Reliability Engineering

Notes 1

Reliability History

- Reliability as a subject gained importance after World War II because a high failure rate of electronic equipment was observed during the war.
- More complicated products were produced, composed of an ever-increasing number of components (television sets, electronic computers, etc.). With automation, the need for complicated control and safety systems also became steadily more pressing.
- The first journal on the subject, Institute of Electrical and Electronics Engineers (IEEE) Transactions on Reliability, came out in 1963. A number of textbooks on the subject were published in the 1960s.

- Industries are increasingly introducing automation for producing goods ranging from the simplest to highly complex systems. However, all processes and products are prone to failure. For example, a television remote control may stop functioning or a malfunction may occur in any other household appliance; an automobile engine starter may fail, an aeroplane may crash due to the failure of some of its component(s), credit card transactions may fail

Reliability

- The reliability of a product (or system) can be defined as the **probability** that a product will perform a **intended function** under **specified conditions** for a certain period of **time**.
- In effect, reliability is a probability .

- The Reliability definition has four important elements:
- **Probability** When a number of identical components operate under similar conditions, the failure times of components generally vary from component to component and we cannot predict their failure times exactly. advance. But we can describe the phenomenon of failure in probabilistic terms. reliability of a component/system at time t is the probability that it performs its function without failure. Being a probability, its value lies between 0 and 1. (A value between 0 and 1, number of times that an event occurs (success) divided by total number trials)
- Suppose that the reliability of the refrigerator of a particular company at 10 years is R(t) = 0.9452. Then this can be interpreted as follows: Out of 10000 refrigerators of this particular company, approximately 9452 refrigerators worked without failure for 10 years.

- **Intended Function:** The intended function of a product must be described in unambiguous terms early on in the design process so that the expected requirements of the customers can be ensured/addressed. Thus, intended function of a product is the function/work/job which it is expected to perform when we put it in operation under stated conditions.
- For example, if a pump is designed to deliver at least 300 gallons of water per minute, the intended function of the pump is to deliver 300 gallons or more water per minute.

- **Time:** Reliability can be meaningful only if it is related to a time interval or period of time. For example, we can say that the reliability of a component is 0.98 for a mission time of 100 hours. But the statement ‘reliability of a component is 0.95’ is meaningless since the time interval is unknown.

$$R(t) = \frac{\text{Number of components performing intended function at time 't'}}{\text{Number of component at start (i.e., when 't' = 0)}}$$

In the above definition, time ‘t’ means the interval $[0, t]$.

- **Operating conditions**: The performance of the product should be observed under normal stated conditions in which it is expected to perform. The environmental conditions (such as temperature, humidity, shock, vibration, altitude, etc.), design loads (such as voltages, pressure, etc.) and operating conditions (such as maintenance,storage, etc.) affect the reliability of a product. So a product will perform better in the field if its design takes into account and is representative of how it is actually used by the customers. For example, a car which is designed for smooth roads will not perform well if a customer uses it on rough roads. These describe the operating conditions (environmental factors, humidity, temperature cycle, operational profile, etc.) that correspond to the stated product life.

iPhone 14 and iPhone 14 Plus are splash, water and dust resistant and were tested under controlled laboratory conditions with a rating of IP68 under IEC standard 60529 (maximum depth of 6 metres up to 30 minutes). Splash, water and dust resistance are not permanent conditions. Resistance might decrease as a result of normal wear.

Reliability Engineering

- Reliability engineering is an engineering field that deals with the study, evaluation, and life-cycle management of reliability.
- Reliability is often measured as probability of failure, frequency of failures, or in terms of availability.
- Maintainability and maintenance are often important parts of reliability engineering.
- Reliability Engineering is concerned with analyzing failures and providing feedback to design and production to prevent future failures.

Reliability Engineering

- Reliability engineers address 3 basic questions:
- • When does something fail?
- • Why does it fail?
- • How can the likelihood of failure be reduced?

Reliability

- Reliability of a product or part is used in two ways.
- 1. Reliability when activated.
- 2. Reliability for a given length of time.
- First one is often used when a product or part must operate for one time, such as an air bag in a car.
- The second of these focuses on the *length of service* , such as most other products e.g., a car.

Reliability Engineering

- The important applications and benefits of reliability engineering may be summarized as follows:
- Implement an integrated reliability engineering and product assurance program in purchasing, engineering, research, development, manufacturing, quality control, testing, packaging, shipping, installation, start up, operation, field service or inspection, and performance feedback.

Reliability Engineering

- Study the types of failures and determine the time-to-failure distribution of parts, components, products and systems in order to minimize failures and be prepared to cope up with them.
- Study the effects of age, mission duration and operation stress levels on reliability to see if the established goals can be met.
- Indicate areas in which design changes would be most beneficial from the reliability improvement and cost reduction point of view. Provide a basis for comparing two or more designs and choosing the best one from the reliability point of view.

Reliability Engineering

- Some of the benefits are as follows:
- a) Reduce warranty costs and reduce inventory costs through correct prediction of spare parts requirements.
- b) It helps in increase of sales as result of increased customer satisfaction and promote them on the basis of reliability indexes and view-points through the sales and marketing departments.
- c) Increase profits or for the same profit we can get more reliable products and systems.

Reliability Engineering

- A reliability engineering department may have various kinds of responsibilities.
- Establishing reliability policy, plans and procedures
- Reliability allocation
- Reliability prediction
- Specification and design reviews with respect to reliability
- Reliability growth monitoring
- Providing reliability related inputs to design specifications and proposals
- Reliability demonstration
- Training reliability manpower and performing reliability-related research and development work

Reliability Engineering

- • Monitoring the reliability activities of subcontractors, if any
- • Auditing the reliability activities
- • Failure data collection and reporting
- • Failure data analysis

Reliability Engineering

- In industry for the control of processes, in computers, in medical electronics, atomic energy, in defence equipments, communications, navigation at sea and in the air, and in many other fields, it is essential that these equipments should operate reliably.

Why Do Engineering Products Fail?

- There are many reasons why a product might fail.
- The reliability engineering effort, during design, development and in manufacture and service should address all of the anticipated and possibly unanticipated causes of failure, to ensure that their occurrence is prevented or minimized.

- The design might be inherently incapable. It might be too weak, consume too much power, suffer resonance at the wrong frequency, and so on.
- The item might be overstressed in some way. If the stress applied exceeds the strength then failure will occur. An electronic component will fail if the applied electrical stress (voltage, current) exceeds the ability to withstand it.

- Failures can be caused by wearout. We will use this term to include any mechanism or process that causes an item that is sufficiently strong at the start of its life to become weaker with age. Well-known examples of such processes are material fatigue, wear between surfaces in moving contact, corrosion, the wearout mechanisms of light bulbs and fluorescent tubes.

- Failures can be caused by errors, such as incorrect specifications, designs or software coding, by faulty assembly or test, by inadequate or incorrect maintenance, or by incorrect use.

$$R(t) = \frac{\textit{number of survivors at time } t}{\textit{number of items put on test at time } t = 0}$$

At time t = 0, the number of survivors is equal to number of items put on test. Therefore, the reliability at t = 0 is

$$R(0) = 1 = 100\%$$

$$R(t \to \infty) = 0$$

- After this, the reliability, R(t), will decline as some components fail

Unreliability

- **Unreliability:**The probability that a device will fail to perform a required or intended function under stated conditions for a specified period of time. It is the complement of reliability
- We denote reliability and unreliability of a component by R and Q, respectively
- $R+Q = 1$

- Failure Distribution Function
- Define the function F(t)
- F(t) = 1- R(t)
- F(0) = 0 $F(\infty) = 1$

- Cumulative failure distribution function and gives unreliability of the component up to time t. So if we want to calculate the probability of failure of a component at time t (known as unreliability of the component), then we have to simply obtain the value of the function F(t).

Failure Rate (λ)

- Rate at which failure occur in a specified time interval.
- Reliability of a system is often specified by the failure rate λ

$$\frac{\text{number of failures}}{\text{total operating hours}}$$

Example

- 10 transformers were tested for 500 h each, and four transformers failed after the following test time periods:
- One failed after 50 h , one failed after 150 h , two failed after 400 h
- What is the failure rate for these types of transformers?

Solution

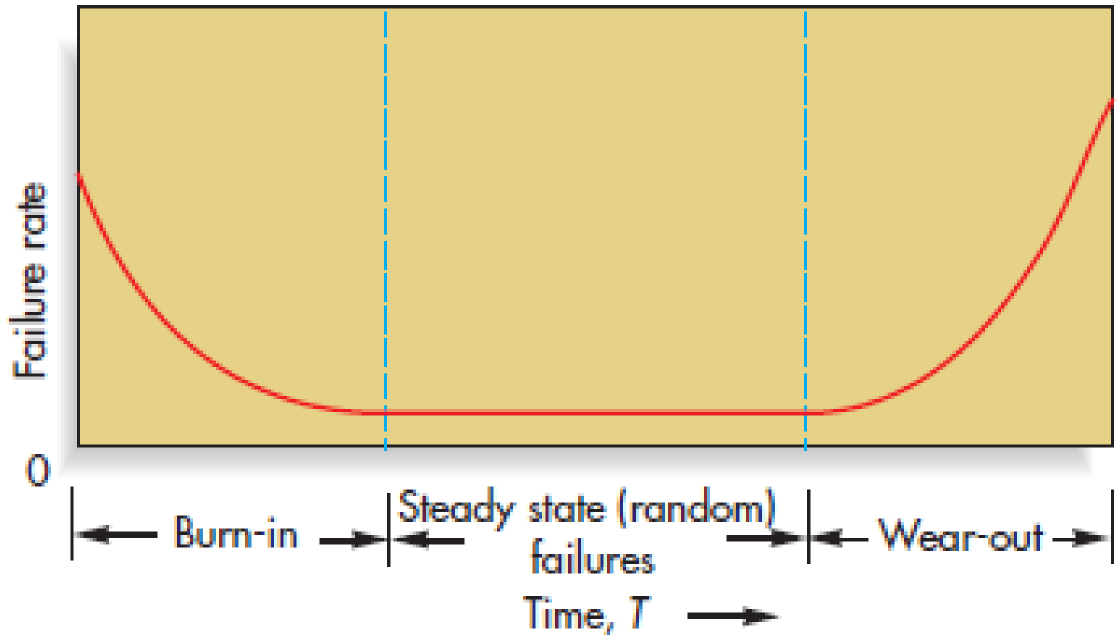
$$\text{Total operating time of units} = (1 \times 50 + 1 \times 150 + 2 \times 400 + \underset{\uparrow}{6} \times 500) \text{ unit h}$$

$$= 4000 \text{ unit h}$$

10-4=6

$$\lambda = \frac{4}{4000} = 0.001 \text{ failures/unit h}$$

Bath tub Curve



Bath tub Curve

- A typical profile of failure rate over time is called the bath tub curve.
- The first period is called burn-in period. Usually, a number of products or parts fail shortly after they are put into service. It is also called infant mortality phase.
- The failures in the beginning are mainly due to defects in design and due to the improper manufacturing techniques, etc.

Bath tub Curve

- During this period, failures occur because engineering did not test products or systems or devices sufficiently, or manufacturing made some defective products.
- Therefore the failure rate at the beginning is high and then it decreases with time after early failures are removed by burn-in or other stress screening methods. Some of the typical early failures are:
- • poor welds
- • poor connections
- • contamination on surface in materials
- • incorrect positioning of parts, etc.

Bath tub Curve

- Second phase is called as the useful life period. During the second phase, random failures occur. In many cases, this phase covers a relatively long period of time (several years).
- In this period the failures are chance failures or random failures.

Bath tub Curve

- Third phase is called the wear-out period and the failures are mainly due to aging effect or lack of maintenance. These failures are called wear-out failures and the failure rate increases. When the failure rate becomes high, repair, replacement of parts etc., should be done.

Bath tub Curve

- The Bath tub curve can be divided into three regions namely
- i) Decreasing hazard rate region
- ii) Constant hazard rate region
- iii) Increasing hazard rate region

Understanding Reliability Keyboard Example

- How might a keyboard key fail? (mechanisms)
- – Material that gives tactile “click” might fatigue and break
- – Electric contacts might corrode or become blocked with dirt
- What might cause these fails? (stresses)
- – Being pressed too many times (wearout)
- – Heat, humidity, dust, dirt, being pressed too hard

- How can we test a key’s entire life? (stress test)
- – Use a machine to press it 1,000,000 times
- – Before that, heat it and shake it with dirt and water
- How can we make it more reliable? (design for rel.)
- – Find what breaks and make that (and only that) stronger

Resources

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Reliability Engineering

Notes 2

Repairable and Non-repairable Items

- Items in a system can be classified into two groups – Repairable items and non-repairable items.
- Repairable items are components which can be repaired upon failure and thus their life histories consist of alternating operating and repair periods.
- For items which are repaired when they fail, reliability is the probability that failure will not occur in the period of interest, when more than one failure can occur.

