# DECISION MAKING TECHNIQUES IN MANAGEMENT INFORMATION SYSTEMS (MIS)

LECTURE -4-
(Group Decision Making)

## SOCIAL CHOICE THEORY

- Voting
- Social Choice Function

## VOTİNG METHODS

- Nonranked Voting System
- Preferential Voting System

## NONRANKED VOTİNG SYSTEM

- One member elected from two candidates
- One member elected from many candidates
- Election of two or more members

## One member elected from two candidates

- Election by simple majority

  Each voter can vote for one candidate

  The candidate with the greater vote total wins the election

## One member elected from many candidates

- The first-past-the-post system
  - Election by simple majority
- Majority representation system
  - Repeated ballots

    Voting goes on through a series of ballots until some candidate obtains an absolute majority of the votes cast
  - The second ballot

    On the first ballot a candidate can't be elected unless he obtains an absolute majority of the votes cast

    The second ballot is a simple plurality ballot involving the two candidates who had been highest in the first ballot

## Election of two or more members

- The single non-transferable vote
  - Each voter has one vote
- Multiple vote
  - Each voter has as many votes as the number of seats to be filled
  - Voters can't cast more than one vote for each candidate
- Limited vote
  - Each voter has a number of votes smaller than the number of seats to be filled
  - Voters can't cast more than one vote for each candidate

## Election of two or more members *cont.*

- Cumulative vote
  - Each voter has as many votes as the number of seats to be filled
  - Voters can cast more than one vote for candidates
- List systems
  - Voter chooses between lists of candidates
    - Highest average (d'Hondt's rule)
    - Greatest remainder

## Election of two or more members *cont.*

- Approval voting

  Each voter can vote for as many candidates as he/she wishes

  Voters can't cast more than one vote for each candidate

## EXAMPLE

- Suppose an constituency in which 200,000 votes are cast for four party lists contesting five seats
- Suppose the distribution of votes is:

| | |
|---|---|
| A | 86,000 |
| B | 56,000 |
| C | 38,000 |
| D | 20,000 |

## Solution with "Highest average" method

- The seats are allocated one by one and each goes to the list which would have the highest average number of votes
- At each allocation, each list's original total of votes is divided **by one more than** the number of seats that list has already won in order to find what its average would be

/2 /3

| | | | | | | |
|---|---|---|---|---|---|---|
| A | **86,000** | 43,000 | **43,000** | 28,667 | **28,667** | **3** |
| B | 56,000 | **56,000** | 28,000 | 28,000 | 28,000 | **1** |
| C | 38,000 | 38,000 | 38,000 | **38,000** | 19,000 | **1** |
| D | 20,000 | 20,000 | 20,000 | 20,000 | 20,000 | **0** |

## Solution with "Greatest remainder" method

- An electoral quotient is calculated by dividing total votes by the number of seats
- Each list's total of votes is divided by the quotient and each list is given as many seats as its poll contains the quotient.
- If any seats remain, these are allocated successively between the competing lists according to the sizes of the remainder

200,000 / 5 = 40,000

| List | Votes | Seats | Remainder | Seats |
|---|---|---|---|---|
| A | 86.000 | 2 | 6.000 | **2** |
| B | 56.000 | 1 | 16.000 | **1** |
| C | 38.000 | 0 | 38.000 | **1** |
| D | 20.000 | 0 | 20.000 | **1** |

## Disadvantages of Nonranked Voting

- Nonranked voting systems arise serious questions as to whether these are fair and proper representations of the voters' will
- Extraordinary injustices may result unless preferential voting systems are used
- Contradictions (3 cases of Dodgson)

## Case 1 of Dodgson

- Contradiction in simple majority: Candidate A and B

| Order of | Voters | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| preference | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 | V9 | V10 | V11 |
| 1 | A | A | A | B | B | B | B | C | C | C | D |
| 2 | C | C | C | A | A | A | A | A | A | A | A |
| 3 | D | D | D | C | C | C | C | D | D | D | C |
| 4 | B | B | B | D | D | D | D | B | B | B | B |

