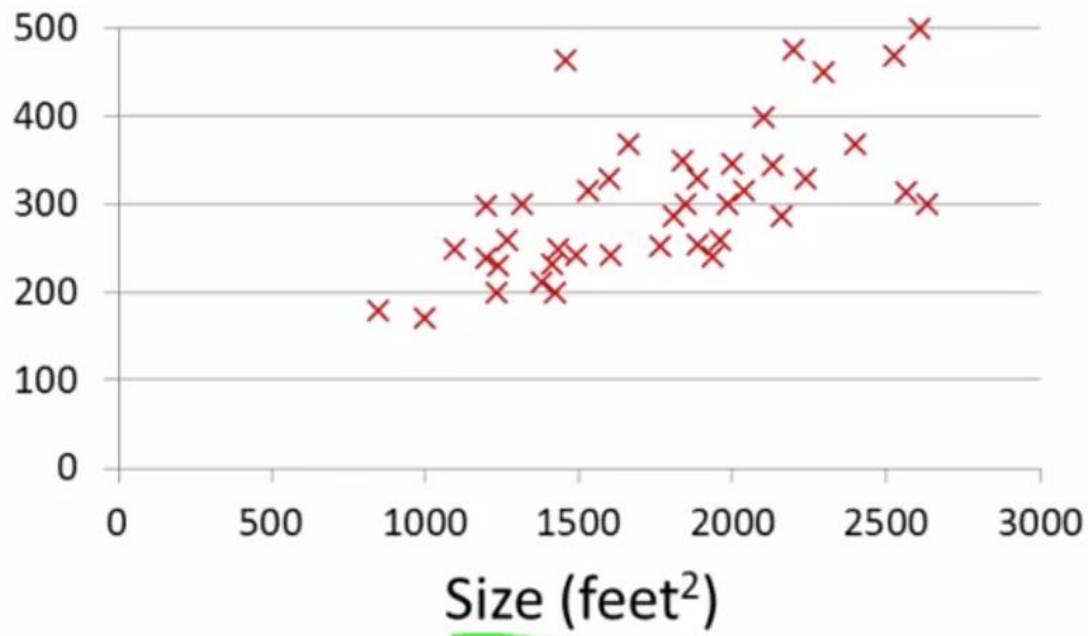


# Model and Cost Function

Model Representation

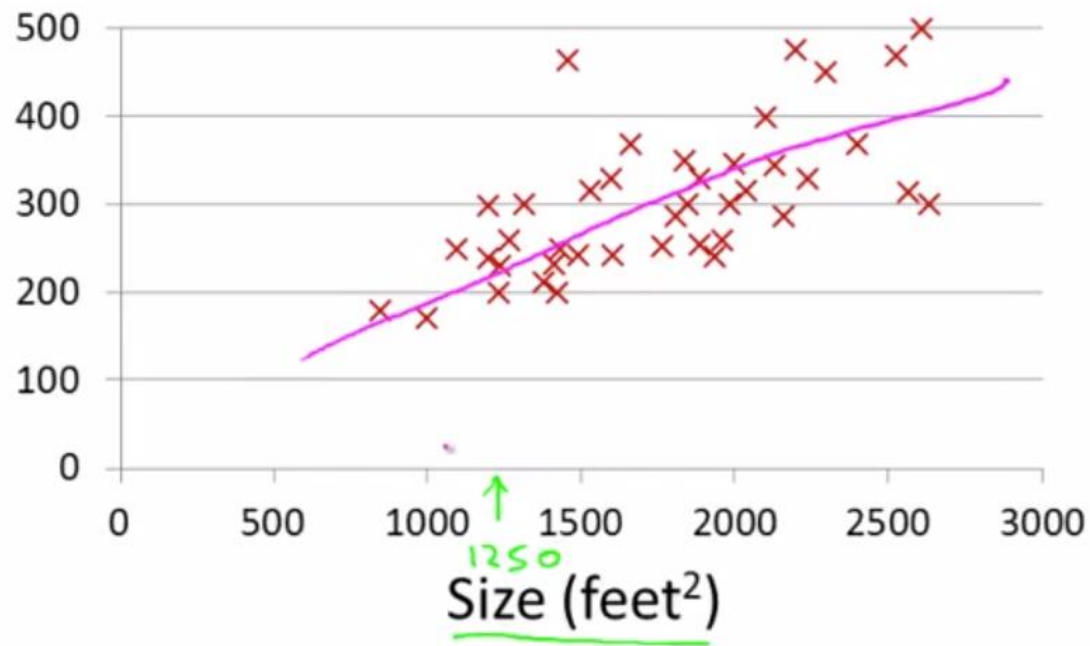
# Housing Prices (Portland, OR)

Price  
(in 1000s  
of dollars)