- Repairable system reliability can also be characterized by the mean time between failures (MTBF), but only under the particular condition of a constant failure rate.

- Non-repairable components are components that cannot be repaired or the repair is uneconomical.
- For a non-repairable item such as a light bulb, reliability is the survival probability over the item’s expected life, or for a period during its life, when only one failure can occur. During the item’s life the instantaneous probability of the first and only failure is called the hazard rate.

- Life values such as the mean life or mean time to failure (MTTF) can be used as reliability characteristics.

Basic Terms

- MEAN TIME TO FAILURE (MTTF): The term is used for non-repairable items or devices. MTTF is a measure that tells us on an average how long the product/component performs its intended function successfully before failure.
- It is calculated by taking the mean of the lifetimes obtained on the basis of results of a sample of such identical products/components tested under stated conditions.

In general, if n components are put to test and $t_i, (i = 1, 2, 3, ..., n)$ denotes the time for which the i^{th} component performs its intended function, then mean time to failure (MTTF) of such components is given by

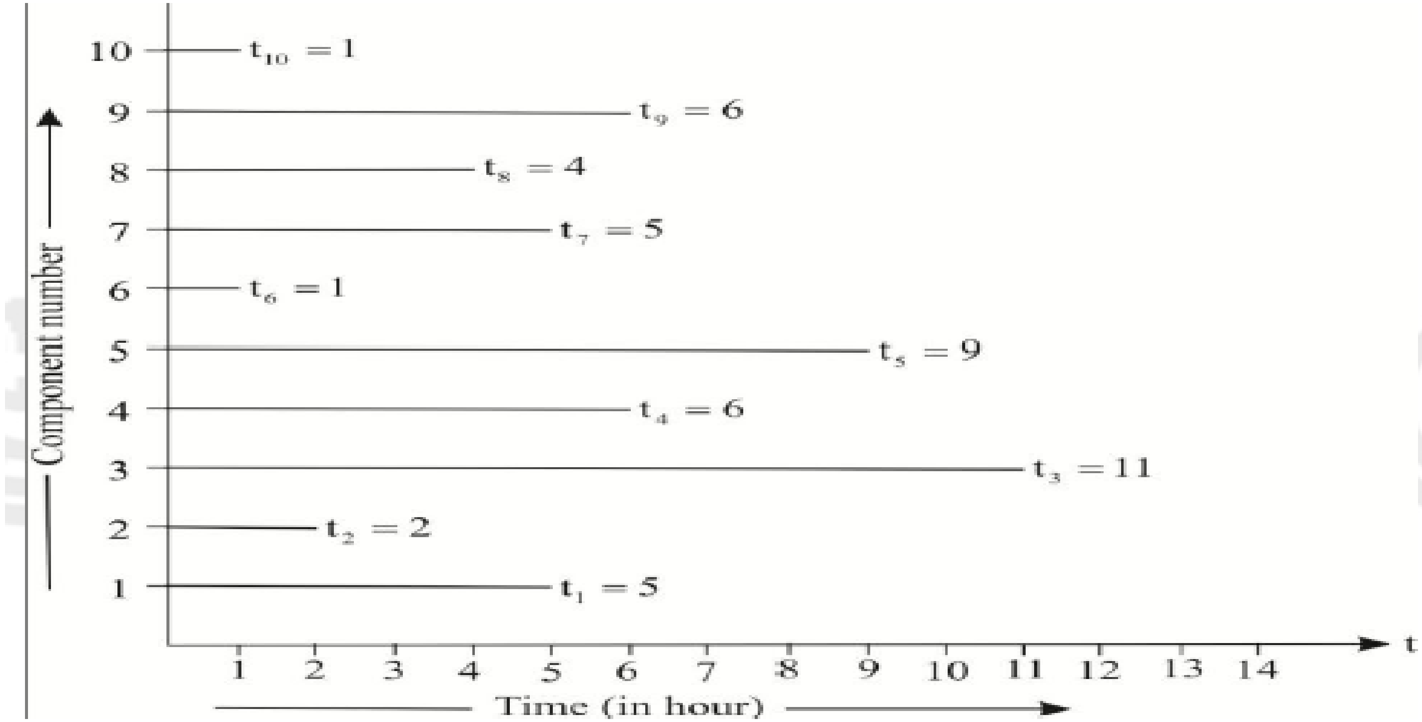
$$MTTF = \frac{1}{n}\sum_{i=1}^{n} t_i$$

- If the failure rate is constant,

$$\boxed{\mathrm{MTTF} = \frac{1}{\text{constant failure rate}}} \quad \text{or} \quad \boxed{\text{Constant failure rate} = \frac{1}{\mathrm{MTTF}}}$$

- For example, four items have lasted 3,000 hours, 4000 hours, 4000 hours and 5,000 hours before failure.
- MTTF is 16,000/4 or 4,000 hours.

Example

- Times for which of each of the 10 components operates successfully are given. What is MTTF?
- 

$$\text{MTTF} = \frac{1}{10}\sum_{i=1}^{10} t_i = \frac{1}{10}(5+2+11+6+9+1+5+4+6+1) = \frac{50}{10} = 5\,\text{hr}$$

Mean time between Failures (MTBF)

- The MTBF is used for repairable products.
- MTBF is the average time from the uptime after the repair following a failure to the next failure.
- Mean time between failures is often used as a synonym for MTTF.
- MTTF and MTBF can often be modelled
- by the Exponential distribution.

$$\text{MTBF} = \frac{\text{Total operating hours for specified units}}{\text{Total failures for those units in a measurement interval}}$$

Basic Terms

- Failure: This is the inability of an item to function within the initially defined guidelines.
- Downtime: This is the time period during which the item is not in a condition to carry out its stated mission.

Basic Terms

- Human reliability: This is the probability of completing a job/task successfully by humans at any required stage in the system operation within a defined minimum time limit (if the time requirement is specified)

Basic Terms

- MEAN TIME BETWEEN MAINTENANCE (MTBM): The average time between all maintenance events that cause downtime, both preventative and corrective maintenance, and also includes any associated logistics delay time.

$$\text{MTBM} = \frac{\text{Total number of life units expended in a given time}}{\text{Total number of scheduled and unscheduled maintenance events due to that item}}$$

- MEAN TIME TO REPAIR (MTTR) : A measure of system maintainability equal to the average system repair time, and this value is the reciprocal of repair rate in the exponential case.

$$MTTR = \frac{\text{Total diagnose, repair, and test hours for specified units}}{\text{Total number of completed repair actions for those units}}$$

Maintainability

- Maintainability is a measure of the speed with which loss of performance is detected, diagnosed and repaired. Maintainability is the probability that a unit or system will be restored to specified conditions within a given period when maintenance action is taken in accordance with prescribed procedures and resources.
- This is the probability that a failed item will be repaired to its satisfactory working state.
- If the reliability is high, frequency of maintenance is low.

Availability

- Availability is defined as the percentage of time that a system is available to perform its required function(s). It is measured in a variety of ways, but it is principally a function of downtime.
- This is the probability that an item is available for application or use when needed.
- Availability can range from zero (never available) to 1.00 (always available). Companies that can offer equipment with high availability have a competitive advantage over companies that offer equipment with lower availability.

$$= \frac{\text{Time available for a specified use during a stated period}}{\text{Total length of stated period}}$$

$$\text{Availability} = \frac{\text{Uptime}}{(\text{Uptime} + \text{Downtime})}$$

$$\text{Availability} + \text{Unavailability} = 1$$

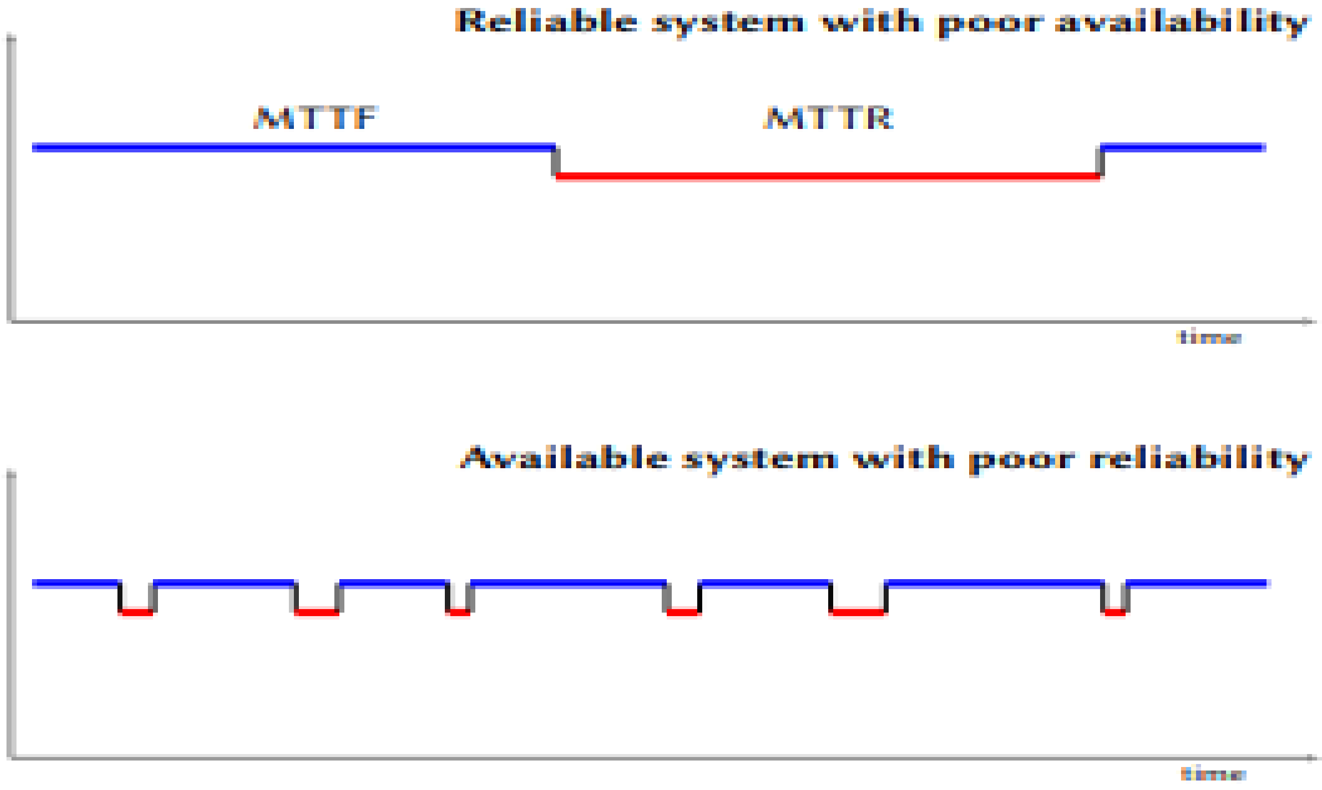
$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

- Maintainability together with reliability determine the availability of a machinery system. Availability is influenced by the time demand made by preventive and corrective maintenance measures.
- To increase availability, designers increase MTBF but also decrease MTTR.

Reliability $\neq$ availability

- Availability measures proportion of time system is alive and well
- Reliability measures probability for failure free operation

Reliability ≠ availability



Example

- A copier is expected to operate for 200 hours after repair, and the mean repair time is expected to be two hours. Determine the availability of the copier.
- MTBF = 200 hours, and MTTR = 2 hours
- Availability = MTBF/MTBF + MTTR
- = 200/(200 + 2) = 0.99

Example

- Equipment is required to meet an availability of Ai = 0.985 and a mean time between failures (MTBF) = 100 hr. What is permissible mean time to repair (MTTR).

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

- 0.985 = 100/(100+MTTR)
- MTTR= 1.52 hours

Example

- 300 cars have accumulated 45000 hours, 10 failures are observed. What is the MTBF? What is the failure rate?

- MTBF = 45000/10 = 4500 hours.
- Average Failure rate = 1/4500 = 0. 00022 per hour.

Example

- Five oil pumps were tested with failure hours of 45, 33, 62, 94 and 105.
- What is the MTTF and failure rate?

- MTTF = (45+33+62+94+105) / 5 = 67. 8 hours
  Failure rate = 1 / 67.8
- = 0. 0147 per hour.

