DECISION MAKING TECHNIQUES IN MANAGEMENT INFORMATION SYSTEMS (MIS)

LECTURE -8-
(AHP)

THE ANALYTİC HİERARCHY PROCESS (AHP)

- Founded by Saaty in 1980.
- It is a popular and widely used method for multi-criteria decision making.
- Allows the use of qualitative, as well as quantitative criteria in evaluation.
- Wide range of applications exists:
  - Selecting a car for purchasing
  - Deciding upon a place to visit for vacation
  - Deciding upon an MBA program after graduation.

AHP-General Idea

- Develop an hierarchy of decision criteria and define the alternative courses of actions.

- AHP algorithm is basically composed of two steps:
  1. Determine the relative weights of the decision criteria
  2. Determine the relative rankings (priorities) of alternatives

! Both qualitative and quantitative information can be compared by using informed judgments to derive weights and priorities.

Typical application areas

- Hiring, evaluating and promoting employees
- Strategic planning
- Relocation decisions
- Vendor selection
- Evaluating mergers and acquisitions
- …

How widespread is its use?

..a few of the thousands of organizations using AHP

IBM	NASA
Goodyear	IRS
Ford Motor Co.	FBI
Citibank	Department of Defense
Xerox	World Bank
Boeing	Texaco
AT&T	Eastman Kodak
General Motors	Inter-American Bank

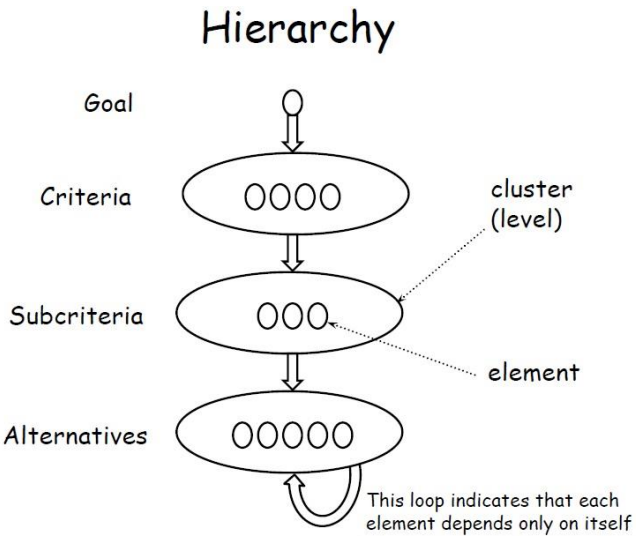
A few of the many Universities using/teaching AHP

Harvard University	Colorado State University
Yale University	University of Cambridge
MIT	Duke University
American University	Purdue University
Naval War College	Katz School of Business
George Washington University	Wharton School of Business
Michigan State University	Johns Hopkins University
Stanford University	University of Maryland

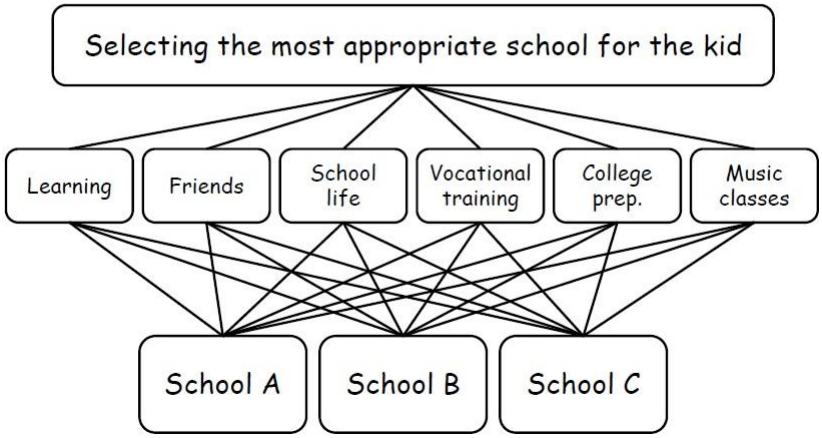
WHY HIERARCHY?

When people faced with a complex problem in order to understand the problem better they must break down the problem into its smaller constituent parts and construct a hierarchical model to represent it.

