Ecuatii cu var separabile

$$x' = f(t)g(x), \quad \text{devine :} \quad \int \frac{dx}{g(x)} = \int f(t)dt$$

Ec omogene

$$x' = h\left(\frac{x}{t}\right)$$

Ec dif ordin I

$$x' = a(t)x + b(t) \qquad x = e^{\int a(t)dt}\left(C + \int e^{-\int a(t)dt} b(t)dt\right)$$

Ec Bernoulli

$$x' = a(t)x + b(t)x^{\alpha}, \quad \text{împărțim la} \quad x^{\alpha}$$

Ec Riccati

$$x' = a(t)x + b(t)x^2 + c(t), \quad x = \varphi(t) + \frac{1}{z}$$

EDE

$$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0, \quad \text{sol:} \quad F(t,x) = c$$

Ec Lagrange si Clairout (mere si cu $t = f(x, x')$)

$$x' = t\varphi(x') + \psi(x'), \quad \text{derivam și} \quad x' = p$$

Ec liniare de ordin n cu coef constanți

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = e^{at}\left(P^1(t)\cos(bt) + P^2(t)\sin(bt)\right)$$
$$\tilde{y}(t) = t^m Q(t) e^{\lambda t}, \quad m = \text{multiplicitatea lui } \lambda, \quad \deg(Q) = \deg(P)$$

Ec Euler

$$t^n y^{(n)} + a_1 t^{n-1} y^{(n-1)} + \cdots + a_n y = f(t), \qquad t = e^s$$

Sist dif liniare cu coef constanți

Metoda substitutiei

Stabilitate $x = \xi$ sol stationara

$$P_{J(\xi)} \text{ Hurwitzian } \iff \text{stabil}$$

PDE

$$\sum f_i(x)\frac{\partial u}{\partial x_i} = g(x) \quad \frac{dx_i}{f_i} = \cdots = \frac{dx_n}{f_n} = \frac{du}{g}$$
$$F(u_1, \cdots) = 0, \quad u_i \text{ integrale prime}$$

Gut

Th inversare locala

$$\frac{d\varphi^{-1}}{dx}(x = \varphi(t)) = \frac{1}{\frac{d\varphi}{dt}(t)}, \qquad \varphi, \varphi^{-1} \in C^1$$

Teorie

(1) EVS

$$\begin{cases} x' = f(t)g(t), \quad f \in C^1(I = (t_1, t_2)), \quad g \in C^1(J = (x_1, x_2)) \\ x(t_0) = x_0 \end{cases}$$

Are sol unica $\forall x_0, t_0$ data de:

$$x = G^{-1}\left(\int_{t_0}^{t} f(s)ds\right), \quad G(x) = \int_{x_0}^{x} \frac{du}{g(u)}$$

(2) Ec dif lin de ord I - FVC - amplificam $\exp(\int a(\tau))$

(3) Lema lui Gronwall, $k \geq 0$

$$x(t) \leq m + \int_a^t k(s)x(s)ds \implies x(t) \leq m \exp(\int_a^t k(s)ds)$$

Derivam, amplificam cu $k(t)$ si apoi cu $\exp(-\int k(s)ds)$

(4) $\exists!$ local - Th Picard

$$f : \Delta = [a, a+h] \times B(\xi, r) \to \mathbb{R}^n \in C^1(\Delta), \quad f \text{ Lipschitz pe } B$$

$$pe[a, a + (\delta = \min\left\{h, \frac{r}{M}\right\}), \quad x' = f(t, x) \quad \text{are sol unica: } \lim$$

$$\begin{cases} x_0(t) = \xi \\ x_{k+1}(t) = \xi + \displaystyle\int_a^t f(\tau, x_k(\tau)d\tau) \end{cases}$$

$$\|x_k(t) - x(t)\| \leq M \frac{L^k \delta^{k+1}}{(k+1)!}$$

Th Peano: f cont $\implies \mathcal{PC}(I, \Omega, f, a, \xi)$ are cel putin o sol locala

(5) PC ec ordin n, $g \in C^0(I \times \Omega)$

$$(PC)y^n = g(y, y', \ldots), \quad g \text{ Lipschitz} \implies \quad \text{sol unica pe } [a - \delta, a + \delta]$$

(6) Depedenta cont de data initiala, $f \in C^0([a.b] \times \mathbb{R}^n)$, lipschitz, $x' = f(t, x)$

Lema: $\forall \xi \in \mathbb{R}^n$, sol saturata $\mathcal{PC}(a, \xi)$ este globala
Th: $\xi \mapsto x(\cdot, \xi)$ lipschitz ($\geq$ ec Voltera, lema Gronwall)

(7) Sist dif liniare $\exists!$ global

$$x' = A(t)x + b(t), x(a) = \xi \text{ are sol unica}$$

dem: f, local apoi global lipschitz, $\|A\| = \max\limits_i\{\sum\limits_j |a_{ij}|\}$ e norma matriceala.

