Chapter 9-10

Confidence Intervals and Hypothesis Testing

One Sided HT for μ and σ^2

Statistics

Mehmet Güray Güler, PhD

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One Sided HT For μ

One Sided HT…

- Until now we deal with ''not equal to'' type of hypothesis like:
  - $H_0: \mu = 5$
  - $H_1: \mu \neq 5$
- We may also have following type of hypothesis:
  - $H_0: \mu \leq 5$ (or $\mu = 5$)
  - $H_1: \mu > 5$
- Or
  - $H_0: \mu \geq 5$ (or $\mu = 5$)
  - $H_1: \mu < 5$
- In general we use the alternative hypothesis to justify our beliefs.
- Test whether the **evidence** is sufficient to suspect the hypothesis.

One sided HT for μ [σ is known]

- **The very very same example☺**
- A random sample of 100 recorded deaths in Turkey during the past year showed an average life span of 71.8 years.
- We want to test the following hypothesis
  - $\mathbf{H_0}: \boldsymbol{\mu \leq 70}$.
  - $\mathbf{H_1}: \boldsymbol{\mu > 70}$.
- Assuming $\sigma = 9$, please perform HT.
- Use a 0.05 level of significance

One sided HT for μ [σ is known]

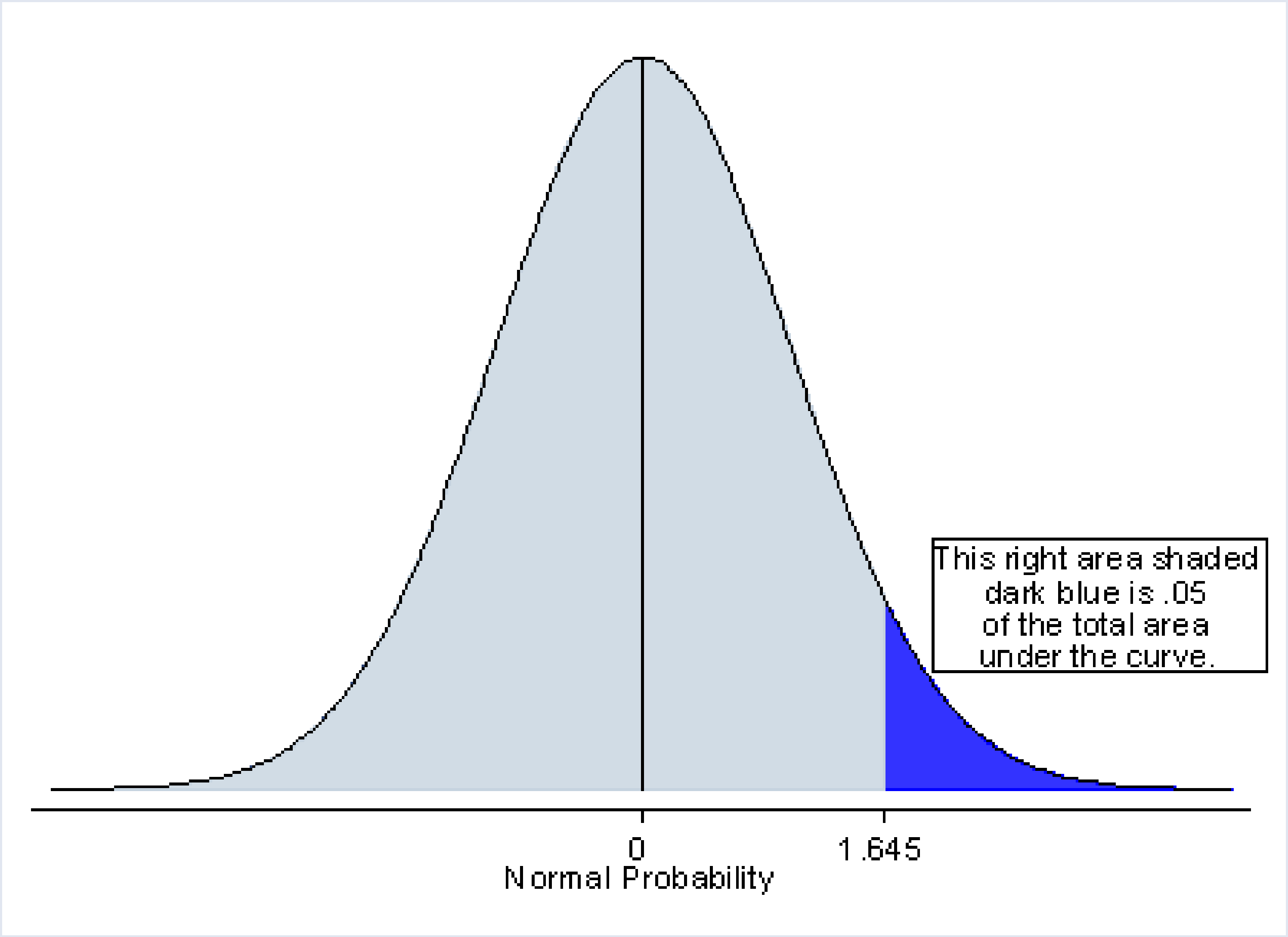
- Since we know σ,

we can use z here.

