Lecture # 16

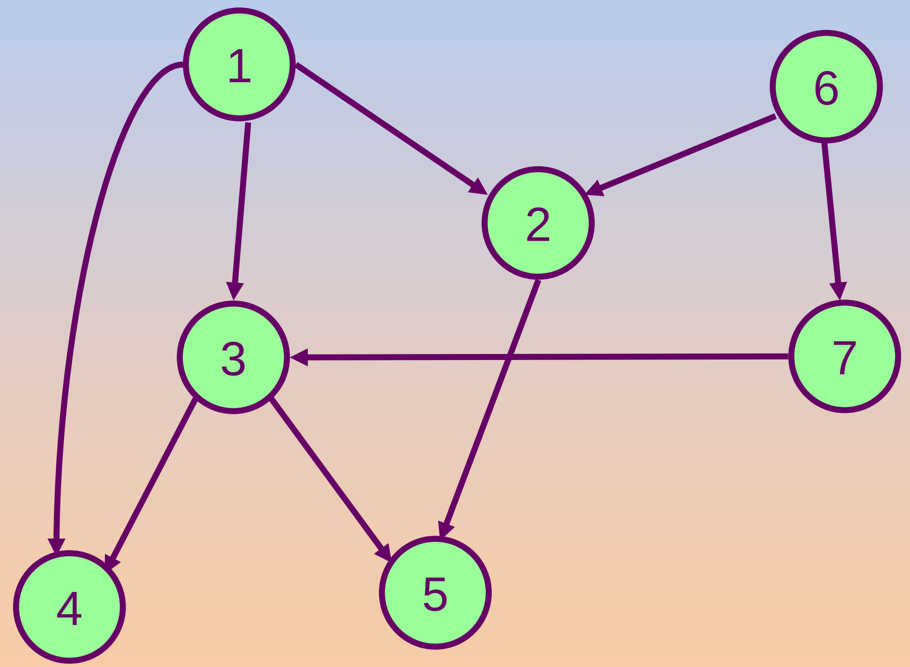
- Topological Sort Algorithm for Directed Acyclic Graph

Outline of Presentation

- What is Directed Acyclic Graph (DAG)?
- Topological Sort
- Existence of Topological Sort
- Topological Sort – Applications
- Topological Sort – Algorithm
- Example 1 – Getting Dressed for Office
- Example 2 – Course prerequisite Plan at University
- Time Complexity

What is Directed Acyclic Graph (DAG)?

- A Graph:
  - Which is Directed
  - and contain *no directed cycles*



Topological Sort

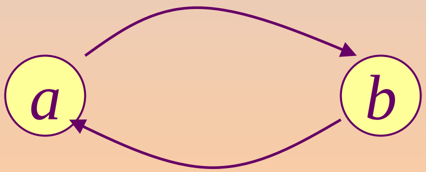
- *Topological sort* of a DAG:
  - Linear ordering of all vertices in graph G such that vertex u comes before vertex v if there is an edge $(u, v) \in G$.
- It is important to note that if the graph is not acyclic, then no linear ordering is possible. That is, we must not have circularities in the directed graph. For example, in order to get a job you need to have work experience, but in order to get work experience you need to have a job.

Existence of Topological Sort

Lemma
A directed graph *G* is <u>acyclic</u> iff a DFS of G yields no back edges.

Lemma
G can be <u>topologically sorted</u> iff it has no cycle, that is, iff it is a ***dag*** (directed acyclic graph).

Proof $\Rightarrow$ If *G* has a cycle, then it cannot be topologically sorted.



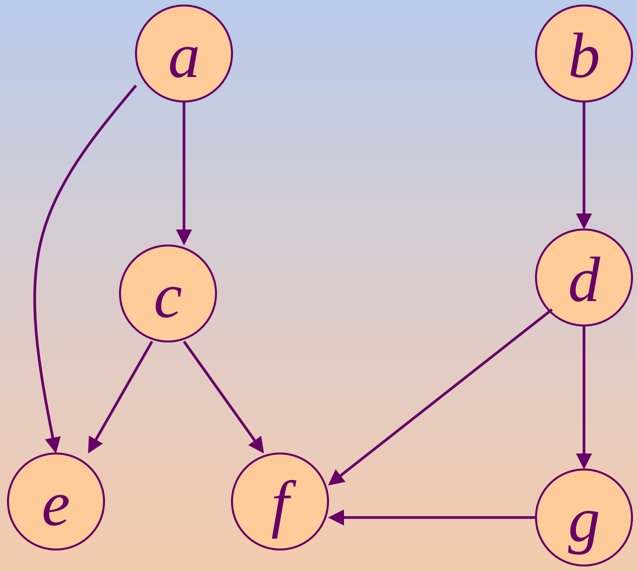
$\Leftarrow$ If *G* has no cycle, then it can be topologically sorted.

Topological Sort

- Each node represents an activity; e.g., taking a class.
- $(u, v) \in E(G)$ implies activity u must be scheduled before activity v.
- Topological sort schedules all activities.
- More than one schedule may exist.

