Example Problem II

Problem Statement: Jack is an aspiring freshman at Ulern University. He realizes that "All work and no play make Jack a dull boy." As a result, Jack wants to apportion his available time of about 10 hours a day between work and play. He estimates that play is twice as much fun as work. He also wants to study at least as much as he plays. However, Jack realizes that if he is going to get all his homework assignments done, he can not play more than 4 hours a day. How should Jack allocate his time to maximize his pleasure from both work and play?

Set up the linear programming problem and solve the linear programming problem graphically.

Questions:

- What do you want to maximize or minimize?

<u>Answer</u>:

X_1 = Play hours per day

X_2 = Work hours per day

Constraints

Objective

<u>Maximize</u>: $z = 2 X_1 + X_2$

X_1 = Play hours per day

X_2 = Work hours per day

What are these constraints?

<u>Subject to:</u>

$X_1 + X_2 <= 10$ (available time)

$X_1 - X_2 <= 0$

$X_1 <= 4$

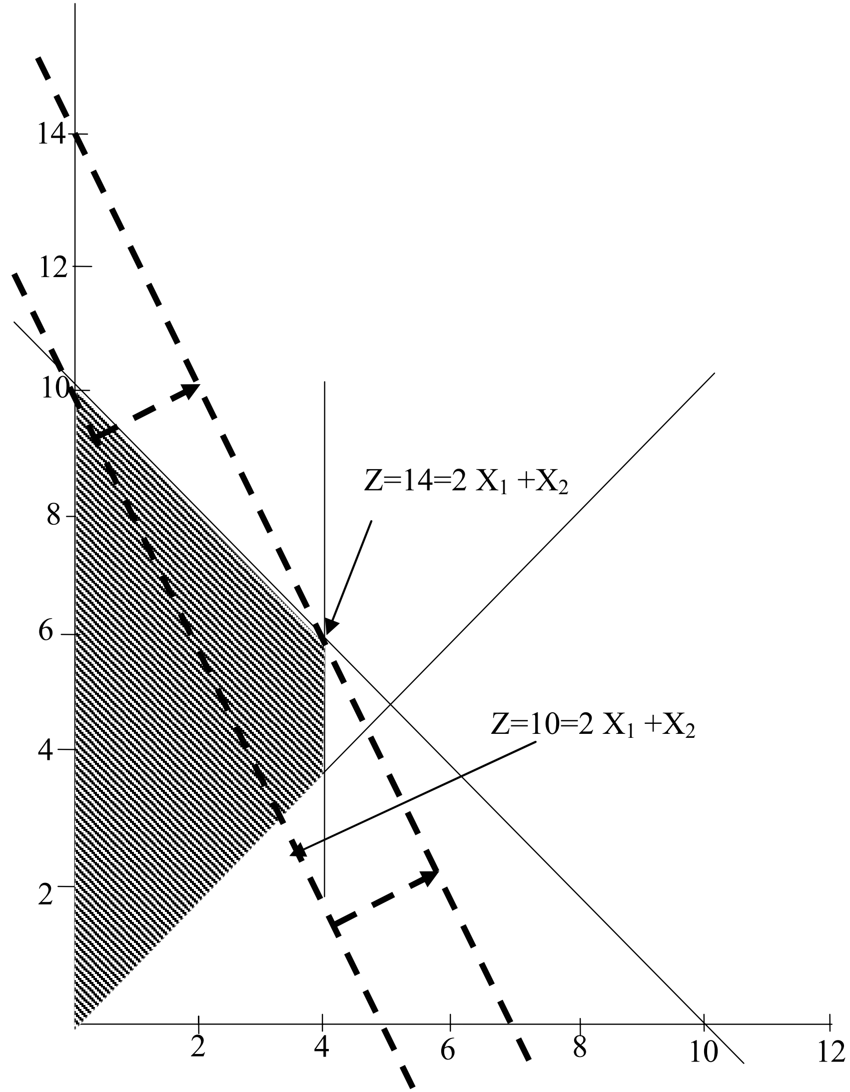
Solve this by graphically

Solving the LP (Prob. II)

- To solve graphically, the constraints can be reformulated as the following straight lines
  $X_2 = 10- X_1$ (available time)
  $X_2 = X_1$
  $X_1 = 4$

- The objective function can be reformulated as
  $X_2 = z - 2 X_1$

  $X_2 = 10 - 2 X_1$



Example Problem III

Problem Statement:

A company makes two types of products, A and B. These products are produced during a 40-hour work-week and then shipped out at the end of the week. They require 20 and 5 kg of raw material per kg of product, respectively, and the company has access to 10000 kg of raw material per week. Only one product can be created at a time with production times for each of 0.05 and 0.15 hrs, respectively. The plant can only store 550 kg of total product per week. Finally, the company makes profits of $45 and $30 on each unit of A and B, respectively.