- Here, since it is single

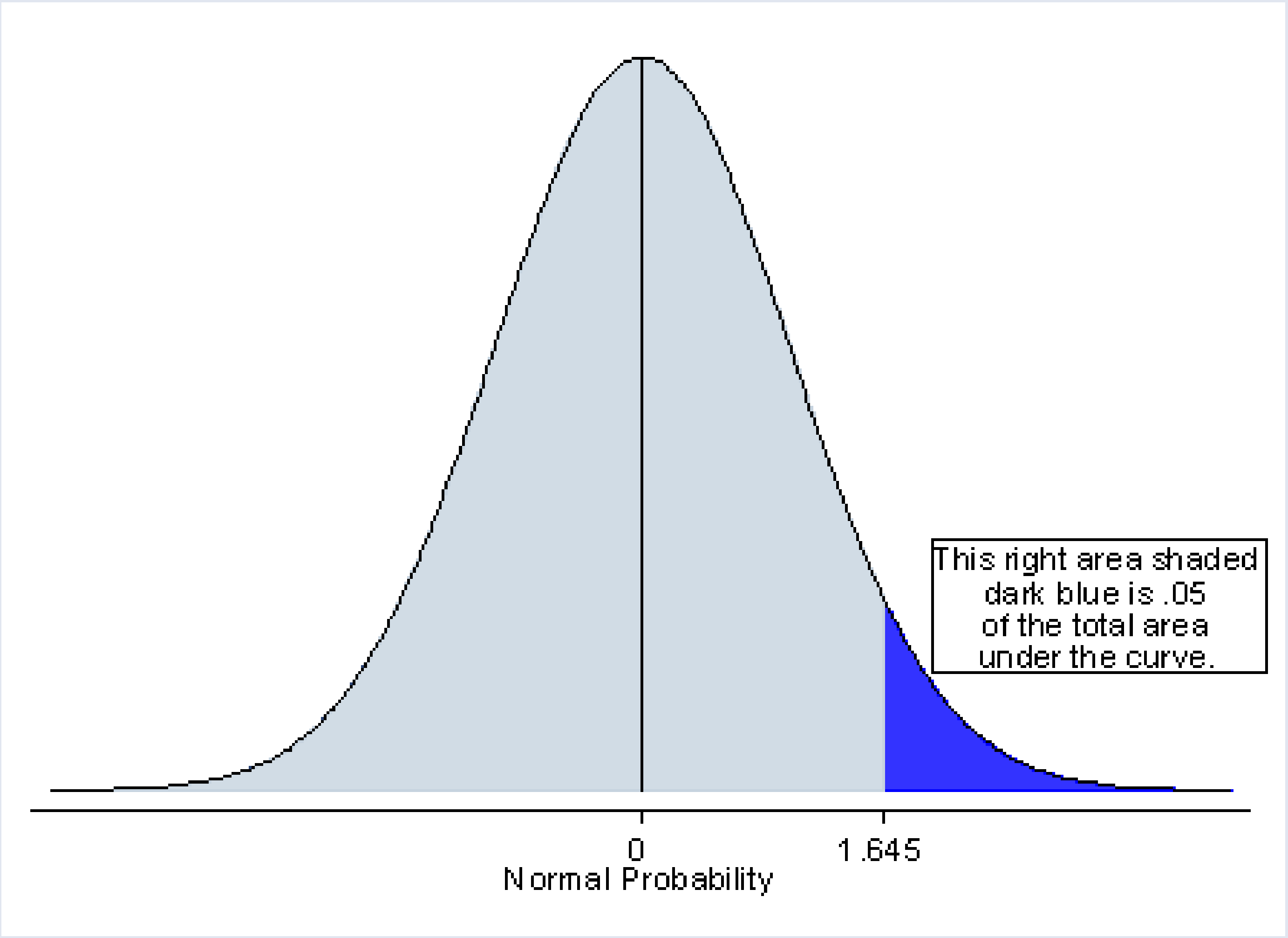
sided we have a rejection

region of this type ➔



One sided HT for μ [σ is known]

- Calculate z_α (**not** $\boldsymbol{z_{\alpha\backslash 2}}$)
- Here we have $z_{0.05} = 1.645$
- Calculate z_{obs}
  - $z_{obs} = \frac{71.8-70}{9/10}$=2>1.645
- Hence we reject H_0



One sided HT for μ [σ is known]

- For the same question, what can you say about testing:
  - $\mathbf{H_0}: \boldsymbol{\mu} \geq \mathbf{70}$.
  - $\mathbf{H_1}: \boldsymbol{\mu} < \mathbf{70}$

One sided HT for μ [σ is known]

- **Step1:** The hypothesis are:
  - $H_0: \mu \geq \mu_0 \ (or\ \mu = \mu_0)$
  - $H_1: \mu < \mu_0$
- **Step2:** The test statistic is given by: $z_{obs} = \frac{\bar{x}_{obs} - \mu_0}{\sigma/\sqrt{n}}$
- **Step 3**: $[-\infty, -z_\alpha]$,
- **Step 4**: Calculate z_{obs} using the formula in step2
- **Step5:** if z_{obs} is in the critical region ($\boldsymbol{z_{obs} \leq -z_\alpha}$), we reject H_0

One sided HT for μ [σ is known]

- **Step1:** The hypothesis are:
  - $H_0: \mu \leq \mu_0 (or\ \mu = \mu_0)$
  - $H_1: \mu > \mu_0$

- **Step2:** The test statistic is given by: $z_{obs} = \frac{\bar{x}_{obs} - \mu_0}{\sigma/\sqrt{n}}$

- **Step 3**: $[z_\alpha, \infty]$,

- **Step 4**: Calculate z_{obs} using the formula in step2

