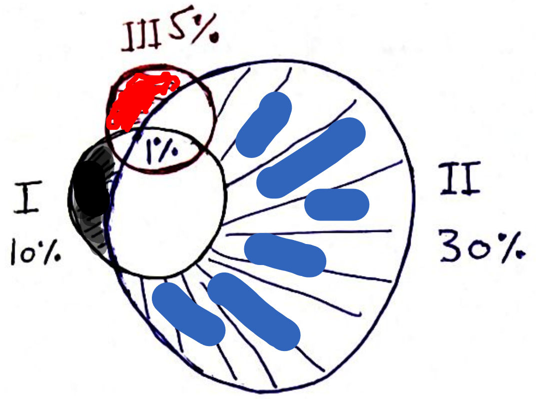
Sol 1



From the venn diagram, we observe that people in the black shaded region reads only newspaper I, people in blue shaded region reads only newspaper II and people in red shaded region reads only newspaper III.

People in black shaded region are:

$10\% - 8\% - 2\% + 1\% = 1\%$ (of total population)

People in blue shaded region are:

$30\% - 8\% - 4\% + 1 = 19\%$ (of total population)

People in red shaded region are:

$5\% - 4\% - 2\% + 1 = 0\%$ (of total population)

So only 20% of the people read only one newspaper.

So The number of people who read only one newspaper $= \frac{20 \times 100,0\cancel{00}}{1\cancel{00}} = 20,000.$

Ans

Similarly the number of people who read atleast two newspaper correspond to the white region in the venn diagram. There are

$$8\% + 4\% + 2\% - 2\% = 12\%$$

So the number of people who read atleast two newspaper $= \frac{12 \times 100,0\cancel{00}}{1\cancel{00}} = 12,000.$

Ans

Sol 2 (a) Let $F \cup G = H$, then

$$P(E \cup F \cup G) = P(E \cup H)$$

$$\Rightarrow P(E \cup F \cup G) = P(E) + P(H) - P(E \cap H) \quad — \star$$

Now

$$P(E \cap H) = P[E \cap (F \cup G)] = P[(E \cap F) \cup (E \cap G)]$$

$$= P(E \cap F) + P(E \cap G) - P(E \cap F \cap E \cap G)$$

$$\Rightarrow P(E \cap H) = P(E \cap F) + P(E \cap G) - P(E \cap F \cap G) \quad -(i)$$

also $P(H) = P(F \cup G) = P(F) + P(G) - P(F \cap G)$ $\quad -(ii)$

Substituting the values from (i) and (ii) in $\star$ we get

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(F \cap G) - P(E \cap F) - P(E \cap G) + P(E \cap F \cap G)$$

proved

(b) $P(\text{Positive} \mid \text{Sterioduser}) = \frac{90}{100}$ (given)

$\Rightarrow P(\text{Negative} \mid \text{Sterioduser}) = \frac{10}{100}$

$P(\text{Positive} \mid \text{Not Sterioduser}) = \frac{2}{100}$ (given)

$\Rightarrow P(\text{Negative} \mid \text{Not Sterioduser}) = \frac{98}{100}$, $P(\text{Sterioduser}) = 5/100$ (given)

$\Rightarrow P(\text{Not Steriodu}) = \frac{95}{100}$

$P(\text{Sterioduser} \mid \text{Negative}) = ?$

By Baye's Rule

$$P(\text{Sterioduser} \mid \text{Negative}) = \frac{P(\text{Negative} \mid \text{sterioduser}) \times P(\text{Sterioduser})}{P(\text{Negative})} \quad — \star$$

By Law of Total Probability

$$P(\text{Negative}) = P(\text{Negative} \mid \text{Sterioduser})\, P(\text{Sterioduser}) + P(\text{Negative} \mid \text{Not Steriodurer})\, P(\text{Not Steriodu})$$

putting values we get

$$P(\text{Negative}) = \frac{10}{100}\left(\frac{5}{100}\right) + \left(\frac{98}{100}\right)\left(\frac{95}{100}\right) = 0.936$$

putting values in $\star$ we get

$$P(\text{sterioduser} \mid \text{Negative}) = \frac{(10/100)(5/100)}{0.936}$$

$$\simeq \boxed{0.00534} \text{ Ans}$$

Sol 3 (a)

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^{\infty} x e^{-(x+y)}\,dy = \int_0^{\infty} x e^{-x} e^{-y}\,dy$$

$$= x e^{-x} \frac{e^{-y}}{-1}\Big|_0^{\infty} = -x e^{-x}\left[e^{-y}\right]_0^{\infty} = -x e^{-x}[0-1]$$

$$\Rightarrow \boxed{f_X(x) = x e^{-x} \quad (\text{PDf of } X)}$$ Ans

(b) $$f_Y(y)=\int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^{\infty} x e^{-(x+y)}\,dy = \int_0^{\infty} x e^{-x}\cdot e^{-y}\,dx$$

$$= e^{-y}\int_0^{\infty} x e^{-x}\,dx = e^{-y}\left[-x e^{-x}\Big|_0^{\infty} + \int_0^{\infty} e^{-x}\,dx\right]$$

$$= e^{-y}\left[-\left(\frac{x}{e^{x}}\right)\Big|_0^{\infty} - e^{-x}\Big|_0^{\infty}\right] = e^{-y}\left[-(0-0)-(0-1)\right]$$

$$\Rightarrow \boxed{f_Y(y) = e^{-y} \quad (\text{PDf of } Y)}$$ Ans

(c) Yes, because

$$f_X(x)\,f_Y(y) = x e^{-(x+y)} = f(x,y)$$

Sol 4 (a)

