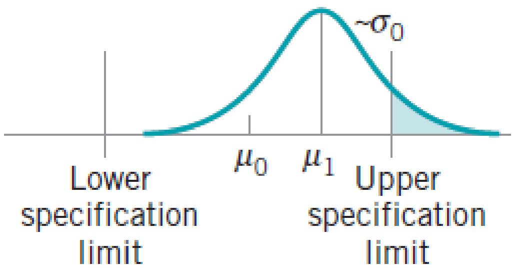
The ***process average or mean quality level*** is usually controlled with the control chart for means, or the **$\bar{x}$ chart.**

The ***process variability*** can be monitored with either a control chart for the standard deviation, called the ***S chart***, or a control chart for the range, called an ***R chart***.

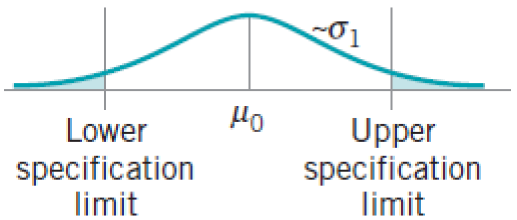
It is important to maintain control over both the process mean and process variability.



Mean and standard deviation at nominal levels



Process mean $\mu_1 > \mu_0$(nominal level)



Process standard deviation $\sigma_1 > \sigma_0$(nominal level)

Control Charts for $\bar{x}$ and R

Suppose that a quality characteristic is normally distributed with mean μ and standard deviation σ, where both μ and σ are known.

$X \sim N(\mu,\sigma^2)$ (X: quality characteristic)

$x_1, x_2, x_3, \ldots, x_n$ random sample of size n

$\Rightarrow \bar{X} \sim N(\mu,\ \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n})$

$\bar{x} = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$

Remember that;

$$P\left(\underbrace{\mu - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{LCL} \leq \bar{X} \leq \underbrace{\mu + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{UCL}\right) = 1-\alpha$$

It is customary to replace $z_{\alpha/2}$ by 3, so that 3-sigma limits are employed. (If $z_{\alpha/2} = 3 \Rightarrow \alpha = ?$)

Charts based on standard values (*μ and σ are known)*

When it is possible to specify standard values for the process mean (μ) and standard deviation (σ), we may use these standards to establish the control charts for $\bar{x}$ and R without analysis of past data.

Control Limits for the $\bar{x}$ chart

$$UCL = \mu + 3\frac{\sigma}{\sqrt{n}} = \mu + A\,\sigma$$

$$CL = \mu$$

$$LCL = \mu - 3\frac{\sigma}{\sqrt{n}} = \mu - A\,\sigma$$

Control Limits for the R chart

$$UCL = \underbrace{d_2\,\sigma}_{E(R)} + 3\ \underbrace{d_3\,\sigma}_{\sigma_R} = \sigma\ \underbrace{(d_2 + 3\,d_3)}_{D_2}$$

$$CL = d_2\,\sigma$$

$$LCL = d_2\,\sigma - 3\,d_3\,\sigma = \sigma\ \underbrace{(d_2 - 3\,d_3)}_{D_1}$$

** Values of A, d_2, d_3, D_1 and D_2 are available for various sample sizes. (See appendix VI, p.702 in the text book)

$$W = \frac{R}{\sigma}$$

W: Relative range

R: Range of a sample from a Normal distribution

σ: Standard deviation of that Normal distribution

E(W) = d_2 ➔ $E\left(\frac{R}{\sigma}\right)$ = d_2 ➔ E(R) = $d_2\,\sigma$

Function of sample size, n

➔ $E\left(\frac{R}{d_2}\right)$ = σ ➔ $E\left(\frac{\bar{R}}{d_2}\right)$ = σ ($\bar{R}$ is the average of m preliminary samples)

➔ $\hat{\sigma} = {}^{\bar{R}}/_{d_2}$ is the estimator of σ.

V(W) = d_3^2 ➔ $V\left(\frac{R}{\sigma}\right) = d_3^2$ ➔ V(R) = $d_3^2\sigma^2$ ➔ $\underbrace{\sqrt{V(R)}}_{\sigma_R} = d_3\sigma$

<u>Charts based on estimated values (*μ and σ are unknown*)</u>

μ and σ must be estimated from preliminary samples or subgroups taken when the process is thought to be **in control.**

Suppose we have m samples, each containing n observations.

Let $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$,…., $\bar{x}_m$ be the average of each sample.

$\bar{\bar{x}} = \frac{\bar{x}_1+\bar{x}_2+\bar{x}_3+\cdots+\bar{x}_m}{m}$ is the best estimator of μ. Thus, CL = $\bar{\bar{x}}$ on the $\bar{x}$ chart.

$(\bar{\bar{x}}: grand\ average)$

If x_1, x_2, x_3,….,x_n is a sample of size n, then

$$R = x_{max} - x_{min}$$

Let R_1, R_2, R_3,…,R_m be the ranges of the m samples

$$\bar{R} = \frac{R_1+R_2+R_3+\cdots+R_m}{m}$$

<u>*Control Limits for the $\bar{x}$ chart*</u>

$$UCL = \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + 3\frac{\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}$$

$$CL = \bar{\bar{x}}$$

$$LCL = \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - 3\frac{\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}$$

⇨ $A_2 = \frac{3}{d_2\sqrt{n}}$ A_2: function of n d_2: function of n

Control Limits for the R chart

$$UCL = \bar{R} + 3\,\hat{\sigma}_R \quad = \quad \bar{R} + 3\ d_3 \frac{\bar{R}}{d_2} \quad = \quad \bar{R}\underbrace{\left(1 + \frac{3\ d_3}{d_2}\right)}_{D_4}$$

$$CL = \bar{R}$$

function of n

$$LCL = \bar{R} - 3\,\hat{\sigma}_R \quad = \quad \bar{R} - 3\ d_3 \frac{\bar{R}}{d_2} \quad = \quad \bar{R}\underbrace{\left(1 - \frac{3\ d_3}{d_2}\right)}_{D_3}$$

These limits are ***trial control limits.***

** Values of A_2, D_3 and D_4 are available for various sample sizes. (See appendix VI, p.702 in the text book)

CLASS EXERCISE 1 (D.C. Montgomery, 6th ed., p252)

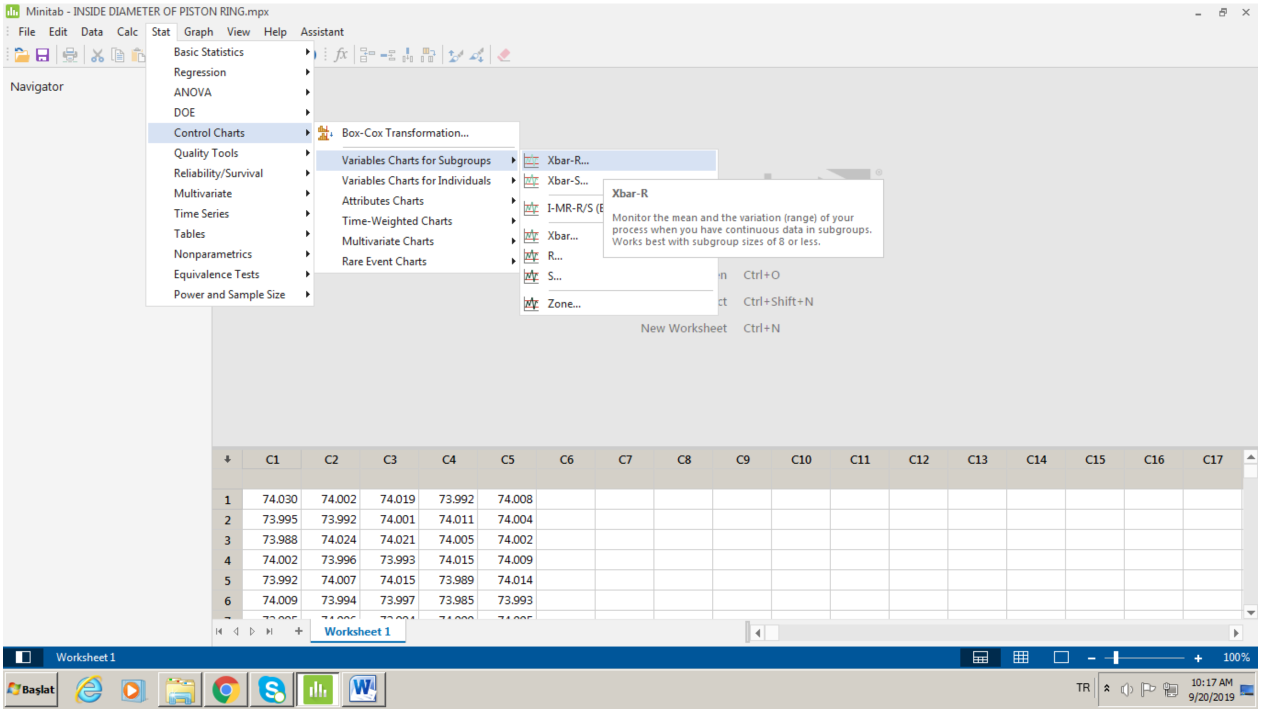
Piston rings for an automotive engine are produced by a forging process. We wish to establish statistical control of the inside diameter of the rings manufactured by this process using $\bar{x}$ and R charts. Twenty-five samples, each of size five, have been taken when we think the process is in control. The inside diameter measurement data from these samples are shown in the following table. (Available in the text book, Montgomery 6th Ed, p252, Table 6.3)

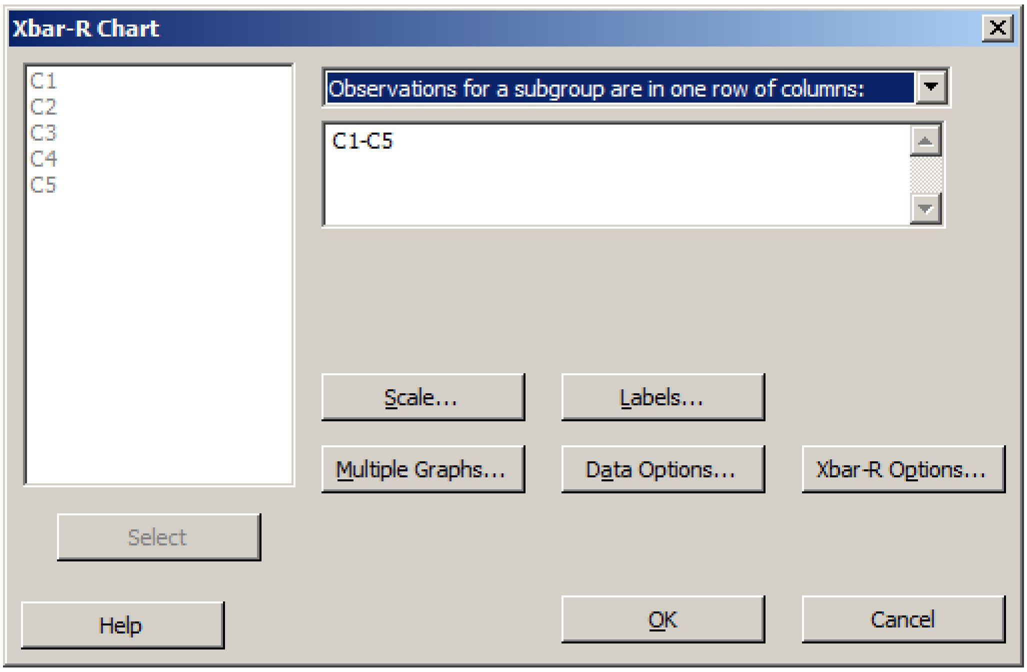
Sample number	**Observations**					$\bar{x}_i$	**R_i**
1	74.030	74.002	74.019	73.992	74.008	**74.010**	**0.038**
2	73.995	73.992	74.001	74.011	74.004	**74.001**	**0.019**
3	73.988	74.024	74.021	74.005	74.002	**74.008**	**0.036**
4	74.002	73.996	73.993	74.015	74.009	**74.003**	**0.022**
5	73.992	74.007	74.015	73.989	74.014	**74.003**	**0.026**
6	74.009	73.994	73.997	73.985	73.993	**73.996**	**0.024**
7	73.995	74.006	73.994	74.000	74.005	**74.000**	**0.012**
8	73.985	74.003	73.993	74.015	73.988	**73.997**	**0.030**
9	74.008	73.995	74.009	74.005	74.004	**74.004**	**0.014**
10	73.998	74.000	73.990	74.007	73.995	**73.998**	**0.017**
11	73.994	73.998	73.994	73.995	73.990	**73.994**	**0.008**
12	74.004	74.000	74.007	74.000	73.996	**74.001**	**0.011**
13	73.983	74.002	73.998	73.997	74.012	**73.998**	**0.029**
14	74.006	73.967	73.994	74.000	73.984	**73.990**	**0.039**
15	74.012	74.014	73.998	73.999	74.007	**74.006**	**0.016**
16	74.000	73.984	74.005	73.998	73.996	**73.997**	**0.021**
17	73.994	74.012	73.986	74.005	74.007	**74.001**	**0.026**
18	74.006	74.010	74.018	74.003	74.000	**74.007**	**0.018**
19	73.984	74.002	74.003	74.005	73.997	**73.998**	**0.021**
20	74.000	74.010	74.013	74.020	74.003	**74.009**	**0.020**
21	73.982	74.001	74.015	74.005	73.996	**74.000**	**0.033**
22	74.004	73.999	73.990	74.006	74.009	**74.002**	**0.019**
23	74.010	73.989	73.990	74.009	74.014	**74.002**	**0.025**
24	74.015	74.008	73.993	74.000	74.010	**74.005**	**0.022**
25	73.982	73.984	73.995	74.017	74.013	**73.998**	**0.035**

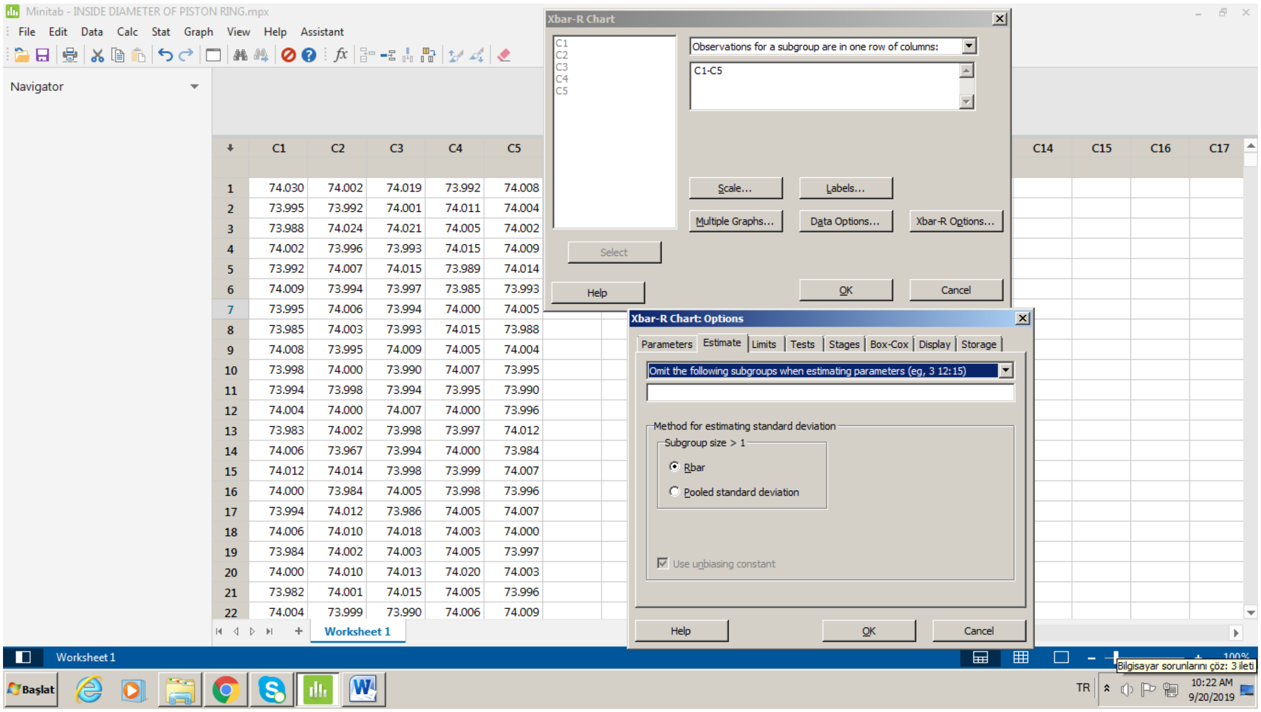
When setting up $\bar{x}$ and R control charts, it is best to begin with the R chart. Because the control limits on the $\bar{x}$ chart depend on the process variability, unless process variability is in control, these limits will not have much meaning.

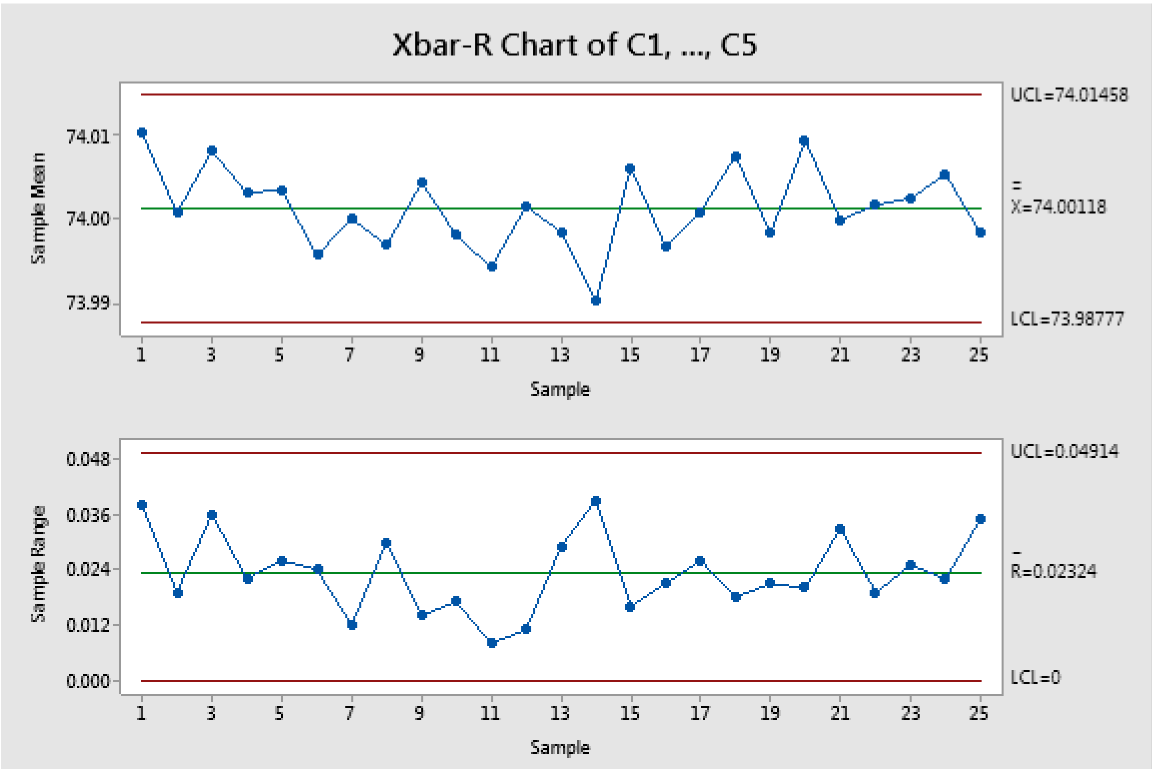
SOLUTION (CLASS EXC 1)

Solution by minitab;









Since both the $\bar{x}$ and R charts exhibit control, we would conclude that the process is in control at the stated levels and adopt the trial control limits for use in on-line statistical process control.

