



## Chapter 5: Bayesian Inference

## Chapter 5

# Bayesian Inference

Bayesian inference techniques specify how one should update one's beliefs upon observing data.





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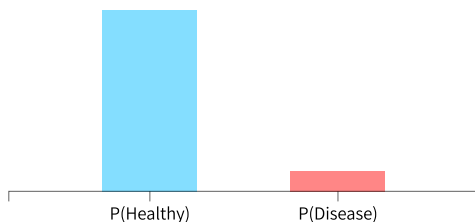
### Bayes' Theorem

Suppose that on your most recent visit to the doctor's office, you decide to get tested for a rare disease. If you are unlucky enough to receive a positive result, the logical next question is, "Given the test result, what is the probability that I actually have this disease?" (Medical tests are, after all, not perfectly accurate.)

Bayes' Theorem tells us exactly how to compute this probability:

$$P(\text{Disease}|+) = \frac{P(+|\text{Disease})P(\text{Disease})}{P(+)}$$

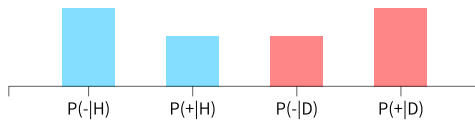
As the equation indicates, the *posterior* probability of having the disease given that the test was positive depends on the *prior* probability of the disease  $P(\text{Disease})$ . Think of this as the incidence of the disease in the general population. Set this probability by dragging the bars below.



The posterior probability also depends on the test accuracy: How often does the test correctly report a negative result for a healthy patient, and how often does it report a positive result for someone with the



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Finally, we need to know the overall probability of a positive result. Use the buttons below to simulate running the test on a representative sample from the population.

Test one patient

Test Remaining

Negative	Positive
0.70	0.30

We now have everything we need to determine the posterior probability that you have the disease. The table below gives this probability among others using Bayes' Theorem.

	Negative	Positive
Healthy	0.96	0.75
Disease	0.04	0.25

Sort

Reset

## Likelihood Function

In statistics, the *likelihood function* has a very precise definition:



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-- select a distribution --

Choose a sample size  $n$  and sample once from your chosen distribution.

$$n = 1$$

Sample

Use the **purple** slider on the right to visualize the likelihood function.

### Prior to Posterior

At the core of Bayesian statistics is the idea that prior beliefs should be updated as new data is acquired. Consider a possibly biased coin that comes up heads with probability  $p$ . This purple slider determines the value of  $p$  (which would be unknown in practice).

$$p = 0.5$$

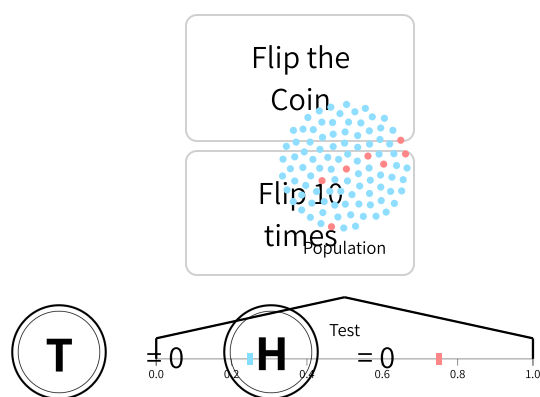
The pink sliders control the shape of the initial  $\text{Beta}(\alpha, \beta)$  prior distribution, the density function of which is also plotted in pink.

$$\alpha = 1$$



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our best guess about the likely values for the bias of the coin. This updated distribution then serves as the prior for future coin tosses.



Negative

Positive

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