Gut

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi} \quad \text{(Gauß)}$$

$$\Gamma(p) = \int_0^{\infty} x^{p-1}e^{-x}dx = (p+1)!, \quad p > 0, \quad \Gamma(\frac{1}{2}) = \sqrt{(\pi)}$$

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}, \quad p > 0, q > 0$$

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin(p\pi)}, \quad p \in (0,1)$$

Coord polare: $f(x,y) = \rho f(\rho\cos\theta, \rho\sin\theta)$
Coord polare generalizate: $f(x,y) = ab\rho f(a\rho\cos\theta, b\rho\sin\theta)$
Coord cilindrice: $f(x,y,z) = \rho f(\rho\cos\theta, \rho\sin\theta)$,
Sferice: $f(x,y,z) = \rho^2 \sin\varphi f(\rho\sin\theta\cos\varphi, \rho\sin\theta\sin\varphi, \rho\cos\theta), \quad \theta \in [0,\pi], \varphi \in [0, 2\pi]$

Integrale curbilinii

De speta I

$$\int_\gamma f(x,y,z)ds = \int_a^b f(x(t),y(t),z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}dt$$

De speta II

$$\int_\gamma \overline{F}d\bar{r} = \int_\gamma P(x,y)dx + Q(x,y)dy = \int_a^b \left[P(x(t),y(t))x'(t) + Q(x(t),y(t))y'(t)\right]dt$$

$$\int_\gamma \frac{\partial P}{\partial x}dx + \frac{\partial Q}{\partial y}dy = f(B) - f(A), \quad \text{unde } A, B \text{ extremitatile curbei (sn forma dif exacta)}$$

Integrale duble

Masura Jordan în plan

Fie $E = \bigcup_i D_i, \quad D_i$ dreptunghiuri disjuncte 2 cate 2.
E sn multime elementara
$\sigma(E) = \sum_i \sigma(D_i), \quad \sigma-$ aria
$\sigma^{\leq}(M) = \sup\{\sigma(E), E \subset M, E - \text{elementara}\}$
$\sigma^{\geq}(M) = \inf\{\sigma(E), E \supset M, E - \text{elementara}\}$
Daca $\sigma^{\leq}(M) = \sigma^{\geq}(M)$ at M este Masurabila Jordan.
Th (caracterizare): O multime $M \subset \mathbb{R}^2$ este măsurabilă Jordan $\iff$ frontiera sa este J-neglijabilă
Prop: $\sigma(D_1 \cup D_2) = \sigma(D_1) + \sigma(D_2) - \sigma(D_1 \cap D_2)$

Simplu in raport cu Oy

$a \leq x \leq b, \quad \alpha(x) \leq y \leq \beta(x), \quad \alpha, \beta$ continue pe $[a,b]$
Analog cu simplu in raport cu Oy
Daca $\gamma : [a,b] \to \mathbb{R}^2$ rectificabil, at $\gamma([a,b])$ J-neglijabilă
Multimi Jordan nemasurabile - fractals and squiggly stuff

Integrale duble

Def: luam diviziune, puncte si $\lim\limits_{\|\Delta \to 0\|} s(f;\Delta,P) = \sum\limits_{i\geq 0} f(P_i)\sigma(D_i) = \iint_D f(M)d\sigma = \iint_D f(x,y)dxdy$
Proprietati ca la integrale Riemann

Schimbare de variabila

$$T : \begin{cases} x = \varphi(u,v) \\ y = \psi(u,v), \end{cases} \quad (u,v) \in \Omega$$

Transformarea $T : \Omega' \to \Omega$ este *regulata* daca $\varphi, \psi \in C^1(\Omega')$, T biunivoca, $J = \frac{D(\varphi,\psi)}{D(u,v)} \neq 0, \quad \forall (u,v) \in \Omega$
Def: f sn *admisibila* daca este marginita si continua cu exceptia unei multimi J-neglijabile
Th: f admisibila, T regulata, $D^* = T^{-1}(D)$ $\displaystyle\iint_D f(x,y)dxdy = \iint_{D^*} f(\varphi(u,v), \psi(u,v))|J|dudv$
def : D domenu standard (poate fi descompus in reuniune finita de dom simple in raport cu ambele axe)
Orientare pozitiva: frontiera este lastata in stanga
Formula Riemann-Green: D domenu standard inchis $P, Q \in C^1(D)$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) = \int_{\mathrm{Fr}D} Pdx + Qdy, \quad \mathrm{Fr}D \text{ orientata pozitiv}$$

Integrale triple

Masura Jordan in spatiu

Same spiel as in 2d, da cu paralelipipede dreptunghice paralele cu axele.

