

# Material Handling Systems

## *Conveyor Analysis*

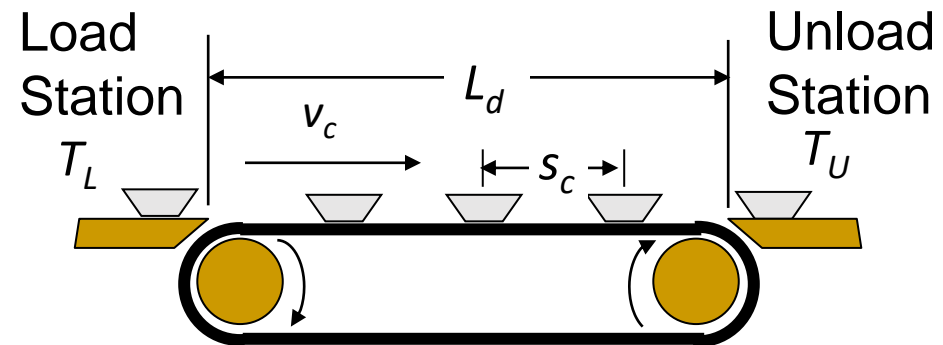
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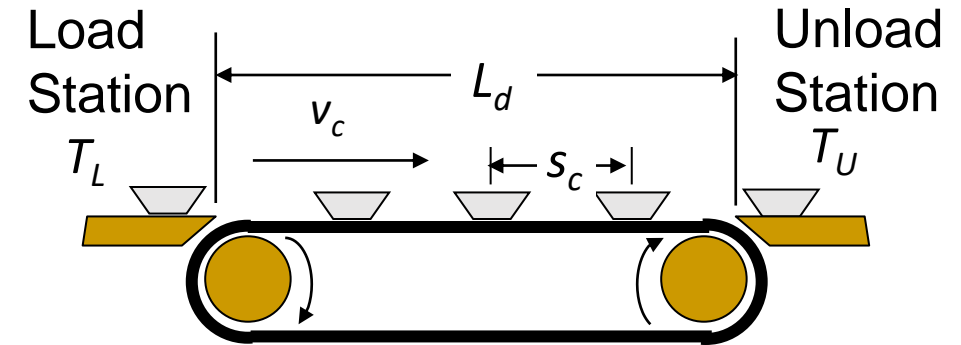
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# Single Direction

- *Three basic types*
  - **A. Single direction**
  - *B. Continuous loop*
  - *C. Recirculating*



# Single Direction



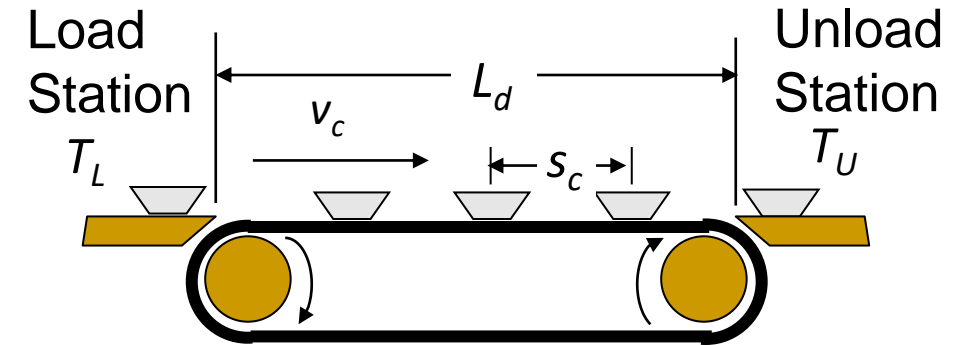
## A. Single Direction:

- transport loads one way from origination point to destination point.
- Appropriate when no need
  - to move loads in both directions or
  - to return containers or carriers from the unloading stations back to the loading stations.
- roller, skate wheel, belt, and chain-in-floor types

# Single Direction

$$T_d = L_d / v_c$$

- $T_d$  the delivery time i.e. the time required to move materials from load station to unload station
- Consider following:
  - if loading time is 0.25 min/carrier,
  - then max loading rate is
    - 4 carr/min



