

# Scheduling

Notes 5

Consider the following problem  
 $7/2/F/C_{\max}$

Jobs	Machine 1	Machine 2
1	6	3
2	2	9
3	4	3
4	1	8
5	7	1
6	4	5
7	7	6

Optimal sequence : ?

$C_{\max}$  = ?

# Extension of Johnson's Algorithm for 3 Machine Flow Shop

- The extension of Johnson's rule does not guarantee an optimal makespan for all three-machine flow shop cases. The extension guarantees an optimal makespan if the following conditions are satisfied.
- To apply Johnson's Algorithm on  $n$  jobs three machines problems , one/both of the following conditions should be verified :
- $\min_{i=1}^n (P_{i1}) \geq \max_{i=1}^n (P_{i2})$   
or/and
- $\min_{i=1}^n (P_{i3}) \geq \max_{i=1}^n (P_{i2})$

# Extension of Johnson's Algorithm for 3 Machine Flow Shop

- This means, maximum processing time of machine 2, should not be greater than the minimum processing time of machine 1 or/and machine 3.
- If this condition is satisfied
- $a_i = P_{i1} + P_{i2}$  (the sum of the processing times of that job on the original machines M1 and M2)
- $b_i = P_{i2} + P_{i3}$  (the sum of the processing times of that job on the original machines M2 and M3)

# Extension of Johnson's Algorithm for 3 Machine Flow Shop

- The 3 machines problem is converted into two machines problem.
- Then the Johnson's rule is applied.
- The jobs on the original three machines are sequenced using the optimal sequence obtained from Johnson's rule.

# Consider 6/3/F/Cmax Problem

Jobs	M1	M2	M3
1	4	1	3
2	6	2	9
3	3	1	2
4	5	3	7
5	8	2	6
6	4	1	1

Jobs	M'1	M'2
1	5	4
2	8	11
3	4	3
4	8	10
5	10	8
6	5	2

Optimal sequence : 2-4-5-1-3-6 or 4-2-5-1-3-6

Cmax= 36

# Johnson's Algorithm for Job Shop

- We assume that there are 4 types of jobs:
- Type A: The jobs that will be processed only on machine 1
- Type B: The jobs that will be processed only on machine 2
- Type C: The jobs that will be processed first on machine 1 and then on machine 2.
- Type D: The jobs that will be processed first on machine 2 and then on machine 1



- To find the optimal sequence these sequences will be created:
- 1.  $S_A$ : Sequence according to Type A
- 2.  $S_B$ : Sequence according to Type B
- 3.  $S_C$ : Sequence according to Type C (Johnson Algorithm)
- 4.  $S_D$ : Sequence according to Type D (Johnson Algorithm)
  
- Optimal sequence : Machine 1: (SC, SA, SD)
- Machine 2: (SD, SB, SC)

# Example

Job	First	Second
1	M1 8	M2 2
2	M1 7	M2 5
3	M1 9	M2 8
4	M1 4	M2 7
5	M2 6	M1 4
6	M2 5	M1 3
7	M1 9	-
8	M2 1	-
9	M2 5	-

- Type A: Job 7
- $S_A$ : 7
- Type B: Job 8 and Job 9
- $S_B$ : (8,9)
- Type C: Job 1, Job 2, Job 3 and Job 4
- $S_C$ : According to Johnson Algorithm (4,3,2,1)
- Type D: Job 5 and Job 6
- $S_D$ : According to Johnson Algorithm (5,6)

Optimal order:

Machine 1 : (4,3,2,1,7,5,6)

Machine 2: (5,6,8,9,4,3,2,1)

$C_{\max} = 44$

# Resources

- Sıralama ve Programlama, Hüseyin Başlıgil
- Çizelgeleme Ders Notları, Prof. Dr. Hüseyin Başlıgil
- [web4.uwindsor.ca/users/b/baki%20fazle/Chapter\\_08\\_Lecture\\_12\\_to\\_19\\_w08\\_431\\_scheduling.ppt](http://web4.uwindsor.ca/users/b/baki%20fazle/Chapter_08_Lecture_12_to_19_w08_431_scheduling.ppt) – Windsor University Operations Scheduling Lecture Notes
- <http://rbutterworth.nfshost.com/Scheduling/1.3-due>
- <http://www.egyankosh.ac.in/bitstream/123456789/20794/1/Unit-7.pdf>