

Assignment #2

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BS (CS) – 5A

jupyter 01-Vector_matrices Last Checkpoint: a few seconds ago (autosaved)



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Code

Vectors

```
In [1]: 1 class Vector:
2       2 def __init__(self, x=0.0, y=0.0):
3         self.x = x
4         self.y = y
5
6       3 def __str__(self):
7         return "{} , {}".format(str(self.x), str(self.y))
8
```

```
In [2]: 1 a = Vector(2,4)
```

```
In [3]: 1 print(a)
[2 , 4]
```

```
In [4]: 1 b = Vector(5,2)
2       2 print(b)
[5 , 2]
```

```
In [5]: 1 def add(self, b):
2       2 c = Vector()
3         c.x = self.x + b.x
4         c.y = self.y + b.y
5         return c
6       3 Vector.add = add
```

```
In [6]: 1 c = a.add(b)
2       2 print(c)
```

```
In [6]: 1 c = a.add(b)
2       2 print(c)
[7 , 6]
```

```
In [7]: 1 def mul(self, s):
2       2 return Vector(s * self.x, s * self.y)
3
4       3 Vector.mul = mul
```

```
In [8]: 1 d = a.mul(2)
2       2 print(d)
[4 , 8]
```

```
In [9]: 1 def sub(self, b):
2       2 return self.add( b.mul(-1))
3
4       3 Vector.sub = sub
```

```
In [10]: 1 d_min_b = d.sub(b)
2        2 print(d_min_b)
[-1 , 6]
```

Matrix Representation (Dense)

```
In [11]: 1 class Matrix:
2
3         def __init__(self, dims, fill):
4             self.rows = dims[0]          # 3
5             self.cols = dims[1]          # 4
6
7             self.A = [
8                 [fill] * self.cols for i in range(self.rows)
9             ]
10
```

```
In [12]: 1 m = Matrix((3,4), 2.0)
```

```
In [13]: 1 print(m)
```

<__main__.Matrix object at 0x7ff81c625940>

```
In [14]: 1 def __str__(self):
2         rows = len(self.A) # getting the first dimension
3         ret = ''
4
5         for i in range(rows):
6             cols = len(self.A[i])
7             for j in range(cols):
8                 ret += str(self.A[i][j]) + "\t"
9             ret += "\n"
10        return ret
11
12 Matrix.__str__ = __str__
```

```
In [15]: 1 print(m)
```

```
2.0    2.0    2.0    2.0
2.0    2.0    2.0    2.0
2.0    2.0    2.0    2.0
```

```
In [16]: 1 %time n = Matrix((100, 100), 0.0)
```

CPU times: user 4.77 ms, sys: 185 µs, total: 4.96 ms
Wall time: 4.56 ms

Memory Usage

```
In [17]: 1 from sys import getsizeof
2         print(getsizeof(m))
3         print(getsizeof(n))
```

```
48
48
```

```
In [18]: 1 from pympler.asizeof import asizeof
```

```
In [19]: 1 asizeof(m), asizeof(n)
```

```
Out[19]: (760, 86880)
```

```
In [20]: 1 dim = 5000
```

```
In [21]: 1 %time m = Matrix((dim, dim), 0.0)
```

CPU times: user 223 ms, sys: 117 ms, total: 340 ms
Wall time: 335 ms

```
In [22]: 1 size = sizeof(m)/(1024*1024)
2         print("{:.2f} MBs".format(size))
```

191.04 MBs

```
In [23]: 1 def get(self, i, j):
2         if i<0 or i>self.rows:
3             raise ValueError("Rows index out of range.")
4         if j<0 or j>self.cols:
5             raise ValueError("Column index out of range.")
6
7         return self.A[i][j]
8         Matrix.get = get
```

```
In [24]: 1 m.get(1,2)
```