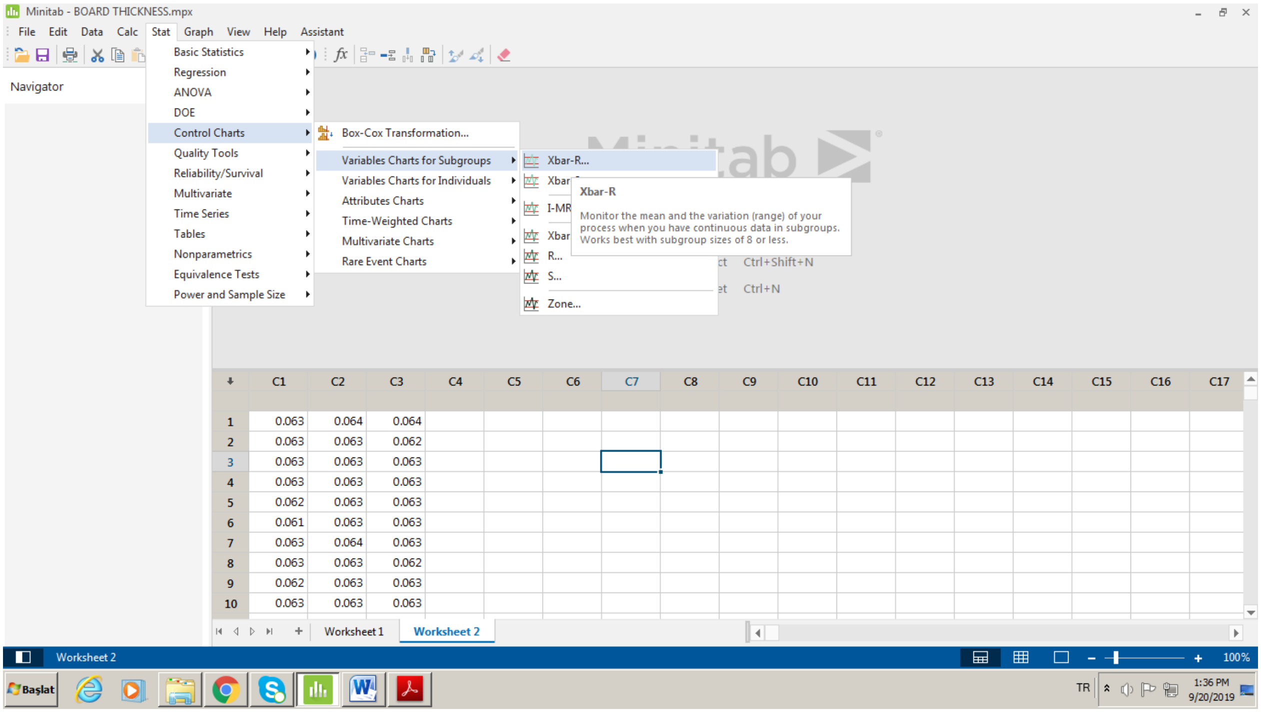
CLASS EXERCISE 2 (D.C. Montgomery, 3rd ed., ex 5.4, p236)

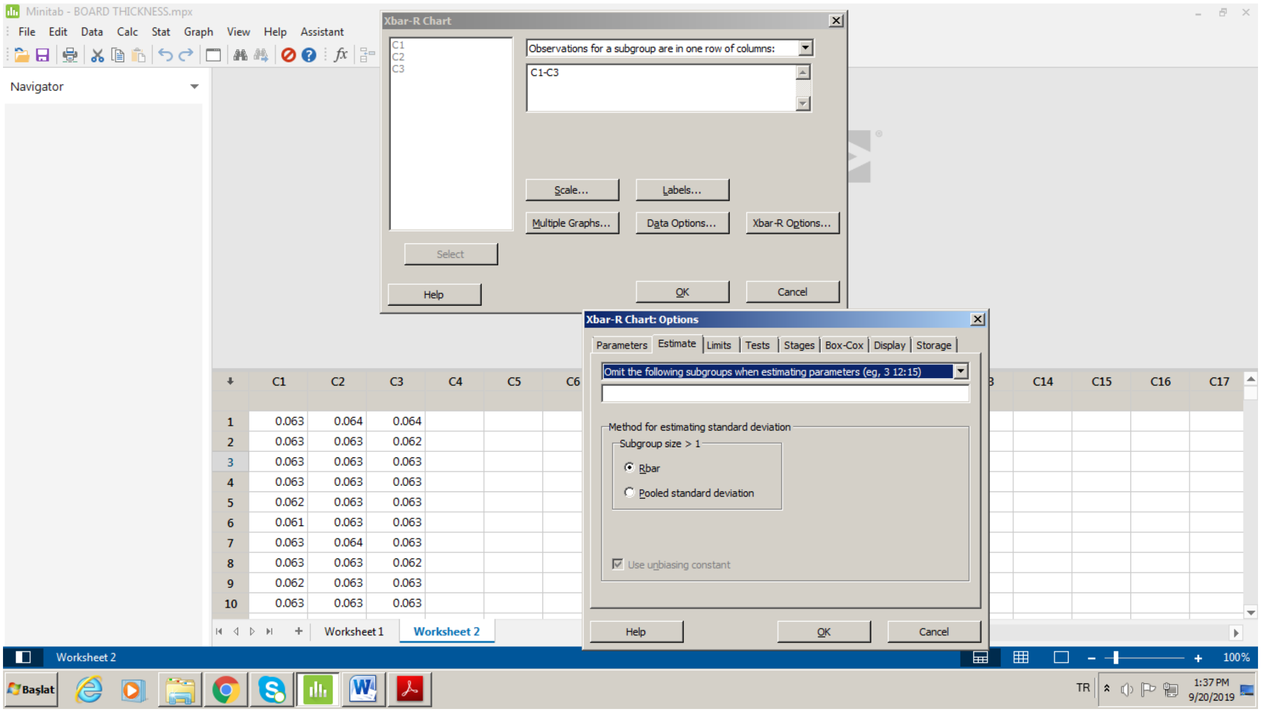
The thickness of a printed circuit board is an important quality parameter. Data on board thickness (in inches) are given for 25 samples of three boards each. Set up $\bar{x}$ and R control charts. Is the process in statistical control?

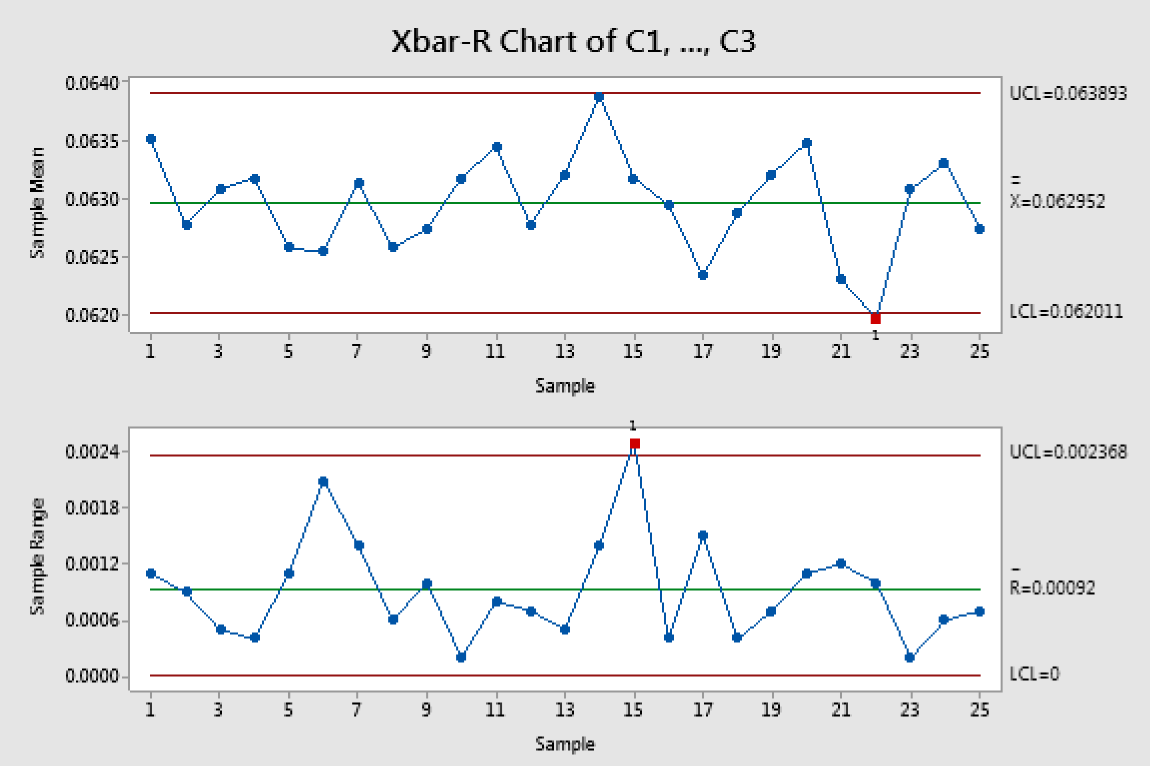
Sample number	**Observations**			$\bar{x}_i$	R_i
1	0.0629	0.0636	0.0640	**0.0635**	**0.0011**
2	0.0630	0.0631	0.0622	**0.0628**	**0.0009**
3	0.0628	0.0631	0.0633	**0.0631**	**0.0005**
4	0.0634	0.0630	0.0631	**0.0632**	**0.0004**
5	0.0619	0.0628	0.0630	**0.0626**	**0.0011**
6	0.0613	0.0629	0.0634	**0.0625**	**0.0021**
7	0.0630	0.0639	0.0625	**0.0631**	**0.0014**
8	0.0628	0.0627	0.0622	**0.0626**	**0.0006**
9	0.0623	0.0626	0.0633	**0.0627**	**0.0010**
10	0.0631	0.0631	0.0633	**0.0632**	**0.0002**
11	0.0635	0.0630	0.0638	**0.0634**	**0.0008**
12	0.0623	0.0630	0.0630	**0.0628**	**0.0007**
13	0.0635	0.0631	0.0630	**0.0632**	**0.0005**
14	0.0645	0.0640	0.0631	**0.0639**	**0.0014**
15	0.0619	0.0644	0.0632	**0.0632**	**0.0025**
16	0.0631	0.0627	0.0630	**0.0629**	**0.0004**
17	0.0616	0.0623	0.0631	**0.0623**	**0.0015**
18	0.0630	0.0630	0.0626	**0.0629**	**0.0004**
19	0.0636	0.0631	0.0629	**0.0632**	**0.0007**
20	0.0640	0.0635	0.0629	**0.0635**	**0.0011**
21	0.0628	0.0625	0.0616	**0.0623**	**0.0012**
22	0.0615	0.0625	0.0619	**0.0620**	**0.0010**
23	0.0630	0.0632	0.0630	**0.0631**	**0.0002**
24	0.0635	0.0629	0.0635	**0.0633**	**0.0006**
25	0.0623	0.0629	0.0630	**0.0627**	**0.0007**

SOLUTION (CLASS EXC. 2)

Solution by minitab;

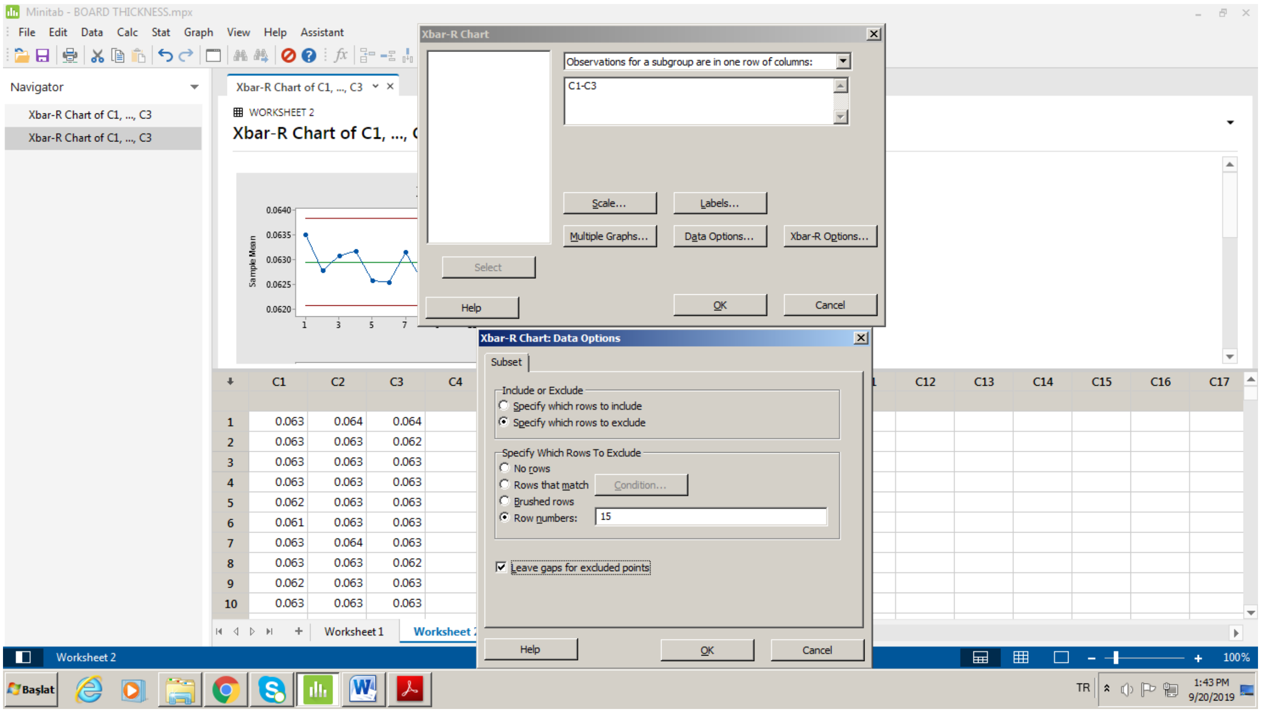


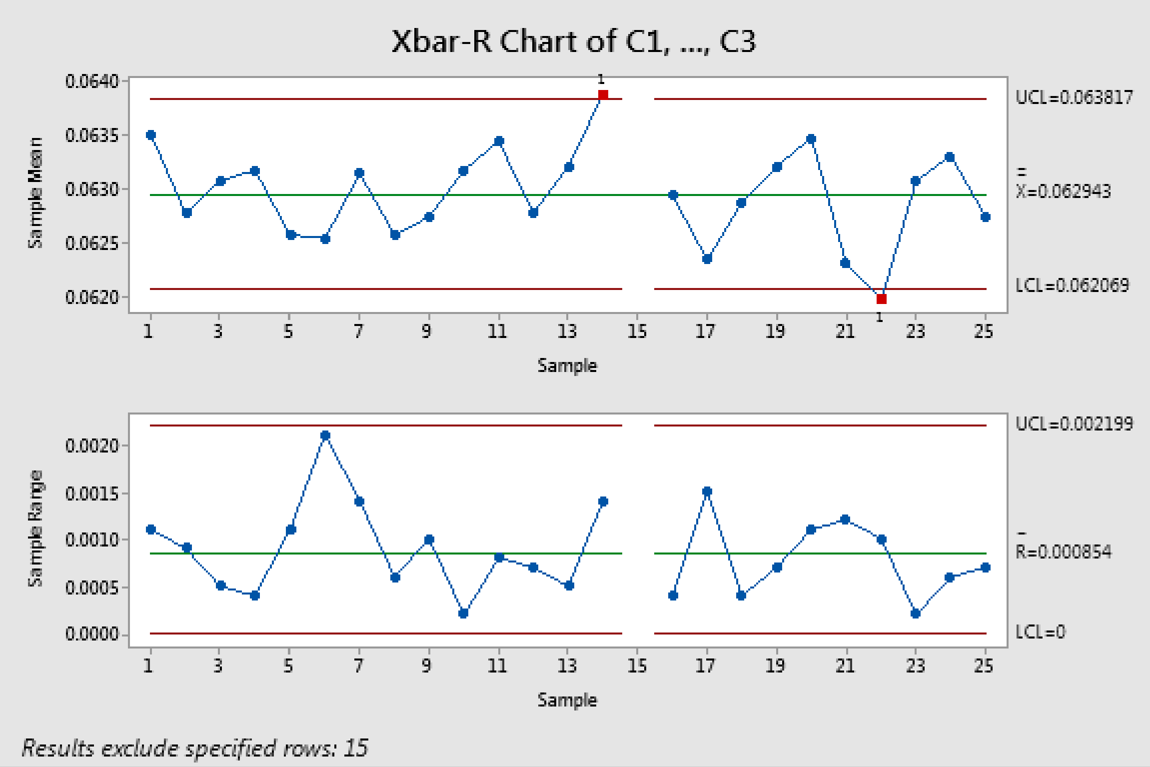




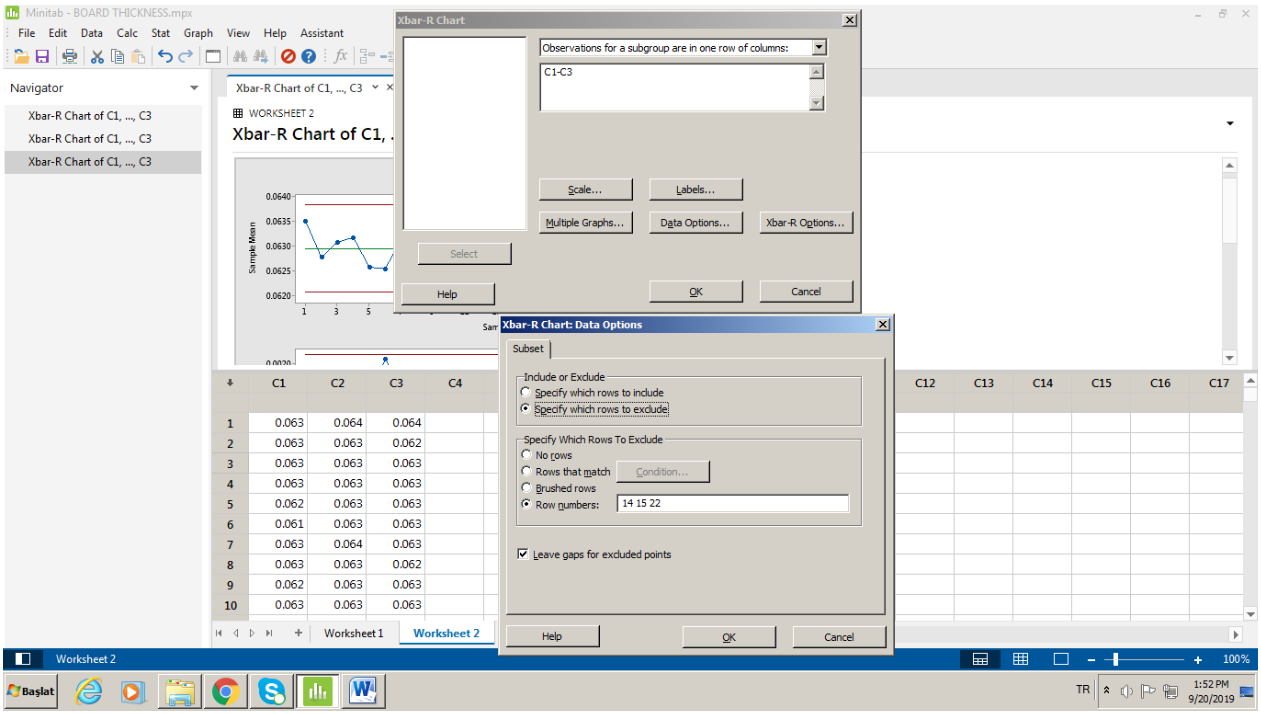
When the R chart is examined it is observed that the 15th point is out of the control limits. That is, the process variability is out of control. We should examine this out of control point, looking for an assignable cause. If an assignable cause is found, the point is discarded and the trail control limits are recalculated, using only the remaining points.

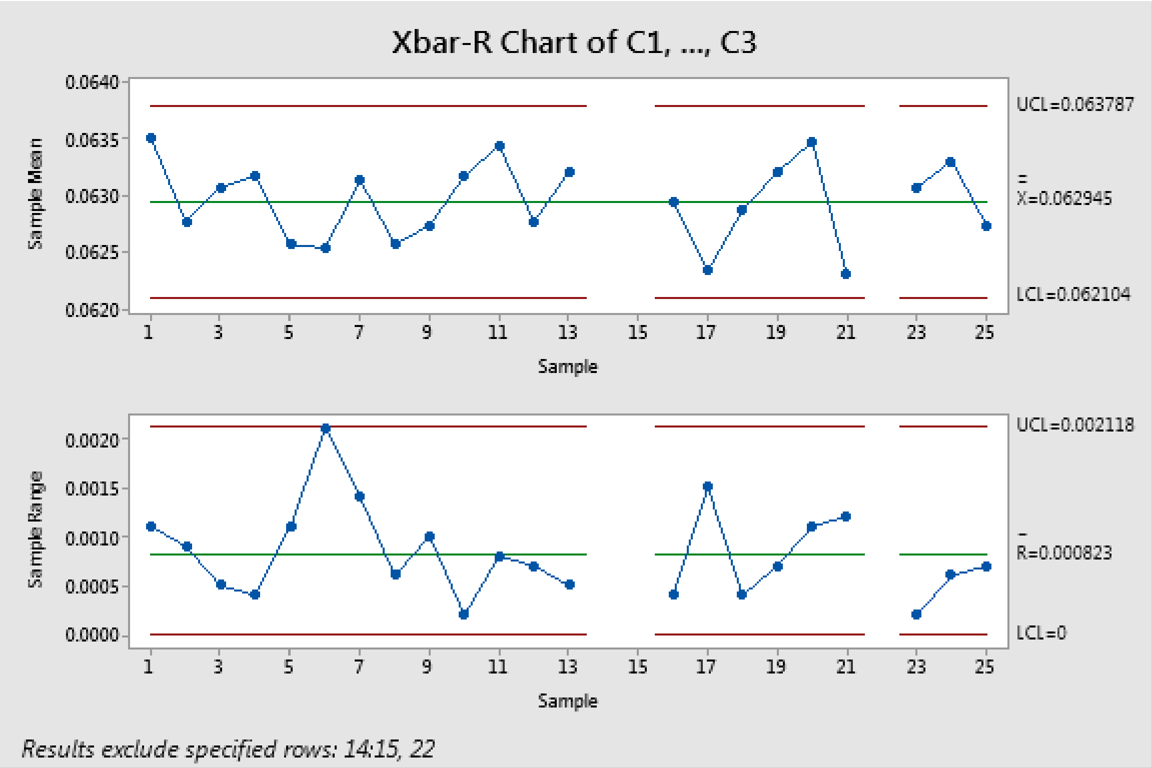
Suppose that an assignable cause is found for point 15. After discarding point 15, revised control limits are recalculated.





Now, process variability is in control. Point 15 is discarded. However, points 14 and 22 are outside the limits in the $\bar{x}$ chart. That is, the process mean is out of control. Suppose that assignable cause is found for both of the out of control points. Then, we discard these points and recalculate the control limits for both $\bar{x}$ and R charts.





The revised $\bar{x}$ chart and R chart indicate that process variability and process mean are in control. Points 14,15 and 22 are discarded assuming that an assignable cause is found for each point.

Control Charts for $\bar{x}$ and S

Generally, $\bar{x}$ and S charts are preferable to their more familiar counterparts, $\bar{x}$ and R charts, when either;

1- The sample size, n is moderately large, say n > 10 or 12. (Recall that the range method for estimating σ loses statistical efficiency for moderate to large samples)

2- The sample size, n is variable.

