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Section:
BSCS-2D

Question # 1:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$$

Since, f is odd on the interval $(-\pi, \pi)$ see its graph. So, $a_0 = 0$, $a_n = 0$ and we expand f in a sine series using equations (4) and (5).

Substituting in Equation (5) with $P = \pi$ and knowing that $f(x) = 1$ for x in $[0, x)$, we have:

$$b_n = \frac{2}{\pi} \int_0^{\pi} 1 \cdot \sin\left(\frac{n\pi}{\pi} x\right) dx$$

Simplify to obtain:

$$= \frac{2}{\pi} \int_0^{\pi} \sin(nx)\, dx$$

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Using substitution method with $u = nx$, $du = n dx \Rightarrow dx = \frac{1}{n} du$, we get:

$$\frac{1}{n}\int \sin(u)\,du = -\frac{1}{n}\cos(u) + C = -\frac{1}{n}$$

$\cos(nx) + C$; hence

$$= \frac{2}{\pi}\left(-\frac{1}{n}\cos(n\cdot\pi) + \frac{1}{n}\cos(n\cdot 0)\right)$$

Simplify using $\cos(0) = 1$ and $\cos(n\pi)$ $= (-1)^n$ for $n = 1, 2, \ldots$

$$= \frac{2}{\pi}\left(-\frac{1}{n}(-1)^n + \frac{1}{n}\right)$$

Rewrite as:

$$= \frac{2}{n\pi}\left(1 - (-1)^n\right)$$

Thus, the substituting into equation (4) with $p = \pi$ and simplifying, we get the expansion of f as:

$$\sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1 - (-1)^n \sin(nx)\right)$$

Question # 2:

$$f(x) = \begin{cases} 1, & -2 < x < -1 \\ 0, & -1 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$$

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$f(x)$ is odd then, $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{P}\right)$

where,

$$b_n = \frac{2}{P}\int_0^P f(x) \sin\left(\frac{n\pi x}{P}\right) dx$$

$f(x)$ is even then,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{P}\right)$$

where,

$$a_0 = \frac{2}{P}\int_0^P f(x) dx$$

$$a_n = \frac{2}{P}\int_0^P f(x) \cos\left(\frac{n\pi x}{P}\right) dx$$

From the graph the function is even.

$P = 2$

$$a_0 = \frac{2}{2}\left[\int_0^1 0 dx + \int_1^2 1 dx\right] = 1$$

$$a_n = \frac{2}{2}\left[\int_0^1 0.\cos\left(\frac{n\pi x}{2}\right) dx + \int_1^2 1.\cos\left(\frac{n\pi x}{2}\right) dx\right]$$

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$$= \frac{-2\sin\left(\frac{n\pi}{2}\right) + 2\sin(n\pi)}{n\pi}$$

$$= \frac{-2\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-2\left(\sin\left(\frac{n\pi}{2}\right)\right)}{n\pi} \cos\left(\frac{n\pi x}{2}\right)$$

Question # 3:

$f(x) = |x|, \quad -\pi < x < \pi$

We have,

$f(x) = |x|$ where $-\pi < x < \pi$

Our aim is to expand $f(x)$ in an appropriate cosine or sine series. From the given definition of $f(x)$, we can find:

$$f(-x) = |-x|$$
$$= |x|$$
$$= f(x)$$

Which implies that the given function is an even one. The fourier series of an even function 'f'

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defined on the interval $(-p, p)$ is the cosine series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{p} x$$

where,

$$a_0 = \frac{2}{p} \int_0^p f(x)\, dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos \frac{n\pi x}{p}\, dx$$

We write the given function as follows:

$$f(x) = \begin{cases} -x & \text{if} \quad -\pi < x < 0 \\ x & \text{if} \quad 0 \leq x < \pi \end{cases}$$

Now we apply the definition and find the coefficients of the series.

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x\, dx$$

$$= \frac{2}{\pi} \left[\frac{x^2}{2} \right]^{x=\pi}$$

$$= \frac{2}{\pi} \left[\frac{\pi^2}{2} \right]^{x=0}$$

$$= \pi$$

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and,

$$a_n = \frac{2}{\pi}\int_0^{\pi} x \cos \frac{n\pi}{\pi} x \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x \cos nx \, dx$$

We do integration by parts, where

$u = x \implies du = dx$ and $dv = \cos nx \, dx$
$\implies v = \frac{1}{n} \sin nx$

Thus,

$$a_n = \frac{2}{\pi}\left(\left[\frac{1}{n} x \sin nx\right]_{x=0}^{x=\pi} - \frac{1}{n}\int_0^{\pi} \sin nx \, dx\right)$$

$$= \frac{-2}{n\pi}\left[\frac{-\cos nx}{n}\right]_{x=0}$$

$$= \frac{-2}{n\pi}\left[\frac{-(-1)^n + 1}{n}\right]_{x=0}$$

$$= \frac{2(-1)^n - 2}{n^2\pi}$$

Therefore, the cosine series of $f(x)$ is

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$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2(-1)^n - 2}{n^2\pi} \cos\frac{n\pi}{\pi}x$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2(-1)^n - 2}{n^2\pi} \cos nx$$

Question #4:

$$f(x) = x, \quad -\pi < x < \pi$$

By the definition f,

$$f(-x) = -x = -(x) = -f(x)$$

This shows that f is an odd function. Since it is defined on the interval $(-\pi, \pi)$, it has period equal to 2π.

