might be useful

$$\frac{d}{dx}\left(\int_0^x f(x,y)\,dy\right) = f\big(x,x\big) + \int_0^x \frac{\partial}{\partial x} f(x,y)\,dy$$

care e obtinuta din formula Leibniz:

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f\big(x,b(x)\big)\cdot\frac{d}{dx}b(x) - f\big(x,a(x)\big)\cdot\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt$$

- laplace operator (laplacian): $\Delta = \nabla^2 = \nabla\cdot\nabla = \sum_i \frac{\partial^2}{\partial x_i^2}$
- field therory shit: (curl=rotor)
  div grad $f \equiv \nabla\cdot\nabla f \equiv \nabla^2 f$
  curl grad $f \equiv \nabla\times\nabla f = \mathbf{0}$
  div curl $\mathbf{A} \equiv \nabla\cdot(\nabla\times\mathbf{A}) = 0$
  curl curl $\mathbf{A} \equiv \nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$
  $\nabla^2(fg) = f\nabla^2 g + 2\nabla f\cdot\nabla g + g\nabla^2 f$
- Gauß-Остроградский:
  $$\int_\Omega \frac{\partial u}{\partial x_i} = \int_{\partial\Omega} u\cdot\nu_i d\sigma$$
  also:
  $$\int_\Omega \operatorname{div} F(x)dx = \int_{\partial\Omega} F\cdot\nu d\sigma$$
- Green formula:
  $$\int_\Omega (\nabla u\cdot\nabla v + v\Delta u)dx = \int_{\partial\Omega} v\sum_i \frac{\partial u}{\partial x_i}\nu_i d\sigma$$
- Green formula 1:
  $$-\int_\Omega \Delta u v dx = \int_\Omega \nabla u\cdot\nabla v dx - \int_{\partial\Omega}\frac{\partial u}{\partial\nu} v d\sigma$$
- Green formula 2:
  $$\int_\Omega (\Delta u v u \Delta v)dx = \int_{\partial\Omega}\frac{\partial u}{\partial\nu}v - u\frac{\partial v}{\partial\nu}d\sigma$$
- convoluție:
  $$f * g(x) = \int_{\mathbb{R}^d} f(x-y)g(y)dy$$

misc

- proiectie pe subspatii inchise:
  $$\exists! Pu \in V \text{ aî } \|Pu - u\| = \inf_{v\in V}\|v-u\|$$
- în plus $(u - Pu)\perp V$ (ie $\langle u - Pu, v\rangle = 0,\ \forall v\in V$)
- bessel inequality: $V = \operatorname{span}\{v_1,\ldots,v_n\}$
  $$\|u\|^2 \geq \|P_V u\|^2 = \sum_{j=1}^n \frac{|\langle u, v_j\rangle|^2}{\|v_j\|^2}$$

- daca are loc ineg parseval și $\{f_j\}$ ortongolală, at e bază Hilbertiană
- scalar product with functions $f : [a,b] \to \mathbb{C} \in L^2([a,b])$:

$$\langle f, g\rangle = \int_a^b f(x)\overline{g(x)}dx$$

- weak convergence:

$$u^n \rightharpoonup u \text{ dacă} \langle u^n - u, v\rangle \to 0, \forall v \in H$$

- subarmonica?

Sturm-Liouville - S1

valori proprii

- c3 - pg 5
- Melnig thing pg 7
- do the $| \cdot \varphi, \int$ to get $\lambda \geq 0$
- if $\varphi = 0$ we ignore that one. we dont want null solutions
- we get the characteristic equation[1] (polinom caracteristic:

$$\varphi''(x) + \lambda\varphi(x) = 0 \text{ becomes } r^2 + \lambda \cdot 1 = 0$$

- and we get the functions of form (may differ depending on the characteristic equation):

$$\{e^{r_i}, \ldots, x^m e^{r_i}\}$$

  or, for our example:

$$\sin(\sqrt{\lambda}x), \cos(\sqrt{\lambda}x)$$

  so

$$\varphi(x) = \alpha \sin(\sqrt{\lambda}x) + \beta \cos(\sqrt{\lambda}x)$$

- with the initial conditions: $\varphi(0) = \varphi(l) = 0$ we get some restrictions for α and β

and, tada, ya get some λ_k, φ_k

Green's function:

- for n-th order differential equations: see green-kurzgesagt

[1] https://en.wikipedia.org/wiki/Characteristic_equation_(calculus)

separation of variables

- see s6 - pg 2
- we have:

$$\begin{cases} -\Delta u = f, & \text{în } \Omega = (a,b) \times (c,d) \\ \text{some condition like } u = 0, & \text{pe } \partial\Omega \end{cases}$$

- we write stuff with respect to x:

$$\begin{cases} -\varphi'' = \lambda\varphi, & \text{în } \Omega = (a,b) \\ \text{some condition like } u(a) = u(b) = 0 \end{cases}$$

- and we get some eigen functions and values: $\{\varphi_k\}, \{\lambda_k\}$
- we write things with the new functions:

$$u(x,y) = \sum_k^{\infty} u_k(y)\varphi_k(x)$$

$$u_{xx} = ..., u_{yy} = ...$$

$$f(x,y) = \sum f_k(y)\varphi_k(x) = \sum \left(\int_a^b f(t,y)\varphi_k(t)dt \right) \varphi_k(x)$$

- then we solve it for some k

$$\begin{cases} -\Delta u_k(y) = f_k(y), & \text{în } (a,b) \\ \text{some condition like } u(a) = u(b) = 0 \end{cases}$$

  and we get

$$u_k(y) = \int_a^b G_k(y,s) f_k(s) ds$$

- sum things together and we get a $G\big((x,y),(t,s)\big)$

TODO test this

toc

course

- C1: basic shit
- C2:
  - basic shit (prod scalar and norm)
  - projections
  - besel inequality
- C3:
  - more besel
  - hilbert basis
  - problem with Green's function

- hilbert spaces examples

- C4:
  - proprietati Green's thing - pg 2
  - Riesz representation theorem - pg 5 (dual stuff)
  - autoadjunct daca $T = T^*$

- C5:
  - weak convergence
  - hilbert basis proprierties & stuff

- C6:
  - more weird abstract shit
  - sturm liouville in general form - pg 11

- C7:
  - differential subvariety stuff
  - green's formulas
  - convolutions
  - that weird Elementary thing

seminaries

S1

- tl;dr normal differential equations

$$\begin{cases} u_k'(t) + \lambda_k u_k(t) = f_k(t), t > 0 \\ u_k(0) = u_k^0 \end{cases}$$

$$u_k(t) = e^{-\lambda_k t} u_k^0 + \int_0^t \exp(-\lambda_k(t-s)) f_k(s) ds$$

- sturm-liouville stuff

S2, s3

- sturm-liouville and fourier exercises

s4:?

- met sep variabilelor pg 4

- fundamental solution pg 10

s5

- green shit

s6

- separation of variabiles for sturm-liouvile problems + green - pg 3

that old book

- green - pg 39

melnig thing

- 7 - val proprii
- 15 - parseval stuff

things to know

- sp Hilbert, serii Fourier, pb Sturm-Liouville
- separarea variabilelor (pb val proprii, hip, parab, eliptice - serii fourier
- fct Green (op laplace+ sturm liouville)
- pp maxim (op eliptici + aplicatii - unicitatea sol si estimari)
- formularea variationala a pb eliptice (si parab si hip) => sep variabilelor
- transformata fourier - calcul + cateva proprietati
- oral: he asks bout some theory bit

todo:

- do some fourier shit
- "sep variabilelor"?
- green shit for liouville and