## Richardson Eutrapolation

in central difference differenceation 
$$f'(x_i) = \frac{f(y_i + y_i) - f(y_i - y_i)}{2y} = D_h i$$

(error | with (n.) kut Rounding error 1)

$$f(x_1-x_1) = f(x_1) - f_{(1)}(x_1)x_1 + \frac{f_{(2)}(x_1)}{2}x_2 + \frac{f_{(3)}(x_1)}{3}x_3 + \frac{f_{(4)}(x_1)}{4!}x_4 + o(x_2)$$

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$$D_{N} = \frac{1}{2h} \left[ 2 f(n_{1})h + 2 \frac{f^{(3)}(n_{1})}{3!} n^{3} + 2 \frac{f^{(3)}(n_{1})}{5!} n^{5} + O(n^{3}) \right]$$

$$D_{\mathbf{h}} = \mathbf{f}^{(1)}(\mathbf{h}_{1}) + \frac{\mathbf{f}^{(3)}(\mathbf{h}_{1})}{3!} \mathbf{h}^{2} + \frac{\mathbf{f}^{(3)}(\mathbf{h}_{1})}{5!} \mathbf{h}^{4} + \delta(\mathbf{h}^{6})$$

$$D_{\mathbf{h}_{2}} = \mathbf{f}^{(1)}(\mathbf{h}_{1}) + \frac{\mathbf{f}^{(3)}(\mathbf{h}_{1})}{3!} \left(\frac{\mathbf{h}_{1}}{2}\right) + \frac{\mathbf{f}^{(3)}(\mathbf{h}_{1})}{5!} \left(\frac{\mathbf{h}_{1}}{2}\right)^{4} + \delta(\mathbf{h}^{6})$$

$$\approx O(\mathbf{h}^{6})$$

$$\sum_{i} D_{i} - D_{i} = (2^{n} - 1) + (\frac{1}{2^{n}} - 1) + (\frac{1}{2^$$

$$= \frac{2^{4}D_{W_{1}} - D_{1}}{(2^{4} - 1)} = f^{(1)}(x_{1}) + \frac{(\frac{1}{2}x_{1} - 1)}{(2^{4} - 1)} + \frac{f^{(6)}(n_{2})}{5!} + \frac{f$$

 $D_n^{(1)} \notin D_{n/2}^{(1)}$  Taken again so that the next  $D_n^{(1)}$  becomes

**Question # 1**: A function is given by  $f\left(x\right)=6e^{-3x}$ . Now Answer the following:

- 1. [1 Mark] Calculate  $f'\left(x\right)$  at x=0.5 with h=0.32 using the central difference formula.
- 2. [1 Mark] Calculate  $f'\left(x\right)$  at x=0.5 with h=0.16 using the central difference formula.
- 3. [3 Marks] Now compute  $D_{0.32}^{(1)}$  at x=0.5 using Richardson extrapolation method. -4.063
- 4. [2 Marks] If the exact value of the derivative,  $f'\left(0.5\right)$  is 1/23288, find the percentage error with extrapolated value found in the previous part.

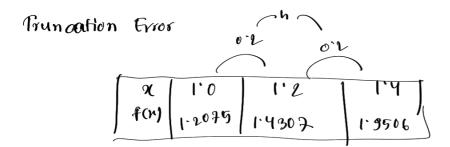
CDD, 
$$f(x_0 + x_1) - f(x_0 - x_1)$$

$$\frac{f'(0.2)^{N=0.35}}{f(0.2+0.35)-f(0.2-0.35)} = -4.6651$$

$$\frac{0.21520 - 3.4064}{5.000} = -4.6651$$

$$= \frac{6.82841 - 2.1636}{2\times0.16} \approx \frac{f(0.2 + 16) - f(0.2 - 0.16)}{2\times0.16}$$

$$\int_{0.32}^{(1)} = \frac{2^2 D_{0.16} - D_{0.32}}{2^2 - 1} = \frac{4(-4.1551) - (-4.6651)}{4 - 1}$$



@ Compute of (12) using central difference methor.

(D); 
$$f(11) \approx \frac{4(11+0.5)-4(1.5-0.5)}{5\times0.5}$$

$$\times 0.5$$