- **Step5:** if z_{obs} is in the critical region ($\boldsymbol{z_{obs} \geq z_\alpha}$), we reject H_0

One sided HT for μ [σ is UNknown]

- **Step1:** The hypothesis are:
  - $H_0: \mu \leq \mu_0 (or\ \mu = \mu_0)$
  - $H_1: \mu > \mu_0$

- **Step2:** The test statistic is given by: $t_{obs} = \frac{\bar{x}_{obs} - \mu_0}{s/\sqrt{n}}$ with v=n-1 d.f

- **Step 3**: $[t_\alpha, \infty]$,

- **Step 4**: Calculate t_{obs} using the formula in step2

- **Step5:** if t_{obs} is in the critical region $(\boldsymbol{t_{obs} \geq t_\alpha})$, we reject H_0

One sided HT for μ [σ is UNknown]

- **Step1:** The hypothesis are:
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Other one sided hypothesis tests:

- All other HTs can be made using similar arguments. You can check:
  - Two means
  - Single variance
  - Two variances

One sided HT for μ: Example

- Example (σ known). A manufacturer of sports equipment has developed a new synthetic fishing line.
- They claim this new synthetic has a
  - mean breaking strength μ of (at least) 8 kg
  - with a standard deviation σ of 0.5 kg.
- A wholesale store (for ex. Decathlon) is interested in this synthetic fishing line but before they can advertise this claim they would like to test it.
- If the new synthetic lines are weaker than 8 kg, they won't buy it. Otherwise they will buy in large quantities.

