DISCLOSURE IN INSURANCE MARKETS WITH LIMITED SCREENING

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JEL Classification: D82; G22

We investigate the impact of information disclosure, via a statistical instrument, on consumer welfare in competitive insurance markets with limited screening. We demonstrate that, under natural constraints on information disclosure, no statistical instrument is "safe" to implement. There always exists a nonnegligible set of prior beliefs about the risk types of consumers, compatible with an observed market situation, under which additional information disclosure strictly worsens welfare.

KEYWORDS: insurance market, adverse selection, information disclosure, screening, regulation, privacy.

1. INTRODUCTION

Regulation of consumer-specific information in insurance markets is a controversial topic, as it balances the concern for consumer protection on one side and the loss of efficiency due to adverse selection on the other side.[1] This tradeoff is mitigated in competitive insurance markets, where insurers have zero profits and the surplus is captured by consumers. In such markets, it may seem intuitive that giving insurers more information about consumers should reduce the adverse selection and improve efficiency. Yet, this paper shows that, when allowing for entry and exit of consumers, regulators cannot dismiss the possibility that disclosure of information related to consumer risk will worsen the welfare. This result highlights the need for regulators to exercise caution when considering the disclosure of consumers' private information.

Date: January 28, 2026.

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The authors thank Amanda Friedenberg, Anton Kolotilin, Vitor Farinha Luz, Thomas Mariotti, anonymous referees, as well as many conference and seminar participants for helpful comments. Author names are in an AEA-certified random order (confirmation code TV-2m3FhAfQG).

[1]A large body of literature finds evidence of varying degrees of adverse selection in insurance markets (e.g., Buchmueller and Dinardo 2002, Schwarze and Wein 2005, Simon 2005, Chiappori et al. 2006, He 2009, Einav et al. 2010b, and Bundorf et al. 2012). Adverse selection can generate significant distortions, as documented in Einav et al. (2010b) who find that the welfare loss due to information asymmetries in the UK annuity market is "£127 million per year or about 2 percent of annuitized wealth." The argument of adverse selection has influenced regulatory decisions, for example, granting to the equal treatment laws significant exceptions that are sometimes exploitable beyond their original purpose (Siegelman, 2004). For surveys of the theory, methodology, and evidence, see Einav et al. (2010a), Chiappori and Salanié (2013), and Geruso and Layton (2017).

We consider a competitive insurance market with a continuum of consumers, as in Bisin and Gottardi (2006), Azevedo and Gottlieb (2017), and Farinha Luz et al. (2023). Consumers are heterogenous in both the underlying risk and their risk preferences, which have been identified as two major sources of private information (Finkelstein and McGarry, 2006, Einav et al., 2007, Cutler et al., 2008). Insurers are risk neutral expected utility maximizers who have limited ability to screen consumer risk types.[2] We model additional information as a *monotone instrument* that classifies consumers into two groups: high-risk and low-risk. Once the instrument is introduced, the insurers are allowed to condition insurance prices on this classification. We show that, for every equilibrium outcome in the initial market, there always exists a nonnegligible set of prior beliefs about consumer risk types compatible with this equilibrium outcome, under which introduction of this instrument reduces welfare.

Our analysis builds on the broader literature on second-best reforms arguing that removing distortions does not always lead to improvements.[3] In the context of competitive insurance markets, a prominent argument for why public information about consumer risks can be detrimental to welfare is that of *price uncertainty* (e.g., Hirshleifer 1971, Boyer et al. 1989, Rothschild and Stiglitz 1997, Schlee 2001, Handel et al. 2015, Farinha Luz et al. 2023, and Veiga 2024). A typical example of price uncertainty in the context of health or life insurance is genetic testing. Risk-averse consumers may refuse a free genetic test, because the test results and the associated health insurance prices are uncertain. Notice, however, that this argument does not apply when dealing with information that consumers already know. E.g., in our example above, consumers know whether they are married or not, thus facing no uncertainty in the event this information is released to the insurers. Our paper is different from the above literature in that we shut down the channel of price uncertainty and isolate a different effect: entry and exit of consumers.

Two related papers, Crocker and Snow (2013) and Farinha Luz et al. (2023), reach the conclusion that more information is always better in competitive insurance markets. They allow for unrestricted screening, where each consumer type reveals themselves by selecting the most suitable contract for that type. Consequently, the reduction in information asymmetry can only be beneficial as it relaxes the incentive compatibility constraints in the contract design. In contrast, in this paper we assume limited screening. This is a practical assumption, as in reality insurance contracts rarely feature a continuum of options that consumers can choose from. Under this assumption, the above conclusion of Crocker and Snow (2013) and Farinha Luz et al. (2023) needs not be valid.

Another related paper is Garcia and Tsur (2021), who characterize optimal information provision in competitive insurance markets with limited screening, addressing the trade-off between risk sharing and contract adaptation. They show that full release of private information

[2]For simplicity, we assume that the insurers can offer only full-coverage insurance contracts. What is essential is that full separation of risk types cannot be achieved. See Section 5 for a discussion.

[3]For example, competitive equilibria are often not Pareto optimal in incomplete markets; but partially completing incomplete markets can make everyone worse off (Hart, 1975).

is generally inefficient, and that optimal design matches consumer risk types in a negative assortative manner. In contrast, we do not take the information design perspective, and consider only monotone information structures, thus ruling out negative assortative matching.

Our paper is related to the literature on information provision in the Akerlof's (1970) framework. Within this framework, Levin (2001) examines how changes in information asymmetry within adverse-selection markets affect trade volumes, demonstrating that increased information asymmetry can have non-monotonic effects on trade.

In addressing inefficiencies linked to information disclosure, we also contribute to the literature on information regulation related to privacy. Typically this literature studies allocations with monopolistic firms, as in Eilat et al. (2021) and Bird and Neeman (2022). In such settings, information disclosure is associated with the loss of consumer information rents. This channel is absent in our setting with competitive insurance markets. Therefore, we see our approach as complementary to information design for pricing within monopolistic markets (see also Bergemann et al., 2015, Roesler and Szentes, 2017, Hidir and Vellodi, 2021).

2. ILLUSTRATIVE EXAMPLE

Consider a homeowners insurance market with risk-averse consumers who face an uncertain loss. Several insurance firms simultaneously offer full-coverage contracts, where the consumer pays an upfront price in exchange for full compensation in the event of a loss. Each consumer chooses the cheapest available contract among those offered or opts out from buying insurance.

