Optimization Objective

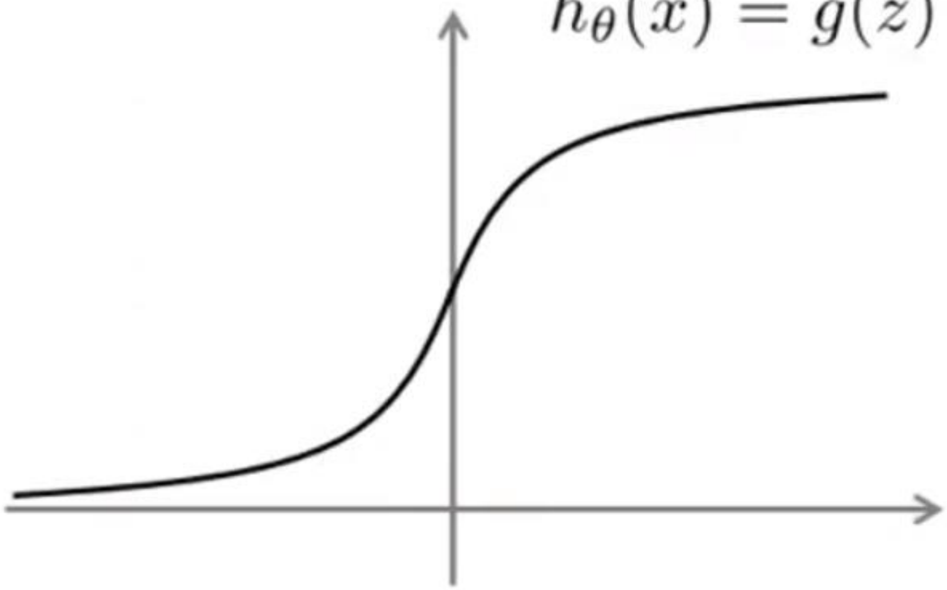
Large Margin Classification

Support Vector Machines

Alternative view of logistic regression

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

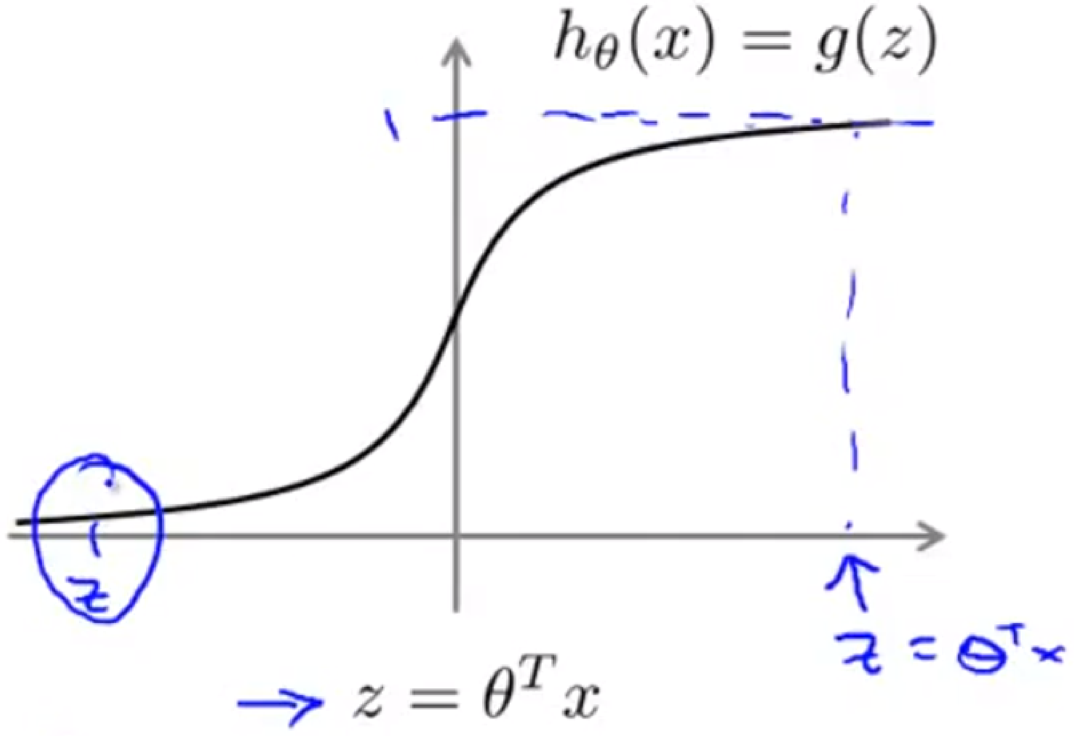
$$h_\theta(x) = g(z)$$



$$z = \theta^T x$$

Alternative view of logistic regression

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$



If $y = 1$, we want $h_\theta(x) \approx 1$, $\theta^T x \gg 0$

If $y = 0$, we want $h_\theta(x) \approx 0$, $\theta^T x \ll 0$