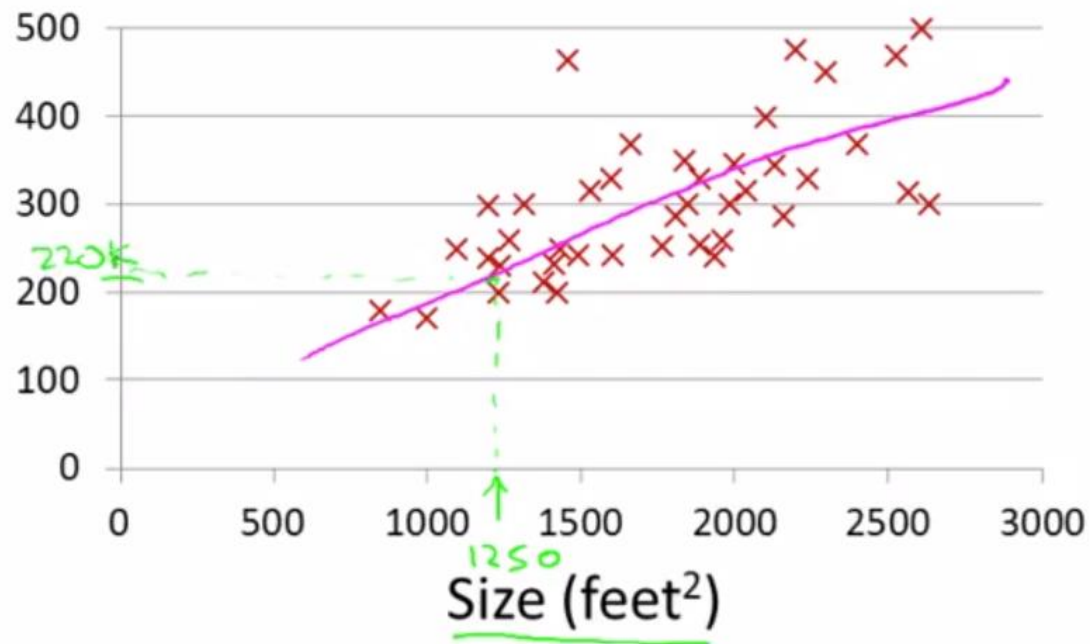
# Housing Prices (Portland, OR)

Price  
(in 1000s  
of dollars)



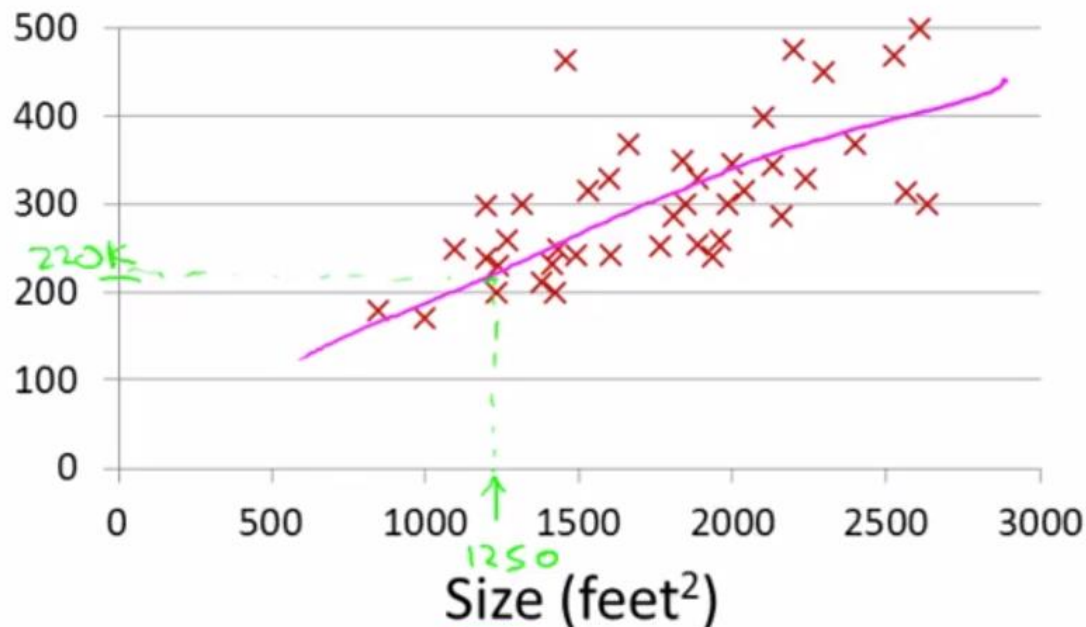
# Housing Prices (Portland, OR)

Price  
(in 1000s  
of dollars)



# Housing Prices (Portland, OR)

Price  
(in 1000s  
of dollars)



## Supervised Learning

Given the “right answer” for each example in the data.

## Regression Problem

Predict real-valued output

**Training set of  
housing prices  
(Portland, OR)**

Size in feet <sup>2</sup> (x)	Price (\$) in 1000's (y)
2104	460
1416	232
1534	315
852	178
...	...

Notation:

**m** = Number of training examples

**x**'s = "input" variable / features

**y**'s = "output" variable / "target" variable

Training set of  
housing prices  
(Portland, OR)

Size in feet <sup>2</sup> (x)	Price (\$) in 1000's (y)
2104	460
1416	232
1534	315
852	178
...	...

} m = 47

Notation:

→ **m** = Number of training examples

**x**'s = "input" variable / features

**y**'s = "output" variable / "target" variable

## Training set of housing prices (Portland, OR)

Size in feet <sup>2</sup> ( $x$ )	Price (\$) in 1000's ( $y$ )
→ 2104	460
1416	232
1534	315
852	178
...	...