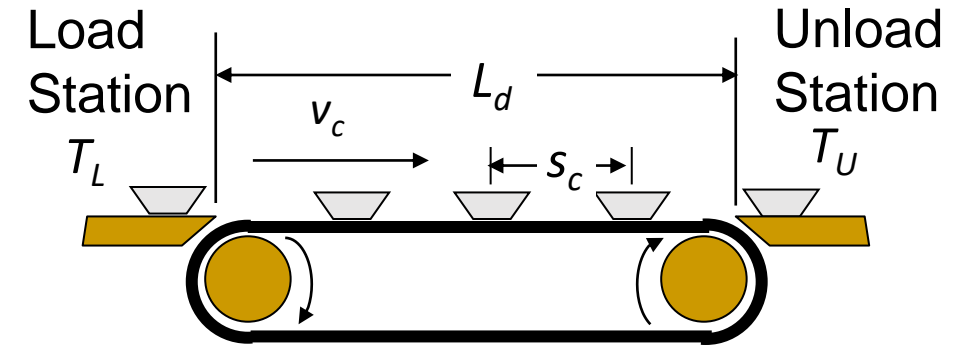
- $T_L$ : loading time(min/carr)
- **max** available loading rate ( $R_L$ ) :

$$R_L \leq 1 / T_L$$

# Single Direction

- The flow (needed) rate  $[R_f]$  of materials on the conveyor is determined by the loading rate  $[R_L]$   
 $(R_f = R_L)$

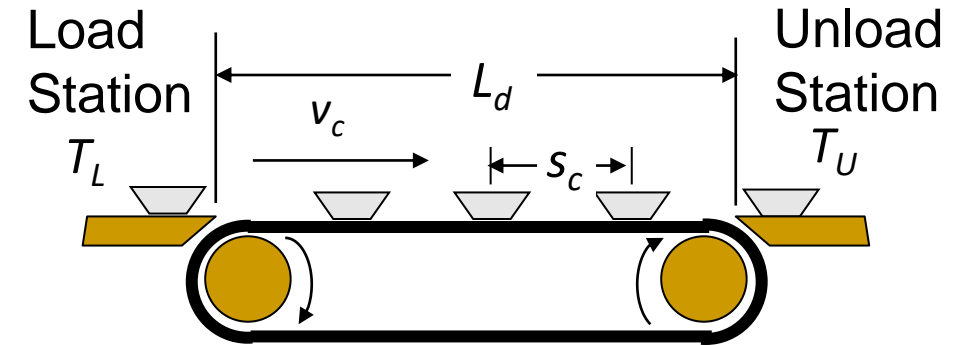
- Let
  - $n_p$ : number of parts per carrier.
  - $s_c$ : Carriers spacing on conveyor(m/carr)
- How to determine  $s_c$ ?



- Ex:
  - $V_c = 50 \text{ m/min}$
  - $R_L = 20 \text{ part/min}$
  - $n_p = 2 \text{ part/carr}$
  - Then
    - $s_c = 2 \times \left(\frac{50}{20}\right) = 5 \text{ mt/carr}$
    - $s_c = \frac{n_p V_c}{R_L}$

# Single Direction

- One might think that  $R_L = n_p / T_L$
- However,
  - $R_L$  is set by the flow rate requirement  $R_f$ ,
  - $T_L$  is determined by ergonomic factors.
- The worker at loading ( $T_L$ ) may be faster than the required flow rate ( $R_f$ ).
- The flow requirement cannot be faster than it is humanly possible to perform the loading task.
- Therefore  $R_L \leq n_p / T_L$



$$R_f = R_L = \frac{n_p v_c}{s_c} \leq \frac{n_p}{T_L}$$

- Additionally we have  $T_u \leq T_L$ , where  $T_U$  is the unloading time (min/carr)

