Derece Temelli Topolojiksel İndeksler

Polynomial formda hesaplanabilen indeksler.

The Randic connectivity index: $R(G)=\sum_{uv\in E(G)}\left(\frac{1}{\sqrt{\deg_G(u)\deg_G(v)}}\right)$.

$\alpha=-\frac{1}{2}$

The general Randic connectivity index: $R_\alpha(G)=\sum_{uv\in E(G)}\left(\deg_G(u)\deg_G(v)\right)^\alpha$

The general sum-connectivity index : $X_\alpha(G)=\sum_{uv\in E(G)}\left(\deg_G(u)+\deg_G(v)\right)^\alpha$

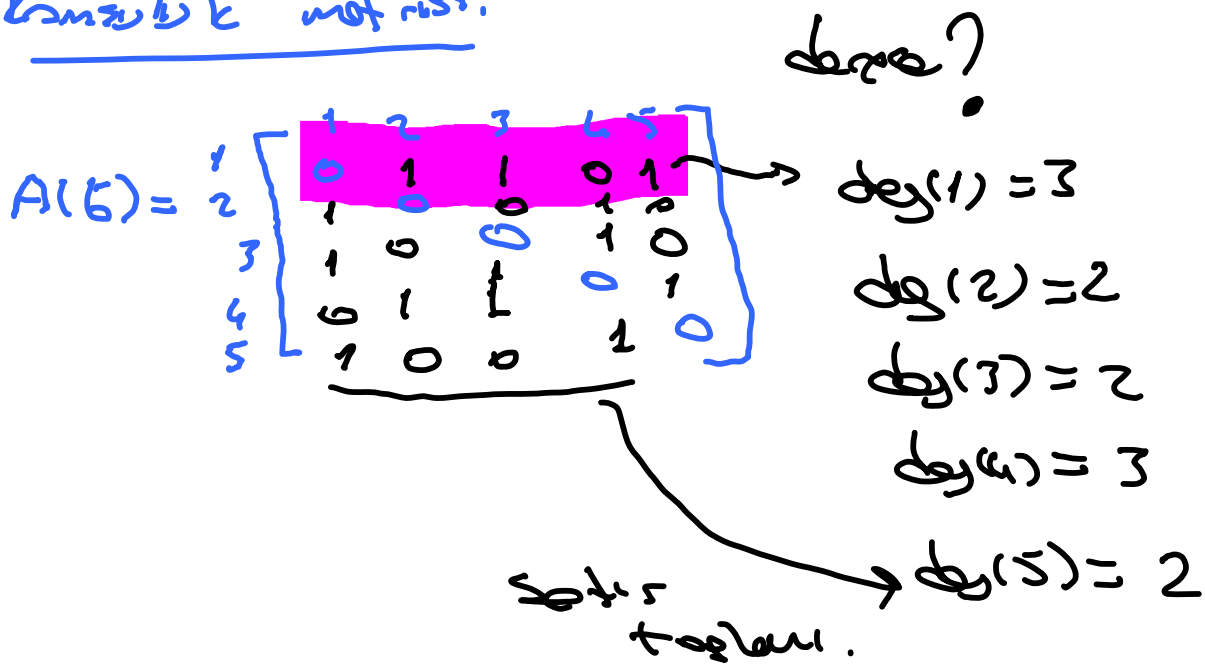
The *first Zagreb index*: $M_1(G)=\sum_{u\in V(G)}\left(\deg_G(u)\right)^2$

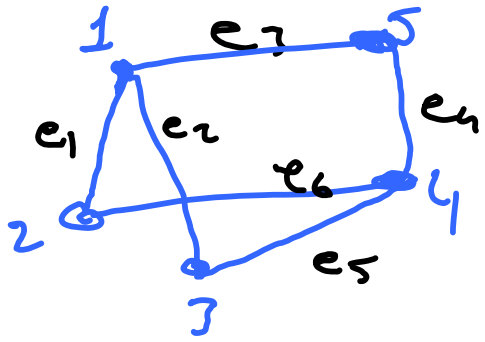
The *second Zagreb index*: $M_2(G)=\sum_{uv\in E(G)}\left(\deg_G(u)\deg_G(u)\right)$.

The *harmonic index*: $H(G)=\sum_{uv\in E(G)}\left(\frac{2}{\deg_G(u)+\deg_G(v)}\right)$.

The *geometric-arithmetic (GA) index*: $GA(G)=\sum_{uv\in E(G)}\left(\frac{2\sqrt{\deg_G(u)\deg_G(v)}}{\deg_G(u)+\deg_G(v)}\right)$.