Topological Sort of Digraphs

Different orderings in Topological Sort Algorithm are as follows



Some topological sorts:

1. *a, c, e, b, d, g, f*
2. *a, b, c, d, g, f, e*
3. *b, d, g, a, c, f, e*

Topological Sort - Applications

- Scheduling a dependent graph.
- Find a feasible course plan for university studies with Course prerequisites.
- Job scheduling (car manufacturing etc.)
- Etc…

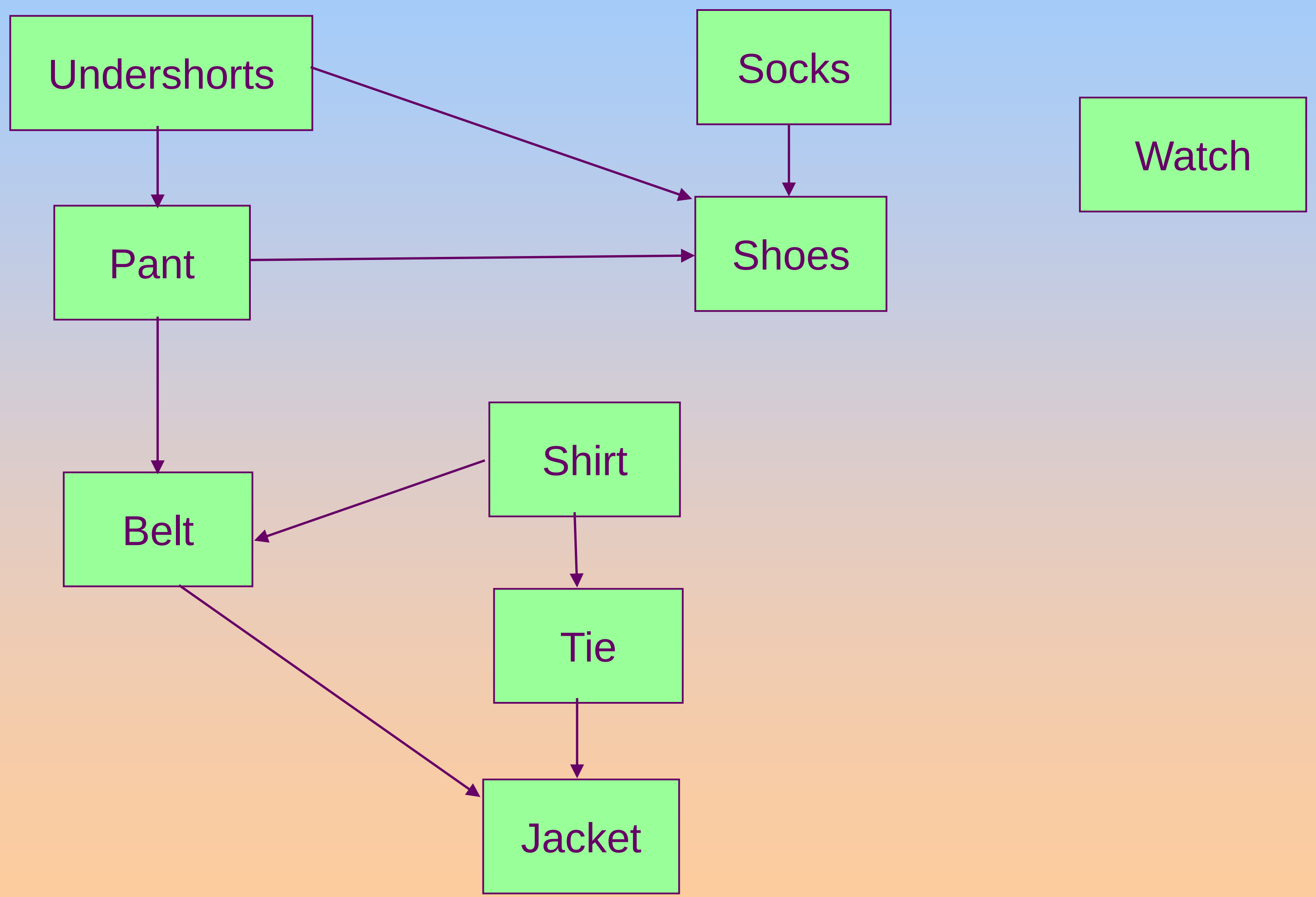
Topological Sort - Algorithm

- Performed on a DAG.
- Linear ordering of the vertices of G such that if $(u, v) \in E$, then u appears somewhere before v.

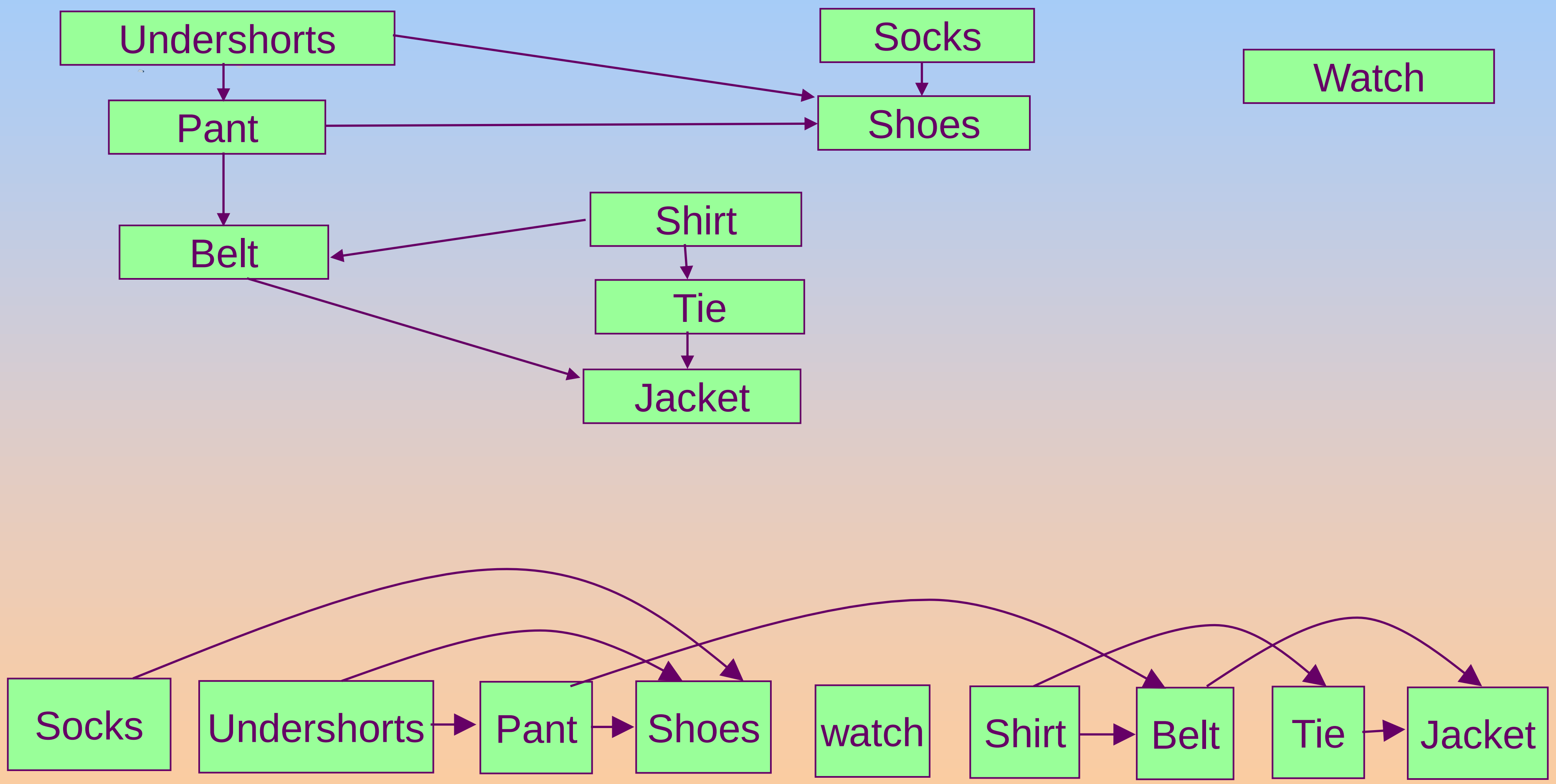
Topological-Sort (G)