# Single Direction: Summary

## Assumptions:

1. Belt moves in one direction
2. One load station at the input end
3. One unload station at the output end

$$T_d = \frac{L_d}{v_c}$$

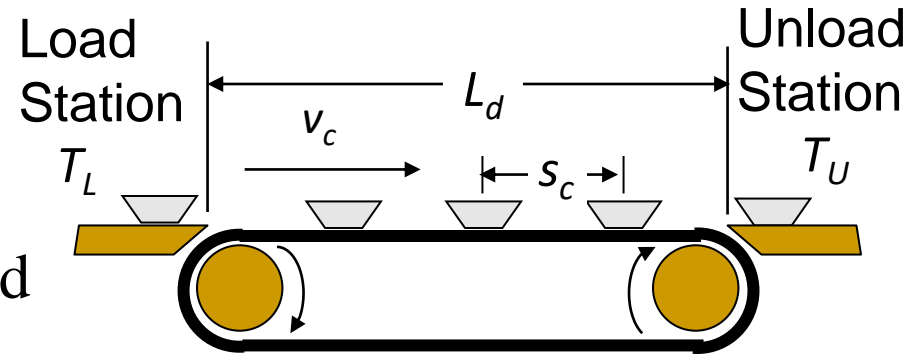
where:

$T_d$  = delivery time (min/carrier)

$L_d$  = conveying distance between load and unload stations (m,ft)

$v_c$  = conveyor speed (m/min, ft/min)

$R_f$  = material flow rate (parts/min)



$$R_f = R_L = \frac{n_p v_c}{s_c} \leq \frac{n_p}{T_L}$$

$R_L$  = loading rate (parts/min)

$n_p$  = number of parts per carrier

$s_c$  = carriers spacing on conveyor (m/carrier, ft/carrier)

$T_L$  = loading time (min/carrier) → Property of the worker

$T_U$  = unloading time (min/carrier) → Property of the worker

$$T_U \leq T_L$$

# Single Direction Conveyors – An Example

- A roller conveyor follows a pathway 35m long between a parts production dept and an assembly dept.
- Velocity of the conveyor is 40m/min.
- Parts are loaded into large boxes.
- Two operators work the loading station.
  - The first worker loads parts into boxes, which takes 20sec.
    - Each box holds 20 parts.
  - The second worker loads boxes onto the conveyor, which takes only 10sec.
- Parts enter the loading station in such a way that a full box can be loaded to conveyor at 25 seconds.



# Single Direction Conveyors

*Determine*

*(a) spacing between boxes along the conveyor,*

*(b) max possible flow rate in parts/min.*

*(c) the minimum time required to unload the box in assembly department.*

# Single Direction Conveyors

## (a) Spacing between boxes

- Loading time for boxes is 25. (*Why?*)  
 $R_L = 60/25 = 2.4 \text{ box/min}$   
 $V_c = 40 \text{ m/min}$
- $s_c = (40 \text{ m/min}) / 2.4 (\text{box/min}) = 16.67 \text{ m/box}$

## (b) Flow rate is given by

- $R_f = \frac{n_p v_c}{s_c}$
- $R_f = \frac{(20)(40)}{16.67} = 48 \text{ parts/min}$

or  $2.4 \text{ box/min} \times 20 = 48 \text{ parts /min}$

# Single Direction Conveyors

*Compare with the loading rate  $T_L$*

- Load parts to a box is 20 sec
- Load a box to conveyor 10 sec
- $T_L = \max(10, 20) = 20 \text{ sec} = 1/3 \text{ min}$
- Max loading rate is  $1/(1/3) = 3 \text{ box/min}$
- *Compare with 2.4 box/min*

