EEE 3005 Signals and Systems

HOMEWORK #4

FAST FOURIER TRANSFORM

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Question 1

```
clc; clear; close all;

fs = 500;
duration = 2;
sampling = fs*duration;

T = 0:1/fs:duration-1/fs;

amp1 = 3; amp2 = 3; amp3 = 1;
freq1 = 30; freq2 = 45; freq3 = 70;
phase1 = 0.6; phase2 = -0.8; phase3 = 2;

f1 = amp1*cos(2*pi*freq1*T+phase1);
f2 = amp2*cos(2*pi*freq2*T+phase2);
f3 = amp3*cos(2*pi*freq3*T+phase3);

f = f1+f2+f3;

Y = fft(f);

Theta = angle(Y);

figure('Name','FFT');
subplot(4,3,1), plot(T,f1),
title('$f_1(t) = 3 \cdot cos(2\pi \cdot 30 \cdot t+0.6)$','Interpreter','latex'),
grid on, xlabel('t'), ylabel('f1(t)')

subplot(4,3,2), plot(T, f2),
title('$f_2(t) = 3 \cdot cos(2\pi \cdot 45 \cdot t-0.8)$','Interpreter','latex'),
grid on, xlabel('t'), ylabel('f2(t)')

subplot(4,3,3), plot(T, f3),
title('$f_3(t) = 1 \cdot cos(2\pi \cdot 70 \cdot t+2)$','Interpreter','latex'),
grid on, xlabel('t'), ylabel('f3(t)')

subplot(4,3,[4;6]), plot(T, f,'r'),
title('$f_1(t)+f_2(t)+f_3(t)$','Interpreter','latex'),
grid on, xlabel('t'), ylabel('f(t)')

subplot(4,3,[7;9]), plot(T, Y,'r'),
title('$FFT, f*$','Interpreter','latex'),
grid on, ylabel('f*(t)')

subplot(4,3,[10;12]), plot(T, Theta,'r'),
title('$Angle$','Interpreter','latex'),
grid on, ylabel('angle')

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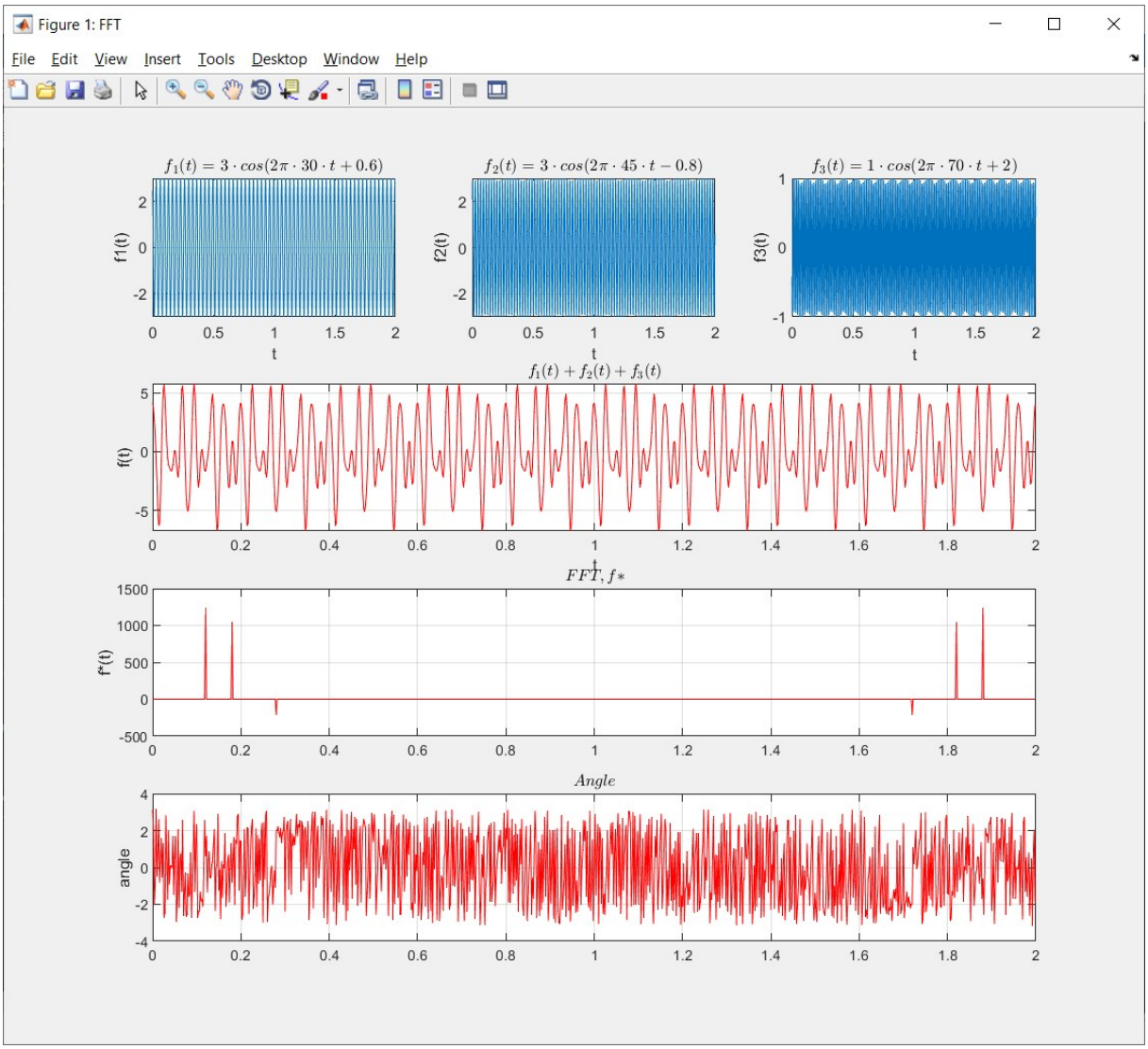


Figure 1: FFT
File Edit View Insert Tools Desktop Window Help