$m = 47$

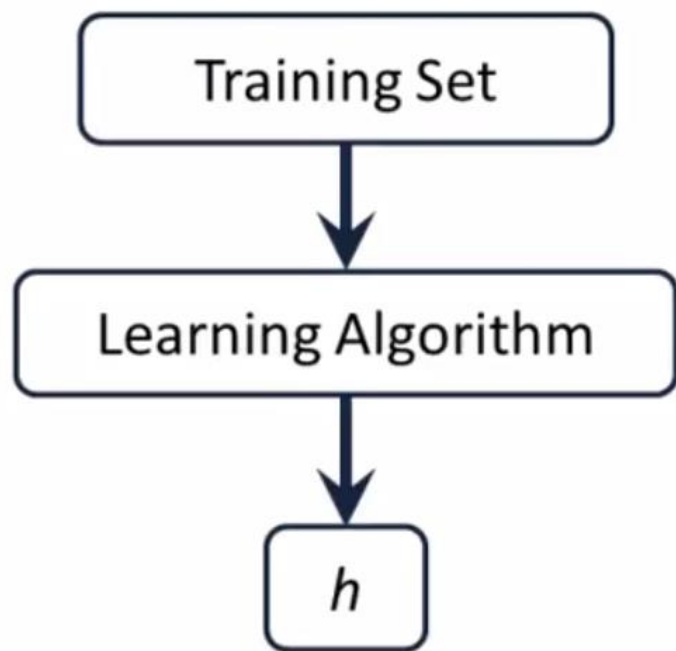
Notation:

- $m$  = Number of training examples
- $x$ 's = "input" variable / features
- $y$ 's = "output" variable / "target" variable

$(x, y)$  - one training example

$(x^{(i)}, y^{(i)})$  -  $i$ th training example





Training Set



Learning Algorithm



Size of  
house



$h$



Estimated  
price

*hypothesis*

Training Set



Learning Algorithm



Size of house



$h$



Estimated price

$x$

*hypothesis*

*(estimated value of  $y$ )*

*$h$  maps from  $x$ 's to  $y$ 's.*

Alt

Training Set



Learning Algorithm



Size of house



$h$



Estimated price

$x$

hypothesis

(estimated value of  $y$ )

$h$  maps from  $x$ 's to  $y$ 's.

How do we represent  $h$  ?

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Shorthand:  $h(x)$

Training Set



Learning Algorithm



Size of house



$h$



Estimated price

(estimated value of  $y$ )

hypothesis

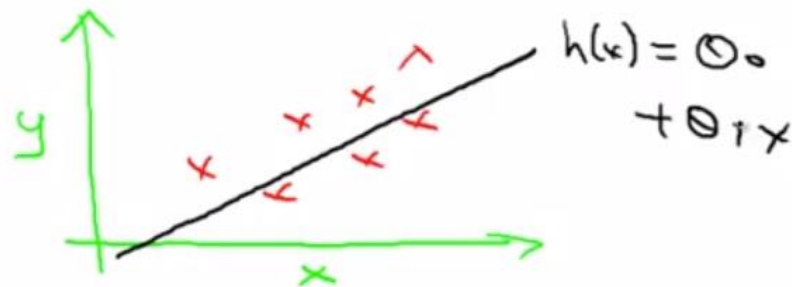
$x$

$h$  maps from  $x$ 's to  $y$ 's.

How do we represent  $h$  ?

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Shorthand:  $h(x)$



Training Set

Learning Algorithm

Size of house

$h$

Estimated price

hypothesis

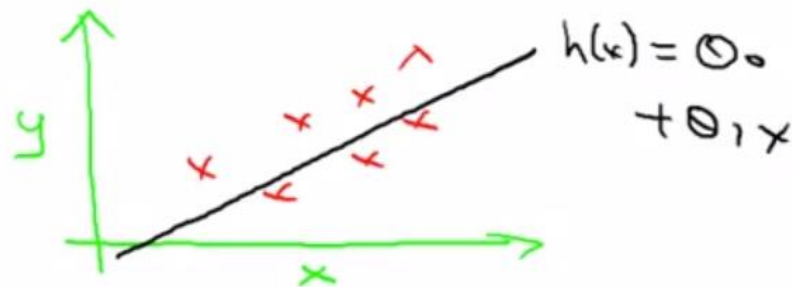
(estimated value of  $y$ )

$h$  maps from  $x$ 's to  $y$ 's.

How do we represent  $h$  ?

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Shorthand:  $h(x)$



Linear regression with one variable. ( $x$ )  
Univariate linear regression.

↳ one variable

Cost Function: How to choose  $\theta$ s?

Training Set	Size in feet <sup>2</sup> (x)	Price (\$) in 1000's (y)
	2104	460
	1416	232
	1534	315
	852	178
	...	...

Hypothesis:  $h_{\theta}(x) = \theta_0 + \theta_1 x$



Training Set

Size in feet <sup>2</sup> (x)	Price (\$) in 1000's (y)
2104	460
1416	232
1534	315
852	178
...	...