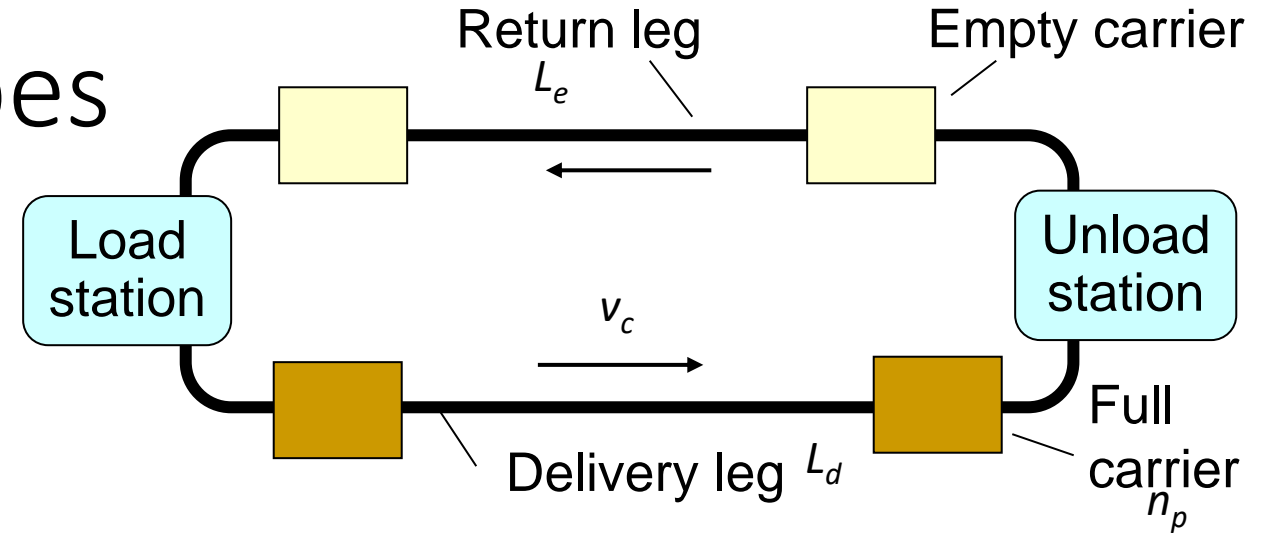
(c) The minimum allowable time to unload a box must be consistent with the flow rate of boxes on the conveyor. Hence:

$$T_U \leq 25$$

# Conveyor Analysis: Types

## B. Continuous loop conveyors

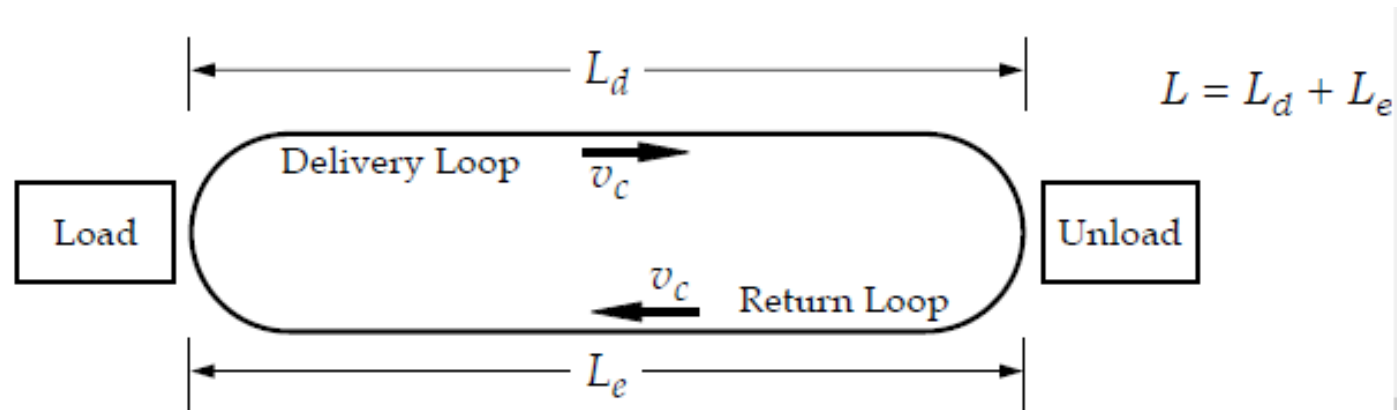
- Form a complete circuit
- An overhead trolley conveyor is an example of this conveyor type.
- Allows materials to be moved between any two stations
- Used when loads are moved in carriers (e.g., hooks, baskets) between load and unload stations
- All carriers are emptied at the unload station.
- The empty carriers are automatically returned from the unload station back to the load station