$f_1(t) = 3 \cdot cos(2\pi \cdot 30 \cdot t + 0.6)$
$f_2(t) = 3 \cdot cos(2\pi \cdot 45 \cdot t - 0.8)$
$f_3(t) = 1 \cdot cos(2\pi \cdot 70 \cdot t + 2)$
f1(t)
f2(t)
f3(t)
t
$f_1(t) + f_2(t) + f_3(t)$
f(t)
$FFT, f*$
f*(t)
Angle
angle

Question 2

given signal :- $f(x) = \begin{cases} \frac{1}{\omega} & , |x| \le \omega \\ 0 & , |x| > \omega \end{cases}$

assuming x = time,

Fourier Transform formula:-

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-jkx}\, dx$$, where k = angular frequency

since $f(x) = 0$ for $|x| > \omega$, so bounds of integration is $-\omega \le x \le \omega$

$$F(k) = \int_{-\omega}^{\omega} \frac{1}{\omega} e^{-jkx}\, dx$$

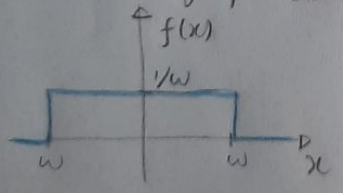
$$= \frac{1}{\omega} \int_{-\omega}^{\omega} e^{-jkx}\, dx = \frac{1}{\omega} \frac{1}{-jk} \left(e^{-jkx} \right)_{-\omega}^{\omega}$$

$$= \frac{1}{-j\omega k} \left[e^{-j\omega k} - e^{j\omega k} \right] = \frac{1}{j\omega k} \left[e^{j\omega k} - e^{-j\omega k} \right]$$

$$= \frac{1}{j\omega k} \left(2j \sin(\omega k) \right) = \because e^{j\theta} - e^{-j\theta} = 2j \sin\theta$$

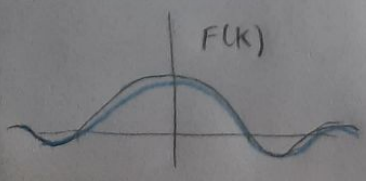
$$F(k) = \frac{2}{\omega k} \sin(\omega k)$$

graphical interpretation:



if ω represents angular frequency such that $\omega = 2\pi x$, then

$$F(\omega) = \frac{2}{\omega^2} \sin\omega^2$$



and also

$$f(x) = \begin{cases} \frac{x}{2a} & , |x| \le \omega \\ 0 & , |x| > \omega \end{cases}$$