Domenii simple

Feliuțe; intre 2 plane: $\displaystyle\iiint_V f(x,y,z)dv = \int_c^d \left(\iint_{D_z} f(x,y,z)dxdy \right) dz$
Bețe; intre 2 suprafete: $\displaystyle\iiint_V f(x,y,z)dv = \iint_D \left(\int_{\varphi_1(x,y)}^{\varphi_2(x,y)} f(x,y,z)dz \right) dxdy$

Schimbarea de variabila

$T : x = \varphi(u,v,w), y = \psi(u,v,w), z = \chi(u,v,w), cu \varphi, \psi, \chi \in C^1$, biunivoce si cu $J \neq 0$,

$$\iiint_V f(x,y,z)dv = \iiint_{V'} f(\varphi(u,v,w), \psi(u,v,w), \chi(u,v,w)) \left| \frac{D(\varphi,\psi,\chi)}{D(u,v,w)} \right| dv'$$

Integrale de suprafață

ec explicita: $z = f(x,y)$
ec implicita: $F(x,y,z) = 0$
ec param: $x = \varphi(u,v) \ldots$

suprafete explicite (aproximam cu diferentiala)

$$d\sigma = \sqrt{p^2 + q^2 + 1}dxdy, \quad p = \frac{\partial f}{\partial x}, q = \frac{\partial f}{\partial y}$$

suprafete param

Functiile de clasa C^1, Matricea jacobiana are rang maximal 2 in orice pct, reprezentarea e biunivoca

$$d\sigma = \sqrt{A^2 + B^2 + C^2}dudv = \sqrt{EG - F^2}dudv$$
$$A = \frac{D(\psi,\chi)}{D(u,v)}, B = \frac{D(\chi,\varphi)}{D(u,v)}, C = \frac{D(\varphi,\psi)}{D(u,v)}$$
$$a = \left(\frac{\partial \varphi}{\partial u}, \frac{\partial \psi}{\partial u}, \frac{\partial \chi}{\partial u} \right), b = \left(\frac{\partial \varphi}{\partial v}, \frac{\partial \psi}{\partial v}, \frac{\partial \chi}{\partial v} \right) \quad E = \langle a,a \rangle, F = \langle a,b \rangle, G = \langle b,b \rangle$$

speta I

def - same spiel as the previous.

$$\iint_\Sigma f(x,y,z)d\sigma = \iint_D f(x(u,v), y(u,v), z(u,v))\sqrt{EG - F^2}dudv$$

speta II

$$\iint_\Sigma \bar{v}\cdot\bar{n}d\sigma, \quad \bar{n} = \text{versorul normalei la suprafață}$$

$$\iint_\Sigma Pdydx + Qdzdx + Rdxdy = \iint_\Sigma Pn_x + Qn_y + Rn_z d\sigma, \quad \bar{n} = (n_x, n_y, n_z)$$

Stokes, Fr = bord, Σ regulata

$$\iint_\Sigma \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy + \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)dydz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)dzdx = \int_{\mathrm{Fr}\Sigma} Pdx + Qdy + Rdz$$

Gauß-Остроградський - domenii simple in raport cu toate axele

$$\iint_{\mathrm{Fr}\Sigma} Pdydz + Qdzdx + Rdxdy = \iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)dxdydz$$

Teoria Campurilor

$$\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$$

$$\nabla\cdot\nabla = \nabla^2 = \Delta = \mathrm{div}(\nabla) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\mathrm{grad}\varphi = \nabla\varphi = (\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z})$$

$$\bar{v}(x,y,z) = (P(x,y,z), Q(x,y,z), R(x,y,z))$$

$$\mathrm{div}_a\bar{v} = \nabla\cdot\bar{v}(a) = \frac{\partial P}{\partial x}(a) + \frac{\partial Q}{\partial y}(a) + \frac{\partial R}{\partial z}(a)$$

$$\mathrm{rot}_a\bar{v} = \nabla\times\bar{v}(a) = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

Def: $\bar{v}$ sn *camp de gradienti* în D daca $\exists\,\varphi \in C^1(D)$ cu $\bar{v} = \mathrm{grad}\varphi$

proprietati

$\nabla c = 0$
$\nabla\cdot\bar{c} = 0$
$\nabla\times\bar{c} = \bar{0}$
Improvisable:
$\mathrm{div}_a(\alpha\bar{v} + \beta\bar{w}) = \alpha\mathrm{div}_a(\bar{v}) + \beta\mathrm{div}_a(\bar{w})$
$\mathrm{rot}_a(\alpha\bar{v} + \beta\bar{w}) = \alpha\mathrm{rot}_a(\bar{v}) + \beta\mathrm{rot}_a(\bar{w})$
$\mathrm{div}_a(\varphi\bar{v}) = \varphi(a)\mathrm{div}_a(\bar{v}) + \bar{v}(a)\cdot\mathrm{grad}_a\varphi$
$\mathrm{rot}_a(\varphi\bar{v}) = \varphi(a)\mathrm{rot}_a(\bar{v}) - \bar{v}(a)\times\mathrm{grad}_a\varphi$
$\mathrm{div}(\bar{v}\times\bar{w}) = \bar{w}\cdot\mathrm{rot}\bar{v} - \bar{v}\cdot\mathrm{rot}\bar{w}$
$\mathrm{div}(\bar{c}\times\bar{r}) = \bar{r}\cdot\mathrm{rot}\bar{c} - \bar{c}\cdot\mathrm{rot}\bar{r}, \quad \bar{r} = \text{vector de pozitie}$

aplicatii

Rieman-Green

$$\int_{\mathrm{Fr}D} \bar{v}\cdot d\bar{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy, \quad \bar{v} = (P(x,y), Q(x,y))$$

Stokes

$$\int_{\mathrm{Fr}S} \bar{v}\cdot d\bar{r} = \iint_S \mathrm{rot}\bar{v}\cdot\bar{N}d\sigma$$

Gauß-Остроградський

$$\iiint_V (\mathrm{div}\bar{v})dxdydz = \iint_{\mathrm{Fr}V} (\bar{v}\cdot\bar{n})d\sigma$$