Out[24]: 0.0

```
In [25]: 1 m.get(15, 0)
```

Out[25]: 0.0

```
In [26]: 1 m.get(1,10)
```

Out[26]: 0.0

Matrix Representation (Sparse)

```
In [27]: 1 class Matrix:
2         def __init__(self, dims):
3             self.rows = dims[0]
4             self.cols = dims[1]
5             self.vals = {}
```

```
In [28]: 1 def set(self, i, j, val):
2         self.vals[ (i,j) ] = val
3
4         Matrix.set = set
```

```
In [29]: 1 def get(self, i, j):
2         if i<0 or i>self.rows:
3             raise ValueError("Rows index out of range.")
4         if j<0 or j>self.cols:
5             raise ValueError("Column index out of range.")
6         if (i,j) in self.vals:
7             return self.vals[(i,j)]
8
9         return 0.0
10
11         Matrix.get = get
```

```
In [30]: 1 m = Matrix((5,5))
```

```
In [31]: 1 print(m.vals)
```

{}

```
In [32]: 1 m.get(1,1)
```

```
Out[32]: 0.0
```

```
In [33]: 1 m.get(10,2)
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-33-2353838e11b2> in <module>
----> 1 m.get(10,2)

<ipython-input-29-41dee7363588> in get(self, i, j)
      1 def get(self, i, j):
      2     if i<0 or i>self.rows:
----> 3         raise ValueError("Rows index out of range.")
      4     if j<0 or j>self.cols:
      5         raise ValueError("Column index out of range.")

ValueError: Rows index out of range.
```

```
In [34]: 1 m.set(1,2,15.0)
```

```
In [35]: 1 m.get(1,2)
```

```
Out[35]: 15.0
```

```
In [36]: 1 m.vals
```

```
Out[36]: {(1, 2): 15.0}
```

```
In [37]: 1 m.set(1,4, 29.9)
```

```
In [38]: 1 m.get(1,4)
```

```
Out[38]: 29.9
```

```
In [39]: 1 dim = 500
          2 m = Matrix((dim, dim))
```

```
In [40]: 1 asizeof(m)
```

```
Out[40]: 416
```

Numpy Intro

```
In [ ]: 1 %matplotlib inline
        2 %run mplimp.py
```

```
In [2]: 1 import numpy as np
```

```
In [3]: 1 np.random.seed(1337)
```

Basics of Matrices

```
In [4]: 1 x = np.array( [1, 4, 3] )
        2 x
```

```
Out[4]: array([1, 4, 3])
```

```
In [5]: 1 y = np.array([ [1, 4, 3],
        2                  [9, 2, 7] ] )
        3 y
```

```
Out[5]: array([[1, 4, 3],
               [9, 2, 7]])
```

```
In [6]: 1 x.shape
```

```
Out[6]: (3,)
```

```

In [7]: 1 y.shape
Out[7]: (2, 3)

In [8]: 1 z = np.array( [ [1, 4, 3] ] )

In [9]: 1 z.shape
Out[9]: (1, 3)

In [10]: 1 z = np.arange(1, 2000, 1) # start, end, step
2 z[:10]
Out[10]: array([ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10])

In [11]: 1 z.shape
Out[11]: (1999,)

In [12]: 1 np.arange(0.5, 3, 0.5)
Out[12]: array([0.5, 1. , 1.5, 2. , 2.5])

In [13]: 1 np.arange(0.5, 10, 1).shape
Out[13]: (10,)

In [14]: 1 np.arange(0.5, 10, 1).reshape(5, 2).shape
Out[14]: (5, 2)

In [15]: 1 np.arange(0.5, 10, 1).reshape(5, 3).shape
-----
ValueError                                Traceback (most recent call last)
<ipython-input-15-7df1cdd41309> in <module>
----> 1 np.arange(0.5, 10, 1).reshape(5, 3).shape

ValueError: cannot reshape array of size 10 into shape (5,3)

In [16]: 1 # Evenly spaced but we don't know the step
2 np.linspace(3, 9, 10)
Out[16]: array([3.        , 3.66666667, 4.33333333, 5.        , 5.66666667,
6.33333333, 7.        , 7.66666667, 8.33333333, 9.        ])

In [17]: 1 print(x)
2 print(x[1])
3 print(x[1:])
[1 4 3]
4
[4 3]

In [18]: 1 print(y)
2 y[0, 1]
[[1 4 3]
 [9 2 7]]

Out[18]: 4

```

```
In [19]: 1 y[:, 1]
```

```
Out[19]: array([4, 2])
```

```
In [20]: 1 y[:, [1, 2]]
```

```
Out[20]: array([[4, 3],  
               [2, 7]])
```