One sided HT for μ: Example

- They want to test:

$$H_0: \mu \geq 8 \ vs \ H_1: \mu < 8$$

- The null hypothesis is the default value, i.e., the fishing line is strong.

- They have tested a sample of n=50 lines
- It turns out that $\bar{X} = 7.85\ kg$.
- What is the result for 0.01 level of significance?

One sided HT for μ: Example

- **Solution**
- $\boldsymbol{H_0: \mu = 8}$ vs $\boldsymbol{H_1: \mu < 8}$
- Decision rule for $\alpha = 0.01$ level of significance will be
  - Reject H_0 if $Z < -z_{0.01} = -2.33$,

  where $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ has the standard normal distribution
- Now, we use data to calculate the observed test statistic.

One sided HT for μ: Example

- (b) Using the sample mean, $\mu_0 = 8$, $\sigma = 0.5$, and $n = 50$, we get

$$z_{obs} = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.85-8}{0.5/\sqrt{50}} = -2.12$$

- **DECISION**: We can't reject H_0. (That means μ is **not smaller** than 8)
- **CONCLUSION**: The company would consider buying the lines. There is no enough evidence that the strength of the fishing line is less than 8 kg.

One sided HT for μ: Example

- **Example (σ unknown):**
- Assume s=0.48 and we are **not given** σ

(a) Decision rule for $\alpha = 0.01$ level of significance will now be

- Reject H_0 if $T <$ -$t_{0.01}$ = -2.40, where T has v= n-1 = 49 d.o.f.

- Since $t_{obs} = \frac{7.85-8}{0.48/\sqrt{50}} = -2.21 > \ -2.40$ we can't reject H_0.

HT = CI!

- *Example. Consider Example 5, but take a two-sided alternative hypothesis.*

$$H_0 : \mu = 8 \quad vs. \quad H_1 : \mu \neq 8.$$

- *Find the decision at the level of significance* $\boldsymbol{\alpha = 0.02}$, $n=50$
- *Solution: Reject* H_0 *if* $T > t_{0.01} = 2.40$, *or* $T < -t_{0.01} = -2.40$
- Since $t_{obs} = \frac{7.85-8}{0.48/\sqrt{50}} = -2.21 > -2.40$, we cannot reject H0 at this level.

HT = CI!

- *OR: A 98% confidence interval for μ:*
- $\left(\bar{X} - t_{\alpha/2}\frac{S}{\sqrt{n}}, \bar{X} + t_{\alpha/2}\frac{S}{\sqrt{n}}\right) = 7.85 \pm 2.40\frac{0.48}{\sqrt{50}} = (7.69, 8.01)$
- *Since 8 is in the CI, we cannot reject H0 at this level.*
- *The two procedures are equivalent*