Set up the linear programming problem to maximize profit and Solve the linear programming problem graphical1y.

	Product		
Resource	**A**	B	**Resource Availability**
Raw material	20 kg/A	5 kg/B	10000 kg /Week
Production time	0.05 hr/A	0.15 hr/B	40 hr /Week
Storage			550 kg/week
Profit	45 /A	30 /B	

Solving the LP (Prob. III)

Note: Although it is not really clear from the problem statement, it should be assumed that each unit of product is equivalent to a kg.

Define: X_a = amount of product A produced, and
X_b = amount of product B produced

- The objective function is to maximize profit,
  $P=45X_a +30X_b$
- Subject to the following constraints
  $20\ X_a\ +5\ X_b \leq 10000$ (raw materials)
  $0.05\ X_a + 0.15\ X_b \leq\ 40$ (production time)
  $X_a + X_b \leq\ 550$ (storage)
  $X_a, X_b \geq 0$ (positivity)

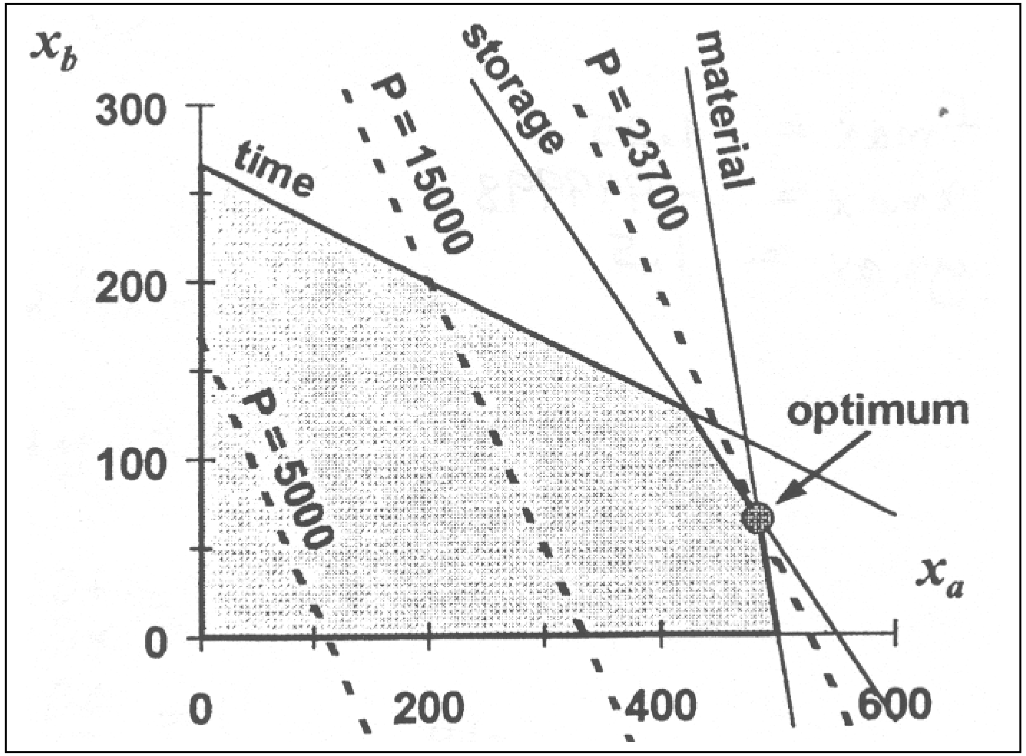
Solving the LP (Prob. III)

- To solve graphically, the constraints can be reformulated as the following straight lines
  $X_b = 2000 - 4\ X_a$ (raw materials)
  $X_b = 266.667 - 0.3333\ X_a$ (production time)
  $X_b = 550 - X_a$ (storage)

- The objective function can be reformulated as
  $X_b = (1/30)P - 1.5\ X_a$

Graphical Solution to the LP (Prob. III)

- The constraint lines can be plotted on the X_b - X_a plane to define the feasible space. Then the objective function line can be superimposed for various values of **P** until it reaches the boundary.