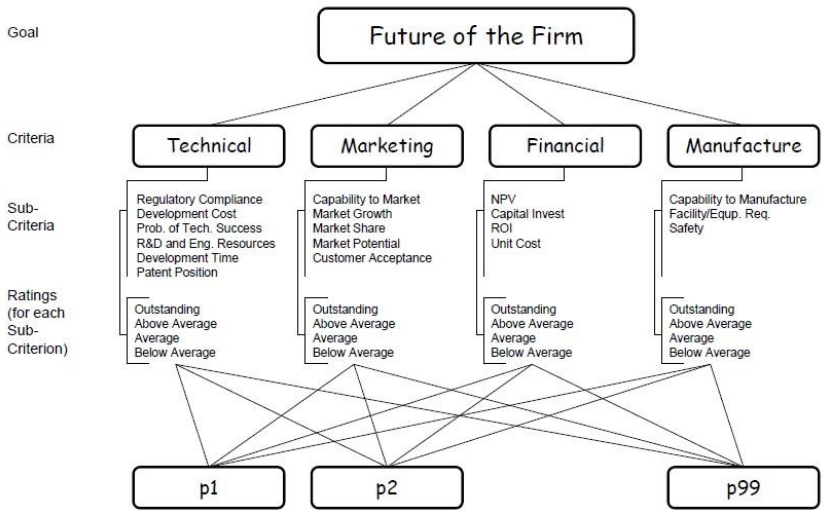
HIERARCHY



School Selection



R&D Project Selection



THE STEPS OF AHP

STEP 1

- Structure a hierarchy. List the overall goal, criteria, and decision alternatives

STEP 2

- Make pairwise comparisons. Rate the relative importance between each pair of decision alternatives and criteria.

THE STEPS OF AHP

- **STEP 2 (Continued):** AHP uses 1-9 scale for the prioritization process.

Numerical ratings	Verbal judgments
1	Equally important (preferred)
3	Moderately more important
5	Strongly more important
7	Very strongly more important
9	Extremely more important

THE STEPS OF AHP

- **STEP 2 (Continued):** Intermediate numeric ratings of 8, 6, 4, 2 can be assigned. A reciprocal rating (i.e. 1/9, 1/8, etc.) is assigned when the second alternative is preferred to the first.

$$A = \begin{bmatrix} 1 & a_{21} & a_{31} & \cdots & a_{n1} \\ 1/a_{21} & 1 & a_{32} & \cdots & a_{n2} \\ 1/a_{31} & 1/a_{32} & 1 & \cdots & a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1/a_{n1} & 1/a_{n2} & 1/a_{n3} & \cdots & 1 \end{bmatrix}_{nxn}$$

THE STEPS OF AHP

- **STEP 3:** Develop a Normalized Matrix. Divide each number in a column of the pair-wise comparison matrix by its column sum.

$$b_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \quad i=1,2,3,\ldots,n \quad j=1,2,3,\ldots,n$$

THE STEPS OF AHP

- **STEP 4:** Develop the Priority Vector. Average each row of the normalized matrix. These row averages form the priority vector of alternative preferences with respect to the particular criterion. The values in this vector sum to 1. This process is performed for:
  - Criteria comparison
  - Alternative comparison with respect to each criterion

$$w_i = \frac{\sum_{j=1}^{n} b_{ij}}{n} \quad i=1,2,3,\ldots,n \quad j=1,2,3,\ldots,n$$

THE STEPS OF AHP

- **STEP 5:** Calculate a Consistency Ratio. The consistency of the subjective input in the pairwise comparison matrix can be measured by calculating a consistency ratio. A consistency ratio of less than .1 is good. For ratios which are greater than .1, the subjective input should be re-evaluated.

THE STEPS OF AHP

- **STEP 6:** In case consistency is satisfied, the results of step 4 are summarized in a priority matrix by listing the decision alternatives horizontally and the criteria vertically (DW). The column entries are the priority vectors for each criterion. The matrix DW is multiplied by the criteria priority vector (W) and the result vector R is obtained.

$$DW = \left[w_{ij}\right]_{mxn} \quad i=1,2,3,\ldots,m \quad j=1,2,3,\ldots,n$$

$$R = DW \times W$$

DETERMINING THE CONSISTENCY RATIO (STEP 5)

- **Step 1:** For each row of the pairwise comparison matrix, determine a weighted sum by summing the multiples of the entries by the priority of its corresponding (column) alternative.

$$i=1,2,3,\ldots,n \quad j=1,2,3,\ldots,n$$

$$D = \left[a_{ij}\right]_{nxn} \times \left[w_i\right]_{nx1} = \left[d_i\right]_{nx1}$$

DETERMINING THE CONSISTENCY RATIO (STEP 5)

- **Step 2:** For each row, divide its weighted sum by the priority of its corresponding (row) alternative and determine the average to obtain $\lambda_{max.}$

$$\lambda_{max} = \frac{\sum_{i=1}^{n} \frac{d_i}{w_i}}{n}$$

DETERMINING THE CONSISTENCY RATIO (STEP 5)

- **Step 3:** Compute the consistency index, CI, of the n alternatives by: $CI = (\lambda_{max} - n)/(n - 1)$.