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator (UE) of σ^2. However, S is not an UE of σ.

If underlying distribution is Normal, E(S) = c_4 σ a constant depending on the sample size, n.

$$\sqrt{V(S)} = \sigma\sqrt{1 - c_4^2} \qquad c_4 = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma[n/2]}{\Gamma[(n-1)/2]}$$

E(S) = c_4 σ $\Longrightarrow$ CL for the S chart when the parameters are known

Control Limits for the S chart (parameters are known)

$$UCL = c_4\sigma + 3\sigma\sqrt{1-c_4^2} = \sigma\underbrace{\left(c_4 + 3\sqrt{1-c_4^2}\right)}_{B_6} = B_6\sigma$$

$$CL = c_4\sigma$$

$$LCL = c_4\sigma - 3\sigma\sqrt{1-c_4^2} = \sigma\underbrace{\left(c_4 - 3\sqrt{1-c_4^2}\right)}_{B_5} = B_5\sigma$$

** Values of B_5 and B_6 are available for various sample sizes. (See appendix VI, p.702 in the text book)

Control Limits for the S chart (parameters are unknown)

$$UCL = \bar{S} + 3\frac{\bar{S}}{c_4}\sqrt{1-c_4^2} = \bar{S}\underbrace{\left(1 + 3\frac{1}{c_4}\sqrt{1-c_4^2}\right)}_{B_4} = B_4\bar{S}$$

$$CL = \bar{S}$$

$$LCL = \bar{S} - 3\frac{\bar{S}}{c_4}\sqrt{1-c_4^2} = \bar{S}\underbrace{\left(1 - 3\frac{1}{c_4}\sqrt{1-c_4^2}\right)}_{B_3} = B_3\bar{S}$$

$E(S) = c_4\,\sigma \;\Rightarrow E(\bar{S}) = c_4\,\sigma$ where $\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_i$ **m:** number of preliminary samples, each of size n

⇨ $E\left(\frac{\bar{S}}{c_4}\right) = \sigma \Rightarrow \frac{\bar{S}}{c_4}$ is an unbiased estimator of σ.

Control Limits on the corresponding $\bar{x}$ chart

$$UCL = \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + 3\frac{\bar{S}/c_4}{\sqrt{n}} = \bar{\bar{x}} + \frac{3}{\underbrace{c_4\sqrt{n}}_{A_3}}\bar{S} = \bar{\bar{x}} + A_3\bar{S}$$

$$CL = \bar{\bar{x}}$$

$$LCL = \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - 3\frac{\bar{S}/c_4}{\sqrt{n}} = \bar{\bar{x}} - \frac{3}{\underbrace{c_4\sqrt{n}}_{A_3}}\bar{S} = \bar{\bar{x}} - A_3\bar{S}$$

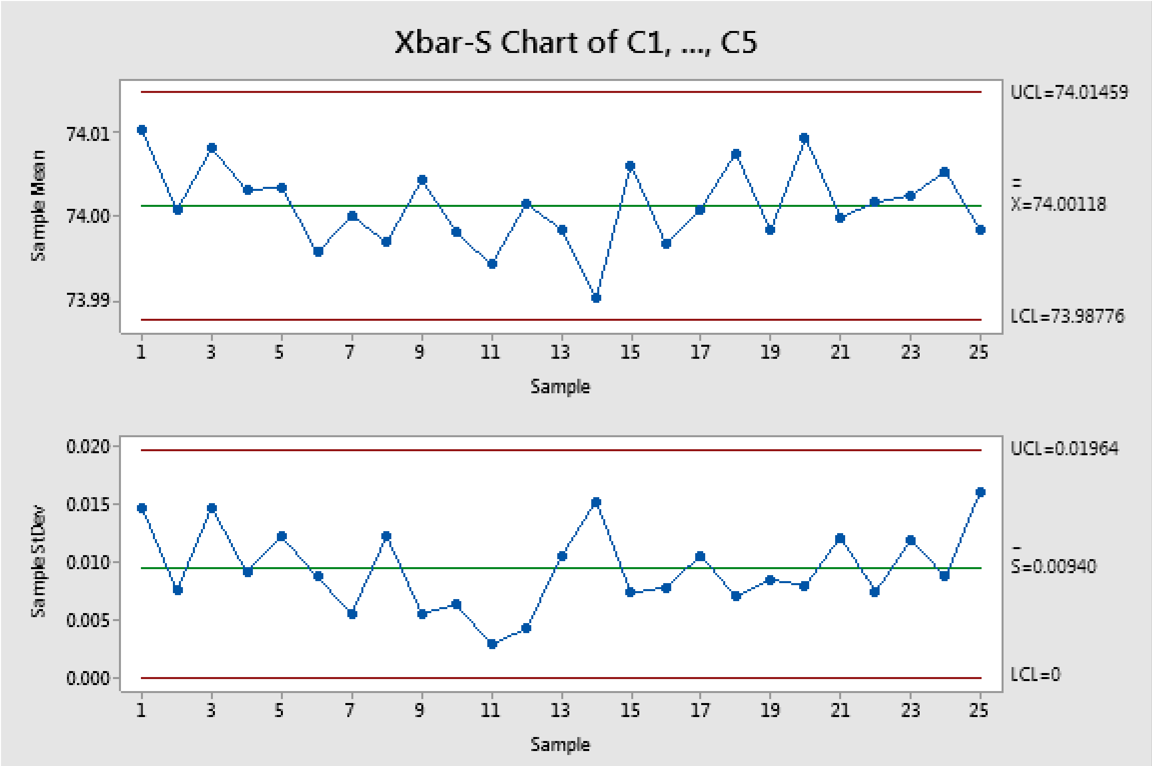
** Values of A_3 are available for various sample sizes. (See appendix VI, p.702 in the text book)

CLASS EXERCISE3(CLASS EXC. 1, with S Chart)

Sample number	**Observations**					$\overline{x}_i$	S_i
1	74.030	74.002	74.019	73.992	74.008	**74.010**	**0.0148**
2	73.995	73.992	74.001	74.011	74.004	**74.001**	**0.0075**
3	73.988	74.024	74.021	74.005	74.002	**74.008**	**0.0147**
4	74.002	73.996	73.993	74.015	74.009	**74.003**	**0.0091**
5	73.992	74.007	74.015	73.989	74.014	**74.003**	**0.0122**
6	74.009	73.994	73.997	73.985	73.993	**73.996**	**0.0087**
7	73.995	74.006	73.994	74.000	74.005	**74.000**	**0.0055**
8	73.985	74.003	73.993	74.015	73.988	**73.997**	**0.0123**
9	74.008	73.995	74.009	74.005	74.004	**74.004**	**0.0055**
10	73.998	74.000	73.990	74.007	73.995	**73.998**	**0.0063**
11	73.994	73.998	73.994	73.995	73.990	**73.994**	**0.0029**
12	74.004	74.000	74.007	74.000	73.996	**74.001**	**0.0042**
13	73.983	74.002	73.998	73.997	74.012	**73.998**	**0.0105**
14	74.006	73.967	73.994	74.000	73.984	**73.990**	**0.0153**
15	74.012	74.014	73.998	73.999	74.007	**74.006**	**0.0073**
16	74.000	73.984	74.005	73.998	73.996	**73.997**	**0.0078**
17	73.994	74.012	73.986	74.005	74.007	**74.001**	**0.0106**
18	74.006	74.010	74.018	74.003	74.000	**74.007**	**0.0070**
19	73.984	74.002	74.003	74.005	73.997	**73.998**	**0.0085**
20	74.000	74.010	74.013	74.020	74.003	**74.009**	**0.0080**
21	73.982	74.001	74.015	74.005	73.996	**74.000**	**0.0122**
22	74.004	73.999	73.990	74.006	74.009	**74.002**	**0.0074**
23	74.010	73.989	73.990	74.009	74.014	**74.002**	**0.0119**
24	74.015	74.008	73.993	74.000	74.010	**74.005**	**0.0087**
25	73.982	73.984	73.995	74.017	74.013	**73.998**	**0.0162**

SOLUTION (CLASS EXC. 1, with S Chart)

Minitab output;



SUMMARY

Standards given;

Chart	**Center Line**	**Control Limits**
x̄ (μ and σ known)	μ	$\mu \pm A\,\sigma$
R (σ known)	$d_2\,\sigma$	$LCL = D_1\,\sigma\,,\; UCL = D_2\,\sigma$
S (σ known)	$c_4\,\sigma$	$LCL = B_5\,\sigma\,,\; UCL = B_6\,\sigma$

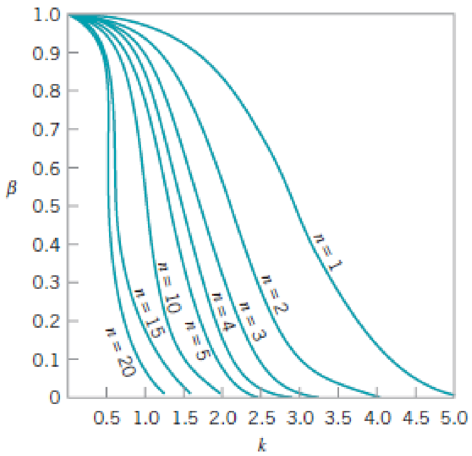
No Standards given (control limits based on past data or estimation);

Chart	**Center Line**	**Control Limits**
x̄ (using R)	$\bar{\bar{x}}$	$\bar{\bar{x}} \pm A_2\bar{R}$
x̄ (using S)	$\bar{\bar{x}}$	$\bar{\bar{x}} \pm A_3\bar{S}$
R	$\bar{R}$	$LCL = D_3\bar{R}\,,\; UCL = D_4\bar{R}$
S	$\bar{S}$	$LCL = B_3\bar{S}\,,\; UCL = B_4\bar{S}$

How we analyze the performance of a control chart?

<u>Operating Characteristic Curve (OC Curve)</u>

OC Curve displays probability of type II error. This curve is an indication of the ability of the control chart to detect process shifts of different magnitudes.



■ **FIGURE 6.13** Operating-characteristic curves for the $\bar{x}$ chart with three-sigma limits. $\beta = P$ (not detecting a shift of $k\sigma$ in the mean on the first sample following the shift).

OC curve is the plot of β risk (or the probability of not detecting a shift) against the magnitude of the shift we wish to detect expressed in standard deviation units.

H_0: $\mu = \mu_0$ (process mean is in control)
H_1: $\mu \neq \mu_0$ (process mean is out of control)

Assumption: $X \sim N(\mu,\sigma^2)$ $\Rightarrow \bar{X} \sim N(\mu, \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n})$ (X: quality characteristic)

$$\beta = P(Type\ II\ Error) = P(\underbrace{cannot\ reject\ H_0}_{not\ detecting\ the\ shift} \mid \underbrace{H_1}_{shift\ occurs})$$

$$\rightarrow \beta = P(LCL \leq \bar{X} \leq UCL \mid \mu = \mu_1 = \mu_0 + k\sigma) = P\left(\frac{LCL - (\mu_0 + k\sigma)}{\frac{\sigma}{\sqrt{n}}} \leq Z \leq \frac{UCL - (\mu_0 + k\sigma)}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(\frac{\left(\mu_0 - L\frac{\sigma}{\sqrt{n}}\right) - (\mu_0 + k\sigma)}{\frac{\sigma}{\sqrt{n}}} \leq Z \leq \frac{\left(\mu_0 + L\frac{\sigma}{\sqrt{n}}\right) - (\mu_0 + k\sigma)}{\frac{\sigma}{\sqrt{n}}}\right) = P(-L - k\sqrt{n} \leq Z \leq L - k\sqrt{n})$$

$$\rightarrow \beta = \Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n})$$

Ex Suppose that we are using an $\bar{x}$-chart with the usual 3-sigma limits. The sample size is five. Determine the probability of not detecting a shift to $\mu_1 = \mu_0 + 2\sigma$ on the first sample following the shift.

$L = 3$; $k = 2$, $n = 5$

$$\beta = \Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n}) = \Phi(3 - 2\sqrt{5}) - \Phi(-3 - 2\sqrt{5})$$

$$= \Phi(-1.47) - \underbrace{\Phi(-7.47)}_{\cong 0}$$

$$= 0.070781$$

$$\beta = P(LCL \leq \bar{x} \leq UCL \mid \mu = \mu_0 + 2\sigma)$$

$$= P\left(\frac{LCL - \mu_0 - 2\sigma}{\sigma/\sqrt{n}} \leq Z \leq \frac{UCL - \mu_0 - 2\sigma}{\sigma/\sqrt{n}}\right)$$

$$= P\left(\frac{\cancel{\mu_0} - 3\sigma/\sqrt{n} - \cancel{\mu_0} - 2\sigma}{\sigma/\sqrt{n}} \leq Z \leq \frac{\cancel{\mu_0} + 3\sigma/\sqrt{n} - \cancel{\mu_0} - 2\sigma}{\sigma/\sqrt{n}}\right)$$

$$= P(-3 - 2\sqrt{n} \leq Z \leq 3 - 2\sqrt{n}) = P(-3 - 2\sqrt{5} \leq Z \leq 3 - 2\sqrt{5})$$

$$= P(-1.47 \leq Z \leq -7.47)$$

$= 0.070781 \rightarrow$ the prob of not detecting the shift on the 1st sample

The prob. that such a shift will be detected on the first subsequent sample is

$$1 - \beta = 1 - 0.070781 = 0.929219$$

The prob. that such a shift will be detected on the second sample $= \beta(1-\beta)$

" " " " third " $= \beta.\beta.(1-\beta) = \beta^2(1-\beta)$

" " " " fourth " $= \beta^3(1-\beta)$

⋮

" " " " r^{th} " $= \underbrace{\beta^{r-1}(1-\beta)}$ prob. fnc. of geometric distr. where $p = 1-\beta$ prob of success

Geometric r.v. : # of trials repeated till we reach success

Here → success : detecting the shift.

Then, the expected number of samples taken until the shift is detected is the expectation of this geometric random variable.

Y ~ Geo(1-β) $\rightarrow E(Y) = \frac{1}{1-\beta} = ARL\ (Average\ Run\ Length)$

Y: number of samples taken till the shift is detected (Run length)

Average Run Length: The average number of points that must be plotted till a point indicates an out of control condition.

For the ex1. $\rightarrow \mathrm{ARL} = \frac{1}{1-\beta} = \frac{1}{0.929219} = 1.076$

<u>In general;</u> $\mathrm{ARL} = \frac{1}{\mathrm{P(one\ point\ plots\ out\ of\ control)}}$

$$\text{In control ARL: } \mathrm{ARL}_0 = \frac{1}{\mathrm{P(one\ point\ plots\ out\ of\ control\ |\ process\ is\ in\ control)}} = \frac{1}{\alpha}$$

$$\text{Out of control ARL: } \quad \mathrm{ARL}_1 = \frac{1}{\mathrm{P(one\ point\ plots\ out\ of\ control\ |\ process\ is\ out\ of\ control)}} = \frac{1}{1-\beta}$$

For the $\bar{x}$ chart with 3-sigma limits (L=3), assuming normality, p=0.0027 is the probability that a single point falls outside the limits when the process is in control. Therefore, the ARL of the $\bar{x}$ chart when the process is in control (ARL_0) is;

$\mathrm{ARL}_0 = \frac{1}{\alpha} = \frac{1}{0.0027} = 370 \rightarrow$ Even if the process remains in control, an out of control signal will be generated every 370 samples, on the average.

α: $\mathrm{P(a\ single\ point\ falls\ outside\ the\ limits\ |}\ \underbrace{\mu = \mu_0}_{\substack{\text{process} \\ \text{is in control}}}\ )$

It is also occasionally convenient to express the performance of the control chart in terms of its **average time to signal (ATS)**. If samples are taken at fixed intervals of time that are h hours apart, then,

ATS = ARL x h

Control Limits, Specification Limits, Natural Tolerance Limits

There is *no relationship between* the *control limits* and *specification limits*.

-The control limits are driven by the natural variability of the process (measured by the process standard deviation, σ), that is, by the natural tolerance limits of the process.

UNTL : 3σ above the process mean

LNTL : 3σ below the process mean

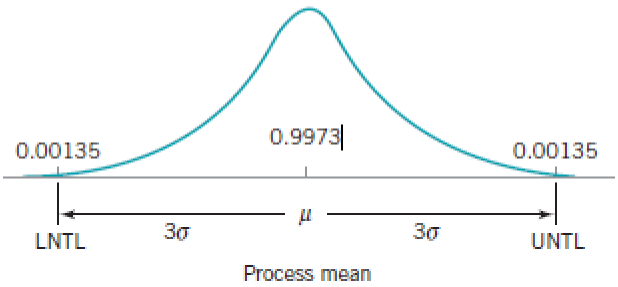
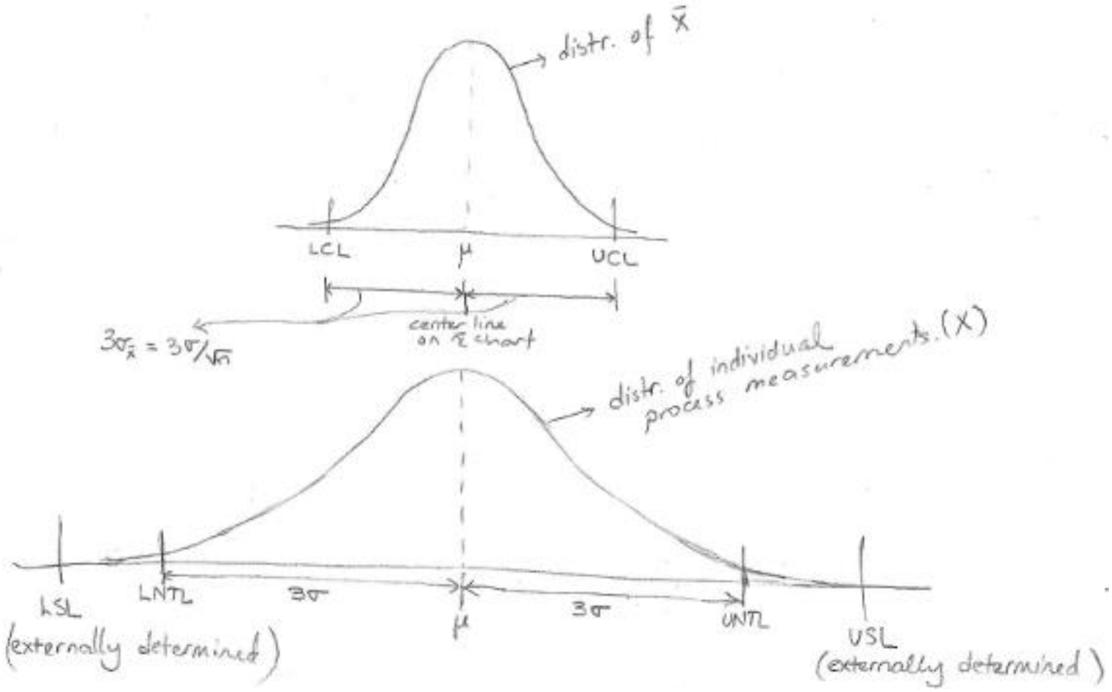


FIGURE 8.1 Upper and lower natural tolerance limits in the normal distribution.

-The specification limits are determined externally. They may be set by the management, the manufacturing engineers, the customer, or by product developers/designers.

Sometimes, we may encounter practitioners who have plotted specification limits on the $\bar{x}$ control chart. This practice is completely incorrect and should not be done. When dealing with plots of individual observations (not averages), it is helpful to plot the specification limits on that chart.



PROCESS CAPABILITY ANALYSIS (D. C. Montgomery, 6th ed., p344)

Statistical techniques can be helpful throughout the product cycle, including development activities prior to manufacturing, in quantifying process variability, in analyzing this variability relative to product requirements or specifications, and in assisting development and manufacturing in eliminating or greatly reducing this variability. This general activity is called **process capability analysis.**

Estimating Process Capability

Process capability ratio: A measure of the ability of the process to manufacture product that meets the specifications.

$$\text{PCR (Cp)} = \frac{\text{USL}-\text{LSL}}{6\sigma}$$

USL: Upper specification limit
LSL: Upper specification limit

Refer to Class Exc. 1. Suppose that the specification limits for the inside diameter of piston rings are $74 \mp 0.05\ mm$

➔ LSL = 73.95 and USL = 74.05

The control chart data may be used to describe the capability of the process to produce piston rings relative to these specifications.

$$\hat{\sigma} = \bar{R}/d_2 = 0.023/2.326 = 0.0099 \;--\rightarrow\; \hat{\text{C}}_{\text{p}} = \frac{\text{USL}-\text{LSL}}{6\hat{\sigma}} = \frac{74.05-73.95}{6(0.0099)} = 1.684$$

This implies that the natural tolerance limits (3σ above and below the mean) in the process are well inside the lower and upper specification limits. Consequently, a relatively low number of nonconforming piston rings will be produced.

Assuming that piston ring diameter is a normally distributed random variable with mean 74.001($\bar{\bar{x}}$) and standard deviation 0.0099($\hat{\sigma}$), we may estimate the fraction nonconforming piston rings produced as;

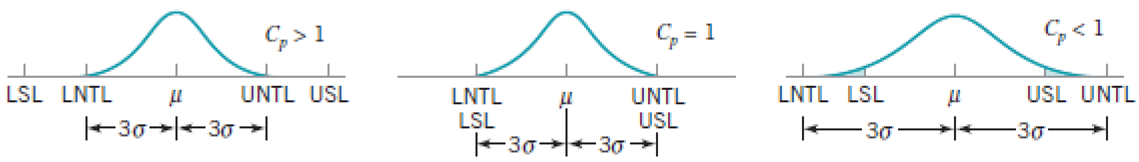
Fraction nonconforming: $\hat{p} = P(X < LSL) + P(X > USL) = P(X < 73.95) + P(X > 74.05)$

$$= \Phi\left(\frac{73.95-74.001}{0.0099}\right) + 1 - \Phi\left(\frac{74.05-74.001}{0.0099}\right) = \Phi(-5.15) + 1 - \Phi(4.04) \cong 0 + 1 - 0.99998 \cong 0.00002$$

That is, 0.002% (20 parts per million (PPM)) of the piston rings produced will be outside of the specifications.

