

Multivariable Calculus Projects

AI-Enhanced Computational Mathematics

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Course: MT1008

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Project Guidelines

- **Teams:**
- **Implementation:** Python (recommended) or C++
- **AI Tools:** ChatGPT/WolframAlpha for verification only
- **Duration:** 7–10 days per project
- **Deliverables:**
 - Functional code with visualizations
 - Trained AI model (where applicable)
 - 2–4 page technical report
 - 10-minute group presentation

Project 1: Gradient Field Analysis with AI Verification

Topics: Partial derivatives, gradient vectors, critical points

Objectives:

1. Compute gradients for at least 3 functions (e.g., $f(x, y) = e^{-(x^2+y^2)}$, $f(x, y) = x^2 - y^2$, $f(x, y) = x^3 + y^3 - 3xy$).
2. Visualize surfaces and gradient fields using `matplotlib`, `Plotly`, or `OpenGL`.
3. Train a model on 80 examples to:
 - Classify critical points (max, min, saddle)
 - Respond to symbolic queries (e.g., “Is (0,0) a saddle point for $x^2 - y^2$?”)
 - Output "Question is beyond my scope." if input doesn't relate to trained topic

Project 2: Smart Solver for Vector Field Integrals

Topics: Line integrals, conservative fields, potential functions

Objectives:

1. Compute work integrals for 2-3 vector fields (e.g., $\vec{F} = (y \cos(xy), x \cos(xy))$, $\vec{F} = (x^2, y^2)$).
2. Develop a trained model (80+ examples) to:
 - Solve line integral queries
 - Check for conservatism via path-independence
 - Generate potential functions if applicable
 - Respond "Question is beyond my scope." for unrelated topics
3. Visualize vector fields and integration paths.

Project 3: Volumetric Analysis with Coordinate Transformations

Topics: Triple integrals, spherical/cylindrical coordinates

Objectives:

1. Calculate volumes for 2 non-trivial regions (e.g., intersection of sphere $x^2 + y^2 + z^2 = a^2$ and cylinder $x^2 + y^2 = r^2$).
2. Implement coordinate transformations and compare results in rectangular, cylindrical, and spherical forms.
3. Train an AI model (80+ examples) to:
 - Suggest optimal coordinate system based on the region description
 - Verify integral setups
 - Estimate volume numerically or return "Question is beyond my scope."

Technical Specifications

- **Programming Languages:** Python 3.10+ or C++17
- **Python Libraries:** NumPy, SymPy, Matplotlib, Plotly, scikit-learn
- **C++ Libraries:** OpenGL, Eigen, SFML (optional)
- **AI Training Data:** Minimum 80 verified examples per project
- **Visualization:** Interactive 2D/3D plots required for gradients, fields, and volumes