Web-Based Medical Decision Support Systems for Three-Way Medical Decision Making With Game-Theoretic Rough Sets

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***Abstract*—The realization of the Web as a common platform, medium, and interface for supporting human activities has attracted many researchers to the study of Web-based support systems (WSS). An important branch of WSS is Web-based decision support systems that provide intelligent support for making effective decisions in different domains. We focus on decision making in Web-based medical decision support systems (WMDSS). Uncertainty is a critical factor that affects decision making and reasoning in the medical field. A three-way decision-making approach is an effective and better choice to lessen the effects of uncertainty. It provides the provision for delaying certain or definite decisions in situations that lack sufficient evidence or accurate information in reaching certain conclusions. Particularly, the option of deferment decisions is added in this approach that provides the flexibility to further examine and investigate the uncertain and doubtful cases. The game-theoretic rough set (GTRS) model is a recent development in rough sets that can be used to determine the three rough set regions in the probabilistic rough sets framework by determining a pair of thresholds. The three regions are used to obtain three-way decision rules in the form of acceptance, rejection, and deferment rules. In this paper, we extend the GTRS model to analyze uncertainty involved in medical decision making. Experimental results with a GTRS-based approach on different health care datasets suggest that the approach may improve the overall quality of decision making in the medical field, as well as other fields. It is hoped that the incorporation of a GTRS component in WMDSS will enrich and enhance its decision-making capabilities.**

***Index Terms*—Game-theoretic rough sets (GTRS), medical decision making, probabilistic rough sets, three-way decision making, Web-based support systems (WSS).**

I. Introduction

The study of Web-based support systems (WSS) aims at developing and transforming existing systems to support and extend various human activities onto the Web [46], [48]. The motivation of WSS research came from the realization of the Web as a common platform, medium, and interface in supporting and assisting activities like managing, planning, searching, and decision making in different fields [45], [51], [62]. An important area of WSS is Web-based decision support systems (WDSS) that provide assistance for decision making in various domains [9], [35]. The WDSS take advantages from opportunities provided by the Web such as increased availability and effective user interfaces along with advances in intelligent decision-making techniques. Some of the recent examples of WDSS are found in the fields of group decision making, business, energy, and waste management [21], [25]–[27], [36], [40]. We consider Web-based medical decision support systems (WMDSS) in this study.

The WMDSS have become a valuable aid for medical practitioners in making effective decisions for selecting a course of action in medical diagnosis and treatment [8], [42]. Komkhao *et al.* view the WMDSS from the viewpoint of a recommender system where suitable decision recommendations are being made by the system [22]. In any case, the WMDSS serve as a platform for integrating evidence-based medicine into effective and efficient care delivery. There are challenges in designing, developing, and deploying WMDSS [37], for example, effective and meaningful user interfaces, recalling and retrieving relevant information, meeting the reliability and accuracy expectations of decision making, and prioritizing and filtering information to users, just to mention a few [37]. We focus on the decision-making aspect of WMDSS.

Uncertainty is one of a critical factor that affects reasoning and decision making in the medical field [17], [38]. A three-way decision-making strategy that includes a delay, deferment, or noncommitment decision option is a better and useful approach for reducing the effect of uncertainty in decision making [24], [53], [54], [56]. A particular model in this regard in the medical field was proposed by Pauker and Kassirer for making treatment decisions [29]. A pair of testing and test-treatment thresholds were used to define the decisions of treatment, no treatment, and delay treatment based on the probability of a disease.

The fuzzy and rough set models and their extensions can provide alternatives for three-way decision making [30], [39], [43], [59]–[61]. The shadowed sets that generalize a fuzzy set into a three-valued approximation provides a compelling approach for three-way decision making [32]. On the other hand, the rough sets and its extensions also lead to three-way decisions by interpreting the rough set-based positive, negative, and boundary regions as regions of acceptance, rejection, and deferment, respectively. The game-theoretic rough set (GTRS) model is a recent development in rough sets that provides yet another approach in this regard [18], [49]. Particularly, the model provides a threshold configuration mechanism for determining effective threshold parameters to define the three probabilistic rough set regions and the implied three-way decisions [3], [5].

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In a recent paper, the applicability of GTRS model was demonstrated for effectively determining the thresholds by setting the minimization of overall uncertainty level of the probabilistic rough set regions as a game objective [5]. A repeated game was defined that iteratively tunes the threshold parameters in order to improve the overall uncertainty level in a game-theoretic learning environment.

In this paper, we extend these initial results to investigate uncertainty in WMDSS. In Section II, we elaborate the fundamentals of three-way decision making and three approaches for three-way decisions, i.e., the GTRS approach, the shadowed sets approach, and the threshold approach. Section III explains an architecture of WMDSS that incorporates a GTRS component for three-way decision making. Section IV describes the uncertainties involved in probabilistic rough set regions and a GTRS-based approach to minimize the overall uncertainty of decision regions. Section V provides a demonstrative example for the use of GTRS in obtaining three-way decisions in medical domain. Finally, Section VI presents experimental results with the proposed approach on different datasets in the medical domain.

II. Three-Way Decision Making

A general framework of three-way decision making was recently outlined by Yao [56].

Considering U as a finite nonempty set of objects and C as a set of criteria, objectives, or constraints, we divide U based on C into three disjoint regions: positive (POS), negative (NEG), and boundary (BND). These three regions are utilized to induce rules for three-way decisions. The POS region generates rules for decisions of acceptance, the NEG region generates rules for rejection, and the BND region generates rules for decision of noncommitment or deferment [53], [54], [56]. The determination of an object for inclusion in a particular region depends on the degree or level to which it satisfies the criteria in C. Particularly, the POS region consists of objects whose satisfiability (for criteria in C) is at or above a certain level of acceptance. The NEG region consists of objects whose satisfiability is at or below a level of rejection. The BND region consists of objects whose satisfiability is above the rejection level but below the acceptance level.

To formally explain the satisfiability of objects and the levels for acceptance and rejection, Yao introduced the notions of evaluations of objects and designated values for acceptance and rejection [56]. It was argued that evaluations provide the degrees of satisfiability, and the designated values for acceptance and rejection are the acceptable degrees of satisfiability and nonsatisfiability of objects. A theory of three-way decisions must consider 1) the construction and interpretation of a set of values for measuring satisfiability and a set of values for measuring nonsatisfiability, 2) the construction and meaning of evaluations, and 3) the determination and interpretation of designated values [56].

There are many approaches to three-way decision making in the literature [7], [16], [32], [44], [57]. We review only three of these approaches, namely, the GTRS-based approach, the shadowed sets approach, and the threshold approach.

A. Game-Theoretic Rough Set Approach

The probabilistic rough set model has been recognized as a major extension and generalization of the Pawlak rough set model [52]. A main result of probabilistic rough sets is that the three regions based on a concept C are defined using a pair of thresholds (α, β) as

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(C) &= \{x \in U | P(C|[x]) \geq \alpha\} \\ \mathrm{NEG}_{(\alpha,\beta)}(C) &= \{x \in U | P(C|[x]) \leq \beta\} \\ \mathrm{BND}_{(\alpha,\beta)}(C) &= \{x \in U | \beta < P(C|[x]) < \alpha\} \end{aligned} \quad (1)$$

where $P(C|[x])$ denotes the conditional probability of an object x to be in C given that the object is in $[x]$, and $0 \leq \beta < \alpha \leq 1$. Moreover, U is the set of objects called universe, and the concept $C \subseteq U$. In the context of decision making, the positive, negative, and boundary regions are sometimes interpreted as regions of acceptance, rejection, and deferment decisions, respectively [52]. The probabilistic thresholds (α, β) are interpreted as levels for acceptance and rejection, and the probability $P(C|[x])$ determines the evaluation of an object given that the object is in $[x]$. How to determine the thresholds is a fundamental issue in probabilistic rough sets [55].