Example

- 10 components were tested for 525 h. The components (not repairable) failed as follows: Component 1, 2, 3, 4, 5 failed after 75, 125, 130, 325, 525 hours.
- Find the failure rate and mean time till failure.

- No. of failures = 5
- Total operating time
- = 75 + 125 + 130 + 325 + 525 + 5*525 = 3805
- Failure rate = 5 / 3805 = 0. 001314
- Mean time to failure =1/0. 001314 = 761. 04 hours.

Resources

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Notes 3

Probability

- A quantitative measure of the likelihood of an event
- Measure of chance
- Quantitative statement about the likelihood of an event or events

Probability

- The likelihood of an occurrence on a scale from 0 (zero chance for an occurrence) to 1 (100% certainty for an occurrence) attached to a random event based on a particular mode for which the event can occur.

- It is the measure of chance which means that the chance of an event to happen

- Generally we can note that (the probability of an event to happing is the number of times that a specific event occurs relative to the sum of all possible events that can occur. This is the classical definition of the probability.
- P (success) = No. of success / No. of possible outcomes; $p = s / (s+f)$
- – q(failure) = No. of failures / No. of possible outcomes; $q = f / (s+f)$
- Where; $p + q = 1$

- Flipping a coin once can results in either a head (H) or a tail (T) ,i.e, 1 out of 2 , or 1/2.

Independent Events:

- Two events are said to be independent if the occurrence of one event does not affect the probability of occurrence of the other event.
- Example: Throwing a dice and tossing coin are independent events.

Mutually exclusive events:

- Two events are said to be mutually exclusive or disjoint if they cannot happen at the same time.
- Example: (i) When throwing a single die, the events 1, 2, 3, 4, 5 and 6 spots are all mutually exclusive because two or more cannot occur simultaneously
- (ii) Similarly success and failure of a device are mutually exclusive events since they cannot occur simultaneously.

Complementary Events:

- Two outcomes of an event are said to be complementary, if when one outcome does not occur, the other must occur.
- If the outcomes A & B have probabilities P(A) and P(B), then
- $P(A) + P(B) = 1 \qquad P(B) = P(\bar{A})$
- Example: When tossing a coin, the outcomes head and tail are complementary since
- $P(\text{head}) + P(\text{tail}) = 1$
- Therefore we can say that two events that are complementary events are mutually exclusive also. But the converse is not necessarily true .

Conditional Events;

- Conditional events are events which occur conditionally on the occurrence of another event or events.
- Consider two events A & B and also consider the probability of event A occurring under the condition that event B has occurred.
- $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Addition rule
- Event (AUB) is the union event and is defined as the event that occurs if A occurs or B occurs or both.
- Mathematically it is the union of the two events and is expressed as (AUB), (A or B)
- If the events are independent but not mutually exclusive then
- $P(A \cup B) = P(A) + P(B) - P(A)*P(B)$
- If the events are dependent then
- $P(A \cup B) = P(A) + P(B) - P(A)*P(B \mid A)$
- if A and B are mutually exclusive
- $P(A \cup B) = P(A) + P(B)$

- Multiplication Rule of Probability
- Event ($A \cap B$) is the intersection event and is defined as the event that occurs if A and B occurs, i.e., the probability of A and B happening together.
- If the events are independent , then the probability of occurrence of each event is not influenced by the probability of occurrence of the other.
- $P(A \cap B) = P(A)*P(B)$

- If two events are not independent, then the probability of occurrence of one event is influenced by the probability of occurrence of the other
- $P(A \cap B) = P(B/A).P(A)$
- $= (PA/B).P(B)$

Example

- An engineer selects two components A & B. The probability that component A is good is 0.9 & the probability that component B is good is 0.95.What is the probability of both components being good.
- P(A good ∩ B good) = P(A good) (B good)
- = 0.9 x 0.95 = 0.85

- There are two lamps in a room. When turned on, one has probability of working of .90
- and the other has probability of working of .80. Only a single lamp is needed to light the room for success. What is the probability of success?

- Solution 1
- $P(A \cup B) = P(A) + P(B) – P(A)*P(B)$
- $= 0.90+0.80-0.90*0.80$
- $= 0.98$
- Solution 2
- $P= 1 - (1 - 0.90)(1 - 0.80) = 0.98.$

Example

- Now suppose 300 of the boys and 100 of the girls are interested in computer games. The school has 400 students out of 800 who like computer games. However, if a student is picked at random, what is the probability of finding a boy who is interested in computer games ?

- Being a boy and being interested in computer games, are not independent

$$
\begin{aligned}
P(\text{boy} \cap \text{likes computer games}) &= P(\text{boy} \mid \text{computer games}) \\
&\quad \times P(\text{likes computer games}) \\
&= 300/400 \times 400/800 \\
&= 0.375
\end{aligned}
$$

Probability Distributions in Reliability Engineering

- Most commonly used distributions are
- Discrete Distribution
- - Binomial Distribution
- Continuous Distribution
- -Weibull distribution
- -Exponential distribution
- -Normal distribution

Binomial Distribution

- To apply binomial distribution:
- -Fixed number of trials
- -Each trial must result in success or failure
- -All trials must have identcal probabilites of success.
- -All trials must be independent

- Consider a random trial having only two possible outcomes, success and failure, such a trial is referred as a “Bernoulli trialé’’
- p = probability of success,
- q= probability of failure
- p+q=1

- Binomial distribution gives the probability of exactly *k* successes in *m* attempts:

$$f(k)=\binom{m}{k}p^k q^{m-k}, \quad 0 \le p \le 1, \quad q=1-p, \quad k=0,1,2,\ldots,m,$$

$$\binom{m}{k} \equiv C_k^m = \frac{m!}{k!(m-k)!},$$

where *p* is the probability of the defined success, *q* (or 1 – *p*) is the probability of failure, *m* is the number of independent trials, *k* is the number of successes in *m* trials, and the combinational formula is defined by

- *F*(*k*), gives the probability of *k* or fewer successes in *m* trials.

$$F(k) = \sum_{i=0}^{k} \binom{m}{i} p^i q^{(m-i)}.$$

- An engineer wants to select four capacitors from a large lot of capacitors in which 10 percent are defective. What is the probability of selecting four capacitors with:
- (a) Zero defective capacitors?
- (b) Exactly one defective capacitor?
- (c) Exactly two defective capacitors?
- (d) Two or fewer defective capacitors?

- a) $f(4)=\binom{4}{4}(0.9)^4(0.1)^0=0.6561.$ or

$$f(0)=\binom{4}{0}(0.1)^0(0.9)^4=0.6561$$

(b) $f(1)=\binom{4}{1}(0.1)^1(0.9)^3=0.2916$

(c) $f(2)=\binom{4}{2}(0.1)^2(0.9)^2=0.0486$

(d) $F(2)=f(0)+f(1)+f(2)=0.9963.$

Continuous Distributions

- **Weibull Distribution**
- Weibull distribution first introduced by W. Weibull in 1937.
- In probability theory and statistics, Weibull distribution is one of most important continuous probability distributions.
- Weibull analysis is leading method for fitting life data.

- The Weibull distribution is very popular among engineers in reliability applications.
- It is valuable in reliability application because it enables to model different failure modes.
- If you want to model all the three phases of the lifecycle curve then one single distribution the Weibull distribution you can use.

- It can be used in events where the Probability of occurrence follows a “Bathtub Curve”
- Describes well the failure rate of real world components.

A random variable T is said to have a **Weibull distribution** with parameter $\beta > 0$ and $\eta > 0$ if its pdf is given by

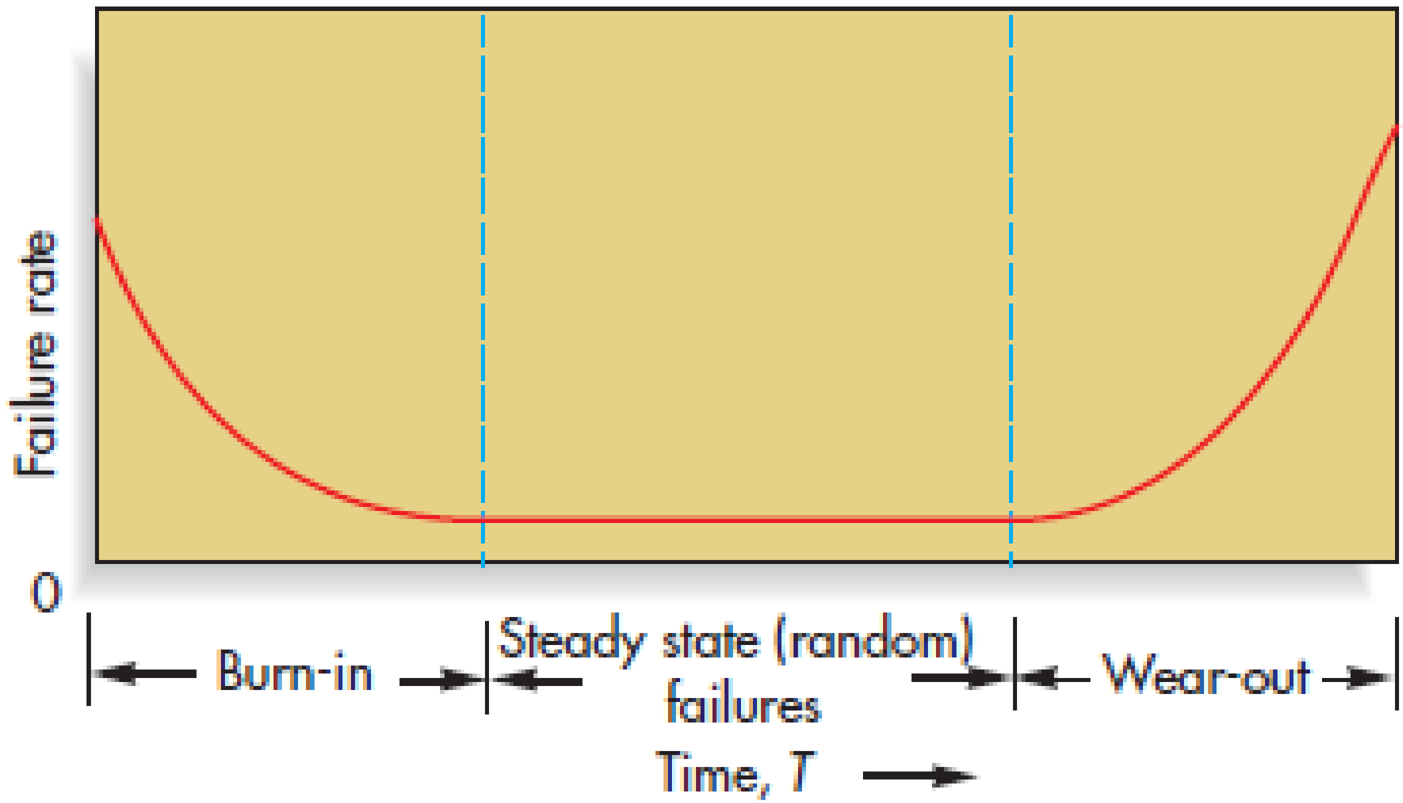
$$f_T(t) = \begin{cases} \dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1} e^{-(t/\eta)^{\beta}}, & t > 0 \\ 0, & \text{otherwise.} \end{cases}$$

- β = shape parameter
- η = scale parameter

- The cdf function
- Failure distribution function

$$F_T(t) = P(T \leq t) = \begin{cases} 1 - e^{-(t/\eta)^\beta}, & t > 0, \\ 0, & t \leq 0. \end{cases}$$