Komşuluk matrisi:





$\Rightarrow$ G graf

Tepe	derece
1	3
2	2
3	2
4	3
5	2

1) Randic index:

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{deg(u) \cdot deg(u)}}$$

$$= \frac{1}{\sqrt{3 \cdot 2}} + \frac{1}{\sqrt{3 \cdot 2}} + \frac{1}{\sqrt{3 \cdot 2}} + \frac{1}{\sqrt{3 \cdot 2}} + \frac{1}{\sqrt{3 \cdot 2}} + \frac{1}{\sqrt{3 \cdot 2}}$$

$$= \frac{6}{\sqrt{6}} = \frac{6\sqrt{6}}{6} = \boxed{\sqrt{6}}$$

2) General Randic index:

$$R_\alpha(G) = \sum_{uv \in E(G)} (deg(u) \cdot deg(u))^{\alpha}$$

$$= 6 \cdot 6^{\alpha} = \boxed{6^{\alpha+1}}$$

$a = -\frac{1}{2} \Rightarrow$ Randic index $= 6^{-\frac{1}{2}+1} = 6^{\frac{1}{2}} = \sqrt{6}$

3) General topological connectivity.

$$\chi_\alpha(G) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))^\alpha$$

$$1 = 6 \cdot 5^\alpha$$

4) Zagreb index

$$M_1(G) = \sum_{v \in V(G)} (d_G(v))^2$$

$$M_1(G) = 3^2 + 2^2 + 2^2 + 3^2 + 2^2$$

$$= 18 + 12 = 30$$

5) İkinci Zagreb indeksi

$$M_2(G) = \sum_{uv \in E(G)} (d_G(u) \cdot d_G(v))$$

$$= 6 \cdot (2 \cdot 5) = 30$$

6) Harmonic indeksi

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_G(u) + d_G(v)}$$

$$H(G) = \left(\frac{2}{2+3}\right) \cdot 6 = \frac{2}{5} \cdot 6 = \boxed{\frac{12}{5}}$$

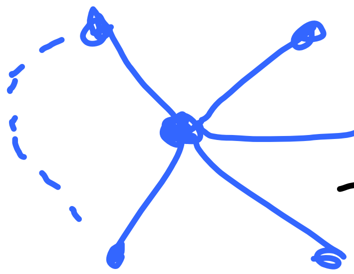
7) Geometric-Arithmetic Index

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_G(u) \cdot d_G(v)}}{d_G(u) + d_G(v)}$$

$$GA(G) = 6 \cdot \left(\frac{2\sqrt{2 \cdot 3}}{2+3} \right)$$

$$= 6 \cdot \frac{(2\sqrt{5})}{5} = \boxed{\frac{12}{5}\sqrt{5}}$$

$K_{1,n}$ yıldız grafının derece temelli topolojik indeks değerlerini elde ediniz.



$K_{1,n}$

→ 1 tane n dereceli
n tane 1 dereceli
tepe var.

→ n tane çizgi var.

uv tepeleri $\to 1$, $\to n$ dereceli

1) Randić indeksi

$$R(K_{1,n}) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\deg(u)\cdot\deg(v)}}$$

$$= n \cdot \left(\frac{1}{\sqrt{1 \cdot n}}\right) = \frac{n}{\sqrt{n}} = \sqrt{n}$$

General Randic index

$$R_\alpha(K_{1,n}) = \sum_{uv \in E(G)} [d_G(u) \cdot d_G(v)]^\alpha$$

$$= n \cdot (1 \cdot n)^\alpha = \boxed{n^{\alpha+1}}$$

$\alpha = -\frac{1}{2}$ için $n^{\frac{1}{2}} = \sqrt{n}$ 'dir.

3) General topological index

$$\chi_\alpha(K_{1,n}) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))^\alpha$$

$$= \boxed{n \cdot (1+n)^\alpha}$$

4) 1. Zagreb index

$$M_1(G) = \sum_{v \in V(G)} (d_G(v))^2$$

$$M_1(K_{1,n}) = 1 \cdot n^2 + n \cdot 1^2$$

$$= n^2 + n$$

5) 2. Zagreb index

$$M_2(G) = \sum_{uv \in E(G)} (d_G(u) \cdot d_G(v))$$

$$M_2(K_{1,n}) = n \cdot (n \cdot 1) = n^2$$

6) Harmonic index

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_G(u) + d_G(v)}$$

$$H(K_{1,n}) = n \cdot \frac{2}{n+1} = \boxed{\frac{2n}{n+1}}$$

7) Geometric-Arithmetic index

$$GA(G) = \sum_{uv \in E(G)} \left(\frac{2\sqrt{d_G(u) \cdot d_G(v)}}{d_G(u) + d_G(v)} \right)$$

$$GA(K_{1,n}) = n \cdot \left(\frac{2\sqrt{n \cdot 1}}{n+1} \right) = \boxed{\frac{2n\sqrt{n}}{n+1}}$$

* Topolojik indeksler

- polinom temelli hesaplanır.
- Özel tipte graf için indeks değeri formülü bulunabilir.

*) P_n, C_n, K_n ve $W_{1,n}$ grafların

derece temelli topolojik indeks değerleri hesaplayınız.