(8) Sist dif liniare omogene, spatiul solutiilor $x = A(t)x$

$$S = \{x : I \to \mathbb{R}^n sol\} \underset{sl}{\subseteq} C^1(I; \mathbb{R}^n); \quad \dim(S) = n; \quad \Gamma_a(x) = x(a), \text{izomorfism de sp lin}$$

Def: $X = (X^i_j)$ sn matrice asociata sist de sol $\{x^1 \ldots x^n\}$
Def: $\{x^1 \ldots x^n\}$ syst fundametal $\iff$ baza in $S \implies X$ mat fundametala
Def: $W(t) = \det X(t)$ wronskian X fundamental $\iff W(t) \neq 0, \ \forall t \iff \exists a \ W(a) \neq 0$ (dem cu baza in $\mathbb{R}^n$)
Th: X mat fundamentala $\implies \forall Y$ mat fundamentala, $Y(t) = X(t)C, \quad C \in M_{n \times n}(\mathbb{R})$.
Lema: $D(t) = (d_{i,j}), \quad D'(t) = \sum\limits_k D_k(t), \quad D_k(t) =$ det obtinut linia derivand doar linia k
Th Liouville: $W(t) = W(a) \exp\left(\int_a^t \mathrm{tr}A(s)ds\right)$

(9) Formula variatiei constantelor

Th: $x_{SGN} = x_{SGO} + \tilde{x}_{SPN}$
Th: Sol $x' = A(t)x + b(t)$ este $x(t) = X(t)\left(X^{-1}(a)\xi + \int_a^t X^{-1}(s)b(s)\right)$ (cautam $\tilde{x} = X(t)c(t)$)

(10) Exponentiala de mat

$x' = Ax(t)$, not $S_A(t) = X(t)$,mat cu$X(0) = I$, avem $S_A(t+s) = S_A(t)S_A(s)$, $S_A(0) = I$, $\lim_{t\to 0} S_A(t)\xi = \xi$
$\frac{d}{dt}e^{tA} = Ae^{tA} = e^{tA}A$

(11) ec lin de ordin n are sol global unica

(12) ec lin omogene de ordin n, sp sol- Totu izomorf cu totu

(13) ec lin de ordin n cu coef constanti: ec omogene -th generare syst fundamental

(14) T stabilitatii: Tipuri

Def $\varphi : \mathbb{R}_+ \to \Omega$ simplu stabila daca $\forall \varepsilon > 0, a \geq 0,\ \exists \delta(\varepsilon, a) > 0$ ai $\forall \xi \in \Omega$ cu $\|\xi - \varphi(a)\| \leq \delta\ x(\cdot, a, \xi)$ def pe $[a, \infty]$, $\|x(t, a, \xi) - \varphi\| \leq \varepsilon$
Def: uniform stabila $\|\xi - \varphi(a)\| \leq \mu(a)$, și $\lim_{t\to\infty} \|x(t, a, \xi) - \varphi(t)\| = 0$
Daca nu deinde de a at uniform stabila

(15) Stabilitatea sist liniare

Th: tot sist (omogen sau neomogen) e la fel ca sol nula
Th: E s. stabil daca are o mat fundamentala marginita, si at toate sunt marginite
Th: E asimpt stabil daca are o mat cu $\lim_{t\to\infty} \|X(t)\| = 0$
Th: uniform stabil daca $\|U(t, a) = X(t)X^{-1}(a)\| \leq M$
Th: uniform asimpt stabil daca $\lim_{t-a\to\infty} \|U(t, a)\| = 0$

(16) Stab sist lin perturbate

Lema: A hurwitziana daca $\|e^{tA}\| \leq Me^{\omega t}$
$x' = Ax + F(t, x)$, $\quad F$ cont si local lipschitz, lipschitz pe $B(0, r) \subset \Omega$
Th Poincare-Liapunov: daca $L < \frac{\omega}{M}$ at sol nula (SLP) e asimpt stab
Th Perron: $\|F(t, x)\| \leq \alpha(\|x\|)$ și $\lim_{\rho\to 0} \frac{\alpha(\rho)}{\rho} = 0$ at sol nula asimpt stab

(17) Sist autonom, sp fazelor

Traiectorie/ orbita $= \mathrm{Im}(\varphi)$ Prop: Multimea sol e inchisa la transaltii in rap cu t
Prop: Prin orice pct trece o singura traiectorie
Prop: Traiectoria unui pct nestationar e o curba regulata de C^1, (cu parametrizare $x = \varphi(t)$)

(18) Integrale prime pt sist autonome

Def: U integrala prima: $\in C^1$, $\nabla U(\xi) = 0$ izolate, $U(x(t)) = c\ \forall x$ sol
Th: conditie echivalenta: $\langle \nabla U(\xi), f(\xi) \rangle = 0\ \forall \xi$
Def $U_1, \ldots U_k$ functional independente $\iff$ rang $\left(\frac{\partial U_i}{\partial x_j}(\xi)\right) = k$
Th: In vecinatatea oricarui pct nestationar exista exact $n-1$ integrale prime functional independente
Th: Fie $U_1 \ldots U_{n-1}$ integrale prime functional independente, at orice alta int prima e de forma $V(x) = F(U_1(x) \ldots U_{n-1}(x))$