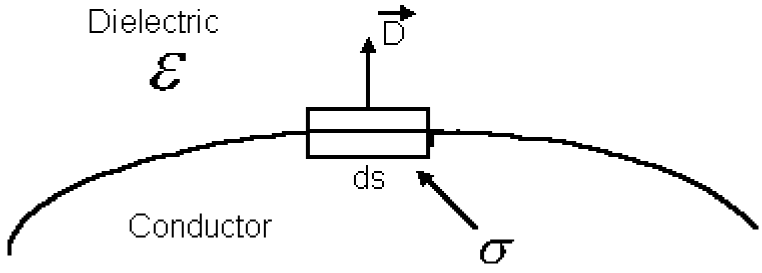
BOUNDARY CONDITIONS

A) Between Conductor and Dielectric:



σ =Surface density of charge

Electric field intensity just off the conductor surface:

The conductor surface is an equipotential. Hence, there cannot be any component of electric field intensity along the conductor surface. In other words, at any point on the conductor surface, the tangential component of electric field intensity is zero. Electric field intensity or flux density vectors are, therefore, always normal to the conductor surface.
Applying Gauss's theorem for the small volume corresponding to the small surface ds, as shown in the figure above,

$$\mathbf{D}.\mathbf{ds} = \sigma\, ds$$

But, the angle between **D** & ds is 0° .

So, $|\mathbf{D}|\ ds = \sigma\ ds$

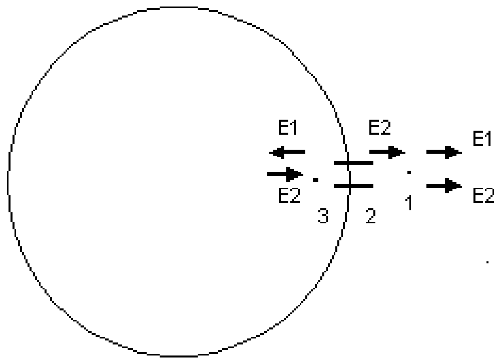
or, $|\mathbf{D}| = \sigma$

Again, $\mathbf{D} = \varepsilon_r\, \varepsilon_0\, \mathbf{E}$

Therefore, $|\mathbf{E}| = \sigma / (\varepsilon_r\, \varepsilon_0)$

This is the electric field intensity just off the conductor surface.

Electric field intensity on the conductor surface :



Consider a very small flat disc surrounding the point 2 on the conductor surface as shown in the figure. Let the electric field intensities at the points 1 and 3 due to the charges on this disc be $\mathbf{E_1}$ and that at the points 1, 2 and 3 due to the rest of the charges lying on the conductor surface be $\mathbf{E_2}$. The magnitudes of the field intensities $\mathbf{E_1}$ and $\mathbf{E_2}$ at these points will be same, as the points 1 and 3 are located extremely close to the point 2, in such a way that the point 1 is outside the conductor and the point 3 is inside the conductor. The direction of $\mathbf{E_2}$ at 1, 2 and 3 will be same as these three points are located almost identically w.r.t. to the charges responsible for $\mathbf{E_2}$. But the directions of $\mathbf{E_1}$ at 1 and 3 will be opposite to each other, as 1 and 3 are lying on the opposite sides of the disc surrounding the point 2, the charges on which are responsible for $\mathbf{E_1}$.
Since, the point 3 is lying within the conductor, electric field intensity at the point 3 is zero.
So, at the point 3,

$$\mathbf{E_1} - \mathbf{E_2} = 0$$

or, $$\mathbf{E_1} = \mathbf{E_2}$$

Again at the point 1 (lying just off the conductor surface),

$$|\mathbf{E_1} + \mathbf{E_2}| = \sigma / (\varepsilon_r \varepsilon_0)$$

or, $$2\ |\mathbf{E_2}| = \sigma / (\varepsilon_r \varepsilon_0)$$

or $$|\mathbf{E_2}| = \sigma / (2\ \varepsilon_r \varepsilon_0)$$

Now, electric field intensity at the point 2 in $\mathbf{E_2}$, as the field intensity due to disc charges $\mathbf{E_1}$ will be zero because point 2 lies on the disc.

