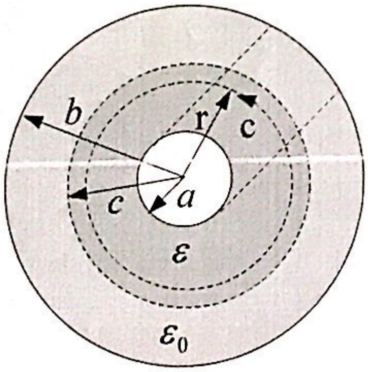
EEE 210 ELECTROMAGNETIC FIELDS THEORY

FINAL EXAMINATION

SOLUTIONS

1.



Applying Gauss's law on closed surface s

$$\oint_s \vec{D}\bullet\vec{ds} = Q\,; \quad D(2\pi r L) = Q \;\Rightarrow \qquad \vec{D} = \frac{Q}{2\pi r L}\hat{a}_r \quad \textbf{(5)}$$

In the region $c > r > a$

$$\vec{D} = \varepsilon\vec{E_1}\,; \qquad \vec{E_1} = \frac{\vec{D}}{\varepsilon} = \frac{Q}{2\pi\varepsilon r L}\hat{a}_r \quad \textbf{(5)}$$

In the region $b > r > c$

$$\vec{D} = \varepsilon_0\vec{E_2}\,; \qquad \vec{E_2} = \frac{\vec{D}}{\varepsilon_0} = \frac{Q}{2\pi\varepsilon_0 r L}\hat{a}_r \quad \textbf{(5)}$$

The potential differences between the conductors is

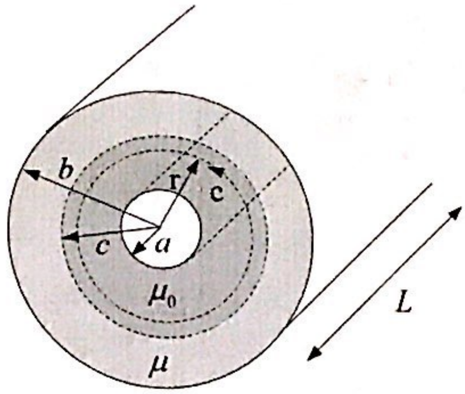
$$V = -\int_b^c \vec{E_2}\bullet\vec{dl} - \int_c^a \vec{E_1}\bullet\vec{dl} = -\frac{Q}{2\pi L}\left(\frac{1}{\varepsilon_0}\int_b^c \frac{dr}{r} + \frac{1}{\varepsilon}\int_c^a \frac{dr}{r}\right) \quad \textbf{(5)}$$

$$V = \frac{Q}{2\pi L}\left(\frac{1}{\varepsilon_0}\ln\frac{b}{c} + \frac{1}{\varepsilon}\ln\frac{c}{a}\right) \quad \textbf{(5)}$$

and the capacitance is

$$C = \frac{Q}{V} = \frac{2\pi L}{\frac{1}{\varepsilon_0}\ln\frac{b}{c} + \frac{1}{\varepsilon}\ln\frac{c}{a}} \qquad \text{and} \qquad \frac{C}{L} = \frac{2\pi\varepsilon_0\varepsilon}{\varepsilon\ln\frac{b}{c} + \varepsilon_0\ln\frac{c}{a}} \quad \textbf{(5)}$$

2.



Applying Ampere's law around the closed contour c;

$$\oint_c \vec{H}\cdot\vec{dl} = I; \quad H(2\pi r) = I \quad \Rightarrow \quad \vec{H} = \frac{I}{2\pi r}\hat{a}_\varphi \quad (5)$$

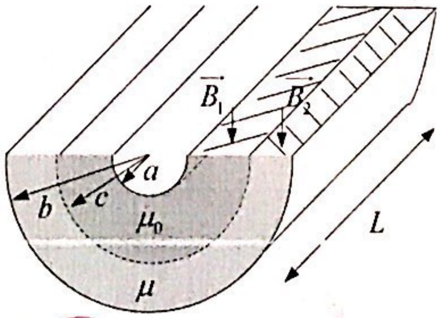
In the region $c > r > a$

$$\vec{B_1} = \mu_0 \vec{H}; \quad \vec{B_1} = \frac{\mu_0 I}{2\pi r}\hat{a}_\varphi \quad (5)$$

In the region $b > r > c$

$$\vec{B_2} = \mu \vec{H}; \quad \vec{B_2} = \frac{\mu I}{2\pi r}\hat{a}_\varphi \quad (5)$$

Total flux between the conductors is evaluated in the area seen below:



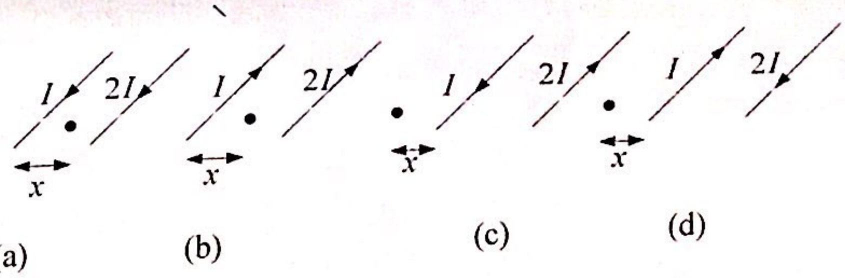
$$\phi = \int_0^L \int_a^c \vec{B_1}\cdot\vec{ds} + \int_0^L \int_c^b \vec{B_2}\cdot\vec{ds}; \quad (5) \qquad \phi = \frac{\mu_0 I}{2\pi}\int_0^L \int_a^c \frac{dr}{r} dz + \frac{\mu I}{2\pi}\int_0^L \int_c^b \frac{dr}{r} dz$$

$$\phi = \frac{\mu_0 I}{2\pi} L \ln\frac{c}{a} + \frac{\mu I}{2\pi} L \ln\frac{b}{c} = \frac{IL}{2\pi}\left(\mu_0 \ln\frac{c}{a} + \mu \ln\frac{b}{c}\right) \quad (5)$$