1. call DFS(G) to compute finishing times $f[v]$ for all $v \in V$
2. as each vertex is finished, insert it onto the front of a linked list
3. return the linked list of vertices

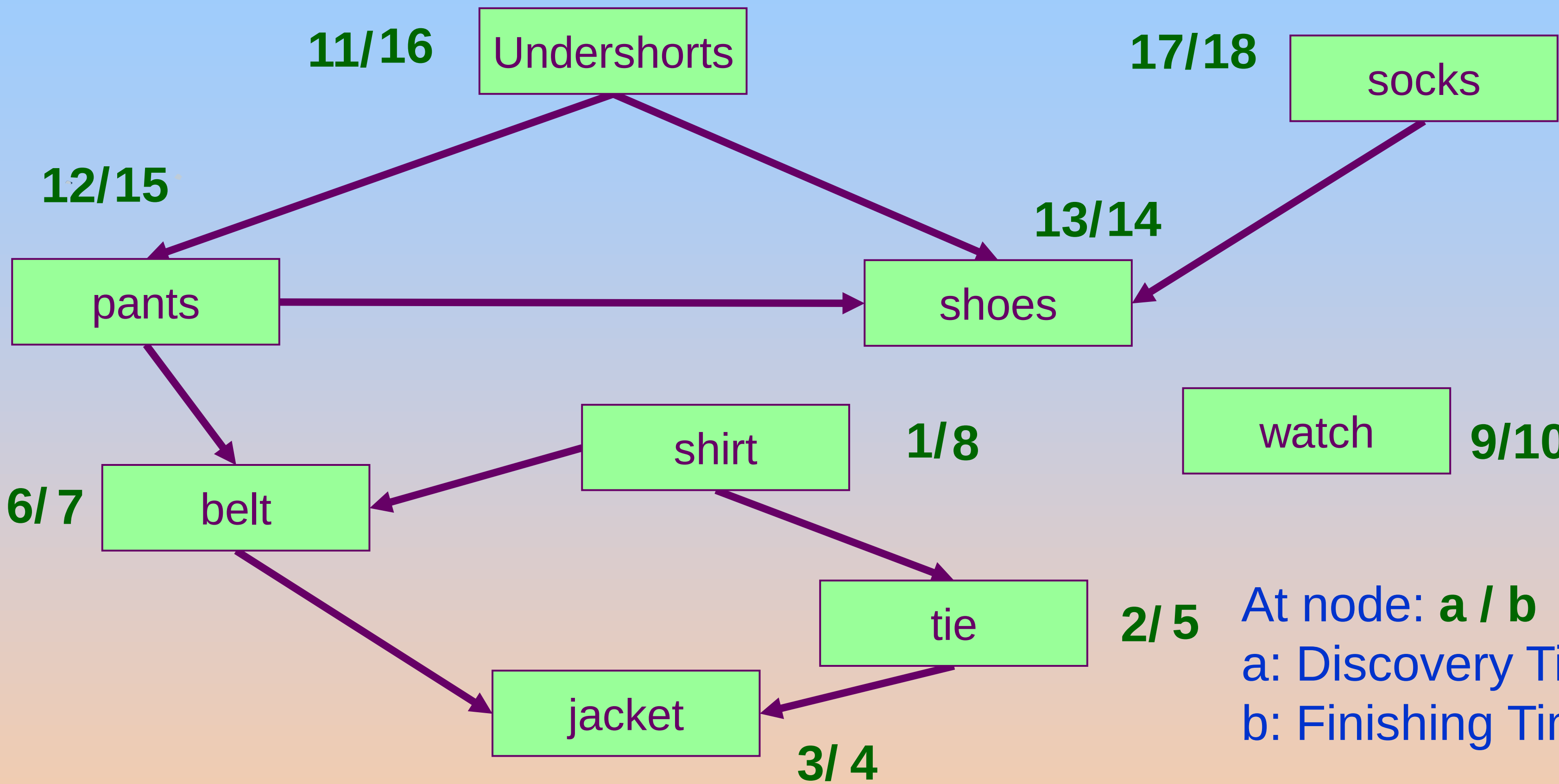
Example 1 – Getting Dressed for Office



Example (Cont.)