The C_p may be interpreted another way. The quantitiy$P = \left(\frac{1}{C_p}\right)100\%$ is the percentage of the specification band that the process uses up.

For the piston ring process, $\hat{P} = \left(\frac{1}{\hat{C}_p}\right)100 = \left(\frac{1}{1.684}\right)100\% = 59.4\%$→**The process uses up 59.4% of the spec. band.**



(a) (b) (c)

See Example 6.1 on page 231-235, D.C.Montgomery, 6^{th} Ed.

EXC: 6.1, 6.3, 6.5, 6.7(a)(b), 6.9, 6.13, 6.19, 6.21, 6.23(a)(b)(d), 6.27, 6.29, 6.33 (student version)

EXC: 6.1, 6.3, 6.5(a)(b), 6.8, 6.11, 6.15, 6.19, 6.23, 6.26(a)(b)(d), 6.30, 6.33, 6.37 (org.copy, pdf)

Ex(Refer to class exc 2): Printed circuit board process.

If the specifications are at 0.063 ∓ 0.0015, what is the process capability ratio (C_p)?

$\bar{R} = 0.00092 \rightarrow \hat{\sigma} = {}^{\bar{R}}/_{d_2} = {}^{0.00092}/_{1.693} = 0.000543$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{0.0645 - 0.0615}{6(0.000543)} = 0.92$$

$\hat{P} = \left(\frac{1}{\hat{C}_p}\right)100 = \left(\frac{1}{0.92}\right)100\% = 108.7\% \rightarrow$Process uses 108.7% of the specification band.

Following exercises from textbook (D.C. Montgomery, 6^{th} ed.) will be solved during the lecture. Please bring your textbook to the class.

6.15, 6.17, 6.30, 6.35, 6.41, 6.42, 6.48 (student version)

6.13, 6.21, 6.31, 6.39, 6.40, 6.41, 6.49 (org. copy, pdf)

C_p (PCR) assume that the process has both upper and lower specification limits. For one sided specifications, **one sided process capability ratios** are used.(D. C. Montgomery, p352)

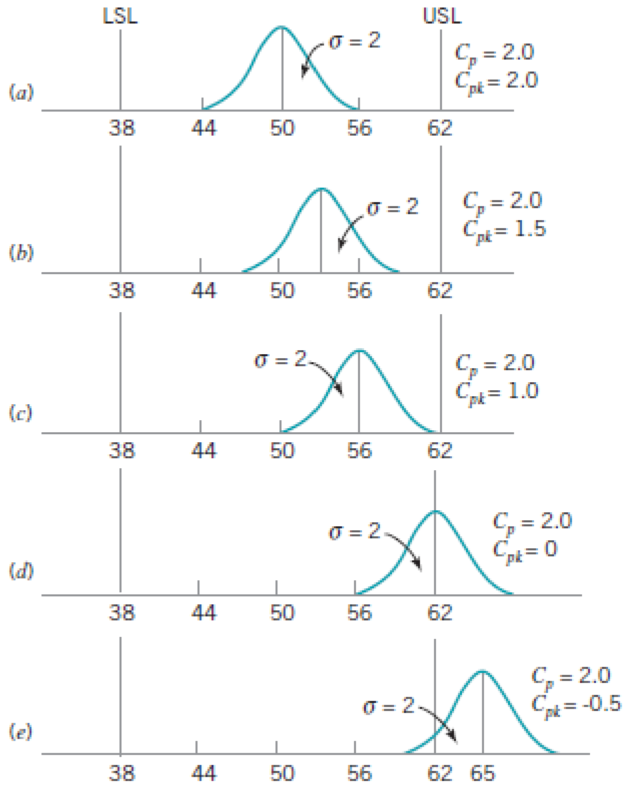
$C_{pu} = \frac{\text{USL}-\mu}{3\sigma}$ (upper specification only) $C_{pl} = \frac{\mu-\text{LSL}}{3\sigma}$ (lower specification only)

Estimates $\hat{C}_{pu}$ and $\hat{C}_{pl}$ would be obtained by replacing μ and σ by estimates $\hat{\mu}$and $\hat{\sigma}$, respectively.

Process Capability Ratio for an Off-Center Process (D. C. Montgomery, p354)

C_p does not take into account where the process mean is located relative to the specifications. C_{pk} takes process centering into account.

$C_{pk} = \min(C_{pu}, C_{pl})$



■ **FIGURE 8.8** Relationship of C_p and C_{pk}.

For the process shown in Figure 8.8(b);

$$C_{pk} = \min(C_{pu}, C_{pl}) = min\left(C_{pu} = \frac{\text{USL} - \mu}{3\sigma}, C_{pl} = \frac{\mu - \text{LSL}}{3\sigma}\right)$$

$$\Rightarrow\ C_{pk} = min\left(C_{pu} = \frac{62-53}{3(2)} = 1.5, C_{pl} = \frac{53-38}{3(2)} = 2.5\right) = 1.5$$

If $C_p = C_{pk}$, the process is centered at the midpoint of the specifications, and when $C_{pk} < C_p$ the process is off-center.

__Process Performance Indices:__ P_p, P_{pk} **(D. C. Montgomery, p363)**

The Automotive Industry Action Group (AIAG) (consists of representatives of the "big three" Ford, General Motors, and Chrysler) and American National Standards Institute recommend using the **process performance indices P_p and P_{pk}** when the process is *not* in control.

$$\hat{P}_p = \frac{\text{USL}-\text{LSL}}{6s} \quad \text{where} \quad s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

However, please note that if the process is not in control, the indices P_p and P_{pk} have no meaningful interpretation relative to process capability, because they cannot **predict** process performance.

<u>Process Capability analysis using a Control Chart</u>

Ex (D.C. Montgomery, 6th ed., p365): Find one sided process capability ratio for the container bursting-strength data below. Suppose that the lower specification limit on bursting strength is 200 psi.

TABLE 8.5
Glass Container Strength Data (psi)

Sample	Data					$\bar{x}$	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	260	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	262	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	260	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70
						$\bar{\bar{x}} = 264.06$	$\bar{R} = 77.3$

$\bar{x}$ Chart

$$\text{Center line} = \bar{\bar{x}} = 264.06$$
$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 264.06 + (0.577)(77.3) = 308.66$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 264.06 - (0.577)(77.3) = 219.46$$

R Chart

$$\text{Center line} = \bar{R} = 77.3$$
$$\text{UCL} = D_4\bar{R} = (2.115)(77.3) = 163.49$$
$$\text{LCL} = D_3\bar{R} = (0)(77.3) = 0$$

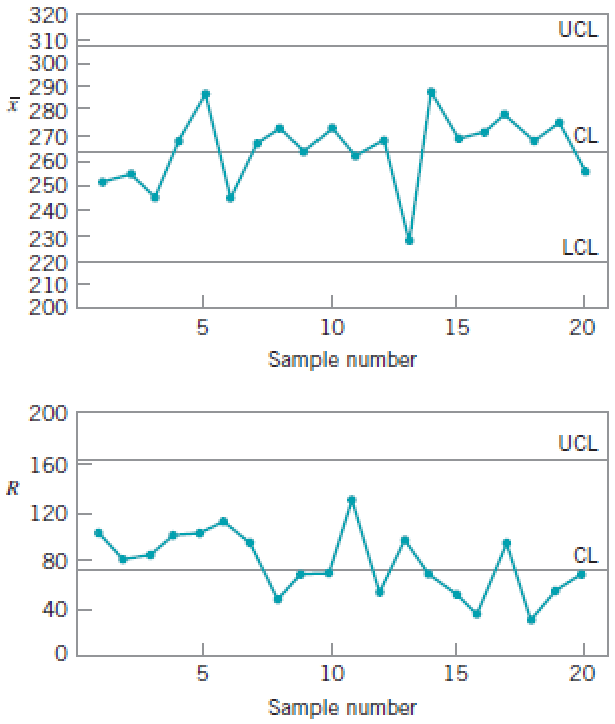


FIGURE 8.12 $\bar{x}$ and R charts for the bottle-strength data.

The Process parameters may be estimated from the control chart as;

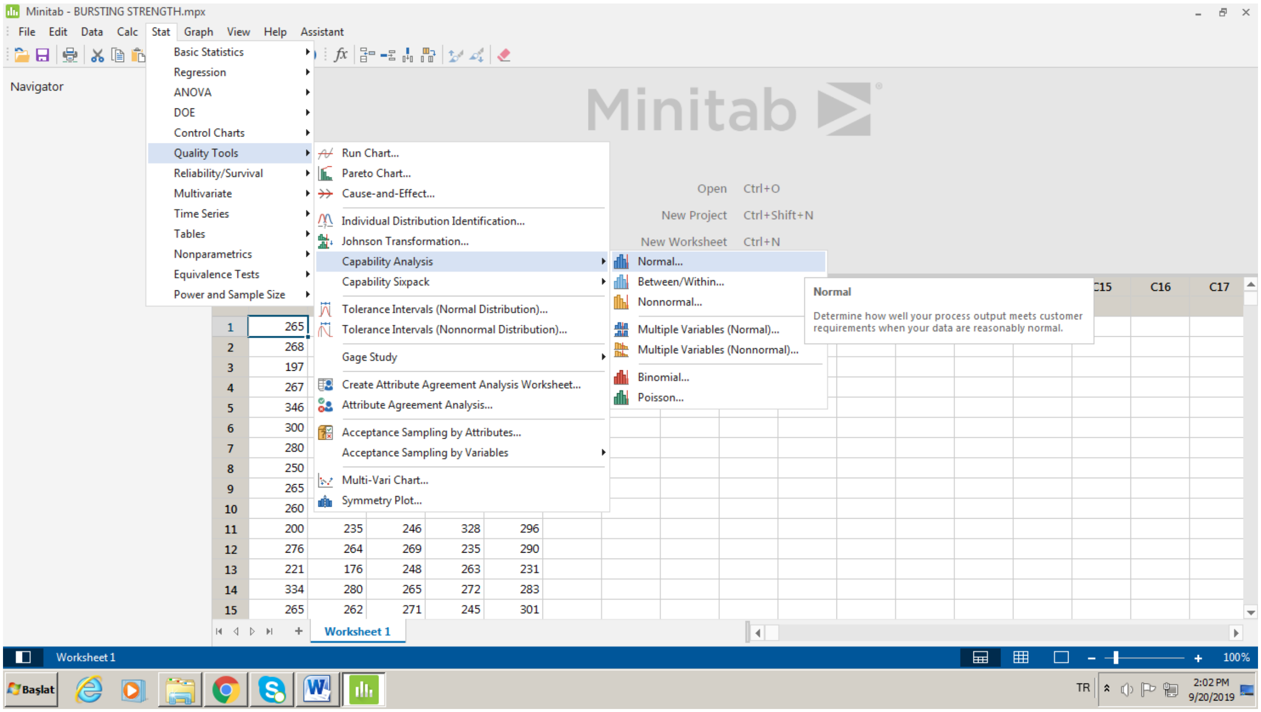
$$\hat{\mu} = \bar{\bar{x}} = 264.06 \qquad and \qquad \hat{\sigma} = \bar{R}/d_2 = 77.3/2.326 = 33.23$$

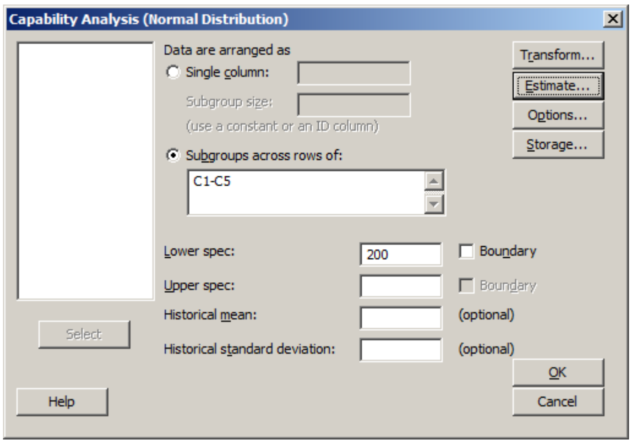
$$\hat{C}_{pl} = \frac{\hat{\mu} - LSL}{3\hat{\sigma}} = \frac{264.06 - 200}{3(33.23)} = 0.64$$

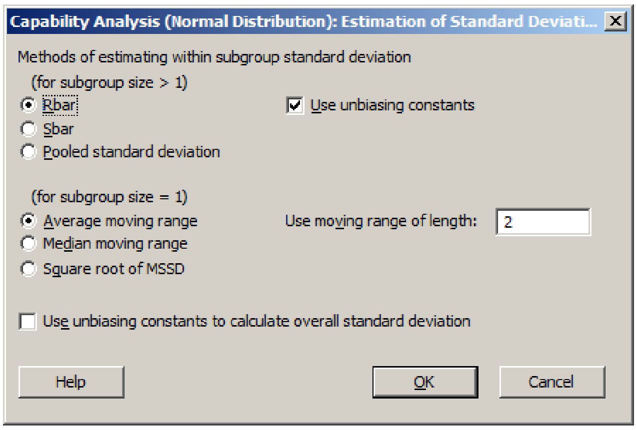
This example illustrates a process that is in control but operating at an unacceptable level. There is no evidence to indicate that the production of nonconforming units is **operator-controllable.** Engineering and/or management intervention will be required either to improve the process or to change the requirements if the quality problems with the bottles are to be solved. The objective of these interventions is to increase the process capability ratio to at least a minimum acceptable level.

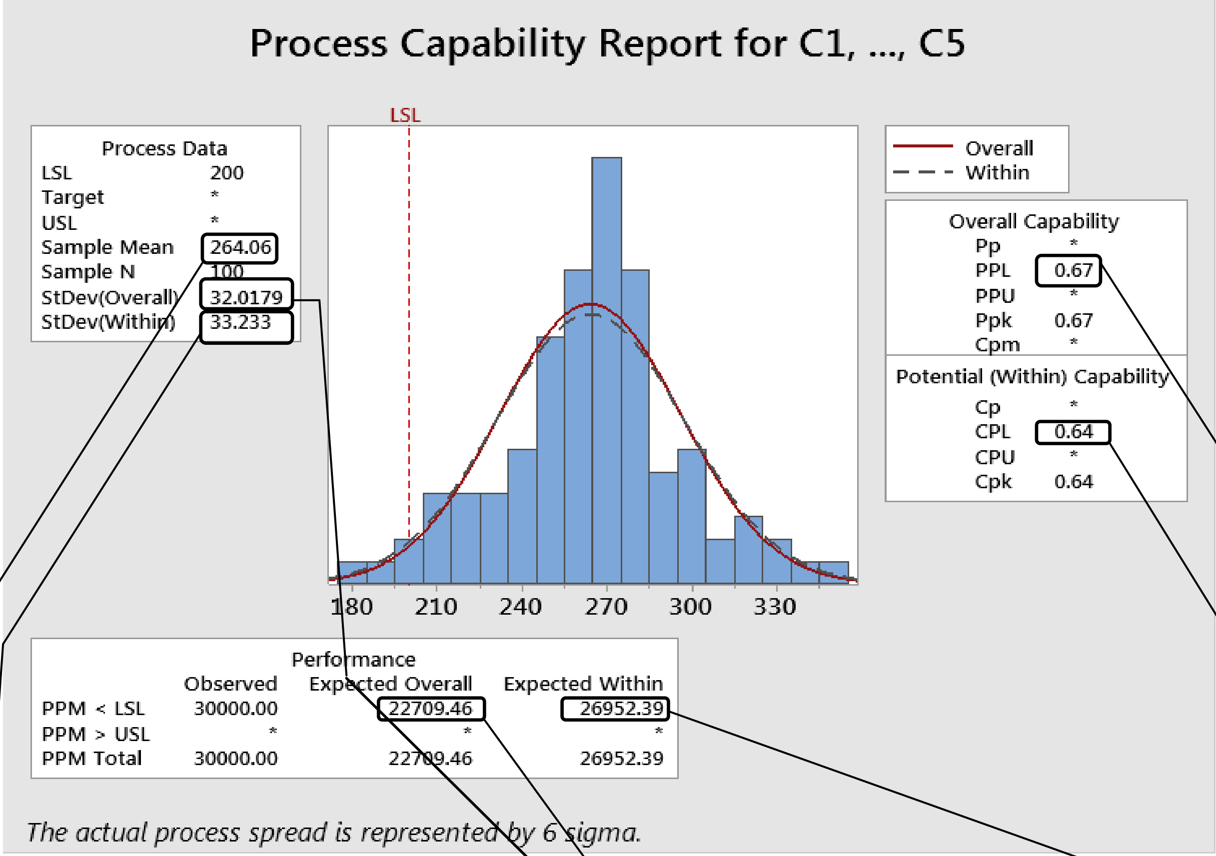
Sometimes the process capability analysis indicates an out-of-control process. It is **unsafe** to estimate process capability in such cases. The process must be stable in order to produce a reliable estimate of process capability. When the process is out of control in the early stages of process capability analysis, the first objective is finding and eliminating the assignable causes in order to bring the process into an in-control state.

Computer Application for Process Capability Analysis (Minitab Application)









Verify the computer output:

$$\bar{x} = \frac{x_1+x_2+x_3+\cdots+x_{100}}{100} = 264.06 \qquad s = \sqrt{\frac{\sum(x_i-\bar{x})^2}{100-1}} = 32.02 \qquad \hat{P}_{pl} = \frac{\hat{\mu}-LSL}{3s} = \frac{264-200}{3(32)} = 0.67$$

$$\hat{\sigma} = \bar{R}/d_2 = 77.3/2.326 = 33.23 \qquad \hat{C}_{pl} = \frac{\hat{\mu}-LSL}{3\hat{\sigma}} = \frac{264.06-200}{3(33.23)} = 0.64$$

Expected overall performance→ fraction of defective containers produced by this process when overall standard deviation (s) is used.

$$\hat{p} = P(X < LSL) = P(X < 200) = P\left(Z < \frac{LSL-\mu}{\sigma}\right) = P\left(Z < \frac{200-264.06}{32.0179}\right)$$

➔ $\hat{p} = P(Z < -2.000756) = 0.022709$ → 22709 PPM

The estimated fallout is about 2.28% defective, or about 22800 nonconforming containers per million.

Expected within performance→ fraction of defective containers produced by this process when within standard deviation $\left(\hat{\sigma} = \bar{R}/d_2\right)$ is used.

$$\hat{p} = P(X < LSL) = P(X < 200) = P\left(Z < \frac{LSL-\mu}{\sigma}\right) = P\left(Z < \frac{200-264.06}{33.233}\right)$$

➔ $\hat{p} = P(Z < -1.9276) = 0.026952$ → 26952 PPM

Note that we assume normality of the quality characteristic (glass container strength) in all computations.

Observed performance → observed fraction of defective containers

There are 3 containers with strength less than LSL =200. Total number of observations is 100.

3/100 = 0.03 → 30000 PPM

CLASS EXERCISE 4 Use the data in Class Exc. 1 (piston ring diameter) in order to perform process capability analysis when the specifications are given as 74 ∓ 0.05 mm

CONTROL CHARTS FOR ATTRIBUTES (D. C. Montgomery, 6th ed., p288)

Many quality characteristics cannot be conveniently represented numerically. In such cases, we usually classify each item inspected as either conforming or nonconforming to the specifications on that quality characteristic. The terminology **defective** or **nondefective** is often used to identify these two classifications of product. More recently, the terminology **conforming** and **nonconforming** has become popular. Quality characteristics of this type are called **attributes.**

Examples of quality characteristics that are attributes:
The number of nonfunctional semiconductor chips on a wafer.
The number of errors or mistakes made in completing a loan application.
The number of medical errors made in a hospital.

There are three widely used attributes control charts.

1- Control chart for fraction nonconforming (*p* chart) and number nonconforming (*np* chart)
2- Control chart for total number of nonconformities in a unit (*c* chart)
3- Control chart for average number of nonconformities per unit (*u* chart)

1- Control Chart for Fraction Nonconforming (*p* chart) and Number Nonconforming (np chart)

The **fraction nonconforming** is defined as the ratio of the number of nonconforming items in a population to the total number of items in that population. The items may have *several* quality characteristics that are examined simultaneously by the inspector. If the item does not conform to standard on one or more of these characteristics, it is classified as nonconforming.

The statistical principles underlying the control chart for fraction nonconforming are based on the binomial distribution.

Suppose the production process is operating in a stable manner and that successive units produced are independent. Then,

***p* :**The probability that any unit will not conform to specifications.(The probability that any unit is defective or nonconforming)

D : The number of units of product that are nonconforming in n units. (Suppose a random sample of *n* units of product is selected)

➔ $D \sim Bin(n,p)$ and $P(D=x)=\binom{n}{x}p^x(1-p)^{n-x},\ x=0,1,2,3,\ldots,n$

➔ $E(D)=np$ and $V(D)=np(1-p)$

$\hat{p}=\frac{D}{n}$ → sample fraction nonconforming

↓

Estimator of p where, $E(\hat{p})=p=\mu_{\hat{p}}$ $V(\hat{p})=\frac{p(1-p)}{n}=\sigma_{\hat{p}}^2$

Suppose that the true fraction nonconforming p in the production process is known or is a specified **standard value.** Then, the center line and control limits of the fraction nonconforming control chart would be as follows:

Fraction Nonconforming Control Chart: Standard Given

$$\text{UCL} = p + 3\sqrt{\frac{p(1-p)}{n}}$$
$$\text{Center line} = p$$
$$\text{LCL} = p - 3\sqrt{\frac{p(1-p)}{n}}$$

Depending on the values of p and n, sometimes the lower control limit LCL < 0. In these cases, we customarily set LCL = 0 and assume that the control chart only has an upper control limit.