- Reliability function
- R(t) = $e^{-(t/\eta)^\beta}$,

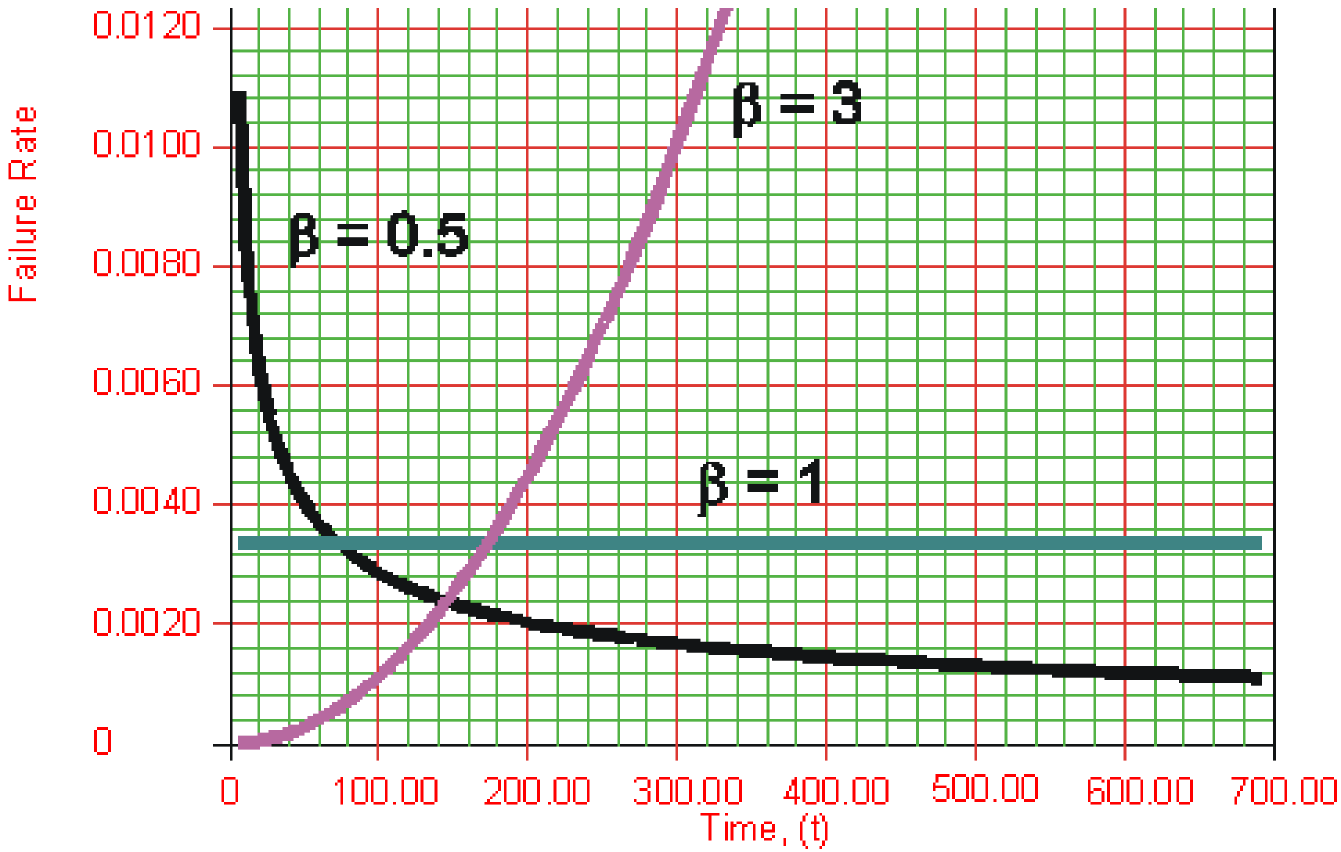


Failure rate
0
Burn-in
Steady state (random)
failures
Wear-out
Time, T

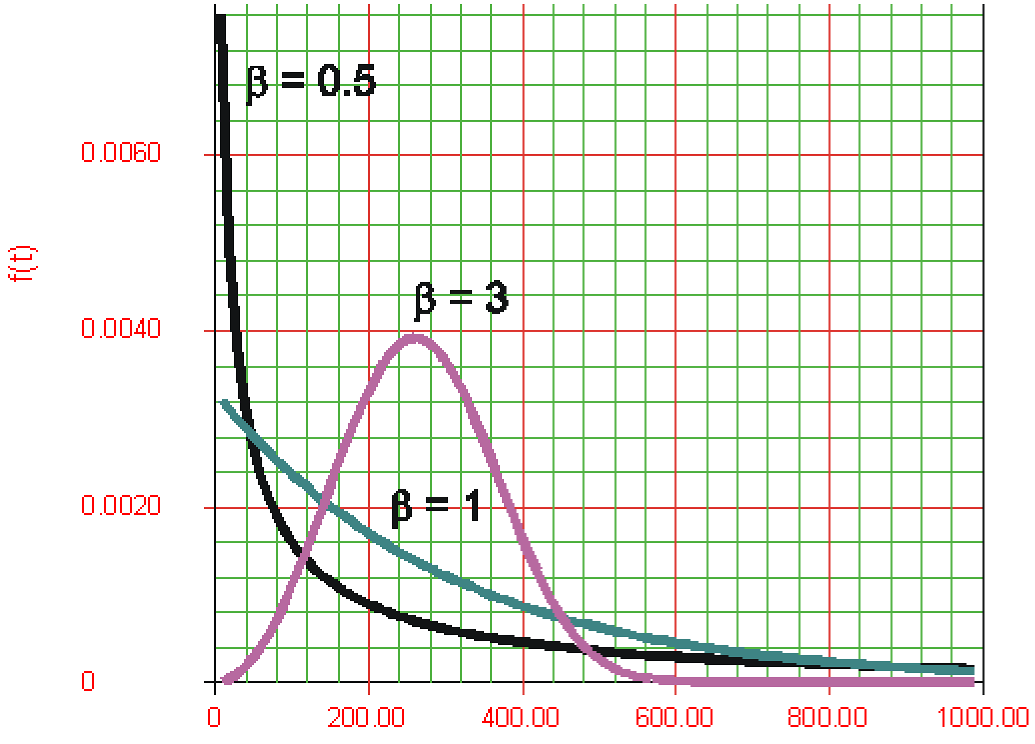
- Beta < 1 - infant mortality
- Beta = 1 - random
- 1 < Beta < 4 - early wear out
- Beta > 4 - old age wear out

- If β = 1 the failure rate is constant over time, and Weibull is identical to the Exponential distribution. If β < 1 failure rate decreases over time, and if β > 1 failure rate increases over
- time. If β takes a value between 3 and 4, then weibull distribution approaches normal distribution.
- Because of its flexibility, Weibull distribution is commonly used to model the time to failure during the burn-in (β < 1) and wear-out (β > 1) phases.

Weibull Failure Rate



Weibull pdf



β = 0.5
β = 3
β = 1
f(t)
0.0060
0.0040
0.0020
0
0
200.00
400.00
600.00
800.00
1000.00

Example

- A mechanical system has demonstrated a Weibull failure pattern, with a shape parameter of 1.4; and the scale parameter of 500 days. Determine the reliability that the system will last for 150 days.

- $\beta = 1.4$
- $\eta = 500$
- $R(150) = e^{-(t/\eta)^{\beta}}$,
- $R(150) = e^{-(\frac{150}{500})^{1.4}}$
- $=0.8308$

Example

- Suppose that the life distribution (life in years of continuous use) of hard disk drives for a computer system follows a two-parameter Weibull distribution with the following parameters: $\beta = 3.10$ and $\eta = 5$ years.
- The manufacturer gives a warranty for 1 year. What is the probability that a disk drive will fail during the warranty period?

- F(1)= 1-R(1) = 1- $e^{-\left(\frac{1}{5}\right)^{3.10}}$ = 1-0.993212
- = 0.006788

Resources

- https://www.philadelphia.edu.jo/academics/mlazim/uploads/PSR%20Lecture%20No.6.pdf, Power System Reliability Lecture No.6 Dr. Mohammed Tawfeeq Lazim
- STATISTICS FOR ENGINEERS Fall 2015 Lecture Notes, Dewei Wang Department of Statistics University of South Carolina.
- Introduction to reliability Lecture Notes (Portsmouth Business School, April 2012)
- https://canmedia.mheducation.ca/college/olcsupport/stevenson/5ce/ste39590_ch04S_001-019.pdf, Supplement to Chapter 4 Reliability
- Quality Design and Control, Design for Reliability- I , Lecture – 43 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- Quality Design and Control, Design for Reliability- I , Lecture – 44 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur

Resources

- https://www.weibull.com/hotwire/issue14/relbasics14.htm
- https://docs.tibco.com/data-science/GUID-E94B660B-73EC-47E7-A4B2-A084AFBC09D5.html
- http://www.stats.ox.ac.uk/~marchini/teaching/L6/L6.slides.pdf
- https://www.mathsisfun.com/data/standard-normal-distribution.html
- http://math.arizona.edu/~rsims/ma464/standardnormaltable.pdf
- https://risk-engineering.org/static/PDF/slides-reliability-engineering.pdf, Overview of reliability engineering, Eric Marsden
- Introduction to Reliability Fundamentals, Donald G. Dunn, 2019 D2 Training
- Ignou The People's University, Unit 11 Reliability Lecture Notes
- Ignou The People's University, Unit 13 Introduction to Reliability Lecture Notes
- **Reliability Engineering,** Kailash C. Kapur , Michael Pecht, 2014 , John Wiley & Sons, Inc

Resources

- Reliability Engineering Lecture Notes, Vardhaman College of Engineering
- https://www.slideshare.net/CharltonInao/reliability-engineering-chapter1csi
- https://extapps.ksc.nasa.gov/Reliability/Documents/210624%20Probability%20Formulas.pdf
- https://slideplayer.com/slide/4707809/
- https://slideplayer.com/slide/5235890/
- http://slideplayer.com/slide/9536322/, Introduction to Reliability Engineering, e-Learning course, CERN
- https://slidetodoc.com/1-introduction-to-reliability-engineering-elearning-course-n
- Power System Reliability, Lecture Notes DR. AUDIH ALFAOURY, 2017- 2018, Al-Balqa Applied University
- **Probability Fundamentals and Models in Generation andBulk System Reliability Evaluation, Roy Billinton Power System Research Group University of Saskatchewan CANADA**
- **Basic Probability and Reliability Concepts, Roy Billinton Power System Research Group University of Saskatchewan CANADA**

Reliability Engineering

Notes 4

Reliability Distributions
Exponential Distribution

Exponential distribution is a special case of Weibull distributions.

If $\beta = 1$ the failure rate is constant over time, and Weibull is identical to the Exponential distribution.

The time we need to wait before an event occurs has an exponential distribution.

How long will a generating unit works without breaking down?

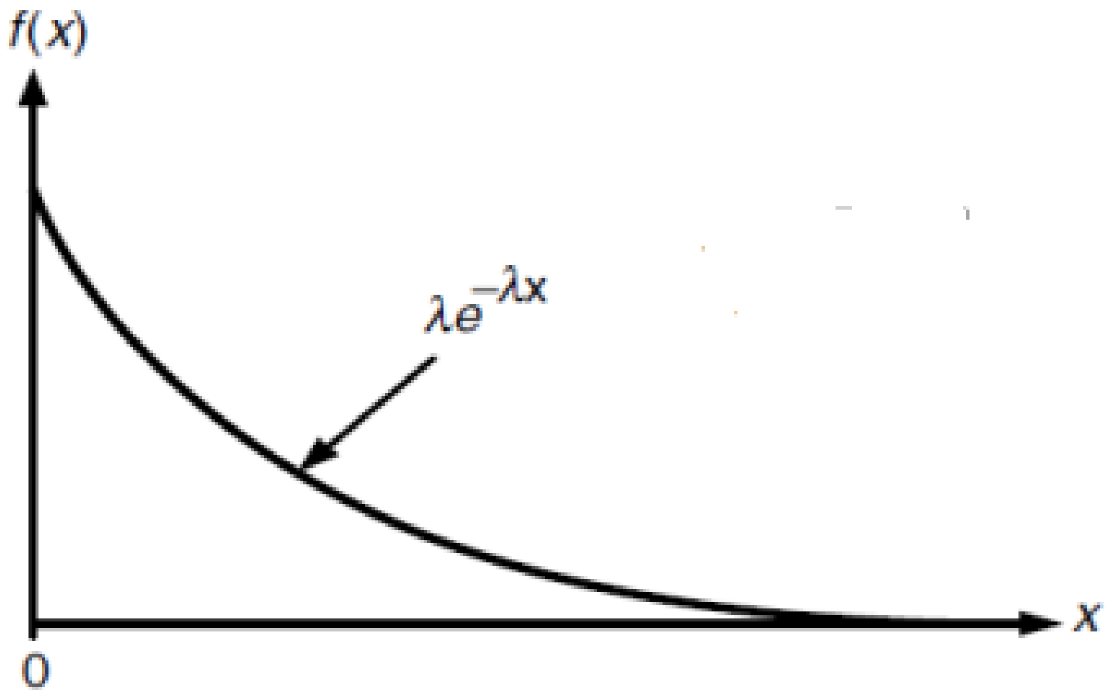
How long will a transformer works without breaking down?

- The exponential distribution is a continuous probability density function.

