lecture 18 :- Equivelence Classes.

$[a]_R = \{ s \mid (a, s) \in R \}$.

Ex 8 : 496. Find the equivelences classes of Integers in

$R = \{(a,b) \mid a = b \text{ or } a = -b\}$ $A = \mathbb{Z}$.

$[1]_R = \{1, -1\}. = (1,1), (1,-1), (-1,-1), (-1,1)$

$[-1] = \{1, -1\}. =$ tuples = ? HW.

Ex 9 :- 496. find the equivelence classes of 0 & 1 for Congruence modulo 4 ?

(a) $\equiv 0 \mod 4$.   (b) $\equiv 1 \mod 4$.

$[0] = \{0, \pm 4, \pm 8, \pm 12, \pm 16, \pm 20, \cdots \infty\}$.

$[1] = \left\{ \begin{matrix} 1, 5, 9, 13, \cdots\cdots \infty \\ -3, -7, -11, -15, -19, \cdots -\infty \end{matrix} \right\}$.

$[2] =$? HW

$[3] =$? HW.

$4\overline{)0}$ quotient 0, -0, remainder 0.

$4\overline{)1}$ quotient 0, ± 0, remainder 1.

$4\overline{)-3}$ quotient -1, ∓ 4, remainder 1.

$4\overline{)5}$ quotient 1, ∓ 4, remainder 1.

Ex 10 496 HW.

EQUIVELENCE CLASSES AND PARTITIONS.

PARTITION :-

def. $P = \{P_1, P_2, P_3, \cdots P_n\}$. is a Partition of the set U.

if (i) $\forall_i \; P_i \neq \phi$.

(ii) $\forall_{ij} \; P_i \cap P_j = \phi$.

(iii) $\bigcup_{i=1}^{n} P_i = U$.

P.

(iii) $\bigcup_{i=1}^{n} P_i = U$.

Ex12 497.

$$\boxed{S = \{1, 2, 3, 4, 5, 6\}}$$

U

P.

$$\left\{ A_1 = \{\underline{1, 2, 3}\},\ A_2 = \{\underline{4, 5}\},\ A_3 = \{\underline{6}\} \right\}$$

$A_1 \neq \phi$, $A_2 \neq \phi$ $A_3 \neq \phi$ $\therefore$ (i) Holds.

$A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, $A_2 \cap A_3 = \phi$ $\therefore$ (ii) Holds.

$A_1 \cup A_2 \cup A_3 = S$

★ Every Equivelence Relation. Creates a partition.

ER $\rightarrow$ EQ $\rightarrow$ Partition.

ER. $\leftarrow$ EQ. $\leftarrow$ Partition.

Ex13 :- 499

$\underline{A_1} = \{1, 2, 3\}$. $\underline{A_2} = \{4, 5\}$. $\underline{A_3} = \{6\}$.

$\underline{S} = \{1, 2, 3, 4, 5, 6\}$.

$R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)$.
$(4,5), (4,4), (5,4), (5,5), (6,6)\}$.

$[1] = \{1, 2, 3\}$. $[4] = \{4, 5\}$. $[6] = \{6\}$.

$[2] =$ " $[5] =$ ".

$[3] =$ ".

Ex14 :- 499 what are the sets of Integers arising from Congruent modulo 4?

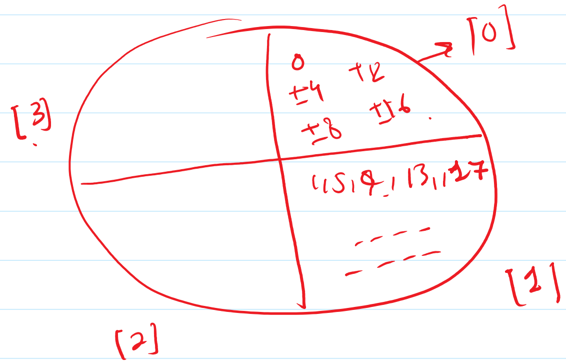
$[0] = \{0, \pm 4, \pm 8, \pm 9, \cdots\}$.