The actual operation of this chart would consist of taking subsequent samples of n units, computing the sample fraction nonconforming $\hat{p}$, and plotting the statistic $\hat{p}$ on the chart. As long as $\hat{p}$ remains within the control limits and the sequence of plotted points does not exhibit any systematic nonrandom pattern, we can conclude that the process is in control at the level p. If a point plots outside of the control limits, or if a nonrandom pattern in the plotted points is observed, we can conclude that the process fraction nonconforming has most likely shifted to a new level and the process is out of control.

When the process fraction nonconforming p is not known, then it must be estimated from observed data. The usual procedure is to select m preliminary samples, each of size n.Then, we compute the fraction nonconforming in the i^{th} sample as;

$$\hat{p}_i = \frac{D_i}{n} \quad , \quad i=1, 2, 3, \ldots, m$$

$\hat{p}_i$: The fraction nonconforming in the i^{th} sample

D_i: The number of nonconforming units in sample i

and the average of these individual sample fractions nonconforming is;

$$\underbrace{\bar{p}}_{\text{Estimator of p}} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$

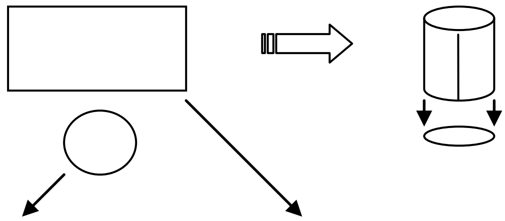
Estimator of p

Fraction Nonconforming Control Chart: No Standard Given

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$
$$\text{Center line} = \bar{p}$$
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Exp. 7.1 (D.C. Montgomery, 6th Ed., p292)

Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel. Set up a control chart to improve the fraction of nonconforming cans produced by this machine.



metal bottom panel　　　　　　cardboard

The data are given in the following Table.

Data for Trial Control Limits, Example 7.1, Sample Size $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

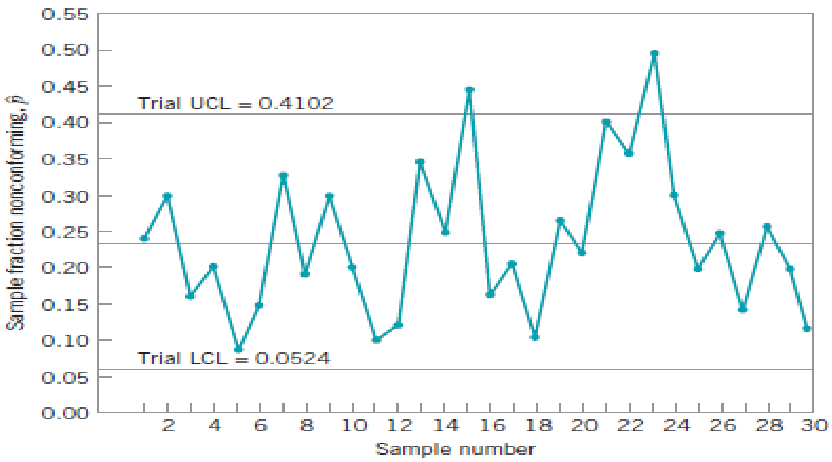
__Solution:__

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

$$\mathrm{UCL} = \bar{\mathrm{p}} + 3\sqrt{\frac{\bar{\mathrm{p}}(1-\bar{\mathrm{p}})}{\mathrm{n}}} = 0.2313 + 3\sqrt{\frac{(0.2313)(0.7687)}{50}} = 0.4102$$

$$\mathrm{CL} = \bar{\mathrm{p}} = 0.2313$$

$$\mathrm{LCL} = \bar{\mathrm{p}} - 3\sqrt{\frac{\bar{\mathrm{p}}(1-\bar{\mathrm{p}})}{\mathrm{n}}} = 0.2313 - 3\sqrt{\frac{(0.2313)(0.7687)}{50}} = 0.0524$$



Note that two points, those from samples 15 and 23, plot above the upper control limit, so the process is not in control. These points must be investigated to see whether an assignable cause can be determined.

Analysis of the data from sample 15 → new batch of cardboard stock was put into production during that half-hour period.

Analysis of the data from sample 23 → a relatively inexperienced operator had been temporarily assigned to the machine

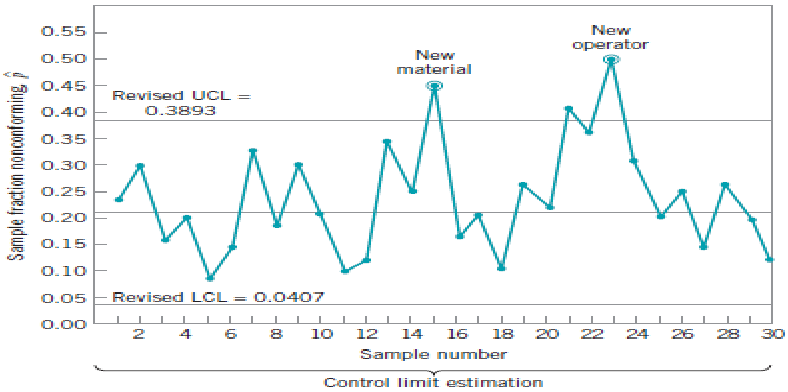
Consequently, samples 15 and 23 are eliminated, and the new center line and revised control limits are calculated as;

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{301}{(28)(50)} = 0.215$$

$$\text{UCL} = \bar{\text{p}} + 3\sqrt{\frac{\bar{\text{p}}(1-\bar{\text{p}})}{\text{n}}} = 0.215 + 3\sqrt{\frac{(0.215)(0.785)}{50}} = 0.3893$$

$$\text{CL} = \bar{\text{p}} = 0.215$$

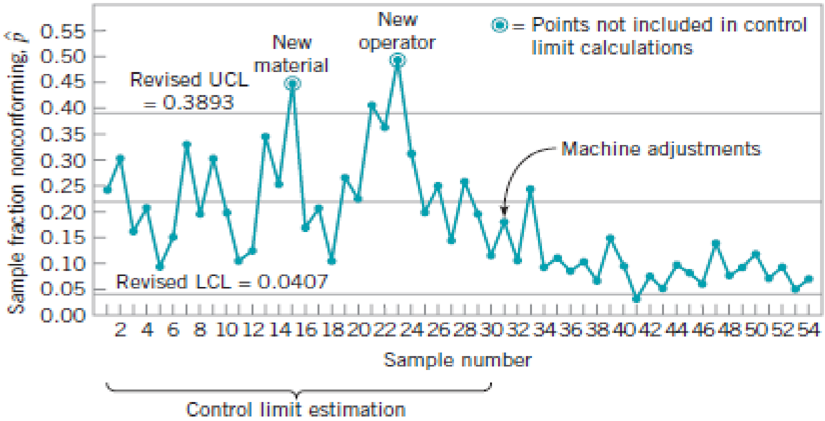
$$\text{LCL} = \bar{\text{p}} - 3\sqrt{\frac{\bar{\text{p}}(1-\bar{\text{p}})}{\text{n}}} = 0.215 - 3\sqrt{\frac{(0.215)(0.785)}{50}} = 0.0407$$



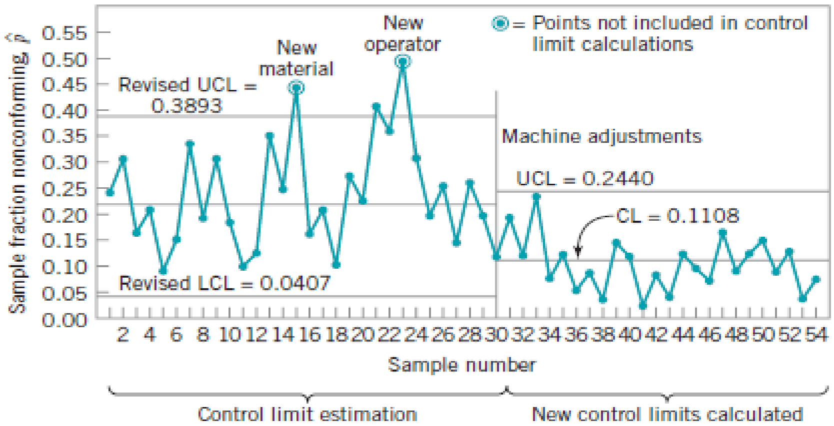
Note also that the fraction nonconforming from sample 21 now exceeds the upper control limit. However, analysis of the data does not produce any reasonable or logical assignable cause for this, and we decide to retain the point. Therefore, we conclude that the new control limits can be used for future samples. Thus, we have concluded the control limit estimation phase (phase I) of control chart usage.

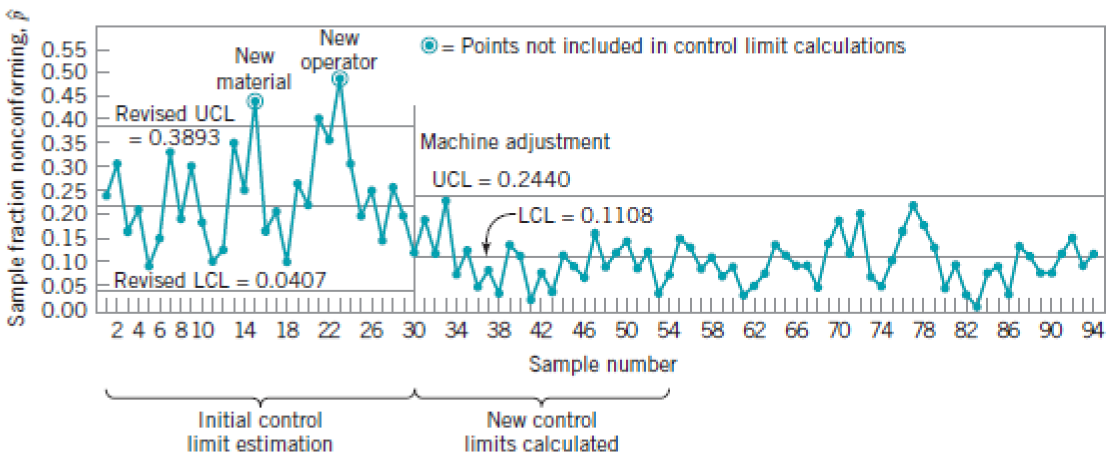
We conclude that the process is in control at the level p =0.215 and that the revised control limits should be adopted for monitoring current production. However, we note that although the process is in control, the fraction nonconforming is much too high. That is, the process is operating in a stable manner, and no unusual **operator-controllable** problems are present. The nonconforming cans produced are **management controllable** because an intervention by management in the process will be required to improve performance.

Engineering staff made several adjustments on the machine that should improve its performance. During the next three shifts following the machine adjustments and the introduction of the control chart, an additional 24 samples of n = 50 observations each are collected.



According to the figure above, our immediate impression is that the process is now operating at a new quality level that is substantially better than the center line level of $\bar{p} = 0.215$.





The Number Nonconforming Control Chart (The np Control Chart): It is also possible to base a control chart on the number nonconforming rather than the fraction nonconforming. This is often called **number nonconforming (*np*) control chart.**

$D \sim Bin(n,p) \Rightarrow E(D) = np$ and $V(D) = np(1-p)$

$\downarrow$

Number of nonconforming items among n items

The *np* Control Chart

$$\text{UCL} = np + 3\sqrt{np(1-p)}$$
$$\text{Center line} = np$$
$$\text{LCL} = np - 3\sqrt{np(1-p)}$$

If a standard value for *p* is unavailable, then $\bar{p}$ can be used to estimate *p*.

Set up an *np* control chart for the orange juice concentrate can process in **Exp. 7.1 (D.C. Montgomery, 6th Ed., p292)**

n=50, $\bar{p} = 0.2313$

$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 50(0.2313) + 3\sqrt{50(0.2313)(0.7687)} = 20.51$

$\text{CL} = n\bar{p} = 50(0.2313) = 11.565$

$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 50(0.2313) - 3\sqrt{50(0.2313)(0.7687)} = 2.62$

Now in practice, the number of nonconforming units in each sample is plotted on the *np* control chart, and the number of nonconforming units is an integer. Thus, if 20 units are nonconforming the process is in control, but if 21 occur the process is out of control. Similarly, if there are three nonconforming units in the sample then the process is in control, but two nonconforming units would imply an out-of-control process.

The Operating Characteristic Function and Average Run Length Calculations

The operating-characteristic (or OC) function of the fraction nonconforming control chart is a graphical display of the probability of incorrectly accepting the hypothesis of statistical control (i.e., a type II error) against the process fraction nonconforming.

$\beta = P(\text{LCL} < \hat{p} < UCL|p) = P(\hat{p} < UCL|p) - P(\hat{p} \leq \text{LCL} \mid p) = P(D < nUCL|p) - P(D \leq nLCL|p)$

$\downarrow$

Number of nonconforming items among n items

Since *D* is a binomial random variable with parameters *n* and *p* ($D \sim Bin(n,p)$), β defined above can be obtained from the cumulative binomial distribution.

Ex. n=50, p chart control limits; LCL=0.0303, UCL=0.3697 and CL=0.2

$$\beta = P(D < 50(0.3697)|p) - P(D \le 50(0.0303)|p) = P(D < 18.49|p) - P(D \le 1.52)|p)$$

$$= P(D \le 18\,|\,p) - P(D \le 1)|p)$$

Following table illustrates the calculations required to generate the OC curve for a control chart for fraction nonconforming with parameters n = 50, LCL = 0.0303, and UCL = 0.3697.

Calculations[a] for Constructing the OC Curve for a Control Chart for Fraction Nonconforming with n = 50, LCL = 0.0303, and UCL = 0.3697

p	$P\{D \le 18\,|\,p\}$	$P\{D \le 1\,|\,p\}$	$\beta = P\{D \le 18\,|\,p\} - P\{D \le 1\,|\,p\}$
0.01	1.0000	0.9106	0.0894
0.03	1.0000	0.5553	0.4447
0.05	1.0000	0.2794	0.7206
0.10	1.0000	0.0338	0.9662
0.15	0.9999	0.0029	0.9970
0.20	0.9975	0.0002	0.9973
0.25	0.9713	0.0000	0.9713
0.30	0.8594	0.0000	0.8594
0.35	0.6216	0.0000	0.6216
0.40	0.3356	0.0000	0.3356
0.45	0.1273	0.0000	0.1273
0.50	0.0325	0.0000	0.0325
0.55	0.0053	0.0000	0.0053

[a]The probabilities in this table were found by evaluating the cumulative binomial distribution. For small p (p < 0.1, say) the Poisson approximation could be used, and for larger values of p the normal approximation could be used.

Find probability of type II error (β) when **p=0.15.**

$$\beta = P(D \le 18\,|\,p = 0.15) - P(D \le 1)|p = 0.15)$$

$$P(D \le 18|p = 0.15) = \sum_{i=0}^{18} \binom{50}{i} (0.15)^i (0.85)^{50-i} = 0.99994 \text{ (by excel)}$$

$$P(D \le 1|p = 0.15) = \sum_{i=0}^{1} \binom{50}{i} (0.15)^i (0.85)^{50-i} = \binom{50}{0} (0.15)^0 (0.85)^{50} + \binom{50}{1} (0.15)^1 (0.85)^{49} = 0.0029$$

⇨ β = 0.99994 – 0.0029 = 0.9970

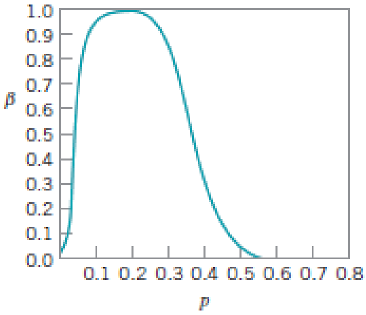


FIGURE 7.11 Operating-characteristic curve for the fraction nonconforming control chart with $\bar{p}$ = 0.20, LCL = 0.0303, and UCL = 0.3697.

We may also calculate average run lengths (ARLs) for the fraction nonconforming control chart.

$$p = 0.15 \rightarrow \text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.997} = 333$$

If the process is in control at the point $p = \bar{p} = 0.2$ find ARL_0.

$$For\ p = 0.2\,;\ \ 1 - \alpha = 0.9973\ (from\ the\ table) \rightarrow \text{ARL}_0 = \frac{1}{\alpha} = \frac{1}{0.0027} \cong 370$$

EXC: 7.4, 7.11, 7.15, 7.17, 7.25(a), 7.30(Use Table 7E.2) → Student version

EXC: 7.2, 7.13, 7.16, 7.19, 7.29(a), 7.33 → Org. copy, pdf

Following exercises from textbook (D.C. Montgomery, 6th ed.) will be solved during the lecture. Please bring your textbook to the class.

7.3, 7.7, 7.13, 7.14, 7.19, 7.26 (Corresponds to questions 7.1, 7.7, 7.15, 7.17, 7.25, 7.27 on pdf)

2- Control chart for total number of nonconformities in a unit (*c* chart)

A nonconforming item is a unit of product that does not satisfy one or more of the specifications for that product. Each specific point at which a specification is not satisfied results in a **defect** or **nonconformity.** Consequently, a nonconforming item will contain at least one nonconformity. However, depending on their nature and severity, it is quite possible for a unit to contain several nonconformities and *not* be classified as nonconforming.

As an example, suppose we are manufacturing personal computers. Each unit could have one or more very minor flaws in the cabinet finish, and since these flaws do not seriously affect the unit's functional operation, it could be classified as conforming. However, if there are too many of these flaws, the personal computer should be classified as nonconforming, since the flaws would be very noticeable to the customer and might affect the sale of the unit.

Here are some practical situations in which we prefer to work directly with the number of defects or nonconformities rather than the fraction nonconforming

- the number of defective welds in 100 m of oil pipeline,
- the number of broken rivets in an aircraft wing,
- the number of functional defects in an electronic logic device,
- the number of errors on a document.

It is possible to develop control charts for either the **total number of nonconformities in a unit (c chart)** or the **average number of nonconformities per unit(u chart).** These control charts usually assume that the occurrence of nonconformities in samples of constant size is well modeled by the ***Poisson distribution***.

$$P(x) = \frac{e^{-c}c^{x}}{x!} \quad , \quad x = 0, 1, 2, 3, \ldots.. \quad , \qquad c > 0$$

X: the number of nonconformities in a unit $\qquad\qquad \mathrm{E(X) = c} \ , \ \mathrm{V(X) = c}$

c : the parameter of the Poisson distribution.

Therefore, a control chart for nonconformities (c chart) with 3 sigma limits would be defined as follows:

Control Chart for Nonconformities: Standard Given

$$\begin{aligned} \mathrm{UCL} &= c + 3\sqrt{c} \\ \text{Center line } &= c \\ \mathrm{LCL} &= c - 3\sqrt{c} \end{aligned}$$

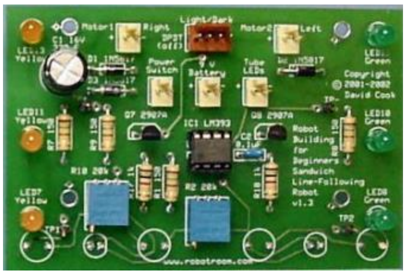
If these calculations yield a negative value for the LCL, set LCL = 0.

Control Chart for Nonconformities: No Standard Given

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{Center line} = \bar{c}$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

$\bar{c} = \frac{\sum_{i=1}^{m} c_i}{m}$ where c_i= the number of nonconformities in the i^{th} inspection unit.

Example 7.3(D. C. Montgomery, 6th Ed., p310) Nonconformities in printed circuit boards



Following table presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Note that, for reasons of convenience, the inspection unit is defined as 100 boards. Set up a *c* chart for these data.

Data on the Number of Nonconformities in Samples of 100 Printed Circuit Boards

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

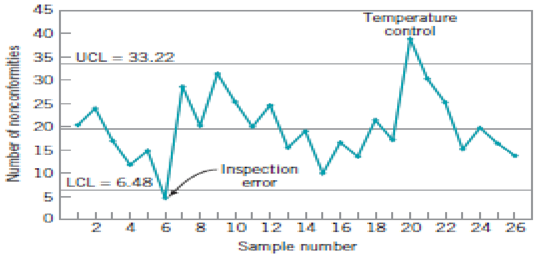
Solution:

$\bar{c} = \frac{\sum_{i=1}^{26} c_i}{26} = \frac{516}{26} = 19.85$

$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$

$CL = \bar{c} = 19.85$

$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$



The number of observed nonconformities from the preliminary samples is plotted on this chart. Two points plot outside the control limits, samples 6 and 20.

Investigation of **sample 6** revealed that a new inspector had examined the boards in this sample and that he did not recognize several of the types of nonconformities that could have been present.

The unusually large number of nonconformities in **sample 20** resulted from a temperature control problem in the wave soldering machine (dalgalı lehimleme makinası), which was subsequently repaired.

Wave Solder Machine

Therefore, it seems reasonable to exclude these two samples and revise the trial control limits.