Matrix Operations

```
In [21]: 1 np.zeros((3, 5))
```

```
Out[21]: array([[0., 0., 0., 0., 0.],  
               [0., 0., 0., 0., 0.],  
               [0., 0., 0., 0., 0.]])
```

```
In [22]: 1 np.ones((5, 3))
```

```
Out[22]: array([[1., 1., 1.],  
               [1., 1., 1.],  
               [1., 1., 1.],  
               [1., 1., 1.],  
               [1., 1., 1.]])
```

```
In [23]: 1 a = np.arange(1, 7)  
2 a
```

```
Out[23]: array([1, 2, 3, 4, 5, 6])
```

```
In [23]: 1 a = np.arange(1, 7)  
2 a
```

```
Out[23]: array([1, 2, 3, 4, 5, 6])
```

```
In [24]: 1 a.shape
```

```
Out[24]: (6,)
```

```
In [25]: 1 a[3] = 7  
2 a
```

```
Out[25]: array([1, 2, 3, 7, 5, 6])
```

```
In [26]: 1 a[:3] = 1  
2 a
```

```
Out[26]: array([1, 1, 1, 7, 5, 6])
```

```
In [27]: 1 a[1:4] = [9, 8, 7]  
2 a
```

```
Out[27]: array([1, 9, 8, 7, 5, 6])
```

```
In [28]: 1 b = np.zeros((2, 2))  
2 b[0, 0] = 1  
3 b[0, 1] = 2  
4 b[1, 1] = 4  
5 b
```

```
Out[28]: array([[1., 2.],  
               [0., 4.]])
```

```
In [29]: 1 b.shape
```

```
Out[29]: (2, 2)
```

Array Operations

```
In [30]: 1 print(b)
```

```
[[1. 2.]  
 [0. 4.]]
```

```
In [31]: 1 b + 2
```

```
Out[31]: array([[3., 4.],  
               [2., 6.]])
```

```
In [32]: 1 2 * b
```

```
Out[32]: array([[2., 4.],  
               [0., 8.]])
```

```
In [33]: 1 b ** 2
```

```
Out[33]: array([[ 1.,  4.],  
               [ 0., 16.]])
```

```
In [34]: 1 sum(b)
```

```
Out[34]: array([1., 6.] )
```

```
In [34]: 1 sum(b)
```

```
Out[34]: array([1., 6.] )
```

```
In [35]: 1 b.sum()
```

```
Out[35]: 7.0
```

```
In [36]: 1 b
```

```
Out[36]: array([[1., 2.],  
               [0., 4.]])
```

```
In [37]: 1 b.sum(axis=0).shape
```

```
Out[37]: (2,)
```

```
In [38]: 1 b.sum(axis=1).shape
```

```
Out[38]: (2,)
```

```
In [39]: 1 b = np.array([[1, 2], [3, 4]])  
2 d = np.array([[3, 4], [5, 6]])
```

```
In [40]: 1 print(b)  
2 print(d)
```

```
[[1 2]  
 [3 4]]  
[[3 4]  
 [5 6]]
```

```
In [41]: 1 b + d
```

```
Out[41]: array([[ 4,  6],  
               [ 8, 10]])
```

```
In [42]: 1 b * d
```

```
Out[42]: array([[ 3,  8],  
               [15, 24]])
```

```
In [43]: 1 b.dot(d)
```

```
Out[43]: array([[13, 16],  
               [29, 36]])
```

```
In [44]: 1 b ** d
```

```
Out[44]: array([[ 1, 16],  
               [243, 4096]])
```

```
In [45]: 1 b.T
```

```
Out[45]: array([[1, 3],  
               [2, 4]])
```

```
In [46]: 1 a
```

```
Out[46]: array([1, 9, 8, 7, 5, 6])
```

```
In [47]: 1 a.shape
```

```
Out[47]: (6,)
```

```
In [48]: 1 a.T
```

```
Out[48]: array([1, 9, 8, 7, 5, 6])
```

```
In [49]: 1 a.T.shape
```

```
Out[49]: (6,)
```

```
In [50]: 1 a.reshape(6,1).T.shape
```

```
Out[50]: (1, 6)
```

```
In [51]: 1 # Numpy has "broadcasting" or "mapping" functions  
2 print(np.sqrt(36))  
3  
4 # works on both scalars and arrays  
5 x = [1, 4, 9, 16]  
6 np.sqrt(x)  
  
6.0
```

```
Out[51]: array([1., 2., 3., 4.])
```

```
In [52]: 1 x = np.array([1, 2, 4, 5, 9, 3])  
2 y = np.array([0, 2, 3, 1, 2, 3])
```

```
In [53]: 1 x > 3
```

```
Out[53]: array([False, False,  True,  True,  True, False])
```



```
In [54]: 1 x > y
```

```
Out[54]: array([ True, False,  True,  True,  True, False])
```

