# Measuring the Length of Coastlines

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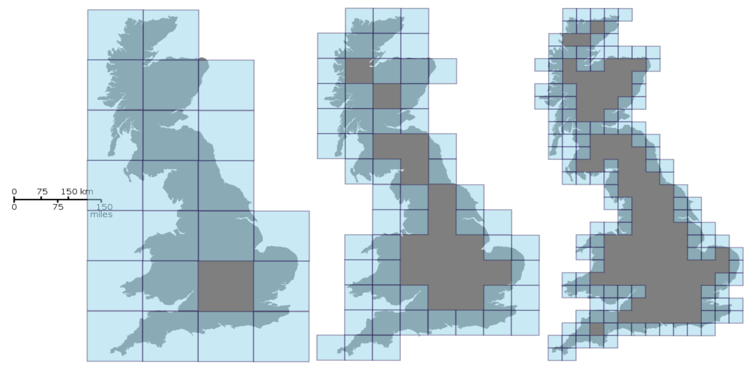
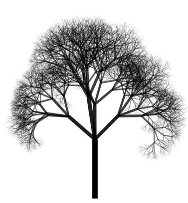


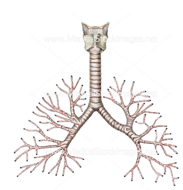
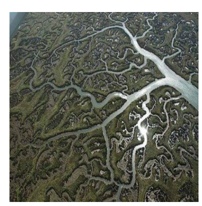

**Abstract**

Through the use of image processing, the Minkowski Box-counting method is experimentally proven by comparing the measured Dimension of the coastline of Great Britain and Australia to the correct value found and agreed upon by other researchers. The results were measured by using the amount of enclosed boxes as a measurement of approximation and comparing it to the percent error between the measured Dimension and the Theoretical Dimension.

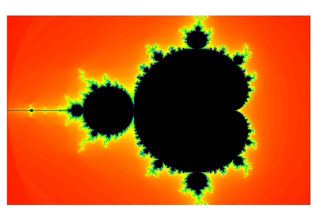
## 1 Introduction

A core differentiator between objects made from nature and objects made from man is level at which similar patterns repeat at different scales[1]. The fractal itself is a shape that exhibits repeating patterns at any scale from which you view it.

For example, the branches of a tree or structure of a snowflake exhibit branching patterns that keep repeating as you zoom in. The structure of a lightning bolt and the bronchi in human lungs also have this same pattern. Nature often creates chaotic phenomena like jagged coastlines, snowflakes, crystal formation, fluid turbulence, and galaxy formations. This is in contrast to man-made objects which are uniform and typically repeat the same pattern once.



However, a problem arises when trying to measure the lengths of fractal objects. When one zooms into a fractal, such as the Mandlebrot set, the patterns repeat themselves infinitely [2]. This means that an attempt to accurately measure the length of these shapes would result in an infinite perimeter which isn't useful.

Instead, fractals are measured by their roughness, a non-integer dimension[1]. This can be calculated using the Minkowski Box counting method: a mathematical approximation of the dimension using boxes at different scales and logarithms.

$$Dimension = \frac{log(Boxes)}{log(Scale)} = \frac{logN^n}{logN} = \frac{nlogN}{logN} = n \tag{1}$$

# 2 Experiment

## 2.1 Hypothesis

The dimension of objects in nature, such as large islands, can be accurately measured through the use of the Minkowski-Bouligand Box counting method with a computer program.

