

The Simplex Algorithm

Why Use a Special Algorithm

- Exhaustive search takes too long
 - Too many feasible solutions
- We want to ask many “*what if*” questions
 - So we run the model over and over
- We want to perform sensitivity analysis
 - What constraints are binding?
 - How much do the constraints cost us?
 - Which products are the most profitable?

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What is The Simplex Algorithm

- Efficient procedure that searches a subset of the feasible solutions
- Hill climbing procedure that finds better and better solutions
- Procedure that calculates the cost of the constraints and value of the products *at the margin*

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Terminology

- Feasible solution
 - Solution where constraints are satisfied
 - Many are possible
- Optimal solution
 - Feasible solution with highest (or lowest) objective function value
 - Usually are one, but ties are possible and happen frequently
- Boundary equation
 - Constraint with inequality replaced by an equality
 - These define the *feasible region*

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The Simplex Method

- This method on the assumption that the optimal solution will be an extreme point.
 - The constraint equations are reformulated as equalities by introducing slack variables.

For Example: From problem I

$$\text{Cons(1)} \quad 7 X_1 + 11 X_2 \leq 77$$

A slack variable S_1 as the amount of **raw** gas that is not used for a particular production level (X_1, X_2). If this quantity is added to the left side of constraint (1), it becomes

$$\text{Cons(1)} \quad 7 X_1 + 11 X_2 + S_1 = 77$$

What S_1 tells us?

- $S_1 > 0$, have some slack for this constrain.
- $S_1 < 0$, have exceeded this constrain.
- $S_1 = 0$, exactly meet this constrain.

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The Simplex Method

A different slack variable is developed for each constraint equation, and form **fully augmented version**.

$$Z = 150 X_1 + 175 X_2$$

$$7 X_1 + 11 X_2 + S_1 = 77 \quad (a)$$

$$10 X_1 + 8 X_2 + S_2 = 80 \quad (b)$$

$$X_1 + S_3 = 9 \quad (c)$$

$$X_2 + S_4 = 6 \quad (d)$$

$X_1, X_2, S_1, S_2, S_3, S_4 \geq 0$, Now we formed a system of linear equations.

- This problem involves solving 4 equations with 6 unknowns.
- The difference between the number of unknown and number of equations equal to 2.
- So every feasible point has **2 variables** out of 6 equal to **zero**.

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The Simplex Method

For example, five corner points of the area ABCDE have the following zero values:

Extreme point	Zero Variables
A	X_1, X_2
B	X_2, S_2
C	S_1, S_2
D	S_1, S_4
E	X_1, S_4

- Thus, extreme points can be determined from the standard form by setting two of variables equal to zero.
 - For example, setting point E, $X_1 = S_4 = 0$, reduces the standard form to

$$11 X_2 + S_1 = 77$$

$$8 X_2 + S_2 = 80$$

$$+ S_3 = 9$$

$$X_2 = 6$$

System of equation can be solved for

$X_2 = 6, S_1 = 11, S_2 = 32, \text{ and } S_3 = 9$

And $X_1 = S_4 = 0$, These values define point E

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The Simplex Method

- To generalize, a basic solution for **m** linear equations with **n** unknowns is developed by setting **n-m** variables to zero.
- Solve the **m** equations for the **m** remaining unknowns.
- The zero variables are formally referred to as *nonbasic variables*, whereas the remaining **m** variables are called *basic variables*.
- If all the basic variables are nonnegative, the result is called a **basic feasible solution**. The optimum will be one of these.
- Now a direct approach to determining the optimal solution would be to calculate all the basic solutions, determine which were feasible, and among those, which had the highest value of Z.
- But this is not a **wise approach**
 - If $m = 10$ with $n = 16$, you have to 8008 [= $16! / (10! 6!)$] 10 x 10 systems of equations to solve!
 - A significant portion of these may be infeasible.

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Implementation of the Simplex Method

The simplex method avoids inefficiencies outlined in the previous section.

- It starts with a basic feasible solution.
 - It moves through a sequence of other basic feasible solutions and
 - Each step improves the value of the objective function.
 - Eventually, the optimal value is reached and the method is terminated.

