ST565: Time Series HW7

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Question 1

$$\gamma(k) = \begin{cases} \beta_1^2\sigma^2 + \sigma^2 & \text{for k} = 0 \\ \beta_1\sigma^2 & \text{for k} = 1 \\ 0 & \text{for k} >= 1 \end{cases}$$

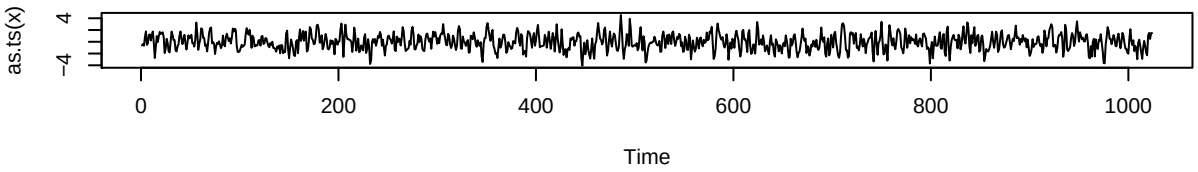
$$f(\omega) = 1/\pi\big[\gamma(0) + 2\Sigma_{k=1}^{\infty}\gamma(k)\cos(\gamma k)\big]$$

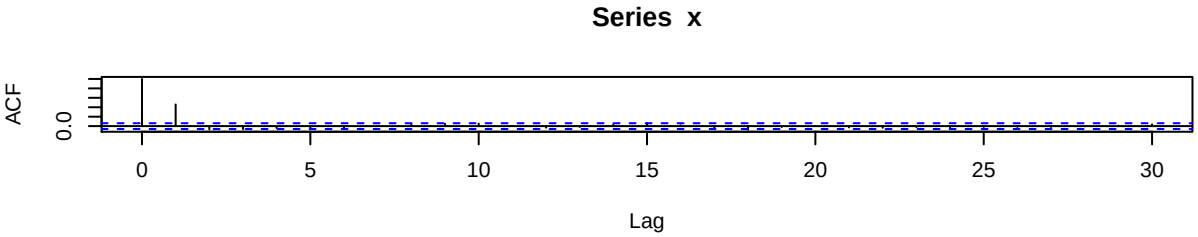
$$f(\omega) = 1/\pi\big[\beta_1^2\sigma^2 + \sigma^2 + 2[\beta_1\sigma^2]\cos(\gamma)\big]$$

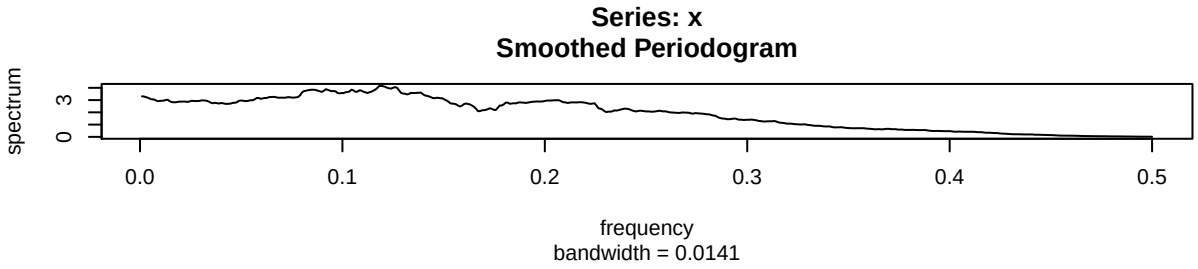
$$f(\omega) = 1/\pi\big[\beta_1^2\sigma^2 + \sigma^2 + 2\beta_1\sigma^2\cos(\gamma)\big]$$