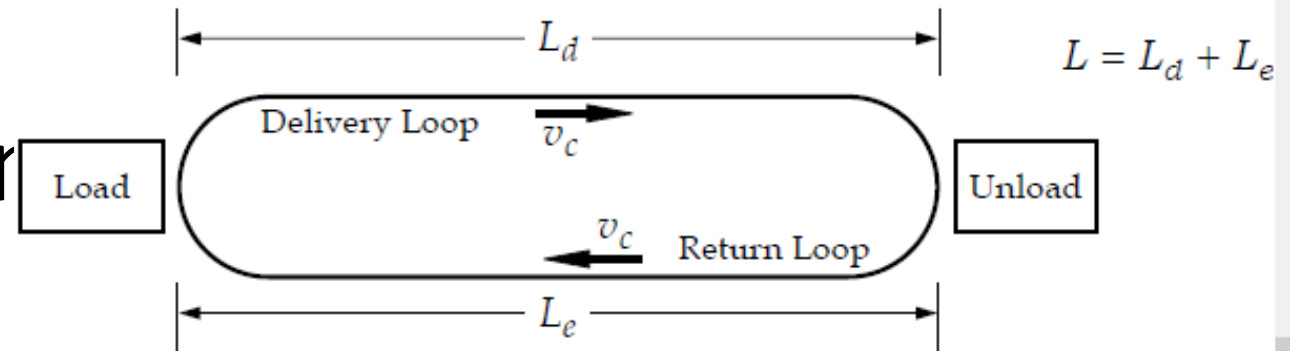
# Continuous Loop Conveyors

- $L_d$ : delivery loop length
- $L_e$ : Return loop length
- The total length

$$L = L_d + L_e .$$



# Continuous Loop Cor



The total time required to travel the whole loop and the time a load spends in the forward loop are

$$T_c = \frac{L}{v_c} \quad T_d = \frac{L_d}{v_c}$$

where  $T_c$  = total cycle time (min)

$T_d$  = delivery time on the forward loop (min)

Carriers are equally spaced along the chain at a distance  $s_c$  apart. Thus the total number of carriers in the loop is given by:

$$n_c = \frac{L}{s_c}$$

where  $n_c$  = number of carriers

$s_c$  = center-to-center distance between carriers (m/carrier, ft/carrier)

The number of carriers must be an integer, and so  $L$  and  $s_c$  must be consistent with that requirement.

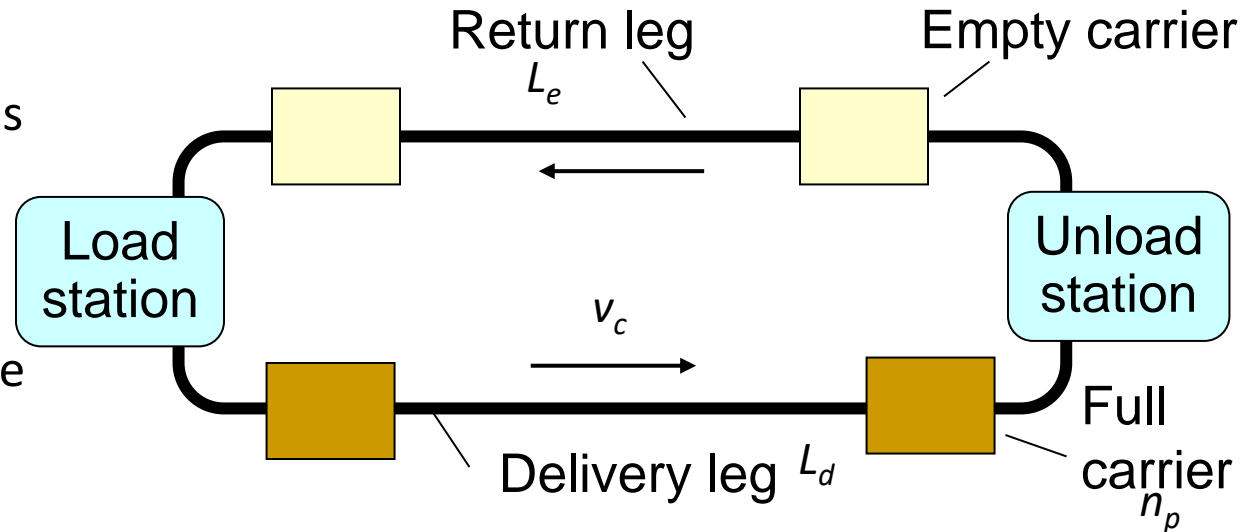
# Continuous Loop Conveyors

- Each carrier
  - capable of holding  $n_p$  parts on the delivery loop
  - holds no parts on the return trip.
- Only those carriers on the forward loop contains parts,
  - Hence the maximum number of parts in the system at any time :
    - $n_p \left[ n_c \left( \frac{L_d}{L} \right) \right]$
- $R_f$  now gives the flow rate (part feed rate) between load and unload stations:

$$R_f = \frac{n_p v_c}{s_c}$$

# Continuous Loop Conveyors – Summary

Consider a continuous closed-loop conveyor, such as an overhead trolley system with one load and one unload station. Assume that all carriers are emptied at the unload station.



