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Section:
BS(CS)-2D

Question # 1

$$2x^2y'' + 5xy' + y = x^2 - x$$

$$2x^2y'' + 5xy' + y = x^2 - x \quad \text{—} (1)$$

Dividing $2x^2$ on both sides to get standard form.

$$y'' + \frac{5}{x}y' + \frac{1}{2x^2}y = \frac{1}{2} - \frac{1}{2x}$$

As associated homogeneous equation is

$$2x^2y'' + 5xy' + y = 0 \quad \text{—} (2)$$

And the auxillary equation is

Let $y = x^m$, so

$$y' = m x^{(m-1)}$$

$$y'' = m(m-1) x^{(m-2)}$$

Putting the values of y, y' & y'' in eq 2

$$2x^2 \left(m(m-1) x^{(m-2)}\right) + 5x\left(m x^{(m-1)}\right) + x^m = 0$$

$$2m(m-1) x^2 x^{(m-2)} + 5 m x^1 x^{m-1} + x^m = 0$$

$$2m(m-1) x^m + 5 m x^m + x^m = 0$$

$$\left(2m(m-1) + 5m + 1\right) x^m = 0.$$

$$(2m^2 - 2m + 5m + 1) x^m = 0$$

$$(2m^2 + 3m + 1) x^m = 0$$

Since x^m can not be equal to 0, so

$$2m^2 + 3m + 1 = 0$$

$$2m^2 + 2m + 1m + 1 = 0$$

$$2m(m+1) + 1(m+1) = 0$$

$$(m+1)(2m+1) = 0.$$

So,

$$m_1 = -1 \quad \& \quad m_2 = -\frac{1}{2}$$

The solution of the homogeneous differential equation is

$$y_c = C_1 x^{-1} + C_2 x^{-\frac{1}{2}}$$

Now we will find the particular solution.

$$y_p = u_1 y_1 + u_2 y_2$$

Since

$$y_1 = x^{-1}$$

$$y_2 = x^{-1/2}$$

So,

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} x^{-1} & x^{-1/2} \\ -x^{-2} & -\frac{1}{2} x^{-3/2} \end{vmatrix}$$

$$W = (x^{-1}) \times \left(\frac{-1}{2} x^{-3/2}\right) - (x^{-1/2}) \times (-x^{-2})$$

$$W = -\frac{1}{2} x^{-5/2} + x^{-5/2}$$

$$W = \frac{1}{2} x^{-5/2}$$

Now we will find W_1 & W_2, so.

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

And $f(x) = \frac{1}{2} x^{-5/2}$ with the help of standard form.

$$W_1 = \begin{vmatrix} 0 & x^{-1/2} \\ \frac{1}{2} - \frac{1}{2x} & \frac{-1}{2} x^{-3/2} \end{vmatrix}$$

$$W_1 = \begin{vmatrix} 0 & x^{-1/2} \\ \frac{1}{2} - \frac{1}{2x} & \frac{-1}{2} x^{-3/2} \end{vmatrix}$$

$$W_1 = 0 - (x^{-1/2}) \times \left(\frac{1}{2} - \frac{1}{2x}\right)$$

$$W_1 = -\frac{1}{2} x^{-1/2} + \frac{1}{2} x^{-3/2}$$

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And

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$W_2 = \begin{vmatrix} x^{-1} & 0 \\ -x^{-2} & \frac{1}{2} - \frac{1}{2x} \end{vmatrix}$$

$$W_2 = (x^{-1}) \times \left(\frac{1}{2} - \frac{1}{2x} \right) - 0$$

$$W_2 = \frac{1}{2} x^{-1} - \frac{1}{2} x^{-2}$$

Now we will find the value of u_1' & u_2', so.

