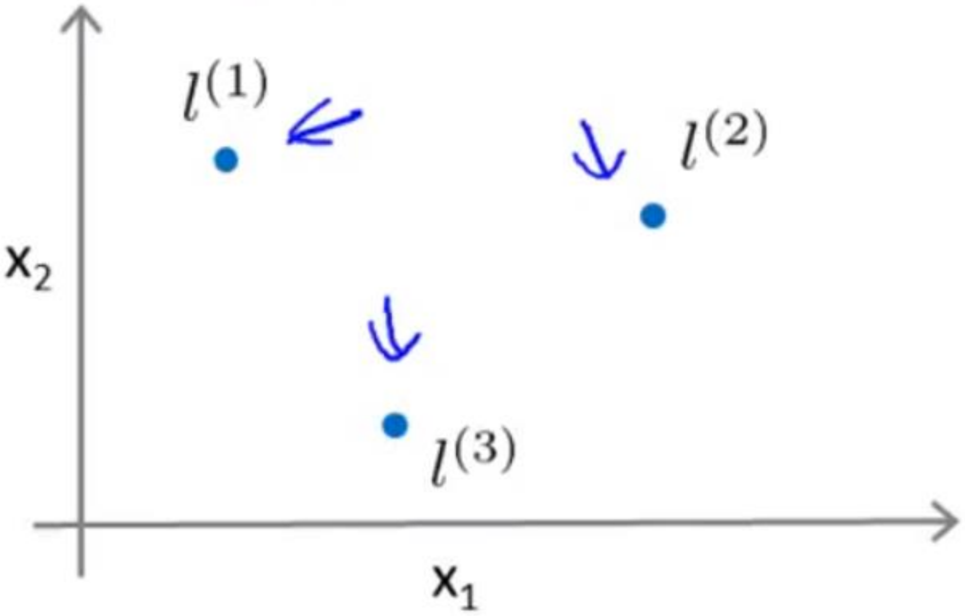
Kernels-2

Kernels

Support Vector Machines

Choosing the landmarks



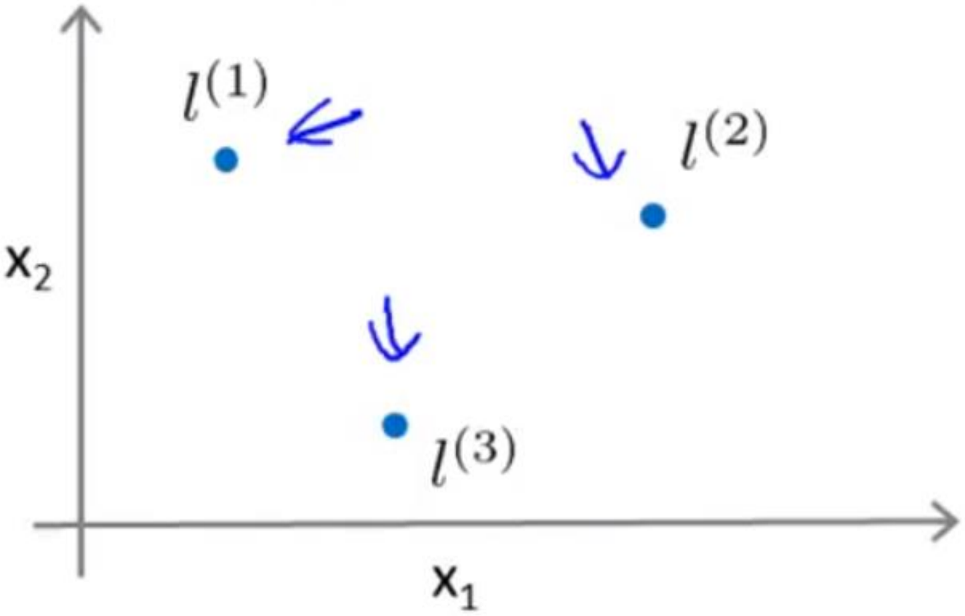
Given x:

$$f_i = \text{similarity}(x, l^{(i)})$$
$$= \exp\left(-\frac{\|x - l^{(i)}\|^2}{2\sigma^2}\right)$$

Predict $y = 1$ if $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 \geq 0$

Where to get $l^{(1)}, l^{(2)}, l^{(3)}, \ldots$?