The GTRS approach formulates a game to determine the required threshold parameters. A typical game is defined as a tuple $\{P, S, u\}$, where we have the following [23]:

1) P is a finite set of n players, indexed by i.
2) $S = S_1 \times \ldots \times S_n$, where S_i is a finite set of strategies for player i. Each vector of the form $s = (s_1, s_2, ..., s_n) \in S$ is called a strategy profile, where player i selects strategy s_i.
3) $u = (u_1, ..., u_n)$, where $u_i : S \longmapsto \Re$ is a real-valued utility or payoff function for player i.

For the sake of convenience, the strategy profile without the ith player strategy is denoted as $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, ..., s_n)$. That is, $s = (s_1, s_2, ..., s_n) = (s_i, s_{-i})$, meaning all the players expect i are committed to play s_{-i}, while player i chooses s_i.

The outcome of a game is usually determined by using the solution concept of Nash equilibrium [41]. A strategy profile $(s_1, s_2, ..., s_n)$ is a Nash equilibrium if the following condition holds:

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}), \text{ where } (s'_i \in S_i \wedge s'_i \neq s_i). \quad (2)$$

In other words, s_i is the best response to s_{-i} for all i. This intuitively means that Nash equilibrium is a strategy profile in which none of the players are benefited by changing their respective strategies, given the other player's chosen strategies.

In a GTRS-based game, the players are considered in the form of multiple criteria. Each criterion represents a particular aspect of interest such as accuracy or applicability of decision rules. Suitable measures are selected to evaluate these criteria in the context of rough set-based approximation and classification. Each criterion is affected by considering different (α, β) threshold configurations. The strategies are, therefore, formulated in terms of changes in probabilistic thresholds [3]. The payoff functions represents possible gains, benefits, or performance levels achieved by adapting different modification of threshold levels. The game is represented using a payoff table that contains the listing of all possible strategies of every player during a game with their respective payoff functions or utilities [4].

The use of GTRS is associated with at least two distinguishing characteristics compared with other models for three-way decision making.

1) Whereas other three-way decision models rely on a separate and external mechanism to model the determination of thresholds, the GTRS model provides a unified formalism for three-way decisions and thresholds determination [5].
2) The ability to determine the threshold levels based on a tradeoff solution between multiple criteria can comparatively lead to more effective and moderate levels for acceptance, rejection, and deferment decisions [47].

These properties of GTRS will become more evident in Section V, when we consider a demonstrative example.

B. Shadowed Set Approach

The shadowed sets introduced by Pedrycz offer a description of a particular fuzzy set by considering three different levels of membership grades [32]. A shadowed set A is formally represented as

$$A : X \longrightarrow \{0, [0, 1], 1\} \tag{3}$$

where X represents the universe, and the range of A consists of three components, i.e., 0, 1, and the unit interval [0,1] representing no membership, complete membership, and uncertain or partial membership, respectively [33].

The shadowed sets define two operations for an object, namely, the elevation and reduction [33]. The former elevates the membership grade of an object to a meaningful and acceptable level of grade, and the later reduces the membership value to another meaningful and acceptable level of grade. These two operations play a key role in interpreting and constructing the three-valued approximation [14]. We elevate the membership grade $A(x)$ for an object $x \in X$ to 1, if $A(x)$ is greater than a certain acceptable level. The membership grade is reduced to 0, if $A(x)$ is lower than a certain rejection level. The membership grade will be either elevated or reduced, if $A(x)$ is between the two levels. An important consideration in this framework is the determination of suitable levels for acceptance and rejection of membership grades. The notion of the so-called α-cut of a fuzzy set was introduced to define the $(\alpha, 1 - \alpha)$ model for interpreting the levels for acceptance and rejection in [33]. A decision-theoretic cost-sensitive approach for determining the two levels was considered in [14]. It is worth pointing out that further insights may be gained into the determination and interpretation of the two levels from granular computing perspective when they are being considered as levels for admitting or excluding in an information granule [31], [50].

The three membership levels or grades in the shadowed sets provide an immediate interpretation from decision-making perspective. For an element, $x \in X$ with its evaluation determined by the membership grade represented as $A(x)$. If $A(x) = 1$, the object is assumed to be compatible with the set or concept conveyed by A and is accepted for A. If $A(x) = 0$, the object is not in accordance with the concept determined by A and is rejected for A. If an element receives a membership grade in the unit interval [0, 1], it is uncertain, and we are not able to allocate any numeric grade. A decision of acceptance or rejection is not being made in this case.

C. Threshold Approach

The threshold approach was proposed by Pauker and Kassirer for making medical treatment decisions [29]. The probability of a disease was used to evaluate a particular decision. A pair of testing and test-treatment thresholds were defined with testing threshold being less than the test-treatment threshold. The testing threshold defines a level for rejecting or refusing to provide a treatment. Particularly, if the probability of a disease is lesser than the testing threshold, the disease is suggested to be absent thereby additional diagnosing tests are avoided, and treatment is withheld. The test-treatment threshold defines an acceptable level for providing a treatment. If the probability of a disease is above the test-treatment threshold, the disease is suggested to be present, and immediate treatment is provided with no further testing. If the probability is between the two thresholds, the presence or absence of a disease is inconclusive. Additional diagnosing tests are performed with a treatment decision depending on the test outcome.

The two thresholds are affected by different factors such as the cost, benefit, availability, and effectiveness of treatment and the accuracy, reliability, and risks associated with diagnostic testing [10]. The determination of threshold values based on multiple and sometimes conflicting aspects is a key challenge in this context. An approximate rather than exact solution may be possible that considers a tradeoff among multiple criteria affecting the two thresholds. An investigation of some soft-computing technique may be acknowledged for finding and reaching such a solution.

III. Architecture of Web-Based Medical Decision Support Systems

The WMDSS contain various components with functionalities ranging from supporting end user activities and interaction through interfaces to maintaining and manipulating the knowledge within the system. We adapt the architecture of rough set-based WMDSS proposed by Yao *et al.* [46], [48]. The architecture is slightly modified to incorporate a GTRS component that will assist in decision making. Fig. 1 represents the general architecture of WMDSS. We describe this architecture as comprising of an interface, management, and data layers. Each of these layers are now briefly discussed.

A. Interface Layer

The Web and the Internet makes up the interface layer. The clients interact with the system that is designed on the server side through an interface that is supported by the Web and Internet.

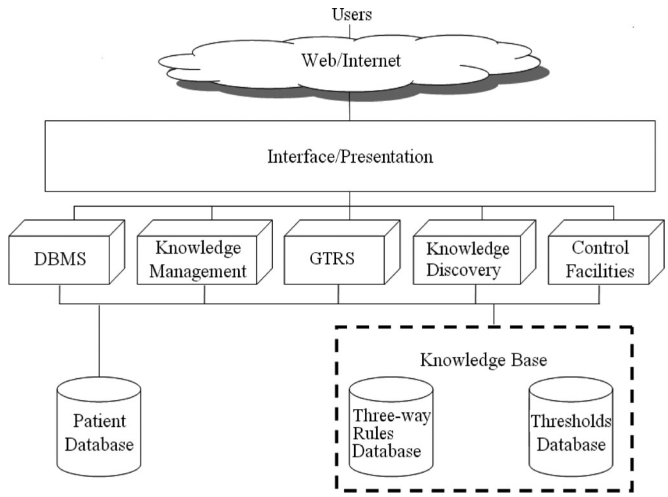

Fig. 1. Architecture of a WMDSS.

