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Section:
BS(CS) -4A

Question # 1

$P\{X = i\}$ $\quad\quad$ $i = 1, 2, 3, \ldots$

Put $i = 1$

$$P(X=1) = \frac{(\text{No. of ways choosing any one out of } 5) \times (\text{No. of ways arranging the rest } 9)}{\text{Total number of ways of ranking 10 different scores.}}$$

$$= \frac{{}^5C_1 \cdot {}^9P_9}{{}^{10}P_{10}} = \frac{5 \cdot 9!}{10!} = \frac{1}{2}.$$

Put $i = 2$

$$P(X=2) = \frac{{}^5P_1 \cdot {}^5C_1 \cdot {}^8P_8}{{}^{10}P_{10}} = \frac{5 \cdot 5 \cdot 8!}{10!} = \frac{5}{18}$$

Put $i = 3$

$$P(X=3) = \frac{{}^{5}P_2 \cdot {}^{5}C_1 \cdot {}^{7}P_7}{{}^{10}P_{10}} = \frac{20 \cdot 5 \cdot 7!}{10!} = \frac{5}{36}$$

Put $i = 4$

$$P(X=4) = \frac{{}^{5}P_3 \cdot {}^{5}C_1 \cdot {}^{6}P_6}{{}^{10}P_{10}} = \frac{60 \cdot 5 \cdot 6!}{10!} = \frac{5}{84}$$

Put $i = 5$

$$P(X=5) = \frac{{}^{5}P_4 \cdot {}^{5}C_1 \cdot {}^{5}P_5}{{}^{10}P_{10}} = \frac{120 \cdot 5 \cdot 5!}{10!} = \frac{5}{252}$$

~~P(X=~~

Put $i = 6$

$$P(X=6) = \frac{{}^{5}P_5 \cdot {}^{5}C_1 \cdot {}^{4}P_4}{{}^{10}P_{10}} = \frac{120 \cdot 5 \cdot 4!}{10} = \frac{1}{252}$$

As there are only 5 boys so the lowest ~~wat~~ value of X can be 6.

$P(X>6) = 0$.

So

$P(X) = \{1/2, 5/18, 5/36, 5/84, 5/252, 1/252, 0, 0, 0, 0\}$.

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Question # 2

We need to find all possible outcomes of X.

In "n" coins tossed tails can be obtained 0, 1, 2, ..., n times. If tails are obtained k times number of heads then number of heads will be

$$n = h + t$$
$$h = n - t \qquad (t = tail, h = head)$$

then
$$X = h - t$$
Put $h = n - t$

$$X = n - t - t.$$
$$= n - 2t.$$

Therefore all possible outcomes of X can be

$$X = \{n - 2t \mid t = 0, 1, \dots, n\}.$$

Question # 3

a) As we see the $f(x)$ is a non-negative function so.

$$\iint_{R} f(x)\,dx\,dy = \int_0^1 \int_0^2 \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\,dx$$

$$= \frac{6}{7} \int_0^1 x^2 y + \frac{x y^2}{4} \Big|_0^2 dx$$

$$= \frac{6}{7} \int_0^1 2x^2 + x \, dx$$

$$= \frac{6}{7} \left(\frac{2x^3}{3} + \frac{x^2}{2} \right) \Big|_0^1$$

$$= \frac{6}{7} \cdot \frac{7}{6}$$

$$= 1$$

Since it is non-negative function that integrates to one, it is a valid density funtion.

b) For $x \in (0,1)$ we have

$$f_x(x) = \int_0^2 f(x) \, dy = \frac{6}{7} \int_0^2 x^2 + \frac{xy}{2} \, dy$$

$$= \frac{6}{7} \left(x^2 y + \frac{x y^2}{4} \right) \Big|_0^2$$

$$= \frac{6}{7} \left(2x^2 + x \right).$$

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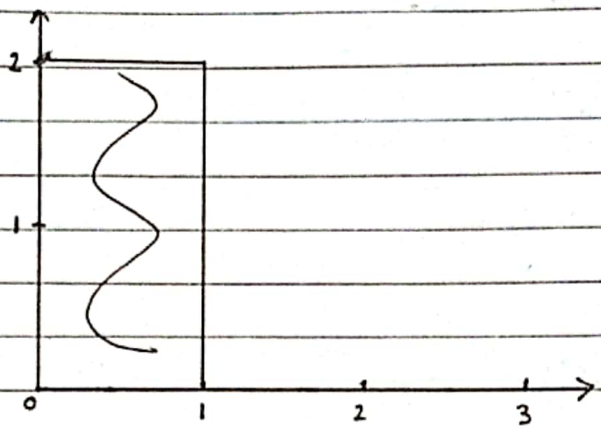
c) In order to find $P(X>Y)$, we have to integrate joint PDF over appropriate region - in rectangle $(0.1) \times (0.2)$ where $x>y$. We have



$$P(X>Y) = \int_0^1 \int_0^x \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\,dx$$

$$= \int_0^1 \frac{6}{7}\left(x^2 y + \frac{x y^2}{4}\right)\Big|_0^x dx$$

$$= \int_0^1 \frac{6}{7}\left(x^3 + \frac{x^3}{4}\right) dx$$

$$= \frac{6}{7}\int_0^1 \left(\frac{5}{4} x^3\right) dx.$$

$$= \frac{15}{56}.$$

Question # 4

$$E(x) = \int_0^{\infty} x f(x)\, dx$$

$$= \int_0^{\infty} x\, a^2 x e^{-ax}\, dx.$$

$$= \int_0^{\infty} a^2 \cdot x^2 \cdot e^{-ax}\, dx$$

$$= -ax^2 e^{-ax} - 2xe^{-ax} - \frac{2e^{-ax}}{a} \Big|_0^{\infty}$$

$$= 0 - \left(-0 - 0 - \frac{2}{a}\right)$$

$$E(X) = \frac{2}{a} \text{ Ans}$$