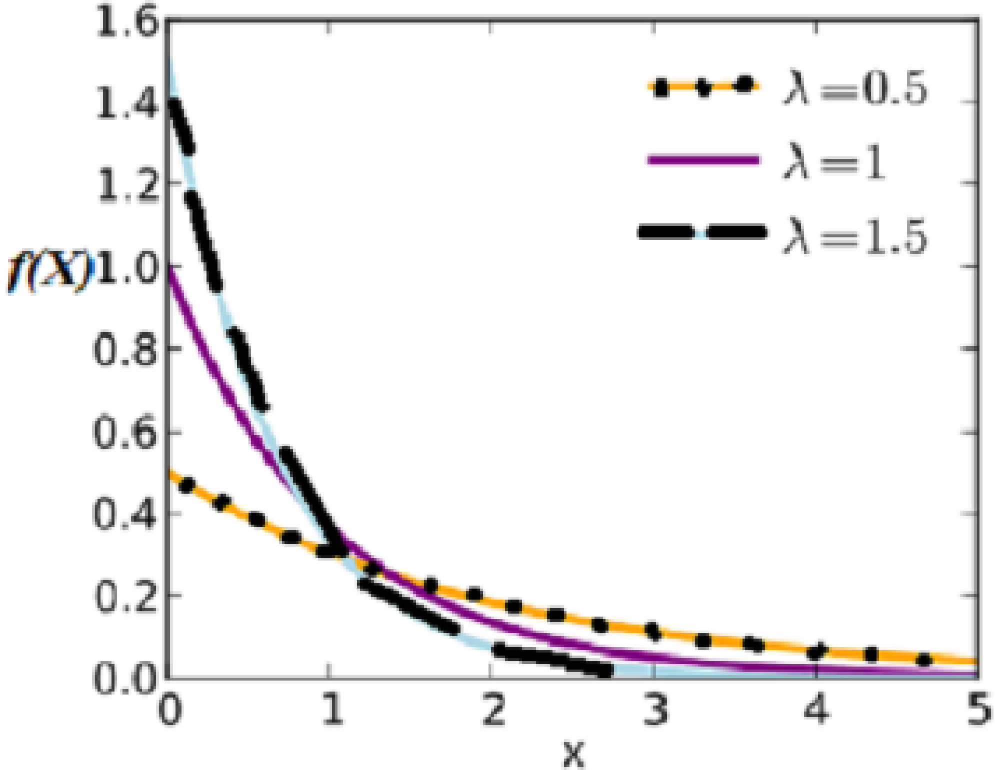
A random variable X is said to have an **exponential distribution** with parameter $\lambda > 0$ if its pdf is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

λ is a parameter of the distribution. The parameter λ is a positive real number, called the constant failure rate. The exponential distribution describes a probability that decreases exponentially with increasing x.



f(x)
λe^−λx
x
0



$f(X)$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
x
$\lambda = 0.5$
$\lambda = 1$
$\lambda = 1.5$

- It is one of the most commonly used distributions in reliability. The exponential distribution applies when the failure rate is constant - the graph is a straight horizontal line. It can be used to analyse the middle phase of a bath tub.

- If λ is the failure rate and t is the time, then the reliability, R, can be determined by the following equation:

$$R(x) = \int_{x}^{\infty} \lambda\, e^{-\lambda x}\, dx = e^{-\lambda x}$$

$$R(t) = e^{-\lambda t}$$

- To see that it gives sensible results, imagine that there are initially 1000 components and that λ is 10% (0.1) per hour. After one hour about 10% of the original 1000 components will have failed - leaving about 900 survivors. After two hours, about 10% of the 900 survivors will have failed leaving about 810. Similarly there will be about 729 survivors after the third hour, which means that the reliability after 3 hours is 0.729.

- Using the exponential distribution the reliability after 3 hours, with λ=0.1, is given by

$$R(t) = e^{-3\lambda} = e^{-0.3} = 0.741$$

Example

- Ten thousand new oil circuit reclosers (OCRs) are put in service. They have a constant failure rate of 0.1 per year. How many units of the original 10,000 will still be in service after 10 years? How many of the original will fail in Year 10?

In 10 years, probability of survival

$$R(10) = e^{-0.1\times 10} = e^{-1.0} = 0.3679$$

Out of 10,000 original units

10,000x0.3679=3679 should survive

Number of failures in Year 10

$$\begin{aligned}
&= (\text{number of survivors after Year 9}) - (\text{number of survivors after Year 10}) \\
&= 10,000 \times e^{-0.1\times 9} - 3679 \\
&= 1000 \times e^{-0.9} - 3679 \\
&= 4066 - 3679 \\
&= 387
\end{aligned}$$

Example

- A weather satellite has expected life of 10 years from the time it is placed into Earth’s orbit.
- Determine its reliability for each of the following lengths of service (assume that Exponential
- distribution is appropriate.)
- **a.** 5 years
- **b.** 12 years
- **c.** 20 years
- **d.** 30 years

- MTTF = 10 years

	t	R(t)
a	5	0.6065
B	12	0.3012
C	20	0.1353
d	30	0.0498

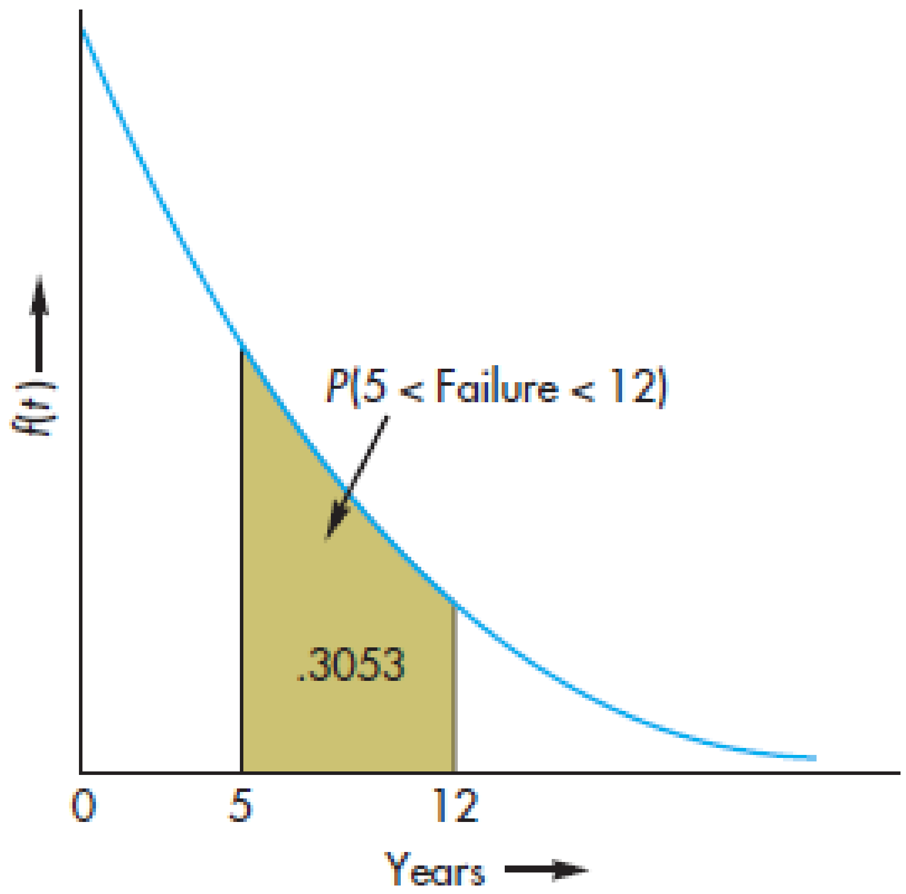
Example

- What is the probability that the satellite described in previous problem will fail between 5 and 12 years after being placed into Earth's orbit?

$$P(5 \text{ years} < \text{failure} < 12 \text{ years}) = P(\text{failure after 5 years}) - P(\text{failure after 12 years})$$

Using the probabilities shown in the previous solution, we obtain:

$$\begin{array}{rr} P(\text{failure after 5 years}) = & .6065 \\ -\ P(\text{failure after 12 years}) = & \underline{.3012} \\ & .3053 \end{array}$$



f(t)
P(5 < Failure < 12)
.3053
0
5
12
Years

Example

- A particular electronic device will only function correctly if two essential components both function correctly. The lifetime of the first component is known to be exponentially distributed with a mean of 5000 hours and the lifetime of the second component (whose failures can be assumed to be independent of those of the first component) is known to be exponentially distributed with a mean of 7000 hours. Find the proportion of devices that may be expected to fail before 6000 hours use.

- First device
- λ= 1/5000

$$R(t) = e^{-\lambda t}$$

- R(t) = $e^{-(\frac{6}{5})}$= 0,301
- Second device
- λ= 1/7000
- R(t) = = $e^{-(\frac{6}{7})}$= 0,424
- Probability that both of the devices will work
- P(1)*P(2)= 0,301*0,424 = 0,1276
- Probability that both of the devices will fail before 6000 hours is %87

Normal Distribution

- The normal distribution occurs whenever a random variable is affected by a sum of random effects, such that no single factor dominates. This motivation is based on central limit theorem, which states that under mild conditions, the sum of a large number of random variables is approximately normally distributed. It has been used to represent dimensional variability in manufactured goods, material properties, and measurement errors. It has also been used to assess product reliability.

- The normal distribution has been used to model various physical, mechanical, electrical, or chemical properties of systems. Some examples are gas molecule velocity, wear, noise, the tensile strength of aluminum alloy steel, the capacity variation of electrical condensers, electrical power consumption in a given area, generator output voltage, and electrical resistance.

Normal Distribution

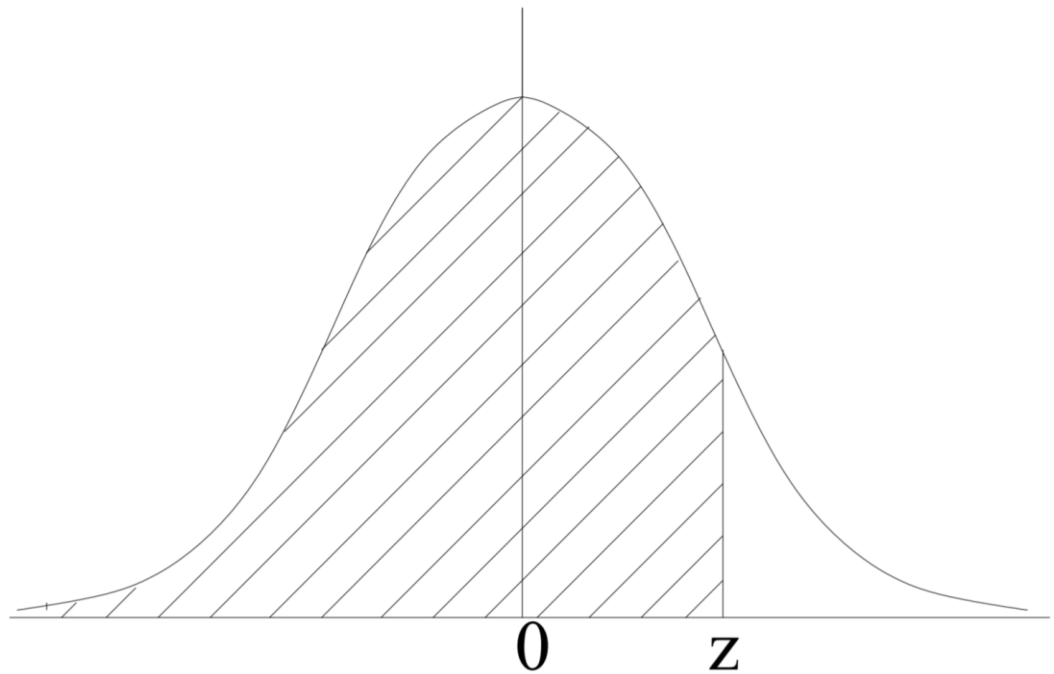
- Symmetric and bell shaped
- Two parameters, μ and σ
- If we want to calculate probabilities from different Normal distributions we convert the probability to one involving the standard Normal distribution. This process is called standardization.

Here is the formula for z-score that we have been using:

$$z = \frac{x - \mu}{\sigma}$$

- **z** is the "z-score" (Standard Score)
- **x** is the value to be standardized
- **μ** is the mean
- **σ** is the standard deviation

The tables allow us to read off probabilities of the form $P(Z < z)$.



- The normal distribution has an increasing hazard rate. The normal distribution has been used to describe the failure distribution for products that show wearout and that degrade with time. The life of tire tread and the cutting edges of machine tools fit this description. Also, mechanical items such as ball bearings, valves, and springs tend to have insignificant burn-in and steady-state phases, and start to wear out right away. In these situations, life is given by a mean value of μ, and the variability about the mean value isdefined through standard deviation.