Examples:

- Start at a basic feasible solution (an extreme corner point of the feasible space).
 - Starting point A; $X_1 = X_2 = 0$.
 - The original 6 equations with 4 unknowns become

$$\begin{array}{rcl} S_1 & & = 77 \\ S_2 & & = 80 \\ S_3 & & = 9 \\ S_4 & & = 6 \end{array}$$

Thus, the starting values for the basic variables are given automatically as being equal to the right-hand sides of the constraints.

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Implementation of the Simplex Method

Summarize key information in a *tableau*.

Basic	Z	X_1	X_2	S_1	S_2	S_3	S_4	Solution	Intercept
Z	1	-150	-175	0	0	0	0	0	
S_1	0	7	11	1	0	0	0	77	11
S_2	0	10	8	0	1	0	0	80	8
S_3	0	1	0	0	0	1	0	9	9
S_4	0	0	1	0	0	0	1	6	∞

- The objective function is $Z - 150 X_1 - 175 X_2 - 0 S_1 - 0 S_2 - 0 S_3 - 0 S_4 = 0$
- The next step involves moving to a new basic feasible solution that leads to an improvement of the objective function.
 - To accomplish this, increase a current nonbasic variable to basic (nonzero) (at this point, X_1 or X_2) so that Z increases. This variable is called the *entering variable*.
 - Extreme points must have 2 zero values. So, one of the current basic variables (S_1 , S_2 , S_3 , or S_4) must also be set to zero (nonbasic). This variable is called the *leaving variable*.

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Develop a mathematical approach for choosing the entering and leaving variables.

- The entering variable can be any variable in the objective function having a negative coefficient (because it will make Z bigger). For our case, X_2 .
- Choose a leaving variable from among basic variables.
 - One way is to calculate the values at which the constraint lines intersect the axis or line corresponding to the leaving variable (in our case, the X_1 axis). We can calculate this value as the ratio of the right-hand side of the constraint (the "Solution" column of the tableau) to the corresponding coefficient of X_1 .
 - For example, for the first constraints slack variable S_1 , the result is, Intercept = $77/7 = 11$,

Intercept (Solution/ X_1)
$S_1 = 11$
$S_2 = 8$
$S_3 = 9$
$S_4 = \infty$

Because 8 is the smallest positive integer, it means that the second constraint line will be reached first as X_1 is increased. Therefore, S_2 should be the entering variable.

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Implementation of the Simplex Method

At this point, we have moved to point B ($X_2 = S_2 = 0$), and the new basic solution becomes

$$\begin{array}{rcl} 7 X_1 + S_1 & = & 77 \\ 10 X_1 & = & 80 \\ X_1 + S_3 & = & 9 \\ & + S_4 = & 6 \end{array}$$

The solution of this system of equations at point B: $X_1 = 8$, $S_1 = 21$, $S_3 = 1$, and $S_4 = 6$.

- The tableau can be used to make the same calculation by employing the Gauss-Jordan method.
- Gauss-Jordan involved converting type pivot element to 1 and then eliminating the coefficients in the same column above and below the pivot element

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Implementation of the Simplex Method

For this example, the pivot row is S_2 (the entering variable) and the pivot element is 10 (the coefficient of the leaving variable, X_1). Dividing the row by 10 and replacing S_2 by X_1 gives

Basic	Z	X_1	X_2	S_1	S_2	S_3	S_4	Solution	Intercept
Z	1	-150	-175	0	0	0	0	0	
S_1	0	7	11	1	0	0	0	77	
X_1	0	1	0.8	0	0.1	0	0	8	
S_3	0	1	0	0	0	1	0	9	
S_4	0	0	1	0	0	0	1	6	

Next, the X_1 coefficients in the other rows can be eliminated, for example the pivot row is multiplied by -150 and subtracted from the first row to give

Z	X_1	X_2	S_1	S_2	S_3	S_4	Solution
1	-150	-175	0	0	0	0	0
-0	$-(-150)$	$-(-120)$	-0	$-(-15)$	0	0	$-(-1200)$
1	0	-55	0	15	0	0	1200