The interface is presented to the clients with the help of Web browsers. It is used by the users to input any relevant information that the system requires. In addition, it is also responsible for providing services and functionalities like searching, storing, or obtaining data analysis results for supporting and making decisions. A carefully designed Web interface is very critical for the success of the whole system. It has to be clear, complete, and consistent and should provide user guidance and autocorrection facilities.

B. Management Layer

This layer serves as the middleware of the three-layer architecture. The information from the top or bottom layers is processed at this layer before being presented to upper or lower layers. Intelligent techniques to analyze data such as logic, inference, and reasoning play a key role at this layer.

Some typical components that are required at this level are Database Management System, Knowledge Management, an intelligent decision-making component such as GTRS, Knowledge Discovery/Data Mining, and Control Facilities. The Database Management System binds the patient database and the GTRS component by making the data available from the database to the GTRS component. The Knowledge Management provides access to the Knowledge Base, which contains the rules database and thresholds database. The GTRS component makes use of a game-theoretic environment in analyzing data to acquire knowledge in the form of three-way decision rules. The Knowledge Discovery component is responsible for mining important information based on patient's data and knowledge base. In some sense, it also serves as an information retrieval component where it searches and retrieves important patterns in data corresponding to different user queries. Finally, the Control Facilities component provides functionalities such as access rights and permissions to patients data.

C. Data Layer

This layer contains data necessary for the operation of the system. A major portion of the data at this layer appears in the form of patient database, which is populated by recording series of questions, observations, and trails performed on patients. From decision-making perspective, the information of symptoms corresponding to a disease is the most important part of this database. The GTRS and Knowledge Discovery components access this database for analysis and retrieval purposes, respectively.

The data layer also contains the Knowledge Base component, which contains information about three-way decision rules in the rules database. Rules are learned from the data using the GTRS component and are utilized to make decisions of acceptance, rejection, and deferment. The decision rules are accompanied by the threshold levels in the threshold database, which provides the levels for inducing and making three-way decisions.

IV. Analyzing Uncertainty in Three-Way Decision Making With Game-Theoretic Rough Set

The GTRS model was recently demonstrated for analyzing the decision uncertainty involved in the three probabilistic rough set regions [5]. A pair of thresholds controls the uncertainties involved in different decision regions [5]. In order to determine effective threshold values, a game was implemented between decision regions with a goal of minimizing the overall uncertainty, resulting in effective and efficient three-way decisions [5]. We briefly review the calculation of uncertainties involved in the probabilistic rough set regions as put forward by Deng and Yao in [13] and [15].

A. Uncertainties in Probabilistic Rough Set Regions

The probabilistic rough set regions are used to create a partition based on a concept C and probabilistic thresholds (α, β) as [13]

$$\pi_{(\alpha,\beta)} = \{\mathrm{POS}_{(\alpha,\beta)}(C), \mathrm{NEG}_{(\alpha,\beta)}(C), \mathrm{BND}_{(\alpha,\beta)}(C)\}. \quad (4)$$

Another partition with respect to concept C is formed as $\pi_C = \{C, C^c\}$. The measure of Shannon entropy is used to calculate the uncertainty in π_C with respect to the three probabilistic regions as follows [13]:

$$\begin{aligned} H(\pi_C|\mathrm{POS}_{(\alpha,\beta)}(C)) = & -P(C|\mathrm{POS}_{(\alpha,\beta)}(C)) \\ & \times \log P(C|\mathrm{POS}_{(\alpha,\beta)}(C)) \\ & - P(C^c|\mathrm{POS}_{(\alpha,\beta)}(C)) \\ & \times \log P(C^c|\mathrm{POS}_{(\alpha,\beta)}(C)) \end{aligned} \quad (5)$$

$$\begin{aligned} H(\pi_C|\mathrm{NEG}_{(\alpha,\beta)}(C)) = & -P(C|\mathrm{NEG}_{(\alpha,\beta)}(C)) \\ & \times \log P(C|\mathrm{NEG}_{(\alpha,\beta)}(C)) \\ & - P(C^c|\mathrm{NEG}_{(\alpha,\beta)}(C)) \\ & \times \log P(C^c|\mathrm{NEG}_{(\alpha,\beta)}(C)) \end{aligned} \quad (6)$$

$$\begin{aligned} H(\pi_C|\mathrm{BND}_{(\alpha,\beta)}(C)) = & -P(C|\mathrm{BND}_{(\alpha,\beta)}(C)) \\ & \times \log P(C|\mathrm{BND}_{(\alpha,\beta)}(C)) \\ & - P(C^c|\mathrm{BND}_{(\alpha,\beta)}(C)) \\ & \times \log P(C^c|\mathrm{BND}_{(\alpha,\beta)}(C)). \end{aligned} \quad (7)$$

Equations (5)–(7) represent the evaluation of uncertainty in π_C with respect to $\text{POS}_{(\alpha,\beta)}(C)$, $\text{NEG}_{(\alpha,\beta)}(C)$, and $\text{BND}_{(\alpha,\beta)}(C)$ regions, respectively. For instance, (5) represents the uncertainty in π_C conditioned upon $\text{POS}_{(\alpha,\beta)}(C)$. This is calculated by considering the two sets or components of the partition π_C, i.e., C and C^c and determining their individual uncertainties with respect to $\text{POS}_{(\alpha,\beta)}(C)$, given by, $-P\left(C|\text{POS}_{(\alpha,\beta)}(C)\right) \times \log P\left(C|\text{POS}_{(\alpha,\beta)}(C)\right)$ and $-P\left(C^c|\text{POS}_{(\alpha,\beta)}(C)\right) \times \log P\left(C^c|\text{POS}_{(\alpha,\beta)}(C)\right)$, respectively. The conditional probabilities in (5) are computed as

$$P(C|\text{POS}_{(\alpha,\beta)}(C)) = \frac{|C \bigcap \text{POS}_{(\alpha,\beta)}(C)|}{|\text{POS}_{(\alpha,\beta)}(C)|} \tag{8}$$

$$P(C^c|\text{POS}_{(\alpha,\beta)}(C)) = \frac{|C^c \bigcap \text{POS}_{(\alpha,\beta)}(C)|}{|\text{POS}_{(\alpha,\beta)}(C)|}. \tag{9}$$

Conditional probabilities for the other regions are similarly obtained. When the three regions are interpreted and related to three-way decisions, the uncertainty due to probabilistic regions are associated with the uncertainty of their respective decisions. For instance, the uncertainty due to positive region, i.e., $H(\pi_C|\text{POS}_{(\alpha,\beta)}(C))$, is associated with uncertainty involved in making the decisions of acceptance. Similarly, the uncertainties $H(\pi_C|\text{NEG}_{(\alpha,\beta)}(C))$ and $H(\pi_C|\text{BND}_{(\alpha,\beta)}(C))$ are associated with decisions of rejection and deferment, respectively.

The overall uncertainty is calculated as an average uncertainty of the three regions. This is also sometimes referred to as conditional entropy of π_C given $\pi_{(\alpha,\beta)}$ and is determined as [13]

$$\begin{aligned} H(\pi_C|\pi_{(\alpha,\beta)}) = \ & P(\text{POS}_{(\alpha,\beta)}(C))H(\pi_C|\text{POS}_{(\alpha,\beta)}(C)) \\ & + P(\text{NEG}_{(\alpha,\beta)}(C))H(\pi_C|\text{NEG}_{(\alpha,\beta)}(C)) \\ & + P(\text{BND}_{(\alpha,\beta)}(C))H(\pi_C|\text{BND}_{(\alpha,\beta)}(C)). \end{aligned} \tag{10}$$

The probability of a particular region, say the positive region, is

$$P(\text{POS}_{(\alpha,\beta)}(C)) = \frac{|\text{POS}_{(\alpha,\beta)}(C)|}{|U|}. \tag{11}$$

The probabilities of the other regions are be similarly defined.