Other Operations with Numpy

```
In [55]: 1 import math
2 def basic_sigmoid(x):
3     """
4     Compute sigmoid of x.
5
6     Arguments:
7     x -- A scalar
8
9     Return:
10    s -- sigmoid(x)
11    """
12
13    s = 1./(1. + math.e ** (-x))
14
15    return s
```

```
In [56]: 1 basic_sigmoid(-1)
```

```
Out[56]: 0.2689414213699951
```

```
In [57]: 1 basic_sigmoid(0)
```

```
Out[57]: 0.5
```

```
In [58]: 1 x = [-1, 0, 3]
2 basic_sigmoid(x)
```

```
-----
TypeError                                 Traceback (most recent call last)
<ipython-input-58-f81e08c4b17c> in <module>
      1 x = [-1, 0, 3]
----> 2 basic_sigmoid(x)

<ipython-input-55-6b445ed2e437> in basic_sigmoid(x)
     11     """
     12
--> 13     s = 1./(1. + math.e ** (-x))
     14
     15     return s

TypeError: bad operand type for unary -: 'list'
```

```
In [59]: 1 import numpy as np
2
3 x = [-1, 0, 3]
4 x = np.array(x)
5 basic_sigmoid(x)
```

```
Out[59]: array([0.26894142, 0.5          , 0.95257413])
```

Broadcasting

```
In [60]: 1 import numpy as np
```

```
In [61]: 1 # What is broadcasting?
2 x = np.array([1, 2, 3])
3 x * 3 # This makes sense
```

```
Out[61]: array([3, 6, 9])
```

```
In [62]: 1 x + 3
```

```
Out[62]: array([4, 5, 6])
```

```
In [63]: 1 x = np.arange(4)
2         xx = x.reshape(4, 1)
3         y = np.ones(5)
4         z = np.ones((3,4))
5
6         print("x = ", x)
7         print("xx = ", xx)
8         print("y = ", y)
9
10        print("Shapes: ")
11        print(x.shape)
12        print(xx.shape)
13        print(y.shape)
```

```
x = [0 1 2 3]
xx = [[0]
      [1]
      [2]
      [3]]
y = [1. 1. 1. 1. 1.]
Shapes:
(4,)
(4, 1)
(5,)
```

```
In [64]: 1 x + y
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-64-cd60f97aa77f> in <module>
----> 1 x + y

ValueError: operands could not be broadcast together with shapes (4,) (5,)
```

```
In [65]: 1 xx.shape, x.shape
```

```
Out[65]: ((4, 1), (4,))
```

```
In [66]: 1 print(y)
2         print(xx)
```

```
[1. 1. 1. 1. 1.]
[[0]
 [1]
 [2]
 [3]]
```

```
In [67]: 1 out = xx + y
```

```
In [68]: 1 out
```

```
Out[68]: array([[1., 1., 1., 1., 1.],
                [2., 2., 2., 2., 2.],
                [3., 3., 3., 3., 3.],
                [4., 4., 4., 4., 4.]])
```

```
In [69]: 1 out.shape
```

```
Out[69]: (4, 5)
```

```
In [70]: 1 np.array([1]) + y
```

```
Out[70]: array([2., 2., 2., 2., 2.])
```

```
In [71]: 1 print(z)
2         z.shape
```

```
[[1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]]
```

```
Out[71]: (3, 4)
```

```
In [72]: 1 x.shape
```

```
Out[72]: (4,)
```

```
In [73]: 1 x
```

```
Out[73]: array([0, 1, 2, 3])
```

```
In [74]: 1 z + x
```

```
Out[74]: array([[1., 2., 3., 4.],
                [1., 2., 3., 4.],
                [1., 2., 3., 4.]])
```

```
In [75]: 1 a = np.array([[ 0.0 , 0.0, 0.0 ],
2                 [ 10.0, 10.0, 10.0 ],
3                 [ 20.0, 20.0, 20.0 ],
4                 [ 30.0, 30.0, 30.0 ]])
5
6 b = np.array( [ 1.0, 2.0, 3.0 ] )
```

```
In [76]: 1 a + b
```

```
Out[76]: array([[ 1.,  2.,  3.],
               [11., 12., 13.],
               [21., 22., 23.],
               [31., 32., 33.]])
```

```
In [77]: 1 a = np.array([[ 0.0 , 0.0, 0.0 , 0.0 ],
2                 [ 10.0, 10.0, 10.0 , 10.0 ],
3                 [ 20.0, 20.0, 20.0 , 20.0 ],
4                 [ 30.0, 30.0, 30.0 , 30.0 ]])
5
6 b = np.array( [ 1.0, 2.0 ] )
```

```
In [78]: 1 a + b
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-78-bd58363a63fc> in <module>
----> 1 a + b

ValueError: operands could not be broadcast together with shapes (4,4) (2,)
```

```
In [79]: 1 a = np.array( [ 0.0, 10.0, 20.0, 30.0 ] )
2 b = np.array( [ 1.0, 2.0, 3.0 ] )
```

```
In [80]: 1 a + b
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-80-bd58363a63fc> in <module>
----> 1 a + b

ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