$$u_1' = \frac{W_1}{W}$$

$$u_1' = \frac{-1/2\, x^{-1/2} + 1/2\, x^{-3/2}}{1/2\, x^{-5/2}}$$

$$u_1' = -x^2 + x$$

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And

$$u_2' = \frac{W_2}{W}$$

$$= \frac{1/2\,x^{-1} - 1/2\,x^{-2}}{1/2\,x^{-5/2}}$$

$$= x^{3/2} - x^{1/2}$$

But we need the value of u_1 & u_2 so we will take integral on b/s.

$$u_1 = \int (-x^2 + x)\,dx$$

$$= -\int x^2\,dx + \int x\,dx$$

$$= -\frac{1}{3}x^3 + \frac{1}{2}x^2$$

Now

$$u_2 = \int (x^{3/2} - x^{1/2})\,dx$$

$$u_2 = \int x^{3/2}\,dx - \int x^{1/2}\,dx$$

$$u_2 = \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2}$$

Now putting the values in equation

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \left(-\frac{1}{3}x^3 + \frac{1}{2}x^2\right) \times (x^{-1}) + \left(\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2}\right) \times (x^{-1/2})$$

$$y_p = -\frac{1}{3}x^2 + \frac{1}{2}x + \frac{2}{5}x^2 - \frac{2}{3}x$$

$$y_p = \frac{1}{15}x^2 - \frac{1}{6}x$$

Now putting the values of y_c & y_p in

$$y = y_c + y_p$$

$$y = C_1 x^{-1} + C_2 x^{-1/2} + \frac{1}{15}x^2 - \frac{1}{6}x \quad \text{Ans}$$

Question # 2

$$x^2y'' + xy' - y = \ln x$$

$$x^2y'' + xy' - y = \ln x \quad \text{—①}$$

Dividing x^2 on both sides to get standard form.

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = \frac{\ln x}{x^2}$$

To find general solution we will find homogeneous differential equation, so

$$x^2 y'' + x y' - y = 0 \quad -② $$

Let

$$y = x^m, \text{ so}$$

$$y' = m x^{(m-1)}$$

$$y'' = m(m-1)x^{(m-2)}$$

Putting the values of y, y' & y'' in eq ②

$$x^2(m(m-1)x^{(m-2)} + x(m x^{(m-1)}) - x^m = 0$$

$$m(m-1)x^2 x^{m-2} + m x^1 x^{(m-1)} - x^m = 0$$

$$m(m-1)x^m + m x^m - x^m = 0$$

$$(m(m-1) + m - 1)x^m = 0$$

$$(m-1)(m+1)x^m = 0$$

Since x^m can not be equal to 0, so

$(m-1)(m+1)=0$

So,

$m_1 = 1$ & $m_2 = -1$

The solution of the homogeneous differential equation is

$$y_c = C_1 x + C_2 x^{-1}$$

Now we will find the particular solution.

$$y_p = u_1 y_1 + u_2 y_2$$

Since

$$y_1 = x$$

$$y_2 = x^{-1}$$

So,

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} x & x^{-1} \\ 1 & -x^{-2} \end{vmatrix}$$

$$W = (x)(-x^{-2}) - (x^{-1})(1)$$

$W = -x^{-1} - x^{-1}$

$W = -2x^{-1}$

Now we will find W_1 & W_2, so

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

And $f(x) = \frac{\ln x}{x^2}$, with the help of standard form.

$$W_1 = \begin{vmatrix} 0 & x^{-1} \\ \frac{\ln x}{x^2} & -x^{-2} \end{vmatrix}$$

$W_1 = 0 - (x^{-1}) \times \left(\frac{\ln x}{x^2} \right)$

$W_1 = -\frac{\ln x}{x^3}$

And

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

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$$W_2 = \begin{vmatrix} x & 0 \\ 1 & \frac{\ln x}{x^2} \end{vmatrix}$$

$$W_2 = (x) \times \left(\frac{\ln x}{x^2}\right) - 0$$

$$W_2 = \frac{\ln x}{x}$$

Now we will find the value of u_1' & u_2'. so

$$u_1' = \frac{W_1}{W}$$

$$u_1' = \frac{-\ln x/x^3}{-2x^{-1}}$$

$$u_1' = \frac{\ln x}{2x^2}$$

And

$$u_2' = \frac{W_2}{W}$$

$$u_2' = \frac{\ln x/x}{-2x^{-1}}$$

$u_2' = -\frac{\ln x}{2}$

But we need the value of u_1 & u_2 so we will take integral on b/s.

