Chapter 9-10

Confidence Intervals and Hypothesis Testing

P Value

Statistics

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Using P-Value in Hypothesis Testing

p-value Approach

- In early practice of hypothesis testing, pre-selected values for α, typically values like 0.01, 0.05, 0.10 were used.
- Scientist don’t have computers, and they just
  - looked at the critical values from the tables.
  - Calculate their observed z or t values (or the others)
  - Compare it with the values from the table and give their decisions.
- This is easy, **however** it is a **yes** or **no** type of decision.
- In modern approach, we prefer not to fix the α value (the level of significance) in advance.

p-value Approach

- **Example**: Test $H_0: \mu = 170$ vs $H_1: \mu > 170$ at $\sigma = 10$ and n=25 with $\alpha = 0.05$
- $z_{0.05} = 1.645$
- What happens if $\bar{X} = \mathbf{173.25}$
- What happens if $\bar{X} = \mathbf{173.50}$

p-value Approach

- DEFINITION. A P-value is the lowest level of significance at which the observed value of the test statistic is significant.
- A small P-value provides evidence **against** H_0.
- Therefore we reject H_0 when P-value is too **small**.
- Hence P-value is a measure of credibility (plausibility or acceptability) for H_0.
- A small P-value discredits H_0 and encourages us to reject it.

p-value Approach

- This gives the decision maker the chance to make a personal judgment about the following:
  - Is this value of the test statistic reasonable (acceptable) and can be assumed to be supporting the null hypothesis, or
  - Is it too extreme to be considered acceptable, thus constitutes strong evidence against H0?
- P-value = P(the observed value of the test statistic can be as extreme as, or more extreme, than the value obtained from the sample, assuming that the null hypothesis H_0 is true)

p-value in Single Sided HT

- Example: A random sample of 100 recorded deaths in Turkey during the past year showed an average life span of 71.8 years.
- We want to test the following hypothesis assuming $\sigma = 9$:
  - $\mathbf{H_0}: \boldsymbol{\mu} \leq \mathbf{70}$.
  - $\mathbf{H_1}: \boldsymbol{\mu} > \mathbf{70}$.
- What is the result for 0.05 level of significance? Use p-value approach.

p-value in Single Sided HT

- Solution: P-value = P(Test statistic is more extreme than observed value)

- p-value = $P(\bar{X} \geq 71.8) = P\left(Z \geq \frac{71.8-70}{9/10}\right) = P(Z > 2) = 0.023$
- p-value = $P(Z \geq z_{obs}) = P(Z \geq 2) = 0.023$

p-value in Single Sided HT

- Example: A manufacturer of sports equipment has developed a new synthetic fishing line.
- They want to test:

$$H_0: \mu \geq 8 \ vs \ H_1: \mu < 8$$

- They have tested a sample of n=50 lines with σ of 0.5 kg.
- It turns out that $\bar{X} = 7.85\ kg$.
- What is the result for 0.01 level of significance? Use p-value approach.

p-value in Single Sided HT

- Solution: P-value = P(Test statistic is more extreme than observed value)
- $z_{obs} = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.85 - 8}{0.5/\sqrt{50}} = -2.12$
- P-value = P(Z<-2.12)=0.017
- The null hypothesis (that the strength has at least 8 kg) does not have much credibility.
- The wholesale store must think twice before they purchase large quantities of this new synthetic fishing line.

p-value in Two Sided HT

- The only difference is **we multiply the proability by 2**.
- Be careful about test statistic and check where it lies in:
  - On the greater side, or
  - on the lower side.
- You will calculate the P value accordingly.

p-value in Two Sided HT

- Example: For the synthetic fishing line find p-value for:.

$$H_0: \mu = 8 \; vs \; H_1: \mu \neq 8$$

- Solution:
- P-value = 2 x P(Z<-2.12)=0.034

- Example: Life time of people in Turkey.

$$H_0: \mu = 70 \; vs \; H_1: \mu \neq 70$$

- Solution:
- p-value = 2 x $P(Z \geq z_{obs}) = 2 \times P(Z \geq 2) = 0.046$

p-value in Two Sided HT

- **Example**. A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation of at most 0.9 year.
- To test if the standard deviation can actually be larger than this claimed value, a random sample of 10 of these batteries is selected and tested.
- The sample standard deviation is found to be 1.2 years.
- Do the data support the manufacturer's claim?
- Use a 0.05 level of significance.

p-value in Two Sided HT

- Solution:
- Recall that $\chi^2_{obs} = \frac{(9)(1.44)}{0.81} = 16.0$
- P-value = $P\left(\chi^2_{(v=9)} > 16.0\right) = 0.067$ *[= 1 – CHISQ.DIST(16;9;1)]*