  - The result is **P** = 23700 with X_a = 483 and X_b = 67 .
  - Notice also that material and storage are the binding constraints and that there is some slack in the time constraint.

Solution of a Minimization Model: Problem I

Problem Statement: John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores: in store 1, John can work between 5 and 12 hours a week, and in store 2, he is allowed to work between 6 and 10 hours. Both stores pay the same hourly wage. John thus wants to base his decision about how many hours to work in each store on a different criterion: work stress factor. Based on interviews with present employees, John estimates that, on a scale of 1 to 10, the stress factor are 8 and 6 at stores 1 and 2, respectively. Because stress mounts by the hour, he presumes that the total stress at the end of the week is proportional to the number of hours he works in the store. How many hours should John work in each store?

Set up the linear programming problem to minimize stress index and solve the linear programming problem graphically.

Constraints (Prob. I)

Objective

X_1 = Number of hours per week in Store 1
X_2 = Number of hours per week in Store 2

Minimize: $\mathbf{z = 8 X_1 + 6 X_2}$

What are these constraints?

Subject to:

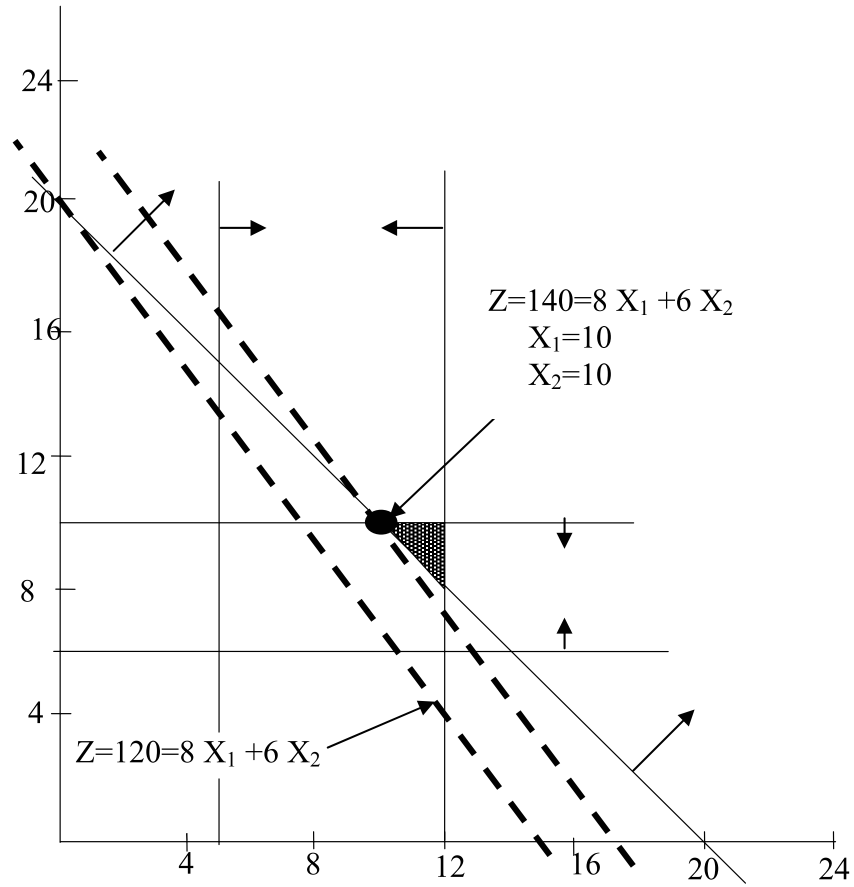
$X_1 + X_2 \Rightarrow 20$ (available time)
$5 <= X_1 <= 12$ (Store 1)
$6 <= X_2 <= 10$ (Store 2)

