

Lecture 18 - Subsequence & subsets

Subarrays: contiguous segment of an array

Non-contiguous segments:

subsequence \swarrow sets
depends on the order

Ex. $[2, -3, 5, 6, 7, 10]$

$-3, 6, 7 \rightarrow$ subsequence

$-3, 7, 6 \rightarrow$ subset

① All subarrays are subsequences

② All subsequences are subsets

Q. How many subsequences for an array of N elements? $\Rightarrow \underline{\underline{2^N}}$

72 Given N elements, check if there exists a subsequence which has sum of its elements as k.

arr = [2, 1, 3, -1, 4]

k = 0

[1, -1] → True

Brute Force

[-2, 6, 4]

0 0 0

- []

0 0 1

- [4]

0 1 0

[6]

1 0 0

[-2]

0 1 1

[6, 4]

1 0 1

[-2, 4]

1 1 0

[-2, 6]

1 1 1

[-2, 6, 4]

```

for i in range (2n * n):
    sum = 0
    for j in range (n):
        if i & (1 << j) != 0:
            sum += A[j]
    if sum == k:
        return True
return False

```

Ex. $[-2, 6, 4]$ $K = 2$

$[-2, 4]$

1 0 1 (bit representation)

for $i = 5 \Rightarrow \underline{\underline{sum = 2}}$

TC: $O(2^N * N)$

Using recursion

```
def check(arr, k, i):
```

if $l == \text{len}(\text{arr})$:

if $k = 0$:

return True

return False

if check(arr, k-arr[l⁰], l+1) or

```
check(arr, k, i+1);
```

return True

return False

$$[-2, 6, 4]$$
$$K \leq 2$$

- True

$[-2, 6, 4]$ $K = 2$ True
check($[-2, 6, 4], 2, 0$) → check(arr, 4, 1)
 → check(arr, 2, 1)

True

check(arr, 4, 1).

```
check(arr, 2, 1)
```

cheek (corr, $-\frac{2}{7}$, 2)

check(²arr, 4, 2)

True

(73) Find the sum of all subsequences

Brute Force

```
all-sum = 0
for i in range(2n):
    sum = 0
    for j in range(n):
        if i & (1 << j) != 0:
            sum += A[j]
    print(sum)
    all-sum += sum
return all-sum
```

TC: $O(2^N * N)$

Optimized

```
ans = 0
for i in range(n):
    ans += (arr[i] * 2n-1)
return ans
```

TC : $O(N)$

(74) Given N array elements, find sum of max of every subsequence

Brute Force

```
maxes = []  
for i in range(2**n):  
    max = -sys.maxsize - 1  
    for j in range(n):  
        if i & (1 << j) != 0:  
            if arr[j] > max:  
                max = arr[j]  
    maxes.append(max)  
return sum(maxes)
```

TC : $O(2^n * N)$

Optimized

In every subsequence where $\max(arr)$ is present

→ $\max(arr)$ will be the max → 2^{N-1} subseq

2nd largest (arr) → 2^{N-2} subseq

& likewise

```
sum = 0
arr.sort()

for i in range(n):
    sum += 2i * arr[i]

return sum
```

TC: $O(n \log n)$

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Find the sum of (max-min) of every subsequence.

```
min-sum = 0
max-sum = 0

for i in range(n):
    max-sum += 2i * arr[i]
    min-sum += 2n-i * arr[i]

return (max-sum - min-sum)
```

TC : $O(n \log n)$