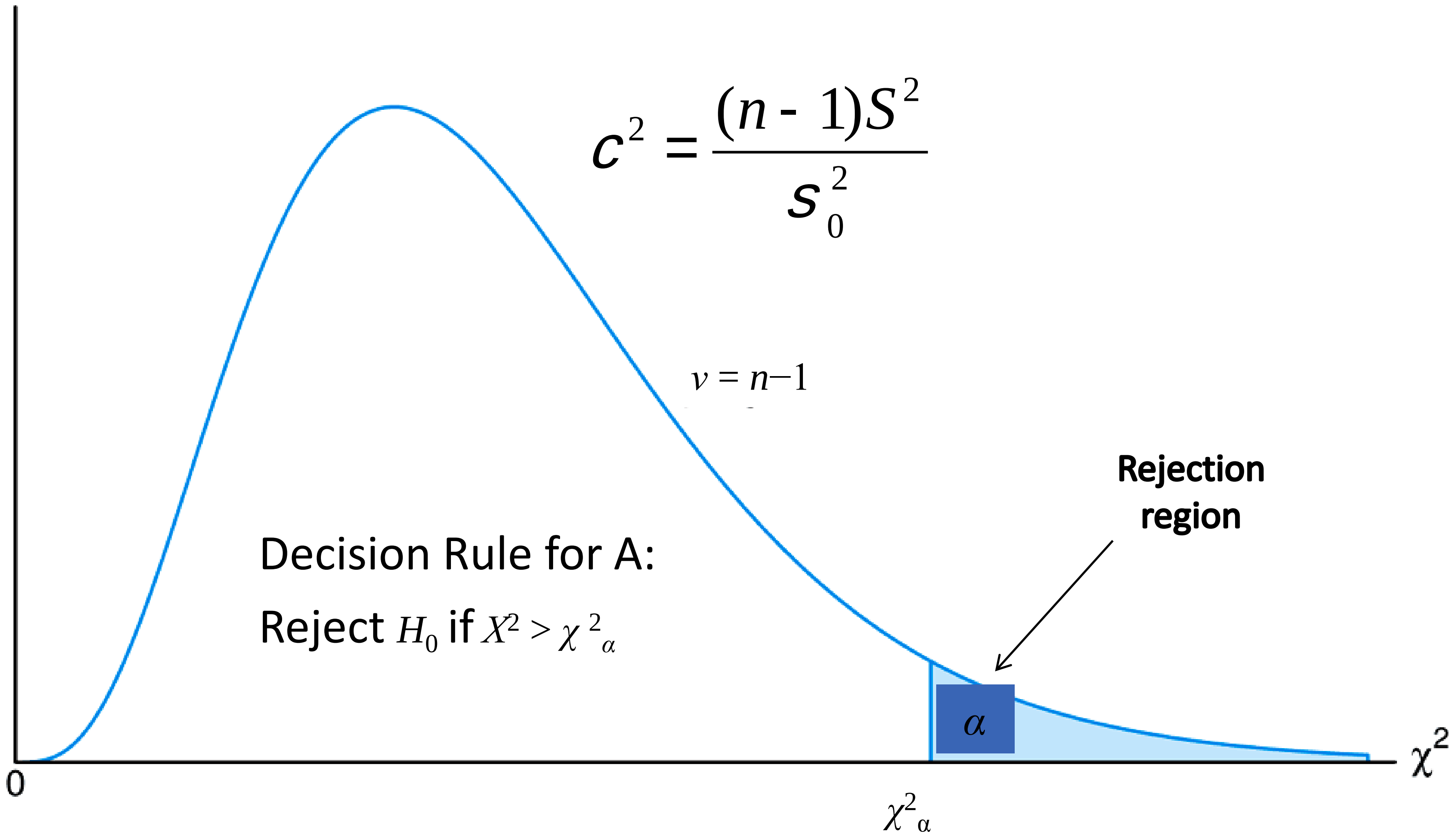
One Sided Tests for Variances

One sided test for σ

1. Write the hypotheses: $H_0: \sigma^2 = {\sigma_0}^2$ vs.
   - A. $H_1: \sigma^2 > {\sigma_0}^2$, B. $H_1: \sigma^2 < {\sigma_0}^2$, or C. $H_1: \sigma^2 \neq {\sigma_0}^2$.
2. Pick your test statistic: $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$ with v = n-1 d.o.f.
3. Find your critical region $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$
4. Calculate χ^2_{obs}
5. Draw appropriate conclusions.