Alternative view of logistic regression

(x, y)

Cost of example: $-(y \log h_\theta(x) + (1-y)\log(1-h_\theta(x)))$ $\leftarrow$

$$= -y \log \frac{1}{1+e^{-\theta^T x}} - (1-y)\log(1 - \frac{1}{1+e^{-\theta^T x}})$$

(x, y)

Alternative view of logistic regression

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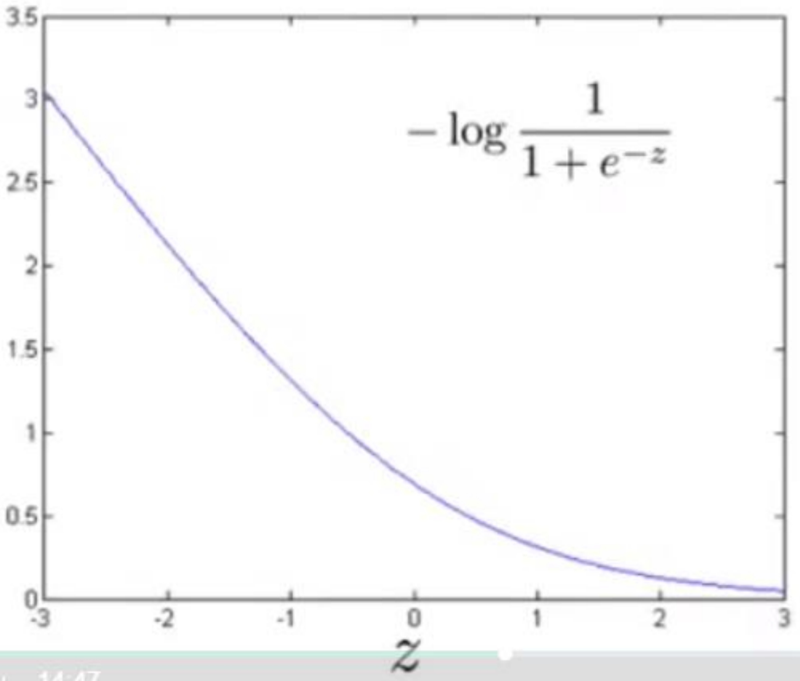
If $y = 1$ (want $\theta^T x \gg 0$):



Alternative view of logistic regression

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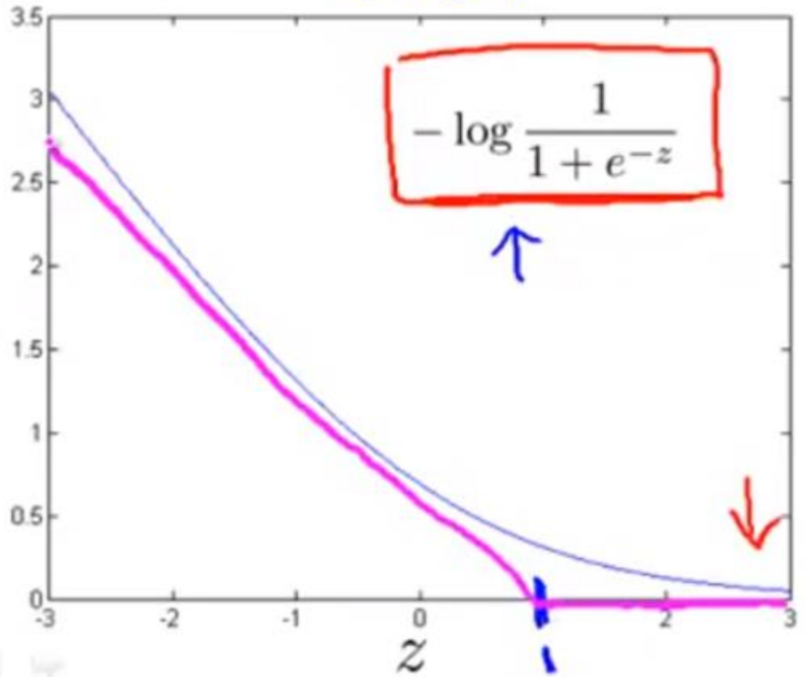