```
In [81]: 1 print(a.shape, b.shape)

(4,) (3,)
```

```
In [82]: 1 a1 = a.reshape(4, 1)
```

```
In [83]: 1 a1 + b
```

```
Out[83]: array([[ 1.,  2.,  3.],
               [11., 12., 13.],
               [21., 22., 23.],
               [31., 32., 33.]])
```

```
In [84]: 1 print(a1.shape, b.shape)

(4, 1) (3,)
```

```
In [85]: 1 print(a1)
```

```
[[ 0.]
 [10.]
 [20.]
 [30.]]
```

Normalizing Rows and Columns

```
In [86]: 1 x = np.array([ [ 0, 3, 4 ],
2                        [ 1, 6, 4 ] ])
```

```
In [87]: 1 x_norm = np.amax(x, axis=1) # get row-wise max
2 print(x_norm)
```

```
[4 6]
```

```
In [88]: 1 x / x_norm
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-88-8a1a7e8482dd> in <module>
----> 1 x / x_norm

ValueError: operands could not be broadcast together with shapes (2,3) (2,)
```

```
In [89]: 1 print(x.shape)
2 print(x_norm.shape)
```

```
(2, 3)
(2,)
```

```
In [90]: 1 x_norm = x_norm.reshape(2, 1)
```

```
In [91]: 1 x_norm
```

```
Out[91]: array([[4],
               [6]])
```

```
In [92]: 1 x / x_norm
```

```
Out[92]: array([[0.        , 0.75       , 1.        ],
               [0.16666667, 1.        , 0.66666667]])
```

```
In [93]: 1 # We can also use another normalization method
2 x = np.array([ [ 0, 3, 4 ],
3               [ 1, 6, 4 ] ])
4
5 # no need to reshape again
6 x_norm = np.linalg.norm(x, ord = 2, axis = 1, keepdims = True)
7 x / x_norm
```

```
Out[93]: array([[0.        , 0.6        , 0.8        ],
               [0.13736056, 0.82416338, 0.54944226]])
```

```
In [94]: 1 x_norm.shape
```

```
Out[94]: (2, 1)
```

Reshaping Revisited

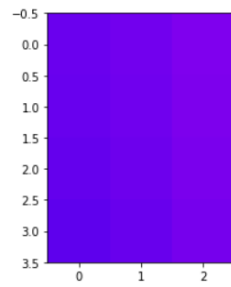
```
In [95]: 1 i = np.array([
2           [ [ 0.1, 0.1, 0.9 ], [ 0.2, 0.1, 0.9 ], [ 0.3, 0.1, 0.9 ] ],
3           [ [ 0.1, 0.2, 0.9 ], [ 0.2, 0.2, 0.9 ], [ 0.3, 0.2, 0.9 ] ],
4           [ [ 0.1, 0.3, 0.9 ], [ 0.2, 0.3, 0.9 ], [ 0.3, 0.3, 0.9 ] ],
5           [ [ 0.1, 0.4, 0.9 ], [ 0.2, 0.4, 0.9 ], [ 0.3, 0.4, 0.9 ] ]
6           ])
7
8
9
10 ])
```