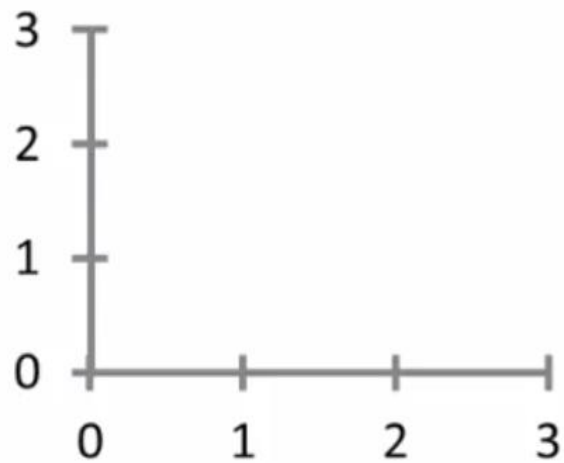
*m = 47*

Hypothesis:  $h_{\theta}(x) = \theta_0 + \theta_1 x$

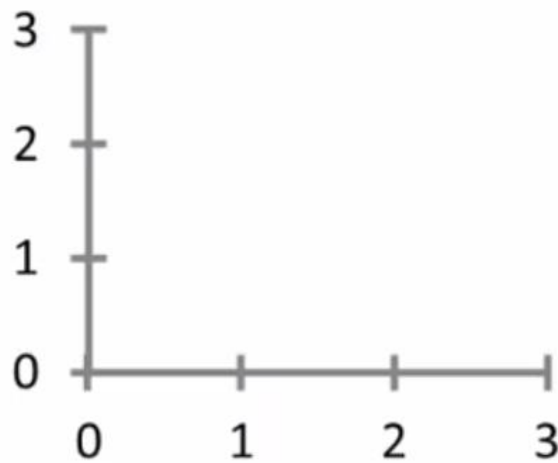
$\theta_i$ 's: Parameters

How to choose  $\theta_i$ 's ?

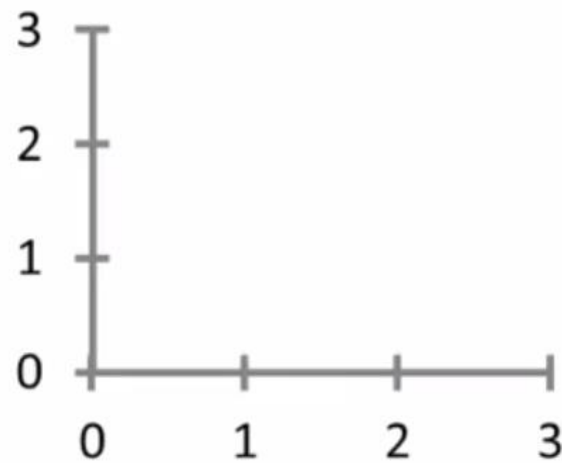
$$h_{\theta}(x) = \theta_0 + \theta_1 x$$