Solve this by graphically

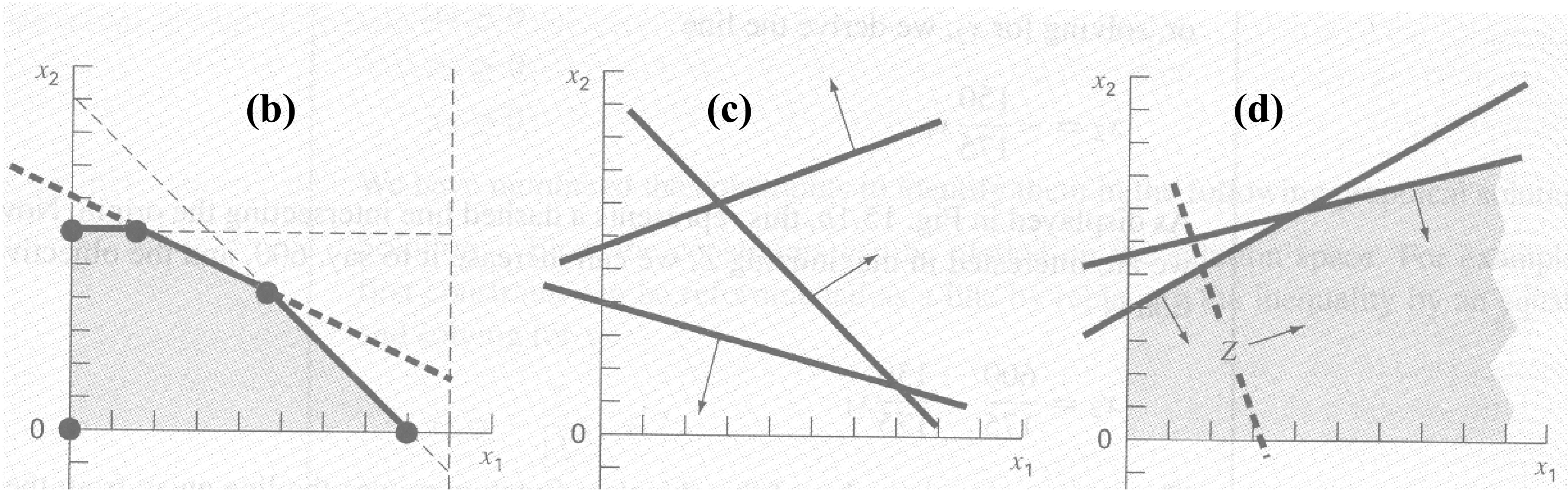
Solving the LP (Prob. I)

- To solve graphically, the constraints can be reformulated as the following straight lines
  $X_2 = 20- X_1$ (available time)

- The objective function can be reformulated as
  $X_2 = z/6 – 8/6\ X_1$



Possible Outcomes to the LP Problem



(a) Unique Solutions
(b) Alternate Solutions
(c) No feasible Solutions
(d) Unbounded Problems

GRAPHICAL SENSITIVITY ANALYSIS

- An LP model is a snapshot of a real situation in which the model parameters assume static values.

The procedure is referred as sensitivity analysis because it studies the sensitivity of the optimal solution to changes made in the model.

Let investigate two cases of sensitivity analysis based on the graphical LP solution:

(1) Changes in the objective coefficients and

(2) Changes in the right-hand side of the constraints.

Change in the Objective Function Coefficients

- The general objective function in a two-variable LP problem can be written as

  Maximize or minimizes $= C_1 X_1 + C_2 X_2$

Changes in the coefficients C_1 and C_2 will change the slope of *z* and, hence, possibly, the optimal corner point.

Example: Electra produces two types of electric motors, each on a separate assembly line. The respective daily capacities of the two lines are 600 and 750 motors. Type 1 motor uses 10 units of a certain electronic component, and type 2 motor uses only 8 units. The supplier of the component can provide 8000 pieces a day. The profits per motor for types 1 and 2, are \$60 and \$40, respectively.

(a) Determine the optimum daily production mix.

(b) Determine the optimality range of the ratio of unit profits that will keep the solution in (a) unchanged.