Step 4: Determine the random index, RI by benefiting from the table below.

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0,58	0,9	1,12	1,24	1,32	1,41	1,45	1,49

Step 5: Determine the consistency ratio, CR, as follows: CR = CI/RI.

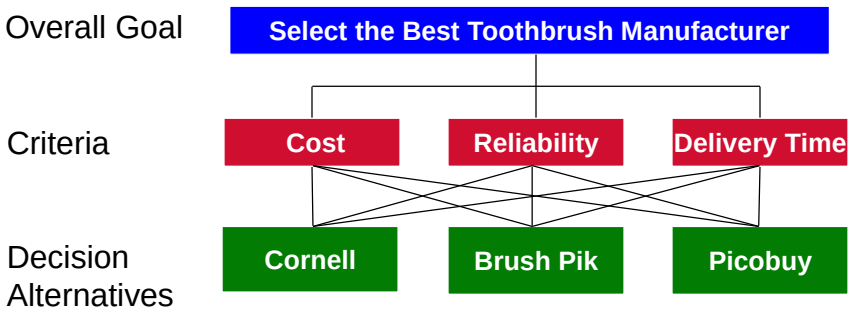
EXAMPLE: GİLL GLASS

Designer Gill Glass must decide which of three manufacturers will develop his "signature" toothbrushes. Three factors seem important to Gill: (1) his costs; (2) reliability of the product; and, (3) delivery time of the orders.

The three manufacturers are Cornell Industries, Brush Pik, and Picobuy. Cornell Industries will sell toothbrushes to Gill Glass for $100 per gross, Brush Pik for $80 per gross, and Picobuy for $144 per gross. Gill has decided that in terms of price, Brush Pik is moderately preferred to Cornell and very strongly preferred to Picobuy. In turn Cornell is strongly to very strongly preferred to Picobuy.

EXAMPLE: GİLL GLASS

- Hierarchy for the Manufacturer Selection Problem



EXAMPLE: GİLL GLASS

- Forming the Pairwise Comparison Matrix For Cost
  - Since Brush Pik is moderately preferred to Cornell, Cornell's entry in the Brush Pik row is 3 and Brush Pik's entry in the Cornell row is 1/3.
  - Since Brush Pik is very strongly preferred to Picobuy, Picobuy's entry in the Brush Pik row is 7 and Brush Pik's entry in the Picobuy row is 1/7.
  - Since Cornell is strongly to very strongly preferred to Picobuy, Picobuy's entry in the Cornell row is 6 and Cornell's entry in the Picobuy row is 1/6.

EXAMPLE: GİLL GLASS

- Pairwise Comparison Matrix for **Cost**

	Cornell	Brush Pik	Picobuy
Cornell	1	1/3	6
Brush Pik	3	1	7
Picobuy	1/6	1/7	1

EXAMPLE: GILL GLASS

- Normalized Matrix for Cost

  Divide each entry in the pairwise comparison matrix by its corresponding column sum. For example, for Cornell the column sum = 1 + 3 + 1/6 = 25/6. This gives:

	Cornell	Brush Pik	Picobuy
Cornell	6/25	7/31	6/14
Brush Pik	18/25	21/31	7/14
Picobuy	1/25	3/31	1/14

EXAMPLE: GILL GLASS

- Priority Vector For Cost

  The priority vector is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cornell: (6/25 + 7/31 + 6/14)/3 = .298
Brush Pik: (18/25 + 21/31 + 7/14)/3 = .632
Picobuy: (1/25 + 3/31 + 1/14)/3 = .069

EXAMPLE: GİLL GLASS

- Checking Consistency
  - Multiply each column of the pairwise comparison matrix by its priority:

$$.298\begin{bmatrix}1\\3\\1/6\end{bmatrix} + .632\begin{bmatrix}1/3\\1\\1/7\end{bmatrix} + .069\begin{bmatrix}6\\7\\1\end{bmatrix} = \begin{bmatrix}.923\\2.009\\.209\end{bmatrix}$$