$u_1 = \int \frac{\ln x}{2x^2} dx$

$u_1 = \frac{1}{2}\int \frac{\ln x}{x^2} dx$

Now we will use integration by parts i.e.

$s = \ln x$, then $ds = \frac{1}{x} dx$, and let $dv = x^{-2} dx$ then $v = -x^{-1}$, so

$u_1 = \frac{1}{2}\left(s \times v - \int v\, ds\right)$

$u_1 = \frac{1}{2}\left(-x^{-1} \ln x - \int -x^{-2} dx\right)$

$u_1 = \frac{1}{2}\left(-x^{-1} \ln x - x^{-1}\right)$

$u_1 = -\frac{\ln x}{2x} - \frac{1}{x}$

Now

$u_2 = \int -\frac{\ln x}{2} dx$

$u_2' = -\frac{1}{2}\int \ln x \, dx$

Now we will use integration by parts i.e.

$s = \ln x$, then $ds = \frac{1}{x} dx$ and let $dv = dx$

then $v = x$, so

$u_2 = -\frac{1}{2}\left(s \times v - \int v \, ds\right)$

$u_2 = -\frac{1}{2}\left(x \ln x - \int \left(x \times \frac{1}{x}\right) dx\right)$

$u_2 = -\frac{1}{2}\left(x \ln x - \int dx\right.$

$u_2 = -\frac{1}{2}(x \ln x - x)$

$u_2 = -\frac{1}{2} x \ln x + \frac{1}{2} x$

Now putting the values in equation

$y_p = u_1 y_1 + u_2 y_2$

$y_p = \left(-\frac{\ln x}{2x} - \frac{1}{2x}\right) \times (x) + \left(-\frac{1}{2} x \ln x + \frac{1}{2} x\right) \times \left(\frac{1}{x}\right)$

$$y_p = -\frac{1}{2}\ln x - \frac{1}{2} - \frac{1}{2}\ln x + \frac{1}{2}$$

$$y_p = -\ln x$$

Now putting the values of y_c & y_p in

$$y = y_c + y_p$$

$$y = c_1 x + c_2 x^{-1} - \ln x \quad \text{Ans:}$$

Question # 3

$$x^2 y'' + x y' - y = \frac{1}{x+1}$$

$$x^2 y'' + x y' - y = \frac{1}{x+1} \quad —①$$

Dividing x^2 on b/s to get standard form.

$$y'' + \frac{1}{x} y' - \frac{1}{x^2} y = \frac{1}{x^2(x+1)}$$

To find general solution we will find homogeneous differential equation, so

$$x^2 y'' + x y' - y = 0 \quad —②$$

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Let

$$y = x^m, \text{ so}$$

$$y' = m x^{(m-1)}$$

$$y'' = m(m-1) x^{(m-2)}$$

Putting the values of y, y' & y'' in eq ②

$$x^2 (m(m-1) x^{(m-2)}) + x(m x^{(m-1)}) - x^m = 0$$

$$m(m-1) x^2 x^{m-2} + m x^{m-1} x^1 - x^m = 0.$$

$$m(m-1) x^m + m x^m - x^m = 0$$

$$(m(m-1) + m - 1) x^m = 0.$$

$$(m+1)(m-1) x^m = 0.$$

Since x^m can not be equal to 0, so

$$(m+1)(m-1) = 0.$$

So,

$$m_1 = -1, \ m_2 = 1$$

The solution of the homogeneous differential equation is

$$y_c = C_1 x^{-1} + C_2 x^{+1}$$

Now we will find the particular solution.