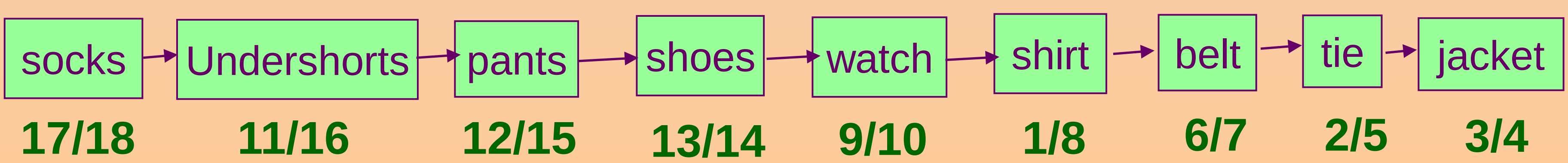


Example – Getting Dressed for Office



At node: **a / b**
a: Discovery Time
b: Finishing Time

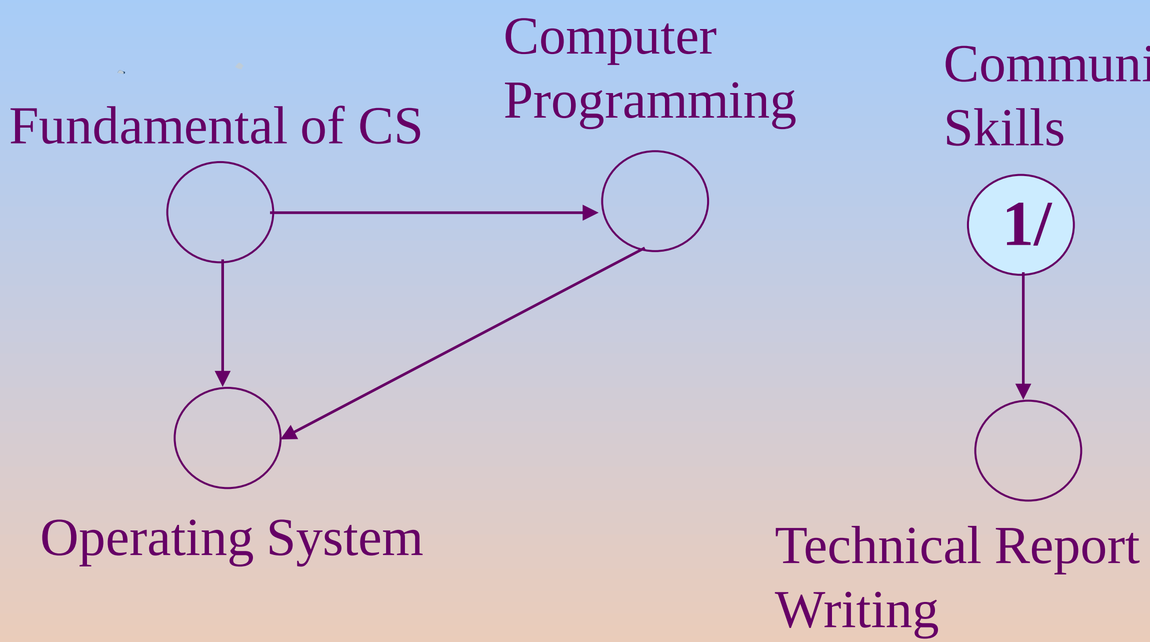
Linked List :-



Another Example

- The CS dept course prerequisites can be represented as a directed acyclic graph (DAG).
  - It must be directed because one course is the prerequisite for another (and not vice versa).
  - There can’t be any cycles because then it would be impossible to meet all the prerequisites.

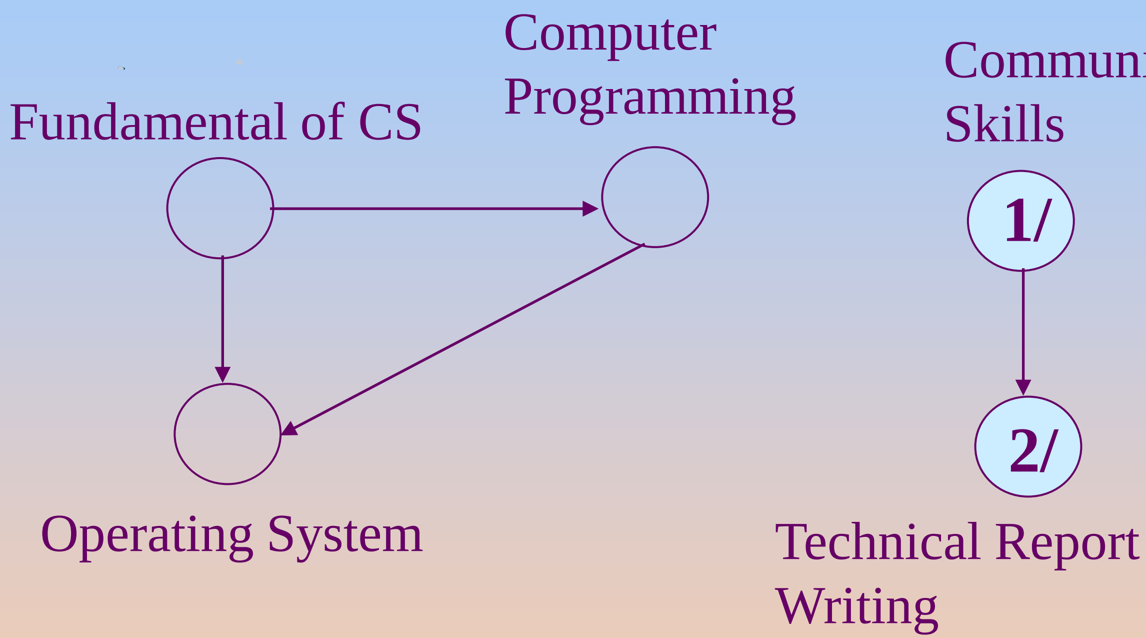
Example 2- Course prerequisite Plan at University