Electric field intensity on the conductor surface = $\sigma / (2\ \varepsilon_r \varepsilon_0)$
= 1/2 x field intensity just off the conductor surface.

Mechanical pressure on the conductor surface :

Mechanical Pressure on the conductor surface
= Electric Field Intensity x Surface Charge Density
= $\sigma / (2\ \varepsilon_r \varepsilon_0) \times \sigma$
= $\sigma^2 / (2\ \varepsilon_r \varepsilon_0)$ N/m^2.

The mechanical pressure will be such that it will try to swell the conductor, irrespective of the polarity of charges. This is due to the fact that the charges on a conductor surface are of same polarity and hence they repel each other, thereby try to increase the distance between the charges.

Problem 1

A metallic sphere of 20cm radius is charged with 1 μC, spread uniformly over the surface and is surrounded by a medium having a relative permittivity of 5. Find the electric field intensity just off the sphere and also on the sphere. Find the mechanical pressure acting on the sphere. What is the electric field intensity inside the sphere?

Solution

Sphere radius = 20 cm = 0.2m
Sphere surface area = $4 \pi \times (0.2)^2 = = 0.5026\ m^2$
Therefore, surface charge density (σ) = $(1\times 10^{-6}) / 0.5026 = 1.989 \times 10^{-6}\ C/m^2$
Electric field intensity just off the sphere surface
$= (1.989 \times 10^{-6}) / (5 \times 8.854 \times 10^{-12}) = 44.93\times10^3\ V/m$
Electric field intensity on the sphere surface
$= (44.93 \times 10^3) / 2 = 22.465 \times10^3\ V/m$
Therefore, mechanical pressure on the sphere surface
$= 1.989 \times 10^{-6} \times 22.465 \times 10^3 = 0.0447\ N/m^2$
Electric field intensity within the sphere = 0

Problem 2

A charged conductor is surrounded by air. Calculate
i) the maximum charge density that the conductor can hold and
ii) the conductor surface mechanical pressure at that charge density.

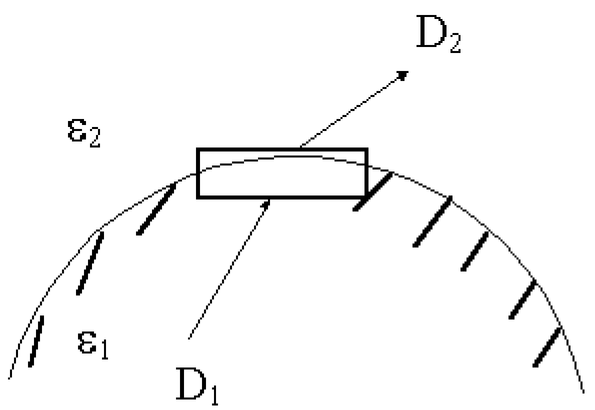
Solution

The conductor can hold the maximum charge density for which the electric field intensity just off the surface is equal to the breakdown strength of air.
Breakdown strength of air at STP = 30 kV /cm
$= 30 \times 10^3 \times 10^2\ V/m = 3 \times 10^6\ V/m$

ε_r for air = 1

So, $\sigma_m / \varepsilon_0 = 3 \times 10^6$
or, $\sigma_m = 3 \times 10^6 \times 8.854 \times 10^{-12} = 26.5\ \mu C/m^2$
Mechanical pressure on the conductor for σ_m
$= (3 \times 10^6 \times 26.5 \times 10^{-6} / 2) = 39.75\ N/m^2$

B) Between two dielectric media:



Boundary Condition for D_n:

Let,
$\mathbf{D_1}$ = Electric flux density at the boundary point just within dielectric 1
$\mathbf{D_2}$ = Electric flux density at the boundary point just within dielectric2
and $\mathbf{D_{1n}}$ and $\mathbf{D_{2n}}$ are the normal components of $\mathbf{D_1}$ and $\mathbf{D_2}$, respectively. Then, applying Gauss's theorem for the small volume corresponding to the small surface ds, as shown in the figure
Net flux leaving the volume = $\mathbf{D_{2n} . ds - D_{1n} . ds} = \sigma\, ds$
where , σ = charge density on the dielectric-dielectric boundary
Therefore, $|\mathbf{D_{2n}}| - |\mathbf{D_{1n}}| = \sigma$
If the dielectric-dielectric boundary is charge free, then

$$\mathbf{D_{2n}} = \mathbf{D_{1n}}$$

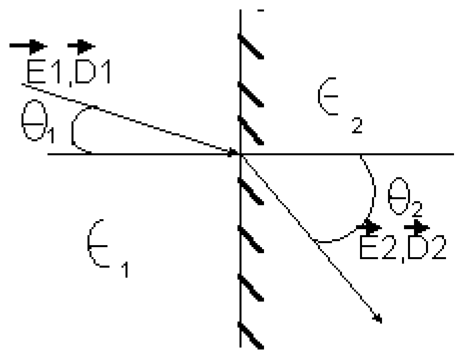
i.e. the normal components of the displacement vector (electric flux density vector) are equal on both sides of the boundary.

Boundary Condition for E_t:

If a unit +ve charge moves along the loop as shown in the figure, then the net work done will be zero. Alternatively, the sum of the potentials in the loop is zero.
Let,
$\mathbf{E_1}$ = Electric field intensity at the boundary point just within dielectric 1
$\mathbf{E_2}$ = Electric field intensity at the boundary point just within dielectric 2
and $\mathbf{E_{1t}}$ and $\mathbf{E_{2t}}$ are the tangential components of $\mathbf{E_1}$ & $\mathbf{E_2}$, respectively.

Then $\mathbf{E_{1t} . dl} - \mathbf{E_{2t} . dl} = 0$
or, $\mathbf{E_{1t}} = \mathbf{E_{2t}}$
i.e. the tangential components of the electric field intensity are equal on both sides of the dielectric-dielectric boundary.

For charge free dielectric-dielectric boundary:



Now, $|\mathbf{D_{1n}}| = |\mathbf{D_{2n}}|$

or, $|\mathbf{D_1}|\cos\theta_1 = |\mathbf{D_2}|\cos\theta_2$

$$\varepsilon_1 |\mathbf{E_1}| \cos\theta_1 = \varepsilon_2 \, |\mathbf{E_2}| \cos\theta_2 \qquad \text{......................(i)}$$

Again, $|\mathbf{E_{1t}}| = |\mathbf{E_{2t}}|$

or, $|\mathbf{E_1}| \sin\theta_1 = |\mathbf{E_2}| \sin\theta_2 \qquad \text{.................(ii)}$

Dividing (ii) by (i),

$$\tan\theta_1 / \varepsilon_1 = \tan\theta_2 / \varepsilon_2$$

or, $\tan\theta_1 / \tan\theta_2 = \varepsilon_1 / \varepsilon_2$

Problem 3

The flux lines of an electric field pass from air into glass, making an angle 25° with the normal to the plane surface separating air and glass at the air-side of the surface. The relative permittivity of glass is 5.0. The field intensity in air is 250 V/m. Calculate the flux density in glass and also the angle, which the flux lines make with the normal on the glass side.

Solution

Given, $\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 5.0$ and $\theta_1 = 25^0$

So, $\tan\theta_1 / \tan\theta_2 = \tan 25° / \tan\theta_2 = 1 / 5$

or, $\tan\theta_2 = 2.331$

or, $\theta_2 = 66.78^0$

Again, $|\mathbf{E_1}| = 250$ V/m

and $|\mathbf{E_1}| \sin\theta_1 = |\mathbf{E_2}| \sin\theta_2$

So, $|\mathbf{E_2}| = (250 \times \sin 25)° / \sin(66.78)° = 114.96$ V/m

Hence, $|\mathbf{D_2}| = \varepsilon_{r2}\,\varepsilon_0\,|\mathbf{E_2}|$

$= 5 \times 8.854 \times 10^{-12} \times 114.96 = 5.089$ nC /m^2