Lecture 17 :- $R = \{(1,3), (1,4), (2,1), (3,2)\}$ $A = \{1,2,3,4\}$.

Missing Elements.

$(1,2), (2,3), (2,4), (3,1)$

$R' = \{(1,3), (1,4), (2,1), (3,2), (1,2), (2,3), (2,4), (3,1)\}$.

b c . a b .

$(3,4) \notin R'$.

PATH IN A GRAPH. $G = (V, E)$.

A path from Vertices a to b $(a, b \in V)$. in G.
if $\exists$ a Sequence of edges Such that $(a, x_1), (x_1, x_2) \cdots$
$\cdots\cdots (x_{n-1}, x_n), (x_n, b)$.

Ex3 :-
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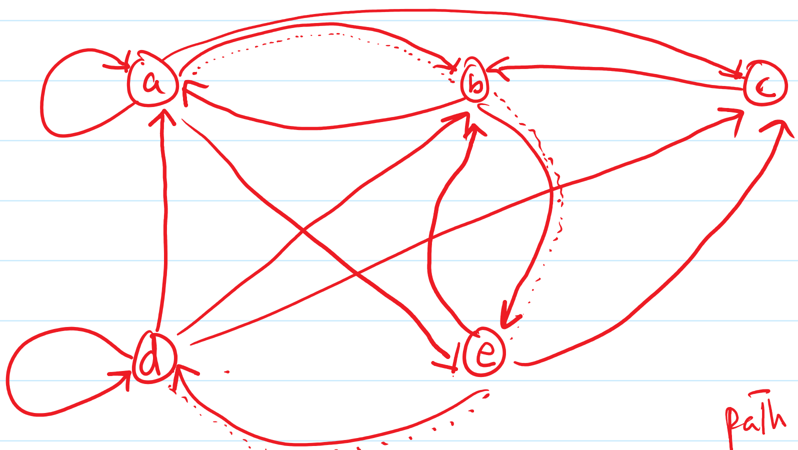

path from a to d.

Length of a path. (Vertices - 1) a b e d $4-1=3$

Number of edges. $(a,b)(b,e)(e,d)$. $=3$.

theorem 1 :-
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Let R be a Relation on A.
$\exists$ a path of length n $(n > Z^+)$
from a to b $(a, b \in V)$. iff and only if.
$(a,b) \in R^n$.

Definition 2. Let R be a Relation on A.

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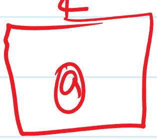
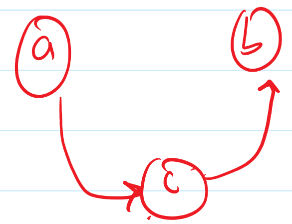
A Connectivity Relation R^* Consists of.
$(a,b) \in R^*$ iff $\exists$ a path from a to b.

$$R^* = \bigcup_{i=1}^{\infty} R^i$$

R



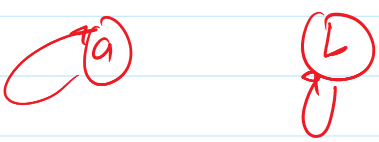
$R^* = \{ \}$.



$R^* = \{(a,c),(c,b),(a,b)\}$.

R

$R^* = \{(a,a)\}$.



$R^* = \{(a,a),(b,b)\}$.

Ex4:- $R = \{(a,b) \mid a$ has met $b\}$. $A =$ Set of people.

what is R^n = ?

what is R^* = ?

Revision.

R	(a,b)	$A \times B$	$a \in A, b \in B$
S	(b,c)	$B \times C$.	$b \in B, c \in C$.

R^n will Contain (a,b)
when $\exists$ $x_1 \dots x_{n-1}$ such that
a has met x_1
x_1 has met x_2
⋮
x_{n-2} has met x_{n-1}
x_{n-1} " " b.

$(a,c) \in S \circ R$ $\quad \exists_b \; (a,b) \in R \wedge (b,c) \in S$.

$(a,c) \in R \circ R$ $\quad \exists_b \; (a,b) \in R \wedge (b,c) \in R$.