```
In [96]: 1 i.shape
```

```
Out[96]: (4, 3, 3)
```

```
In [97]: 1 %matplotlib inline
2 import matplotlib.pyplot as plt
3 import numpy as np
```

```
In [98]: 1 _ = plt.imshow(i)
```



```
In [99]: 1 i.reshape(36)
```

```
Out[99]: array([0.1, 0.1, 0.9, 0.2, 0.1, 0.9, 0.3, 0.1, 0.9, 0.1, 0.2, 0.9, 0.2,  
               0.2, 0.9, 0.3, 0.2, 0.9, 0.1, 0.3, 0.9, 0.2, 0.3, 0.9, 0.3, 0.3,  
               0.9, 0.1, 0.4, 0.9, 0.2, 0.4, 0.9, 0.3, 0.4, 0.9])
```

```
In [100]: 1 print(i.shape)  
          2 i_sh = i.shape  
          3 i.reshape(i_sh[0] * i_sh[1] * i_sh[2], 1)  
  
(4, 3, 3)
```

```
(4, 3, 3)
```

```
Out[100]: array([[0.1],  
                [0.1],  
                [0.9],  
                [0.2],  
                [0.1],  
                [0.9],  
                [0.3],  
                [0.1],  
                [0.9],  
                [0.1],  
                [0.2],  
                [0.9],  
                [0.2],  
                [0.2],  
                [0.9],  
                [0.3],  
                [0.2],  
                [0.9],  
                [0.1],  
                [0.3],  
                [0.9],  
                [0.2],  
                [0.3],  
                [0.9],  
                [0.3],  
                [0.3],  
                [0.9],  
                [0.1],  
                [0.4],  
                [0.9],  
                [0.2],  
                [0.4],  
                [0.9],  
                [0.3],  
                [0.4],  
                [0.9]])
```

Vectorization

```
In [101]: 1 dim = 100
```

```
In [102]: 1 A = np.random.rand(dim, dim)
          2 B = np.random.rand(dim, dim)
```

```
In [103]: 1 A[0].size, A[1].size
```

```
Out[103]: (100, 100)
```

```
In [104]: 1 def add_arrays(A, B):
          2     C = np.zeros((A[0].size, A[1].size))
          3
          4     for i in range(A[0].size):
          5         for j in range(A[1].size):
          6             C[i, j] = A[i, j] + B[i, j]
          7     return C
```

```
In [105]: 1 %time C = add_arrays(A, B)
```

```
CPU times: user 5.38 ms, sys: 262 µs, total: 5.64 ms
Wall time: 5.53 ms
```

```
In [106]: 1 %time C = A + B
```

```
CPU times: user 171 µs, sys: 35 µs, total: 206 µs
Wall time: 155 µs
```

```
In [107]: 1 import time
          2
          3 # Non-vectorized time
          4 start = time.time()
          5
          6 C = add_arrays(A, B)
          7
          8 end = time.time()
          9 non_vec_time = end - start
         10
         11 ## Vectorized time
         12 start = time.time()
         13
         14 C = A + B
         15
         16 end = time.time()
         17 vec_time = end - start
         18
```

```
In [108]: 1 vec_time / non_vec_time * 100
```

```
Out[108]: 2.956498207157838
```

```
In [109]: 1 %%time
          2 total = 0
          3 for i in np.arange(100_000_000):
          4     total += i
          5 print(total)
```

```
49999999500000000
CPU times: user 14.6 s, sys: 79.3 ms, total: 14.7 s
Wall time: 14.7 s
```

```
In [110]: 1 %time sum(np.arange(100_000_000))
```

```
CPU times: user 7.4 s, sys: 100 ms, total: 7.5 s
Wall time: 7.47 s
```

```
Out[110]: 49999999500000000
```

```
In [ ]: 1
```