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Alternative view of logistic regression

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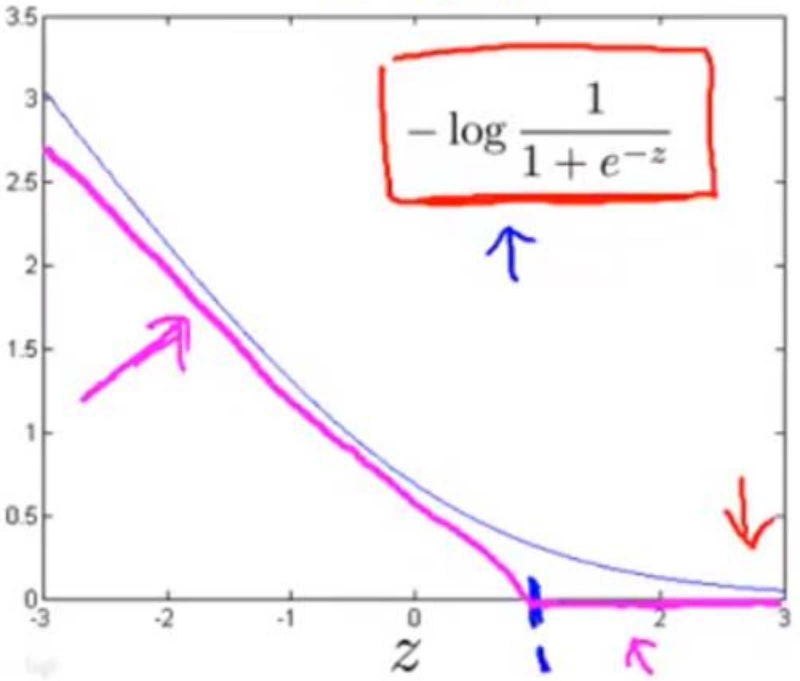
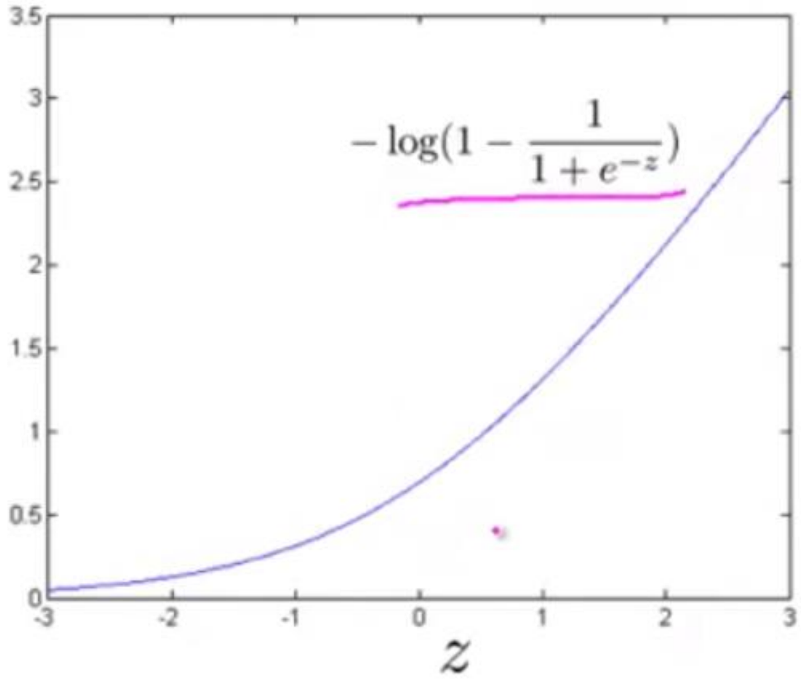


If $y = 0$ (want $\theta^T x \ll 0$):



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Alternative view of logistic regression

(x, y)