Equation (10) is reformulated in a more readable form. Considering the notations of $\Delta_P(\alpha,\beta)$, $\Delta_N(\alpha,\beta)$, and $\Delta_B(\alpha,\beta)$ for representing the uncertainties of positive, negative, and boundary regions, respectively, i.e.,

$$\Delta_P(\alpha,\beta) = P(\text{POS}_{(\alpha,\beta)}(C))H(\pi_C|\text{POS}_{(\alpha,\beta)}(C)) \tag{12}$$

$$\Delta_N(\alpha,\beta) = P(\text{NEG}_{(\alpha,\beta)}(C))H(\pi_C|\text{NEG}_{(\alpha,\beta)}(C)) \tag{13}$$

$$\Delta_B(\alpha,\beta) = P(\text{BND}_{(\alpha,\beta)}(C))H(\pi_C|\text{BND}_{(\alpha,\beta)}(C)). \tag{14}$$

Using (12)–(14), (10) is rewritten as

$$\Delta(\alpha,\beta) = \Delta_P(\alpha,\beta) + \Delta_N(\alpha,\beta) + \Delta_B(\alpha,\beta). \tag{15}$$

Equation (15) represents the overall uncertainty with respect to a particular (α,β) threshold pair.

Different probabilistic rough set models represented by various (α,β) thresholds will lead to different overall uncertainties. An effective model is obtained by considering a suitable tradeoff among uncertainties of the three regions controlled by the (α,β) thresholds. We intend to highlight the role of GTRS model in determining balanced and optimal thresholds that will lead to effective and efficient levels of uncertainties.

B. Game-Theoretic Rough Set Approach for Analyzing Uncertainty

We briefly review the GTRS-based approach for analyzing region uncertainties in this section [5].

The conventional Pawlak model has zero uncertainties in its positive and negative regions while nonzero uncertainty in the boundary region [13]. The overall uncertainty is, however, not necessarily minimum in this model. To decrease the uncertainty of the boundary, objects must be moved from the boundary to either positive or negative regions. Doing this will reduce the uncertainty of the boundary at the cost of an increase in uncertainty in either positive or negative regions. In some sense, the boundary region is considered as a common opponent for both the positive and negative regions. This leads to the formation of a two-player competitive game where the positive and negative regions jointly play against the boundary region. We will refer to the player representing the positive and negative regions as immediate decision region, denoted as I, and the player representing the boundary region as deferred decision region, denoted as D.

The two players in the game are affected by different threshold values. Selecting suitable thresholds will allow the players to compete and guard their personal interests. The strategies are, therefore, formulated in terms of different threshold levels. We consider three types of strategies, namely, $s_1 = \alpha_\downarrow$ (decrease α), $s_2 = \beta_\uparrow$ (increase β), and $s_3 = \alpha_\downarrow\beta_\uparrow$ (decrease α and increase β). This means that the strategy sets for the players are the same, i.e., $S_1 = S_2 = \{s_1, s_2, s_3\}$. Although the strategies may be formulated in different ways, the ones considered here are based on the initial threshold configuration of $(\alpha,\beta) = (1, 0)$. An interesting issue in this formulation is to determine how much a threshold based on a certain strategy should be decreased or increased. This is addressed later in the same section.

The utilities or payoff functions should reflect possible benefits or gains of a particular player. We consider the uncertainties associated with different regions to define the payoff functions. However, an uncertainty value represents a level of loss or disadvantage measured in the range of 0–1 [5]. An uncertainty of 1 means an extremely undesirable condition or a minimum possible gain, and an uncertainty of 0 means the maximum gain. In order to calculate the payoff functions in terms of gains or benefits for the players, the notion of certainty is used, which is considered as 1 − uncertainty. The payoff functions corresponding to a particular strategy profile, say (s_i, s_j), with associated thresholds (α,β) are defined as

$$u_I(s_i, s_j) = (C_P(\alpha,\beta) + C_N(\alpha,\beta))/2 \tag{16}$$

$$u_D(s_i, s_j) = C_B(\alpha,\beta) \tag{17}$$

TABLE I
PAYOFF TABLE FOR THE GAME

		D		
		$s_1 = \alpha_\downarrow$	$s_2 = \beta_\uparrow$	$s_3 = \alpha_\downarrow\beta_\uparrow$
I	$s_1 = \alpha_\downarrow$	$u_I(s_1, s_1), u_D(s_1, s_1)$	$u_I(s_1, s_2), u_D(s_1, s_2)$	$u_I(s_1, s_3), u_D(s_1, s_3)$
	$s_2 = \beta_\uparrow$	$u_I(s_2, s_1), u_D(s_2, s_1)$	$u_I(s_2, s_2), u_D(s_2, s_2)$	$u_I(s_2, s_3), u_D(s_2, s_3)$
	$s_3 = \alpha_\downarrow\beta_\uparrow$	$u_I(s_3, s_1), u_D(s_3, s_1)$	$u_I(s_3, s_2), u_D(s_3, s_2)$	$u_I(s_3, s_3), u_D(s_3, s_3)$

where

$$C_P(\alpha, \beta) = 1 - \Delta_P(\alpha, \beta) \tag{18}$$

$$C_N(\alpha, \beta) = 1 - \Delta_N(\alpha, \beta) \tag{19}$$

$$C_B(\alpha, \beta) = 1 - \Delta_B(\alpha, \beta). \tag{20}$$

The functions u_I and u_D represent the payoff functions corresponding to immediate and deferred decision regions, respectively, and $\Delta_P(\alpha, \beta)$, $\Delta_P(\alpha, \beta)$, and $\Delta_P(\alpha, \beta)$ are defined as in (12).

Table I represents the game in a payoff table. The rows represent the strategies of player I, and the columns represent the strategies of player D. Each cell of the table represents a strategy profile of the form (s_i,s_j), which means that player I has selected strategy s_i, and player D has selected s_j. The payoff functions corresponding to a strategy profile (s_i,s_j) is represented as $<u_I(s_i, s_j)$,$u_D(s_i,s_j)>$. During the game, a particular player would prefer strategy s_i over s_j, if it provides more payoff. A strategy profile (s_i, s_j) would be the Nash equilibrium or the game solution, if the following conditions are satisfied for the players:

$$\begin{aligned} &\text{For } I: \quad \forall s_i' \in S_1, u_I(s_i, s_j) \geq u_I(s_i', s_j), \text{with } (s_i' \neq s_i) \\ &\text{For } D: \quad \forall s_j' \in S_2, u_D(s_i, s_j) \geq u_D(s_i, s_j'), \text{with } (s_j' \neq s_j). \end{aligned} \tag{21}$$

In other words, none of the players are benefitted by switching to a different strategy.

We now examine the issue of threshold changes based on a certain strategy during the game. We note from Table I that there are four types of fundamental or elementary changes based on the two thresholds. These changes are defined as

$$\alpha^- = \text{ single player suggests to decrease } \alpha \tag{22}$$

$$\alpha^{--} = \text{ both the players suggest to decrease } \alpha \tag{23}$$

$$\beta^+ = \text{ single player suggests to increase } \beta \tag{24}$$

$$\beta^{++} = \text{ both the players suggest to increase } \beta. \tag{25}$$

A threshold pair during a game corresponding to a strategy profile, such as $(s_1, s_1) = (\alpha_\downarrow, \alpha_\downarrow)$, is determined as (α^{--}, β), since both the players intend to decrease threshold α [see (23)]. In the same way, the profile $(s_3, s_3) = (\alpha_\downarrow\beta_\uparrow, \alpha_\downarrow\beta_\uparrow)$ is determined as $(\alpha^{--}, \beta^{++})$ based on (23) and (25).