Probability of Class (lecture) cancellation $= p = 0.05$ (Probability of success)

Total number of classes $= n = 15$

Probability of failure $= q = 1 - 0.05 = 0.95$ (Probability of not cancellation)

$P\{X \geq 4\} = ?$

$$P\{X \geq 4\} = 1 - P\{X < 4\}$$

$$= 1 - [P\{X=0\} + P\{X=1\} + P\{X=2\} + P\{X=3\}]$$

$$= 1 - \left[\binom{15}{0}(0.05)^0(0.95)^{15} + \binom{15}{1}(0.05)^1(0.95)^{14} + \binom{15}{2}(0.05)^2(0.95)^{13} + \binom{15}{3}(0.05)^3(0.95)^{12}\right]$$

$$\Rightarrow \boxed{P\{X \geq 4\} = 0.0055} \text{ Ans}$$

(b)

$$E[X_1] = 0 \cdot P\{X_1 = 0\} + 1 \cdot P\{X_1 = 1\} + 2 \cdot P\{X_1 = 2\} + 3 \cdot P\{X_1 = 3\}$$

X_1	X_2	1	2	$P\{X_1 = x\}$
0		$\frac{1}{8}$	$\frac{1}{16}$	$\frac{3}{16}$
1		$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$
2		$\frac{3}{16}$	$\frac{1}{8}$	$\frac{5}{16}$
3		$1/8$	$1/4$	$\frac{3}{8}$
	$P\{X_2 = x_2\}$	$\frac{8}{16}$	$\frac{8}{16}$	

So $E[X_1] = 0 + 1\left(\frac{1}{8}\right) + \cancel{2}^{1}\cdot\left(\frac{5}{\cancel{16}_{8}}\right) + 3\left(\frac{3}{8}\right) = \frac{15}{8} \Rightarrow \boxed{E[X_1] = \frac{15}{8} = 1.875}$ Ans

$$Var(X_2) = E[X_2^2] - (E[X_2])^2$$

Now $E[X_2^2] = 1^2 P\{X_2 = 1\} + 2^2 P\{X_2 = 2\}$

$$= 1 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow E[X_2^2] = \frac{5}{2}$$

and $E[X_2] = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} = \frac{3}{2}$

So $Var(X_2) = \frac{5}{2} - \left(\frac{3}{2}\right)^2 = \frac{5}{2} - \frac{9}{4} = \frac{1}{4}$

$$\Rightarrow \boxed{Var(X_2) = \frac{1}{4}}$$ Ans

and

$$Cov(X_1, X_2) = E[X_1 X_2] - E[X_1]E[X_2]$$

$$E[X_1 X_2] = (0)(1)\frac{1}{8} + (0)(2)\frac{1}{16} + (1)(1)\frac{1}{16} + (1)(2)\frac{1}{16}$$

$$+ (2)(1)\frac{3}{16} + (2)(2)\frac{1}{8} + (3)(1)\left(\frac{1}{8}\right) + (3)(2)\left(\frac{1}{4}\right)$$

$$= \frac{1}{16} + \frac{2}{16} + \frac{6}{16} + \frac{4}{8} + \frac{3}{8} + \frac{6}{4} = \frac{1+2+6+8+6+24}{16}$$

$$E[X_1 X_2] = \frac{47}{16}$$

So $Cov(X_1, X_2) = \frac{47}{16} - \frac{15}{8} \cdot \frac{3}{2} = \frac{47}{16} - \frac{45}{16} = \frac{2}{16}$

$$\Rightarrow \boxed{Cov(X_1, X_2) = \frac{1}{8} = 0.125}$$ Ans

Sol 5 Let's first arrange the data from smallest to largest.

49, 50, 57, 59, 60, 61, 68, 71, 73, 73, 74, 76, 78, 80, 80, 82, 84, 87, 88, 90, 90, 92, 93, 99

$IQR = Q_3 - Q_1$

For Q_1, $p = 1/4$ $n = 24$, $np = \frac{24}{4} = 6$ (integer)

So Q_1 is the average of the values in positions 6 & 7.

So $Q_1 = \frac{61 + 68}{2} = 64.5$

For Q_3, $P = 3/4$, $n = 24$, $np = \frac{3}{4} \cdot 24 = 18$, so

$Q_3 = \frac{87 + 88}{2} = 87.5$

So $\boxed{IQR = 87.5 - 64.5 = 23}$ ~~Ans~~

Sol 7

$\mu = 50{,}000$, $\sigma = 20{,}000$ (Not standard Normal)

(a) $P\{0 < X < 30{,}000\} = ?$

(where X is the monthly salary of an employ

$$P\{0 < X < 30{,}000\} = P\left\{\frac{0-50{,}000}{20{,}000} < \frac{X-50{,}000}{20{,}000} < \frac{30{,}000-50{,}000}{20{,}000}\right\}$$

$$\Rightarrow P\{0<X<30{,}000\} = P\left\{-\frac{5}{2} < Z < -1\right\}$$

$$= P\{-2.5 < Z < -1\}$$

$$= \phi(-1) - \phi(-2.5)$$

$$= P\{Z \leq -1\} - P\{Z \leq -2.5\}$$

$$= 0.1587 - 0.0062 \quad \text{(using Table)}$$

$$P\{0<X<30{,}000\} = 0.1525$$

Approximately 15.25 % employees earn less than \$30,000.

(b) $P\{40{,}000 < X < 75{,}000\} = P\{-0.5 < Z < 1.25\}$

$$= \phi(1.25) - \phi(-0.5)$$

$$= 0.8944 - 0.3085$$

$$= \boxed{0.5859} \quad A$$

Approx 58.59 % empolyees has salary b/w \$40,000 & \$75,000