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Similar operations can be performed on the remaining rows to give the new **Tableau**

Basic	Z	X ₁	X ₂	S ₁	S ₂	S ₃	S ₄	Solution	Intercept
Z	1	0	-55	0	15	0	0	1200	
S ₁	0	0	5.4	1	-0.7	0	0	21	3.889
X ₁	0	1	0.8	0	0.1	0	0	8	10
S ₃	0	0	-0.8	0	-0.1	1	0	1	-1.25
S ₄	0	0	1	0	0	0	1	6	6

The objective function has increased to $Z = 1200$.

- Next, and in this case final step.
- Only one more variable, X_2 , has a negative value in the objective function, and it is therefore chosen as the leaving variable.
 - According to the intercept values (now calculated as the solution column over the coefficients in the X_2 column), the first constraint has the smallest positive value, and therefore, S_1 is selected as the entering variable.

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Implementation of the Simplex Method

Thus, the simplex method moves us from points B to C in Figure. Finally, the Gauss-Jordan elimination can be implemented to solve the simultaneous equations.

Basic	Z	X ₁	X ₂	S ₁	S ₂	S ₃	S ₄	Solution	Intercept
Z	1	0	-55	0	15	0	0	1200	
X ₂	0	0	5.4/5.4	1/5.4	-0.7/5.4	0	0	21/5.4	
X ₁	0	1	0.8	0	0.1	0	0	8	
S ₃	0	0	-0.8	0	-0.1	1	0	1	
S ₄	0	0	1	0	0	0	1	6	

Next, the X_2 coefficients in the other rows can be eliminated, for example the pivot row is multiplied by -55 and subtracted from the first row to give

Z	X ₁	X ₂	S ₁	S ₂	S ₃	S ₄	Solution
1	0	-55	0	15	0	0	1200
-0	-0	$-(-55 \times 1)$	$-(-55 \times 0.1852)$	$-(-55 \times (-0.1296))$	0	0	$-(-55 \times (3.889))$
1	0	0	10.186	7.8704	0	0	1413.895

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Implementation of the Simplex Method

The result is the final **Tableau**.

Basic	Z	X ₁	X ₂	S ₁	S ₂	S ₃	S ₄	Solution	Intercept
Z	1	0	0	10.186	7.8704	0	0	1413.895	
X ₂	0	0	1	0.1852	-0.1296	0	0	3.889	
X ₁	0	1	0	-0.1481	0.2037	0	0	4.889	
S ₃	0	0	0	0.1481	-0.2037	1	0	4.111	
S ₄	0	0	0	-0.1852	0.1296	0	1	2.111	

- Result is final, since there is no negative coefficient remains in the objective function row.
- The final solution is X₁= 4.889, X₂= 3.889 which gives the maximum objective function of Z= 1413.895

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Example Problem I

Problem Statement:

Suppose that a gas processing plant receives a fixed amount of raw gas each week. The raw gas is processed into three grades of heating gas, regular, premium and supreme quality. These grades of gas are in high demand (that is, they are guaranteed to sell) and yield different profits to the company. However, their production involves both time and on-site storage constraints. For example, only one of the grades can be produced at a time, and the facility is open for only 80 hrs/week. Further, there is limited on-site storage for each of the products. All these factors are listed below (note that a metric ton is equal to 1000 kg):

	Product			
<u>Resource</u>	<u>Regular</u>	<u>Premium</u>	<u>Supreme</u>	<u>Resource Availability</u>
Raw gas	7 m ³ /Ton	11 m ³ /Ton	15 m ³ /Ton	154 m ³ /Week
Production time	10 hr /Ton	8 hr /Ton	12 hr /Ton	80 hr /Week
Storage	9 Ton	6 Ton	5 Ton	
Profit	150 /Ton	175 /Ton	250 /Ton	

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Solving the LPP I with Simplex Method

Develop a linear programming formulation to maximize the profits for this operation.