The firms initially classify the consumers into broad categories based on observable characteristics, such as location and property value. Let us focus on one such category. Suppose that there are three consumer types within that category. For each type $j = A, B, C$, a consumer of that type faces a lottery whose expected loss is μ_j and is willing to pay ν_j for the full-coverage insurance of the loss. The values of (μ_j, ν_j) and their measures are as follows:

Consumer Type, j	A	B	C
Measure, $P(j)$	$1/3$	$1/3$	$1/3$
μ_j	100	120	140
ν_j	110	131	152

This is a standard setting of adverse selection with firms competing for consumers. Because the firms compete for consumers, they offer the same price. Types B and C purchase insurance at that price, while type A does not participate due to adverse selection. To see this, notice that the expected loss of types B and C together is

$$p_* = \left(\frac{1}{3}\mu_B + \frac{1}{3}\mu_C\right) \Big/ \left(\frac{1}{3} + \frac{1}{3}\right) = 130.$$

Thus, p_* is the lowest price such that the firms make nonnegative profit when both types B and C are willing to buy insurance (as both ν_B and ν_C are above p_*), while type A is not (as ν_A

is strictly below p_*). The firms cannot profitably deviate by undercutting p_* to attract type A. Indeed, to attract type A, the price must be lowered to at most $\nu_a = 110$. But in that case the expected loss from all three types would be $(\mu_A + \mu_B + \mu_C)/3 = 120$, which strictly exceeds the price.

Let us interpret the difference between consumer type j's willingness to pay, ν_j, and the expected loss, μ_j, as type j's surplus from buying insurance, evaluated from the perspective of a risk-neutral regulator. As a measure of welfare, we add up this surplus across all consumer types who buy insurance. Since firms make zero profit in equilibrium, the equilibrium price p^* is equal to the average loss across all types who buy insurance at that price. Thus, the welfare measure we use is given by the weighted willingness to pay net of the price across the consumer types who buy insurance at this price:

$$W(p_*) = \frac{1}{3}(\nu_B - p_*) + \frac{1}{3}(\nu_C - p_*) = \frac{23}{3} \approx 7.7.$$

Now, suppose that a regulator is concerned that only two thirds of the consumer base purchase insurance. Understanding the issue of adverse selection, the regulator considers whether to allow the firms to use additional information about consumers' marital status $t \in \{t_u, t_m\}$ (unmarried/married), with the goal to reduce adverse selection. Suppose that the measures of unmarried consumers conditional on their types A, B, and C, are $1/4$, $1/2$, and $3/4$, respectively. Thus, being unmarried signals a higher expected loss. The joint distribution over the type and the marital status is as follows:

	A	B	C
t_u	$1/12$	$1/6$	$1/4$
t_m	$1/4$	$1/6$	$1/12$

With this information, the firms offer different prices to the unmarried (t_u) and married (t_m) categories. As a result, in the married category, types B and C purchase insurance, while type A does not participate; in the unmarried category (t_u), only type C purchases insurance, while the other types do not participate. To see this, notice that the expected loss of types B and C the married category together is

$$p_*^m = \left(\frac{1}{6}\mu_B + \frac{1}{12}\mu_C\right) \Big/ \left(\frac{1}{6} + \frac{1}{12}\right) = \frac{380}{3} \approx 127.$$

Thus, p_*^m is the lowest price offered to the married category such that the firms make nonnegative profit when both types B and C are willing to buy insurance (as both ν_B and ν_C are above p_*^m), while type A is not (as ν_A is strictly below p_*^m).

Next, the expected loss of type C in the unmarried category is given by $p_*^u = \mu_C = 140$. Thus, p_*^u is the lowest price offered to the unmarried category such that the firms make nonnegative profit when type C is willing to buy insurance (as ν_C is above p_*^u), while types A and B are not (as both ν_A and ν_B are strictly below p^u_*). Finally, it is straightforward to verify that the firms cannot profitably deviate by undercutting the prices in either category.

Observe that introducing information about marital status strictly worsens the outcome: Type A remains excluded, and type B exits the market in the unmarried category. The new welfare is equal to:

$$W(p^u_*, p^m_*) = \frac{1}{4}(152 - 140) + \frac{1}{6}\left(131 - \frac{380}{3}\right) + \frac{1}{12}\left(152 - \frac{380}{3}\right) = \frac{35}{6} \approx 5.8,$$

which is strictly smaller than the original welfare of approximately 7.7. This example illustrates the main result of the paper: More information may reduce welfare. The unmarried consumers face a higher price, causing some of them to exit the market, while the married consumers face a lower price, but not low enough to attract any of them to enter the market.

3. BASELINE MODEL

This section presents a stylized model of a competitive insurance market. For ease of presentation, we make several simplifying assumptions. A discussion of the role of the assumptions is deferred to Section 5.

Consider an insurance market with $n \geq 2$ firms, denoted by the set $N = \{1, \ldots, n\}$, and a continuum of risk-averse consumers. Each consumer is characterized by a type $(\mu, \nu) \in [0, \bar{\nu}]^2$, with $\bar{\nu} > 0$, where μ denotes the expected uninsured loss and ν denotes the willingness to pay for the full-coverage insurance. We interpret the difference $\nu - \mu$ as a measure of the surplus of consumer type (μ, ν) from buying insurance, evaluated from the perspective of a risk-neutral regulator. This type specification is quite general, allowing for risk attitude and wealth heterogeneity, as well as a rich set of distributions of losses.

Let $\mathcal{P}$ be the set of measures $P \in \Delta([0, \bar{\nu}]^2)$ that have no mass point on any specific value of ν, and have a strictly positive mass of consumers with a strictly positive expected loss μ:

$$P([0, \bar{\nu}] \times \{\nu\}) = 0 \text{ for each } \nu \in [0, \bar{\nu}], \tag{A$_1$}$$

$$P((0, \bar{\nu}] \times [0, \bar{\nu}]) > 0. \tag{A$_2$}$$

Let $P \in \mathcal{P}$ be a commonly known measure. It describes the empirical distribution of types within the population of consumers. The firms and consumers are engaged in the following game, denoted by $\Gamma(P)$. First, each consumer privately observes their own type. Second, each firm $i \in N$ chooses a price $p_i \in \mathbb{R}_+$ of full-coverage insurance. The firms choose prices simultaneously. Finally, each consumer chooses whether to buy insurance, and if so, from which firm to buy it. The payoffs associated with each consumer type $(\mu, \nu) \in [0, \bar{\nu}]^2$ are as follows. If the consumer with type (μ, ν) buys insurance from a firm i at price p_i, then the consumer's payoff is $\nu - p_i$, firm i's payoff is $p_i - \mu$, and all other firms' payoffs are zero. Otherwise, if this consumer does not buy insurance, then everyone gets zero.