$$y_p = u_1 y_1 + u_2 y_2$$

Since,

$$y_1 = x$$

$$y_2 = x^{-1}$$

So,

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} x & x^{-1} \\ 1 & -x^{-2} \end{vmatrix}$$

$$W = -x^{-1} - (x^{-1})$$

$$W = -2x^{-1}$$

$$W = -\frac{2}{x}$$

Now we will find W_1 & W_2, so

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

And $f(x) = \frac{1}{x^2(x+1)}$, with the help of standard form.

$$W_1 = \begin{vmatrix} 0 & x^{-1} \\ \frac{1}{x^2(x+1)} & -x^{-2} \end{vmatrix}$$

$$W_1 = -\frac{1}{x^2(x+1)} \cdot x^{-1} - 0$$

$$W_1 = -\frac{1}{x^3(x+1)}$$

And

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & \frac{1}{x^2(x+1)} \end{vmatrix}$$

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$$W_2 = \frac{1}{x^2(x+1)} \cdot x - 0$$

$$W_2 = \frac{1}{x(x+1)}$$

Now we will find the value of u_1' & u_2', so

$$u_1' = \frac{W_1}{W}$$

$$u_1' = -\frac{1}{x^3(x+1)} \cdot \left(\frac{-x}{2}\right)$$

$$u_1' = \frac{1}{2x^2(x+1)}$$

And

$$u_2' = \frac{W_2}{W}$$

$$u_2' = \frac{1}{x(x+1)} \cdot \left(\frac{-x}{2}\right)$$

$$u_2' = -\frac{1}{2(x+1)}$$

But we need the value of u_1 & u_2 so we will take integral on b/s.

$$u_1 = \int \frac{1}{2x^2(x+1)} dx$$

Now we will right integrand in partial form.

$$\frac{1}{2x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\frac{1}{2x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\frac{1}{2} = Ax(x+1) + B(x+1) + Cx^2$$

$$\frac{1}{2} = Ax^2 + Ax + Bx + B + Cx^2$$

$$\frac{1}{2} = (A+C)x^2 + (A+B)x + B$$

Comparing the co-efficients

$$B = \frac{1}{2}$$

So

$$A + B = 0$$

$$A = -B$$

$$A = -\frac{1}{2}$$

$A + C = 0$

$C = -A$

$C = \frac{1}{2}$

So,

$$\frac{1}{2x^2(x+1)} = -\frac{1}{2x} + \frac{1}{2x^2} + \frac{1}{2(x+1)}$$

Hence we have.

$$u_1 = \int -\frac{1}{2x} + \frac{1}{2x^2} + \frac{1}{2(x+1)}\, dx.$$

$$u_1 = -\frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{1}{x^2}\,dx + \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$$u_1 = -\frac{1}{2}\ln x - \frac{1}{2x} + \frac{1}{2}\ln(x+1)$$

Now,

$$u_2 = -\int \frac{1}{2(x+1)}\,dx$$

$$u_2 = -\frac{1}{2}\ln(x+1)$$

Now putting the values in equation

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \left[-\frac{1}{2}\ln x - \frac{1}{2x} + \frac{1}{2}\ln(x+1)\right]\cdot(x) + \left[-\frac{1}{2}\ln(x+1)\right]\cdot(x^{-1})$$

$$y_p = -\frac{1}{2}x\ln x - \frac{1}{2} + \frac{1}{2}x\ln(x+1) - \frac{1}{2x}\ln(x+1)$$

$$y_p = -\frac{1}{2}x\ln x - \frac{1}{2} + \frac{1}{2}x\ln(x+1) - \frac{1}{2x}\ln(x+1)$$

$$y_p = -\frac{1}{2} - \frac{1}{2}x\ln x + \frac{1}{2}x\ln(x+1) - \frac{1}{2x}\ln(x+1)$$

Now putting the values of y_c & y_p in

$$y = y_c + y_p$$

$$y = c_1 x^{-1} + c_2 x - \frac{1}{2} - \frac{1}{2}x\ln x + \frac{1}{2}x\ln(x+1) - \frac{1}{2x}\ln(x+1) \text{ Ans.}$$