$$n_c = \frac{L_d + L_e}{s_c}$$

$$N_p = \frac{n_p L_d}{s_c} = \frac{n_p n_c L_d}{L_d + L_e}$$

$$R_f = \frac{n_p v_c}{s_c} \leq \frac{n_p}{T_L}$$

$n_c$  = number of carriers in the system  
 $L_d$  = length of the delivery leg (ft or m)  
 $L_e$  = length of the return leg (ft or m)  
 $s_c$  = carriers spacing (ft or m/carrier)

$N_p$  = total number of parts in the system  
 $n_p$  = number of parts in each carrier  
 $R_f$  = part feed rate (parts/min)  
 $v_c$  = conveyor speed (ft/min or m/min)



# Conveyor Analysis: Types

## C. Recirculating Conveyors

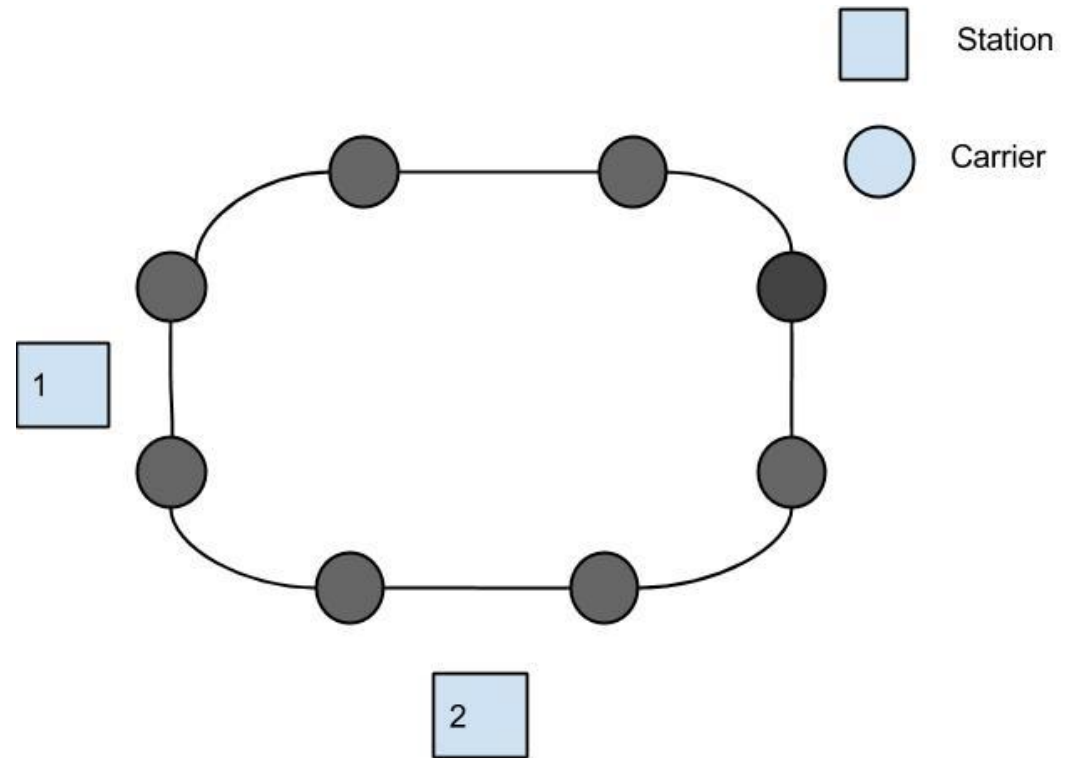
- Continuous loop conveyors overlooks an important opportunity
  - to store as well as deliver parts.
- Conveyor systems that allow parts to remain on the return loop for one or more revolutions are called **recirculating conveyors**.
- In providing a storage function, the conveyor system can be used to accumulate parts to smooth out effects of loading and unloading variations at stations in the conveyor.