## Case 2 of Dodgson

- Contradiction in absolute majority: Candidate A and B

| Order of | Voters | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Preference | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 | V9 | V10 | V11 |
| 1 | B | B | B | B | B | B | A | A | A | A | A |
| 2 | A | A | A | A | A | A | C | C | C | D | D |
| 3 | C | C | C | D | D | D | D | D | D | C | C |
| 4 | D | D | D | C | C | C | B | B | B | B | B |

## Case 3 of Dodgson

- Contradiction in second ballot: Elimination of A

| Order of | Voters | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Preference | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 | V9 | V10 | V11 |
| 1 | B | B | B | C | C | C | C | D | D | A | A |
| 2 | A | A | A | A | A | A | A | A | A | B | D |
| 3 | D | C | D | B | B | B | D | C | B | D | C |
| 4 | C | D | C | D | D | D | B | B | C | C | B |

# Preferential Voting System

The voter places 1 on the ballot paper against the name of the candidate whom he considers most suitable

He/she places a figure 2 against the name of his second choice, and so on…

The votes are counted and the individual preferences are aggregated with the principle of simple majority rule

- Strict Simple Majority $xPy: \#(i:xP_iy) > \#(i:yP_ix)$
- Weak Simple Majority $xRy: \#(i:xP_iy) \geq \#(i:yP_ix)$
- Tie $xIy: \#(i:xP_iy) = \#(i:yP_ix)$

# EXAMPLE

- Suppose the 100 voters' preferential judgments are as follows:

| | |
|---|---|
| 38 votes: | a P c P b |
| 32 votes: | b P c P a |
| 27 votes: | c P b P a |
| 3 votes: | c P a P b |

- All candidates are compared two by two:

$$\left.\begin{array}{l}\text{a P b: 41 votes; b P a } \mathbf{59} \text{ votes}\\ \text{a P c: 38 votes; c P a } \mathbf{62} \text{ votes}\\ \text{b P c: 32 votes; c P b } \mathbf{68} \text{ votes}\end{array}\right\} \textbf{c} \text{ P b P a}$$

## Advantages of Preferential Voting

- If nonranked voting is utilized for the previous example:

Simple Majority

38 votes: a P c P b
32 votes: b P c P a
27 votes: c P b P a
3 votes: c P a P b

**a: 38 votes**
b: 32 votes
c: 27+3=30 votes

Second ballot

Absolute majority is 51 votes: **c** is eliminated
The second ballot is a simple plurality ballot
(Suppose preferential ranks are not changed)

a: 41 votes
**b: 59 votes**

## Disadvantages of Preferential Voting

- Committee would have a circular preference among the alternatives: would not be able to arrive at a transitive ranking

23 votes: a P b P c
17 votes: b P c P a
2 votes: b P a P c
10 votes: c P a P b
8 votes: c P b P a

b P c (42>18), c P a (35>25), a P b (33>27)

$\Rightarrow$ Intransitivity (paradox of voting)

## Disadvantages of Preferential Voting *cont.*

- Aggregate judgments can be incompatible

| Voters | Order of preference 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| V1 | A | B | C | D |
| V2 | D | A | B | C |
| V3 | B | C | D | A |

| | | | Winner |
|---|---|---|---|
| B*P* D | A*P* B | A*P* C | A |
| D*P* A | B*P* D | B*P* C | B |
| A*P* B | D*P* A | C*P* D | C |
| A*P* B | A*P* C | D*P* A | D |

## Social Choice Functions

- Condorcet's
- Borda's
- Copeland's
- Nanson's
- Dodgson's
- Eigenvector

# EXAMPLE

- Suppose the 100 voters' preferential judgments are as follows:

  38 votes: 'a P b P c'

  28 votes: 'b P c P a'

  17 votes: 'c P a P b'

  14 votes: 'c P b P a'

  3 votes: 'b P a P c'

# CONDERCET'S FUNCTION

- The candidates are ranked in the order of the values of $f_C$

  $f_C(x) = \min_{y \in A \setminus \{x\}} \#(i: x\ P_i\ y)$

  'a P b' 55 votes & 'b P a' 45 votes
  'a P c' 41 votes & 'c P a' 59 votes
  'b P c' 69 votes & 'c P b' 31 votes

| | a | b | c | $f_C$ |
|---|---|---|---|---|
| a | - | 55 | 41 | 41 |
| b | 45 | - | 69 | 45 |
| c | 59 | 31 | - | 31 |

b P a P c

# Borda's Function

- The candidates are ranked in the order of the values of $f_B$

  $f_B(x) = \sum_{y \in A \setminus \{x\}} \#(i;\ x\ P_i\ y)$

| | a | b | c | $f_B$ |
|---|---|---|---|---|
| a | - | 55 | 41 | 96 |
| b | 45 | - | 69 | 114 |
| c | 59 | 31 | - | 90 |

**b** P a P c

# Borda's Function (alternative approach)

A rank order method is used.