One sided test for σ

$H_0: \sigma^2 = \sigma_0^2$ vs. A. $H_1: \sigma^2 > \sigma_0^2$



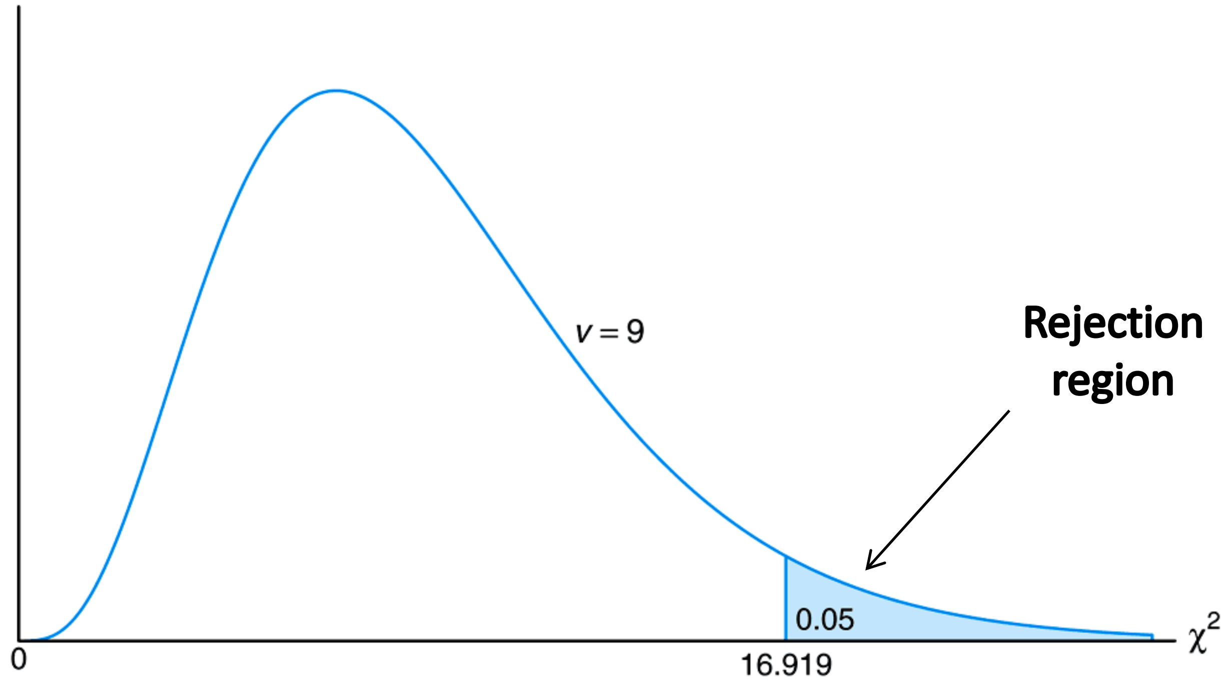
One sided test for σ

- **Example**. A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation of at most 0.9 year.
- To test if the standard deviation can actually be larger than this claimed value, a random sample of 10 of these batteries is selected and tested.
- The sample standard deviation is found to be 1.2 years.
- Do the data support the manufacturer's claim?
- Use a 0.05 level of significance.

One sided test for σ

Solution. We begin with the hypotheses and the decision rule.

1. H_0: $\sigma^2 = 0.81$ vs. H_1: $\sigma^2 > 0.81$.
2. For $\alpha = 0.05$, the decision rule: Reject H_0 when $\chi^2 > 16.919$, with $v = 9$ degrees of freedom.



One sided test for σ

Solution(ctd.) H_0: $\sigma^2 = 0.81$ vs. H_1: $\sigma^2 > 0.81$

3. Computations: $s^2 = 1.44$ and $n = 10$. This gives:

$$\chi^2_{obs} = \frac{(9)(1.44)}{0.81} = 16.0$$

4. Decision: The χ^2-statistic is not significant at the 0.05 level.
   We can't reject H_0.

5. P-value $= P(\chi^2 > 16.0) \approx 0.07$. (later)

Based on the P-value of 0.07, H_0 does not have high credibility.

A bigger sample size may be taken to get a better picture.