Question # 4

$$x^2y'' - 2xy' + 2y = x^4e^x$$

$$2\ x^2y'' - 2xy' + 2y = x^4e^x \quad -①$$

Dividing x^2 on b/s to get standard form.

$$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = x^2e^x$$

To find general solution we will find homogeneous differential equation, so

$$x^2y'' - 2xy' + 2y = 0 \quad -(2)$$

Let

$y = x^m$, so

$y' = m(x)^{m-1}$

$y'' = m(m-1)\,x^{(m-2)}$

Putting the values of y, y' & y'' in eq (2)

$x^2 m(m-1)x^{m-2} - 2x\,mx^{m-1} + 2x^m = 0$

$m(m-1)\,x^2 x^{m-2} - 2m\,x^1 x^{m-1} + 2x^m = 0$

$m(m-1)x^m - 2mx^m + 2x^m = 0$

$(m^2 - m - 2m + 2)\,x^m = 0$

$(m(m-1) - 2(m-1))\,x^m = 0$

$(m-1)(m-2)\,x^m = 0$

Since x^m can not be equal to 0, so

$(m-1)(m-2) = 0$

So,

$m_1 = 1$ & $m_2 = 2$.

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The solution of the homogeneous differential equation is

$$y_c = C_1 x + C_2 x^2$$

Now we will find the particular solution.

$$y_p = u_1 y_1 + u_2 y_2$$

Since

$$y_1 = x$$

$$y_2 = x^2$$

So,

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix}$$

$$W = (x)(2x) - (1)(x^2)$$

$$W = 2x^2 - x^2$$

$$W = x^2$$

Now we will find W_1 & W_2, so

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

And $f(x) = x^2 e^x$, with the help of standard form.

$$W_1 = \begin{vmatrix} 0 & x^2 \\ x^2 e^x & 2x \end{vmatrix}$$

$$W_1 = 0 - x^2 \cdot x^2 e^x$$

$$W_1 = -x^4 e^x.$$

And

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & x^2 e^x \end{vmatrix}$$

$$W_2 = x \cdot x^2 e^x - 0$$

$$W_2 = x^3 e^x$$

Now we will find the value of u_1' & u_2', so

$$u_1' = \frac{W_1}{W}$$

$u_1' = \frac{W_1}{W}$

$u_1' = \frac{-x^4 e^x}{x^2}$

$u_1' = -x^2 e^x$

And

$u_2' = \frac{W_2}{W}$

$u_2' = \frac{x^3 e^x}{x^2}$

$u_2' = x e^x$

But we need the value of u_1 & u_2, so we will take integral on b/s.

$u_1 = \int -x^2 e^x \, dx$

$u_1 = -\int x^2 e^x \, dx$

Now we will use integration by parts i.e.

$u = x^2$, then $du = 2x dx$, and let $dv = e^x dx$ then $v = e^x$, so.

$u_1 = -\left(u \times v - \int v \, du\right)$

$$u_1 = -x^2 e^x + \int e^x \cdot 2x \, dx$$

Again we will use integration by parts i.e.

$s = 2x$, then $ds = 2dx$, and let $dv = e^x dx$ then $v = e^x$, so.

$$u_1 = -x^2 e^x + \left(s \times v - \int v \, ds \right)$$

$$u_1 = -x^2 e^x + 2xe^x - 2e^x$$

$$u_1 = -e^x (x^2 - 2x + 2)$$

Now

$$u_2 = \int x e^x \, dx$$

Now we will use integration by parts i.e.

$s = x$, then $ds = 1dx$, and let $dv = e^x dx$ then $v = e^x$, so.

$$u_2 = s \times v - \int v \, ds$$

$$u_2 = xe^x - \int e^x \, dx$$

$u_2 = xe^x - e^x$

$u_2 = (x-1)e^x$

Now putting the values in equation

$y_p = u_1 y_1 + u_2 y_2$

$y_p = -e^x(x^2 - 2x + 2) \cdot (x) + (x-1)e^x \cdot x^2$

$y_p = e^x(-\cancel{x^3} + 2x^2 - 2x + \cancel{x^3} - x^2)$

$y_p = e^x(x^2 - 2x)$

Now putting the values of y_c & y_p in

$y = y_c + y_p$

$y = c_1 x + c_2 x^2 + e^x(x^2 - 2x)$ Ans.

