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Section:
BS(CS)-3C

Assignment # 4

Question # 1

i) $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\lambda I - A = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\det(\lambda I - A) = \begin{vmatrix} \lambda & 0 & -2 & 0 \\ -1 & \lambda & -1 & 0 \\ 0 & -1 & \lambda+2 & 0 \\ 0 & 0 & 0 & \lambda-1 \end{vmatrix}$

Expanding by row 4.

$$-(\lambda-1)\begin{vmatrix} \lambda & 0 & -2 \\ -1 & \lambda & -1 \\ 0 & -1 & \lambda+2 \end{vmatrix} = 0$$

Expanding by coloumn 1

$$-(\lambda-1)\left[\lambda\begin{vmatrix} \lambda & -1 \\ -1 & \lambda+2 \end{vmatrix} + 1\begin{vmatrix} 0 & -2 \\ -1 & \lambda+2 \end{vmatrix} + 0\right] = 0$$

$$(1-\lambda)\left[\lambda(\lambda(\lambda+2)-1) + 1(0-2)\right] = 0$$

$$(1-\lambda)\left[\lambda(\lambda^2+2\lambda-1) - 2\right] = 0$$

$$(1-\lambda)(\lambda^3+2\lambda^2-\lambda-2) = 0$$

$1-\lambda = 0$

$\lambda = 1$ $\qquad\qquad$ $\lambda^3+2\lambda^2-\lambda-2 = 0.$

$\lambda^3+2\lambda^2-\lambda-2 = 0. \quad — (1)$

Divisors of -2 that statisfies (1) are $1, -1, -2$

So

Eigen values are

$\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = -1$, $\lambda_4 = -2$

For $\lambda = 1$.

$(\lambda I - A)x = 0.$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ -1 & 1 & -3 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R \sim \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 + R_1$$

$$R \sim \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 + R_2$$

Let $x_4 = t$, $x_3 = s$.

$x_2 = 3s$, $x_1 = 2s$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2s \\ 3s \\ s \\ t \end{bmatrix} \Rightarrow \begin{bmatrix} 2s \\ 3s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ t \end{bmatrix}$$

$$\Rightarrow s \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda = -1$

$$\begin{bmatrix} -1 & 0 & -2 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} R_2 - R_1$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} R_3 - R_2$$

$x_4 = 0$, $x_3 = t$, $x_2 = t$, $x_1 = 2t$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{bmatrix} -2 & 0 & -2 & 0 \\ -1 & -2 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\underset{\sim}{R} \begin{bmatrix} -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} R_2 - \frac{1}{2}R_1$$

$$\underset{\sim}{R} \begin{bmatrix} -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} R_3 - \frac{1}{2}R_2$$

$x_4 = 0, \quad x_3 = t, \quad x_2 = 0, \quad x_1 = -t$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

So the eigen vectors are:

$$\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

ii) $\begin{bmatrix} 8 & -9 & 4 \\ 3 & -4 & 3 \\ -3 & 3 & 1 \end{bmatrix}$

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$$(\lambda I - A) = \begin{bmatrix} \lambda-8 & 9 & -4 \\ -3 & \lambda+4 & -3 \\ 3 & -3 & \lambda-1 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-8 & 9 & -4 \\ -3 & \lambda+4 & -3 \\ 3 & -3 & \lambda-1 \end{vmatrix}$$

$$\underset{\sim}{R} \begin{vmatrix} \lambda-8 & 9 & -4 \\ -3 & \lambda+4 & -3 \\ 0 & \lambda+1 & \lambda-4 \end{vmatrix}$$

Expanding by column 1

$$= (\lambda-8) \begin{vmatrix} \lambda+4 & -3 \\ \lambda+1 & \lambda-4 \end{vmatrix} + 3 \begin{vmatrix} 9 & -4 \\ \lambda+1 & \lambda-4 \end{vmatrix} + 0$$

$$= (\lambda-8)\left[(\lambda^2-16+3\lambda+3)\right] + 3(9\lambda-36+4\lambda+4)$$

$$= \lambda^3 - 16\lambda + 3\lambda^2 + 3\lambda - 8\lambda^2 + 128 - 24\lambda - 24 +$$
$$27\lambda - 108 + 12\lambda + 12$$

$$= \lambda^3 - 5\lambda^2 + 2\lambda + 8$$

$\det(\lambda I - A) = 0.$

$\lambda^3 - 5\lambda^2 + 2\lambda + 8 = 0. \quad — (1)$

Divisors of 8 that satisfies (1) are
$-1, 2, 4$

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So, eigen values are

$\lambda_1 = -1$, $\lambda_2 = 2$, $\lambda_3 = 4$.