- Obtaining Normal probabilities involves the use of the standard Normal table. The table provides areas under a Normal curve up to a specified point z , where z is a *standardized* value calculated using the formula

$$z = \frac{T - \text{Mean wear-out time}}{\text{Standard deviation of wear-out time}}$$

$$F(t) = \Phi(z) = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right)$$

Normal Distribution Table

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361

Resim şu anda görüntülenemiyor.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831

Resim şu anda görüntülenemiyor.

Example

- The mean life of a certain ball bearing can be modelled using a Normal distribution with a mean of six years and a standard deviation of one year. Determine:
- **a.** The probability that a ball bearing will fail *before* seven years of service.
- **b.** The probability that a ball bearing will fail *after* seven years of service (i.e., find its reliability).
- **c.** The service life that will provide a failure probability of 10 percent.

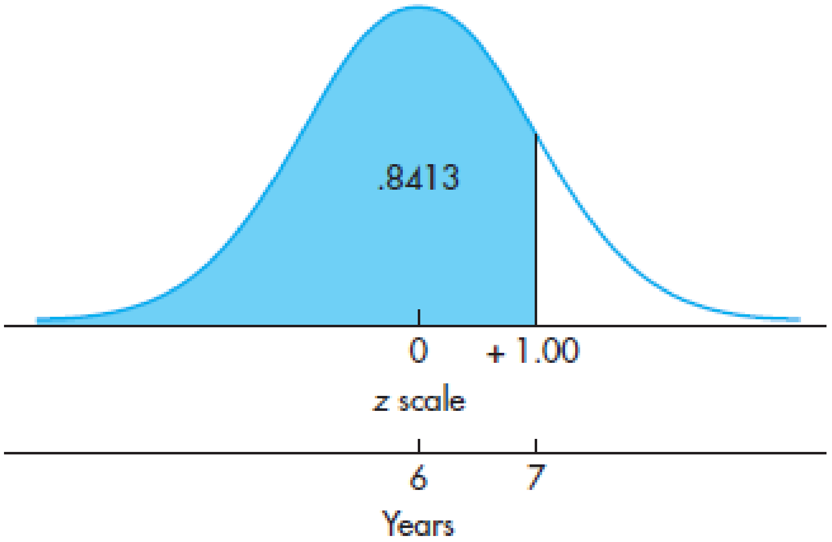
Wear-out mean = 6 years

Wear-out standard deviation = 1 year

Wear-out is Normally distributed

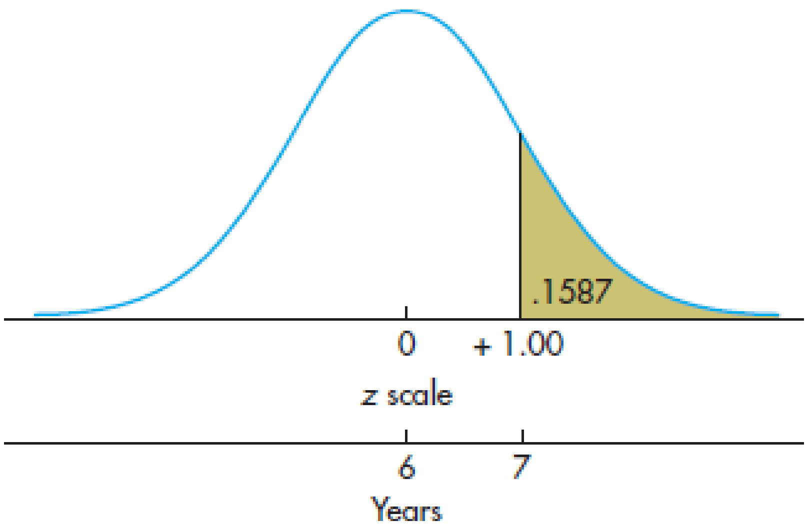
a. Calculate z using the above formula:

$$z = \frac{7 - 6}{1} = +1.00$$



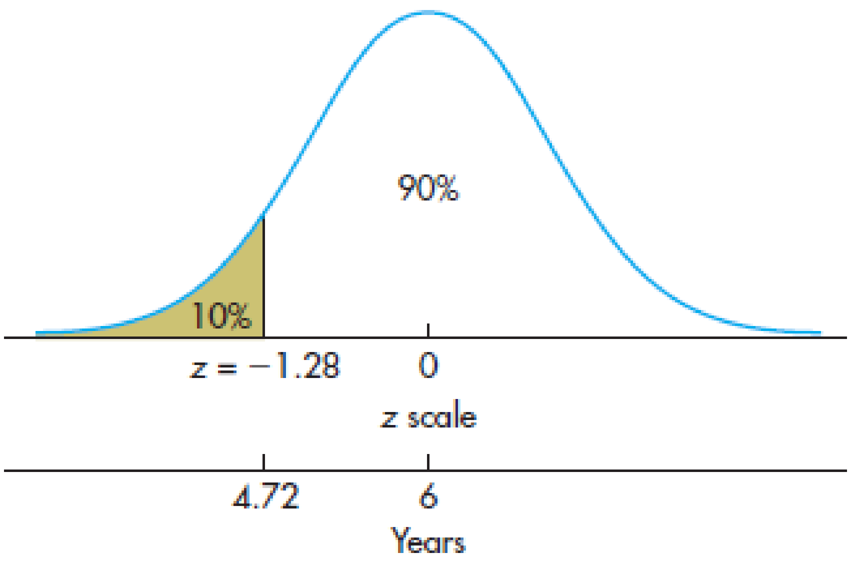
b. Subtract the probability determined in part *a* from 1.00

$1.00 - .8413 = .1587$



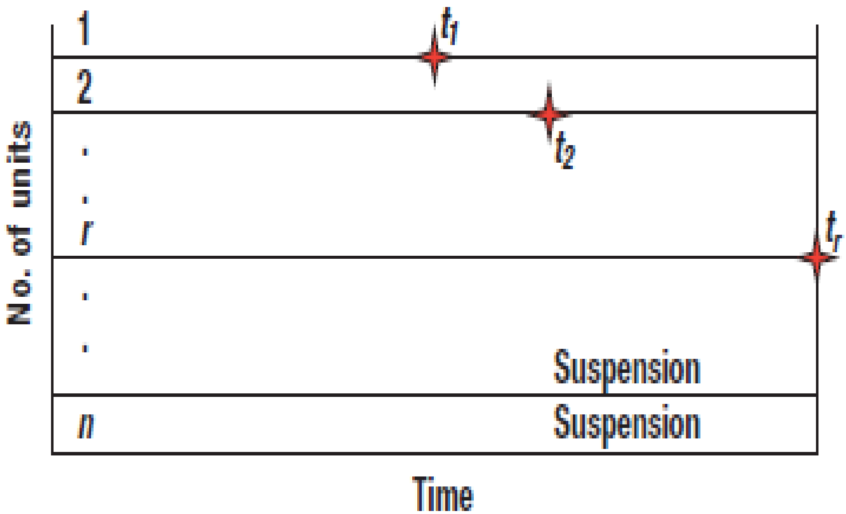
- z= -1.28 = (t-6)/1 = 4.72

c. Use the standard Normal table in reverse, i.e., find the value of z that corresponds to an area under the curve (starting from the left side) of .10. Thus, $z = -1.28$.
Now, insert this in the z formula above:



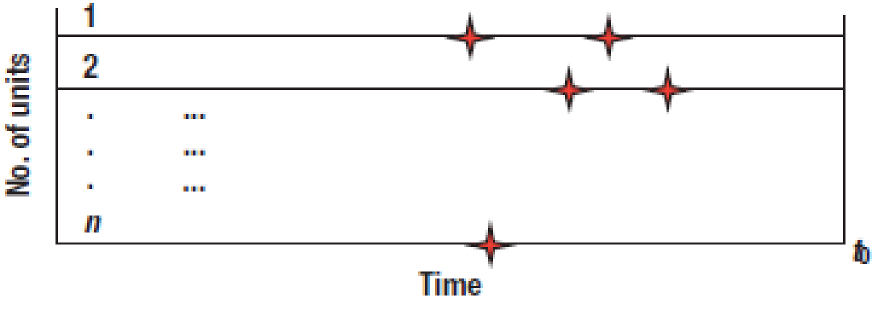
Life Testing

- For reliability tests , the time to failure can be assumed to follow an exponential distribution, the constant failure rate can be estimated by life testing. There are various ways to test the items.
- One of them is a failure-truncated test, in which n items on individual test stands are monitored to failure. The test ends as soon as there are r failures (without replacement $r \leq n$)
- The total time on test, T, considering both failed and unfailed units



Failure-truncated test. Failures are denoted by ✦.

- Another test situation is called time-truncated testing.
- Testing for these units continues until some predetermined time.



Time-truncated test. ✦, Failures.

- For life testing, two different types of tests can be conducted.
- Replacement or without replacement.
- In replacement, failed items replaced as soon as they fail.

- Estimate the MTBF for the following reliability test situations:
- (a) Failure terminated, with no replacement. Twelve items were tested until the fourth failure occurred, with failures at 200, 500, 625, and 800 hours.
- (b) Time terminated, with no replacement. Twelve items were tested up to 1000 hours, with failures at 200, 500, 625, and 800 hours.
- (c) Failure terminated, with replacement. Eight items were tested until the third failure occurred, with failures at 150, 400, and 650 hours.
- (d) Time terminated, with replacement. Eight items were tested up to 1000 hours, with failures at 150, 400, and 650 hours.
- (e) Mixed replacement/nonreplacement. Six items were tested through 1000 hours on six different test stands. The first failure on the test stand occurred at 300 hours, and its replacement failed after an additional 400 hours. On the second test stand, failure occurred at 350 hours, and its replacement failed after an additional 500 hours. On the third test stand, failure occurred at 600 hours, and its replacement did not fail up to the completion of the test. The items on the other three test stands did not fail for the duration of the test.

(a) $\mathrm{MTBF}(e) = (200 + 500 + 625 + 800 + 8(800))/4 = 2{,}131$ hours

(b) $\mathrm{MTBF}(e) = (200 + 500 + 625 + 800 + 8(1000))/4 = 2{,}531$ hours

(c) $\mathrm{MTBF}(e) = (8)(650)/3 = 1{,}733$ hours

(d) $\mathrm{MTBF}(e) = (8)(1000)/3 = 2{,}667$ hours

(e) $\mathrm{MTBF}(e) = (700 + 850 + 1000 + (3)(1000))/5 = 1{,}110$ hours.

Resources

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- STATISTICS FOR ENGINEERS Fall 2015 Lecture Notes, Dewei Wang Department of Statistics University of South Carolina.
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- https://canmedia.mheducation.ca/college/olcsupport/stevenson/5ce/ste39590_ch04S_001-019.pdf, Supplement to Chapter 4 Reliability
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- Introduction to Reliability Fundamentals, Donald G. Dunn, 2019 D2 Training
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- https://www.weibull.com/hotwire/issue14/relbasics14.htm
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- Power System Reliability, Lecture Notes DR. AUDIH ALFAOURY, 2017- 2018, Al-Balqa Applied University
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- **Basic Probability and Reliability Concepts, Roy Billinton Power System Research Group University of Saskatchewan CANADA**

Reliability Engineering

Notes 5

System Reliability

- Most products are made up of a number of components. The reliability of each component and the configuration of the system consisting of these components determines the system reliability.
- System reliability is a function of the reliabilities of the (sub reliabilities of the (sub -) components and of) components and of the relationships between the components.

- The components may be in
- Series system
- Parallel system
- Combination of both
- K out of n system

- The reliability evaluation of engineering systems can be obtained by drawing Reliability Block Diagram (RBD).
- The reliability block diagram (RBD) is a pictorial way of showing the success or failure combinations for a system.
- A system reliability block diagram presents a logical relationship of the system, subsystems, and components.
- In a RBD each of the engineering system components is indicated by a block.
- Any closed path through the diagram is a success path. it is closed when the component it represents is working and is opened when the component has failed.

System Reliability

- Series System
- A system may consist of one or more of the sub-systems and each subsystem may have different sub subsystems.
- In series or chain structure, all components must function for the system to function.
- Example, lights (bulbs or LEDs) used in festivals and weddings.

Series System

- It consists of several components and all the components must run simultaneously.
- All the components must be working that it must not fail at a particular point in time, if just one component fails entire system breaks down.
- Series system component failures are statistically independent. That means, for each of these components it has its own time to failure time.

