problema:

$A = \{ y(x) \mid x \in \{ 2, -1, 1/2, 1 \} \}$

4) Aproximati $\int_{-1}^{2} y(x)dx$ cu o formula Gauss pe 3 noduri. Estimati restul. Se poate itera formula?

BTW, y de aici e y-ul aflat la ex 1
avem ponderea $p(x) = 1$

pentru ca iti cere $\displaystyle\int y(x)dx$ aka $\displaystyle\int y(x) \cdot 1dx$

daca cerea de ex $\displaystyle\int e^{-x} y(x)dx$, atunci ponderea era e^{-x}

si cum ponderea e 1, "o formula gauss" inseamna Gauss-Legendre
de ex daca era e^{-x} era gauss-laguerre

si acum ar trebui sa convertim functia de la una pe $[-1, 2]$ la una pe $[-1, 1]$ (practic e schimbare de variabila)
(Gauss-Legendre cere ca functia sa fie pe intervalul $[-1, 1]$)
`https://en.wikipedia.org/wiki/Gauss%E2%80%93Legendre_quadrature`

deci folosim:

$[-1,1]$:
$x = \frac{a+b}{2} + t\frac{b-a}{2}$, $x \in [a,b]$, $t \in [-1,1]$.

deci avem dupa schimbarea de variabila:

$$\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f(t)dt$$

formula e (noi lucram cu $\int_{-1}^{1} f(t)dt$, nu $\int_{-1}^{1} f(x)$ dx)

$[-1, 1]$, the rule takes the form:

$$\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

where

- n is the number of sample points used,
- w_i are quadrature weights, and
- x_i are the roots of the nth Legendre polynomial.

aici e un tabel pt alte ponderi (am vazut doar ponderea 1 in exercitii)
(sau newton cotes- dar asta e alta treaba)
`https://en.wikipedia.org/wiki/Gaussian_quadrature`

Interval	$\omega(x)$	Orthogonal polynomials	A & S	For more information, see ...
$[-1, 1]$	1	Legendre polynomials	25.4.29	§ Gauss–Legendre quadrature
$(-1, 1)$	$(1-x)^\alpha(1+x)^\beta, \quad \alpha,\beta > -1$	Jacobi polynomials	25.4.33 ($\beta = 0$)	Gauss–Jacobi quadrature
$(-1, 1)$	$\frac{1}{\sqrt{1-x^2}}$	Chebyshev polynomials (first kind)	25.4.38	Chebyshev–Gauss quadrature
$[-1, 1]$	$\sqrt{1-x^2}$	Chebyshev polynomials (second kind)	25.4.40	Chebyshev–Gauss quadrature
$[0, \infty)$	e^{-x}	Laguerre polynomials	25.4.45	Gauss–Laguerre quadrature
$[0, \infty)$	$x^\alpha e^{-x}, \quad \alpha > -1$	Generalized Laguerre polynomials		Gauss–Laguerre quadrature
$(-\infty, \infty)$	e^{-x^2}	Hermite polynomials	25.4.46	Gauss–Hermite quadrature

cum cere pe 3 noduri, $n = 3$
deci avem nevoie de al 3-lea polinom lagrange - notat de profa cu T_3

pe care-l calculam asa

The Rodrigues representation provides the formula

$$P_l(x) = \frac{1}{2^l\, l!} \frac{d^l}{d\,x^l} (x^2 - 1)^l,$$

which yields upon expansion

si ar trebui sa dea atat

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

(sunt mai multe valori aici:
`https://mathworld.wolfram.com/LegendrePolynomial.html`)

si trebuie sa afli radacinile polinomului

aici are o expresie mai simpla si se vede aproape automat ca radacinile sunt: $0, \pm\sqrt{5/3}$

deci in formula asta stim $n = 3$, stim $x_i =$ radacinile, stim functia definita pe $[-1, 1]$

$[-1, 1]$, the rule takes the form:

$$\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

where

- n is the number of sample points used,
- w_i are quadrature weights, and
- x_i are the roots of the nth Legendre polynomial.

deci mai avem nevoie doar de w_i

care au minunata forma:

weights are given by the formula

$$w_i = \frac{2}{\left(1 - x_i^2\right)\left[P_n'(x_i)\right]^2}.$$

cu $P_n = T_3$ notatia profei
adica asta:

$$P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right)$$

si acum ai tot ce-ti trebuie, doar inlocuiesti (si speri la ce e mai bun :))))

si asta e restul:

The error of a Gaussian quadrature rule can be stated as follows (Stoer & Bulirsch 2002, Thm 3.6.24). For an integrand which has $2n$ continuous derivatives,

$$\int_a^b \omega(x)\, f(x)\, dx - \sum_{i=1}^{n} w_i\, f(x_i) = \frac{f^{(2n)}(\xi)}{(2n)!}\,(p_n, p_n)$$

for some ξ in (a, b), where p_n is the monic (i.e. the leading coefficient is 1) orthogonal polynomial of degree n and where

$$(f, g) = \int_a^b \omega(x) f(x) g(x)\, dx.$$

aici calculam cu functia pe care am facut-o pe $[-1, 1]$ si acel p_n (cred că) e polinomul lagrange la care impartim prin coef lui x^3 (adica $\frac{5}{2}$):

$$x^3 - \frac{6}{5}x$$

(de aici `https://en.wikipedia.org/wiki/Gaussian_quadrature#Error_estimates`)

si acum ultima parte din problema:
se poate itera daca $w_i > 0$, $\forall i = 1..n$

atentie, x_i urile sunt radacinile polinomului, astea nu se mai iau de la ex1
ca iti si zice (sau mai degraba nu zice) "pe 3 noduri", nu "pe nodurile din A"
sanity check:
calculeaza integrala (cu wolfram alpha eventual) si verifica daca „seamană” cu ce ti-a dat