作業一手寫

Prove that $\omega_{n+1}'(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$ where ω_{n+1} is the nodal polynomial (8.6)

Then, check (8.5).

Note: (8.5) $\Pi_n(x) = \sum_{i=0}^{n} \frac{\omega_{n+1}(x)}{(x - x_i)\,\omega_{n+1}'(x_i)} y_i$

(8.6) $\omega_{n+1}(x) = \prod_{i=0}^{n} (x - x_i)$

①

$\Rightarrow$ Consider $\omega_{n+1} = \prod_{i=0}^{n} (x - x_i)$

Fix an index $i \in \{0, \dots, n\}$.

$\Rightarrow \omega_{n+1}(x) = (x - x_i)\, g_i(x)$

where $g_i(x) := \prod_{\substack{j=0 \\ j\neq i}}^{n} (x - x_j)$

$\Rightarrow \omega_{n+1}'(x) = 1 \cdot g_i(x) + (x - x_i)\, g_i'(x)$

If $x = x_i$,

then $\omega_{n+1}'(x_i) = g_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$

Thus $\omega_{n+1}'(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$

② Since $\omega_{n+1}'(x_i) = \prod_{j\neq i} (x_i - x_j)$

and $\frac{\omega_{n+1}(x)}{x - x_i} = \prod_{j\neq i} (x - x_j)$

therefore

$$\frac{\omega_{n+1}(x)}{\quad} = \frac{\prod_{j\neq i} (x - x_j)}{\quad} := l_i(x)$$

$$\frac{\omega_{n+1}(x)}{(x-x_i)\,\omega_{n+1}(x_i)} = \frac{\prod_{j\neq i}(x-x_j)}{\prod_{j\neq i}(x_i-x_j)} := l_i(x)$$

Note : $l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j}$

Thus $\prod^{n}(x) = \sum_{i=0}^{n} l_i(x)\, y_i$