Homework 3

Due in class Jan 28

1.

1.7 Shumway & Stoffer (2nd Ed.)

Consider the following time series model:

$$x_t = w_{t-1} + 2w_t + w_{t+1}$$

where the w_t are i.i.d with mean zero and variance σ^2. Determine the autocovariance and autocorrelation functions of x_t.

2.

1.6 Shumway & Stoffer (2nd Ed.) Consider the time series

$$x_t = \beta_1 + \beta_2 t + w_t$$

where w_t are i.i.d with mean zero and variance σ^2.

a. Determine whether x_t is weakly stationary.
b. Show that the process $y_t = x_t - x_{t-1}$ is weakly stationary. (Charlotte says: this is known as taking a first difference of the series)
c. Show the mean of the moving average

$$v_t = \frac{1}{2q+1} \sum_{r=-q}^{q} x_{t-r}$$

   is $\beta_1 + \beta_2 t$ and give a simplified expression for the autocovariance funcion of v_t.

3.

Chatfield 3.9

For each of the following processes:
a) $X_t = 0.3X_{t-1} + Z_t$
b) $X_t = Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$
c) $X_t = 0.5X_{t-1} + Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$

express the model using B notation and determine whether the model is stationary and/or invertible. For model (a) find the equivlant MA representation.

4.

Chatfield 3.11

Show that the ac.f. of the ARMA(1,1) model

$$X_t = \alpha X_{t-1} + Z_t + \beta Z_{t-1}$$

where $|\alpha| < 1$, and $|\beta| < 1$ is given by

$$\rho(1) = \frac{(1 + \alpha\beta)(\alpha + \beta)}{(1 + \beta^2 + 2\alpha\beta)}$$
$$\rho(h) = \alpha\rho(h - 1)$$

5.

a. Use the `ARMAacf` function in R to find the **theoretical** autocorrelation function of the following AR(2) model, out to lag 10.

$$x_t = 0.8x_{t-1} - 0.2x_{t-2} + Z_t$$

where $Z_t \sim_{i.i.d} N(0, 1)$.

b. Simulate a time series of length 30 from the same model, and use the `acf` function to **estimate** the autocorrelation coefficent at lag 1.

c. Repeat the simulation 1000 times. Does the sample autocorrelation coefficient appear to be an unbiased estimate of the true autocorrelation at lag 1?

d. Does the answer to 3. change with longer time series?

e. What is the relationship between the variance of the sample autocorrelation coefficent and the time series length?

If you haven't seen the `replicate` function in R, you should check it out.

Challenge Question

Use simulation to illustrate the following properties of a random walk with zero drift:

- The mean function, μ_t, is zero for all t.
- The variance function, σ_t^2 increases as a function of t.
- The autocovariance, $Cov(x_t, x_{t+h})$ depends only on t and not on h.