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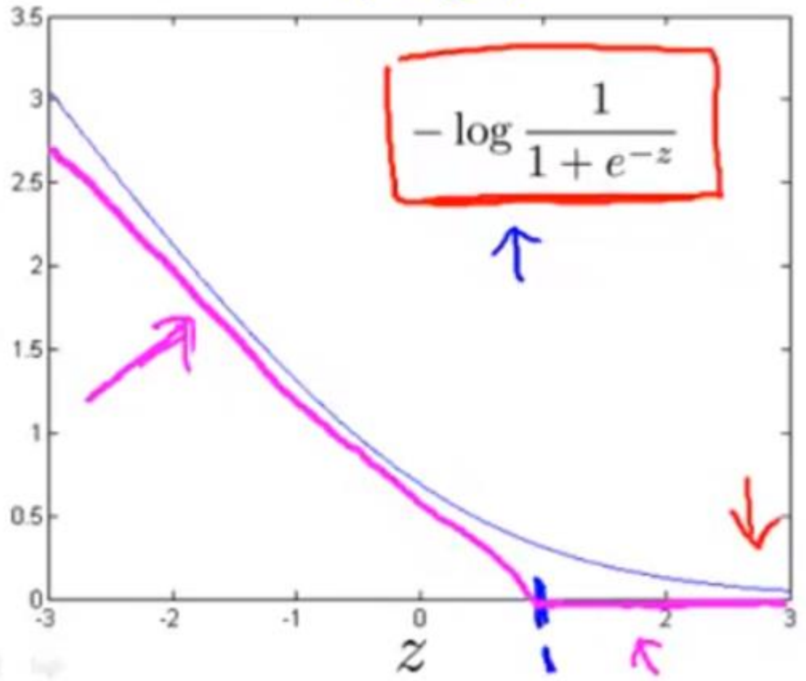

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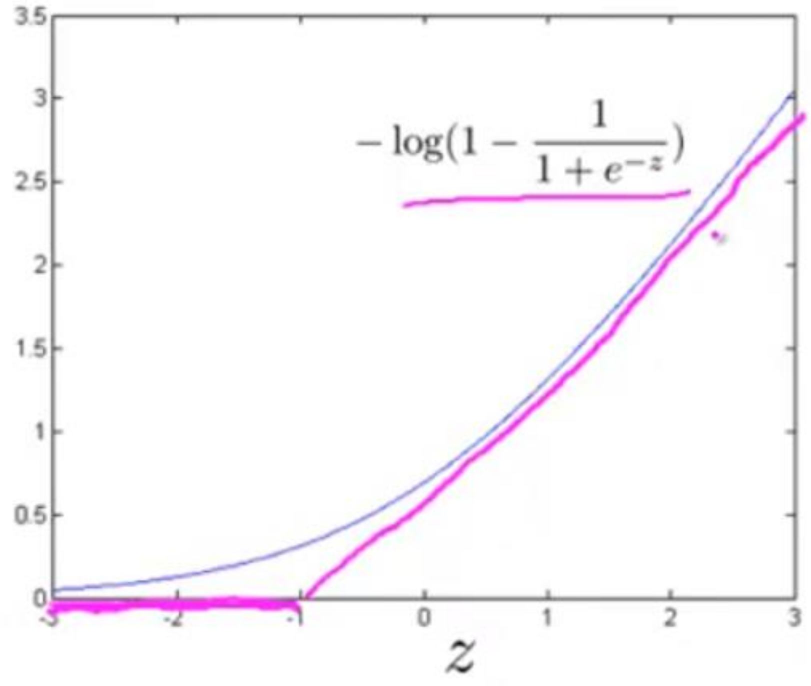

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Alternative view of logistic regression