Solving the LP Problem and Sensitivity Analysis

Objective

X_1 = Number of Type 1 motors per day
X_2 = Number of Type 2 motors per day

Maximize:

$$z = 60 X_1 + 40 X_2$$

Subject to:

$10 X_1 + 8 X_2 <= 8000$ (available component)
$X_1 <= 600$ (Type 1 Motor)
$X_2 <= 750$ (Type 2 Motor)
$X_1, X_2 => 0$

Solving the LP Problem and Sensitivity Analysis

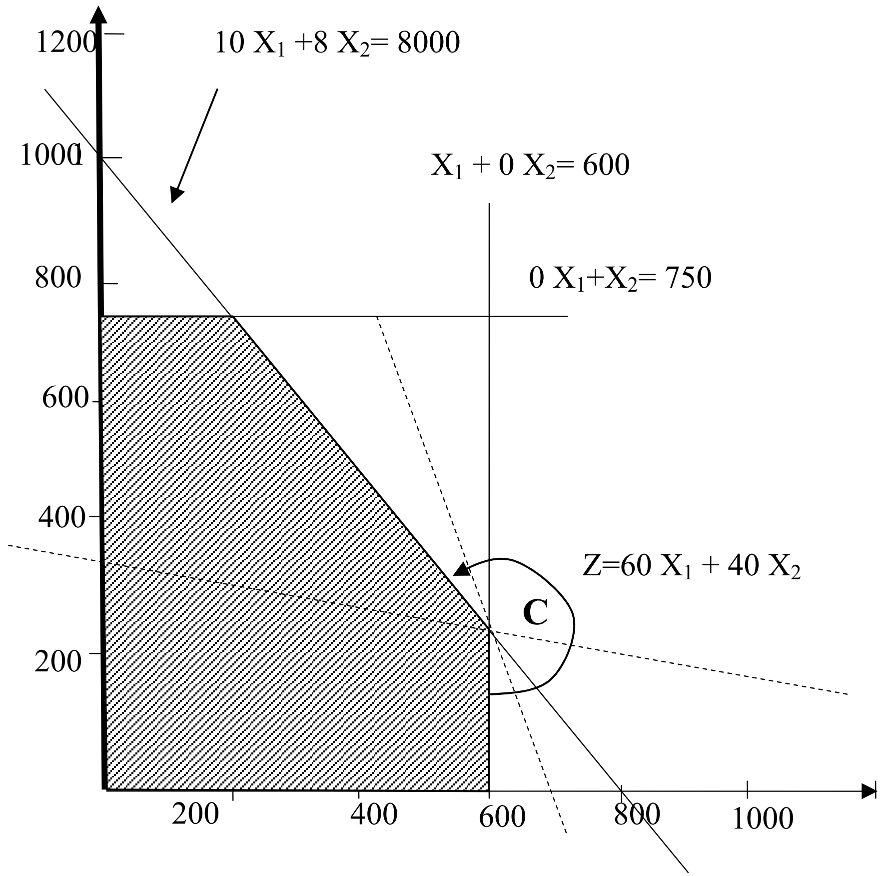
- To solve graphically, the constraints can be reformulated as the following straight lines
  $X_2 = 1000 – 5/4\ X_1$ (available part)

- The objective function can be reformulated as
  $X_2 = z/40 – 60/40\ X_1$

(a) Optimum
  $X_1=600$
  $X_2 = 250$ and $Z= 46\ 000$

(b) The solution C will remain optimum so long as the slope of Z lies between the slope of two lines intersecting at C, namely, $X_1 + 0\ X_2= 600$, and $10\ X_1 +8\ X_2= 8000$,

So $0\leq C_2/C_1 \leq 8/10$



Change in the Availability of Resources

- LP models, constrains, directly or indirectly, represent the usage of limited resources.
- Effect of changes in the amount of available resources on the optimum solution.
- Unit worth of a resource: rate of a change in the optimum objective value that result from making changes in the available amount of a resource.

Let $\mathbf{y_i}$ represent the worth per unit resource i,

$$y_i = \frac{\text{Change in volume of z corresponding to feasible range of resource } i}{\text{feasible range of resource } i}$$

Change in the Availability of Resources

Example: A company produces two *products, A* and *B*. The sales volume for *A* is at least 80% of the total sales of both *A* and *B*. However the company cannot sell more than 100 units of *A* per day. Both products use one raw material, of which the maximum daily availability is 240 lb. The usage rates of the raw material are 2 lb per unit of *A* and 4 lb per unit of *B*.
The unit prices for *A* and *B* are \$20 and \$50, respectively.