Modifying the thresholds repeatedly to improve the utility levels of the players will lead to a learning mechanism. The learning principle employed in such a mechanism is based on the relationship between modifications in threshold values and their impact on improving the utilities of players (or uncertainties of regions). We wish to exploit this relationship to define the variables $(\alpha^-, \alpha^{--}, \beta^+, \beta^{++})$ and to continuously tune the thresholds in the aim of reaching effective thresholds. A repeated or iterative game is considered for this purpose.

Consider each iteration of a repeated game that is being played with initial thresholds of (α, β). The game will use equilibrium analysis to determine the output strategy profile and the corresponding threshold pair, say (α', β'). Based on (α, β) and (α', β'), we define the four variables that appeared in (22)–(25) as

$$\alpha^- = \alpha - (\alpha \times (C_B(\alpha', \beta') - C_B(\alpha, \beta))) \tag{26}$$

$$\alpha^{--} = \alpha - c(\alpha \times (C_B(\alpha', \beta') - C_B(\alpha, \beta))) \tag{27}$$

$$\beta^{++} = \beta - (\beta \times (C_B(\alpha', \beta') - C_B(\alpha, \beta))) \tag{28}$$

$$\beta^{++} = \beta - c(\beta \times (C_B(\alpha', \beta') - C_B(\alpha, \beta))). \tag{29}$$

The threshold values for the next iteration are updated to (α', β'). The constant c in (27) and (29) is introduced to reflect the desired level of change in thresholds and should be greater than 1. It is argued that a lower value of c allows us to fine tune the thresholds based on the data but involves more computations [5]. On the other hand, a higher value of c results in lesser computations; however, fine tuning may not be possible. The iterative process stops when either the deferred decision region becomes empty, or the positive region size exceeds the prior probability of the concept C, or the utility of deferred decision region exceeds that of immediate decision region. These three conditions are mathematically expressed as

$$P(\text{BND}_{(\alpha,\beta)}(C)) = 0 \tag{30}$$

$$P(\text{POS}_{(\alpha,\beta)}(C)) > P(C) \tag{31}$$

$$u_D(\alpha, \beta) < u_I(\alpha, \beta). \tag{32}$$

An important consideration in the above framework is the initial values for the variables α^-, α^{--}, β^+, and β^{++}. One may set them based on the data itself. If the initial thresholds lead to positive and negative regions that cover majority of the objects, then one would like to slightly modify the thresholds. The values of α^- and α^{--} are set closer to 1.0, and the value of β^+ and β^{++} are set closer to 0.0. If the initial thresholds lead to positive and negative regions that cover only small portion of the objects, then significant modifications in the thresholds are needed. The values of α^- and α^{--} are set significantly away from 1.0, and the values of β^+ and β^{++} are set farther from 0.0.

TABLE II
INFORMATION TABLE CONTAINING PATIENTS DATA

Patient	S_1	S_2	S_3	Treatment
P_1	1	1	1	Yes
P_2	1	1	0	Yes
P_3	1	0	1	Yes
P_4	0	1	1	Yes
P_5	1	1	0	Yes
P_6	0	1	1	No
P_7	0	1	0	Yes
P_8	0	0	1	Yes
P_9	0	1	1	Yes
P_{10}	1	0	1	Yes
P_{11}	0	1	1	Yes
P_{12}	1	0	0	Yes
P_{13}	1	0	0	No
P_{14}	0	0	0	No
P_{15}	1	0	1	Yes
P_{16}	1	0	1	Yes
P_{17}	0	1	0	Yes
P_{18}	1	0	0	No
P_{19}	0	0	0	No
P_{20}	1	0	1	No
P_{21}	0	1	0	No
P_{22}	0	0	1	No
P_{23}	0	0	1	No
P_{24}	1	0	0	No
P_{25}	0	0	0	No
P_{26}	1	0	0	No
P_{27}	0	1	0	No

The properties of risks or cost of misclassifications can also be considered while setting the initial values of these variables.

V. Example of Three-Way Treatment Decisions With Game-Theoretic Rough Set

To illustrate how the GTRS component affects decision making in WMDSS, we consider a case of treating a possible disease. The system will make a treatment decision based on some properties recorded in the form of symptoms or medical tests corresponding to a patient. Table II presents information about previous patients along with the past treatments. This information may be retrieved form the patient database. The rows of the table represent the information that is recorded against each patient. The columns S_1, S_2, and S_3 are different characteristics shown by the human body that help in making a treatment decision. They may be of discrete values such as binary, if the symptoms are evaluated based on presence or absence. Alternatively, they may be real numbers reporting some properties like blood pressure or cholesterol levels. We consider the case of binary values for the sake of simplicity. The last column represents successful treatments in the past.

In order to do meaningful analysis using a rough set-based technique, such as GTRS, we need to have data in the form of an information table. Table II is essentially an information table, since the set of objects (perceived as patients) is described by a set of attributes (perceived as set of symptoms and treatment) [58]. Let X_i represent an equivalence class, which is the set of patients having the same description. The following equivalence classes are formed based on Table II:

$$X_1 = \{P_1\}, \quad X_2 = \{P_2, P_5\}$$
$$X_3 = \{P_3, P_{10}, P_{15}, P_{16}, P_{20}\}, \quad X_4 = \{P_4, P_6, P_9, P_{11}\}$$
$$X_5 = \{P_7, P_{17}, P_{21}, P_{27}\}, \quad X_6 = \{P_8, P_{22}, P_{23}\}$$
$$X_7 = \{P_{12}, P_{13}, P_{18}, P_{24}, P_{26}\}, \quad X_8 = \{P_{14}, P_{19}, P_{25}\}.$$

The concept of interest in this case is Treatment = Yes. We want to approximate this concept in the probabilistic rough set framework using (1). The association of each equivalence class X_i with the concept, i.e., $P(C|X_i)$, needs to be determined for this purpose in order to approximate the concept by three regions. The conditional probability is

$$\begin{aligned} P(C|X_i) = &\ P(\text{Treatment = Yes} \mid X_i) \\ = &\ \frac{|\text{ Treatment = Yes} \bigcap X_i|}{|X_i|}. \end{aligned} \tag{33}$$

The conditional probabilities of equivalence classes $X_1, ..., X_8$ based on (33) are calculated as 1.0, 1.0, 0.8, 0.75, 0.5, 0.33, 0.2, and 0.0, respectively. The probability of an equivalence class X_i is determined as $P(X_i) = |X_i|/|U|$, which means that the probability of X_1 is $|X_1|/|U| = 1/27 = 0.037$. The probabilities of other equivalence classes $X_2, ..., X_8$ are similarly calculated as 0.074, 0.1851, 1481, 0.1481, 0.111, 0.1851, and 0.111, respectively.

We now analyze the uncertainties involved in making decisions based on the information given in Table II and the uncertainties involved in probabilistic rough set regions discussed in Section IV-A. The uncertainties of the regions based on a certain (α, β) thresholds are calculated by determining the $\text{POS}_{(\alpha,\beta)}(C)$, $\text{NEG}_{(\alpha,\beta)}(C)$, and $\text{BND}_{(\alpha,\beta)}(C)$ regions. Considering the thresholds $(\alpha, \beta) = (1, 0)$, the three regions based on (1) and conditional probabilities $P(C|X_1), ..., P(C|X_8)$ are $\text{POS}_{(1,0)}(C) = \bigcup\{X_1, X_2\}$, $\text{BND}_{(1,0)}(C) = \bigcup\{X_3, X_4, X_5, X_6, X_7\}$, and $\text{NEG}_{(1,0)}(C) = \{X_8\}$. The probability of positive region is $P(\text{POS}_{(\alpha,\beta)}(C)) = |X_1 \bigcup X_2|/|U| = 3/27 = 0.111$. The probabilities for the negative and boundary regions are similarly obtained.