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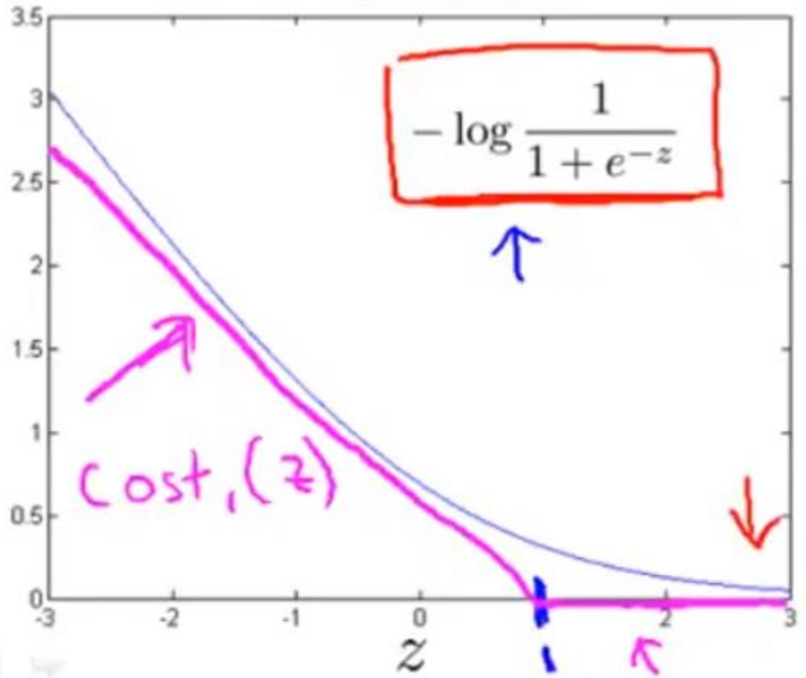

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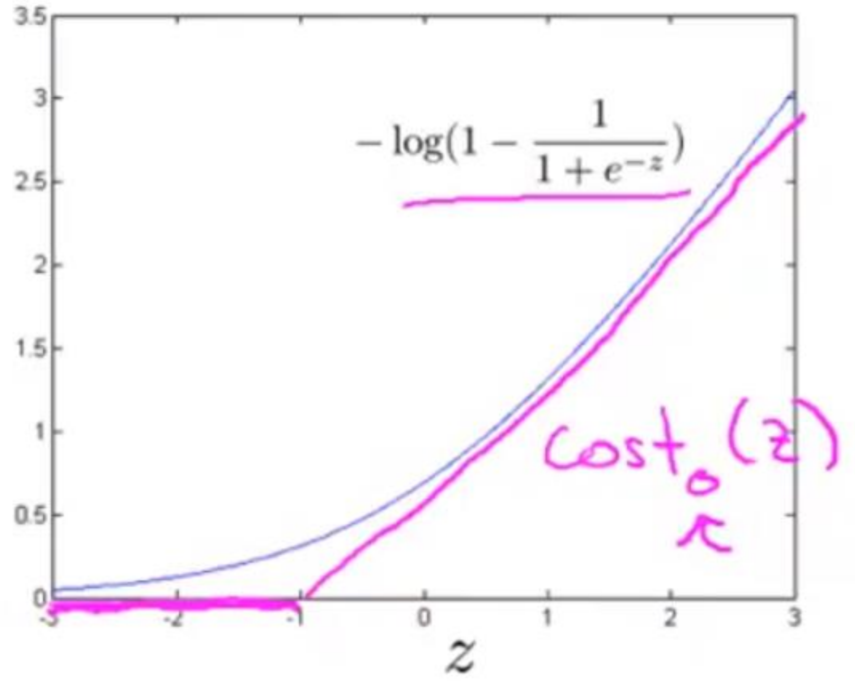

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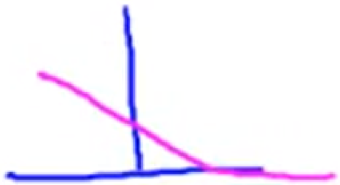
Support vector machine

Logistic regression:

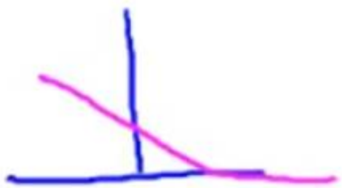


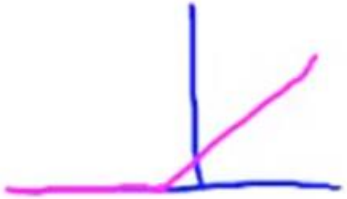
$$\min_\theta \frac{1}{m}\left[\sum_{i=1}^m y^{(i)}\underbrace{\left(-\log h_\theta(x^{(i)})\right)}_{\text{cost}_1(\theta^T x^{(i)})} + (1-y^{(i)})\underbrace{\left((-\log(1-h_\theta(x^{(i)}))\right)}_{\text{cost}_0(\theta^T x^{(i)})}\right] + \frac{\lambda}{2m}\sum_{j=1}^n \theta_j^2$$