$\bar{c} = \frac{516-5-39}{26-2} = \frac{472}{24} = 19.67$

$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97$

$CL = \bar{c} = 19.67$

$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.36$

Since all the points are between the LCL and UCL, no lack of control is indicated. Therefore these limits become the standard values against which production in the next period can be compared. *(Although no lack of control is indicated, the number of nonconformities per board is still unacceptably high. In order to improve the process, further analysis of nonconformities is necessary. For the further analysis* ***see p312-314, D. C. Montgomery, 6th Ed****.)*

3- <u>Control chart for average number of nonconformities per unit (*u* chart)</u>

Example 7.3 illustrates a control chart for nonconformities with the sample size exactly equal to one inspection unit (100 printed circuit boards). In fact, we would often prefer to use *several* inspection units in the sample, thereby increasing the area of opportunity for the occurrence of nonconformities.

Suppose we decide to base the control chart on a sample size of n inspection units. Note that n does not have to be an integer. To illustrate this, suppose that in Example 7.3 we were to specify a subgroup size of n = 2.5 inspection units. Then the sample size becomes (2.5)(100) = 250 boards. There are two general approaches to constructing the revised chart once a new sample size has been selected.

1- Simply to redefine a new inspection unit that is equal to *n* times the old inspection unit. In this case;

The center line: $n\bar{c}$

The control limits: $n\bar{c} \pm 3\sqrt{n\bar{c}}$,

where $\bar{c}$ is the observed mean number of nonconformities in the *original* inspection unit.

Suppose that in Example 7.3, after revising the trial control limits, we decided to use a sample size of *n*=2.5 inspection units. Then;

UCL= $n\bar{c} + 3\sqrt{n\bar{c}} = 49.18 + 3\sqrt{49.18} = 70.22$

CL= $n\bar{c}$ = (2.5)(19.67) = 49.18

LCL= $n\bar{c} - 3\sqrt{n\bar{c}} = 49.18 - 3\sqrt{49.18} = 28.14$

2- The second approach involves setting up a control chart based on the *average number of nonconformities per inspection unit*.

If we find x total nonconformities in a sample of *n* inspection units, then the *average* number of nonconformities per inspection unit is;

$$u = \frac{x}{n}$$

x: The total number of nonconformities in a sample of *n* inspection units

$X \sim \text{Poisson}(\lambda) \rightarrow E(X) = \lambda\ ,\ \ V(X) = \lambda$

$$E(U) = E\left(\frac{X}{n}\right) = \frac{E(X)}{n} = \frac{\lambda}{n} \quad \longrightarrow \quad \text{estimated by } \bar{u} = \frac{\sum_{i=1}^{m} u_i}{m}$$

$$V(U) = V\left(\frac{X}{n}\right) = \frac{V(X)}{n^2} = \frac{\lambda}{n^2} \quad \longrightarrow \quad \text{estimated by } \frac{\bar{u}}{n}$$

Control limits for u chart

$UCL = E(U) + 3\sqrt{V(U)}$

$CL = E(U)$

$LCL = E(U) - 3\sqrt{V(U)}$

Control Chart for Average Number of Nonconformities per Unit

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{Center line} = \bar{u}$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

Example 7.4 (D. C. Montgomery, 6th Ed., p315) Control Charts in Supply Chain Operations

A supply chain engineering group monitors shipments of materials through the company distribution network. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors recorded. Data for twenty weeks are shown in the following table. Set up a *u* control chart to monitor this process.

Data on Number of Shipping Errors in a Supply Chain Network

Sample Number (week), i	Sample Size, n	Total Number of Errors (Nonconformities), x_i	Average Number of Errors (Nonconformities) per Unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48

Solution:

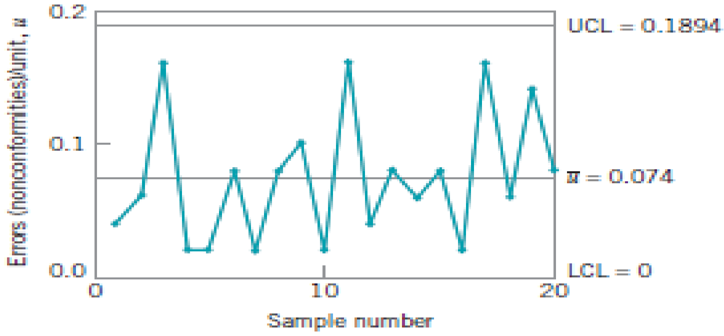
We estimate the number of errors (nonconformities) per unit (shipment) to be:

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.074$$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.074 + 3\sqrt{\frac{0.074}{50}} = 0.1894$$

$$CL = \bar{u} = 0.074$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.074 - 3\sqrt{\frac{0.074}{50}} = -0.0414 \quad \Rightarrow \quad LCL = 0$$



The preliminary data do not exhibit lack of statistical control; therefore, the trial control limits given here would be adopted for phase II monitoring of future operations.

The Operating Characteristic Function for c-chart and u-chart

The operating-characteristic (OC) curves for both the *c* chart and the *u* chart can be obtained from the Poisson distribution.

For c-chart;

X: The total number of nonconformities in an inspection unit.

$X \sim \text{Poisson}(\lambda = c)$

$\beta = P(LCL < X < UCL|c) = P(X < UCL|c) - P(X \leq LCL \mid c)$

For u-chart;

X: The total number of nonconformities in a sample of *n* inspection units $\left(u = \frac{X}{n}\right)$

$X \sim \text{Poisson}(\lambda = nu)$

$$\beta = P\left(LCL < \frac{X}{n} < UCL \middle| \lambda = nu\right) = P\left(\frac{X}{n} < UCL \middle| \lambda = nu\right) - P\left(\frac{X}{n} \leq LCL \middle| \lambda = nu\right)$$

$\Rightarrow \quad \beta = P(X < nUCL|\lambda = nu) - P(X \leq nLCL|\lambda = nu)$

Ex: Generate the OC curve for the *c* chart in Example 7.3

Calculation of the OC Curve for a *c* Chart with UCL = 33.22 and LCL = 6.48

c	$P\{x \leq 33 \mid c\}$	$P\{x \leq 6 \mid c\}$	$\beta = P\{x \leq 33 \mid c\} - P\{x \leq 6 \mid c\}$
1	1.000	0.999	0.001
3	1.000	0.966	0.034
5	1.000	0.762	0.238
7	1.000	0.450	0.550
10	1.000	0.130	0.870
15	0.999	0.008	0.991
20	0.997	0.000	0.997
25	0.950	0.000	0.950
30	0.744	0.000	0.744
33	0.546	0.000	0.546
35	0.410	0.000	0.410
40	0.151	0.000	0.151
45	0.038	0.000	0.038

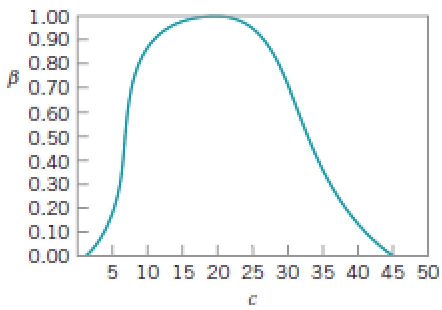


FIGURE 7.19 OC curve of a *c* chart with LCL = 6.48 and UCL = 33.22.

Find the probability of type II error (β) when **c = 7.**

$\beta = P(LCL < X < UCL|c) = P(X < UCL = 33.22|c = 7) - P(X \leq LCL = 6.48|c = 7)$

$= P(X \leq 33 \mid c = 7) - P(X \leq 6|c = 7)$

$= \sum_{x=0}^{33} \frac{e^{-c}c^x}{x!} - \sum_{x=0}^{6} \frac{e^{-c}c^x}{x!} = \sum_{x=0}^{33} \frac{e^{-7}7^x}{x!} - \sum_{x=0}^{6} \frac{e^{-7}7^x}{x!} = 1 - 0.45 = 0.55$ (by excel)

Find the probability of type II error (β) when **c = 15.**

$\beta = P(LCL < X < UCL|c) = P(X < UCL = 33.22|c = 15) - P(X \leq LCL = 6.48|c = 15)$

$= P(X \leq 33 \mid c = 15) - P(X \leq 6|c = 15)$

$= \sum_{x=0}^{33} \frac{e^{-c}c^x}{x!} - \sum_{x=0}^{6} \frac{e^{-c}c^x}{x!} = \sum_{x=0}^{33} \frac{e^{-15}15^x}{x!} - \sum_{x=0}^{6} \frac{e^{-15}15^x}{x!} = 0.999 - 0.008 = 0.991$ (by excel)

EXC: 7.35, 7.37, 7.45 (student version)

EXC: 7.41, 7.43, 7.50 (org. copy, pdf)

Following exercises from textbook (D.C. Montgomery, 6th ed.) will be solved during the lecture.

Ex 7.44, 7.50, 7.54, (corresponds to questions 7.49, 7.56, 7.59 in org. copy, pdf)

ACCEPTANCE SAMPLING

A typical application of acceptance sampling is as follows: A company receives a shipment of product from a supplier. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot, and some quality characteristic of the units in the sample is inspected. On the basis of the information in this sample, a decision is made regarding **lot disposition.** Usually, this decision is either to accept or to reject the lot. Sometimes we refer to this decision as **lot sentencing.** Accepted lots are put into production; rejected lots may be returned to the supplier or may be subjected to some other **lot disposition action.**

Although it is customary to think of acceptance sampling as a receiving inspection activity, there are other uses of sampling methods. For example, frequently a manufacturer will sample and inspect its own product at various stages of production. Lots that are accepted are sent forward for further processing, and rejected lots may be reworked or scrapped.

Generally, there are three approaches to lot sentencing:

(1) Accept with no inspection

Useful in situations where either the supplier's process is so good that defective units are almost never encountered or where there is no economic justification to look for defective units. For example, if the supplier's process capability ratio is 3 or 4, acceptance sampling is unlikely to discover any defective units.

(2) 100% inspection—that is, inspect every item in the lot, removing all defective units found (defectives may be returned to the supplier, reworked, replaced with known good items, or discarded)

Useful in situations where the component is extremely critical and passing any defectives would result in an unacceptably high failure cost at subsequent stages, or where the supplier's process capability is inadequate to meet specifications.

(3) Acceptance sampling

Useful in situations

1. When testing is destructive

2. When the cost of 100% inspection is extremely high

3. When 100% inspection is not technologically feasible or would require so much calendar time that production scheduling would be seriously impacted

4. When there are many items to be inspected and the inspection error rate is sufficiently high that 100% inspection might cause a higher percentage of defective units to be passed than would occur with the use of a sampling plan

5. When the supplier has an excellent quality history, and some reduction in inspection from 100% is desired, but the supplier's process capability is sufficiently low as to make no inspection an unsatisfactory alternative

6. When there are potentially serious product liability risks, and although the supplier's process is satisfactory, a program for continuously monitoring the product is necessary

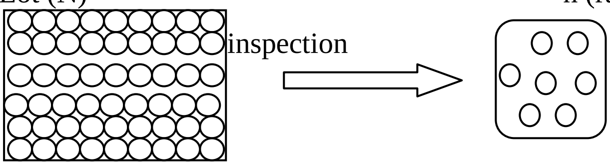
Types of Sampling Plans

Variables **(will not be discussed in this course)** | Attributes

Acceptance-sampling Plans for Attributes

1. Single-sampling plan

   Lot (N) | n (random sample)

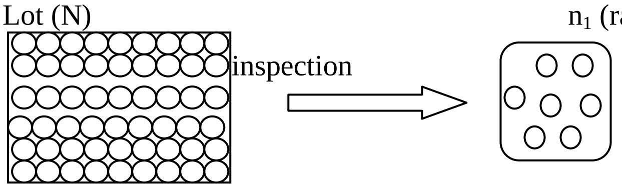
   

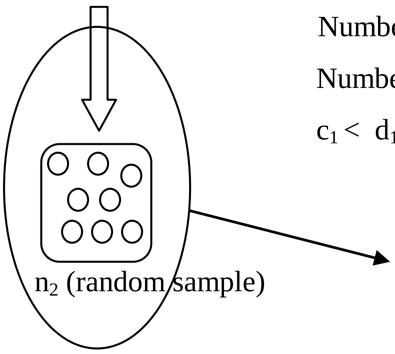
   Number of defective items in the sample (d) $\leq$ c $\Rightarrow$ Accept the lot

   Number of defective items in the sample (d) > c $\Rightarrow$ Reject the lot

2. Double-sampling plan

   Lot (N) | n_1 (random sample)

   Number of defective items in the 1st sample (d_1) $\leq$ c_1 $\Rightarrow$ Accept the lot

   Number of defective items in the 1st sample (d_1) > c_2 $\Rightarrow$ Reject the lot

   $c_1 < d_1 \leq c_2$ $\Rightarrow$ Take a second random sample of size n_2

   Taken if necessary

   Number of defective items in the 2nd sample : d_2

   $d_1 + d_2 \leq c_2$ $\Rightarrow$ Accept the lot

   $d_1 + d_2 > c_2$ $\Rightarrow$ Reject the lot

3. Multiple-sampling plan

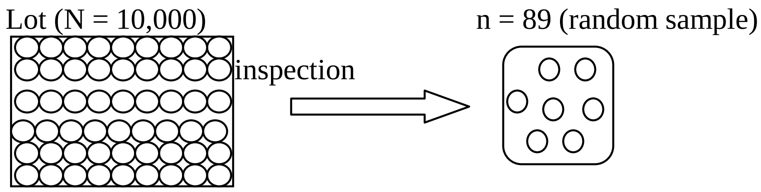
   More than two samples may be required in order to reach a decision regarding the disposition of the lot.

4. Sequential-sampling plan

   Units are selected from the lot one at a time, and following inspection of each unit, a decision is made either to accept the lot, reject the lot, or select another unit.

1. Single-Sampling Plans

Suppose that a lot of size N has been submitted for inspection. A **single-sampling plan** is defined by the sample size n and the acceptance number c. Thus, if the lot size is N = 10,000, then the sampling plan n = 89 and c = 2 means that from a lot of size 10,000 a random sample of n = 89 units is inspected and the number of nonconforming or defective items d observed.



$d \leq c = 2 \quad \Rightarrow \quad$ Accept the lot

$d > c = 2 \quad \Rightarrow \quad$ Reject the lot

The OC Curve

An important measure of the performance of an acceptance-sampling plan is the **operating characteristic (OC) curve.** This curve plots the probability of accepting the lot versus the lot fraction defective. Thus, the OC curve displays the discriminatory power of the sampling plan.

The OC curve of the sampling plan n = 89, c = 2 is given below:

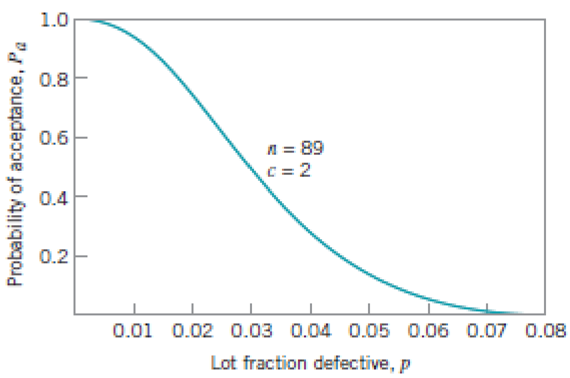


FIGURE 15.2 OC curve of the single-sampling plan $n = 89$, $c = 2$.

How the points on this curve are obtained?

Suppose that the lot size N is large (theoretically infinite).

D: The number of defectives in a random sample of n items

D ~Bin(n, p)$\Rightarrow P(D = d) = \binom{n}{d} p^d (1-p)^{n-d}, \;\; d = 0, 1, 2, \dots, n$

where p: The fraction of defective items in the lot.

$$P_a = P(\text{Accepting the lot}) = P(D \leq c) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d}$$

Following table displays the calculated acceptance probabilities for the single sampling plan n = 89 and c = 2.

Probabilities of Acceptance for the Single-Sampling Plan $n = 89$, $c = 2$

Fraction Defective, p	Probability of Acceptance, P_a
0.005	0.9897
$\boxed{0.010}$	$\boxed{0.9397}$
$\boxed{0.020}$	$\boxed{0.7366}$
0.030	0.4985
0.040	0.3042
0.050	0.1721
0.060	0.0919
0.070	0.0468
0.080	0.0230
0.090	0.0109

- $p = 0.01$, $n = 89$, $c = 2$ ➔ $P_a = ?$

$$P_a = P(D \leq 2) = \sum_{d=0}^{2} \binom{89}{d} 0.01^d (1-0.01)^{89-d}$$

$$= \binom{89}{0} 0.01^0 (1-0.01)^{89-0} + \binom{89}{1} 0.01^1 (1-0.01)^{89-1} + \binom{89}{2} 0.01^2 (1-0.01)^{89-2} = 0.9397 \cong 0.94$$

If the lots are 1% defective (p = 0.01), the probability of acceptance is approximately 0.94. This means that if 100 lots from a process that manufactures 1% defective product are submitted to this sampling plan, we will expect to accept 94 of the lots and reject 6 of them.

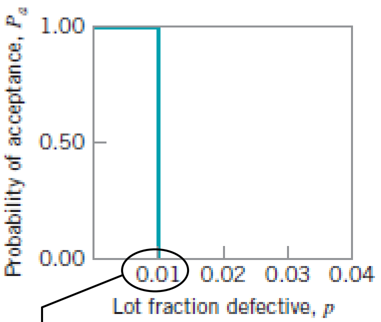
- $p = 0.02$, $n = 89$, $c = 2$ ➔ $P_a = ?$

$$P_a = P(D \leq 2) = \sum_{d=0}^{2} \binom{89}{d} 0.02^d (1-0.02)^{89-d}$$

$$= \binom{89}{0} 0.02^0 (1-0.02)^{89-0} + \binom{89}{1} 0.02^1 (1-0.02)^{89-1} + \binom{89}{2} 0.02^2 (1-0.02)^{89-2} = 0.7366 \cong 0.74$$

If the lots are 2% defective (p = 0.02), the probability of acceptance is approximately 0.74. This means that if 100 lots from a process that manufactures 2% defective product are submitted to this sampling plan, we will expect to accept 74 of the lots and reject 26 of them.

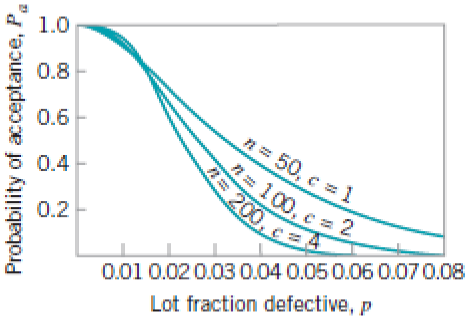
<u>Ideal OC curve:</u> An OC curve for a sampling plan that discriminated perfectlybetween good and bad lots.



p≥ 0.01 ⇒level of lot quality is considered as "bad"

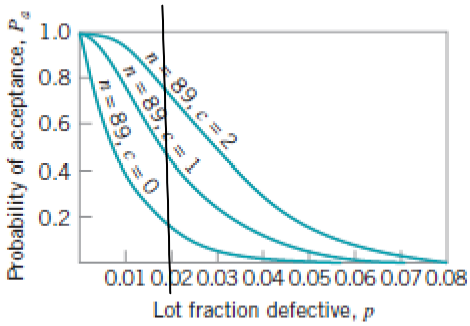
If such a sampling plan could be employed, all lots of “bad” quality would be rejected, and all lots of “good” quality would be accepted. Unfortunately, the **ideal OC curve** can almost never be obtained in practice.
The ideal OC curve shape can be approached, however, by increasing the sample size.

Effect of n on OC Curves:



The OC curve becomes more like the idealized OC curve shape as the sample sizeincreases. (Note that the acceptance number c is kept proportional to n.) The greater is the slope of the OC curve, the greater is the discriminatory power.

Effect of c on OC Curves:



Generally, changing the acceptance number (c) does not dramatically change the slope of the OC curve. As the acceptance number is decreased, the OC curve is shifted to the left.

When the lot size (N) is small, hypergeometric distribution is used for calculating the probability of lot acceptance and accordingly plotting the OC curve.

Exp.: N = 500, n= 50, c = 1, p: The fraction of defective items in the lot.

$\mathrm{P_a} = \mathrm{P(D \leq c|p) = P(D \leq 1|p) = P(D = 0|p) + P(D = 1|p)}$

$$= \frac{\binom{500p}{0}\binom{500(1-p)}{50}}{\binom{500}{50}} + \frac{\binom{500p}{1}\binom{500(1-p)}{49}}{\binom{500}{50}}$$

For p = 0.01; $\mathrm{P_a} = \frac{\binom{500(0.01)}{0}\binom{500(0.99)}{50}}{\binom{500}{50}} + \frac{\binom{500(0.01)}{1}\binom{500(0.99)}{49}}{\binom{500}{50}} = 0.589 + 0.3303 = 0.9193$

Suppose that N = 500 is considered as large and probability of lot acceptance is found by using binomial distribution;

$$P_a = P(D \leq 1|p = 0.01) = \sum_{d=0}^{1} \binom{50}{d} 0.01^d (1-0.01)^{50-d}$$

$$= \binom{50}{0} 0.01^0 (1-0.01)^{50-0} + \binom{50}{1} 0.01^1 (1-0.01)^{50-1} = 0.9106$$

<u>*Type-A OC Curves:*</u>Suppose that the lot size is *N (small)*, the sample size is *n*, and the acceptance number is *c*. The exact sampling distribution of the number of defective items in the sample for calculating the probability of lot acceptance is the **hypergeometric distribution.**

<u>*Type-B OC Curves:*</u>In the construction of the OC curve it was assumed that the samples came from a large lot or that we were sampling from a stream of lots selected at random from a process. In this situation, the **binomial distribution** is the exact probability distribution for calculating the probability of lot acceptance.

Note:$\frac{n}{N} \leq 0.10 \Rightarrow$ Type A and Type B OC Curves are almost same.

