lecture 6:- Nested Quantifiers.

Ex 20:- P39:- "There is an honest politician".

Quantifier. predicate subject.

there exist x, x is a politician, x is honest.

let $P(x)$: x is honest.

$x \in$ Politician.

$\neg(\exists x P(x)) = \forall x \neg P(x)$.

for all x, x is a politician, x is not honest.

$\forall x \forall y P(x,y)$ $\quad x, y \in \{1,2,3,\cdots,N\}$.

Quantifiers. predicate.

$\forall x P(x)$: $P(1) \wedge P(2) \wedge \cdots$

$\forall x (P(x,1) \wedge P(x,2) \wedge P(x,3) \wedge \cdots \wedge P(x,N))$

$= \forall x P(x,1) \wedge \forall x P(x,2) \wedge \cdots \wedge \forall x P(x,N)$.

$$\left\{ \begin{array}{l} (P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1)) \wedge \\ (P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2)) \wedge \\ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ (P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N)) \end{array} \right\}$$

$\forall x \exists y P(x,y)$ $\quad x, y \in \{1,2,3,\cdots N\}$

$\forall x (P(x,1) \vee P(x,2) \vee P(x,3) \vee \cdots \vee P(x,N))$

$= \forall x P(x,1) \vee \forall x P(x,2) \vee \cdots \vee \forall x P(x,N)$.

$= (P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1)) \vee$

$(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2)) \vee$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N))$.

$\exists x \forall y P(x,y) = ?$ ✓ $\quad \neg \forall x \forall y P(x,y)$.

$\exists x \exists y P(x,y) = ?$ ✓. $\quad = ?$

$\exists x \exists y \neg P(x,y)$.

Ex 4 p48. $Q(x,y) =$ $x + y \geq 0$.

$\exists y \forall x \, Q(x,y) = ?$ $\quad x, y \in \mathbb{R}$.

There exist y, such that for all x, y and x are real numbers $x + y \geq 0$.

$\forall x \exists y \, Q(x,y) = ?$

Ex 5 p49. $Q(xy, z) =$ $x + y = z$.

$\forall x \forall y \exists z \, Q(xy,z) = ?$ T $\quad x, y, z \in \mathbb{R}$.

$\exists z \forall x \forall y \, Q(x,y,z) = ?$ F

Ex 9 51: $\forall x (C(x)) \vee \exists y (C(y) \wedge F(x,y))$.

$C(x) =$ x has a Computer.

$F(x,y) =$ x and y are friends. $\quad x, y$ Epersons.

P(x,y) : x and y are friends.

for all x, x is a person, x has a Computer.
or there exit y, y is a person, y has a Computer
and x and y are friends.

$\forall \rightarrow$.

Ex11 p51. "If a person is a female and is a parent then this person is someones mother".

$\wedge$

$\exists$

for all x, x is person. if x is a female and x is a parent then there exist y, x is the mother of y.

$x, y \in$ persons.

$f(x)$: x is a female.

$P(x)$: x is a parent.

$M(x,y)$: x is the mother of y.

$\forall x \, (f(x) \wedge P(x)) \rightarrow \exists y M(x,y)$.

" Everyone has exactly one best friend".

$B(x,y)$: x is the best friend of y.

$\forall x \exists y \, (B(x,y) \wedge \forall z \, (z \neq y) \rightarrow \neg B(x,z))$.

Ex28 p57 (c) $\forall x \exists y \, (x^2 = y)$. $x, y \in \mathbb{R}$.

(b) $\forall x \exists y \, (x = y^2)$ " "

Ex29 p57 (a) $\forall x \forall y \, P(x,y)$. $x, y \in \{1,2,3\}$.

Ex 29 P57 (a) $\forall x \forall y\, P(x,y)$. $x, y \in \{1,2,3\}$.

Ex 11 - P55 Some students has asked every faculty member a Question.

There exist x, x is a student, for all y, y is a faculty member. x has asked y a Question.

$Q(x,y)$: x has asked y a Question.

$x \in$ student.

$y \in$ faculty.

$\exists x \forall y\, Q(x,y)$.