

## 作業一手寫

Prove that  $\omega'_{n+1}(x_i) = \prod_{\substack{j=0 \\ j \neq i}}^n (x_i - x_j)$  where  $\omega_{n+1}$  is the nodal polynomial (8.6)

Then, check (8.5).

Note:

$$(8.5) \quad \prod_{i=0}^n \frac{\omega_{n+1}(x)}{(x - x_i) \omega'_{n+1}(x_i)} y_i$$
$$(8.6) \quad \omega_{n+1}(x) = \prod_{i=0}^n (x - x_i)$$

①

$\Rightarrow$  Consider  $\omega_{n+1} = \prod_{i=0}^n (x - x_i)$

Fix an index  $i \in \{0, \dots, n\}$ .

$$\Rightarrow \omega_{n+1}(x) = (x - x_i) g_i(x)$$

$$\text{where } g_i(x) := \prod_{\substack{j=0 \\ j \neq i}}^n (x - x_j)$$

$$\Rightarrow \omega'_{n+1}(x) = 1 \cdot g_i(x) + (x - x_i) g'_i(x)$$

If  $x = x_i$ ,

$$\text{then } \omega'_{n+1}(x_i) = g_i(x_i) = \prod_{\substack{j=0 \\ j \neq i}}^n (x_i - x_j)$$

$$\text{Thus } \omega'_{n+1}(x_i) = \prod_{\substack{j=0 \\ j \neq i}}^n (x_i - x_j)$$

② Since  $\omega'_{n+1}(x_i) = \prod_{\substack{j=0 \\ j \neq i}}^n (x_i - x_j)$

$$\text{and } \frac{\omega_{n+1}(x)}{x - x_i} = \prod_{\substack{j=0 \\ j \neq i}}^n (x - x_j)$$

therefore

$$\frac{\omega_{n+1}(x)}{(x-x_i)\omega_{n+1}(x_i)} = \frac{\prod_{j \neq i} (x-x_j)}{\prod_{j \neq i} (x_i-x_j)} := l_i(x)$$

$$\text{Note: } l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x-x_j}{x_i-x_j}$$

$$\text{Thus } \prod_{i=0}^n (x) = \sum_{i=0}^n l_i(x) \gamma_i$$