Support vector machine:

Support vector machine

Logistic regression:





$$\min_\theta \frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \underbrace{\left(-\log h_\theta(x^{(i)})\right)}_{\mathrm{cost}_1(\theta^T x^{(i)})} + (1-y^{(i)}) \underbrace{\left((-\log(1-h_\theta(x^{(i)}))\right)}_{\mathrm{cost}_0(\theta^T x^{(i)})}\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

Support vector machine:

$$\min_\theta \frac{1}{m} \sum_{i=1}^{m} y^{(i)} \mathrm{cost}_1(\theta^T x^{(i)}) + (1-y^{(i)}) \mathrm{cost}_0(\theta^T x^{(i)}) + \frac{\lambda}{2m}\sum_{i=0}^{n}\theta_j^2$$

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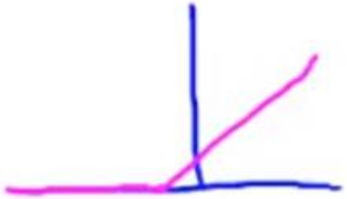
Support vector machine

Logistic regression:



$$\min_\theta \underbrace{\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\underbrace{\left(-\log h_\theta(x^{(i)})\right)}_{\text{cost}_1(\theta^T x^{(i)})} + (1-y^{(i)})\underbrace{\left((-\log(1-h_\theta(x^{(i)}))\right)}_{\text{cost}_0(\theta^T x^{(i)})}\right]}_{A} + \underbrace{\frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2}_{B}$$

Support vector machine:

$$\min_\theta \cancel{\frac{1}{m}}\, C\sum_{i=1}^{m} y^{(i)}\,\text{cost}_1(\theta^T x^{(i)}) + (1-y^{(i)})\,\text{cost}_0(\theta^T x^{(i)}) + \frac{1\cancel{\lambda}}{2\cancel{m}}\sum_{i=0}^{n}\theta_j^2$$

$$\min_u \left((u-5)^2+1\right) \times 10 \rightarrow u=5$$

$$\min_u 10(u-5)^2+10 \rightarrow u=5$$

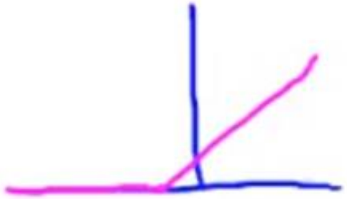
$$A+\lambda B$$

$$CA+B$$

$$C=\frac{1}{\lambda}$$

Support vector machine

Logistic regression:



$$\min_\theta \underbrace{\frac{1}{m}\left[\sum_{i=1}^m y^{(i)} \underbrace{\left(-\log h_\theta(x^{(i)})\right)}_{cost_1(\theta^T x^{(i)})} + (1-y^{(i)}) \underbrace{\left((-\log(1-h_\theta(x^{(i)}))\right)}_{cost_0(\theta^T x^{(i)})}\right]}_{A} + \underbrace{\frac{\lambda}{2m}\sum_{j=1}^n \theta_j^2}_{B}$$

Support vector machine:

$$\min_\theta \cancel{\frac{1}{m}}\, C \sum_{i=1}^m y^{(i)} cost_1(\theta^T x^{(i)}) + (1-y^{(i)}) cost_0(\theta^T x^{(i)}) + \frac{1\cancel{\lambda}}{2\cancel{m}} \sum_{i=0}^n \theta_j^2$$

$$\min_u \left((u-5)^2+1\right) \rightarrow u=5 \qquad \times 10$$

$$\min_u 10(u-5)^2+10 \rightarrow u=5$$

$$A + \lambda B$$

$$C A + B$$

$$C = \frac{1}{\lambda}$$

$$\rightarrow \min_\theta C \sum_{i=1}^m \left[y^{(i)} cost_1(\theta^T x^{(i)}) + (1-y^{(i)}) cost_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{i=1}^n \theta_j^2$$

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SVM hypothesis

$$\min_{\theta} C \sum_{i=1}^{m} \left[y^{(i)} cost_1(\theta^T x^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T x^{(i)}) \right] + \frac{1}{2} \sum_{i=1}^{n} \theta_j^2$$

Hypothesis:

$$h_\theta(x) \begin{cases} 1 & \text{if } \theta^T x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$