CHAPTER II || Polynomial Interpolation || Part 2

Lagrange form

	x	$f(x)$
0	2	30
1	5	40
2	9	20

⇒ for this case the polynomial will have degree

#of node − 1

= 3 − 1

= 2

∴ So, the polynomial would be like

requires n^3 time to solve

$$P_2(x) = a_0 + a_1 x + a_2 x^2 \rightarrow \boxed{\text{vandermonende matrix}}$$

Instead of the $\{1, x, x^2\}$ basis now we take a different basis to solve this issue.

Let's define $P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$

$$\text{Lagrange Basis} = \{ l_0(x),\ l_1(x),\ l_2(x) \}$$

Now, how to calculate these basis.

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2}$$

0 skip
1
2

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2}$$

0
1 skip
2

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1}$$

0
1

$x_2 - x_1$ (2) skip

Kronecker Delta

x	$f(x)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$

$$\rightarrow P_2(x) = l_0(x)\, P(x_0) + l_1(x)\, P(x_1) + l_2(x)\, f(x_2)$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2} \quad \begin{cases} x_0 = 1 \\ x_1 = 0 \\ x_2 = 0 \end{cases}$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_2 - x_2} \quad \begin{cases} x_0 = 0 \\ x_1 = 1 \\ x_2 = 0 \end{cases}$$

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_0 - x_1} \quad \begin{cases} x_0 = 0 \\ x_1 = 0 \\ x_2 = 1 \end{cases}$$

Generalizing this, $l_i(x_j) = \begin{cases} 0, \text{ when } i \neq j \\ 1, \text{ when } i = j \end{cases}$

$$\Rightarrow l_i(x_j) = \delta_{ij}$$

$\downarrow$

Kronecker Delta

Practice Problem

	x	$f(x)$
0	-1	2.2
1	0	10.6
2	1	17.0
3	2	[illegible]

use both Lagrange Polynomial and Vandermonde Matrix to get the appropriate polynomial and find the value at point $x =$ [illegible]

VM

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1^2 & -1^3 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1^2 & 1^3 \\ 1 & 2 & 2^2 & 2^3 \end{bmatrix}^{-1} \begin{bmatrix} 2.2 \\ 10.6 \\ 17 \\ 22.4 \end{bmatrix} = \begin{bmatrix} 10.6 \\ 7.2333 \\ -1 \\ 0.1666 \end{bmatrix}$$

$$P_3(x) = 10.6 + 7.233x - x^2 + 0.1666x^3$$

$$P_3(2.5) = 25.0375$$

Lagrange

$$P_3(x) = l_0(x) f(x_0) + \dots + l_3(x) f(x_3)$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2} \times \frac{x - x_2}{x_0 - x_2}$$

$$= \frac{1}{-1 \times -2 \times -3} (x)(x-1)(x-2)$$

$$= \frac{1}{-6} (x)(x-1)(x-2)$$

$$l_1(x) =$$