$$f(\omega) = \sigma^2/\pi\big[\beta_1^2 + 1 + 2\beta_1\cos(\gamma)\big]$$

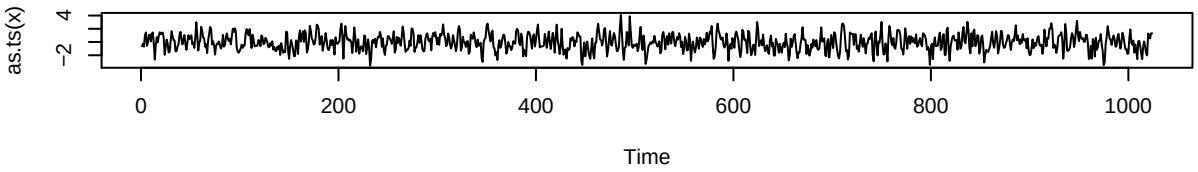
```
# WN
# set.seed(1)
# x <- w <- rnorm(1024)
# for (t in 2:1024) x[t]<- w[t]
# layout(1:3)
# plot(as.ts(x))
# acf(x)
# spectrum(x, span = 51, log = c("no"))
#
# # AR(1)
# set.seed(1)
# x <- w <- rnorm(1024)
# for (t in 2:1024) x[t]<- 0.9 * x[t-1] + w[t]
# layout(1:3)
# plot(as.ts(x))
# acf(x)
# spectrum(x, span = 51, log = c("no"))

# MA(1)
set.seed(1)
x <- w <- rnorm(1024)
# beta = 0.9
for (t in 2:1024) x[t]<- 0.9 * w[t-1] + w[t]
layout(1:3)
plot(as.ts(x))
acf(x)
spectrum(x, span = 51, log = c("no"))
```

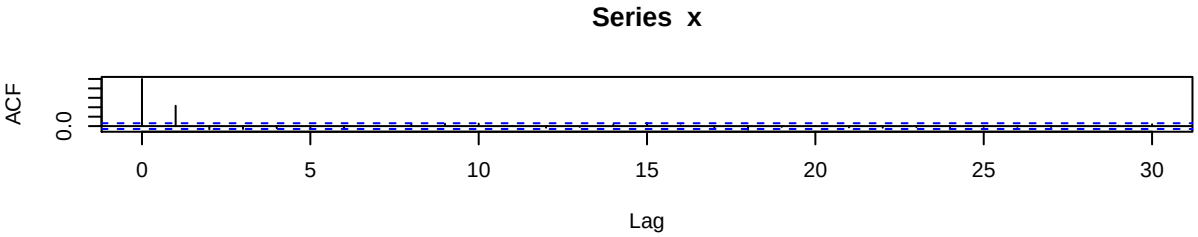


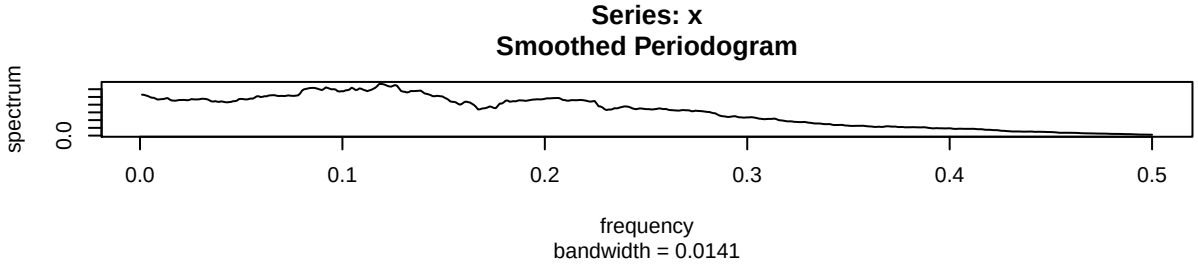




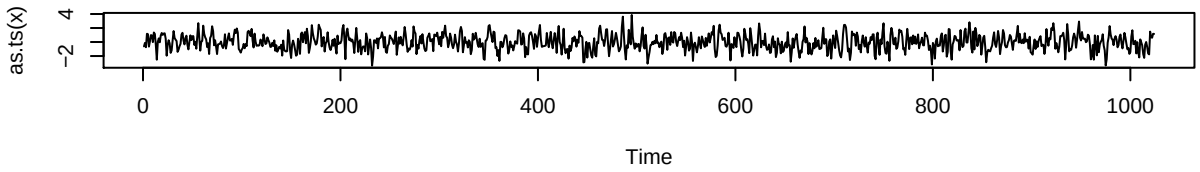

```
# beta = 0.7
for (t in 2:1024) x[t]<- 0.7 * w[t-1] + w[t]
layout(1:3)
plot(as.ts(x))
acf(x)
spectrum(x, span = 51, log = c("no"))
```