Solution. The engineer operating this plant must decide how much of each gas to produce to maximize profits. If the amounts of regular, premium and supreme produced weekly are designated as X_1 , X_2 , and X_3 respectively, the total weekly profit can be calculated as

- Total profit $Z = 150 X_1 + 175 X_2 + 250 X_3$
- Subject to
 - $7 X_1 + 11 X_2 + 15 X_3 \leq 154 \text{ m}^3 / \text{Week}$ Material constraint
 - $10 X_1 + 8 X_2 + 12 X_3 \leq 80 \text{ hr /tonne}$ Time constraint
 - $X_1 \leq 9$ "Regular" storage constraint
 - $X_2 \leq 6$ "Premium" storage constraint
 - $X_3 \leq 5$ "Supreme" storage constraint
 - $X_1, X_2, X_3 \geq 0$ Positivity constraint

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Simplex Solution to the LPP I

The simplex tableau for the problem can be set up and solved as

- Start at a basic feasible solution (an extreme corner point of the feasible space).
 - Starting point A; $X_1 = X_2 = X_3 = 0$.
 - The original 5 equations with 5 unknowns become

$$\begin{array}{rcl}
 S_1 & & = 154 \\
 & S_2 & = 80 \\
 & & S_3 = 9 \\
 & & S_4 = 6 \\
 & & S_5 = 5
 \end{array}$$

Summarize key information in a **tableau**.

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	Solution	Intercept
P	1	-150	-175	-250	0	0	0	0	0	0	
S_1	0	7	11	15	1	0	0	0	0	154	10.2667
S_2	0	10	8	12	0	1	0	0	0	80	6.66667
S_3	0	1	0	0	0	0	1	0	0	9	∞
S_4	0	0	1	0	0	0	0	1	0	6	∞
S_5	0	0	0	1	0	0	0	0	1	5	5

The objective function is $Z - 150 X_1 - 175 X_2 - 250 X_3 - 0 S_1 - 0 S_2 - 0 S_3 - 0 S_4 - 0 S_5 = 0$

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Simplex Solution to the LPP I

- The next step involves moving to a new basic feasible solution that leads to an improvement of the objective function.
 - The **entering variable** can be any variable in the objective function having a negative coefficient (because it will make Z bigger). For our case, X_3 .
 - Choose a **leaving variable** from among basic variables with the smallest positive integer value of intercept, for our case, S_5 .

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	Solution	Intercept
P	1	-150	-175	0	0	0	0	0	250	1250	
S_1	0	7	11	0	1	0	0	0	-15	79	7.18182
S_2	0	10	8	0	0	1	0	0	-12	20	2.5
S_3	0	1	0	0	0	0	1	0	0	9	∞
S_4	0	0	1	0	0	0	0	1	0	6	6
X_3	0	0	0	1	0	0	0	0	1	5	∞

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Simplex Solution to the LPP I

- The next step involves moving to a new basic feasible solution that leads to an improvement of the objective function.
 - The **entering variable** can be X_2 .
 - Choose a **leaving variable** can be, S_2 .

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	Solution	Intercept
P	1	68.75	0	0	0	21.88	0	0	-12.5	1687	
S_1	0	-6.75	0	0	1	-1.375	0	0	1.5	51.5	34.3334
X_2	0	1.25	1	0	0	0.125	0	0	-1.5	2.5	-1.6667
S_3	0	1	0	0	0	0	1	0	0	9	∞
S_4	0	-1.25	0	0	0	-0.125	0	1	1.5	3.5	2.3334
X_3	0	0	0	1	0	0	0	0	1	5	5

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Simplex Solution to the LPP I

- The next step involves improvement of the objective function.
 - The *entering variable* can be S_5 .
 - Choose a *leaving variable*, that can be, S_4 .

