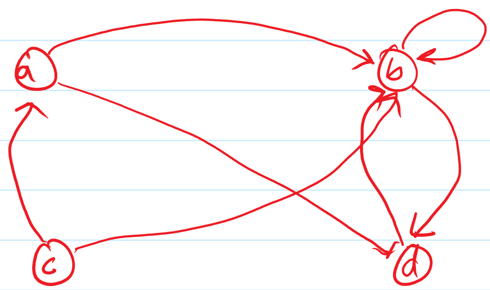
Lecture 16:- REPRESENTING RELATIONS.

GRAPHS:- ○ Vertices.
→ Edges/Arcs.

Two Sets.
Set of Vertices.
Set of Edges.

Ex7/479. Set of Vertices = {a, b, c, d}.
Set of Edges = {(a,b), (a,d), (b,b), (b,d), (c,a), (c,b), (d,b)}.

Make a Graph.



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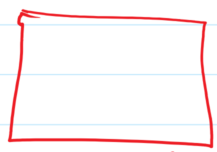


empty Graph.

Representing Relation Using Graphs.

Set of Vertices = the Set on which the Relation is defined.
Set of Edges = R.

Ex8/480 :- R = {(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)}.
A = {1, 2, 3, 4}.

RELATION → Graph.

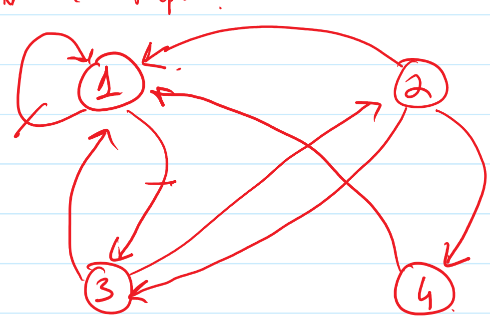


Figure 4
480.

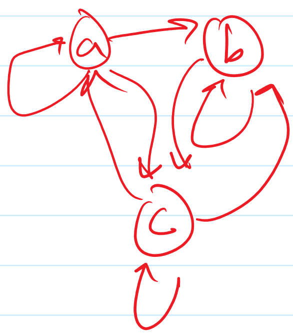
Graph → Relation

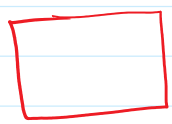
Graph → Relation

$R = \{(1,1), (1,3), (2,1), \ldots$ HW $\ldots\}$ $\qquad A = \{1,2,3,4\}$

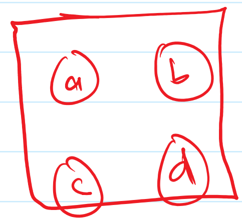
Three Equivalent forms
1- Relation in Set.
2- " " Matrix.
3- " " Graph.

1- Reflexive. $\forall a \in A \quad (a,a) \in R$.



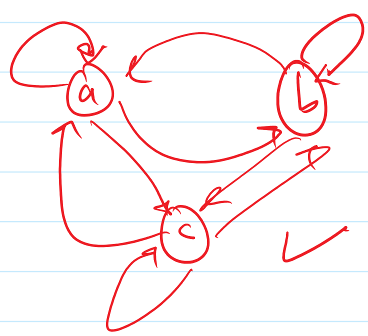


$A = \{\}$. Reflexive ✓



Reflexive = X.
$A = \{a,b,c,d\}$.

2- Symmetric $\forall_{a,b} \in A \quad (a,b) \in R \rightarrow (b,a) \in R$.



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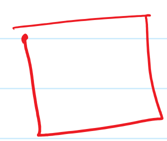
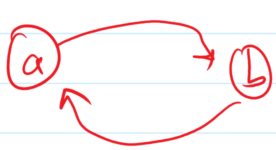
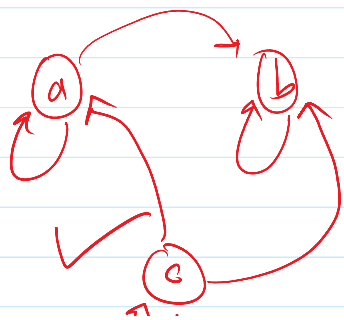


✓

3- AntiSymmetric. $\forall a,b \in A \quad$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$



X.