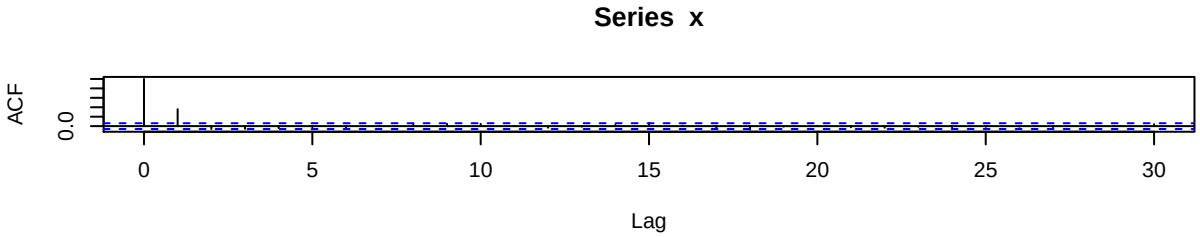


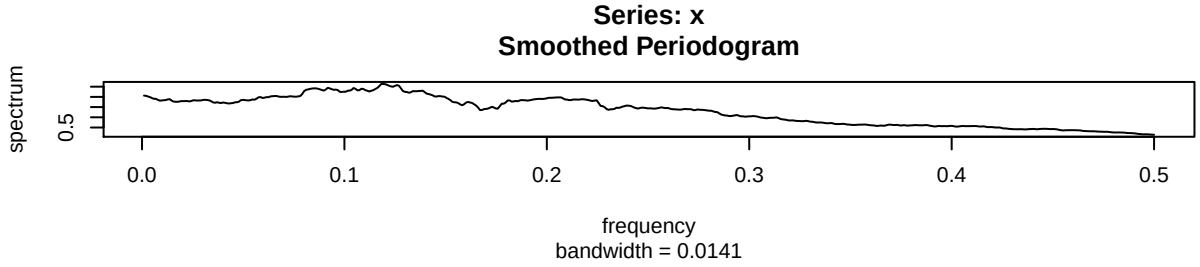




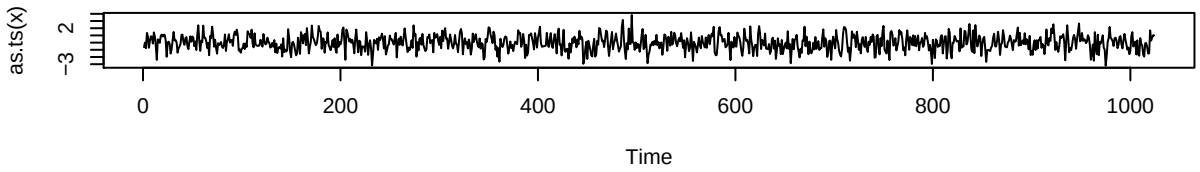

```
# beta = 0.5
for (t in 2:1024) x[t]<- 0.5 * w[t-1] + w[t]
layout(1:3)
plot(as.ts(x))
acf(x)
spectrum(x, span = 51, log = c("no"))
```