For each $i \in N$, firm i's strategy is a price $p_i \in \mathbb{R}_+$. A consumer's strategy is a mapping $\sigma : [0,\bar{\nu}]^2 \times \mathbb{R}_+^n \to \Delta(\{0,1,...,n\})$ that associates with each type $(\mu,\nu) \in [0,\bar{\nu}]^2$ and each profile of prices $(p_1,...,p_n) \in \mathbb{R}_+^n$ offered by the firms a probability distribution over choices in the set $\{0,1,...,n\}$. The choice of $i \in \{1,...,n\}$ is interpreted as a purchase of insurance from firm i, and $i = 0$ is interpreted as no insurance. Assume that σ is P-measurable with respect to (μ,ν) for each $(p_1,...,p_n) \in \mathbb{R}_+^n$.

Denote $p_{-i} := (p_j)_{j \in N\setminus\{i\}}$. Let $\beta_i(\mu,\nu,p_i,p_{-i}) \in [0,1]$ be a belief of firm i about the probability that consumer with type (μ,ν) buys insurance from i given a profile of prices (p_i,p_{-i}). Let $\boldsymbol{\beta} = (\beta_1,...,\beta_n)$ and $\boldsymbol{p} = (p_1,...,p_n)$.

Given a belief mapping β_i and a price profile p_{-i} of firms other than i, firm i's profit from offering price p_i is

$$\pi_i(p_i,p_{-i},\beta_i) = \int_{(\mu,\nu)\in[0,\bar{\nu}]} (p_i - \mu)\beta_i(\mu,\nu,p_1,...,p_n)P(\mathrm{d}\mu,\mathrm{d}\nu).$$

Given a price profile $\boldsymbol{p}$, the payoff of consumer with type (μ,ν) is

$$u(\sigma,\mu,\nu,\boldsymbol{p}) = \sum_{i\in N} \sigma_i(\mu,\nu,\boldsymbol{p})(\nu - p_i).$$

A tuple $(\boldsymbol{p},\sigma,\boldsymbol{\beta})$ is a *perfect Bayesian equilibrium (PBE)* if the following conditions hold:
(a) Firms make profit-maximizing choices:

$$p_i \in \arg\max_{p_i' \in \mathbb{R}_+} \pi_i(p_i',p_{-i},\beta_i), \quad \text{for all } i \in N.$$

(b) Consumers make payoff-maximizing choices:

$$\sigma_i(\mu,\nu,\boldsymbol{p}) > 0 \ \text{ implies } \ \nu - p_i = \max\left\{\max_{j\in N}(\nu - p_j), 0\right\},$$
$$\text{for all } i \in N \text{ and all } (\mu,\nu) \in [0,\bar{\nu}]^2.$$

(c) Firms' beliefs are consistent:

$$\beta_i(\mu,\nu,\boldsymbol{p}) = \sigma_i(\mu,\nu,\boldsymbol{p}), \quad \text{for all } i \in N \text{ and all } (\mu,\nu) \in [0,\bar{\nu}]^2.$$

In a PBE, a consumer with type $(\mu,\nu) \in [0,\bar{\nu}]^2$ buys the insurance at the lowest available price p' if $\nu > p'$, does not buy it if $\nu < p'$, and is indifferent if $\nu = p'$. Given a profile of equilibrium prices $(p_1,...,p_n)$, the insurance is traded only at the lowest price $p_* = \min\{p_1,...,p_n\}$, which will be referred to as the *equilibrium price*. By (A_1), the set of consumer types who are indifferent between buying and not buying insurance at p_* has measure zero, so w.l.o.g. the set of consumers types who buy at p_* is $[0,\bar{\nu}] \times [p_*,\bar{\nu}]$. The equilibrium price p_* has a property that it yields zero profit to any firm that charges it, and no firm can make a strictly positive profit by undercutting it with a lower price, $p < p_*$.

The next lemma shows that a PBE exists, and presents its key property that we will use to prove our main result. To state the lemma, we introduce the following notation. Let $\pi^M(p)$ be the profit of a monopoly firm when it offers price p and believes that consumers buy the insurance at this price if and only if their willingness to pay ν satisfies $\nu > p$:

$$\pi^M(p) = \int_{[0,\bar{\nu}]\times(p,\bar{\nu}]} (p-\mu)P(\mathrm{d}\mu, \mathrm{d}\nu). \tag{1}$$

LEMMA 1: *For each $P \in \mathcal{P}$:*

(i) There exists a PBE of game $\Gamma(P)$.

(ii) For each $p_* \in \mathbb{R}_+$, there exists a PBE of game $\Gamma(P)$ whose equilibrium price is equal to p_* if and only if p_* satisfies

$$\pi^M(p_*) = 0, \quad \textit{and} \quad \pi^M(p) \le 0 \ \textit{for all}\ p \in [0, p_*). \tag{2}$$

Let $p \in \mathbb{R}_+$ be an equilibrium price and let $z \in [0,1]$ be a mass of consumers who buy at that price. Given $(p,z) \in \mathbb{R}_+ \times [0,1]$, write $\mathcal{S}(p,z)$ for the set of $P \in \mathcal{P}$ such that there exists a PBE $(\boldsymbol{p}, \sigma, \boldsymbol{\beta})$ of game $\Gamma(P)$ that satisfies $p = \min\{p_1, ..., p_n\}$ and $z = P([0,\bar{\nu}] \times [p,\bar{\nu}])$. Endow $\mathcal{S}(p,z)$ with the weak topology.

DEFINITION 1: Say that $(p,z) \in \mathbb{R}_+ \times [0,1]$ is an *observable situation* if $\mathcal{S}(p,z) \neq \emptyset$.

An observable situation is a pair (p,z) of a price and a quantity of sales that the regulator could possibly observe in some equilibrium under some prior.

4. REGULATION OF INFORMATION

We are interested in the welfare implications of providing information to the firms. Suppose that, in addition to type (μ,ν), each consumer has a characteristic $t \in \{H, L\}$ that is irrelevant for the consumer's payoff but may be informative about the consumer's type. Let $\lambda : [0,\bar{\nu}]^2 \to [0,1]$ be a Borel function. For each type $(\mu,\nu) \in [0,\bar{\nu}]^2$, $\lambda(\mu,\nu)$ and $1-\lambda(\mu,\nu)$ are conditional measures of consumers with characteristics $t = H$ and $t = L$, respectively, among the consumers with type (μ,ν). We refer to $\tau = (\{H,L\},\lambda)$ as an *instrument*. We call an instrument $\tau = (\{H,L\},\lambda)$ *monotone* if $\lambda(\mu,\nu)$ is strictly increasing in μ for each ν. The monotonicity of $\lambda(\mu,\nu)$ in μ means that the characteristic t is more likely to be H when the consumer has a higher expected loss. Let T be the set of monotone instruments.

