作業九

1. Let $u(x,t)=1$ be the exact solution of (1). ($f\equiv 0$.)

Explain why the energy estimate (2) does not hold here.

Note : (1) $u_t=\nu u_{xx}+f(x,t)$

(2) $E(t)\le e^{-\gamma t}E(0)+\frac{1}{r}\int_0^t e^{r(s-t)}F(s)\,ds$

where $F(t)=\int_0^1 f^2(x,t)\,dx$

$\Rightarrow$ Consider $u_t=\nu u_{xx}+f(x,t)$ and $u(x,t)=1$

Suppose $f\equiv 0$ (which implies $F(t)\equiv 0$), then $E(t)\le e^{-rt}E(0)$

Since $E(t)=\int_0^1 u^2\,dx=\int_0^1 1^2\,dx=x\Big|_0^1=1 \quad \forall t$

$\Rightarrow E(t)=1\le e^{-rt}E(0)=e^{-rt}\cdot 1=e^{-rt}$

$\rightarrow 1\le e^{-rt}$

But if $r>0$ and $t>0$, then $1\ge e^{-rt}$ $\rightarrow\leftarrow$

Thus the energy estime (2) does not hold ※