# Multiple Features

LR with Multiple Variables

- Linear regression with multiple variables is also known as "multivariate linear regression".
- We now introduce notation for equations where we can have any number of input variables.

# Multiple features (variables).

| Size (feet²) | Price ($1000) |
|---|---|
| $x$ | $y$ |
| 2104 | 460 |
| 1416 | 232 |
| 1534 | 315 |
| 852 | 178 |
| ... | ... |

$$h_\theta(x) = \theta_0 + \theta_1 x$$

# Multiple features (variables).

| Size (feet$^2$) | Number of bedrooms | Number of floors | Age of home (years) | Price ($1000) |
|---|---|---|---|---|
| 2104 | 5 | 1 | 45 | 460 |
| 1416 | 3 | 2 | 40 | 232 |
| 1534 | 3 | 2 | 30 | 315 |
| 852 | 2 | 1 | 36 | 178 |
| ... | ... | ... | ... | ... |

# Multiple features (variables).

| Size (feet²) $x_1$ | Number of bedrooms $x_2$ | Number of floors $x_3$ | Age of home (years) $x_4$ | Price ($1000) $y$ |
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Notation:

- $n$ = number of features $n = 4$
- $x^{(i)}$ = input (features) of $i^{th}$ training example.
- $x_j^{(i)}$ = value of feature $j$ in $i^{th}$ training example.

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$n = 4$

$x^{(2)} = \begin{bmatrix} 1416 \\ 3 \\ 2 \\ 40 \end{bmatrix}$

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$x^{(2)} = \begin{bmatrix} 1416 \\ 3 \\ 2 \\ 40 \end{bmatrix}$

$x_3^{(2)} = 2$

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Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

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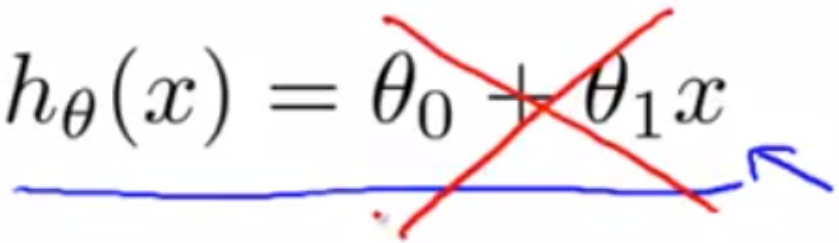

Hypothesis:

Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$

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E.g. $h_\theta(x) = 80 + 0.1 x_1 + 0.01 x_2 + 3 x_3 - 2 x_4$

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For convenience of notation, define $x_0 = 1$.

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

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$$= \theta^T x.$$

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$$h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \cdots + \theta_n x_n \quad (x_0 = 1)$$

$$= \theta^T x$$

$\theta^T$: $(n+1) \times 1$ matrix

$$[\theta_0 \ \theta_1 \cdots \theta_n] \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$\theta^T x$ $\quad x$

Multivariate linear regression.

- This is a vectorization of our hypothesis function for one training example

$$h_\theta(x) = [\theta_0 \quad \theta_1 \quad \dots \quad \theta_n] \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \theta^T x$$