Let $P \in \mathcal{P}$ be a commonly known measure representing prior beliefs about (μ,ν) and let $\tau \in T$ be a commonly known monotone instrument. In the baseline game (Section 3), characteristic t is not present. We interpret it as if the firms do not observe t or are not allowed to condition the contracts on t. We compare the baseline game with the following game where

t is publicly observable.[4] First, as in the baseline game, each consumer privately observes their own type (μ,η). In addition, each consumer's characteristic t is observed by everyone. Second, each firm $i \in N$ chooses a pair of prices $(p_i^H, p_i^L) \in \mathbb{R}_+^2$, where p_i^t is the price of full-coverage insurance offered to every consumer with characteristic $t \in \{H, L\}$. The firms choose prices simultaneously. Finally, each consumer chooses whether to buy insurance, and if so, from which firm to buy it. Consumers with characteristic $t = H$ can only choose among the prices in $\{p_1^H, ..., p_n^H\}$, and consumers with characteristic $t = L$ can only choose among the prices in $\{p_1^L, ..., p_n^L\}$.

An equivalent formulation of the above is that the population of consumers is divided into two categories, $t = H$ (high-risk) and $t = L$ (low-risk). Then, there are two independent games, one for category $t = H$ and one for category L that are identical to the original game but with different distributions of the consumers in the population. The game for the category $t \in \{H, L\}$ is $\Gamma(\hat{P}^t(P,\lambda))$, where $\hat{P}^t(P,\lambda)$ is the posterior distribution conditional on t obtained from P and λ by Bayes' rule.[5]

Our question is whether the regulator could guarantee to increase the welfare by using any given monotone instrument. The difficulty that the regulator faces is that the welfare may go up or down, depending on the prior. However, we assume that the regulator does not know the prior. The regulator only sees the instrument τ and the observable situation (p_*, z_*) describing an equilibrium price and a mass of consumers who buy at that price in an equilibrium of the game where the instrument τ is not introduced and the characteristic t is not observed.

We compare the welfare before and after the instrument τ is introduced, in every possible equilibrium of the "before" game that is consistent with a given observable situation (p_*, z_*) and every possible equilibrium of the "after" game. To evaluate the welfare, let $F_P(\nu) = P([0,\bar{\nu}] \times [0,\nu])$ be the marginal CDF of ν under P. Let $P(\mu|\nu)$ be (a version of) the conditional distribution of μ given ν (e.g., Billingsley, 2017, Section 33). The original welfare (without the instrument) under equilibrium price p_* is given by

$$W_P(p_*) = \int_{[0,\bar{\nu}]^2} \max\{\nu - p_*, 0\} P(\mathrm{d}\mu, \mathrm{d}\nu) = \int_{p_*}^{\bar{\nu}} (\nu - p_*) F_P(\mathrm{d}\nu). \tag{3}$$

Next, for each $t \in \{H, L\}$, consider an arbitrary equilibrium of the game $\Gamma(\hat{P}^t(P,\lambda))$ (which exists by Lemma 1), and let p_*^t be the corresponding equilibrium price. The welfare with the

[4] This is realistic in many applications, such as the one presented in Section 2 where a consumer's marital status is public knowledge. In Section 5 we discuss the assumption that t is observed only by the firms but not by consumers.

[5] For each $X \subset [0,\bar{\nu}]^2$, the posterior probability of X conditional on $t = H$, denoted by $\hat{P}^H(P,\lambda)[X]$, is given by $\int_X \lambda(\mu,\nu) P(\mathrm{d}\mu,\mathrm{d}\nu) / \int_{[0,\bar{\nu}]^2} \lambda(\mu,\nu) P(\mathrm{d}\mu,\mathrm{d}\nu)$. The expression for $\hat{P}^L(P,\lambda)[X]$ is analogous. Since $\lambda(\mu,\nu)$ is strictly increasing in μ, it is in $(0,1)$ for P-almost all (μ,ν). Hence, the posteriors $\hat{P}^H(P,\lambda)$ and $\hat{P}^L(P,\lambda)$ are well defined, and properties (A_1) and (A_2) are easily verified for both of them.

instrument τ under the equilibrium pair of prices (p_*^H, p_*^L) is given by

$$W_P^\tau(p_*^H, p_*^L) = \int_{[0,\bar{\nu}]^2} \Big(\max\{\nu - p_*^H, 0\}\lambda(\mu,\nu) + \max\{\nu - p_*^L, 0\}(1-\lambda(\mu,\nu)) \Big) P(\mathrm{d}\mu, \mathrm{d}\nu)$$

$$= \int_{p_*^H}^{\bar{\nu}} (\nu - p_*^H)\hat{\lambda}(\nu) F_P(\mathrm{d}\nu) + \int_{p_*^L}^{\bar{\nu}} (\nu - p_*^L)(1-\hat{\lambda}(\nu)) F_P(\mathrm{d}\nu), \tag{4}$$

where

$$\hat{\lambda}(\nu) = \int_0^{\bar{\nu}} \lambda(\mu,\nu) P(\mathrm{d}\mu|\nu). \tag{5}$$

DEFINITION 2: Suppose that (p,z) is an observable situation. Say, instrument $\tau = (\{H,L\},\lambda) \in T$ is *potentially welfare damaging in* (p,z) if there exists a nonempty open set $U \subseteq \mathcal{S}(p,z)$ such that for all priors $P \in U$ and all equilibria of games $\Gamma(\hat{P}^H(P,\lambda))$ and $\Gamma(\hat{P}^L(P,\lambda))$ with the corresponding equilibrium prices p_*^H and p_*^L, we have $W_P(p) > W_P^\tau(p_*^H, p_*^L)$.

Our main result shows that no monotone instrument is "safe" to implement. For any observable situation on the insurance market that has a strictly positive amount of sales (i.e., $z > 0$), every monotone instrument can potentially reduce the total welfare.

THEOREM 1: *For every observable situation* (p,z) *with* $z > 0$*, every instrument* $\tau \in T$ *is potentially welfare damaging in* (p,z).