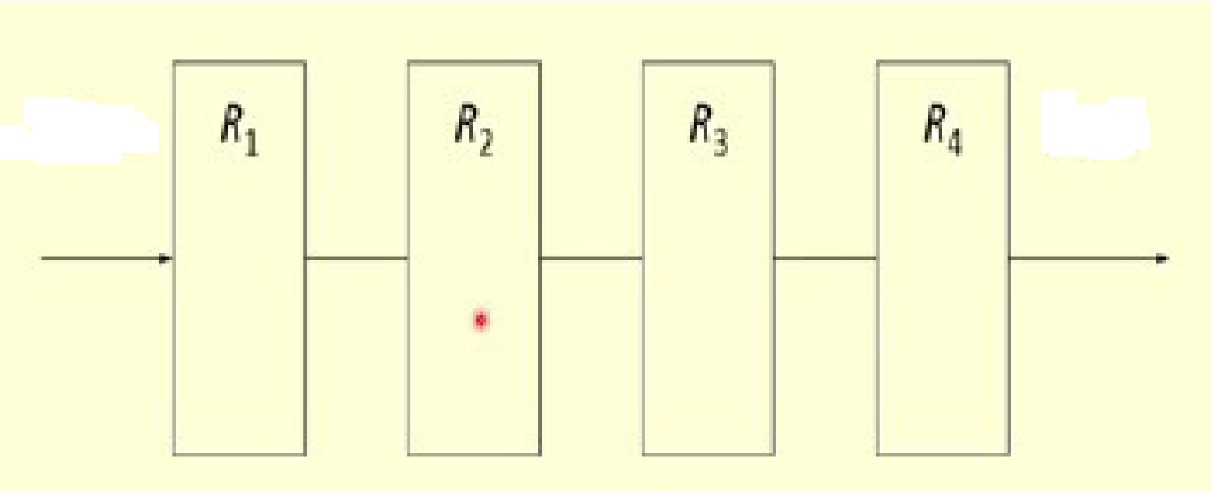
Series System

- The function of one component no way is affected by the function of the other components. So, we may assume that they are all statistically independent.
- Even though the individual components of a series system (product) might have high reliability, the series system (product) as a whole can have considerably less reliability because all its components must function (i.e., the system is dependent on each of its components). As the number of components in a series system (product) increases, the system (product) reliability decreases.

Series System

- Reliability of the series system can never be better than the reliability of the worst component in the system.
- Serial system reliability is smaller than any individual reliability of the components.
- All the reliabilites should be given for a common time period.

Series System



Series System Reliability

If R denotes the reliability of the series system, then, from the definitions of reliability and series system, we have

$$R = P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n)$$

In equation , we take the intersection of events because reliability is a probability and the definition of series system implies that for the successful operation of the system, all components must work.

Series System Reliability

The probability of success of the system is

$$P(S) = P(X_1 \cap X_2 \cap X_3 \cap \dots\dots\dots\dots\dots\dots\dots\dots\dots. \cap X_n)$$

$$or \quad P(S) = P(X_1).P(X_2).P(X_3) \dots\dots\dots\dots\dots\dots\dots\dots\dots. P(X_n)$$

Where $P(X_1)$ = Probability of success of X_1 = Reliability R_1(t)

$P(X_2)$ = Probability of success of X_2 = Reliability R_2(t)

$P(X_3)$ = Probability of success of X_3 = Reliability R_3(t)

. . . .

$P(X_n)$ = Probability of success of X_n = Reliability R_n(t)

$$R_{\text{system}}(t) = R_1(t) \times R_2(t) \times R_3(t) \times \cdots \times R_n(t)$$

$$R_{syst}(t) = \prod_{i=1}^{n} R_i\,(t)$$

Example

- A system is composed of 3 independent serially connected components
- R 1 = 0.95
- R 2 = 0.87
- R 3 = 0.82
- What is the system reliability?
- Rs = 0.95 x 0.87 x 0.82 = 0.6777

Example

- A two component series system contains identical components each having a reliability of 0.99. Evaluate the unreliability of the system.
- $Rs= (0,99)^2 = 0,9801$
- Unreliability= $Qs = 1-0,9801 = 0,0199$

Example

- A system design required 200 identical components in series. If the overall reliability must not be less than 0.99, what is the minimum reliability of each component.
- $Rs = R^{200} = 0.99$
- $R = 0.99^{1/200} = 0.99995$

Example

- A system has three components connected in series having reliabilities 0.40, 0.70, 0.80, respectively, for a mission of 400 hours. What is the percentage increase in the reliability of the system in each of the following cases?
- (i) Reliability of the first component is increased by 0.1 and that of the second and third components remains the same.
- (ii) Reliability of the second component is increased by 0.1 and that of the first and third components remains the same.
- (iii) Reliability of the third component is increased by 0.1 and that of the first and second components remains the same.

- System Reliability
- R = 0.4x0.7x0.8
- R = 0.224

	Reliability of the First Component	Reliability of the Second Component	Reliability of the Third Component	Reliability of the System	Percentage Increase in the Reliability of the System
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
	0.4	0.7	0.8	0.224	–
i)	0.5	0.7	0.8	0.280	$\frac{0.280-0.224}{0.224}\times 100 = 25\%$
ii)	0.4	0.8	0.8	0.256	$\frac{0.256-0.224}{0.224}\times 100 \quad 14.29\%$
iii)	0.4	0.7	0.9	0.252	$\frac{0.252-0.224}{0.224}\times 100 = 12.5\%$

- If a system engineer wants to improve the reliability of the series system, he/she should **concentrate** on the **improvement** of the reliability of the **poorest component**.

Parallel Systems

- Only one of N components is needed to make the system function properly.
- If all of the components fail, the system fails.
- System is composed of n independent components connected in parallel.
- Failure of all components results in the failure of the whole system.
- Example : a laptop with a power source and a battery

- If R denotes the reliability of the parallel system for the mission of t units of time, then by definition of reliability and parallel system, we have

$$R = P(E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$$

We take the union of events in equation because the definition of reliability implies that reliability is a probability and the definition of parallel system implies that for the successful operation of the parallel system only one component needs to work.

$$R = 1 - P\left(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \ldots \cap \bar{E}_n\right)$$

$$R = 1 - P\left(\bar{E}_1\right)P\left(\bar{E}_2\right)P\left(\bar{E}_3\right)\ldots P\left(\bar{E}_n\right)$$

$$= 1 - \left(1 - P\left(E_1\right)\right)\left(1 - P\left(E_2\right)\right)\left(1 - P\left(E_3\right)\right)\ldots\left(1 - P\left(E_n\right)\right)$$

$$R = 1 - \left(1 - R_1\right)\left(1 - R_2\right)\left(1 - R_3\right)\ldots\left(1 - R_n\right)$$

$$\text{or } R = 1 - \prod_{i=1}^{n}\left(1 - R_i\right)$$

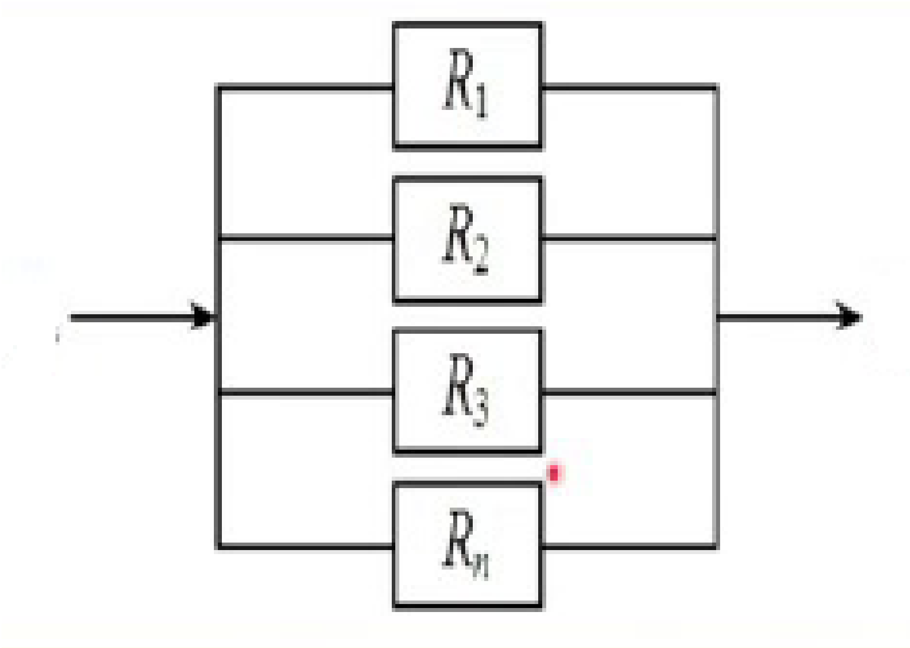
Parallel Systems

- The reliability is higher than the highest among the component reliabilities.

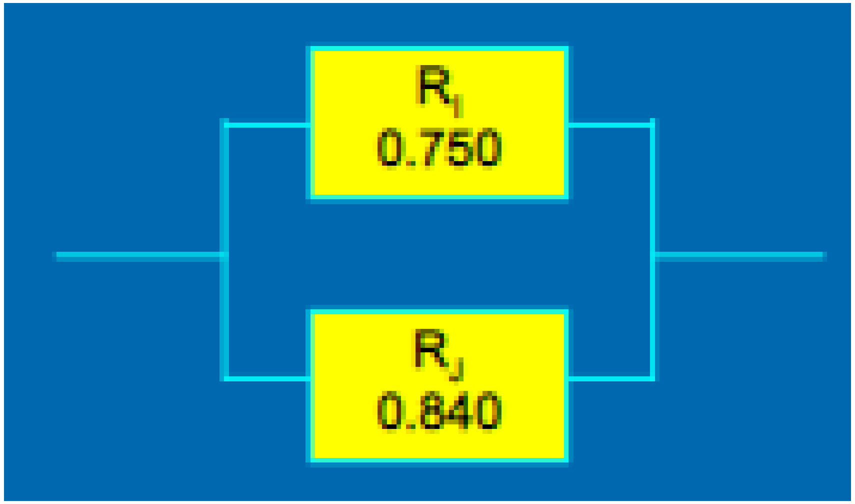
$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i)$$

- n means number of components
- R_i means the reliability of the ith component.
- $R_p = 1-(1-R_A)(1-R_B)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1-R_n)$

Parallel Systems



Example

- A system consists of 2 independent parallel connected components.
- 

- Rp = 1 – (1-0.750)(1- 0.840)
- = 1- 0.04
- = 0.960

Example

- A system has three components connected in parallel from a reliability point of view having reliabilities 0.20, 0.40, 0.50, respectively, for a 400 hours period of time. What is the percentage increase in the reliability of the system in the following cases?

a) Reliability of the first component is increased by 0.1 and that of the second and third components remains the same.

b) Reliability of the third component is increased by 0.1 and that of the first and second components remains the same.

- System reliability
- R1 = 0.2 R2= 0.4 R3 = 0.5
- Rp = 1- (1-0.2)(1-0.4)(1-0.5)
-    = 1- 0.8x0.6x0.5
-    = 1-0.240
-    = 0.760

- a) Reliability of the first component is increased by 0.1
- $R_p = 1- (1-0.3)(1-0.4)(1-0.5)$
- $\quad = 1- 0.7x0.6x0.5$
- $\quad = 1-0.21$
- $\quad = 0.79$
- System reliability increases
- [(0.790-0.760)/0.760]x100 = %3.95

