Lecture 15:- Properties of Relations

→ when relations is given in matrix form.

1:- REFLEXIVE: $\forall a \in A \quad (a,a) \in R$.

$\forall a_i \in A \quad (a_i,a_i) \in R$. $\quad A = \{a_1, a_2, a_3, \ldots, a_n\}$.

$\forall i \in \{1,2,3 \ldots n\} \quad m_{ii} = 1$.

$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad [1]$. $\quad [0]$. $\quad [\,]$ 0×0.

$|A| = 0$.

2:- SYMMETRIC. $\forall a,b \in A \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a_i, b_j \in A, B$. if $(a_i,b_j) \in R \rightarrow (b_j,a_i) \in R$. $\quad A = \{a_1, a_2, \ldots, a_n\}$. $B = \{b_1, b_2, \ldots, b_n\}$.

$\forall i,j \in \{1,2,3,4 \cdots n\}$. if $\underbrace{m_{ij} = 1}$ $\rightarrow m_{ji} = 1$.

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ✗.

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ m_{13}. m_{23}. m_{31} m_{32}.

$[1]$.

Antisymmetric:- $\forall a,b \in A \quad$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a_i, b_j \in A, B \quad$ if $(a_i,b_j) \in R \wedge (b_j,a_i) \in R \rightarrow a_i = b_j$ $\quad A = \{a_1, a_2, \ldots a_n\}$. $B = \{b_1, b_2, \ldots b_n\}$.

$\forall i,j \in \{1,2, \ldots n\}$. if $m_{ij} = 1 \wedge m_{ji} = 1 \rightarrow i = j$.

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ m_{23}. m_{32}.

SOME OPERATIONS ON RELATIONS. (MATRICES)

Union. Intersection.

$[1 \quad 0 \quad 1]$ $\quad$ $[1 \quad 0 \quad 1]$

Union. Intersection.

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$M_{R_1 - R_2} = \begin{bmatrix} \; \end{bmatrix}$ HW. $\qquad M_{R_2 - R_1} = \begin{bmatrix} \; \end{bmatrix}$ HW.

INVERSE :- R^{-1} is computed. M_R^T

COMPLEMENT :- $\bar{R}$ is u by subtracting $1 - M_R$.

COMPOSITE :- R (a,b) $A \times B$ $a \in A$ $b \in B$.
S (b,c) $B \times C$ $b \in B$ $c \in C$.

$S \circ R$ (a,c). $a \in A$ $c \in C$.

$(a,c) \in S \circ R$ iff $\exists_b$ $(a,b) \in R \land (b,c) \in S$.

$(a_i, c_j) \in S \circ R$ iff $\exists_{b_k}$ $(a_i, b_k) \in R \land (b_k, c_j) \in S$.

$A = \{a_1, a_2, a_3, \ldots, a_m\}$.
$B = \{b_1, b_2, b_3, \ldots, b_n\}$.
$C = \{c_1, c_2, c_3, \ldots, c_p\}$.

$t_{ij} = 1$. if $\exists_k$ $r_{ik} = 1 \land s_{kj} = 1$.
if $\exists_k$ $r_{1k} = 1 \land s_{k3} = 1$.

$R = M_R = [r_{ij}]$.
$S = M_S = [s_{ij}]$.

$S \circ R = M_{S \circ R} = [t_{ij}]$.

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$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{S\circ R} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ HW & HW & HW \\ HW & HW & HW \end{bmatrix}$$

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$A = \{1, 2, \ldots, 100\}$.

$R = \{(a,b) \mid a > b\}$. How many Non zero entries in R.

Ex Questions 1-29 p 481 482 HW.