The conditional probability of C with positive region is

$$\begin{aligned} P(C|\text{POS}_{(1,0)}(C)) = &\ \frac{\sum_{i=1}^{2} P(C|X_i) * P(X_i)}{\sum_{i=1}^{2} P(X_i)} \\ = &\ \frac{1 * 0.037 + 1 * 0.074}{0.037 + 0.0740} = 1. \end{aligned} \tag{34}$$

The probability $P(C^c|\text{POS}_{(1,0)}(C))$ is computed as $1 - P(C|\text{POS}_{(1,0)}(C)) = 1 - 1 = 0$. The Shannon entropy of the positive region based on (5) is, therefore, calculated as

$$H(\pi_C|\text{POS}_{(1,0)}(C)) = -1 * \log 1 - (0 * \log 0) = 0. \tag{35}$$

The average uncertainty of the positive region according to (12) is $\Delta_P(1,0) = P(\text{POS}_{(1,0)}(C))H(\pi_C|\text{POS}_{(1,0)}(C)) = 0$. In the same way, the uncertainty of the negative region is $\Delta_N(1,0) = P(\text{NEG}_{(1,0)}(C))H(\pi_C|\text{NEG}_{(1,0)}(C)) = 0$, and the uncertainty of the boundary region is $\Delta_B(1,0) =$

TABLE III
REGION UNCERTAINTIES CORRESPONDING TO DIFFERENT THRESHOLD VALUES

		β		
		0.0	0.2	0.4
		Δ_P, Δ_B, Δ_N	Δ_P, Δ_B, Δ_N	Δ_P, Δ_B, Δ_N
	1.0	(0.0,0.78,0.0)	(0.0,0.57,0.16)	(0.0,0.43,0.28)
α	0.8	(0.16,0.59,0.0)	(0.16,0.41,0.16)	(0.16,0.28,0.28)
	0.6	(0.29,0.41,0.0)	(0.29,0.26,0.16)	(0.29,0.15,0.28)

TABLE IV
PAYOFF TABLE FOR THE EXAMPLE GAME

		D		
		$\alpha_\downarrow$	$\beta_\uparrow$	$\alpha_\downarrow\beta_\uparrow$
	$\alpha_\downarrow$	$0.855, 0.59$	$0.84, 0.59$	$0.775, 0.74$
I	$\beta_\uparrow$	$0.84, 0.59$	$0.86, 0.57$	$\mathbf{0.78, 0.72}$
	$\alpha_\downarrow\beta_\uparrow$	$0.775, 0.74$	$0.78, 0.72$	$0.715, 0.85$

$P(\text{BND}_{(1,0)}(C))H(\pi_C|\text{BND}_{(1,0)}(C)) = 0.78$. It should be noted that the procedure for calculating uncertainties will remain the same in case of real- or integer-valued attributes. This is because individual attribute values only take part in determining equivalence classes. Once the equivalence classes are determined, the uncertainties involved in different regions are calculated based on the equivalence classes contained in different regions. In other words, the attribute values are not directly used in calculating region uncertainties. The only difference one may expect in case of real- or integer-valued attributes is having more equivalence classes due to more variety in the data values.

We highlight the association between thresholds (α, β) and the uncertainties of the probabilistic rough set regions. Table III is constructed for this purpose. Each cell in the table represents three values of the form $\Delta_P(\alpha, \beta), \Delta_B(\alpha, \beta), \Delta_N(\alpha, \beta)$ corresponding to a particular threshold pair. For instance, the cell at the top left corner that corresponds to $(\alpha, \beta) = (1.0, 0.0)$ has associated region uncertainties of $\Delta_P(1, 0) = 0.0$, $\Delta_B(1, 0) = 0.78$, and $\Delta_N(1, 0) = 0.0$. It is noted that different (α, β) values lead to different region uncertainties. A decrease in α reduces the uncertainty of the boundary region at a cost of an increase in the uncertainty of the positive region. In the same way, an increase in β also reduces the uncertainty of the boundary at a cost of an increase in the uncertainty of the negative region. The (α, β) threshold pair controls the tradeoff among uncertainties of the three regions. One may expect to minimize the overall uncertainty by considering a suitable configuration of thresholds. The GTRS model is used for this purpose.

Considering a GTRS-based game outline in Section IV-B, we play the game by considering initial threshold values of $\alpha^- = 0.8$, $\alpha^{--} = 0.6$, $\beta^+ = 0.2$, and $\beta^{++} = 0.4$. The payoff table corresponding to the game is shown in Table IV. The cell in bold, i.e., $(\mathbf{0.78}, \mathbf{0.72})$, with corresponding strategy pair $(\beta_\uparrow, \alpha_\downarrow\beta_\uparrow)$ is the Nash solution of the game defined in (2). The threshold pair corresponding to this strategy profile is $(\alpha, \beta) = (0.8, 0.4)$. The calculated thresholds with GTRS are used to generate three-way decisions according to (1).

Let us interpret the GTRS results from a medical practitioner perspective. Considering a patient x belonging to an equivalence class X_i with conditional probability $P(C|X_i)$, i.e., level of confidence in making a treatment decision, we have 78% certainty in making a correct treatment decision when the level of confidence for treating x is set to be greater than or equal to 0.8, and the level of confidence for not treating x is set to be less than or equal to 0.4. When the level of confidence is less than 0.8 but greater than 0.4, we are unable to decide whether to treat or not, thereby deferring a certain treatment decision. The deferred decisions have 72% certainty, which can be further explored and utilized when additional information is made available. The threshold levels affect the certainty level of the two players. The game-theoretic formulation aims to achieve a suitable tradeoff between the two types of decisions.

VI. EXPERIMENTAL RESULTS AND DISCUSSION

We investigate the usage of the GTRS-based approach on four different health care datasets obtained from the UCI machine-learning repository [6]. We briefly describe these datasets, namely, contraceptive method choice (CMC), haberman's survival (HS), thyroid disease (TD), and pima indian diabetes (PID).

A. UCI Datasets

1) Contraceptive Method Choice: The data in this dataset are taken from the 1987 National Indonesia Contraceptive Prevalence Survey. The records correspond to married women who were either not pregnant or did not know if they were pregnant at the time of data collection. The problem is to determine the CMC of the women as *No-use*, *Long-term*, or *Short-term* methods based on demographic and socioeconomic characteristics. There are 1473 records and ten attributes, including the class attribute.

2) Haberman's Survival: This dataset contains cases from a study that was conducted between 1958 and 1970 at the University of Chicago's Billings Hospital. The study considers the survival of patients who had undergone surgery for breast cancer. The problem is to decide and predict whether the patient survived five years or longer ($\geq$5 years) or the patient died within five years ($<$5 years). The dataset contains 306 instances and four attributes.

3) Thyroid Disease: There are different datasets in the UCI repository under this name. We used the new thyroid dataset. It contains information about five lab tests to determine a patient thyroid as *Euthyroidism*, *Hypothyroidism,* or *Hyperthyroidism*. There are 215 patient records with six attributes including the class attribute.

4) Pima Indian Diabetes: This dataset contains information about female patients with at least 21 years of age at Pima Indian heritage living near Phoenix, AZ, USA. The problem is to diagnose diabetes as *Positive* or *Negative* given a number of physiological measurements and medical test results. The dataset consists of 768 records with nine attributes.