For $\lambda = -1$

$$\begin{bmatrix} -9 & 9 & -4 \\ -3 & 3 & -3 \\ +3 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\underset{\sim}{R} \begin{bmatrix} -9 & 9 & -4 \\ 0 & 0 & -5/3 \\ 0 & 0 & -10/3 \end{bmatrix} \begin{array}{l} \\ R_2 - 1/3\, R_1 \\ R_3 + 1/3\, R_1 \end{array}$$

Let

$x_2 = t$, $x_3 = 0$, $x_1 = t$.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$.

$$\begin{bmatrix} -6 & 9 & -4 \\ -3 & 6 & -3 \\ 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\underset{\sim}{R} \begin{bmatrix} -6 & 9 & -4 \\ 0 & 3/2 & -1 \\ 0 & 3/2 & -1 \end{bmatrix} \begin{array}{l} \\ R_2 - 1/2\, R_1 \\ R_3 + 1/2\, R_1 \end{array}$$

$$\sim_{R}\begin{bmatrix} -6 & 9 & -4 \\ 0 & 3/2 & -1 \\ 0 & 0 & 0 \end{bmatrix} R_3 - R_2$$

$x_3 = t$, $x_2 = 2/3\, t$, $x_1 = \frac{1}{3} t$.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/3t \\ 2/3t \\ t \end{bmatrix} = t \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}$$

For $\lambda = 4$

$$\begin{bmatrix} -4 & 9 & -4 \\ -3 & 8 & -3 \\ 3 & -3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim_{R}\begin{bmatrix} -4 & 9 & -4 \\ 0 & 5/4 & 0 \\ 0 & 15/4 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - 3/4 R_1 \\ R_3 + 3/4 R_1 \end{matrix}$$

$$\sim_{R}\begin{bmatrix} -4 & 9 & -4 \\ 0 & 5/4 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_3 - 3R_2$$

$x_3 = t$, $x_2 = 0$, $x_1 = -t$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So the eigen vectors are

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$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Question # 2

i) $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

To find eigen values and eigen space of A^{25} we will first diagonalize it

$$\lambda I - A = \begin{bmatrix} \lambda+1 & 2 & 2 \\ -1 & \lambda-2 & -1 \\ 1 & 1 & \lambda \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} \lambda+1 & 2 & 2 \\ -1 & \lambda-2 & -1 \\ 0 & \lambda-1 & \lambda-1 \end{bmatrix} R_3 + R_2$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda+1 & 2 & 2 \\ -1 & \lambda-2 & -1 \\ 0 & \lambda-1 & \lambda-1 \end{vmatrix}$$

Expanding by col 1:

$$(\lambda+1)\begin{vmatrix} \lambda-2 & -1 \\ \lambda-1 & \lambda-1 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ \lambda-1 & \lambda-1 \end{vmatrix} - 0 = 0$$

$(\lambda+1)(\lambda^2-2\lambda+1)=0.$

$\lambda+1=0 \qquad (\lambda-1)^2=0.$

$\lambda=-1 \qquad \lambda=1$

So eigenvalues are

$\lambda_1=-1, \ \lambda_2=1, \ \lambda_3=1.$

For $\lambda=-1$

$(\lambda I-A)x=0.$

$$\begin{bmatrix} 0 & 2 & 2 \\ -1 & -3 & -1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 0 & 2 & 2 \\ -1 & -3 & -1 \\ 0 & -2 & -2 \end{bmatrix} R_3+R_2$$

$$\overset{R}{\sim}\begin{bmatrix} 0 & 2 & 2 \\ -1 & -3 & -1 \\ 0 & 0 & 0 \end{bmatrix} R_3+R_1$$

$x_3=t, \ x_2=-t, \ x_1=2t$

$$x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 2t \\ -t \\ t \end{bmatrix}=t\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

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For $\lambda = 1$

$$\begin{bmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$R \to \begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2 + 1/2 R_1 \\ R_3 - 1/2 R_1 \end{matrix}$$

$x_3 = t$, $x_2 = s$, $x_1 = -s-t$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -s-t \\ s \\ t \end{bmatrix} = \begin{bmatrix} -s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix}$$

$$\Rightarrow s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So the eigenspace is

$$P = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Finding P^{-1}

$$\det(P) = 2 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$$

$|P| = 2 + 1(-1) - 1(-1)$

$|P| = 2$

Finding Co-Factors:

$C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$C_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$C_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3$

$C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 \\ -1 & 0 \end{vmatrix} = 1$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 1$

So.