$$\theta_0 = 1.5$$
$$\theta_1 = 0$$

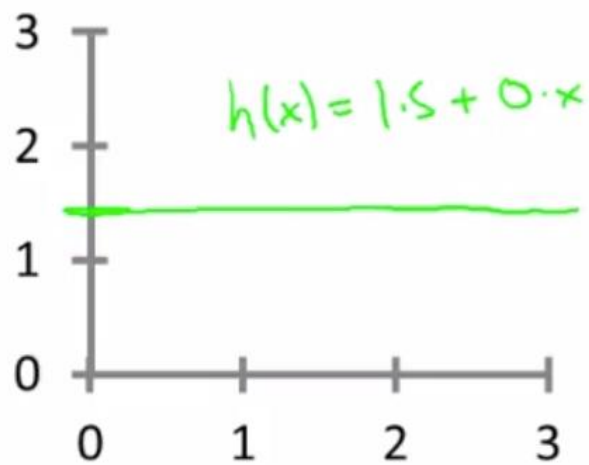


$$\theta_0 = 0$$
$$\theta_1 = 0.5$$



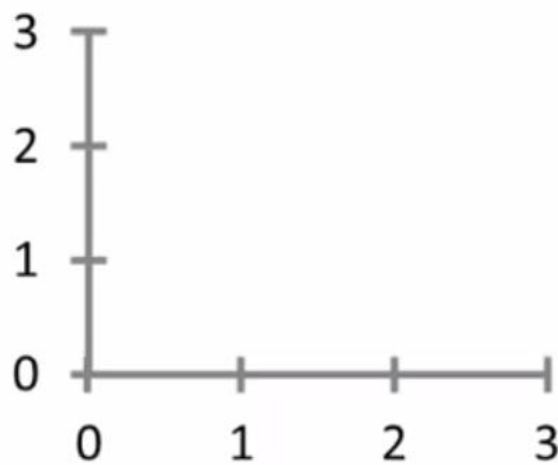
$$\theta_0 = 1$$
$$\theta_1 = 0.5$$

$$\underline{h_{\theta}(x)} = \theta_0 + \theta_1 x$$



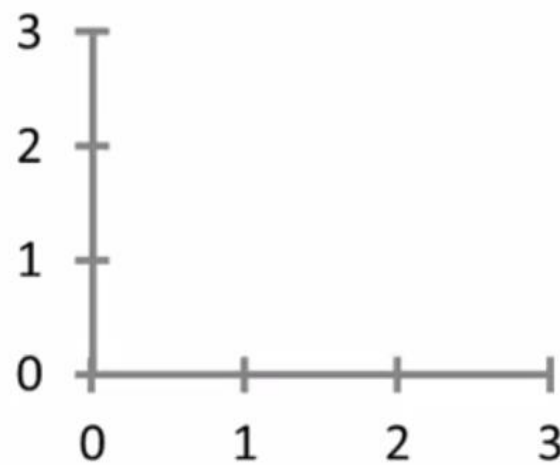
→  $\theta_0 = 1.5$

→  $\theta_1 = 0$



→  $\theta_0 = 0$

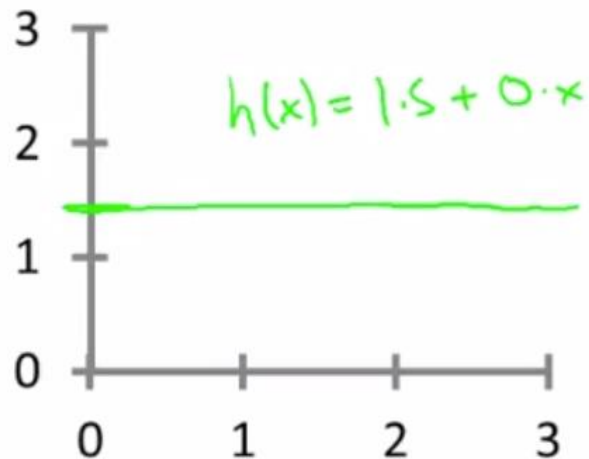
$\theta_1 = 0.5$



$\theta_0 = 1$

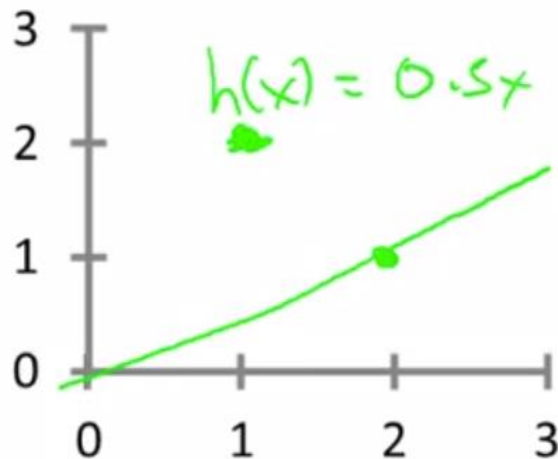
$\theta_1 = 0.5$

$$\underline{h_{\theta}(x)} = \theta_0 + \theta_1 x$$



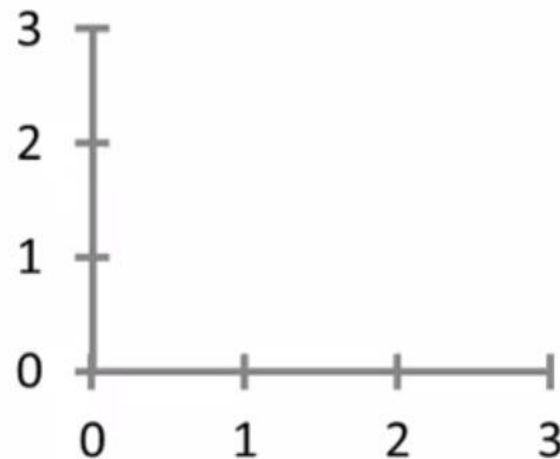
$$\rightarrow \theta_0 = 1.5$$

$$\rightarrow \theta_1 = 0$$



$$\rightarrow \theta_0 = 0$$