  - Divide these number by their priorities to get:

$$.923/.298 = 3.097$$
$$2.009/.632 = 3.179$$
$$.209/.069 = 3.029$$

EXAMPLE: GİLL GLASS

- Checking Consistency
  - Average the above results to get λ_{max}.

$$\lambda_{max} = (3.097 + 3.179 + 3.029)/3 = 3.102$$

  - Compute the consistence index, CI, for two terms by:

$$CI = (\lambda_{max} - n)/(n - 1) = (3.102 - 3)/2 = .051$$

  - Compute the consistency ratio, CR, by CI/RI, where RI = .58 for 3 factors:

$$CR = CI/RI = .051/.58 = .088$$

Since the consistency ratio, CR, is less than .10, this is well within the acceptable range for consistency.

EXAMPLE: GİLL GLASS

Gill Glass has determined that for **reliability**, Cornell is very strongly preferable to Brush Pik and equally preferable to Picobuy. Also, Picobuy is strongly preferable to Brush Pik.

EXAMPLE: GİLL GLASS

- Pairwise Comparison Matrix for Reliability

	Cornell	Brush Pik	Picobuy
Cornell	1	7	2
Brush Pik	1/7	1	1/3
Picobuy	1/2	3	1

EXAMPLE: GILL GLASS

- Normalized Matrix for Reliability

  Divide each entry in the pairwise comparison matrix by its corresponding column sum. For example, for Cornell the column sum = 1 + 1/7 + 1/2 = 23/14. This gives:

	Cornell	Brush Pik	Picobuy
Cornell	14/23	7/11	3/5
Brush Pik	2/23	1/11	1/10
Picobuy	7/23	3/11	3/10

EXAMPLE: GILL GLASS

- Priority Vector For Reliability

  The priority vector is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cornell: (14/23 + 7/11 + 3/5)/3 = .615
Brush Pik: (2/23 + 1/11 + 1/10)/3 = .093
Picobuy: (7/23 + 3/11 + 3/10)/3 = .292

- Checking Consistency

  Gill Glass' responses to reliability could be checked for consistency in the same manner as was cost.

EXAMPLE: GİLL GLASS

Gill Glass has determined that for **delivery time**, Cornell is equally preferable to Picobuy. Both Cornell and Picobuy are very strongly to extremely preferable to Brush Pik.

EXAMPLE: GİLL GLASS

- Pairwise Comparison Matrix for Delivery Time

	Cornell	Brush Pik	Picobuy
Cornell	1	8	1
Brush Pik	1/8	1	1/8
Picobuy	1	8	1

EXAMPLE: GILL GLASS

- Normalized Matrix for Delivery Time

  Divide each entry in the pairwise comparison matrix by its corresponding column sum.

	Cornell	Brush Pik	Picobuy
Cornell	8/17	8/17	8/17
Brush Pik	1/17	1/17	1/17
Picobuy	8/17	8/17	8/17

EXAMPLE: GILL GLASS

- Priority Vector For Delivery Time

  The priority vector is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cornell: (8/17 + 8/17 + 8/17)/3 = .471
Brush Pik: (1/17 + 1/17 + 1/17)/3 = .059
Picobuy: (8/17 + 8/17 + 8/17)/3 = .471

- Checking Consistency

  Gill Glass' responses to delivery time could be checked for consistency in the same manner as was cost.

EXAMPLE: GİLL GLASS

The accounting department has determined that in terms of criteria, cost is extremely preferable to delivery time and very strongly preferable to reliability, and that reliability is very strongly preferable to delivery time.

EXAMPLE: GİLL GLASS

- Pairwise Comparison Matrix for Criteria

	Cost	Reliability	Delivery
Cost	1	7	9
Reliability	1/7	1	2
Delivery	1/9	1/2	1

EXAMPLE: GİLL GLASS

- Normalized Matrix for Criteria

  Divide each entry in the pairwise comparison matrix by its corresponding column sum.