Designing a Single-Sampling Plan with a Specified OC Curve

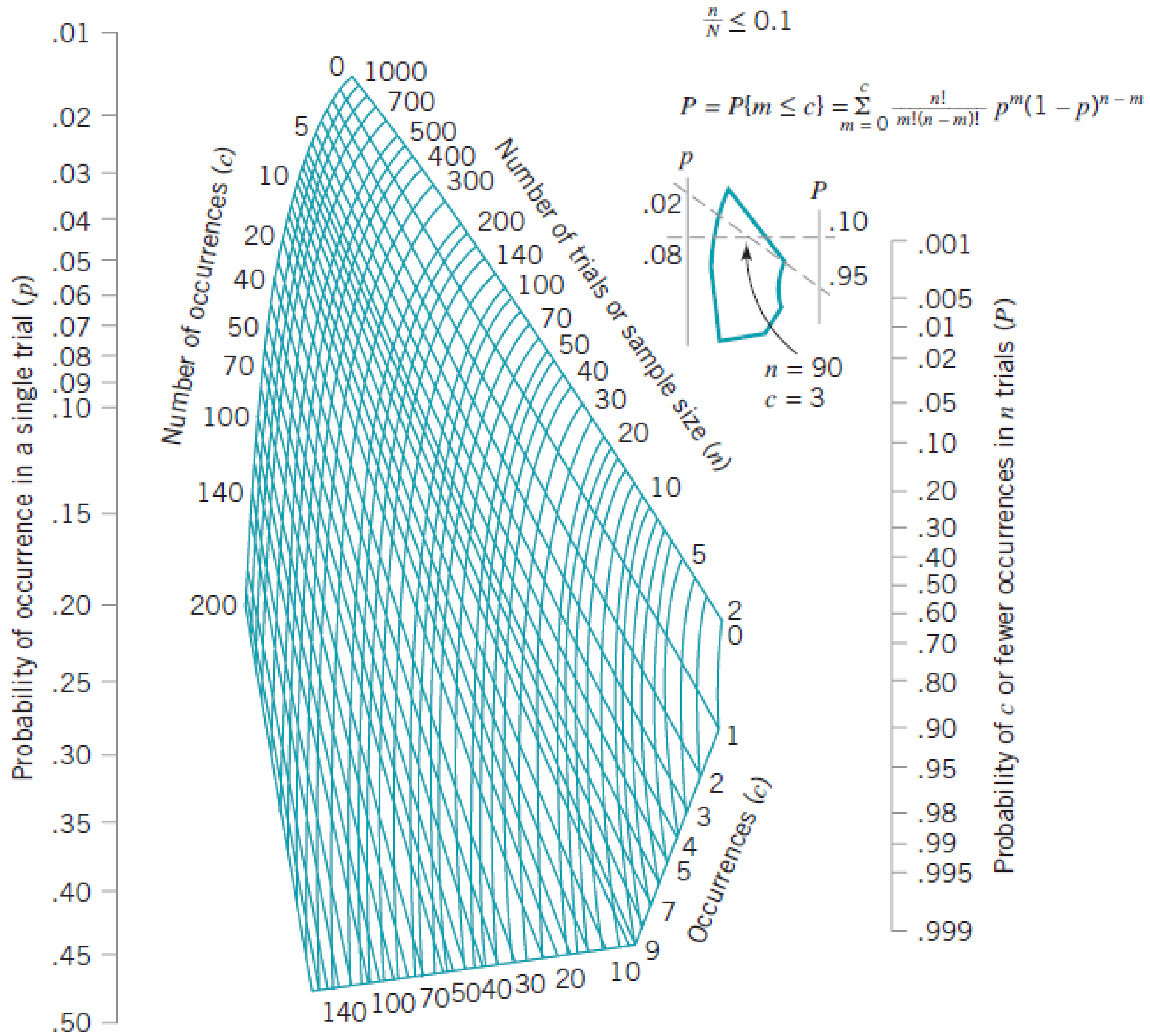
A common approach to the design of an acceptance-sampling plan is to require that the OCcurve pass through two designated points. Note that one point is not enough to fully specifythe sampling plan; however, two points are sufficient.

Suppose that we wish to construct a sampling plan such that the probability of acceptanceis 1-α for lots with fraction defective p_1, and the probability of acceptance is β for lotswith fraction defective p_2. Assuming that binomial sampling (with type-B OC curves) isappropriate, we see that the sample size *n* and acceptance number *c* are the solution to,

$$1-\alpha = P(D \leq c) = \sum_{d=0}^{c} \binom{n}{d} p_1^d (1-p_1)^{n-d}$$

$$\beta = P(D \leq c) = \sum_{d=0}^{c} \binom{n}{d} p_2^d (1-p_2)^{n-d}$$

These equations are nonlinear, and there is no simple, direct solution. The following nomograph can be used for solving these equations.



Suppose we wish to construct a sampling plan for which p_1 = 0.01, α=0.05, p_2 = 0.06, and β= 0.10. Locating the intersection of the lines connecting (**p_1 = 0.01,1-α= 0.95**) and (**p_2 = 0.06, β= 0.10**) on the nomograph indicates that the plan ***n* = 89, *c* =2** is very close to passing through these two points on the OC curve.

Although any two points on the OC curve could be used to define the sampling plan, it is customary in many industries to use the **acceptable quality level (AQL)** and **lot tolerance percent defective (LTPD)** points for this purpose.

AQL (considered as p_1): The poorest level of quality for the supplier's process that the consumer would consider to be acceptable as a process average. It is hoped that the supplier's process will operate at a fallout level that is considerably better than the AQL.

LTPD (considered as p_2): The poorest level of quality that the consumer is willing to accept in an individual lot. Alternate names for the LTPD are the **rejectable quality level (RQL)** and the **limiting quality level (LQL).**

When the levels of lot quality specified are p_1 = **AQL**andp_2 = **LTPD**, the corresponding points on the OC curve are usually referred to as theproducer's risk point and the consumer's risk point, respectively.

α:The **probability of rejecting a lot** with an **acceptable quality level** (Producer's risk)

β:The **probability of accepting a lot** with a **rejectable quality level** (Consumer's risk)

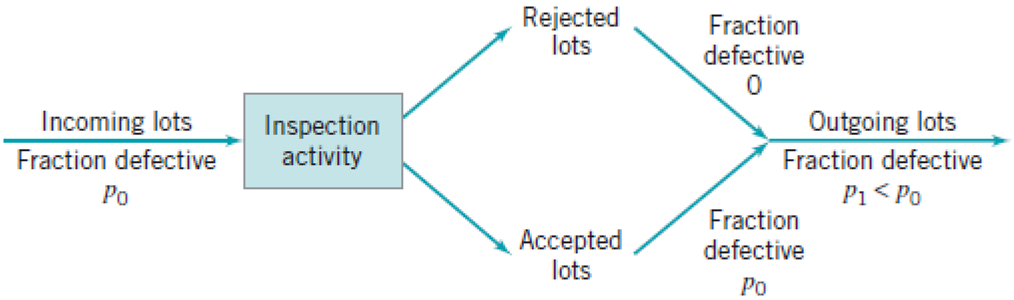
The consumer will often design the sampling procedure so that the OC curve gives a high probability of acceptance at the AQL and low probability of acceptance at the LTPD point.

Ex 15.4 from textbook (D.C. Montgomery, 6th ed.) will be solved during the lecture

(corresponds to questions 15.5 in org. copy, pdf)

EXC: 15.5, 15.6 **(corresponds to questions 15.4, 15.6 in org. copy, pdf)**

Rectifying Inspection



Rectifying inspection programs are used in situations where the manufacturer wishes to know the **average level of quality** that is likely to result at a given stage of the manufacturing operations. Thus, rectifying inspection programs are used either at receiving inspection, inprocess inspection of semifinished products, or at final inspection of finished goods.

Average outgoing quality is widely used for the evaluation of a rectifying sampling plan.

Average outgoing quality (AOQ): The quality in the lot that results from the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective *p*.

If we assume that all discovered defectives are replaced with good units → $\quad \mathrm{AOQ} = \frac{P_a p(N-n)}{N}$

where N: the lot size

p: fraction defective of incoming lots

P_a: probability of acceptance of a lot

n: sample size (after inspection, n items contain no defectives, because all discovered defectives are replaced)

1. After inspection, *n* items in the sample contain no defectives, because all discovered defectives are replaced

2. If the lot is rejected, $N - n$ items also contain no defectives

3. If the lot is accepted, $N - n$ items contain $p(N - n)$ defectives

Thus, lots in the outgoing stage of inspection have an expected number of defective units equal to $P_a p(N - n)$.

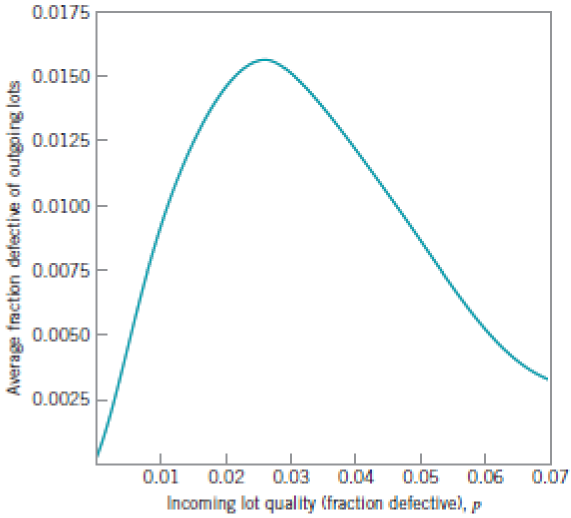
Exp: N=10000, n=89, c=2

p = 0.01 ➔P_a= 0.9397 (found previously)

$$AOQ = \frac{P_a p(N-n)}{N} = \frac{(0.9397)(0.01)(10000-89)}{10000} = 0.0093$$ ➔the average outgoing quality is 0.93% defective

Note that as the lot size *N* becomes large relative to the sample size *n;* $AOQ \cong P_a p$

Average outgoing quality will vary as the fraction defective of the incoming lots varies. The curve that plots average outgoing quality against incoming lot quality is called an *AOQ curve*. The AOQ curve for the sampling plan *n* = 89, *c*=2 is shown below.



Average outgoing quality curve for n=89, c=2

When the incoming quality is very good, the average outgoing quality is also very good. In contrast, when the incoming lot quality is very bad, most of the lots are rejected and screened, which leads to a very good level of quality in the outgoing lots. In between these extremes, the AOQ curve rises, passes through a maximum, and descends. The maximum ordinate on the AOQ curve represents the worst possible average quality that would result from the rectifying inspection program, and this point is called the **average outgoing quality limit (AOQL).**

According to the figure **AOQL=0.0155**. That is, no matter how bad the fraction defective is in the incoming lots, the outgoing lots will never have a worse quality level on the average than 1.55% defective.

Note that, this AOQL is an *average level of quality, across a large stream of lots*. It does not give assurance that an isolated lot will have quality no worse than 1.55% defective.

Another important measure relative to rectifying inspection is the total amount of inspection required by the sampling program and **Average Total Inspection (ATI)** is computed by the following formula.

$$ATI = n + (1 - P_a)(N - n)$$

If *p=0* (the lots contain no defective items) → no lots will be rejected $(P_a = 1)$→ ATI = n (the amount of inspection per lot)

If $p=1$ (the items are all defective) → every lot will be submitted to 100% inspection ($P_a = 0$)→ ATI = N

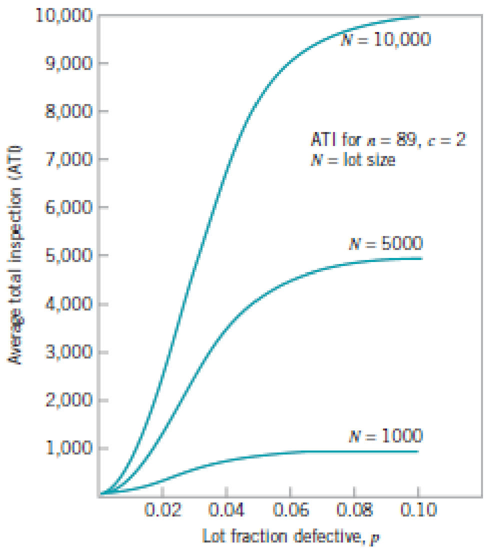
Exp: N=10000, n=89, c=2

p = 0.01 ➔ P_a = 0.9397 (found previously)

$$ATI = n + (1 - P_a)(N - n) = 89 + (1 - 0.9397)(10000 - 89) \cong 687$$

ATI=687 is the average number of units inspected over *many* lots with fraction defective p = 0.01.

It is possible to draw a curve of average total inspection as a function of lot quality (p). Average total inspection curves for the sampling plan n = 89, c = 2, for lot sizes of 1000, 5000, and 10000, are shown below.



Ex 15.7 from textbook (D.C. Montgomery, 6th ed.) will be solved during the lecture (corresponds to question 15.10 in org. copy, pdf)

2. **Double-Sampling Plans**

A double-sampling plan is a procedure in which, under certain circumstances, a second sample is required before the lot can be sentenced. A double-sampling plan is defined by four parameters.

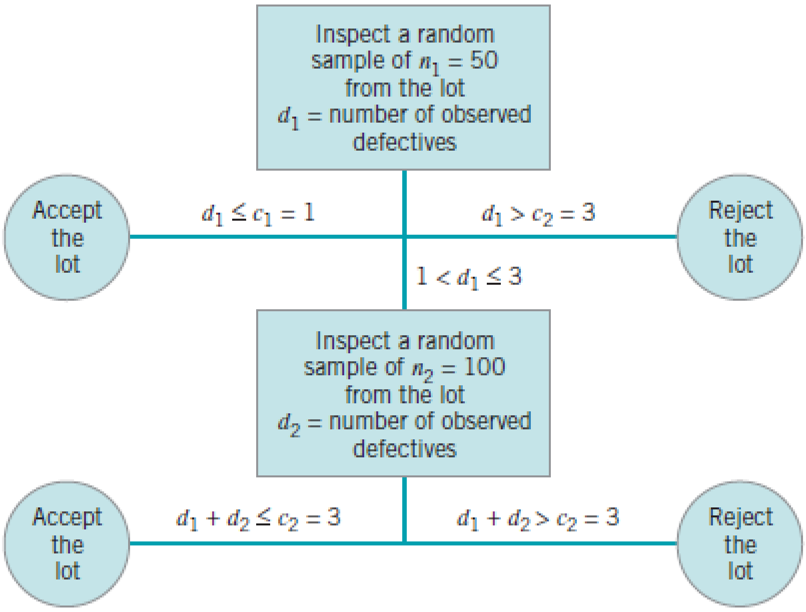
n_1 = sample size on the first sample

c_1 = acceptance number of the first sample

n_2 = sample size on the second sample

c_2 = acceptance number for both sample

Exp: Suppose $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, and $c_2 = 3$



<u>The OC Curve</u>

The performance of a double-sampling plan can be conveniently summarized by means of its OC curve where it gives the probability of acceptance as a function of lot or process quality (p).

P_a: The probability of acceptance on the combined samples

P_a^I: The probability of acceptance on the first sample $\Rightarrow$ $P_a = P_a^I + P_a^{II}$

P_a^{II}: The probability of acceptance on the second sample

For the previous example where $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, and $c_2 = 3$;

$$P_a^I = P(d_1 \leq c_1 = 1|p) = \sum_{d_1=0}^{1} \binom{50}{d_1} p^{d_1}(1-p)^{50-d_1}$$

$$P_a^{II} = P(d_1 = 2, d_2 \leq 1|p) + P(d_1 = 3, d_2 = 0|p) = \underbrace{P(d_1 = 2|p)P(d_2 \leq 1|p)}_{1} + \underbrace{P(d_1 = 3|p)P(d_2 = 0|p)}_{2}$$

1- $P(d_1 = 2|p)P(d_2 \leq 1|p) = \left[\binom{50}{2} p^2(1-p)^{48}\right]\left[\sum_{d_2=0}^{1} \binom{100}{d_2} p^{d_2}(1-p)^{100-d_2}\right]$

2- $P(d_1 = 3|p)P(d_2 = 0|p) = \left[\binom{50}{3} p^3(1-p)^{47}\right]\left[\binom{100}{0} p^0(1-p)^{100}\right]$

If $p = 0.05$ is the fraction defective in the incoming lot, then

$$P_a^I = P(d_1 \leq c_1 = 1|p = 0.05) = \sum_{d_1=0}^{1} \binom{50}{d_1} 0.05^{d_1}(1-0.05)^{50-d_1} = 0.279$$

$$P_a^{II} = \underbrace{P(d_1 = 2|p = 0.05)P(\, d_2 \leq 1|p = 0.05)}_{1} + \underbrace{P(d_1 = 3|p = 0.05)P(\, d_2 = 0|p = 0.05)}_{2}$$

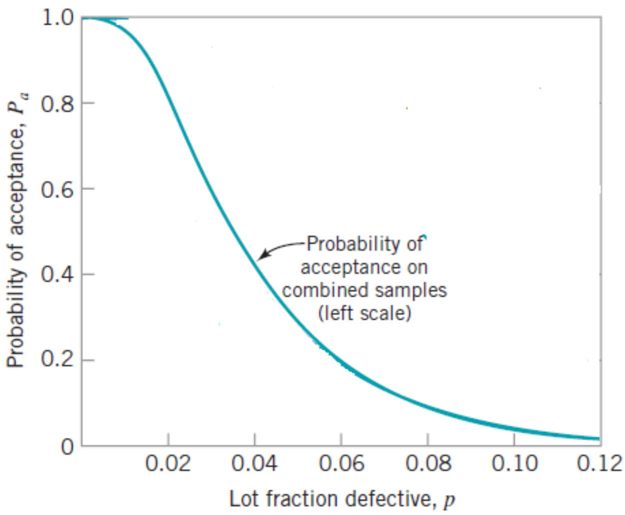
1- $\left[\binom{50}{2} 0.05^2(0.95)^{48}\right]\left[\sum_{d_2=0}^{1} \binom{100}{d_2} 0.05^{d_2}(0.95)^{100-d_2}\right] = 0.0097$

2- $\left[\binom{50}{3} p^3(1-p)^{47}\right]\left[\binom{100}{0} p^0(1-p)^{100}\right] = 0.001$

$\Rightarrow \quad P_a^{II} = 0.0097 + 0.001 = 0.0107$

$\Rightarrow \quad P_a = P_a^I + P_a^{II} = 0.279 + 0.0107 = 0.2897$

Other points on the OC curve are calculated similarly and following OC curve is obtained.



OC curve for the double-sampling plan, $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, and $c_2 = 3$.

The average sample number (ASN) curve of a double-sampling plan is also usually of interest to the quality engineer. In single-sampling, the size of the sample inspected from the lot is always constant, whereas in double-sampling, the size of the sample selected depends on whether or not the second sample is necessary.

With complete inspection of the second sample;

$$ASN = n_1 P_I + (n_1 + n_2)(1 - P_I)$$

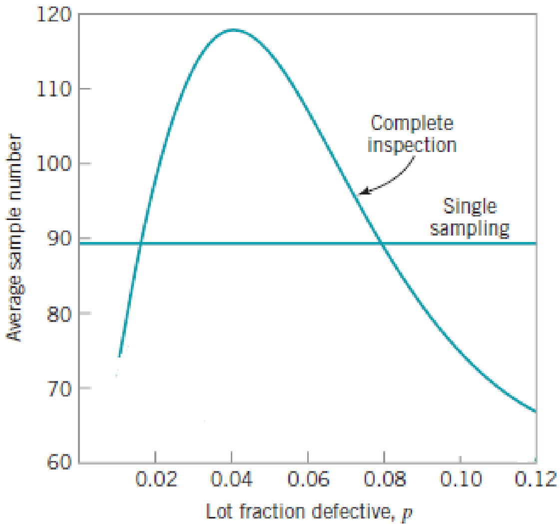
where P_I: the probability of making a lot-dispositioning decision on the *first* sample

P_I = P(lot is accepted on the first sample) + P (lot is rejected on the first sample)

$P_I = P(d_1 \leq c_1|p) + P(d_1 > c_2|p)$

If ASN values are computed for various values of lot fraction defective p, the plot of ASN versus p **(average sample number curve)** is obtained.

Following figure gives **the average sample number curve** for the **double-sampling plan n_1 = 60, c_1 = 2, n_2= 120, c_2 = 3**, and **the average sample number** for the **single-sampling with n = 89, c = 2**. Obviously, the sample size in the single-sampling plan is always constant. This double-sampling plan has been selected because it has an OC curve that is nearly identical to the OC curve for the single-sampling plan.



Rectifying Inspection

When rectifying inspection is performed with double sampling, the AOQ curve is given by

$$\mathrm{AOQ} = \frac{[\mathrm{P_a^I}(\mathrm{N} - \mathrm{n_1}) + \mathrm{P_a^{II}}(\mathrm{N} - \mathrm{n_1} - \mathrm{n_2})]\mathrm{p}}{\mathrm{N}}$$

assuming that all defective items discovered, either in sampling or 100% inspection, are replaced with good ones.

The average total inspection curve is given by

$$ATI = n_1 P_a^I + (n_1 + n_2) P_a^{II} + N(1 - P_a)$$

3. Multiple-Sampling Plans

A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot. An example of a multiple-sampling plan with five stages follows.