The proof is in the Appendix. The intuition is as follows. The introduction of a monotone instrument raises the price for consumers classified as high-risk, from an initial value p up to some new value p_*^H. At the same time, it lowers the price for consumers classified as low-risk, from p down to some new value p_*^L. High-risk consumers whose willingness to pay is between p and p_*^H no longer buy insurance, thus contributing to welfare loss, whereas low-risk consumers whose willingness to pay is between p_*^L and p switch from not buying to buying insurance, thus contributing to welfare gain. However, the observable situation does not provide any information about the relative mass of the consumers who contribute to welfare gain to those who contribute to welfare loss. There always exists an open set of priors P under which the latter dominates the former. By a symmetric argument, it can also be claimed that, for every observable situation (p,z) with $z < 1$ and every monotone instrument, there exists a non-empty open set of priors such that this instrument is welfare improving.

5. DISCUSSION

We now discuss several assumptions and their role in our results.

Informed firms. A natural extension of our analysis concerns the case where firms possess prior information about consumers and are allowed to offer contracts conditional on this information. This can be modeled as a publicly observable signal $k \in \{1,...,K\}$ that carries information about consumer type, where each k occurs with a strictly positive probability, and a posterior distribution of the consumer type conditional on each k is in $\mathcal{P}$. The game begins by Nature drawing k. The subgame after each possible realization of k is as described in Section 3. Theorem 1 then applies separately for each k.[6]

Partial coverage contracts. We have only considered the simplest contracts that offer full coverage to consumers, and do not permit any screening, except for the consumers' participation decision. However, the crucial modeling feature that Theorem 1 relies on is that some types are pooled in equilibrium. This feature remains unchanged if we allow for a finite menu of contracts $\{(p, x)\}$, where p is a price and x is an insurance cover, under the assumption of a parametric family of the consumers' utility functions (e.g., CARA).[7] Pooling of risk types in competitive equilibrium can emerge under other modeling assumptions, such as transaction costs (Allard et al., 1997)[8] or under other solution concepts, such as the equilibrium concept *E2* proposed by Wilson (1977).

Observability of instruments. We have assumed that the realized characteristic t of the instrument is publicly observable. Suppose instead that t is observed only by the firms. This does not change the equilibrium analysis, because the consumers' payoffs are independent of t and their decisions to buy or not to buy insurance are made ex post, after they receive price offers from the firms. However, it may affect the evaluation of welfare from the consumers' perspective. Ex ante the consumers don't know their characteristic t and, thus, they face another layer of uncertainty, namely, the uncertainty about price they will be offered.[9] As the consumers are risk averse and dislike uncertainty, this would reinforce the main message of the paper that additional information disclosure may strictly worsen welfare.

Non-monotone instruments. We assume strict monotonicity of instruments for Theorem 1 to hold. If we allow for weakly monotone instruments, we obtain a weaker result: for every observable situation, every weakly monotone instrument is potentially *weakly* welfare damaging, in the sense that the welfare reduces only weakly. This is because the key part of the proof is that, after the introduction of the instrument, a strictly positive mass of consumers who previously bought insurance are now categorized as high-risk, face a higher price, and decide

[6]Specifically, for each k, each observable situation (p_k, z_k), and each monotone instrument τ_k, the proof of Theorem 1 constructs a distribution P_k that yields (p_k, z_k) in a PBE, such that the welfare conditional on k strictly goes down after the introduction of τ_k. Denoting by q_k the probability of k, the prior P is then constructed from the posteriors $(P_k)_{k\in K}$ as follows: $P(X) = \sum_{k=1}^{K} P_k(X) q_k$ for each $X \subset [0, \bar{\nu}]^2$.

[7]The proof follows the same steps, but the construction of the prior distribution of consumer types is more intricate. The support of types is concentrated in small intervals to the right of the threshold types (who are indifferent between adjacent contracts in the menu). As a result, when the instrument is introduced, the consumers switch only downwards, to contracts with less coverage, thus causing the welfare to decrease.

[8]See also Chade and Schlee (2020) who study pooling with a monopolist insurer.

[9]Unless they have a special kind of preferences that is neutral to the price uncertainty (see Segal, 1990).

not buy insurance. Such a mass of consumers need not exist if the instrument is only weakly monotone, so that it could treat all types in some interval identically.

The assumption of instrument monotonicity is reasonable for many applications. If we dispose of the monotonicity assumption entirely, then our result need not hold. Garcia and Tsur (2021) show that a welfare improvement can always be achieved by an instrument that pools high-risk and low-risk types pairwise in a negative assortative fashion.

Non-binary instruments. Our results extend to instruments that categorize consumers into $m \geq 2$ categories, with finite m, as long as the likelihood ratio of the underlying risk between each pair of categories is weakly monotone, and for some pair it is strictly monotone. Our proof applies verbatim for each pair of categories, and the intuition for this result does not change.

Restrictions on priors. We assume that the set of admissible priors $\mathcal{P}$ contains only distributions that satisfy (A_1) and (A_2). The latter rules out the trivial case where all consumers have zero loss ($\mu = 0$) almost surely. The former stipulates that the marginal distribution of ν has no mass points. On the one hand, it is natural to assume that consumers have diverse preferences that do not cluster at any specific point. On the other hand, this assumption simplifies the exposition while making no difference on our results, ceteris paribus.[10] Indeed, Lemma 1(i) and the "if" part of Lemma 1(ii) do not depend on (A_1) at all, and the "only if" part of Lemma 1(ii) can be easily extended.[11] Theorem 1 continues to hold as its proof relies on Lemma 1, and its key argument is constructive and does not require (A_1).

Sufficient conditions for welfare improvement. We allow for all priors that satisfy (A_1) and (A_2). A natural question is that if there are sufficient conditions on the set of priors so that a given instrument or a subclass of instruments are welfare improving for all these priors. This is an open question that we leave for future research.

Pareto incomparability. We would like to point out that the equilibrium payoffs of consumers before and after the introduction of an instrument are not Pareto comparable. When a monotone instrument is introduced, the equilibrium price for consumers with $t = H$ goes up and the equilibrium price for consumers with $t = L$ goes down, so the former are worse off and the latter are better off. However, if the instrument increases (decreases) the welfare and monetary transfers

[10]The key conceptual issue that arises if (A_1) is not assumed is that firms may have strictly positive profits in equilibrium. This may occur if the equilibrium price p_* coincides with a mass point of ν. But, our definition of observable situation does not capture the firms' profits. If we include the producer surplus to the observable situation, then a new channel of effect emerges. An instrument (even negligibly informative one) may be used to introduce noise that breaks apart the mass point in the consumers' preferences, and by doing so, redistributes the surplus.