	Cost	Reliability	Delivery
Cost	63/79	14/17	9/12
Reliability	9/79	2/17	2/12
Delivery	7/79	1/17	1/12

EXAMPLE: GİLL GLASS

- Priority Vector For Criteria

  The priority vector is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cost:	(63/79 + 14/17 + 9/12)/3	=	.790
Reliability:	(9/79 + 2/17 + 2/12)/3	=	.133
Delivery:	(7/79 + 1/17 + 1/12)/3	=	.077

EXAMPLE: GILL GLASS

- Overall Priority Vector

  The overall priorities are determined by multiplying the priority vector of the criteria by the priorities for each decision alternative for each objective.

Priority Vector for Criteria	[.790	.133	.077]
	Cost	Reliability	Delivery
Cornell	.298	.615	.471
Brush Pik	.632	.093	.059
Picobuy	.069	.292	.471

EXAMPLE: GILL GLASS

- Overall Priority Vector (continued)

  Thus, the overall priority vector is:

Cornell: (.790)(.298) + (.133)(.615) + (.077)(.471) = .353
Brush Pik: (.790)(.632) + (.133)(.093) + (.077)(.059) = .516
Picobuy: (.790)(.069) + (.133)(.292) + (.077)(.471) = .130

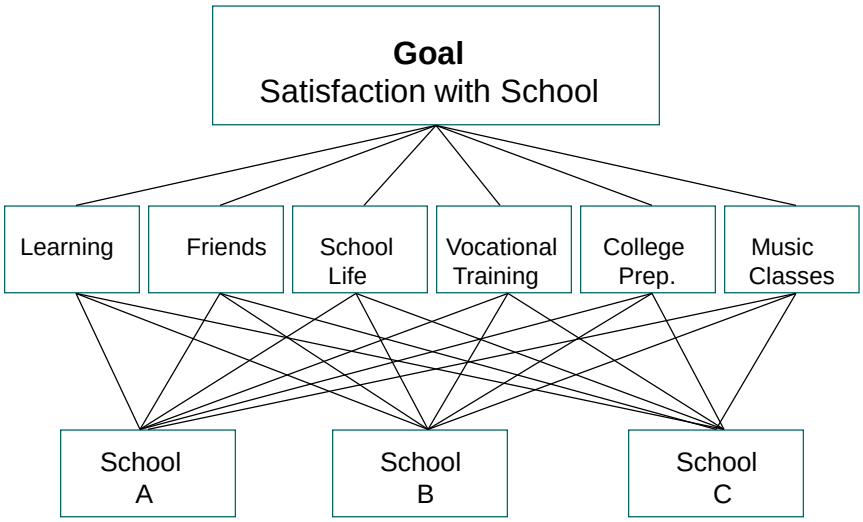
Brush Pik appears to be the overall recommendation.

AN EXAMPLE WİTH AHP

CHOOSİNG THE MOST SATİSFİED SCHOOL

- **Goal:** To select the most satisfied school.
- **Criteria:** learning, friends, school life, vocational training, college prep. and music classes.
- **Alternatives:** School A, school B, and school C.

Hierarchy:



Pairwise comparisons:

School Selection

	L	F	SL	VT	CP	MC	Weights
Learning	1	4	3	1	3	4	.32
Friends	1/4	1	7	3	1/5	1	.14
School Life	1/3	1/7	1	1/5	1/5	1/6	.03
Vocational Trng.	1	1/3	5	1	1	1/3	.13
College Prep.	1/3	5	5	1	1	3	.24
Music Classes	1/4	1	6	3	1/3	1	.14

Comparison of Schools with Respect to the Six Characteristics

	Learning A	B	C	Priorities
A	1	1/3	1/2	.16
B	3	1	3	.59
C	2	1/3	1	.25

	Friends A	B	C	Priorities
A	1	1	1	.33
B	1	1	1	.33
C	1	1	1	.33

	School Life A	B	C	Priorities
A	1	5	1	.45
B	1/5	1	1/5	.09
C	1	5	1	.46

	Vocational Trng. A	B	C	Priorities
A	1	9	7	.77
B	1/9	1	1/5	.05
C	1/7	5	1	.17

	College Prep. A	B	C	Priorities
A	1	1/2	1	.25
B	2	1	2	.50
C	1	1/2	1	.25

	Music Classes A	B	C	Priorities
A	1	6	4	.69
B	1/6	1	1/3	.09
C	1/4	3	1	.22

Composition and Synthesis

Impacts of School on Criteria

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	**Composite Impact of Schools**
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

School A: .16*.32+.33*.14+.45*.03+.77*.13+.25*.24+.69*.14= .37

REFERENCES

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