We shall expand this function into an odd function over R.

Observe that here $L = \pi$

Since f is odd, $a_0, a_1, a_2 \cdots = 0$

We shall find out $b_1, b_2, \dots, b_n \dots$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \sin(nx)\, dx$$

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$$= \frac{2}{\pi}\left[-\frac{x\cos(nx)}{n}+\frac{\sin(nx)}{n^2}\right]_0^{\pi}$$

$$= -\frac{2\cos(n\pi)}{n}$$

Once we have found out bn's , we can write a periodic function f over R as follows:

$$f(x) = \sum_{n=1}^{\infty}\left(-\frac{2\cos(n\pi)}{n}\right)\sin(nx)$$

Once we have found out bn's we can write a periodic formula f over R as follows:

$$f(x) = \sum_{n=1}^{\infty}\left(-\frac{2\cos(n\pi)}{n}\right)\sin(nx)$$

Question # 15:

$$f(x) = x^2, \quad -1 < x < 1$$

We have,

$$f(x) = x^2, \quad -1 < x < 1$$

We can expand f(x) as fourier series:

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\frac{n\pi x}{c} +$$

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$$+ \sum b_n \sin \frac{n\pi x}{c} \text{ ——— (i)}$$

where,

$$a_0 = \frac{2}{c} \int_{-c}^{c} f(x) dx$$

So,

$$a_0 = \frac{2}{1} \int_{-1}^{1} x^2 . dx$$

$$= 2 \left[\frac{x^3}{3} \right]_{-1}^{1}$$

$$= 2 \left[\frac{1}{3} + \frac{1}{3} \right]$$

$$= 2 \left(\frac{2}{3} \right)$$

$$a_0 = \frac{4}{3}$$

$$a_n = \frac{2}{c} \int_{-c}^{c} f(x) \cos \frac{n\pi x}{c} . dx$$

$$a_n = \frac{2}{1} \int_{-1}^{1} x^2 . \cos n\pi x . dx$$

Integral is even function. so,

$$a_n = 2 \times 2 \int_{0}^{1} x^2 . \cos n\pi x . dx$$

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$$a_n = 4\int_0^1 x^2 \cos n\pi x \, dx$$

Using by parts

$$a_n = 4\left[\cancel{x^2 \cdot \frac{\sin n\pi x}{n\pi}}^{\,0} \Big|_0^1 - \int_0^1 2x \cdot \frac{\sin n\pi x}{n\pi} \, dx \right]$$

$$a_n = \frac{-4 \times 2}{n\pi} \int_0^1 x \sin n\pi x \, dx$$

By using parts again:

$$a_n = \frac{-8}{n\pi}\left[\frac{x(-\cos n\pi x}{n\pi}\Big|_0^1 - \int_0^1 \frac{-\cos n\pi x}{n\pi} \, dx \right]$$

$$a_n = \frac{-8}{n\pi}\left[\frac{-\cos n\pi}{n\pi} + \frac{1}{n\pi}\int_0^1 \cos n\pi x \, dx \right]$$

$$= \frac{-8}{n\pi}\left[\frac{-\cos n\pi}{n\pi} + \frac{1}{n\pi}\left[\frac{\sin n\pi x}{n\pi} \right]_0^1 \right]$$

$$a_n = \frac{-8}{n\pi}\left[\frac{-\cos n\pi}{n\pi} + \frac{1}{n\pi}\left[\frac{\cancel{\sin n\pi}^{\,0}}{\cancel{n}\pi} - \frac{0}{n\pi} \right] \right]$$

$$a_n = \frac{8 \cos n\pi}{(n\pi)^2} \qquad \text{But } \cos n\pi = (-1)^n$$

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$$a_n = \frac{8(-1)^n}{(n\pi)^n}$$

$$b_n = \frac{2}{c} \int_{-c}^{c} f(x) \sin n\pi x \cdot dx$$

$$b_n = \frac{2}{1} \int_{-1}^{1} x^2 \cdot \sin n\pi x \cdot dx$$

Integral is odd function

$$\int_{-a}^{a} g(x) = 0 \quad \text{if } f(x) \text{ is odd.}$$

$$b_n = 2 \times 0$$

$$b_n = 0$$

Now from (i)