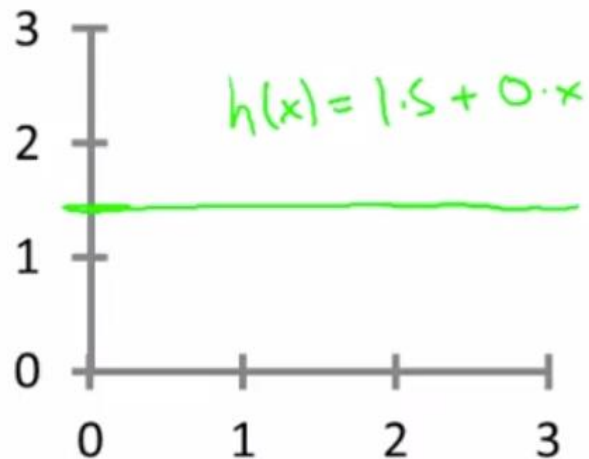
$$\rightarrow \theta_1 = 0.5$$



$$\theta_0 = 1$$

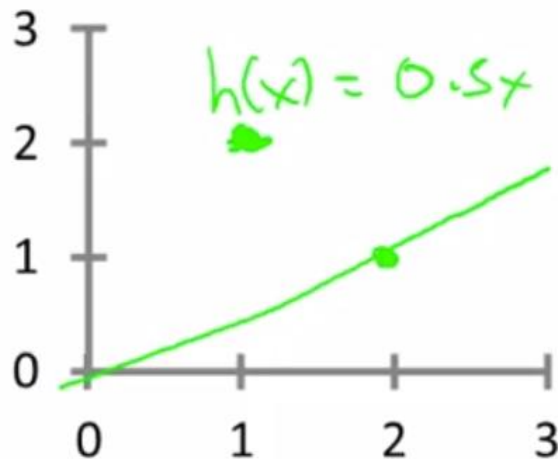
$$\theta_1 = 0.5$$

$$\underline{h_{\theta}(x)} = \theta_0 + \theta_1 x$$



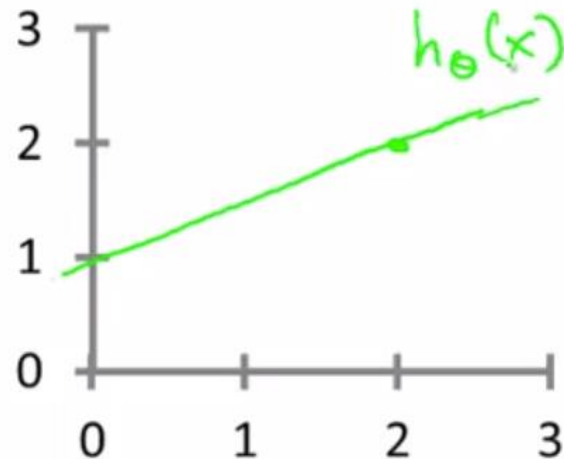
$$\rightarrow \theta_0 = 1.5$$

$$\rightarrow \theta_1 = 0$$



$$\rightarrow \theta_0 = 0$$

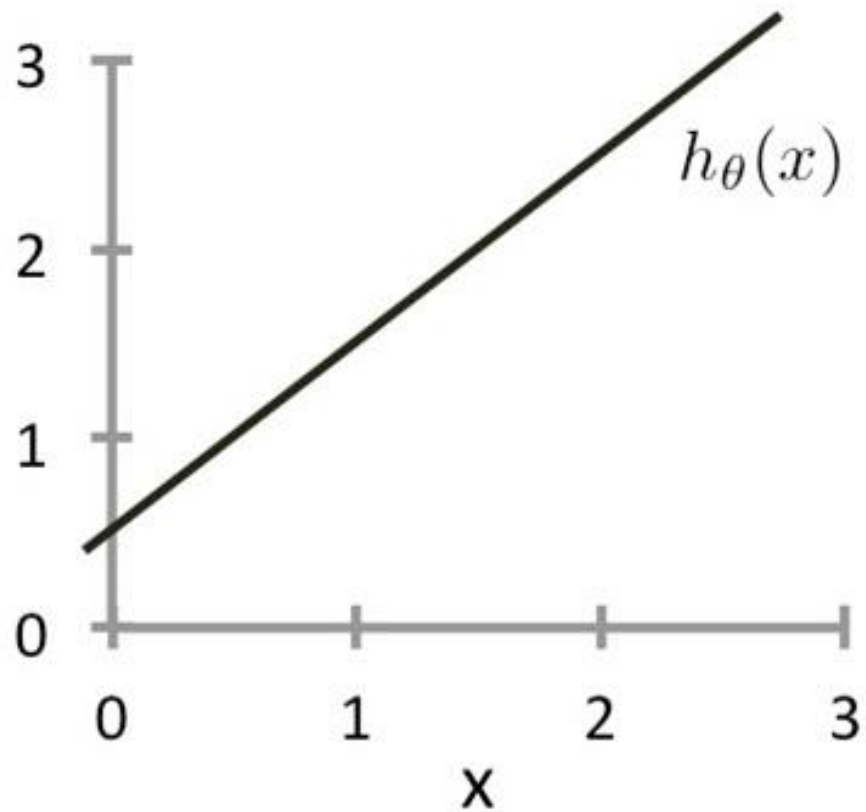
$$\rightarrow \theta_1 = 0.5$$

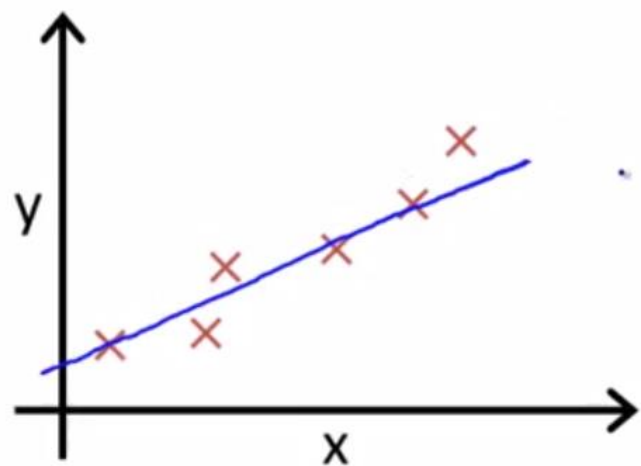


$$\rightarrow \theta_0 = 1$$

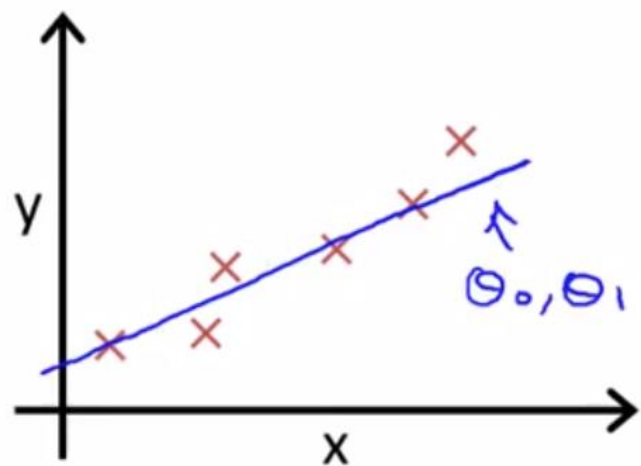
$$\rightarrow \theta_1 = 0.5$$

Consider the plot below of  $h_{\theta}(x) = \theta_0 + \theta_1 x$ . What are  $\theta_0$  and  $\theta_1$ ?