Question # 6

$$x^2 y'' + 10xy' + 8y = x^2$$

$$x^2 y'' + 10x y' + 8y = x^2 \quad \text{—} (1)$$

Now first we will make substitution

$$x = e^t$$

So,

$$x = e^t$$

Taking ln on b/s.

$$\ln x = \ln e^t$$

$$\ln x = t.$$

Now differentiating w.r.t "x", so.

$$\frac{dt}{dx} = \frac{d}{dx}\ln x$$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$dt = \frac{1}{x}dx$$

But according to chain rule

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

So, putting the value of $\frac{dt}{dx}$ in above equation.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x}\frac{dy}{dt}$$

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$$\frac{dy}{dx} = \frac{1}{x}\frac{dy}{dt}$$

$$y' = \frac{1}{x}\frac{dy}{dt}$$

Again taking $\frac{d}{dx}$ on b/s of the above equation.

$$\frac{d}{dx} y' = \frac{d}{dx}\left(\frac{1}{x}\cdot\frac{dy}{dt}\right)$$

Using product rule.

$$\frac{d}{dx} y' = \frac{1}{x} \times \frac{d}{dx}\left(\frac{dy}{dt}\right) + \frac{dy}{dt} \times \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$y'' = \frac{1}{x}\frac{dy}{dx}\left(\frac{dy}{dt}\right) - \frac{1}{x^2}\frac{dy}{dt}$$

Putting the value of $\frac{dy}{dx}$ in above equation.

$$y'' = \frac{1}{x}\frac{dy}{dt}\left(\frac{1}{x}\frac{dy}{dt}\right) - \frac{1}{x^2}\frac{dy}{dt}$$

$$y'' = \frac{1}{x^2}\frac{d^2y}{dt^2} - \frac{1}{x^2}\frac{dy}{dt}$$

$$y'' = \frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

Putting $x = e^t$ and the values of y' & y'' in eq ①.

$$\cancel{x^2} \cdot \frac{1}{\cancel{x^2}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + 10x \times \frac{1}{x}\frac{dy}{dt} + 8y = e^{2t}$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 10\frac{dy}{dt} + 8y = e^{2t}$$

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 8y = e^{2t}$$

Now to find general solution we will find homogeneous differential equation, so

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 8y = 0 \quad - ②$$

Let $y = \cancel{m^n} e^{mt}$, so

$$y' = me^{mt}$$

$$y'' = m^2 e^{mt}$$

Putting the values of y, y' & y'' in eq ②

$$m^2 e^{mt} + 9me^{mt} + 8e^{mt} = 0$$

$$(m^2 + 9m + 8)e^{mt} = 0$$

Since e^{mt} can not be equal to 0, so

$$(m+1)(m+8)=0$$

So,

$$m_1=-1 \quad \& \quad m_2=-8$$

The solution of the homogeneous differential equation is

$$y_c = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$y_c = C_1 e^{-t} + C_2 e^{-8t}$$

Now we will find the particular solution so let,

$$y_p = Ae^{2t}$$

Taking $\frac{d}{dt}$ on b/s

$$\frac{dy_p}{dt} = 2Ae^{2t}$$

Similarly,

$$\frac{d^2y_p}{dt^2} = 4Ae^{2t}$$

Putting $y_p = Ae^{2t}$ and the value of $\frac{dy_p}{dt}$ & $\frac{d^2y_p}{dt^2}$ in below equation.