# Conveyor Analysis

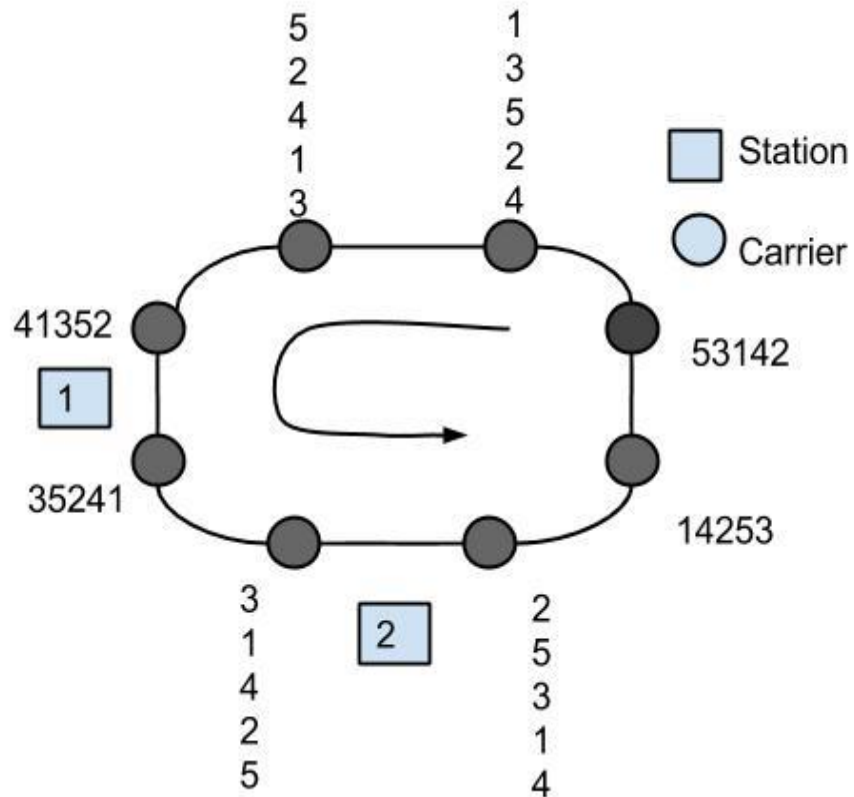
- Continuous loop conveyors can be used for storage, if loaded carriers are allowed to flow back on the return leg
- However, we have the following possibilities:
  - Empty carriers are not available when needed for loading
  - Full carriers are not immediately available for unloading (*airport conveyor*)

# Recirculating Conveyors

- Assume closed-loop recirculating conveyors with equally spaced carriers.
- **Example:**
- Consider the conveyor system
- The load and unload **cycle length** has period 5
- The load and unload sequences for the two stations:
  - $F_1 = \{ 2, 0, 2, 0, 3 \}$
  - $F_2 = \{ -2, 0, 0, -1, -4 \}$
- Determine the conveyor capacity



# Recirculating Conveyors



Round Num.	Position	Loads		Cum. Loads		Adjusted Cum.L	
		St. 1	St.2	St1	St2	St1	St2
1	1	2	-2	2	0	4	2
2	4	0	-1	0	-1	2	1
3	2	0	0	-1	-1	1	1
4	5	3	-4	2	-2	4	0
5	3	2	0	0	0	2	2

$F1 = \{ 2, 0, 2, 0, 3 \}$

$F2 = \{ -2, 0, 0, -1, -4 \}$

# Recirculating Conveyors

- General Model

- **m** stations
- **n** carriers
- $f_{ij}$  = denote the amount of material loaded onto the  $j$ th carrier as it passes station  $i$ .
- If material is unloaded then  $f_{ij} < 0$
- **p**: load and unload cycle period
- $F_i = \{f_{i1}, f_{i2}, \dots, f_{ip}\}$
- No need for  $p=n$
- Recall
  - $F1 = \{2, 0, 2, 0, 3\}$
  - $F2 = \{-2, 0, 0, -1, -4\}$

