Quiz 01

$\{-3, -2, -1, 0, 1, 2\}$

Question 01 [2 + 1 = 3 Marks]

Consider the following number system, $\beta = 2,\ m = 5\ and\ -3 \le e \le 2$ and answer the following questions.

a. What is the *maximum non-negative* number that can be represented in this number system? Provide your answer in the original format.

Standard Format	*Normalized Format*
$+(0.11111)_2 \times 2^2$	$+(1.11111)_2 \times 2^2$

b. How many different numbers can be represented through this system when using the denormalized convention?

$$\underset{2}{\pm}\,(0.1\underbrace{\_\ \_\ \_\ \_\ \_}_{2^5})_2 \times 2^{\underset{6}{[\ ]}}$$

$2 \times 2^5 \times 6$

Question 02 [1 + 2 = 3 Marks]

Consider the following quadratic equation, $x^2 - 70x + 9 = 0$. Below calculate up to 5 significant figures.

a. Find the actual roots of the quadratic equations, using the quadratic formula.

$$x = \frac{-(-70) \pm \sqrt{70^2 - 4.1.9}}{2 \times 1} = 35 \pm 8\sqrt{19}$$

$\alpha, \beta = 35 \pm 8\sqrt{19}$

$\boxed{\alpha = 69.871 \quad \beta = 0.12881}$ upto 5 sf

b. Show if the roots evaluated in the previous part satisfies the solution if we consider S.F. = 5

According to the polynomial of degree 2,

we know,

$$\alpha \times \beta = \frac{c}{a} = \frac{9}{1} = 9$$

But, $\alpha \times \beta = 9.0001 \neq 9$

so there exists a loss of significance.

Quiz 02

QUESTION 01

Consider a function, $f(x) = 2\sqrt{3}\,sin(x)$ and you want to interpolate the function with the nodes $x_i = \{0.3,\ \pi/2,\ 2.45\}$.

a. Using the given nodes, determine the degree of the interpolating polynomial using a matrix method and write its general structure accordingly. [1+1 = 2]

degree, n = 2

$P_n(x) = a_0 + a_1 x + a_2 x^2$

b. Consider if the degree was 92, what would be the dimensions of the matrix? [1]

(93 × 93)

c. Find the polynomial, $P_n(x)$. [2]

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 0.3 & 0.3^2 \\ 1 & \pi/2 & (\pi/2)^2 \\ 1 & 2.45 & 2.45^2 \end{bmatrix}^{-1} \begin{bmatrix} 1.02371 \\ 3.4641 \\ 2.20928 \end{bmatrix}$$

$a_0 = -0.286126$

$a_1 = 4.83322$

$a_2 = -1.55702$

QUESTION 02

Consider the same function $f(x)$ and nodes x_i.

$$f(x) = 2\sqrt{3}\,sin(x)\ \&\ x_i = \{0.3,\ \pi/2,\ 3\}$$

Now you want to improve the previous method in terms of time complexity.

a. What would be the Method of Polynomial Interpolation? Why? [1]

Lagrange or Newton's devided diff. $O(n^2) < O(n^3)$

choose any

b. Define the polynomial and calculate it accordingly. [4]

degree 02

x	$f(x)$
0.3	1.0237
$\pi/2$	$2\sqrt{3}$
3	0.48885

$$l_0(x) = \frac{x - \pi/2}{0.3 - \pi/2} \times \frac{x - 3}{0.3 - 3}$$

$$P_2(x) = 1.0237\, l_0(x) + 2\sqrt{3}\ l_1(x) + 0.48885\, l_2(x)$$

$$l_1(x) = \frac{x - 0.3}{\frac{\pi}{2} - 0.3} \times \frac{x - 3}{\frac{\pi}{2} - 3}$$

$$l_2(x) = \frac{x - 0.3}{3 - 0.3} \times \frac{x - \pi/2}{3 - \pi/2}$$

Quiz 03

1.1 If a Hermite interpolating polynomial is constructed using x distinct points with both function and derivative values given, what is the maximum possible degree of the polynomial?

node → x

So, $2x-1$

A) x B) 2x - 1 C) 2x D) x+1

1.2 What is the main advantage of Hermite interpolation over Lagrange interpolation?

A) Hermite interpolation uses a lower-degree polynomial.

B) Hermite interpolation provides better approximation for differentiable functions.

C) Hermite interpolation does not require divided differences.

D) Lagrange interpolation is always more accurate.

1.3 The divided difference method used in Newton interpolation is extended in Hermite interpolation by incorporating which additional values?

A) Function values at additional points B) Higher-order derivatives

C) Tangent line equations D) Lagrange basis polynomials

Question 02:

	x_0	x_1
x	1	2
f(x)	2	3
f'(x)	1	2

Calculate the hermite basis: $h_1(x)$ (3 Marks)

$$l_1(x) = \frac{x-1}{2-1} = x-1$$

$$h_1(x) = \{1-2(x-x_1)\,l_1'(x_1)\}\,l^2(x)$$

$$l_1'(x) = 1$$

$$= \{1-2(x-1)\times 1\}(x-1)^2$$

$$l_1'(1) = 1$$

$$= \{1-2(x-1)\}(x-1)^2$$

Question 03: Consider the following dataset:

	0.2	0.2	
x	2.0	2.2	2.4
f(x)	1.6212	1.9800	2.5349

(a) (3 marks) Using the above data, compute $f'(2.2)$ using the central difference method.

$h = 0.2$

(b) (1 Marks) How would the truncation error react if I increased the value of h?

(a) $$f'(2.2) = \frac{f(\overset{2.4}{\cancel{2.2+h}}) - f(\overset{2.0}{\cancel{2.2-h}})}{2\underset{0.2}{\cancel{h}}} = \frac{2.5349 - 1.6212}{2\times 0.2}$$

$$= 2.2842$$

(b) As TE↑ h↑ → TE↑