- With m candidates competing, assign marks of m–1, m–2, ..., 1, 0 to the first ranked, second ranked, ..., last ranked but one, last ranked candidate for each voter.
- Determine the Borda score for each candidate as the sum of the voter marks for that candidate

a: 2 * 38 + 0 * 28 + 1 * 17 + 0 * 14 + 1 * 3 = 96

b: 2 * ( 28 + 3 ) + 1 * ( 38 + 14 ) + 0 * 17 = 114

c: 2 * ( 17 + 14 ) + 1 * 28 + 0 * ( 38 + 3 ) = 90

# Copeland's Function

- The candidates are ranked in the order of values of $f_{CP}$
- $f_{CP}(x)$ is the number of candidates in A that x has a strict simple majority over, minus the number of candidates in A that have strict simple majorities over x

$f_{CP}(x) = \#(y: y \in A \wedge x\ P\ y) - \#(y: y \in A \wedge y\ P\ x)$

$\#(i: a\ P_i\ b) = 55 > \#(i: b\ P_i\ a) = 45 \Rightarrow$ 'a P b'

$\#(i: a\ P_i\ c) = 41 < \#(i: c\ P_i\ a) = 59 \Rightarrow$ 'c P a'

$\#(i: b\ P_i\ c) = 69 > \#(i: c\ P_i\ b) = 31 \Rightarrow$ 'b P c'

$f_{CP}(a) = 1 - 1 = 0$; $f_{CP}(b) = 1 - 1 = 0$; $f_{CP}(c) = 1 - 1 = 0$

- Indifference among three candidates

# Copeland's Function (another example)

38 votes: a P c P b

32 votes: b P c P a

27 votes: c P b P a

3 votes: c P a P b

Judgments of simple majority: 'b P a', 'c P a' and 'c P b'

→ $f_{CP}(a) = 0 - 2 = -2$; $f_{CP}(b) = 1 - 1 = 0$; $f_{CP}(c) = 2 - 0 = 2$

The ranking of alternatives: 'c P b P a'

# Nanson's Function

Let $A_1 = A$ and for each $j \geq 1$ let

$A_{j+1} = A_j \setminus \{x \in A_j: f_B(x) \leq f_B(y)$ for all $y \in A_j$,

and $f_B(x) < f_B(y)$ for some $y \in A_j\}$

where $f_B(x)$ is the Borda score

Then $f_N(x) = \lim_{j\to\infty} A_j$ gives the winning candidate

$A_1 = A = \{a, b, c\}$

$f_B(a) = 96$

$f_B(b) = 114$

$f_B(c) = 90$

Candidate c is eliminated as s/he has the lowest score:

$A_2 = \{a, b\}$

38 votes: 'a P b'

28 votes: 'b P a'

17 votes: 'a P b'

14 votes: 'b P a'

3 votes: 'b P a'

$f_B(a) = 55$

$f_B(b) = 45$

Candidate b is eliminated and candidate a is the winner:

**a** P b P c

# Dodgson's Function

- Based on the idea that the candidates are scored on the basis of the smallest number of changes needed in voters' preference orders to create a simple majority winner (or nonloser).

| | a | b | c | change |
|---|---|---|---|---|
| a | - | 55/45 | 41/59 | 9 |
| b | 45/55 | - | 69/31 | 5 |
| c | 59/41 | 31/69 | - | 19 |

**b** P a P c

# Eigenvector Function

- The preferential judgment of each decision maker is assessed by asking them to make pairwise comparisons of the candidates
- Geometric means of the entries in each matrix are calculated and an aggregated pairwise comparison matrix is revealed
- The ranking of candidates is based on the eigenvector values of this matrix.

## REFERENCES

- Lecture notes of “Prof. Dr. Y. İlker Topçu”, http://web.itu.edu.tr/topcuil/