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	Solution
P	1	58.33	0	0	0	20.83	0	8.33	0	1716.7
S_1	0	-5.5	0	0	1	-1.25	0	-1	0	48
X_2	0	0	1	0	0	0	0	1	0	6
S_3	0	1	0	0	0	0	1	0	0	9
S_5	0	-0.833	0	0	0	-0.083	0	0.67	1	2.3334
X_3	0	0.833	0	1	0	0.083	0	-0.67	0	2.6667

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Example Problem II

Problem Statement:

A chemical plant makes three major products on a weekly basis. Each of these products requires a certain quantity of raw chemical, different production times, and yields different profits. The pertinent information is summarized below:

	Product 1	Product 2	Product 3	Resource Availability
Raw chemical	5 kg/kg	4 kg/kg;	10 kg/kg	3000 kg
Production time	0.05 hr/kg	0.1 hr/kg	0.2 hr/kg	55 hr/week
New Profit	\$30/kg	\$30/kg	\$35/kg	

Note that there is sufficient warehouse space at the plant to store a total of 450 kg/week.

Maximize $Z = 30 X_1 + 30 X_2 + 35 X_3$ Maximize Profits

Subject to

$$5 X_1 + 4 X_2 + 10 X_3 \leq 3000$$

$$0.05 X_1 + 0.1 X_2 + 0.2 X_3 \leq 55$$

$$X_1 + X_2 + X_3 \leq 450$$

$$X_1, X_2, X_3 \geq 0$$

Material constraint

Time constraint

storage constraint

Positivity constraint

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Simplex Solution to the LPP II

The simplex tableau for the problem can be set up and solved as

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	Solution	Intercept
P	1	-30	-30	-35	0	0	0	0	
S_1	0	5	4	10	1	0	0	3000	300
S_2	0	0.05	0.1	0.2	0	1	0	55	275
S_3	0	1	1	1	0	0	1	450	450

- The next step involves moving to a new basic feasible solution that leads to an improvement of the objective function.
 - The **entering variable** can be X_3 .
 - Choose a **leaving variable** can be, S_2 .

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	Solution	Intercept
P	1	-21.25	-12.5	0	0	175	0	9625	
S_1	0	2.5	-1	0	1	-50	0	250	100
X_3	0	0.25	0.5	1	0	5	0	275	1100
S_3	0	0.75	0.5	0	0	-5	1	175	233.3334

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Simplex Solution to the LPP II

- The next step involves moving to a new basic feasible solution that leads to an improvement of the objective function.
 - The **entering variable** can be X_1 .
 - Choose a **leaving variable** can be, S_1 .

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	Solution	Intercept
P	1	0	-21	0	8.5	-250	0	11750	
X_1	0	1	-0.4	0	0.4	-20	0	100	-250
X_3	0	0	0.6	1	-0.1	10	0	250	416.6667
S_3	0	0	0.8	0	-0.3	10	1	100	125

- The next step involves improvement of the objective function.
 - The **entering variable** can be X_2 .
 - Choose a **leaving variable** can be, S_3 .

Basis	P	X_1	X_2	X_3	S_1	S_2	S_3	Solution
P	1	0	0	0	0.625	12.5	26.25	14375
X_1	0	1	0	0	0.25	-15	0.5	150
X_3	0	0	0	1	0.125	2.5	-0.75	175
X_2	0	0	1	0	-0.375	12.5	1.25	125

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Simplex Method: Minimization Problem

- In maximization problem, one of the corner points of the feasible region is at the origin.
 - By setting basic variables equal to zero, an initial feasible solution can be obtained.
- However, in case of minimization, the origin lies outside of the feasible region.
 - “What is the way of starting with a basic feasible solution in Simplex method”
 - Include *surplus variables* (analogous to the *slack variables* added to less than inequalities) to absorb or subtract any surplus of any of the minimum conditions represented in the “greater than” inequality constraints.
 - Add other variables to the problem; “*Artificial variables*” for every “greater than” constrain inequality to start with a basic feasible solution.
 -

For Example: From following problem Minimize

$$Z = 10 X_1 + 12 X_2$$

Subject to

$$X_1 + X_2 \geq 3$$

$$X_1 + 2X_2 \geq 4$$

$$X_1, X_2 \geq 0$$

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Simplex Method: Minimization Problem

After adding *surplus* and *artificial* variables to the problem. Then we have following formulation: Minimize

$$Z = 10 X_1 + 12 X_2$$

Subject to

$$X_1 + X_2 - X_3 + X_5 = 3$$

$$X_1 + 2X_2 - X_4 + X_6 = 4$$

$$X_j \geq 0$$

X_3 and X_4 are *surplus* and X_5 and X_6 are *artificial* variables.

