lecture 13:- Operations on Relations.

Ex17
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$A = \{1,2,3\}$ $B = \{1,2,3,4\}$

$R_1 = \{(1,1),(2,2),(3,3)\}$ $R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$.

$R_1 \cup R_2 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(1,4)\}$. — 1

$R_1 \cap R_2 = \{(1,1)\}$. — 2.

$R_1 - R_2 = R_1 - (R_1 \cap R_2) = \{(2,2),(3,3)\}$.

$R_2 - R_1 = R_2 - (R_1 \cap R_2) = \{(1,2),(1,3),(1,4)\}$.

$R_1 \oplus R_2 = \underbrace{R_1 \cup R_2}_{R_1} - \underbrace{R_1 \cap R_2}_{R_2}$. from 1 & 2.

$= R_1 \cup R_2 - ((R_1 \cup R_2) \cap (R_1 \cap R_2))$. = HW.

Ex19 :- $R_1 = \{(x,y) \mid x < y\}$. $R_2 = \{(x,y) \mid x > y\}$. $A = \mathbb{R}$.
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$R_1 \cup R_2 = \{(x,y) \mid x \neq y\}$.

$R_1 \cap R_2$ Φ.

$R_1 - R_2 = R_1 - (R_1 \cap R_2) = R_1$.

$R_2 - R_1 = R_2 - (R_1 \cap R_2) = R_2$

$R_1 \oplus R_2 = R_1 \cup R_2 - R_1 \cap R_2 = R_1 \cup R_2 = \{(x,y) \mid x \neq y\}$.

COMPOSITE RELATION:-

R	$A \times B$	(a,b).	$a \in A$	$b \in B$.
S	$B \times C$	(b,c).	$b \in B$	$c \in C$.

$(a,c) \in S \circ R$. if $\exists_b$ $(a,b) \in R \wedge (b,c) \in S$.

Ex20
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$A = \{1,2,3\}$ $B = \{1,2,3,4\}$. $C = \{0,1,2\}$.

$R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$. $A \times B$.

$S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$ $B \times C$.

$S \circ R = \{(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)\}$

$R \circ S =$ HW.

$R \circ R = R^2 \circ R = R^3 \circ R = R^4$.

$S \circ S = S^2 \circ S = S^3 \circ S = S^4$

$R \circ R = R^2 \circ R = R^3 \circ R = R^4$.
$S \circ S = S^2 \circ S = S^3 \circ S = S^4$

theorem :- 466. A relation R on a Set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3 \ldots$

Exercise Questions:-
Q4. 467 A relation R is defined on A = Set of people.
a) a is taller than b.
$R = \{(a,b) \mid a$ is taller than $b\}$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$.
$\forall a \in$ Set of people, a is taller than a. X.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$. X.
$\forall a,b \in$ Set of people if a is taller than b $\rightarrow$ b is taller than a.

Anti Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$. ✓
$\forall a,b \in$ Set of people. if a is taller than b $\wedge$ b is taller than a $\rightarrow a = b$.

Transitive:- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in$ Set of people if a is taller than b $\wedge$ b is taller than c $\rightarrow$ a is taller than c. ✓.

Exercise Question 6:- 466. $x = 2y$. $R = \{(x,y) \mid x = 2y\}$. $A = \mathbb{R}$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$.
$\forall a \in \mathbb{R} \quad x = 2x$. X.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{R}$. if $x = 2y \rightarrow y = 2x$. X.

Anti Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.
$\forall a,b \in$ ✓ HW ✓HW ✓HW ✓HW

Anti Symmetric :- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a=b$.

$\forall a,b \in$ ✓ HW ✓ HW ✓ HW ✓ HW.

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$. X.

$\forall a,b,c \in \mathbb{R}$ if $x=2y \wedge y=2z \rightarrow x=2z$.

$(4,2) \in R \wedge (2,1) \in R \not\rightarrow (4,1) \notin R$.