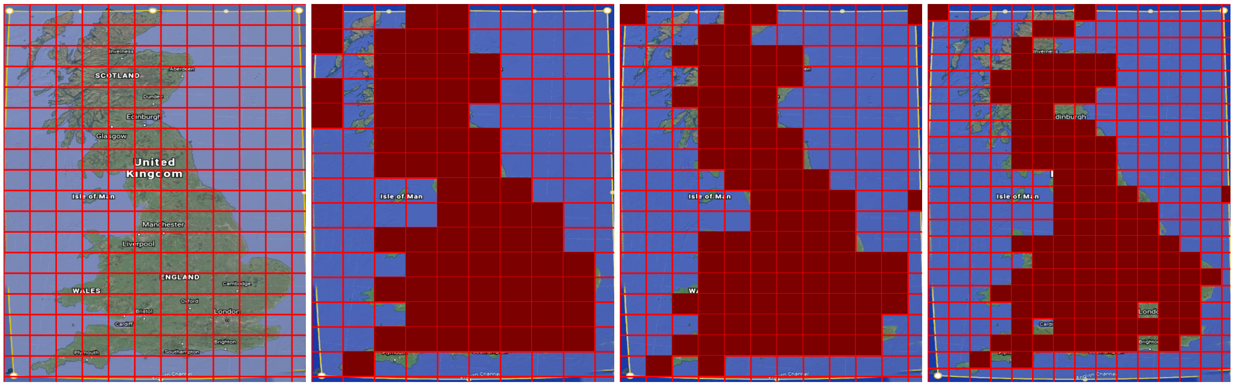
## 2.2 Materials

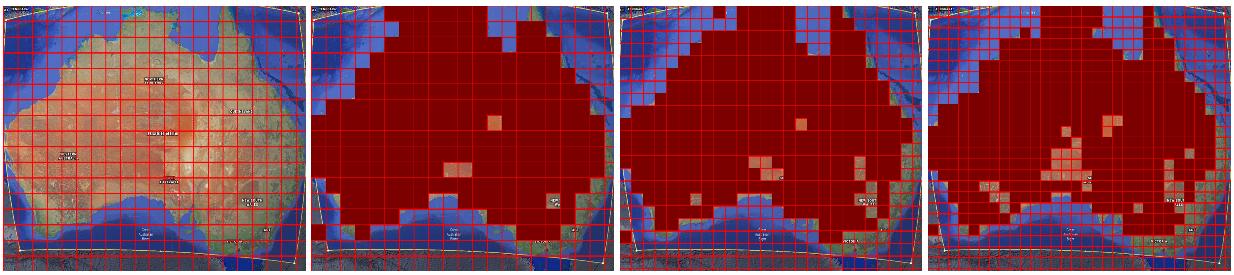
- Access to the Internet
- Google Earth
- Python and its OpenCV Libraries

## 2.3 Procedure

An image of a continent or country is downloaded from Google Earth or Google Maps. The two following images of Great Britain and Australia are used


A Python program then draws a grid, with each square being of a certain size on the image, and then counts and displays the number of squares that bound the landmass. An accurate length of one square is then measured using Google Earth. Then, the amount of squares that enclose the landmass is recorded by the program.





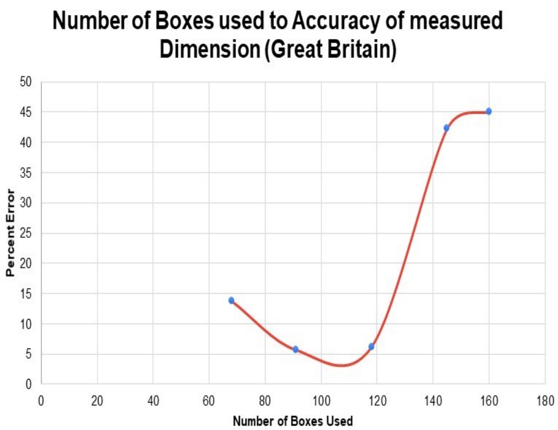
A spreadsheet then takes the logarithm of the length of each square and divides it by the logarithm of the box count to obtain the empirical dimension. This is repeated with 5 scales of different box lengths.

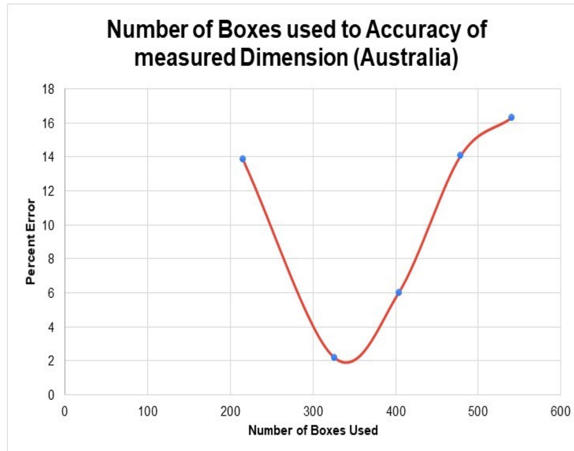
$$Dimension = \frac{log(Boxes)}{log(Scale)} \tag{2}$$

```
box_size = 25

threshold = 95

enclose_count = 0
for h in range(int(image_height+1 / (box_size))):
    for w in range(int(image_width / box_size+1)):
        cv2.rectangle(image, (box_size*w, box_size*h), (box_size + box_size*w, 1+box_size + box_size*h), color=(0,0,255), thickness=2)
        #We use the green channel as it shows the highest contrast from ocean(blue) to islands(green)
        selection = image[box_size*h:1+box_size + box_size*h,box_size*w:box_size + box_size*w, 1]
        blue_mean = np.mean(selection)
        if(blue_mean >= threshold):
            cv2.rectangle(image, (box_size*w, box_size*h), (box_size + box_size*w, 1+box_size + box_size*h), color=(0,0,125), thickness=-1)
            enclose_count +=1
```

Listing 1: Main Drawing Code:

# 3 Analysis of Results




At first, the results show an inverse relationship between the number of boxes used to accurately measure the dimension of the coastline, and the percent error between the theoretical and experimental dimension calculated. Then, after a certain amount of boxes used, the error is minimized and then increases with a positive relationship.