Inside Node: a/b
Where
a: Discovery Time of Node and
b: Finishing Time of Node

Linked List:

Example - 2



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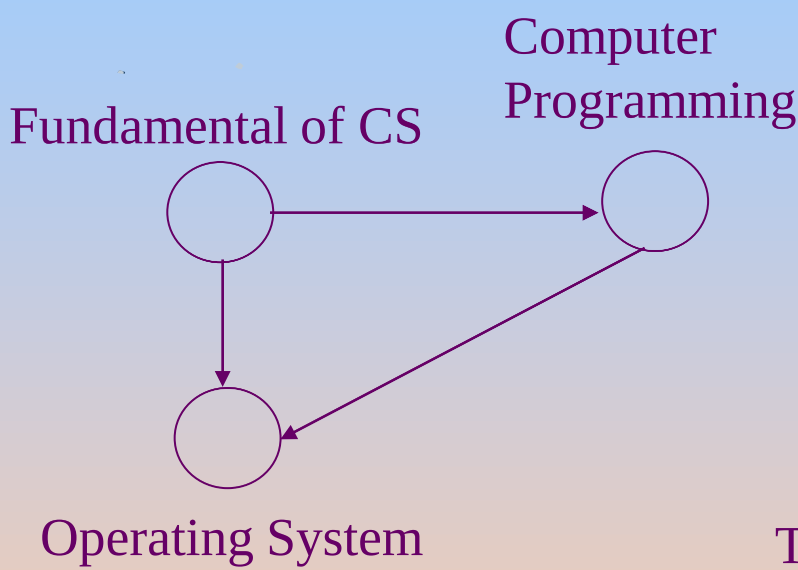

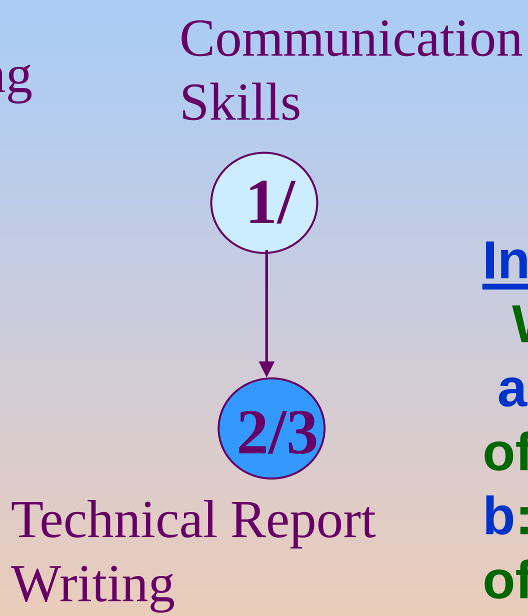

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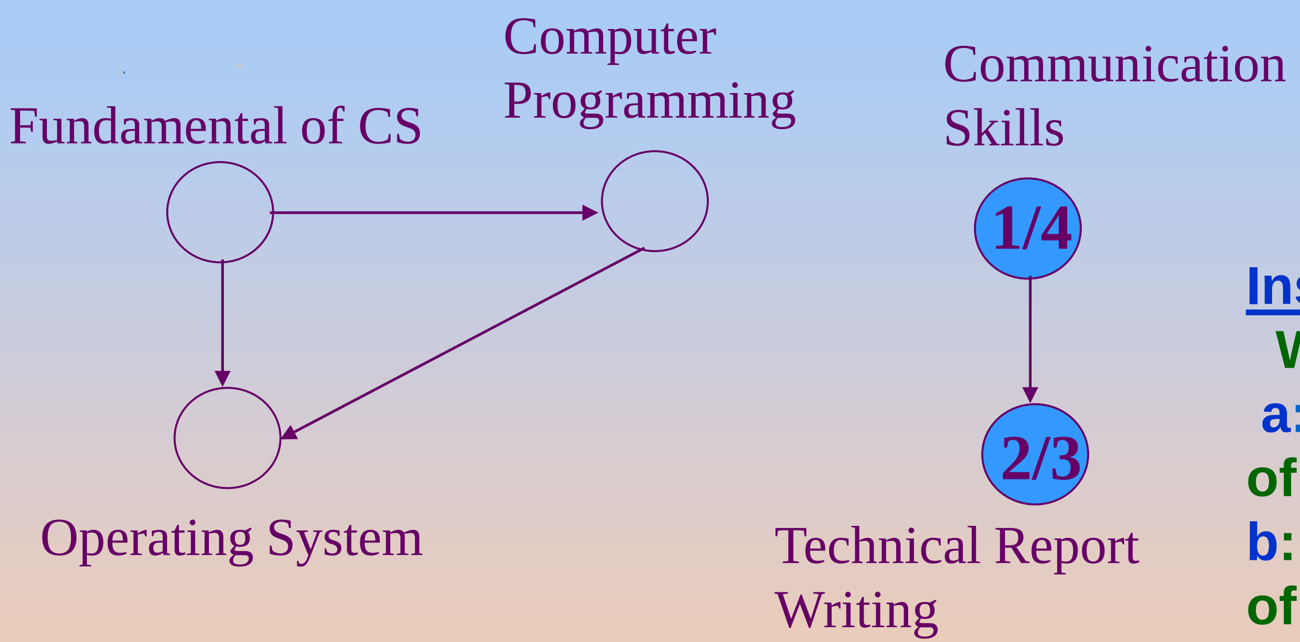