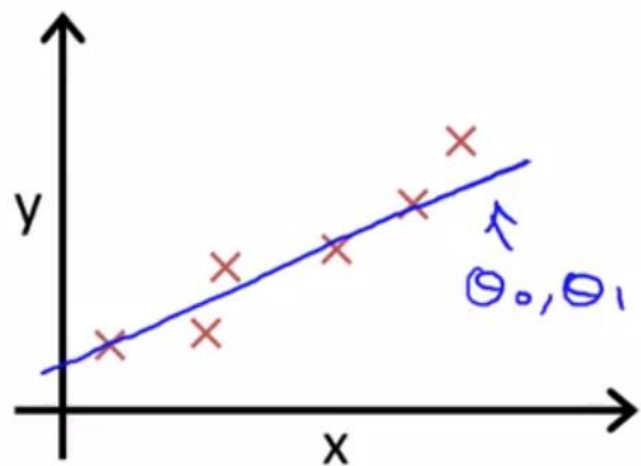


How to find a good fit?



Idea: Choose  $\theta_0, \theta_1$  so that  $h_{\theta}(x)$  is close to  $y$  for our training examples  $(x, y)$



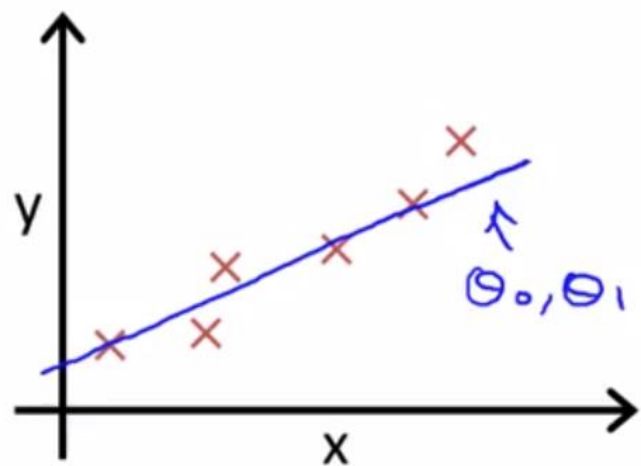


minimize  
 $\theta_0, \theta_1$

$$h_{\theta}(x) - y$$

Idea: Choose  $\theta_0, \theta_1$  so that  
 $h_{\theta}(x)$  is close to  $y$  for our  
 training examples  $(x, y)$

$x, y$

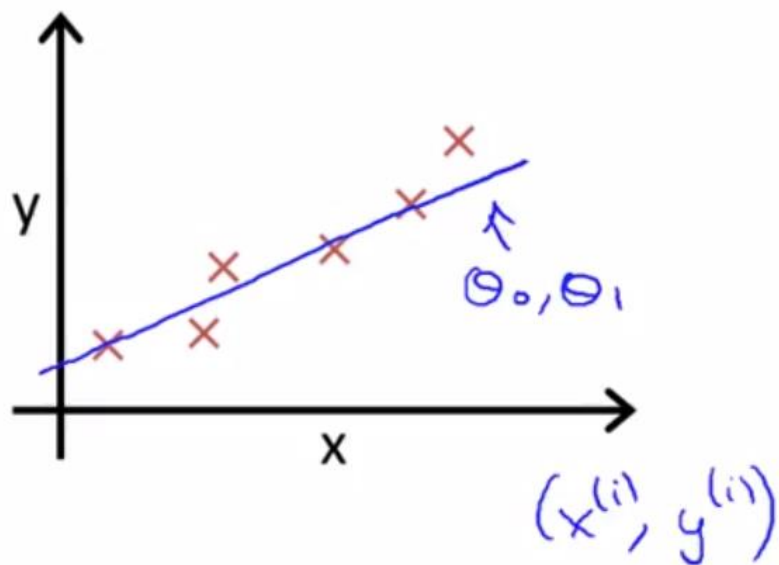


minimize  
 $\theta_0, \theta_1$

$$(h_{\theta}(x) - y)^2$$

Idea: Choose  $\theta_0, \theta_1$  so that  
 $h_{\theta}(x)$  is close to  $y$  for our  
 training examples  $(x, y)$