(a) Determine the optimal product mix for the company.

(b) Determine the worth per unit change in the availability of the raw material and its range of applicability.

(c) Determine the revenue range corresponding to the feasibility range of the raw material resource.

(d) Use the unit worth per unit to determine the effect of changing the maximum demand for product *A* by ±10 units.

Solving The LP Problem and Sensitivity Analysis

Objective

X_1 = Number of Units of A
X_2 = Number of Units of B

Maximize:

$$\mathbf{z = 20\ X_1 + 50\ X_2}$$

Subject to:

$\dfrac{X_1}{X_1 + X_2} \geq 0.8$ or $-0.2\ X_1 + 0.8\ X_2 <= 0$ (Sales of A)

$X_1 <= 100$

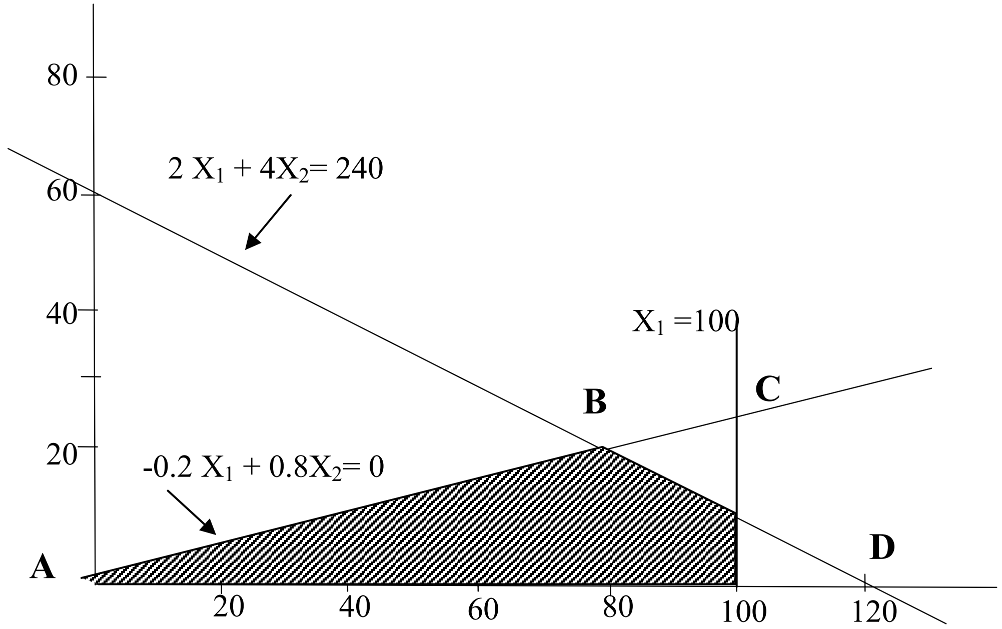
$2\ X_1 + 4\ X_2 <= 240$ (Raw material usage)

$X_1, X_2 => 0$

Solving the LP Problem and Sensitivity Analysis

- To solve graphically, the constraints can be reformulated as the following straight lines
  $X_2 = z/40 - 0.5\ X_1$
  $X_2 = 0.25\ X_1$

(a) Optimum solution at B
$X_1 = 80$ units,
$X_2 = 20$ units and $Z = 2\,600$



(b) Let R= units of raw material
A=(0, 0) C=(100, 25)

R: A 2x0+4x0=0
C 2x100+4x25=300

z: A 20x0+50x0=0
C 20x100+50x25=3250

$$\text{Worth per unit} = \frac{3250-0}{300-0} = 10.8$$

$$\text{validity range } 0 \le R \le 300$$

Solving the LP Problem and Sensitivity Analysis

(c) Let d= maximum demand for A

B=(80, 20)   D=(120, 0)

d: B 80

D 120

z: B 20x80+50x20=2600

D 20x120+50x20=2600

$$\text{Worth per unit} = \frac{2600 - 2600}{120 - 80} = 0$$

$$\text{validity range} \qquad 80 \le d \le 120$$

(d) Given d=100 units, A±10%,

Change is equivalent to

$90 \le d \le 110$

So the range falls entirely in the validity range.

Since the worth per unit =0,

Opimum z remains the same for the entire range; i.e. z= 2600