Table 1: Collected Data

| | **Scale:** | **Box Length (km) :** | **Amount of Boxes:** | **Calculated Dimension:** | **Expected:** | **% Error** |
|---|---|---|---|---|---|---|
| Great Britain | 1 | 57 | 68 | 1.043644373 | 1.21 | 13.74839893 |
| Great Britain | 2 | 52 | 91 | 1.141630289 | 1.21 | 5.650389377 |
| Great Britain | 3 | 41 | 118 | 1.284661921 | 1.21 | 6.170406683 |
| Great Britain | 4 | 18 | 145 | 1.721831709 | 1.21 | 42.30014121 |
| Great Britain | 5 | 18 | 160 | 1.755889636 | 1.21 | 45.11484599 |
| Australia | 1 | 250 | 216 | 0.973524668 | 1.13 | 13.8473745 |
| Australia | 2 | 150 | 326 | 1.154922887 | 1.13 | 2.205565253 |
| Australia | 3 | 150 | 404 | 1.197735322 | 1.13 | 5.994276293 |
| Australia | 4 | 120 | 479 | 1.289130286 | 1.13 | 14.08232615 |
| Australia | 5 | 120 | 541 | 1.314554597 | 1.13 | 16.33226523 |

## 3.1 Discussion

The number of boxes used is generally inversely proportional to accuracy up to a certain point: the point at which the Minkowski Box Counting formula converges to a single dimension. For the Great Britain graph, this occurs at around 110 boxes, 40-50 kilometers in length, and for the Australia graph at around 340 boxes, 150 kilometers in length.

## 3.2 Sources of Error

Possible sources of error include inaccuracy in measuring the length of the boxes on Google Earth or a miscalculation in counting the number of boxes used. The program used to automate the calculation process uses a rudimentary image processing technique of comparing the average amount of green pixels to blue pixels which often leaves out a few unwanted boxes. The images themselves were also screenshots of Google Earth that that included white titles for each country that also likely added to the inaccuracy.

### 3.3 Conclusion

It's a strange to think about certain objects in nature, that can be physically held in the three dimensions of space, yet at the same time exist in a fractional dimension themselves. Through the experiment, the Minkowski-Bouligand Box counting method was proven experimentally with the hypothesis which further proves how Fractal dimension remains a powerful measurement.

# 4 Applications of Results:

The most useful part of Fractal dimension is that is allows for the measurement of roughness. Roughness is an important natural measurement that can be used to compare the trees of different ecological systems, the health of bronchi in a patient's lungs, or the quality of snowflakes affected by pollution in a region.[3]

Imagine a candy company owner wants to test out the taste of a new candy bar in comparison to his competitors' products. A study could be set up using unlabeled candy bars given to participants, and a hypothesis could be formed of how the smoothness of a candy bar affects its taste. Fractal dimension would be used to compare the roughness of the different candy bars and see which one holds the better taste.

Perhaps an ecologist wants to compare the trees in the African Savanna to the trees in the Amazon Forest. Trees, are by nature fractal as they infinitely repeat their branching patterns, and the roughness of different trees could reveal important qualities about the environment from which they're grown. If the trees in the Amazon were found to have more roughness than the trees in the African Savanna, then that would shed light on the effects of certain types of pollution in those areas.

The possibilities of application are endless, and all it takes is a creative, observant mind to benefit the world using this measurement tool.

# References


[1] Grant Sanderson, 3Blue1Brown *Fractals are typically not self-similar* [Video]. YouTube. https://www.youtube.com/watch?v=gB9n2gHsHN4, 2017, January 27

[2] Yuliya Klochan, MITK12Videos *What Is A Fractal (and what are they good for)?* [Video]. YouTube. https://www.youtube.com/watch?v=WFtTdf3I6Ug

[3] Fractal Foundation *Fractal Applications* [Article]. http://fractalfoundation.org/OFC/OFC-12-1.html