[11]If (A_1) is not assumed, then $\pi^M(p)$ need not be continuous. Thus, it may no longer be true that $\pi^M(p^*) = 0$ in equilibrium, so (2) need not hold. However, $\pi^M(p)$ is right-continuous by construction. The "only if" part of the proof of Lemma 1 (ii) can therefore be adjusted to show that there exists a PBE whose equilibrium price is p_* only if $\pi^M(p_*) \geq 0$ and $\pi^M(p) \leq 0$ for all $p < p_*$, and such $p_* \in [0, \bar{\nu}]$ exists by the right-continuity of π^M, $\pi^M(0) < 0$, and $\pi^M(\bar{\nu}) \geq 0$.

are feasible, then the regulator can design a transfer scheme that increases (decreases, respectively) the utility of each type of consumers when the instrument is introduced, provided the consumers' underlying preferences are of the Gorman form (Gorman, 1953), which includes CARA and CRRA families of utility functions as special cases.

APPENDIX A: PROOFS

A.1. *Proofs of Lemma 1*

Fix $P \in \mathcal{P}$. By construction, $\pi^M(p)$ is right-continuous in p.[12] Clearly, $\pi^M(\bar{\nu}) = 0$. Since P satisfies (A_2), $\pi^M(0) < 0$. Hence, there exists $p_* \in (0, \bar{\nu}]$ that satisfies (2).

Part (i). Because there exists p_* that satisfies (2), part (i) follows from part (ii).

Part (ii), if. Let p_* satisfy (2). A PBE $(\boldsymbol{p}, \sigma, \boldsymbol{\beta})$ is constructed as follows. For each profile of prices $\boldsymbol{p} = (p_1, ..., p_n)$, denote

$$p_{min}(\boldsymbol{p}) = \min\{p_1, ..., p_n\} \quad \text{and} \quad J(\boldsymbol{p}) = \{j \in \{1, ..., N\} : p_j = p_{min}(\boldsymbol{p})\}.$$

For each type $(\mu, \nu) \in \mathbb{R}_+$, each profile of prices $\boldsymbol{p} = (p_1, ..., p_n)$, and each $i \in N$, let

$$\sigma_i(\mu, \nu, \boldsymbol{p}) = \begin{cases} 1/|J(\boldsymbol{p})|, & \text{if } \nu > p_{min}(\boldsymbol{p}) \text{ and } i \in J(\boldsymbol{p}), \\ 0, & \text{otherwise.} \end{cases}$$

That is, consumers buy insurance at the smallest offered price $p_{min}(\boldsymbol{p})$ whenever they strictly prefer buying (as compared to not buying) it. Moreover, whenever they buy, they randomize with equal probabilities among all firms that offer $p_{min}(\boldsymbol{p})$.

Next, let $\boldsymbol{\beta}$ be given by $\beta_i = \sigma_i$ for each $i \in N$, so the firms' beliefs are correct. Finally, let $\boldsymbol{p}$ be given by $p_i = p_*$ for all $i \in N$.

Clearly, σ and $\boldsymbol{\beta}$ satisfy conditions (b) and (c) of PBE by construction. To see that $p_1 = ... = p_n = p_*$ are equilibrium prices, observe that the firms equally share the monopoly profit, so $\pi_i(p_*, p_{-i}) = \pi^M(p_*)/N$, which is zero by (2). For each $i \in N$, firm i's deviation to $p_i' > p_*$ is not profitable, because, according to β_i, firm i would make no sales. Firm i's deviation to $p_i' < p_*$ is not profitable either, because, according to β_i, firm i would become a monopolist seller at price p_i', and $\pi^M(p_i') \leq 0$ by (2). Consequently, $(\boldsymbol{p}, \sigma, \boldsymbol{\beta})$ is a PBE.

Part (ii), only if. Let $(\boldsymbol{p}, \sigma, \boldsymbol{\beta})$ be a PBE, and let $p_* = \min\{p_1, ..., p_n\}$ be its equilibrium price. By conditions (b) and (c) of PBE, the consumers buy insurance at the lowest offered price, and the firms have correct beliefs about it. By (A_1), π^M is continuous. If $\pi^M(p_*) > 0$, then any firm i can profitably deviate by offering p_i' that marginally undercuts p_*. If $\pi^M(p_*) < 0$, then there

[12] Discontinuity of π^M can only occur at points p where P has an atom at $\nu = p$, i.e., $P([0, \bar{\nu}] \times \{p\}) > 0$. This is ruled out by ($A_1$). However, right-contimuity of $\pi^M(p)$ holds even without (A_1). To see this, consider an atom at p and a sequence $p_k \downarrow p$. Then the area of integration, $[0, \bar{\nu}] \times (p_k, \bar{\nu}]$, excludes this atom for all k, and thus $\pi^M(p_k) \to \pi^M(p)$ as $p_k \downarrow p$.

exists firm i that offers $p_i = p_*$ and gets strictly negative profit. This firm can profitably deviate by offering $p_i' = \bar{\nu}$, thus guaranteeing at least zero profit. It follows that in equilibrium we must have $\pi^M(p_*) = 0$, and all firms get zero profit. Finally, if $\pi^M(p) > 0$ for some $p < p_*$, than any firm i can profitably deviate by offering $p_i' = p$, thus becoming a monopolist seller and getting a strictly positive profit. We conclude that (2) holds. *Q.E.D.*

A.2. *Proof of Theorem 1*

As a first step, we show that if $(p_*, z_*) \in \mathbb{R}_+ \times (0,1]$ is an observable situation, then price p_* satisfies

$$0 < p_* < \bar{\nu}. \tag{6}$$

Indeed, let $(p_*, z_*) \in \mathbb{R}_+ \times (0,1]$ be an observable situation, let $P \in \mathcal{S}(p_*, z_*)$, and let $\pi^M(p)$ be the profit of a monopoly firm given by (1).

By Lemma 1(ii), the equilibrium price p_* satisfies $\pi^M(p_*) = 0$. By (A_2), we have $\pi^M(0) < 0$. Since $z_* > 0$ by assumption, we have $F_P(p_*) = 1 - z_* < 1$. Thus, by (A_1), we have $p_* < \bar{\nu}$. We obtain (6).