TABLE V
DATASET CMC, CATEGORY = NO-USE

Region Size			Thresholds		Certainty	
POS	BND	NEG	α	β	u_I	u_D
19.3	57.9	22.7	1.000	0.000	1.000	0.437
22.0	53.0	25.0	0.800	0.100	0.975	0.487
24.2	50.2	25.6	0.720	0.120	0.959	0.517
24.6	49.8	25.6	0.676	0.127	0.956	0.523
24.6	49.8	25.6	0.668	0.129	0.956	0.523
27.7	46.7	25.6	0.653	0.130	0.934	0.559
28.2	43.2	28.5	0.606	0.149	0.918	0.588
31.2	40.3	28.5	0.536	0.149	0.896	0.620
31.2	38.7	30.1	0.501	0.168	0.889	0.633
42.4	27.4	30.1	0.476	0.173	0.809	0.754

TABLE VI
DATASET CMC, CATEGORY = LONG TERM

Region Size			Thresholds		Certainty	
POS	BND	NEG	α	β	u_I	u_D
4.5	45.8	49.8	1.000	0.000	1.000	0.557
4.9	44.5	50.6	0.800	0.100	0.994	0.568
4.9	44.5	50.6	0.782	0.102	0.994	0.568
4.9	44.5	50.6	0.764	0.105	0.994	0.568
4.9	44.5	50.6	0.745	0.107	0.994	0.568
4.9	44.5	50.6	0.727	0.109	0.994	0.568
4.9	43.3	51.8	0.709	0.114	0.988	0.578
5.6	42.6	51.8	0.681	0.114	0.983	0.586
5.6	42.6	51.8	0.671	0.115	0.983	0.586
9.0	39.2	51.8	0.649	0.117	0.962	0.625
10.7	35.8	53.4	0.598	0.136	0.944	0.657
14.6	30.5	54.9	0.560	0.153	0.916	0.711

B. Experimental Results With Game-Theoretic Rough Set-Based Method

In order to apply the GTRS-based repetitive threshold method, we started with initial values for the variables defined in (27) as $\alpha^{-} = 0.9$, $\alpha^{--} = 0.8$, $\beta^{+} = 0.1$, and $\beta^{++} = 0.2$. The next values of these variables were automatically obtained as mentioned in Section IV-B. The initial thresholds were set to $(\alpha, \beta) = (1, 0)$ in all experiments for starting the method.

Table V shows the results for category *No-use* of CMC dataset. Each row of the table represents a single iteration of the repetitive game. The first row has the initial threshold settings of (1, 0) which corresponds to the Pawlak model. We have maximum utility for player I, i.e., 1.0, and moderate utility for player D, i.e., 0.437. As the method continues, the thresholds are modified based on game-theoretic analysis and game outcomes. The probabilistic positive, negative, and boundary regions change in size based on configuration of thresholds, which ultimately leads to different uncertainty levels and utilities for players. The size of positive and negative regions keeps on increasing, while the boundary region keeps on decreasing. The method stops at the ninth iteration as one of the stop conditions described in (30)–(32) was reached. The certainty of definite decision region decreased by 19.1%, while the certainty of deferred decision region increased by 31.7%. The definite decision region increased by 30.5%, which means that we have more immediate decisions. An intuitive interpretation of these results is that by decreasing our expectation of making 100% certain decisions to 80.9% certain decisions, we included 30.5% of additional cases to our definite or immediate decision region that were previously unknown to us. The final threshold configuration is $(\alpha, \beta) = (0.476, 0.168)$.

Table VI shows the results for category *Long term* of CMC dataset. The method reaches the stop criteria in slightly more iterations. The configured thresholds in this case correspond to $(\alpha, \beta) = (0.56, 0.154)$. The certainty of the deferred decision region increased by 15.4% at a cost of 8.4% decrease in the certainty of definite decision region. Table VII presents the results for the category short term of the same dataset.

Tables VIII and IX show the results for the categories of HS dataset. The deferred decision regions of the two categories are

TABLE VII
DATASET CMC, CATEGORY = SHORT TERM

Region Size			Thresholds		Certainty	
POS	BND	NEG	α	β	u_I	u_D
10.5	58.0	31.4	1.000	0.000	1.000	0.431
12.8	55.1	32.1	0.800	0.100	0.985	0.464
14.7	53.2	32.1	0.696	0.107	0.974	0.486
18.3	49.6	32.1	0.633	0.111	0.950	0.529
20.6	46.2	33.2	0.579	0.130	0.930	0.563
23.2	43.6	33.2	0.501	0.139	0.913	0.592

TABLE VIII
DATASET HS, CATEGORY = ≥5 YEARS

Region Size			Thresholds		Certainty	
POS	BND	NEG	α	β	u_I	u_D
7.8	92.2	0.0	1.000	0.000	1.000	0.203
20.6	79.4	0.0	0.900	0.200	0.972	0.281
49.0	48.4	2.6	0.759	0.263	0.860	0.535

TABLE IX
DATASET HS, CATEGORY = <5 YEARS

Region Size			Thresholds		Certainty	
POS	BND	NEG	α	β	u_I	u_D
0.0	92.2	7.8	1.000	0.000	1.000	0.203
0.0	79.4	20.6	0.900	0.100	0.972	0.281
0.0	73.2	26.8	0.759	0.131	0.956	0.324
2.6	65.7	31.7	0.695	0.154	0.930	0.394
9.8	54.2	35.9	0.570	0.175	0.882	0.513

successfully increased in a few iterations. For both the categories, the certainty increases of deferred decision regions are much more and greater than the certainty decreases of immediate decision regions. This is acceptable as the overall certainty or utility of the two players has improved. It seems relevant in this case to investigate a suitable value of constant c [used in

TABLE X
RESULTS FOR DATASETS TD AND PID

Dataset	Category	Iterations	Config.	Region Size			Thresholds		Certainty	
				POS	BND	NEG	α	β	u_I	u_D
TD	*Euthy.*	6	Initial	58.6	22.8	18.6	1.000	0.000	1.000	0.772
			Final	64.7	11.2	24.2	0.636	0.278	0.935	0.889
	Hypothy.	16	Initial	7.0	14.9	78.1	1.000	0.000	1.000	0.858
			Final	14.9	7.0	78.1	0.404	0.000	0.967	0.932
	Hyperthy.	9	Initial	11.6	7.9	80.5	1.000	0.000	1.000	0.931
			Final	11.6	5.1	83.3	0.502	0.275	0.979	0.952
PID	*Negative*	14	Initial	59.5	12.0	28.5	1.000	0.000	1.000	0.881
			Final	61.1	9.2	29.7	0.503	0.264	0.968	0.908
	Positive	23	Initial	28.5	12.0	59.5	1.000	0.000	1.000	0.881
			Final	32.2	6.8	61.1	0.501	0.431	0.942	0.932

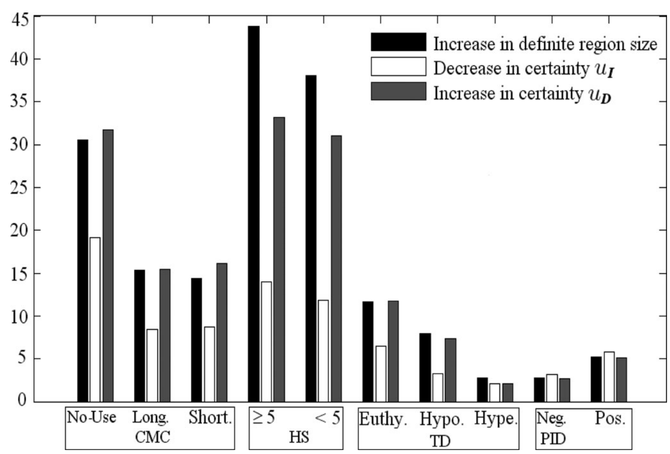

Fig. 2. Summary of changes in certainty and the size of immediate decision region.