$$\frac{d^2y_p}{dt^2} + 9\frac{dy_p}{dt} + 8y_p = e^{2t}$$

$$4Ae^{2t} + 9 \times 2Ae^{2t} + 8Ae^{2t} = e^{2t}$$

$$(4A + 18A + 8A)e^{2t} = e^{2t}$$

$$30A = 1$$

$$A = \frac{1}{30}$$

So the particular solution of is

$$y_p = \frac{1}{30}e^{2t}$$

So, Putting the values in given equation.

$$y = y_c + y_p \text{ , so}$$

$$y = C_1e^{-t} + C_2e^{-8t} + \frac{1}{30}e^{2t}$$

Now putting back $t = \ln x$ in above equation.

$$y = C_1e^{-\ln x} + C_2e^{-8\ln x} + \frac{1}{30}e^{2\ln x}$$

$$y = C_1x^{-1} + C_2x^{-8} + \frac{1}{30}x^2 \text{ Ans}$$

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Question # 5

$$4x^2y'' + y = 0, \quad y(-1) = 2, \quad y'(-1) = 4$$

Using the substitution $t = -x$, it follows from the Product Rule & Chain Rule that

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = -\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} = -\frac{d}{dx}\left(\frac{dy}{dt}\right)$$

$$= -\frac{d}{dt}\left(\frac{dy}{dt}\right)\frac{dt}{dx}$$

$$= \frac{d^2y}{dt^2}$$

Substituting in the given differential equation and simplifying yield

$$4t^2y'' + y = 0$$

The y' term is missing in the given Cauchy-Eular equation; nevertheless, the substitution $y = t^m$ yields.

$$4t^2y'' + y = 0$$

$$4t^2(m(m-1)t^{m-2}) + t^m = 0$$

$$4m(m-1)t^m + t^m = 0$$

$$t^m(4m^2 - 4m + 1) = 0$$

Since t^m is not 0, so

$$4m^2 - 4m + 1 = 0$$

Using quadratic formula,

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$$

$$m = \frac{+4 \pm \sqrt{16 - 16}}{8}$$

$$m = \frac{4 \pm \sqrt{0}}{8}$$

$$m = \frac{\cancel{4}\,1}{\cancel{8}\,2}$$

$$m = +\frac{1}{2}$$

$$m_1 = m_2 = \frac{1}{2}$$

Since roots are repeated and real, so.

$$y = c_1 t^{1/2} + c_2 t^{1/2} \ln t$$

Now, the derivative for solution is

$$y' = \frac{1}{2} c_1 t^{-1/2} + C_2 \left(\frac{1}{2} t^{-1/2} \ln t + t^{1/2} \frac{1}{t} \right)$$

$$y' = \frac{c_1 t^{-1/2}}{2} + C_2 \left(\frac{1}{2} t^{-1/2} \ln t + t^{-1/2} \right)$$

$$y' = t^{-1/2} \left(\frac{c_1}{2} + C_2 \left(\frac{\ln t}{2} + 1 \right) \right)$$

Since $t = -x$, the initial conditions become $y(1) = 2$ and $y'(1) = 4$.

Substituting $y(1) = 2$ in the solution gives

$$c_1 (1)^{\frac{1}{2}} + C_2 (1)^{1/2} \ln 1 = 2$$

$$C_1 + C_2 (0) = 2$$

$$C_1 = 2 \quad \text{—} \quad 1$$

Substituting $y'(1) = -4$ in the derivative equation gives.

$$(1)^{-1/2}\left[\frac{c_1}{2}+c_2\left(\frac{\ln 1}{2}+1\right)\right]=-4$$

$$\frac{c_1}{2}+c_2(0+1)=-4$$

Put $c_1=2$

$$\frac{2}{2}+c_2=-4$$

$$1+c_2=-4$$

$$c_2=-5 \quad \text{—} ②$$

Hence, the general solution is

$$y=2t^{1/2}-5t^{1/2}\ln t$$

and upon resubstituting $t=-x$ gives the general solution

$$y=2(-x)^{1/2}-5(-x)^{1/2}\ln(-x), \quad x<0$$

Here x is negative because logrithm of negative numbers is not defined.