$(a,b) \in R \circ R$ $\quad \exists_{x_1} \; (a,x_1) \in R \wedge (x_1,b) \in R$.

$(a,b) \in R^2$ $\quad \exists_{x_1}$ a has met x_1 $\wedge$ x_1 has met b.

R^* will Contain (a,b) if $\exists$ any Number people between a, b. and.

Ex 6 :- 486 $R = \{(a,b) \mid$ a and b has a Common border$\}$. A = set of states in US.

What is $R^n = ?$

What is $R^* = ?$

?

?

ANS.

Revision.

R	(a,b)	$A \times B$	$a \in A, b \in B$
S	(b,c)	$B \times C$.	$b \in B, c \in$

$(a,c) \in S \circ R$ $\quad \exists_b \ (a,b) \in R \wedge (b,c) \in S$.

$(a,c) \in R \circ R$ $\quad \exists_b \ (a,b) \in R \wedge (b,c) \in R$

$(a,b) \in R \circ R$ $\quad \exists_{x_1} \ (a,x_1) \in R \wedge (x_1,b) \in R$

$(a,b) \in R^2$ $\quad \exists_{x_1}$ a and x_1 has a Common border $\wedge$ x_1 and b has a Com border

The Algorithm for transitive Closure = WARSHAL ALGO.

P 491-492-493 Ex Q 1-30.

EQUIVELENCE RELATION. $\quad a \sim_R b. = (a,b) \in R$.

1- Reflexive.
2- Symmetric.
3- Transitive.

Ex :- 494 $\quad R = \{(a,b) \mid a - b \in \mathbb{Z}\}$ $\quad A = \mathbb{R}$.

1- Reflexive. $\quad \forall a \in A \quad (a,a) \in R$.

1- Reflexive. $\forall a \in A \quad (a,a) \in R$.
$\forall a \in \mathbb{R} \quad a - a \in \mathbb{Z}$

2- Symmetric $\forall a,b \in A \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{R} \quad$ if $a - b \in \mathbb{Z} \rightarrow b - a \in \mathbb{Z}$.

3- Transitive $\forall a,b,c \in A \quad$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{R} \quad$ if $a-b \in \mathbb{Z} \wedge b-c \in \mathbb{Z} \rightarrow a-c \in \mathbb{Z}$.

Hence Equivalence Relation

Ex4 :- $R = \{(a,b) \mid a \equiv b \mod m\} \quad m > 1 \wedge m \in \mathbb{Z}^+$
494 $\quad A = \mathbb{Z}$.

1- Reflexive. $\forall a \in A \quad (a,a) \in R$.
$\forall a \in \mathbb{Z} \quad a \equiv a \mod m$.

2- Symmetric $\forall a,b \in A \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{Z} \quad$ if $a \equiv b \mod m \rightarrow b \equiv a \mod m$ ✓

3- Transitive $\forall a,b,c \in A \quad$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z} \quad$ if $a \equiv b \mod m \wedge b \equiv c \mod m \rightarrow$
$a \equiv c \mod m$.

Hence Equivalence.

Ex6 $\quad R = \{(a,b) \mid a$ divides $b\} \quad A = \mathbb{Z}$.

1- Reflexive. $\forall a \in A \quad (a,a) \in R$.
$\forall a \in \mathbb{Z} \quad a$ divide a ✓.

2- Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$. X

$\forall a,b \in \mathbb{Z}$ if a divides b $\rightarrow$ b divides a.

3- Transitive $\forall a,b,c \in A$ if $(a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c$

Not Equivalence.

__Ex7__ :- $R = \{(a,b) \mid |a-b| < 1\}$. $A = \mathbb{R}$.

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1- Reflexive. $\forall a \in A$ $(a,a) \in R$.

$\forall a \in \mathbb{R}$ $|a-a| < 1$. ✓

2- Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in \mathbb{R}$. if $|a-b| < 1 \rightarrow |b-a| < 1$. ✓

3- Transitive $\forall a,b,c \in A$ if $(a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$ if $|a-b| < 1 \land |b-c| < 1 \rightarrow |a-c| < 1$.

0.2 0.9 0.9 1.5 0.2 − 1.5

0.7 ✓ ✓ X.

✓

Not Equivalence.