

Demonstrați că $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

Știm: $\forall A, B \in X$

- (1) $\overline{cA} = c\overset{\circ}{A}$
- (2) $c(A \cap B) = cA \cup cB$
- (3) $\overline{c(A \cap B)} = \overset{\circ}{cA} \cap \overset{\circ}{cB}$

Fie $Y, Z \subseteq X$.

Fie $A = cY, Z = cB$.

Demonstrăm că $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

$$\begin{aligned} A \cup B &= cY \cup cZ \stackrel{(2)}{=} c(Y \cap Z) \implies \overline{A \cup B} = \overline{c(Y \cap Z)} \stackrel{(1)}{=} c(\overline{Y \cap Z}) \stackrel{(3)}{=} c(\overset{\circ}{Y} \cap \overset{\circ}{Z}) \stackrel{(2)}{=} c\overset{\circ}{Y} \cup c\overset{\circ}{Z} = \\ &= \overline{cY} \cup \overline{cZ} = \overline{A} \cup \overline{B}. \end{aligned}$$

q.e.d.