As a second step, for an arbitrary observable situation (p_*, z_*) with $z_* > 0$, we construct a measure $P \in \mathcal{S}(p_*, z_*)$ such that p_* is the unique equilibrium price under P. Define P through its marginal CDF $F_P(\nu)$ and its conditional distribution $P(\mu|\nu)$ as follows. Let

$$m(\nu) = \left(1 - \frac{p_*}{\bar{\nu}}\right)\frac{p_*}{2} + \frac{p_*}{\bar{\nu}}\nu \quad \text{for each } \nu \in [0, \bar{\nu}]. \tag{7}$$

Note that, by (6), $m(\nu)$ is strictly increasing in ν, and $m(\nu) \in (0, \bar{\nu})$ for all $\nu \in [0, \bar{\nu}]$. Define

$$F_P(\nu) = \begin{cases} \frac{2(1-z_*)}{p_*}\nu, & \text{if } \nu \in [0, \frac{p_*}{2}), \\ 1 - z_*, & \text{if } \nu \in [\frac{p_*}{2}, p_*), \\ 1 - z_* + \frac{z_*}{\bar{\nu} - p_*}(\nu - p_*), & \text{if } \nu \in [p_*, \bar{\nu}], \end{cases} \tag{8}$$

$$P(\mu|\nu) \text{ assigns probability 1 to } \mu = m(\nu) \text{ for each } \nu \in [0, \bar{\nu}], \tag{9}$$

By (6), F_P is well defined and m is strictly increasing. By construction, F_P is continuous, so (A_1) is satisfied. Also, $P((0, \bar{\nu}] \times (0, \bar{\nu}]) = 1 - F_P(0) = 1$, so ($A_2$) is satisfied. To see that $P \in \mathcal{S}(p_*, z_*)$, observe that z_* is the equilibrium quantity under P given price p_*:

$$P([0, \bar{\nu}] \times (p_*, \bar{\nu}]) = 1 - F_P(p_*) = z_*.$$

Also, p_* is an equilibrium price under P for the following reason. By (9), substitute $\mu = m(\nu)$ into (1) and obtain

$$\pi^M(p) = \int_p^{\bar{\nu}} (p - m(\nu))\, F_P(\mathrm{d}\nu). \tag{10}$$

Evaluating (10) using (8) and (7) yields

$$\pi^M(p) = \begin{cases} -z_*(p_*-p) - \frac{2(1-z_*)}{\bar{\nu} p_*}\left(\bar{\nu} - \frac{p_*}{2}\right)\left(\frac{p^*}{2} - p\right)^2, & p \in [0, \frac{p_*}{2}), \\ -z_*(p_*-p), & p \in [\frac{p_*}{2}, p_*), \\ \frac{z_*}{\bar{\nu}(\bar{\nu}-p_*)}(p-p_*)(\bar{\nu}-p)\left(\bar{\nu} - \frac{p_*}{2}\right), & p \in [p_*, \bar{\nu}]. \end{cases} \tag{11}$$

We thus obtain that $\pi^M(p_*) = 0$, $\pi^M(p) < 0$ for all $p < p_*$, and $\pi^M(p) > 0$ for all $p \in (p_*, \bar{\nu})$. Hence, by Lemma 1(ii), p_* is the unique equilibrium price under P.

Let $\tau = (\{H, L\}, \lambda) \in T$. We show that, for all equilibria that emerge under signals H and L, the associated equilibrium prices, p_*^H and p_*^L, respectively, satisfy $p_*/2 \le p_*^L < p_* < p_*^H$. To do this, we use Lemma 1(ii). Let

$$\pi^H(p) = \int_{[0,\bar{\nu}]\times(p,\bar{\nu}]} (p-\mu)\lambda(\mu,\nu)P(\mathrm{d}\mu, \mathrm{d}\nu),$$

$$\pi^L(p) = \int_{[0,\bar{\nu}]\times(p,\bar{\nu}]} (p-\mu)(1-\lambda(\mu,\nu))P(\mathrm{d}\mu, \mathrm{d}\nu).$$

Note that π^H and π^L are the monopoly profit functions under the posteriors induced by signals H and L, up to multiplication by positive constants $\int_{[0,\bar{\nu}]^2} \lambda(\mu,\nu)P(\mathrm{d}\mu, \mathrm{d}\nu)$ and $\int_{[0,\bar{\nu}]^2}(1 - \lambda(\mu,\nu))P(\mathrm{d}\mu, \mathrm{d}\nu)$, respectively. By (5) and (9), we obtain

$$\pi^H(p) = \int_p^{\bar{\nu}} (p - m(\nu))\hat{\lambda}(m(\nu))F_P(\mathrm{d}\nu), \tag{12}$$

$$\pi^L(p) = \int_p^{\bar{\nu}} (p - m(\nu))(1-\hat{\lambda}(\nu))F_P(\mathrm{d}\nu). \tag{13}$$

Consider an arbitrary equilibrium under H, and let p_*^H be the corresponding equilibrium price. By Lemma 1 (ii), p_*^H is an equilibrium price if and only if $\pi^H(p_*^H) = 0$ and $\pi^H(p) \le 0$ for all $p < p_*^H$. Thus, to prove $p_*^H > p_*$, it suffices to show that $\pi^H(p) < 0$ for all $p \le p_*$. For $p \in [0, p_*/2)$, we obtain $\pi^H(p) < 0$, since, by (7), we have $p - m(\nu) < 0$ for all $\nu \ge p$. For $p \in [p_*/2, p_*)$, we obtain

$$\begin{aligned} \pi^H(p) &= \int_p^{m^{-1}(p)} (p-m(\nu))\hat{\lambda}(m(\nu))F_P(\mathrm{d}\nu) + \int_{m^{-1}(p)}^{\bar{\nu}} (p-m(\nu))\hat{\lambda}(m(\nu))F_P(\mathrm{d}\nu) \\ &< \hat{\lambda}(p)\left(\int_p^{m^{-1}(p)} (p-m(\nu))F_P(\mathrm{d}\nu) + \int_{m^{-1}(p)}^{\bar{\nu}} (p-m(\nu))F_P(\mathrm{d}\nu)\right) \\ &= \hat{\lambda}(p)\,\pi^M(p) \le 0, \end{aligned}$$

where the first equality is by rearrangement of (12), the first inequality is because $p - m(\nu) > 0$ when $\nu \in [p, m^{-1}(p))$, $p - m(\nu) < 0$ when $\nu \in (m^{-1}(p), \bar{\nu}]$, and $\hat{\lambda}$ is strictly increasing, the second equality is by (10), and the second inequality is by (11) and $p \in [p_*/2, p_*)$.