(27)] to fine tune the thresholds and to achieve and improve the utilities of the players.

The results obtained with the datasets TD and PID are summarized and presented together in Table X. The results are quite similar to those obtained with the datasets CMC and HS.

Fig. 2 summarizes the changes in immediate decision region size and the certainties of the two players. A set of three bars is shown for each category of the considered datasets. The black bars correspond to the percentage increases in the size of immediate decision regions. The white bars represent the increases in certainty of immediate decision region, i.e., u_I. The gray bars represent the decreases in certainty of deferred decision regions, i.e., u_D. Considering the CMC dataset, the results are quite encouraging for the three categories. The sizes of definite decision regions increased by 30.5%, 15.3%, and 14.4% for the three categories as indicated by the black bars. Consider the white and grey bars for the three categories of this dataset. The increases in certainty of deferred decision regions are more than decreases in certainty of the immediate decision regions for the three categories. This means that the overall uncertainty is improved to a certain extent. The HS dataset showed the most significant improvements. The black bars for the two categories of this dataset indicate that the immediate decision regions increased by 43.8% and 38% for the two categories. A considerable improvement in certainty of deferred decision regions is also noticed from the grey bars indicated in the figure. The figure also suggests some improvements, although not very significant on the TD and PID datasets.

C. Further Analysis and Discussion

We further analyze the decision-making capabilities of GTRS by considering the quality of obtained decision regions. The property of classification accuracy of decision regions is considered for this purpose and is defined as [5]

$$\text{Accuracy}(\alpha,\beta) = \frac{\text{Correctly classified by } \text{POS}_{(\alpha,\beta)} \text{ and } \text{NEG}_{(\alpha,\beta)}}{\text{Total classified by } \text{POS}_{(\alpha,\beta)} \text{ and } \text{NEG}_{(\alpha,\beta)}}. \quad (36)$$

The option of deferred decision in GTRS reduces some misclassifications, which will lead to an improved accuracy compared with other methods. In order to have a better insight and comparable results, we also include the deferred decision rate, i.e., the percentage of decisions that are being deferred (also called as noncommitment rate [15]). The deferred or noncommitment rate is the relative size of the boundary region with respect to the universe, i.e.,

$$\text{non-commitment}(\alpha,\beta) = \frac{|\text{BND}_{(\alpha,\beta)}(C)|}{|U|}. \quad (37)$$

Let us first consider the results for the CMC dataset. The GTRS-based approach was used to learn the thresholds which were used to determine the decision regions and the associated decision accuracy according to (36). A decision accuracy of 90.57% was noted with noncommitment rate of 34.3%. This means that the GTRS was able to classify 65.7% with a success or accuracy rate of 90.57%. Additional information is needed to make certain decisions about the remaining 34.3% of the objects.

An important issue while considering the GTRS based performance is that it requires additional information for some objects. In the absence of such information, one may feel uncertain with remaining doubts about its performance. Let us look at the GTRS performance in a worst-case scenario where we have no access to additional information, and we are asked to make certain decisions about the deferred cases. Suppose that random decisions are being made with a 50% chance of being correct and another 50% of being incorrect. In this case, the GTRS provides 90.57% correct decisions for 65.7% of the objects and 50% correct decisions for the remaining 34.3% of the objects. The average accuracy in this case is $(90.57 \times 0.657) + (50 \times 0.34) = 76.65\%$. This is comparable and, to a certain extent, quite encouraging when we consider some of the previous results for the same dataset. For instance, a fuzzy decision tree approach was reported to have 76.2% accuracy [12]. The same research reported accuracies of 63.86%, 50.8%, and 40.85% for the neural network, support vector machine, and k-nearest neighbour approaches. Some other known approaches like C4.5 and k-means were reported to have 61.67% and 66.66% accuracies, receptively [1].

Another important characteristic of the GTRS is that it requires additional information about a proportion of the objects in order to improve its performance. For example, in case of the CMC dataset, 34.3% of the objects need additional information to be certainly classified. There is no explicit mechanism in conventional two-way decision methods to identify the cases that require additional investigations. They require additional information for all objects in order to further improve their performance.

We now consider the case of the HS dataset. The GTRS-based method achieved a decision accuracy of 83.5% with a noncommitment rate of 42.9%. The deferred decision region in this case is slightly larger compared with the CMC dataset, where it was found to be 34.3%. The size of this region is expected to decrease, if we lower the expectation of being highly accurate or when additional information is being made available. Once again, we consider the constraints of no additional information and forced two-way certain decisions. Under these constraints, the GTRS makes 83.5% correct decisions for 57.1% objects and a 50% chance of correct decisions for the remaining 42.9% objects. The average accuracy in this case is 69.12%, which is the worst but is still comparable with some other approaches in the literature. For instance, C4.5 was reported to have 71.7% accuracy [11]. The Naive Bayesian and SVM were reported to have accuracies of 73.83% and 73.52%, respectively [28].

The results for the datasets TD and PID were quite encouraging. The decision accuracies for the TD and PID datasets were 98.64% and 98.66% with noncommitment rates of 7.7% and 8.1%, respectively. The deferred decision regions for these datasets were comparatively small implying many certain decisions. This means that correct decisions are confidently made for majority of the objects. If we consider the constraints as discussed above, i.e., no additional information and forced two-way certain decisions, the average accuracies are 94.89% and 94.72% for the TD and PID datasets, respectively. These results are quite promising when compared with some of the existing results. For instance, an information gain-based approach and Fuzzy C-Means were reported to have accuracies of 95.9% and 83.7%, respectively, for the TD dataset [2], [20]. On the other hand, an approach based on combining regression with neural networks and another approach based on discriminant analysis with support vector machine demonstrated accuracies of 80.21% and 79.16%, respectively, for the TD dataset [19], [34]. Further improvements in the GTRS performance is expected by providing additional information for the remaining 7.7% and 8.1% of the objects in the deferred decision regions of the TD and PID datasets, respectively.

The results presented in this section advocate for the use of a GTRS component in WMDSS as an alternative way for obtaining effective decision support for different medical decision-making problems.

VII. Conclusion

We have examined and extended the decision-making capabilities of WMDSSs. Decision making in the medical field is typically associated with a level of uncertainty. A three-way decision-making approach is a useful and better choice to reduce the impact of uncertainty in medical decision making. The concept of three-way decisions was recently incorporated in the rough set theory for interpreting the three rough set regions as regions of acceptance, rejection, and deferment. We consider three-way decisions in the probabilistic rough set model, which is the generalized representation of rough sets. A fundamental problem in this model is the interpretation and determination of decision thresholds that define the division between the three rough set regions. The game-theoretic rough set or GTRS model has been demonstrated for determining effective thresholds for probabilistic rough sets by analyzing the uncertainties involved in the probabilistic rough set regions and the implied three-way decisions. We incorporated a GTRS component in a WMDSS architecture with the aim of enriching its decision-making capabilities. The benefits from a GTRS component is demonstrated by considering a GTRS-based method for improving the overall uncertainty level of the three-way classification. The method iteratively modifies the thresholds in order to improve the overall uncertainty of rough set-based regions and their respective decisions. The method provides encouraging results on four different medical datasets.

The uncertainty analysis of decision making sets up the motivation for analyzing different decision-making aspects with GTRS, such as, the risks, errors, costs, and benefits associated with medical decision making. Different competitive or cooperative games may be setup to determine cost effective and balanced thresholds based on these aspects.

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