Choosing the landmarks



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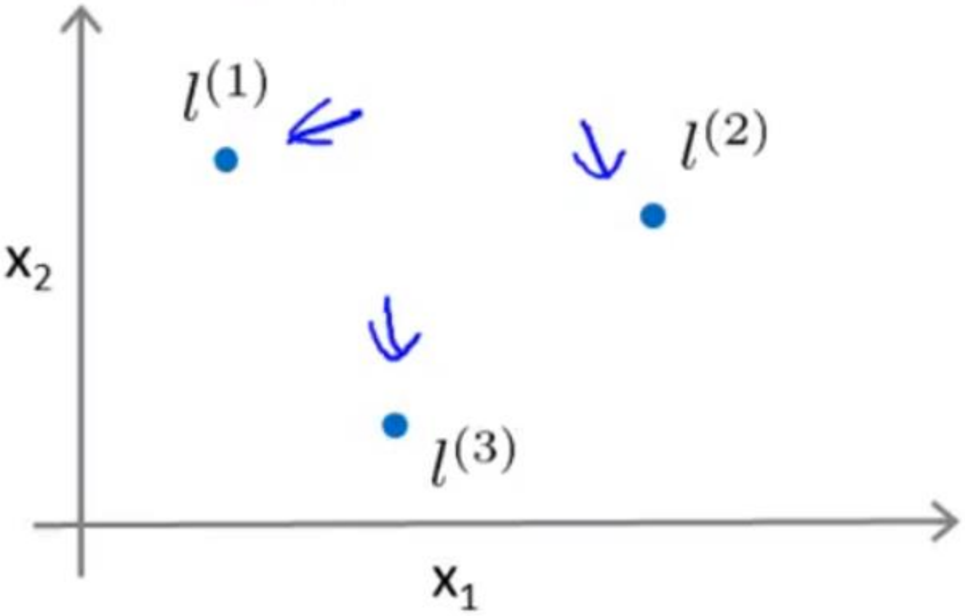

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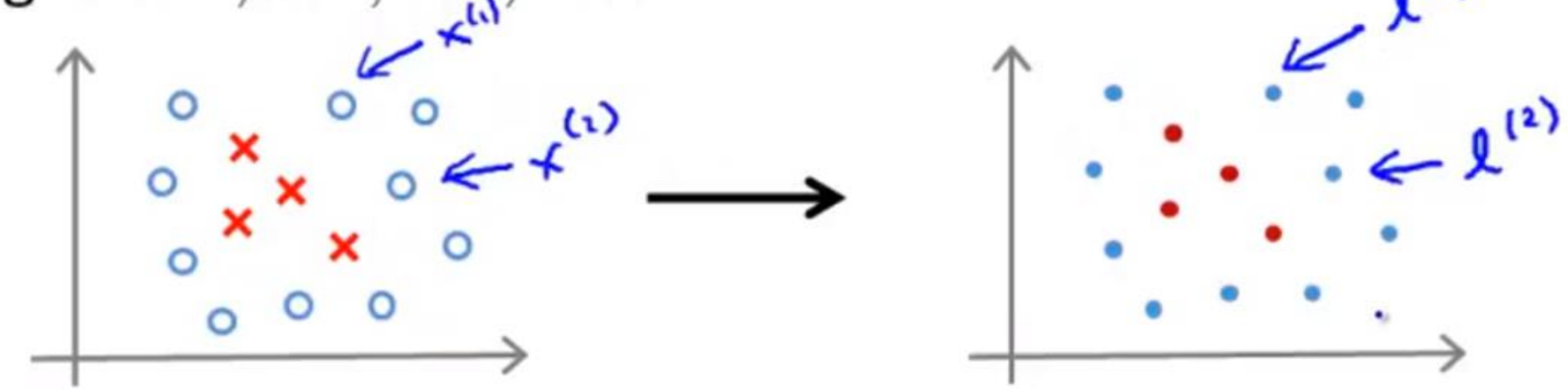