Next, consider an arbitrary equilibrium under L, and let p_*^L be the corresponding equilibrium price. By Lemma 1 (ii), $\pi^L(p_*^L) = 0$ and $\pi^L(p) \leq 0$ for all $p < p_*^L$. Thus, to prove that $p_*/2 \leq p_*^L < p_*$, it suffices to show that $\pi^L(p) < 0$ for all $p < p_*/2$, and $\pi^L(p_*) > 0$.

Let us show that $\pi^L(p) < 0$ for all $p < p_*/2$. Let $p \in [0, p_*/2)$. By (7), $p - m(\nu) < 0$ for all $\nu \in [p, \bar{\nu}]$. Then, by (13), it is immediate that $\pi^L(p) < 0$.

Next, let us show that $\pi^L(p_*) > 0$. Evaluating (13) at $p = p_*$, we obtain

$$\begin{aligned}
\pi^L(p_*) = &\int_{p_*}^{m^{-1}(p_*)} (p_* - m(\nu))(1 - \hat{\lambda}(m(\nu)))F_P(\mathrm{d}\nu) \\
&+ \int_{m^{-1}(p_*)}^{\bar{\nu}} (p_* - m(\nu))(1 - \hat{\lambda}(m(\nu)))F_P(\mathrm{d}\nu) \\
&> (1 - \hat{\lambda}(p_*)) \left(\int_{p_*}^{m^{-1}(p_*)} (p_* - m(\nu))F_P(\mathrm{d}\nu) + \int_{m^{-1}(p_*)}^{\bar{\nu}} (p_* - m(\nu))F_P(\mathrm{d}\nu) \right) \\
&= (1 - \hat{\lambda}(p_*))\pi^M(p_*) = 0,
\end{aligned}$$

where the first equality is by rearrangement of (13) with $p = p_*$, the inequality is because $p_* - m(\nu) > 0$ when $\nu \in [p_*, m^{-1}(p_*))$, $p_* - m(\nu) < 0$ when $\nu \in (m^{-1}(p_*), \bar{\nu}]$, and $1 - \hat{\lambda}$ is strictly decreasing, the second equality is by (10), and third equality is by (11).

Next, denote $b = z_*/(\bar{\nu} - p_*)$. Note that b is the density of F_P for $p > p_*$. We have

$$\begin{aligned}
&W_P^\tau(p_*^H, p_*^L) - W_P(p_*) = \\
&= \int_{p_*^H}^{\bar{\nu}} (\nu - p_*^H)\hat{\lambda}(m(\nu))F_P(\mathrm{d}\nu) + \int_{p_*^L}^{\bar{\nu}} (\nu - p_*^L)(1 - \hat{\lambda}(m(\nu)))F_P(\mathrm{d}\nu) - \int_{p_*}^{\bar{\nu}} (\nu - p_*)F_P(\mathrm{d}\nu) \\
&= \int_{p_*^H}^{\bar{\nu}} (\nu - m(\nu))\hat{\lambda}(m(\nu))F_P(\mathrm{d}\nu) + \int_{p_*^L}^{\bar{\nu}} (\nu - m(\nu))(1 - \hat{\lambda}(m(\nu)))F_P(\mathrm{d}\nu) \\
&- \int_{p_*}^{\bar{\nu}} (\nu - m(\nu))F_P(\mathrm{d}\nu) \\
&= \int_{p_*^H}^{\bar{\nu}} (\nu - m(\nu))\hat{\lambda}(m(\nu))b\mathrm{d}\nu + \int_{p_*}^{\bar{\nu}} (\nu - m(\nu))(1 - \hat{\lambda}(m(\nu)))b\mathrm{d}\nu - \int_{p_*}^{\bar{\nu}} (\nu - m(\nu))b\mathrm{d}\nu \\
&= -b\left(1 - \frac{p_*}{\bar{\nu}}\right) \int_{p_*}^{p_*^H} \left(\nu - \frac{p_*}{2}\right) \hat{\lambda}(m(\nu))\mathrm{d}\nu < 0.
\end{aligned} \tag{14}$$

The first equality is by (3), (4), and (9). The second equality is by (10), (12), (13), and the fact that $\pi^M(p_*) = 0$, $\pi^H(p_*^H) = 0$, and $\pi^L(p_*^L) = 0$ by Lemma 1(ii). The third equality is because $p_*/2 \leq p_L^* < p_*$ and, by (8), F_P has zero density on $(p_*/2, p_*)$, and thus we can increase the lower integration bound from p_*^L to p_* in the middle integral. The fourth equality is by substitution of $m(\nu)$ from (7) and rearrangement. The inequality is because $p_* < p_H^* \leq \bar{\nu}$, $\hat{\lambda}$ is strictly positive, and, by (6), $b = z_*/(\bar{\nu} - p_*) > 0$.

Finally, let $(P_k)_{k=1,2,\ldots}$ be a sequence of priors in $\mathcal{S}(p_*, z_*)$ that converges to P (in weak topology). Let π_k^H and π_k^L be given by (12) and (13), respectively, but under P_k instead of P. Let p_k^H be the smallest price that satisfies $\pi_k^H(p_k^H) = 0$ and $\pi_k^H(p) \leq 0$ for all $p < p_k^H$. Let p_k^L be the largest price that satisfies $\pi_k^L(p_k^L) = 0$ and $\pi_k^L(p) \leq 0$ for all $p < p_k^L$. Observe that the graphs $\{(p, \pi_k^H(p)) : p \in [0, \bar{\nu}]\}$ and $\{(p, \pi_k^L(p)) : p \in [0, \bar{\nu}]\}$ converge pointwise to $\{(p, \pi^H(p) : p \in [0, \bar{\nu}]\}$ and $\{(p, \pi^L(p) : p \in [0, \bar{\nu}]\}$, respectively, as $k \to \infty$. Thus, as $k \to \infty$, we have $p_k^H \to p_*^H$, and $p_k^L \to p_*^L$, where p_*^H and p_*^L are the smallest and the largest equilibrium price under prior P and signals H and L, respectively. Moreover, for $\varepsilon = \min\{p_* - p_*^L, p_*^H - p_*\}/2$, there exists large enough k_0 such that

$$p_k^L + \varepsilon < p_* < p_k^H - \varepsilon \ \text{ for all } k > k_0. \tag{15}$$

By (14), (15), and the continuity of W_P and W_P^τ in P, we obtain $W_{P_k}^\tau(p_k^H, p_k^L) < W_{P_k}(p_*)$ for all $k > k_0$. This proves that the welfare strictly goes down due to signal τ for all priors in a small enough neigborhood of P in $\mathcal{S}(p_*, z_*)$. *Q.E.D.*

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