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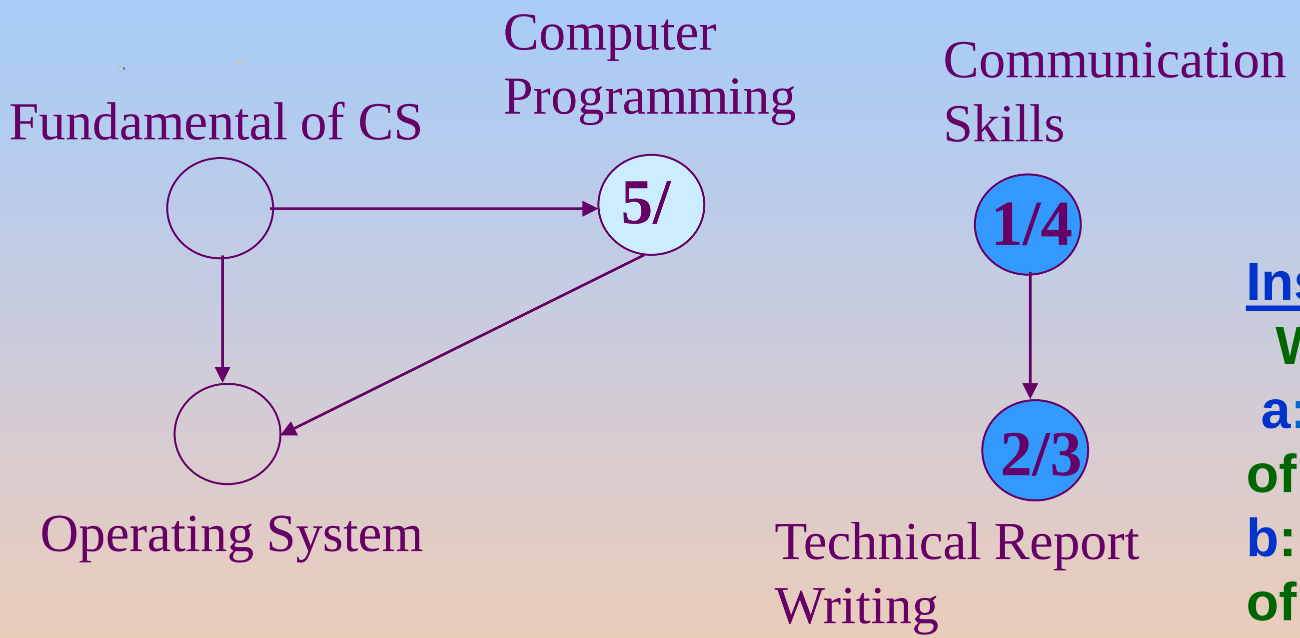

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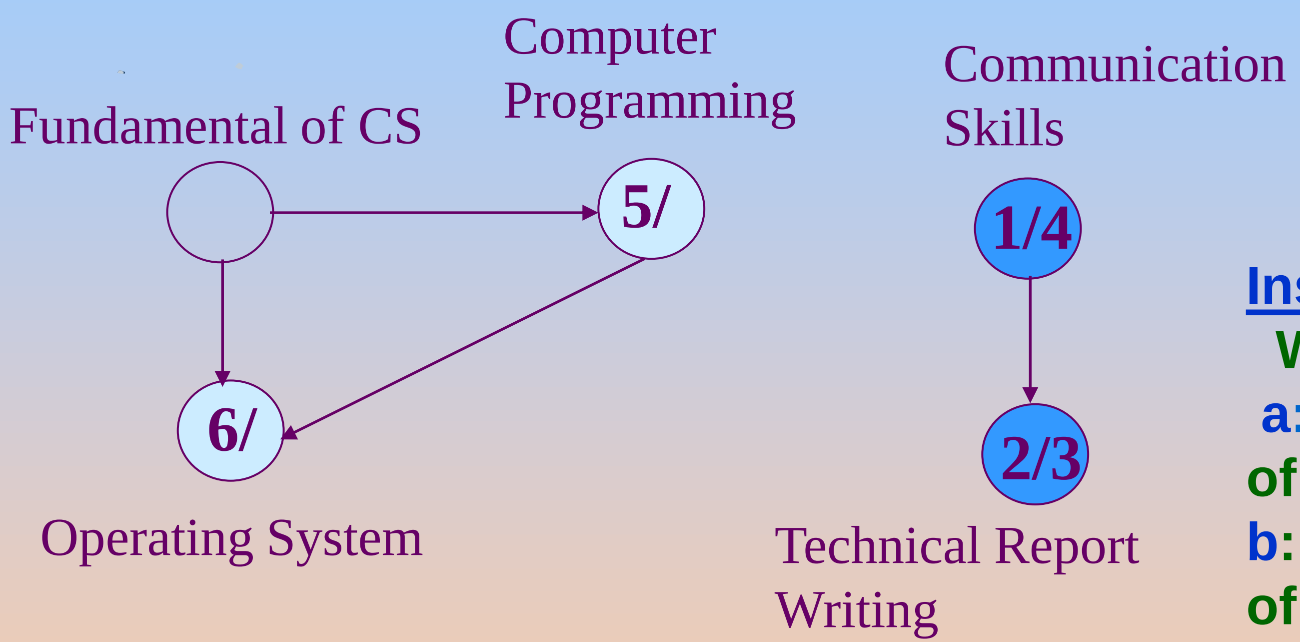