- Set the X_1, X_2, X_3 and X_4 equal to zero, and assign extremely high values to the coefficients of the *artificial* variables in the objective function.
- Forcing X_5 and X_6 to be equal to zero would assure that solution would be optimal feasible solution.

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Simplex Method: Minimization Problem

- Let assign a value of 100 to each of *artificial* variables.

This gives us following problem.

$$Z = 10X_1 + 12X_2 + 0X_3 + 0X_4 + 100X_5 + 100X_6$$

Subject to

$$1X_1 + 1X_2 - 1X_3 + 0X_4 + 1X_5 + 0X_6 = 3$$

$$X_1 + 2X_2 + 0X_3 - 1X_4 + 0X_5 + 1X_6 = 4$$

$$X_j \geq 0$$

This is seen in the Simplex tableau

			Choice variable		Surplus variable		Artificial variable		Right hand-side constants
Basic Variables	Eq. Number	Z	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	
Z	0	1	-10	-12	0	0	-100	-100	0
	1	0	1	1	-1	0	1	0	3
	2	0	1	2	0	-1	0	1	4

- The Row 0 has coefficients of -100 for X₅, and X₆. These will have to be replaced by zeros in the order for the initial basic solution to the problem to be read from RHS column.

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Simplex Method: Minimization Problem

- Add 100 times to Row 1 to Row 0, and add 100 times to Row 2 to Row 0, gives us.

			Choice variable		Surplus variable		Artificial variable		Right hand-side constants
Basic Variables	Eq. Number	Z	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	
Z	0	1	190	288	-100	-100	0	0	700
X ₅	1	0	1	1	-1	0	1	0	3
X ₆	2	0	1	2	0	-1	0	1	4

- In minimization problems, the criterion is just the reverse of what it is for maximization problems.
 - The *entering variable* can be X₂ (largest coefficient).
 - Choose a *leaving variable* can be, X₆ (from 3/1 and 4/2, min. of these ratios just as max. case).

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Simplex Method: Minimization Problem

Making the following necessary changes due to new basic variables

			Choice variable		Surplus variable		Artificial variable		Right hand-side constants
Basic Variables	Eq. Number	Z	X ₁ ↓	X ₂	X ₃	X ₄	X ₅	X ₆	
Z	0	1	46	0	-100	44	0	-144	124
← X ₅	1	0	1/2	0	-1	1/2	1	-1/2	1
X ₂	2	0	1/2	1	0	-1/2	0	1/2	2

- The next step involves moving to a new basic feasible solution that leads to an improvement of the objective function.
 - The **entering variable** can be X₁ (largest coefficient).
 - Choose a **leaving variable** can be, X₅ (from 1/0.5 and 2/0.5, min. of these ratios).

			Choice variable		Surplus variable		Artificial variable		Right hand-side constants
Basic Variables	Eq. Number	Z	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	
Z	0	1	0	0	-8	-2	-92	-98	32
X ₁	1	0	1	0	-2	1	2	-1	2
X ₂	2	0	0	1	1	-1	-1	1	1

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Simplex Method: Minimization Problem

			Choice variable		Surplus variable		Artificial variable		Right hand-side constants
Basic Variables	Eq. Number	Z	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	
Z	0	1	0	0	-8	-2	-92	-98	32
X ₁	1	0	1	0	-2	1	2	-1	2
X ₂	2	0	0	1	1	-1	-1	1	1

Now make a optimality test: No positive coefficient appears in Row 0. Thus, the right hand column shows us an optimal feasible solution with

$$Z=32, X_1=2, X_2=1, X_3=0, X_4=0, X_5=0, \text{ and } X_6=0$$

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