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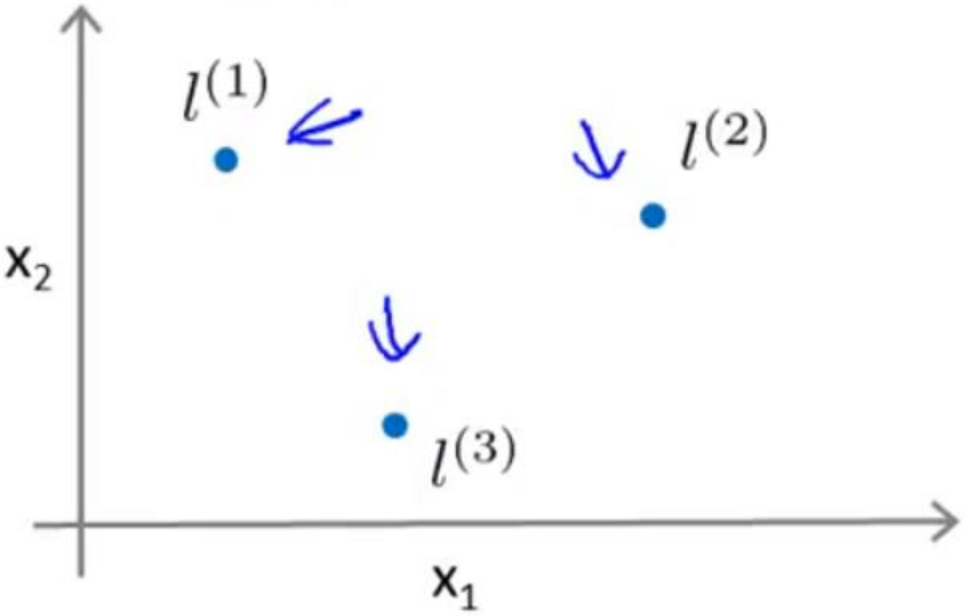

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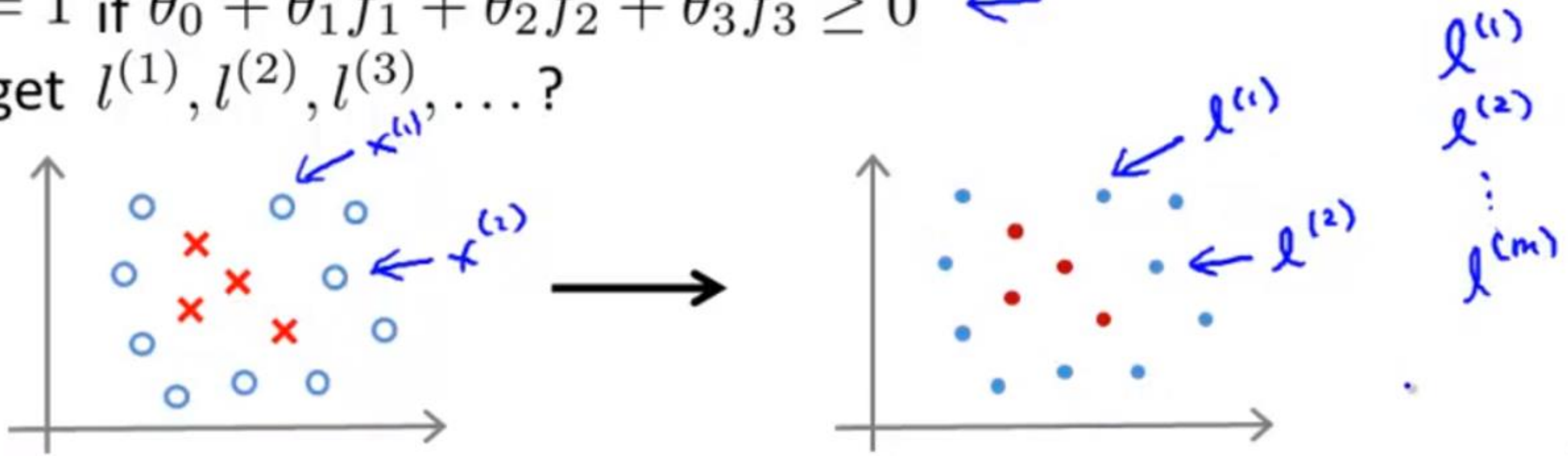

SVM with Kernels

- Given $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})$,
  choose $l^{(1)} = x^{(1)}, l^{(2)} = x^{(2)}, \ldots, l^{(m)} = x^{(m)}$.