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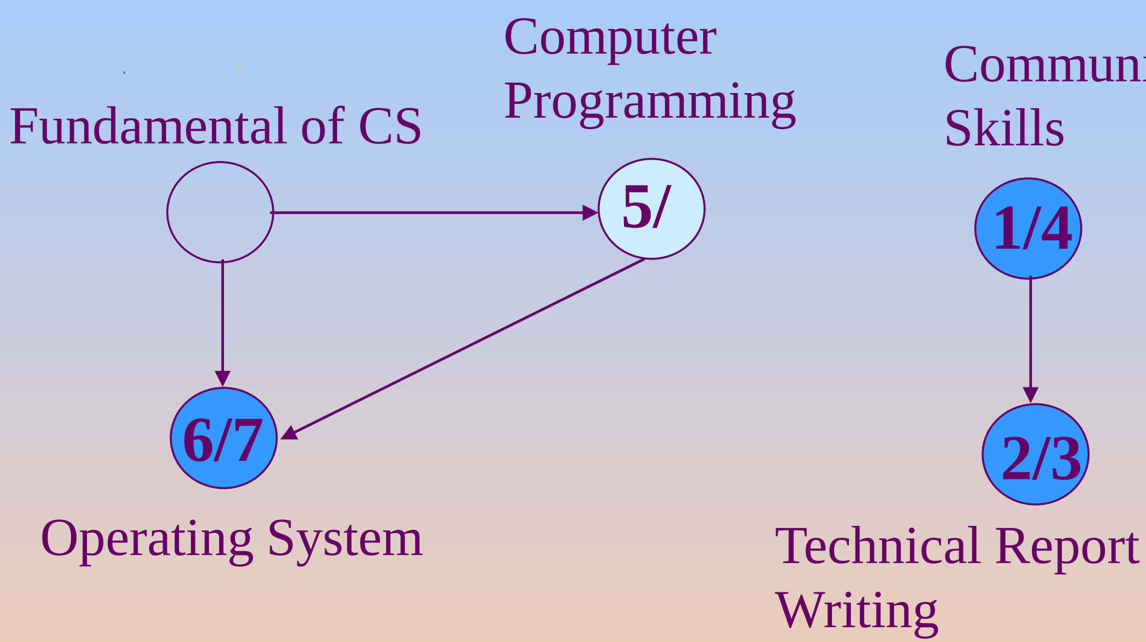

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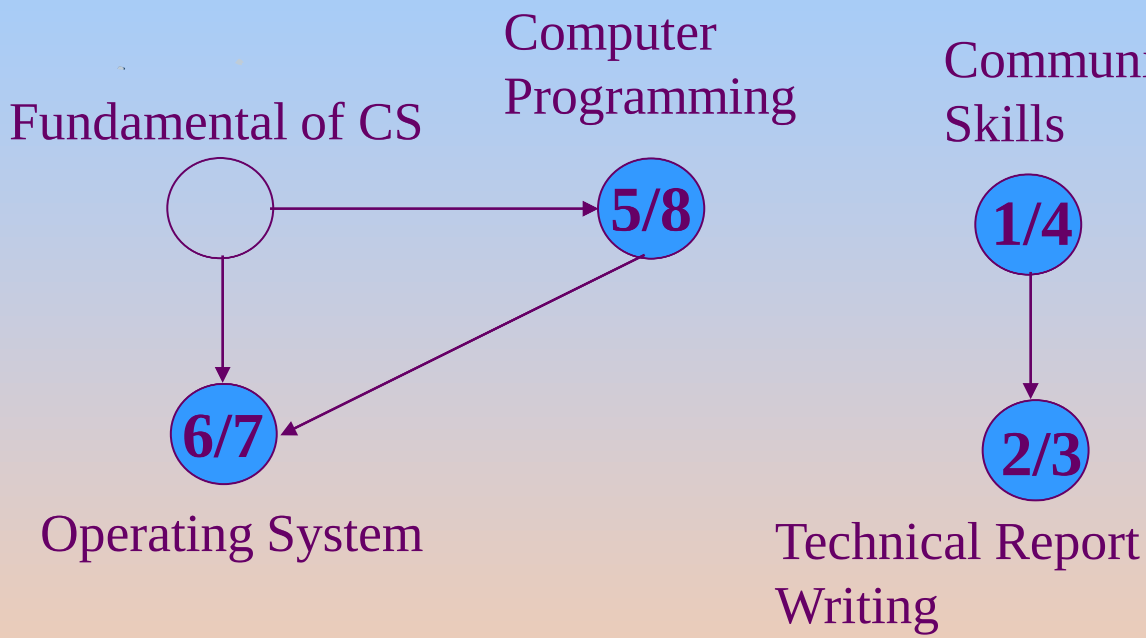