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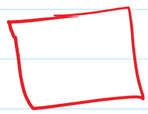
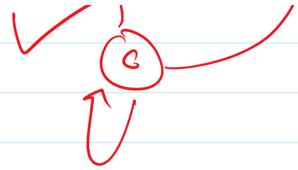
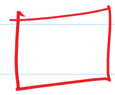
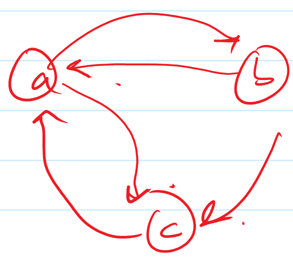




4- Transitive $\forall a,b,c \in$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.





Ex 481 - 482 . Q 1 - 3s.

CLOSURE OF A RELATION.

$R = \{(1,1),(2,1)\}$. $A = \{1,2\}$.

REFLEXIVE CLOSURE.

$R \cup \Delta$

$\{(1,1),(2,1)\} \cup \{(1,1),(2,2)\}$.

$= \{(1,1),(2,1),(2,2)\}$ ✓.

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$\Delta = \{(a,a) \mid a \in A\}$.

$\Delta = \{(1,1),(2,2)\}$.

Hw. 16 Relations on $A = \{1,2\}$. and then Reflexive.

Symmetric Closure.

$A = \{1,2\}$.

$R = \{(1,2)\}$.

$R \cup R^{-1}$

Hw. 16 Relation on $A = \{1,2\}$ and make it Symmetric.

Ex1 :- 483 $R = \{(a,b) \mid a \leq b\}$. $A = Z$.

$\Delta = \{(a,a) \mid a \in Z\}$

Ex1 :- 483

$R = \{(a,b) \mid a \le b\}$. $A = \mathbb{Z}$.

$\Delta = \{(a,a) \mid a \in \mathbb{Z}\}$.

$R \cup \Delta = \{(a,b) \mid a \le b\} \cup \{(a,a) \mid a \in \mathbb{Z}\} = R$.

Ex2: 480.

$R = \{(a,b) \mid a > b\}$ $A = \mathbb{Z}^+$

$R \cup R^{-1} = \{(a,b) \mid a > b\} \cup \{(a,b) \mid b > a\}$.

$= \{(a,b) \mid a \ne b\}$.

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$

$= \{(a,b) \mid (b,a) \in R\}$.

Transitive Closure:-

$R = \{(1,3),(1,4),(2,1),(3,2)\}$ $A = \{1,2,3,4\}$.

missing $\Rightarrow$ $(1,2), (2,3), (2,4), (3,1)$.

$R = \{(1,3),(1,4),(2,1),(3,2),(1,2),(2,3),(2,4),(3,1)\}$.

b c a b.

$(3,1) \wedge (1,4)$

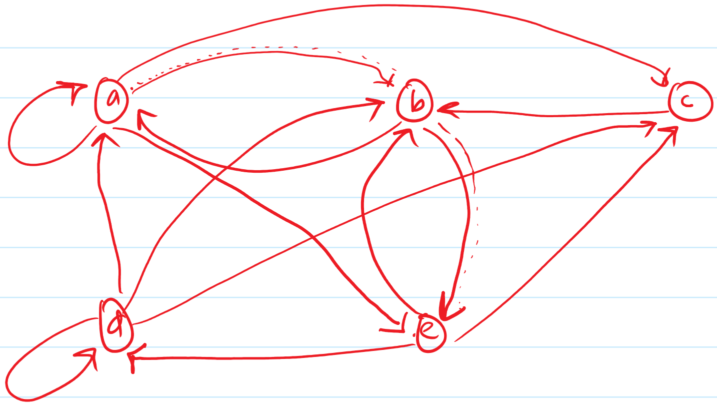
$(3,4) \notin R$.

=

Still Not Transitive.

PATH:- A path from a to b in graph G is a sequence of edges $(a,x_1),(x_1,x_2),(x_2,x_3)$ $\ldots (x_{n-1},x_n),(x_n,b)$. of length n.

Ex3 484.



a – e.

length of path . a → b → e . (a,b) (b,e).

theorem 1:- R be the Relation on A.
there is a path of length n $n \in \mathbb{Z}^+$
from a to b iff $(a,b) \in R^n$.

Find. R^2 of the Relation Given above.
and Confirm length 2 Paths.