Algebraic Topology

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Chapter 1

Categories.

Definition of a Category.

Definition. A **category** $\mathcal{C}$ is a collection of a class of **objects**, denoted $\operatorname{obj}\mathcal{C}$ a collection of sets of **morphisms** $\operatorname{Hom}(A, B)$ for each $A, B \in \operatorname{obj}\mathcal{C}$ and a binary operation $\circ : \operatorname{Hom}(A, B) \times \operatorname{Hom}(B, C) \to \operatorname{Hom}(A, C)$, defined by $(f, g) \to g \circ f$, called **composition** such that:

(1) Each $\operatorname{Hom}(A, B)$ is pairwise disjoint for all $A, B \in \operatorname{obj}\mathcal{C}$.

(2) $\circ$ is associative when defined; that is if either $(g \circ f) \circ h$ or $g \circ (f \circ h)$ are defined, then $(g \circ f) \circ h = g \circ (f \circ h)$, for morphisms f, g, h.

(3) For each $A \in \operatorname{obj}\mathcal{C}$, there exists an **identity** morphism $1_A \in \operatorname{Hom}(A, A)$ such that for each $B, C \in \operatorname{obj}\mathcal{C}$, $1_A \circ f = f$ and $g \circ 1_A = g$ for each morphism $f \in \operatorname{Hom}(B, A)$ and $g \in \operatorname{Hom}(A, C)$.

Bibliography

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