- Material Balance Equation:

- $\sum_{i=1}^m (f_{i1} + f_{i2} + \dots + f_{ip}) = 0$
- One round is completed when all  $n$  carriers have gone around the conveyor once.
- $P_{ijk}$  = denote the load and on load sequence position of a carrier  $j$ , with respect to station  $i$  in round  $k$ .
- Given  $P_{ij1}$  one can determine  $P_{ijk}$
- $P_{ijk} = [P_{ijk-1} + n \bmod p] \bmod p$

# Recirculating Conveyors – Kwo Analysis

- There are two problems complicating analysis of the operation of a recirculating conveyor system:
  - (1) the possibility that no empty carriers are immediately available at the loading station when needed
  - (2) the possibility that no loaded carriers are immediately available at the unloading station when needed.
- According to Kwo analysis, there are three basic principles that must be obeyed in designing a conveyor system with one load station and one unload station:

# Recirculating Conveyors – Kwo Analysis

## ***Speed Rule:***

Speed of the conveyor ( $v_c$ ) must be within a certain range.

- 1. Carrying rate capacity must be greater than loading and unloading rates

$$\frac{n_p v_c}{s_c} \geq \max(R_L, R_U)$$

- 2. The speed of the conveyor must be lower than that is allowed by the loading and unloading times.

$$\frac{v_c}{s_c} \leq \min\left(\frac{1}{T_L}, \frac{1}{T_U}\right)$$

- $T_L$  and  $T_U$  are the loading and unloading rates for the carrier

# Recirculating Conveyors – Kwo Analysis

## ***Capacity Rule***

- flow rate capacity of the conveyor must be at least equal to the flow rate requirement to accommodate reserve stock and allow for the time elapsed between loading and unloading due to delivery distance.

$$\frac{n_p v_c}{s_c} \geq R_f$$

- $R_f$  must be interpreted as a system specification required of the recirculating conveyor.
- Flow rate capacity > Flow rate requirement



# Recirculating Conveyors – Kwo Analysis

## ***Uniformity Rule:***

- The parts should be uniformly distributed throughout the length of the conveyor so that
  - there will be no sections of the conveyor in which every carrier is full
  - while other sections are virtually empty.
- Or, empty and full carriers should be uniformly distributed along the line to avoid excessive waiting for carriers
- We should check whether
  - flow rate capacity is substantially greater than the required loading and unloading rate.
  - Loading rate = unloading rate

# Recirculating Conveyors – Kwo Analysis Ex.

- **Example:** A recirculating conveyor has a length of 300m. Its speed is 60m/min, and the spacing of part carriers along its length is 12m.
- Each carrier can hold two parts.
- The task time required to load two parts into each carrier is 0.20min and the unload time is the same.
- The required loading and unloading rates are both defined by the specified flow rate, which is 4parts/min.
- Evaluate the conveyor system design with respect to Kwo's three principles.

# Recirculating Conveyors – Kwo Analysis Ex.

- **Speed Rule:**
- The *lower limit* on speed is set by the required loading and unloading rates, which is 4 parts/min.
- $\frac{n_p v_c}{s_c} \geq \max(R_L, R_U)$
- $\Rightarrow \frac{(2)(60)}{12} \geq 4 \Rightarrow 10 \geq 4$
- The *Upper limit* is set by the loading rate of the carriers.
- $\frac{v_c}{s_c} \leq \min\left(\frac{1}{T_L}, \frac{1}{T_U}\right)$
- $\frac{60}{12} \leq \min\left(\frac{1}{0.2}, \frac{1}{0.2}\right)$
- $5 \leq 5$ . Hence the speed rule is satisfied.

# Recirculating Conveyors – Kwo Analysis Ex.

- ***Capacity Constraint:***

- The conveyor flow rate capacity 10 parts/min.
- This is greater than the required delivery rate of 4 part/min
- Hence, the capacity constraint is satisfied.

- ***Uniformity Principle:***

- The loading and unloading rates are equal
- The flow rate capacity (10parts/min) is substantially greater than the load/unload rate (4parts/min)
- Hence uniformity rate is satisfied.