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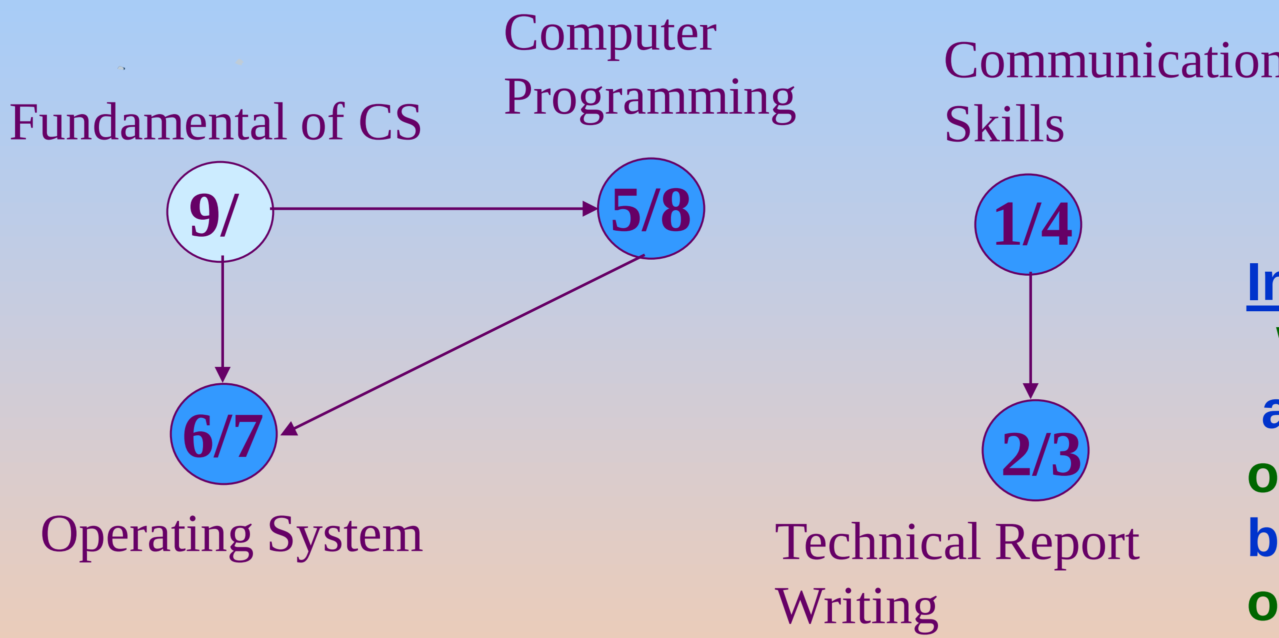

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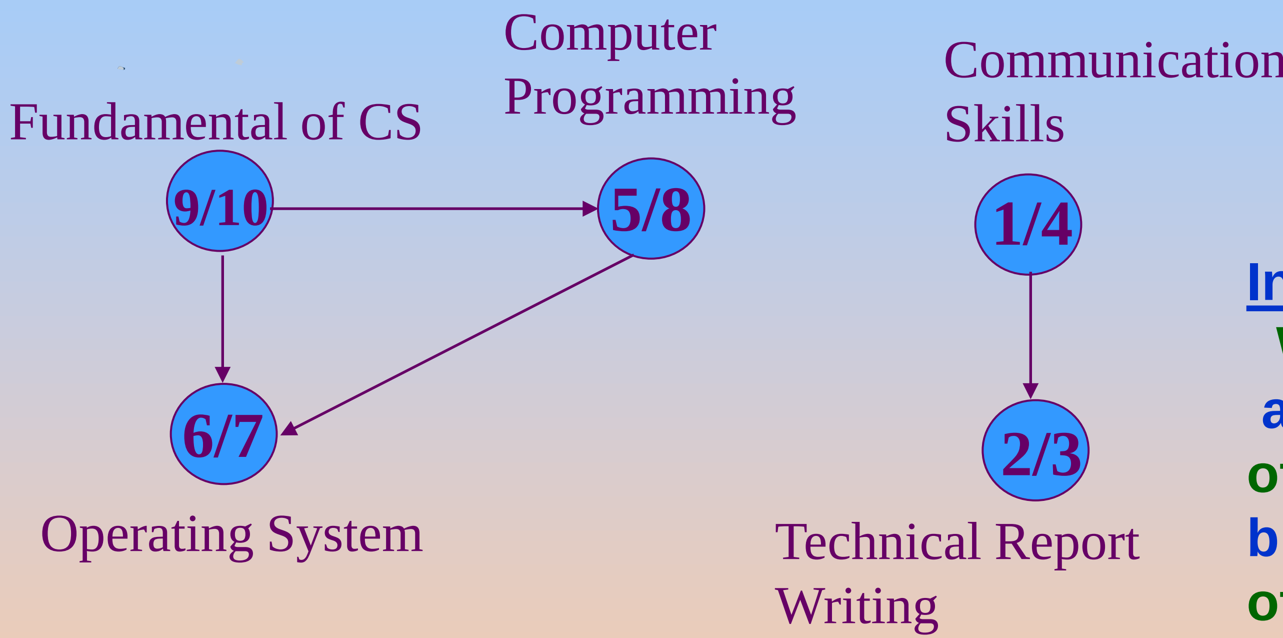

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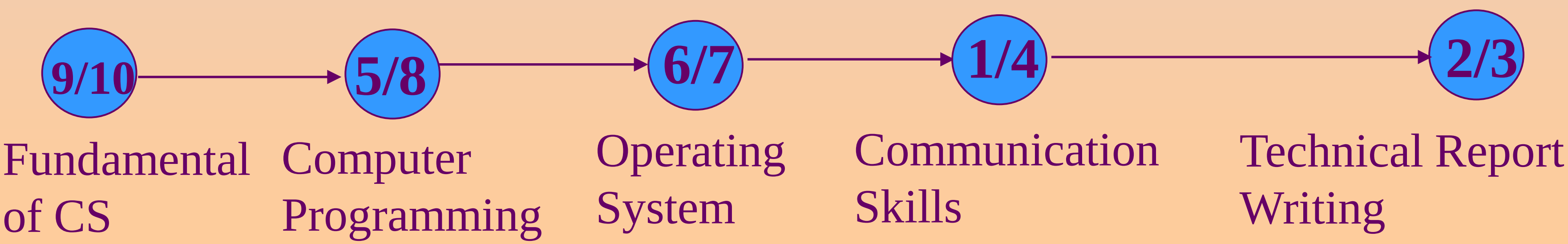

Time Complexity

- It takes O(1) time to insert each of the |V| vertices onto the front of the linked list.
- Total running time of topological sort is θ(V+E) . Since DFS(G) search takes θ(V+E) time