$x, y$



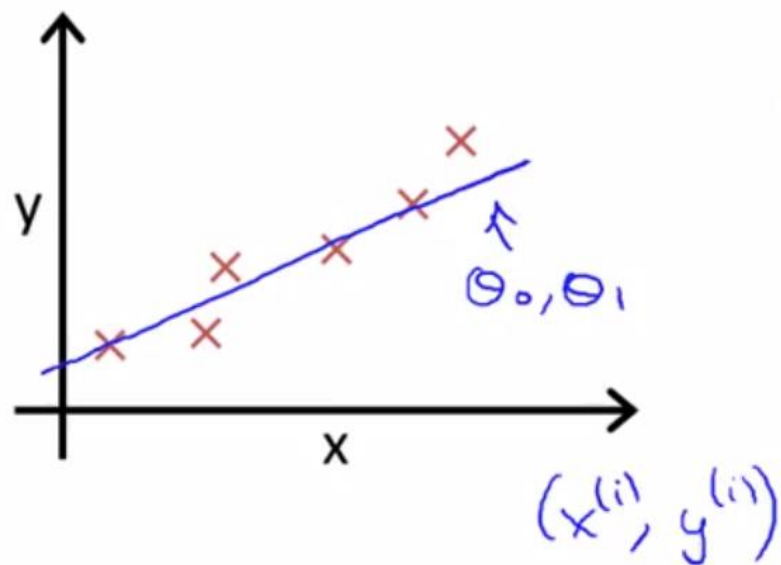
minimize  
 $\theta_0, \theta_1$

$m$  — #training examples

$$\sum_{i=1}^m \left( \underline{h_{\theta}(x^{(i)})} - \underline{y^{(i)}} \right)^2$$

Idea: Choose  $\underline{\theta_0}, \underline{\theta_1}$  so that  
 $h_{\theta}(x)$  is close to  $y$  for our  
 training examples  $(x, y)$

$x, y$

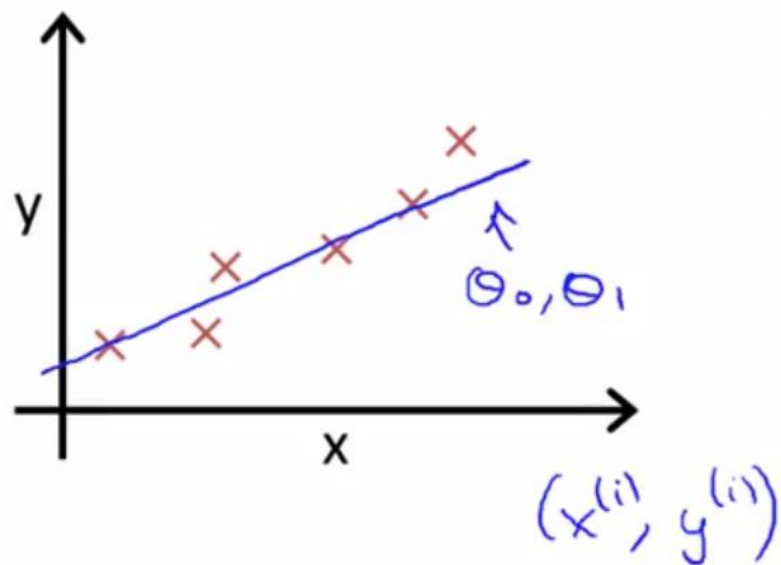


minimize  $\theta_0, \theta_1$   $\frac{1}{2m} \sum_{i=1}^m \left( \underline{h_{\theta}(x^{(i)})} - \underline{y^{(i)}} \right)^2$

*#training examples*

Idea: Choose  $\underline{\theta_0}, \underline{\theta_1}$  so that  $\underline{h_{\theta}(x)}$  is close to  $\underline{y}$  for our training examples  $\underline{(x, y)}$

$x, y$



minimize  $\theta_0, \theta_1$

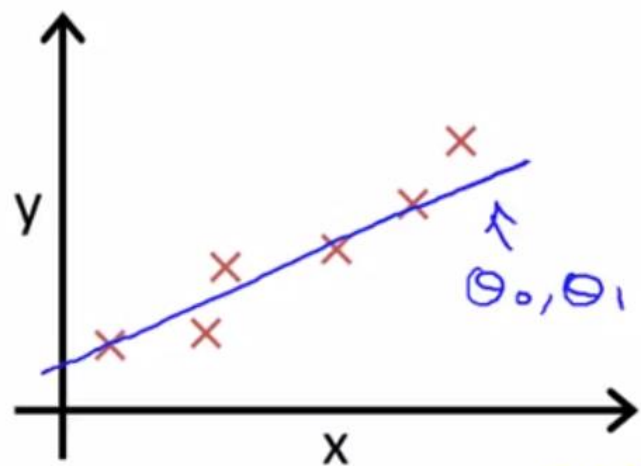
$\frac{1}{2m} \sum_{i=1}^m \left( \underbrace{h_{\theta}(x^{(i)})}_{h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}} - y^{(i)} \right)^2$

#training examples

---

Idea: Choose  $\theta_0, \theta_1$  so that  $\underline{h_{\theta}(x)}$  is close to  $\underline{y}$  for our training examples  $\underline{(x, y)}$

$x, y$



$(x^{(i)}, y^{(i)})$

Idea: Choose  $\theta_0, \theta_1$  so that  $h_{\theta}(x)$  is close to  $y$  for our training examples  $(x, y)$

$x, y$

minimize  $\theta_0, \theta_1$

$\frac{1}{2m} \sum_{i=1}^m \left( \underbrace{h_{\theta}(x^{(i)})}_{h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}} - y^{(i)} \right)^2$

# training examples

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

minimize  $\theta_0, \theta_1$   $J(\theta_0, \theta_1)$

Cost function

Squared error function