$$adj(P) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = D$$

Now.

$$A^k = PD^kP^{-1}$$

$k = 25$

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$$A^{25} = P D^{25} P^{-1}$$

$$= \begin{bmatrix} 2 & -1 & -1 \\ -1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 & -4 & -4 \\ 2 & 4 & 2 \\ -2 & -2 & 0 \end{bmatrix}$$

$$A^{25} = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$A^{25} = A$, so.

eigen values are $\lambda_1 = -1$ & $\lambda_2 = 1$.

eigen space $\begin{bmatrix} -2 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

ii) A^{-2301}

$$A^{-2301} = P D^{-2301} P^{-1}$$

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$$A^{-2301} = \begin{bmatrix} -2 & -1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-2301} = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

Question # 3.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda-1 & -2 & 2 \\ 3 & \lambda-4 & 0 \\ 3 & -1 & \lambda-3 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & -2 & 2 \\ 3 & \lambda-4 & 0 \\ 3 & -1 & \lambda-3 \end{vmatrix}$$

$$B \begin{bmatrix} \lambda-1 & -2 & 2 \\ +3 & \lambda-4 & 0 \\ 0 & 3-\lambda & \lambda-3 \end{bmatrix}$$

Expanding by col 1:

$$(\lambda-1)\begin{vmatrix} \lambda-4 & 0 \\ 3-\lambda & \lambda-3 \end{vmatrix} - 3\begin{vmatrix} -2 & 2 \\ 3-\lambda & \lambda-3 \end{vmatrix} + 0 = 0$$

$(\lambda-1)(\lambda-4)(\lambda-3)-3(-2(\lambda-3)-2(3-\lambda))=0.$

$(\lambda-1)(\lambda-4)(\lambda-3)=0.$

$\lambda_1=1$, $\lambda_2=4$, $\lambda_3=3.$

For $\lambda=1$

$\lambda I-Ax=0.$

$$\begin{bmatrix} 0 & -2 & 2 \\ 3 & -3 & 0 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 0 & -2 & 2 \\ 3 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_3-R_2 \text{ and } R_3-R_1$$

$x_3=t$, $x_2=t$, $x_1=t.$

$$x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} t \\ t \\ t \end{bmatrix}=t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda=4.$

$$\begin{bmatrix} 3 & -2 & 2 \\ 3 & 0 & 0 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & 2 \\ 3 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} R_1-R_2 \\ \\ R_3-R_2 \end{matrix}$$

$$R_3 \begin{bmatrix} 0 & -2 & 2 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_3 - \frac{1}{2} R_1$$

$x_3 = t$, $x_2 = t$, $x_1 = 0$.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda = 3$.

$$\begin{bmatrix} 2 & -2 & 2 \\ 3 & -1 & 0 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \begin{bmatrix} 2 & -2 & 2 \\ 3 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_3 - R_2$$

$x_2 = t$, $x_1 = \frac{t}{3}$, $x_3 = \frac{2t}{3}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t/3 \\ t \\ 2t/3 \end{bmatrix} = t \begin{bmatrix} 1/3 \\ 1 \\ 2/3 \end{bmatrix}$$

So

$$P = \begin{bmatrix} 1 & 0 & 1/3 \\ 1 & 1 & 1 \\ 1 & 1 & 2/3 \end{bmatrix}$$

$$\det(P) = 1 \begin{vmatrix} 1 & 1 \\ 1 & 2/3 \end{vmatrix} + 0 + \frac{1}{3} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= -\frac{1}{3}$$

Co-factors:

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 1 \\ 1 & 2/3 \end{vmatrix} = -1/3$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 1 & 2/3 \end{vmatrix} = 1/3$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 1/3 \\ 1 & 2/3 \end{vmatrix} = 1/3$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1/3 \\ 1 & 2/3 \end{vmatrix} = 1/3$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 1/3 \\ 1 & 1/3 \end{vmatrix} = -1/3$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1/3 \\ 1 & 1 \end{vmatrix} = -2/3$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

So

$$\text{adj } P = \begin{bmatrix} -1/3 & 1/3 & -1/3 \\ 1/3 & 1/3 & -2/3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$P^{-1} = -3 \begin{bmatrix} -1/3 & 1/3 & -1/3 \\ 1/3 & 1/3 & -2/3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & +1 \\ -1 & -1 & 2 \\ 0 & 3 & -3 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 2 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1/3 \\ 1 & 1 & 1 \\ 1 & 1 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ -4 & -4 & 8 \\ 8 & 9 & -9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1/3 \\ 1 & 1 & 1 \\ 1 & 1 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ Ans*