$$f(x) = \frac{4}{3 \times 2} + \sum \frac{8(-1)^n}{(n\pi)^2} \cos n\pi x +$$

$$\sum 0 \cdot \cancelto{0}{\sin n\pi x}$$

$$f(x) = \frac{2}{3} + \frac{8}{\pi^2} \left[\frac{-1}{1^2} \cos \pi x + \frac{1}{2^2} \cos 2\pi x \right.$$

$$\left. \frac{-1}{3^2} \cos 3\pi x + \dots \right]$$

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f(a) in form of cos

Question # 6:

$$f(x) = x|x|, \quad -1 < x < 1$$

$$f(x) = \begin{cases} x(-x) \longrightarrow & -1 < x < 0 \\ 0 \longrightarrow & x = 0 \\ x(x) \longrightarrow & 0 < x < 1 \end{cases}$$

$$f(x) \Rightarrow \begin{cases} -x^2 \longrightarrow & -1 < x < 0 \\ 0 \longrightarrow & x = 0 \\ x^2 \longrightarrow & 0 < x < 1 \end{cases}$$

So, $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} x + b_n$

$\sin \frac{\pi}{2} x.$

$$a_0 = \frac{1}{2} \int_{-1}^{1} f(x)\, dx$$

$$a_n = \frac{1}{2} \int_{-1}^{1} f(x) \sin \frac{n\pi}{2} x\, dx$$

$$a_0 = \frac{1}{2}\left[\int_{-1}^{0} -x^2 dx + \int_{0}^{1} x^2 dx\right] \rightarrow \frac{1}{2}\left[\left(\frac{-x^3}{3}\right)_{-1}^{0} + \left(\frac{x^3}{3}\right)_{0}^{1}\right]$$

$$a_0 = \frac{1}{2}\left[-\left(+\frac{1}{3}\right) + \frac{1}{3}\right] = 0$$

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$$a_n = \frac{1}{2}\left[\int_{-1}^{0} (-x^2)\cos\left(\frac{n\pi}{2}\right)x\,dx + \int_{0}^{1} x^2 \cos\frac{n\pi}{2}x\,dx\right]$$

$$a_n = \frac{1}{2}\left[\int_{-1}^{0} (-x^2)\cos\left(\frac{n\pi}{2}\right)x\,dx + \int_{0}^{1} x^2 \sin\frac{n\pi}{2}x\,dx\right]$$

$$a_n = \frac{1}{2}\left[\int_{-1}^{0} -x^2\cos\frac{n\pi}{2}\,dx + \int_{0}^{1} x^2 \cos\frac{n\pi}{2}x\,dx\right]$$

replace x with $-x$

then,

$$\int_{-1}^{0} -x^2\cos\frac{n\pi x}{2}\,dx = -\int_{0}^{1} -x^2\cos\frac{n\pi x}{2}(-dx) =$$

$$-\int_{0}^{1} x^2\cos\left(\frac{n\pi x}{2}\right)d$$

So, $a_n = 0$

$$b_n = \frac{1}{2}\left[\int_{-1}^{0} -x^2\sin\frac{n\pi x}{2}\,dx + \int_{0}^{1} +x^2\sin\frac{n\pi x}{2}\,dx\right]$$

replace x with $-x$

$$b_n = \frac{1}{2}\int_{-1}^{0} -x^2\sin\frac{n\pi x}{2}\,dx = -\int_{0}^{1} -x^2 - \sin\frac{n\pi x}{2}.$$

$$(-dx) = +\int_{0}^{1} x^2 \sin\frac{\pi x n}{2}\,dx$$

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So,

$$b_n = \int_0^1 x^2 \sin\left(\frac{n\pi x}{2}\right) dx.$$

$$b_n = 8\pi \sin\left(\frac{n\pi}{2}\right) + (16 - 2\pi^2 n^2)\cos\left(\frac{\pi n}{2}\right)$$

$$-16$$

So,

$$b_n = \int_0^1 x^2 \sin\left(\frac{n\pi x}{2}\right) dx.$$

$$b_n = \frac{8\pi n \sin\left(\frac{n\pi}{2}\right) + (16 - 2\pi^2 n^2)\cos\left(\frac{\pi n}{2}\right) - 16}{\pi^3 n^3}$$

So,

$$f(x) = 0 + \sum_{n=1}^{\infty} 0 + \left[\frac{8\pi n \left(\sin \frac{n\pi}{2}\right) + (16 - 2\pi^2 n^2) \cdot}{2} \right.$$

$$\left. \frac{\cos\left(\frac{\pi n}{2}\right) - 16}{} \right] \sin \frac{n\pi x}{2}$$

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