$[1] = \{1, 5, 9, 13, 17, \cdots, -3, -7, -11, -15, \cdots\}$.

$[2] = ?$ HW

$[3] = ?$ HW.

0 ± 4 ± 8 $\rightarrow [0]$



Ex Q 1-30 HW.
500

Find $[n]_5$ $n=3$. modulo5.

$R=\{(a,b) \mid a \equiv b \mod 5\}$.

$a \equiv 3 \mod 5$

$[3]=\{3,8,13,18,23,28,\dots\}$.

-ve HW.

PARTIAL ORDERINGS:-

1- Reflexive.
2- Anti Symmetric.
3- Transitive.

$a \sim_R b \Leftrightarrow (a,b)\in R$.

$a \preceq_R b \Leftrightarrow (a,b)\in R$.

Ex1
504

$R=\{(a,b) \mid a \geq b\}$. $A=\mathbb{Z}$.

1) Reflexive. $\forall a\in A$ $(a,a)\in R$.

$\forall a \in \mathbb{Z}$ $a \geq a$ ✓

2) Anti Symmetric $\forall a,b \in A$ if $(a,b)\in R \land (b,a)\in R \rightarrow a=b$.

$\forall a,b\in\mathbb{Z}$ if $a\geq b \land b \geq a \rightarrow a=b$. ✓

3). Transitive $\forall a,b,c \in A$ if $(a,b)\in R \land (b,c)\in R \rightarrow (a,c)\in R$.

$\forall a,b,c\in\mathbb{Z}$ if $a\geq b \land b\geq c \rightarrow a \geq c$. ✓

Ex2. 504

$R = \{(a, b) \mid a \text{ divides } b\}$. $A = \mathbb{Z}^+$.

1) Reflexive. $\forall a \in A \quad (a, a) \in R$.
$\forall a \in \mathbb{Z}^+ \quad a$ divides a ✓.

2) Anti Symmetric $\forall a, b \in A$ if $(a, b) \in R \wedge (b, a) \in R \rightarrow a = b$.
$\forall a, b \in \mathbb{Z}^+$ if a divides b $\wedge$ b divides a $\rightarrow a = b$ ✓.

3). Transitive $\forall a, b, c \in A$ if $(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R$
$\forall a, b, c \in \mathbb{Z}^+$ if a divides b $\wedge$ b divides c $\rightarrow$ a divides c.
✓

POSET. $(S, \preceq)$.

$(\mathbb{Z}, \geq)$.

$R = \{(a, b) \mid a \geq b\}$ $A = \mathbb{Z}$

$\preceq = \geq$
$\preceq = \mid$
$\preceq = \leq$
$\preceq = \subseteq$.

$(\mathbb{Z}^+, \mid) \Leftrightarrow R = \{(a, b) \mid a \mid b\}$.
$A = \mathbb{Z}^+$.

Definitions:

1 Comparable.

two elements a and b in the Poset $(S, \preceq)$.
if either $a \preceq b$ or $b \preceq a$.
$(a, b) \in R$ $\quad$ $(b, a) \in R$.

Ex: $(3, 5)$ $\quad (3, 7)$, $\quad (5, 7)$. are they Comparable in $(\mathbb{Z}^+, \mid)$.

3/5 or 5/3 3/7 or 7/3 5/7 or 7/5
$a \preceq b$ or $b \preceq a$.
Not Comparable. Not Comparable. Not Comparable.

(3, 9)
3/9 or 9/3 ✓.
Comparable.

Total Order: A Partial Order $(S, \preceq)$ is a total order.
when for all $a, b \in S$ a is comparable to b.

Exs 504:- Show that Inclusion or $\subseteq$ is a partial order
on the powerset of any set. $(P(S), \subseteq)$.

$R = \{ (a, b) \mid a \subseteq b \}$.

$S = \{a, b, c\}$.