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Given example x:

$$f_1 = \text{similarity}(x, l^{(1)})$$
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$$\ldots$$

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Given example $\underline{x}$:

$f_1 = \text{similarity}(x, l^{(1)})$ $\leftarrow x^{(1)}$

$f_2 = \text{similarity}(x, l^{(2)})$

$\ldots$

The vector f calculates the proximity of the point x with m different landmarks:

$$f = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \qquad f_0 = 1$$

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$x^{(1)}$

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$\boxed{\ldots}$

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For training example $(x^{(i)}, y^{(i)})$:

$x^{(i)} \rightarrow$

$f_1^{(i)} = \text{sim}(x^{(i)}, l^{(1)})$

$f_2^{(i)} = \text{sim}(x^{(i)}, l^{(2)})$

$x^{(i)}$

$\vdots \leftarrow f_i^{(i)} = \text{sim}(x^{(i)}, l^{(i)}) = \exp(-\frac{0}{2\sigma^2}) = 1$

$f_m^{(i)} = \text{sim}(x^{(i)}, l^{(m)})$

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$f_m^{(i)} = \text{sim}(x^{(i)}, l^{(m)})$

$x^{(i)} \in \mathbb{R}^{n+1}$ (or $\mathbb{R}^n$)

$$\rightarrow f^{(i)} = \begin{bmatrix} f_0^{(i)} \\ f_1^{(i)} \\ \vdots \\ f_m^{(i)} \end{bmatrix}$$

$f_0^{(i)} = 1$

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Hypothesis: Given x, compute features $f \in \mathbb{R}^{m+1}$

Predict "y=1" if $\theta^T f \geq 0$

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Training:

$$\min_\theta C \sum_{i=1}^{m} y^{(i)} cost_1(\theta^T f^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T f^{(i)}) + \frac{1}{2} \sum_{j=1}^{n} \theta_j^2$$

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$\cancel{\Theta^T x^{(i)}}$ $\quad \Theta^T f^{(i)}$

WE used to make predictions by looking at an x.
Now, we calculate f^i's for each x and then predict using the f^i's.

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Training:

$n = m$

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$\cancel{n} = m$

$\cancel{\theta^T x^{(i)}}$ $\theta^T f^{(i)}$

→ θ_0

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$\cancel{\theta^T x^{(i)}}$ $\theta^T f^{(i)}$ $\rightarrow \theta_0$

$$- \sum_j \theta_j^2 = \theta^T \theta \leftarrow \|\theta\|^2 \qquad \theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_m \end{bmatrix} \quad (\text{ignore } \theta_0)$$

$$- \quad \theta^T M \theta$$

SVM parameters:

C ($= \frac{1}{\lambda}$). → Large C: Lower bias, high variance. (small λ)

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σ^2 Large σ^2: Features f_i vary more smoothly.

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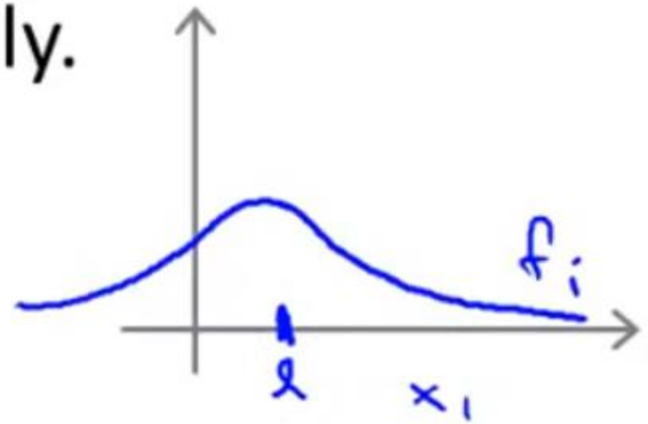
$\exp\left(-\frac{\|x-l^{(i)}\|^2}{2\sigma^2}\right)$



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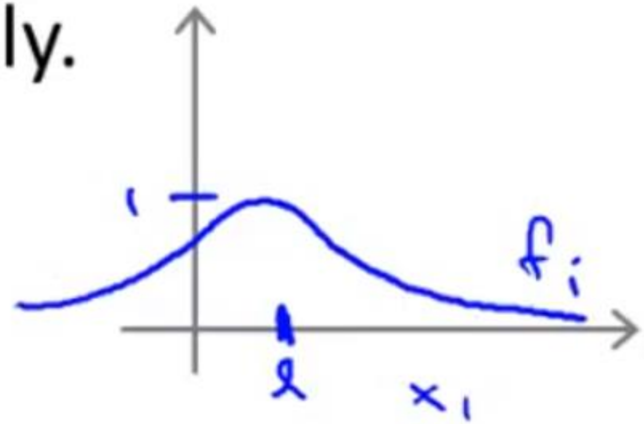

Small σ^2: Features f_i vary less smoothly.

Lower bias, higher variance.