The self inductance is

$$L_{11} = \frac{\phi}{I} = \frac{L}{2\pi}\left(\mu_0 \ln\frac{c}{a} + \mu \ln\frac{b}{c}\right) \quad \text{and} \quad \frac{L_{11}}{L} = \frac{\phi}{I} = \frac{1}{2\pi}\left(\mu_0 \ln\frac{c}{a} + \mu \ln\frac{b}{c}\right) \quad (5)$$

3. Possible current directions and corresponding locations of point P for each case:



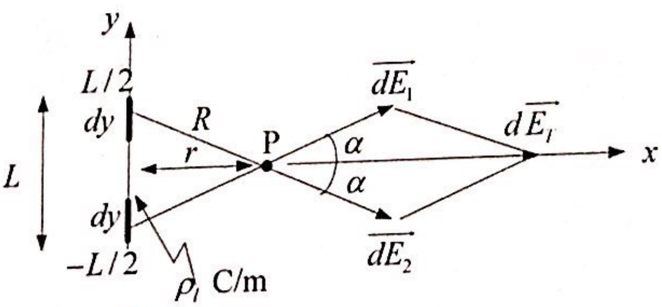
a) $\left|\dfrac{\mu_0 I}{2\pi x}\right| = \left|\dfrac{\mu_0 (2I)}{2\pi(2d-x)}\right|$; $\quad \dfrac{1}{x} = \dfrac{2}{2d-x} \quad \Rightarrow \quad 2x = 2d - x$; $\quad x = \dfrac{2d}{3}$ **(10)**

b) same with (a) **(5)**

c) $\left|\dfrac{\mu_0 I}{2\pi x}\right| = \left|\dfrac{\mu_0 (2I)}{2\pi(2d+x)}\right|$; $\quad \dfrac{1}{|x|} = \dfrac{2}{2d+|x|} \quad \Rightarrow \quad x = -2d$ **(10)**

d) same with (c) **(5)**

4. Consider the following geometry and find $\vec{E}$ at point P:



$$\left|d\vec{E_1}\right| = \left|d\vec{E_2}\right| = \frac{\rho_l dy}{4\pi\varepsilon_0 R^2} \ \textbf{(3)}$$

$$d\vec{E_T} = 2\frac{\rho_l dy}{4\pi\varepsilon_0 R^2}\cos\alpha \ \hat{a}_x \ \textbf{(3)}$$

$$\tan\alpha = \frac{y}{r} \quad \Rightarrow \quad dy = \frac{r d\alpha}{\cos^2\alpha} \ \textbf{(3)}; \qquad R\cos\alpha = r \quad \Rightarrow \quad R = \frac{r}{\cos\alpha} \ \textbf{(3)}$$

$$d\vec{E_T} = 2\frac{\rho_l\left(\dfrac{r d\alpha}{\cos^2\alpha}\right)}{4\pi\varepsilon_0\left(\dfrac{r}{\cos\alpha}\right)^2}\cos\alpha \ \hat{a}_x; \qquad d\vec{E_T} = \frac{\rho_l d\alpha}{2\pi\varepsilon_0 r}\cos\alpha \ \hat{a}_x \ \textbf{(3)}$$

$$\overrightarrow{E_T} = \frac{\rho_l}{2\pi\varepsilon_0 r}(\sin\alpha)\Big|_0^{\frac{L/2}{\sqrt{r^2+(L/2)^2}}}\,\hat{a}_x \quad (3)$$

$$\overrightarrow{E_T} = \frac{\rho_l}{2\pi\varepsilon_0 r}\frac{L/2}{\sqrt{r^2+(L/2)^2}}\hat{a}_x = \frac{\rho_l}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{4r^2+L^2}}\hat{a}_x \quad (3)$$

So, for the vertical charge distribution $r = d$, $L = d$

$$\overrightarrow{E_1} = \frac{\rho_l}{2\sqrt{5}\pi\varepsilon_0 d}\hat{a}_x \quad (3)$$

For the horizontal charge distribution $r = d/2$, $L = 2d$

$$\overrightarrow{E_2} = \frac{\rho_l(2d)}{2\pi\varepsilon_0 d/2}\frac{1}{\sqrt{4d^2+d^2}}\hat{a}_y = \frac{2\rho_l}{\sqrt{5}\pi\varepsilon_0 d}\hat{a}_y \quad (3)$$

and the total field is

$$\overrightarrow{E_T} = \overrightarrow{E_1} + \overrightarrow{E_2} = \frac{\rho_l}{\sqrt{5}\pi\varepsilon_0 d}\left(0.5\hat{a}_x + 2\hat{a}_y\right) \quad (3)$$