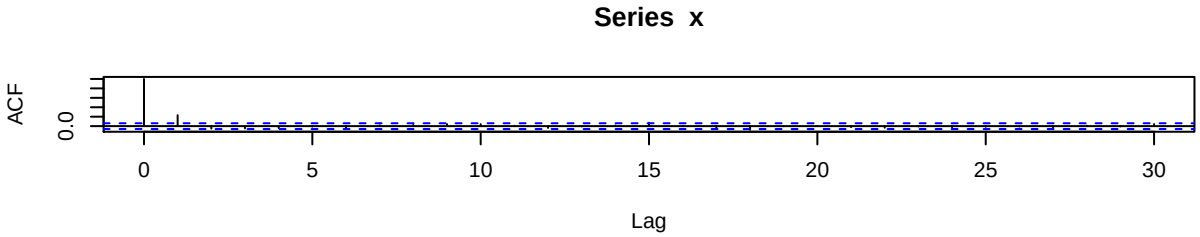


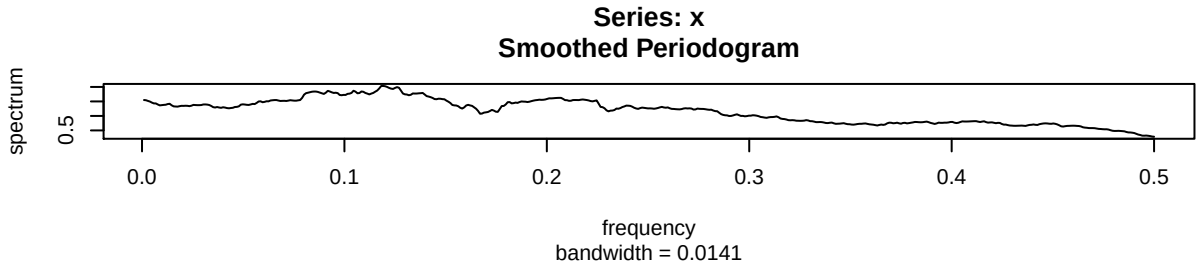




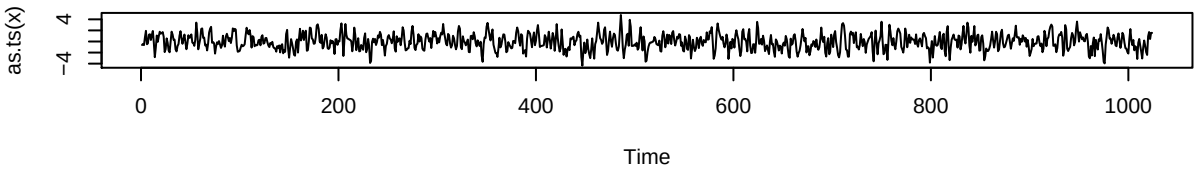

```
# beta = 0.3
for (t in 2:1024) x[t]<- 0.3 * w[t-1] + w[t]
layout(1:3)
plot(as.ts(x))
acf(x)
spectrum(x, span = 51, log = c("no"))
```