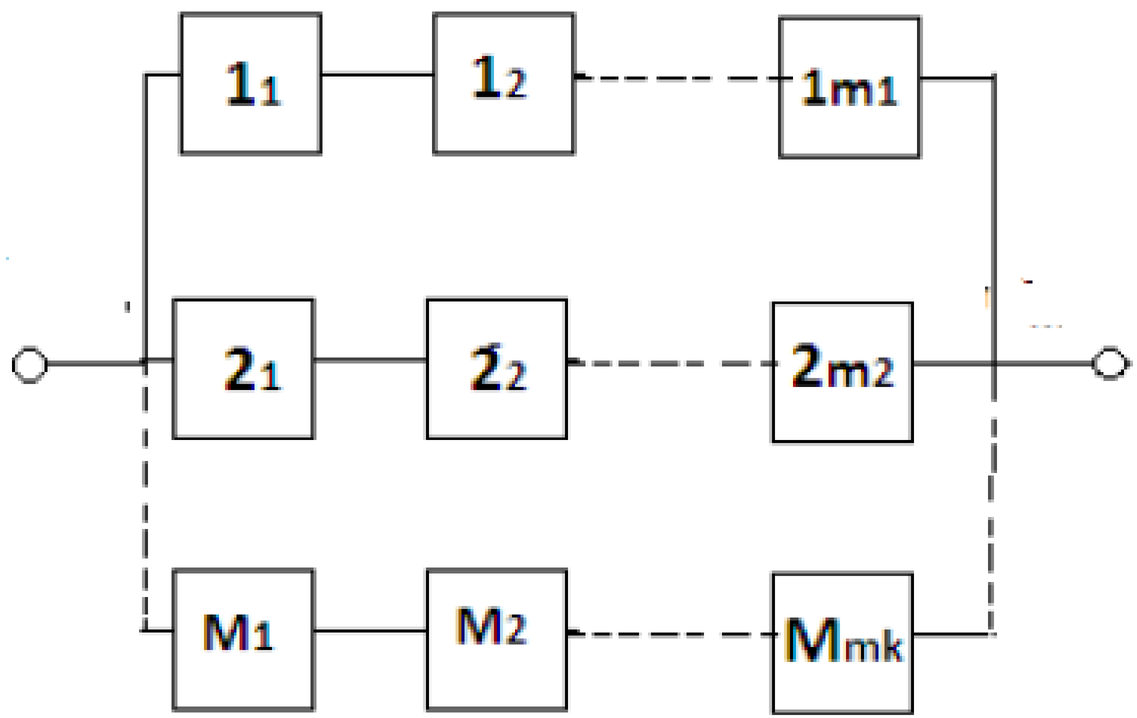
- b) Reliability of the third component is increased by 0.1
- $R_p = 1 - (1-0.2)(1-0.4)(1-0.6)$
- $\quad = 1 - 0.8 \times 0.6 \times 0.4$
- $\quad = 1 - 0.192$
- $\quad = 0.808$
- System reliability increases
- $[(0.808-0.760)/0.760]\times 100 = \% \ 6.32$

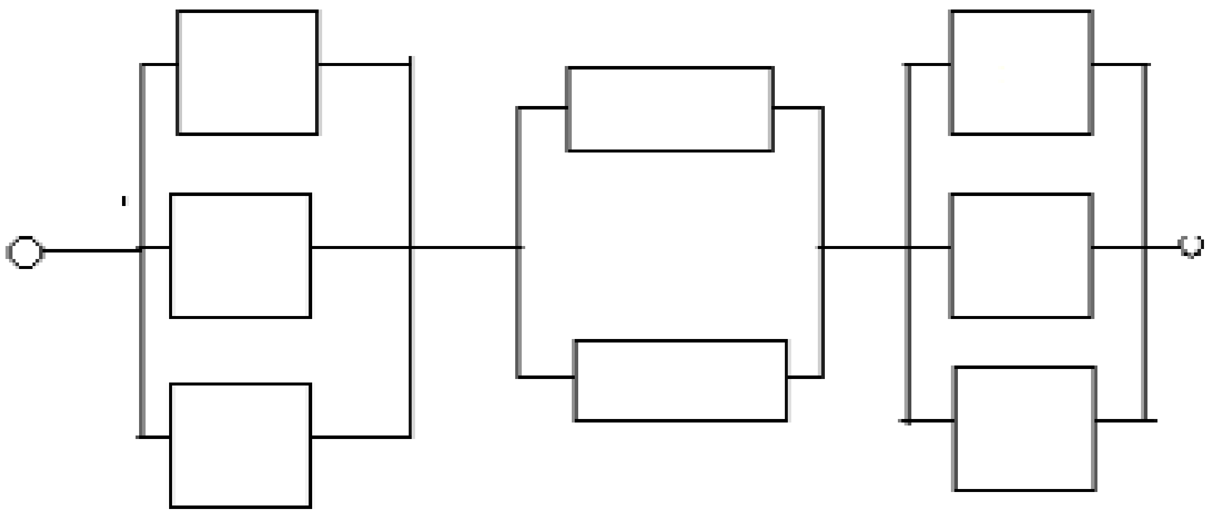
- The improvement in the reliability of the system (in percentage) is higher when reliability of the best component is increased by 0.1.
- If a system engineer wants to improve the reliability of a parallel system, he/she should concentrate on the improvement of the reliability of the best component.

Mixed Systems

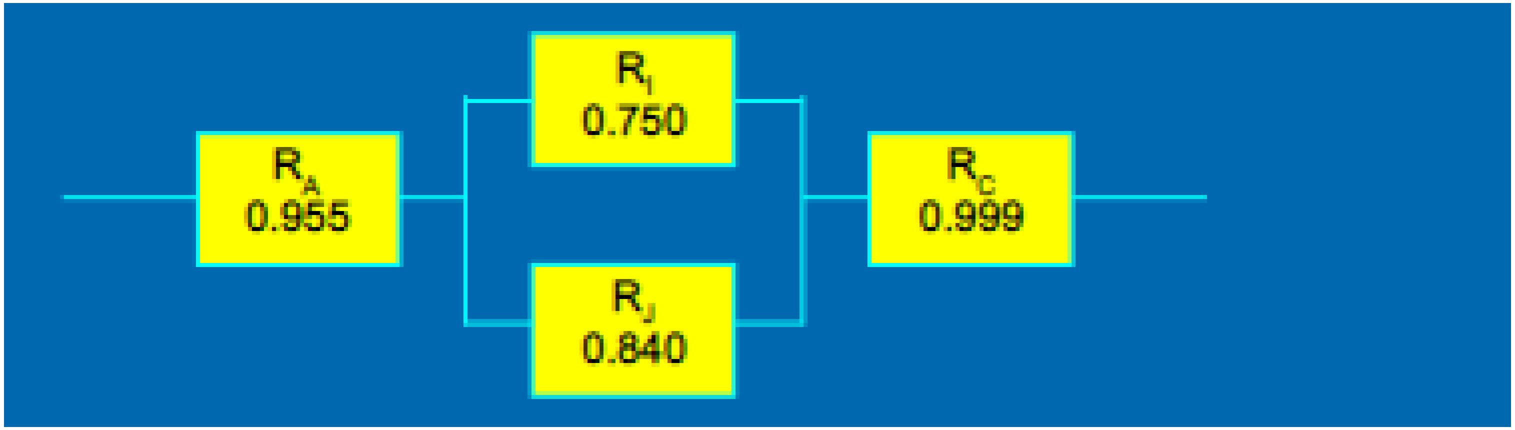
- A system is said to be a mixed system if the components of the system are connected both in series and in parallel configurations.
- To evaluate the reliability of a mixed system, we first break the reliability block diagram into series or parallel subsystems. Then we evaluate the reliability of each subsystem. Finally, we evaluate the reliability of the given mixed system by combining the reliabilities of the subsystems. It is called reduction method.

Mixed System





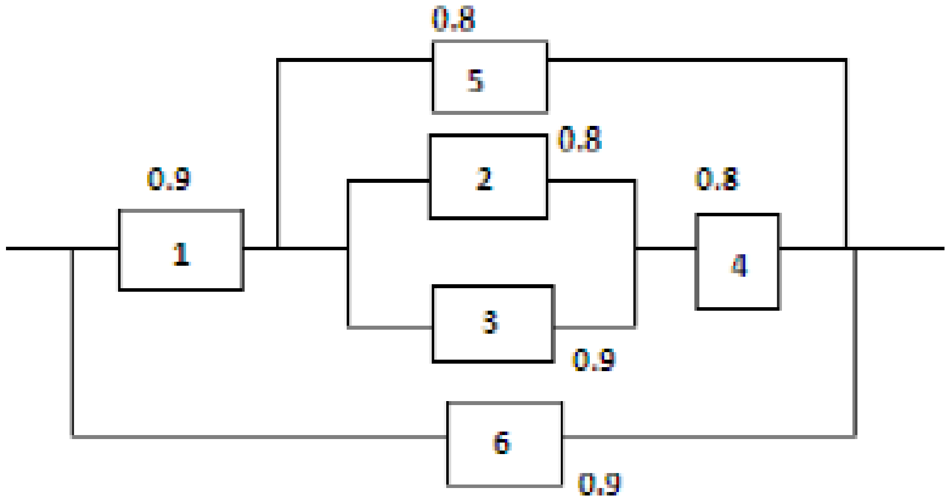
Example

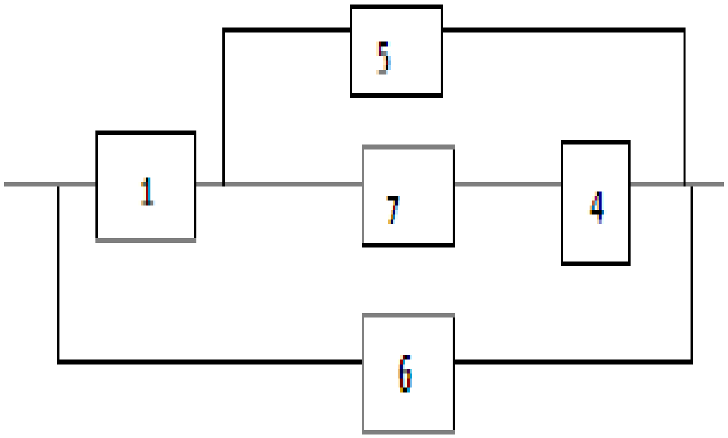


- First of all, we find the parallel system reliability.
- $R_{ıj}$ = 1- (1- 0.750)(1-0.840) =
- = 1- 0.04
- = 0.96
- Then, the system is reduced to series system
- $R_s = (R_A)(R_{ıj})(R_c)$
- = (0.955)(0.960)(0.999)
- = 0.916

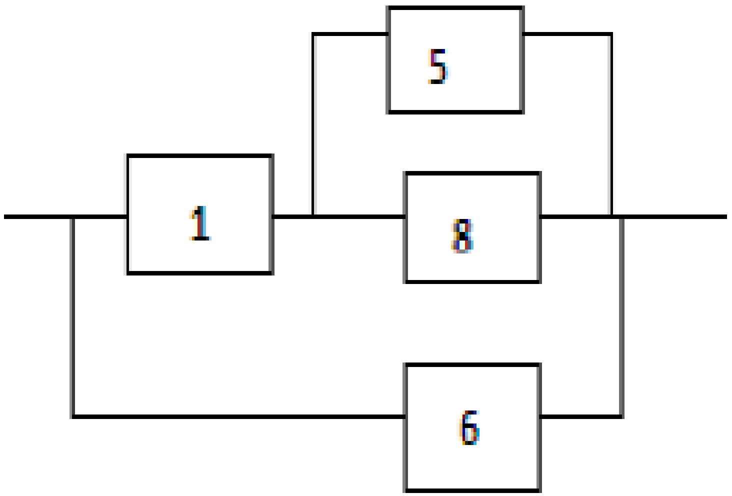
Example

Calculate the reliability of the system shown using network reduction technique?

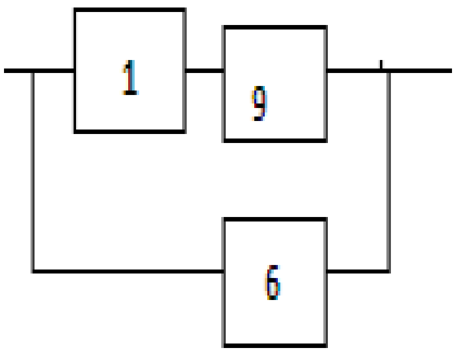




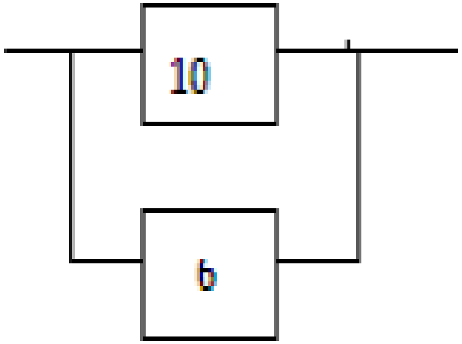
$$R_7 = 1-\{(1-0.8)(1-0.9)\} = 1-0.02 = 0.98$$



$R_8 = R_7 R_4 = 0.98 \times 0.8 = 0.784$

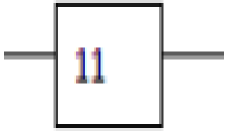


$R_9 = 1-\{(1-R_5)(1-R_8) = 1- (0.2x\ 0.216) = 1-0.0432 = 0.9568$



$R_{10}=R_1R_9=0.9x0.9568 = 0.86112$

$$R_{11} 1- \{(1-0.86112)(1-0.9)\} = 1-(0.13888 x 0.1) = 1-0.01388 = 0.98612$$



Resources

- Quality Design and Control, Design for Reliability- I , Lecture – 44 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
- Quality Design and Control, Design for Reliability- II , Lecture – 46 Notes , Prof. Pradip Kumar Ray, Department of Industrial and Systems Engineering Indian Institute of Technology, Kharagpur
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