Cumulative-Sample Size	Acceptance Number	Rejection Number
20	0	3
40	1	4
60	3	5
80	5	7
100	8	9

d_1: the number of defective items in the first sample

d_2: the number of defective items in the second sample

d_3: the number of defective items in the third sample

d_4: the number of defective items in the fourth sample

d_5: the number of defective items in the fifth sample

According to the given multiple sampling plan;

$\mathrm{d}_1 \leq 0 \quad \Rightarrow \quad$ lot is accepted

$\mathrm{d}_1 \geq 3 \quad \Rightarrow \quad$ lot is rejected

$0 < \mathrm{d}_1 < 3 \;\Rightarrow\;$ 2nd sample is drawn

$\mathrm{d}_1 + \mathrm{d}_2 \leq 1 \quad \Rightarrow \quad$ lot is accepted

$\mathrm{d}_1 + \mathrm{d}_2 \geq 4 \quad \Rightarrow \quad$ lot is rejected

$1 < d_1 + \mathrm{d}_2 < 4 \;\Rightarrow\; 3rd\ sample\ is\ drawn$

$\mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 \leq 3 \quad \Rightarrow \quad$ lot is accepted

$\mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 \geq 5 \quad \Rightarrow \quad$ lot is rejected

$3 < \mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 < 5 \;\Rightarrow\;$ 4th sample is drawn

----

$\mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 + \mathrm{d}_4 \leq 5 \quad \Rightarrow \quad$ lot is accepted

$\mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 + \mathrm{d}_4 \geq 7 \quad \Rightarrow \quad$ lot is rejected

$5 < \mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 + \mathrm{d}_4 < 7 \;\Rightarrow\;$ 5th sample is drawn

----

$\mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 + \mathrm{d}_4 + \mathrm{d}_5 \leq 8 \quad \Rightarrow \quad$ lot is accepted

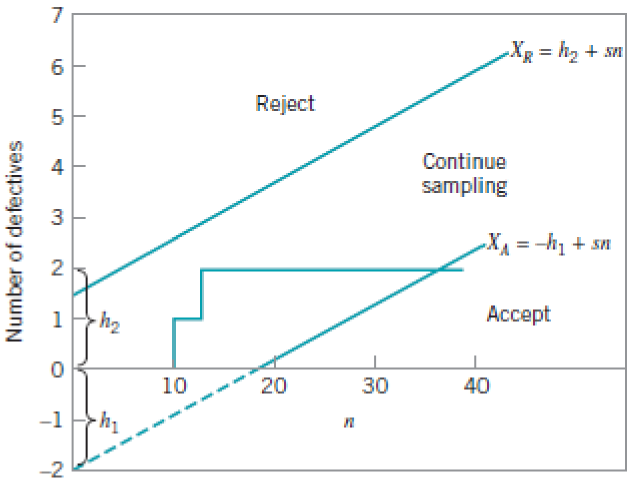
$\mathrm{d}_1 + \mathrm{d}_2 + \mathrm{d}_3 + \mathrm{d}_4 + \mathrm{d}_5 \geq 9 \quad \Rightarrow \quad$ lot is rejected

Ex 15.12(b), 15.13from textbook (D.C. Montgomery, 6^{th} ed.) will be solved during the lecture (corresponds to questions 15.11(b), 15.12 in org. copy, pdf)

4. **Sequential-Sampling Plans**

In sequential-sampling, we take a sequence of samples from the lot and allow the number of samples to be determined entirely by the results of the sampling process. In practice, sequential-sampling can theoretically continue indefinitely, until the lot is inspected 100%. If the sample size selected at each stage is greater than one, the process is usually called *group* sequential-sampling. If the sample size inspected at each stage is one, the procedure is usually called **item-by-item sequential-sampling.**

The operation of an item-by-item sequential-sampling plan is illustrated in the following figure.



The equations for the two limit lines for specified values of p_1, $1 - \alpha$, p_2, and β are;

$X_A = -h_1 + sn$ (acceptance line)

$X_R = h_2 + sn$ (rejection line)

where

$h_1 = \left(log\frac{1-\alpha}{\beta}\right)/k$ $h_2 = \left(log\frac{1-\beta}{\alpha}\right)/k$

$k = log\frac{p_2(1-p_1)}{p_1(1-p_2)}$ $s = \left(log\frac{1-p_1}{1-p_2}\right)/k$

Exp: Find a sequential-sampling plan for which $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.06$, and $\beta = 0.10$.

$$k = log\frac{p_2(1-p_1)}{p_1(1-p_2)} = log\frac{0.06(1-0.01)}{0.01(1-0.06)} = 0.80066$$

$$h_1 = \left(log\frac{1-\alpha}{\beta}\right)/k = \left(log\frac{1-0.05}{0.10}\right)/0.80066 = 1.22 h_2 = \left(log\frac{1-\beta}{\alpha}\right)/k = \left(log\frac{1-0.10}{0.05}\right)/0.80066 = 1.57$$

$$s = \left(log\frac{1-p_1}{1-p_2}\right)/k = \left(log\frac{1-0.01}{1-0.06}\right)/0.80066 = 0.028$$

$\Longrightarrow X_A = -h_1 + sn = -1.22 + 0.028n$ (acceptance line)

$X_R = h_2 + sn = 1.57 + 0.028n$ (rejection line)

n=1 → $X_A = -1.22 + 0.028n = -1.22 + 0.028(1) = -1.192$ → lot cannot be accepted

$X_R = 1.57 + 0.028n = 1.57 + 0.028(1) = 1.598$ → lot cannot be rejected

n=2 → $X_A = -1.22 + 0.028n = -1.22 + 0.028(2) = -1.164$ → lot cannot be accepted

$X_R = 1.57 + 0.028n = 1.57 + 0.028(2) = 1.626$ → If both 1st and 2nd items are defective, lot is rejected

⋮

n=45 → $X_A = -1.22 + 0.028n = -1.22 + 0.028(45) = 0.04$ → If all of the 45 items are nondefective, the lot is accepted.

$X_R = 1.57 + 0.028n = 1.57 + 0.028(45) = 2.83$ → If 3 out of 45 items are defective, the lot is rejected.

Item-by-Item Sequential-Sampling Plan $p_1 = 0.01, \alpha = 0.05, p_2 = 0.06, \beta = 0.10$ (first 46 units only)

Number of Items Inspected, *n*	Acceptance Number	Rejection Number	Number of Items Inspected, *n*	Acceptance Number	Rejection Number
1	a	b	24	a	3
2	a	2	25	a	3
3	a	2	26	a	3
4	a	2	27	a	3
5	a	2	28	a	3
6	a	2	29	a	3
7	a	2	30	a	3
8	a	2	31	a	3
9	a	2	32	a	3
10	a	2	33	a	3
11	a	2	34	a	3
12	a	2	35	a	3
13	a	2	36	a	3
14	a	2	37	a	3
15	a	2	38	a	3
16	a	3	39	a	3
17	a	3	40	a	3
18	a	3	41	a	3
19	a	3	42	a	3
20	a	3	43	a	3
21	a	3	44	0	3
22	a	3	45	0	3
23	a	3	46	0	3

"a" means acceptance not possible.
"b" means rejection not possible.

Military Standard 105E (MIL STD 105E)

The sampling plans discussed in previous sections of this chapter are individual sampling plans. A sampling scheme is an overall strategy specifying the way in which sampling plans are to be used. MIL STD 105E is a collection of sampling schemes; therefore, it is an acceptancesampling system. Our discussion will focus primarily on MIL STD 105E.

The standard provides for three types of sampling: single-sampling, double-sampling, and multiple-sampling. For each type of sampling plan, a provision is made for either normal inspection, tightened inspection, or reduced inspection.

Normal inspection: Used at the start of the inspection activity

Tightened inspection: Instituted when the supplier's recent quality history has deteriorated

Reduced inspection: Instituted when the supplier's recent quality history has been exceptionally good.

The primary focal point of MIL STD 105E is the acceptable quality level (AQL). The AQL is generally specified in the contract or by the authority responsible for sampling. Different AQLs may be designated for different types of defects. For example, the standard differentiates critical defects, major defects, and minor defects. It is relatively common practice to choose an AQL of 1% for major defects and an AQL of 2.5% for minor defects. No critical defects would be acceptable.

The only control over the discriminatory power of the sampling plan (i.e., the steepness of the OC curve) is through the choice of inspection level.

The sample size used in MIL STD 105E is determined by the lot size and by the choice of inspection level.

<u>General levels of inspection:</u>

Level I→ Requires about one-half the amount of inspection as Level II and may be used when less discrimination is needed.

Level II→ Normal

Level III→ Requires about twice as much inspection as Level II and should be used when more discrimination is needed.

<u>Special inspection levels:</u>

The special inspection levels use very small samples, and should be employed only when the small sample sizes are necessary and when greater sampling risks can or must be tolerated. (*S-1, S-2, S-3,* and *S-4*)

A step-by-step procedure for using MIL STD 105E is as follows:

1. Choose the AQL.

2. Choose the inspection level.

3. Determine the lot size.

4. Find the appropriate sample size code letter from Table 15.4.

5. Determine the appropriate type of sampling plan to use (single, double, multiple).

6. Enter the appropriate table to find the type of plan to be used.

7. Determine the corresponding normal and reduced inspection plans to be used when required.

Tables 15.5, 15.6, and 15.7 (D. C. Montgomery, p658-660) present the single-sampling plans for normal inspection, tightened inspection, and reduced inspection, respectively. The standard also contains tables for double-sampling plans and multiple-sampling plans for normal, tightened, and reduced inspection.

Exp: Suppose that a product is submitted in lots of size N = 2000. The acceptable quality level is 0.65%. Use MIL STD 105E to generate normal, tightened, and reduced single-sampling plans under general inspection level II.

N=2000, General inspection level II ➔ Sample size code letter = K (Table 15.4)

Single-sampling plan(c: acceptance number, r: rejection number)

Normal inspection (Table 15.5) ➔ n = 125, c = 2

Tightenedinspection (Table 15.6) ➔ n = 125, c = 1

Reducedinspection (Table 15.7) ➔ n = 50, c = 1, r = 3 (if two defectives were encountered, the lot would be accepted, but the next lot would be inspected under normal inspection)

For a specified AQL and inspection level and a given lot size, MIL STD 105E provides a normal sampling plan that is to be used as long as the supplier is producing the product at AQL quality or better. It also provides a procedure for switching to tightened and reduced inspection whenever there is an indication that the supplier's quality has changed.

<u>The switching procedures between normal, tightened, and reduced inspection</u>

1. Normal to tightened. When normal inspection is in effect, tightened inspection is instituted when two out of five consecutive lots have been rejected on original submission.

2. Tightened to normal. When tightened inspection is in effect, normal inspection is instituted when five consecutive lots or batches are accepted on original inspection.

3. Normal to reduced. When normal inspection is in effect, reduced inspection is instituted provided all of the following conditions are satisfied.

- **a.** The preceding ten lots have been on normal inspection, and none of the lots has been rejected on original inspection.
- **b.** Production is at a steady rate; that is, no difficulty such as machine breakdowns, material shortages, or other problems have recently occurred
- **c.** Reduced inspection is considered desirable by the authority responsible for sampling

4. Reduced to normal. When reduced inspection is in effect, normal inspection is instituted when any of the following conditions occur.

- **a.** A lot or batch is rejected.
- **b.** When the sampling procedure terminates with neither acceptance nor rejection criteria having been met, the lot or batch will be accepted, but normal inspection is reinstituted starting with the next lot.
- **c.** Production is irregular or delayed

5. Discontinuance of inspection. In the event that ten consecutive lots remain on tightened inspection, inspection under the provision of MIL STD 105E should be terminated, and action should be taken at the supplier level to improve the quality of submitted lots.

Ex 15.14, 15.15 from textbook (D.C. Montgomery, 6th ed.) will be solved during the lecture
(corresponds to questions 15.16, 15.17 in org. copy, pdf)

The Dodge–Romig Sampling Plans

H. F. Dodge and H. G. Romig developed a set of sampling inspection tables for lot-by-lot inspection of product by attributes using two types of sampling plans:

1.Plans that provide a specified average outgoing quality limit (AOQL) (**AOQL Plans)**

2.Plans for lot tolerance percent defective (LTPD) protection (**LTPD Plans)**

For each of these approaches to sampling plan design, there are tables for single- and double-sampling.

1. **AOQL Plans** are designed so that the average total inspection (ATI) for a given AOQL and a specified process average *p*(the most likely level of incoming lot quality)will be minimized.
2. **LTPD Plans** are designed so that the probability of lot acceptance at the specified LTPD is 0.1 (β=0.1) and the average total inspection (ATI) is minimum.

AOQL Plans

The Dodge–Romig (1959) tables give AOQL sampling plans for AOQL values of 0.1%, 0.25%, 0.5%, 0.75%, 1%, 1.5%, 2%, 2.5%, 3%, 4%, 5%, 7%, and 10%. For each of these AOQL values, six classes of values for the process average are specified. Tables are provided for both single- and double-sampling.D.C. Montgomery, 6^{th} ed. provides only one of these tables as an example on page 665. (**Table 15.8.** Dodge-Romig inspection table-single sampling plan for AOQL=3%)

Exp: Suppose that we are inspecting LSI memory elements for a personal computer and that the elements are shipped in lots of size N = 5000. The supplier’s process average fallout is 1% nonconforming. We wish to find a single-sampling plan with an AOQL = 3%.

n = 65, c = 3 (Table 15.8)

LTPD = 10.3% (This is the point on the OC curve for which P_a= 0.10)

Therefore, the sampling plan n = 65, c = 3 gives an AOQL of 3% nonconforming and provides assurance that 90% of incoming lots that are as bad as 10.3% defective will be rejected.

Assuming that incoming quality is equal to the process average (p=0.01);

$$\mathrm{P_a} = \mathrm{P}(\mathrm{D} \leq 3|\mathrm{p} = 0.01) = \sum_{\mathrm{d}=0}^{3} \binom{65}{\mathrm{d}} 0.01^{\mathrm{d}}(1-0.01)^{65-\mathrm{d}} = 0.9958$$

The average total inspection (ATI) for this plan is;

$$ATI = n + (1 - P_a)(N - n) = 65 + (1 - 0.9958)(5000 - 65) \cong 86$$

Thus, we will inspect approximately 86 units, on the average, in order to sentence a lot.

LTPD Plans

The Dodge–Romig LTPD tables are provided for LTPD values of 0.5%, 1%, 2%, 3%, 4%, 5%, 7%, and 10%. **Table 15.9 (D.C.Montgomery, 6th ed.)** for an LTPD of 1% is representative of these Dodge–Romig tables.

Exp: Suppose that LSI memory elements for a personal computer are shipped from the supplier in lots of size N=5000. The supplier's process average fallout is 0.25% nonconforming, and we wish to use a single-sampling plan with an LTPD of 1%.

n = 770, c = 4 (Table 15.9)

If we assume that rejected lots are screened 100% and that defective items are replaced with good ones, the AOQL for this plan is approximately 0.28%.

Estimation of ProcessAverage

Selection of a Dodge–Romig plan depends on knowledge of the supplier's process average fallout or percent nonconforming. An estimate of the process average can be obtained using a fraction defective control chart, based on the first 25 lots submitted by the supplier. Until results from 25 lots have been accumulated, the recommended procedure is to use the largest process average in the appropriate table.

Ex 15.18, 15.20 from textbook (D.C. Montgomery, 6th ed.) will be solved during the lecture

(corresponds to questions 15.22, 15.20 in org. copy, pdf)

TABLE 15.4

Sample Size Code Letters (MIL STD 105E, Table 1)

	Special Inspection Levels				General Inspection Levels		
Lot or Batch Size	**S-1**	**S-2**	**S-3**	**S-4**	**I**	**II**	**III**
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1200	C	C	E	F	G	J	K
1201 to 3200	C	D	E	G	H	K	L
3201 to 10000	C	D	F	G	J	L	M
10001 to 35000	C	D	F	H	K	M	N
35001 to 150000	D	E	G	J	L	N	P
150001 to 500000	D	E	G	J	M	P	Q
500001 and over	D	E	H	K	N	Q	R

TABLE 15.5

Master Table for Normal Inspection for Single-Sampling (U.S. Dept. of Defense MIL STD 105E, Table II-A)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (normal inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

TABLE 15.6

Master Table for Tightened Inspection—Single-Sampling (U.S. Dept. of Defense MIL STD 105E, Table II-B)

Sample Size Code Letter	Sample Size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150			1 2																							

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

■ TABLE 15.7

Master Table for Reduced Inspection—Single-Sampling (U.S. Dept. of Defense MIL STD 105E, Table II-C)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (reduced inspection)†																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	2 4	3 5	5 6	7 8	10 11	14 15	21 22	30 31
C	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑
D	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑
E	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑	↑
F	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑
G	13	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑
H	20	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑
J	32	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	50	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	80	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	125	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	200	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	315	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	500	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	800	↑	↑	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

† = If the acceptance number has been exceeded, but the rejection number has not been reached, accept the lot, but reinstate normal inspection.

■ **TABLE 15.8**

Dodge–Romig Inspection Table—Single-Sampling Plans for AOQL = 3.0%

	Process Average																	
	0–0.06%			0.07–0.60%			0.61–1.20%			1.21–1.80%			1.81–2.40%			2.41–3.00%		
			LTPD			LTPD			LTPD			LTPD			LTPD			LTPD
Lot Size	n	c	%	n	c	%	n	c	%	n	c	%	n	c	%	n	c	%
1–10	All	0	—	All	0	—	All	0	—	All	0	—	All	0	—	All	0	—
11–50	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0
51–100	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	22	1	16.4
101–200	12	0	17.0	12	0	17.0	12	0	17.0	25	1	15.1	25	1	15.1	25	1	15.1
201–300	12	0	17.0	12	0	17.0	26	1	14.6	26	1	14.6	26	1	14.6	40	2	12.8
301–400	12	0	17.1	12	0	17.1	26	1	14.7	26	1	14.7	41	2	12.7	41	2	12.7
401–500	12	0	17.2	27	1	14.1	27	1	14.1	42	2	12.4	42	2	12.4	42	2	12.4
501–600	12	0	17.3	27	1	14.2	27	1	14.2	42	2	12.4	42	2	12.4	60	3	10.8
601–800	12	0	17.3	27	1	14.2	27	1	14.2	43	2	12.1	60	3	10.9	60	3	10.9
801–1000	12	0	17.4	27	1	14.2	44	2	11.8	44	2	11.8	60	3	11.0	80	4	9.8
1,001–2,000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	80	4	9.8	100	5	9.1
2,001–3,000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	100	5	9.1	140	7	8.2
3,001–4,000	12	0	17.5	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	165	8	7.8
4,001–5,000	28	1	13.8	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	210	10	7.4
5,001–7,000	28	1	13.8	45	2	11.8	65	3	10.3	105	5	8.8	145	7	8.1	235	11	7.1
7,001–10,000	28	1	13.9	46	2	11.6	65	3	10.3	105	5	8.8	170	8	7.6	280	13	6.8
10,001–20,000	28	1	13.9	46	2	11.7	85	4	9.5	125	6	8.4	215	10	7.2	380	17	6.2
20,001–50,000	28	1	13.9	65	3	10.3	105	5	8.8	170	8	7.6	310	14	6.5	560	24	5.7
50,001–100,000	28	1	13.9	65	3	10.3	125	6	8.4	215	10	7.2	385	17	6.2	690	29	5.4

TABLE 15.9

Dodge–Romig Single-Sampling Table for Lot Tolerance Percent Defective (LTPD) = 1.0%

	Process Average																	
	0–0.01%			0.011%–0.10%			0.11–0.20%			0.21–0.30%			0.31–0.40%			0.41–0.50%		
			AOQL			AOQL			AOQL			AOQL			AOQL			AOQL
Lot Size	n	c	%	n	c	%	n	c	%	n	c	%	n	c	%	n	c	%
1–120	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0
121–150	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06
151–200	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08
201–300	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10
301–400	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12
401–500	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13
501–600	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	305	1	0.14
601–800	200	0	0.14	200	0	0.14	200	0	0.14	330	1	0.15	330	1	0.15	330	1	0.15
801–1000	205	0	0.14	205	0	0.14	205	0	0.14	335	1	0.17	335	1	0.17	335	1	0.17
1,001–2,000	220	0	0.15	220	0	0.15	360	1	0.19	490	2	0.21	490	2	0.21	610	3	0.22
2,001–3,000	220	0	0.15	375	1	0.20	505	2	0.23	630	3	0.24	745	4	0.26	870	5	0.26
3,001–4,000	225	0	0.15	380	1	0.20	510	2	0.23	645	3	0.25	880	5	0.28	1,000	6	0.29
4,001–5,000	225	0	0.16	380	1	0.20	520	2	0.24	770	4	0.28	895	5	0.29	1,120	7	0.31
5,001–7,000	230	0	0.16	385	1	0.21	655	3	0.27	780	4	0.29	1,020	6	0.32	1,260	8	0.34
7,001–10,000	230	0	0.16	520	2	0.25	660	3	0.28	910	5	0.32	1,150	7	0.34	1,500	10	0.37
10,001–20,000	390	1	0.21	525	2	0.26	785	4	0.31	1,040	6	0.35	1,400	9	0.39	1,980	14	0.43
20,001–50,000	390	1	0.21	530	2	0.26	920	5	0.34	1,300	8	0.39	1,890	13	0.44	2,570	19	0.48
50,001–100,000	390	1	0.21	670	3	0.29	1,040	6	0.36	1,420	9	0.41	2,120	15	0.47	3,150	23	0.50

APPENDIX VI

Factors for Constructing Variables Control Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits A	A_2	A_3	Chart for Averages: Factors for Center Line c_4	$1/c_4$	Chart for Standard Deviations: Factors for Control Limits B_3	B_4	B_5	B_6	Factors for Center Line d_2	$1/d_2$	Chart for Ranges: Factors for Control Limits d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$.

$$A = \frac{3}{\sqrt{n}} \qquad A_3 = \frac{3}{c_4\sqrt{n}} \qquad c_4 \cong \frac{4(n-1)}{4n-3}$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}} \qquad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}} \qquad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$