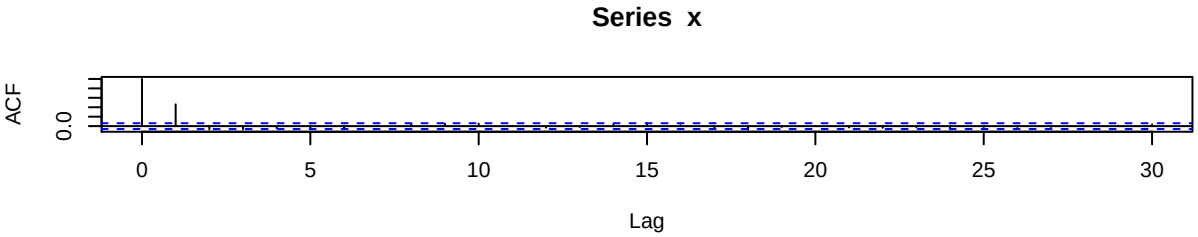


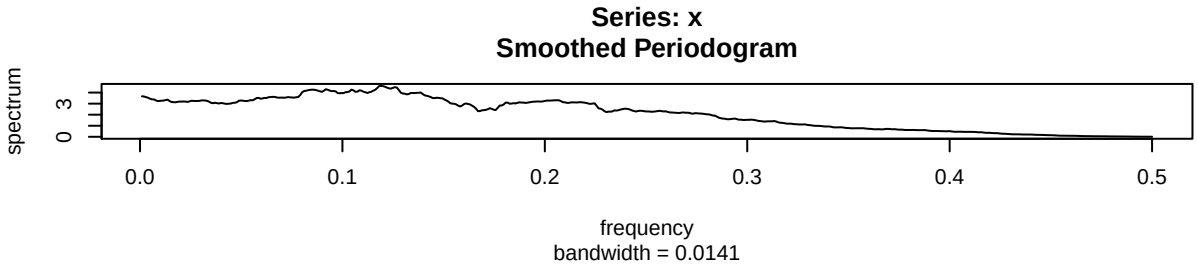




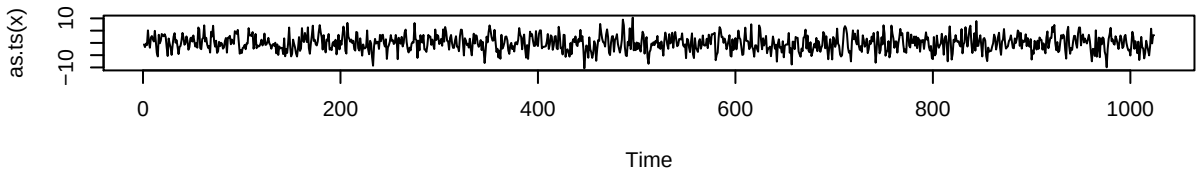

```
# beta = 1
for (t in 2:1024) x[t]<- 1 * w[t-1] + w[t]
layout(1:3)
plot(as.ts(x))
acf(x)
spectrum(x, span = 51, log = c("no"))
```

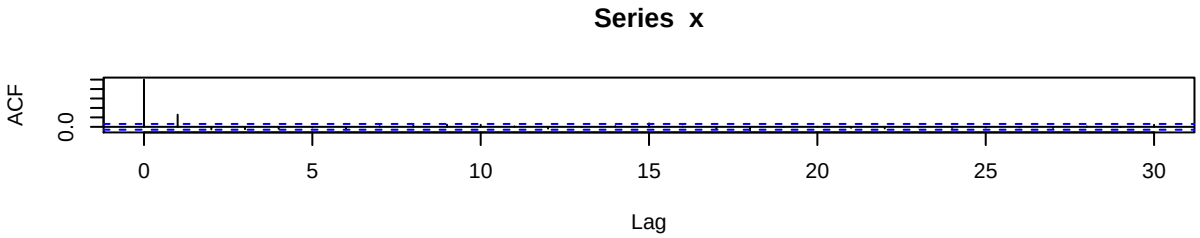


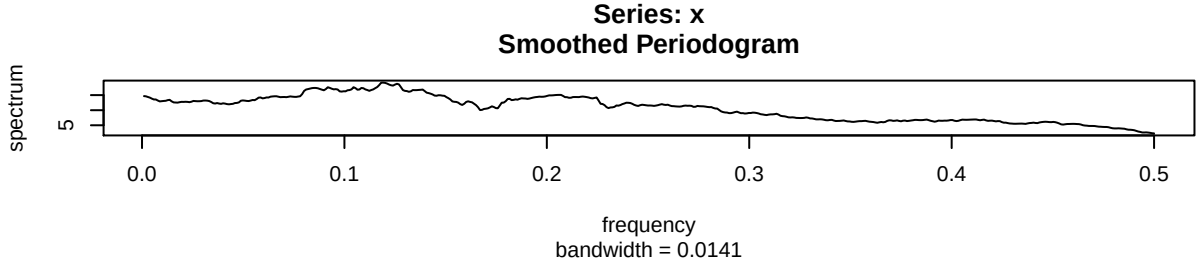





```r
# beta = 3
for (t in 2:1024) x[t]<- 3 * w[t-1] + w[t]
layout(1:3)
plot(as.ts(x))
acf(x)
spectrum(x, span = 51, log = c("no"))
```


as.ts(x)
10
−10
0
200
400
600
800
1000
Time


Series x
ACF
0.0
0
5
10
15
20
25
30
Lag


Series: x
Smoothed Periodogram
spectrum
5
0.0
0.1
0.2
0.3
0.4
0.5
frequency
bandwidth = 0.0141