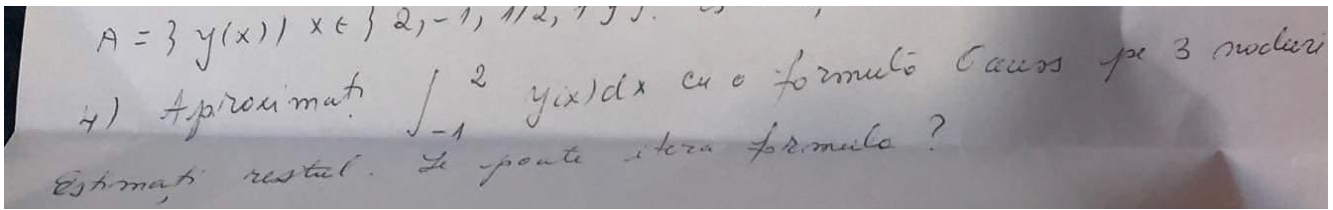


btw, this file is subject to change (although i don't plan on changing it), so check for updates  
problema:



btw,  $y$  de aici e  $y$ -ul aflat la ex 1

avem ponderea  $p(x) = 1$

pentru ca iti cere  $\int y(x) dx$  aka  $\int y(x) \cdot 1 dx$

daca cerea de ex  $\int e^{-x} y(x) dx$ , atunci ponderea era  $e^{-x}$

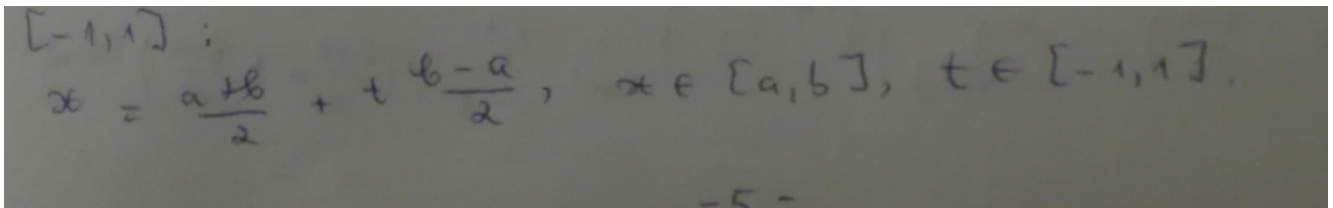
si cum ponderea e 1, "o formula gauss" inseamna Gauss-Legendre  
de ex daca era  $e^{-x}$  era gauss-laguerre

si acum ar trebui sa convertim functia de la una pe  $[-1, 2]$  la una pe  $[-1, 1]$  (practic e schimbare de variabila)

(Gauss-Legendre cere ca functia sa fie pe intervalul  $[-1, 1]$ )

[https://en.wikipedia.org/wiki/Gauss%E2%80%93Legendre\\_quadrature](https://en.wikipedia.org/wiki/Gauss%E2%80%93Legendre_quadrature)

deci folosim:



deci avem dupa schimbarea de variabila:

$$\int_a^b f(x) dx = \frac{b-a}{2} \int_{-1}^1 f(t) dt$$

formula e (noi lucram cu  $\int_{-1}^1 f(t) dt$ , nu  $\int_{-1}^1 f(x) dx$ )

$[-1, 1]$ , the rule takes the form:

$$\int_{-1}^1 f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

where

- $n$  is the number of sample points used,
- $w_i$  are quadrature weights, and
- $x_i$  are the roots of the  $n$ th Legendre polynomial.

aici e un tabel pt alte ponderi (am vazut doar ponderea 1 in exercitii)  
(sau newton cotes- dar asta e alta treaba)

[https://en.wikipedia.org/wiki/Gaussian\\_quadrature](https://en.wikipedia.org/wiki/Gaussian_quadrature)

Interval	$w(x)$	Orthogonal polynomials	A & S	For more information, see ...
$[-1, 1]$	1	<a href="#">Legendre polynomials</a>	25.4.29	<a href="#">§ Gauss-Legendre quadrature</a>
$(-1, 1)$	$(1-x)^\alpha(1+x)^\beta, \quad \alpha, \beta > -1$	<a href="#">Jacobi polynomials</a>	25.4.33 ( $\beta = 0$ )	<a href="#">Gauss-Jacobi quadrature</a>
$(-1, 1)$	$\frac{1}{\sqrt{1-x^2}}$	<a href="#">Chebyshev polynomials</a> (first kind)	25.4.38	<a href="#">Chebyshev-Gauss quadrature</a>
$[-1, 1]$	$\sqrt{1-x^2}$	<a href="#">Chebyshev polynomials</a> (second kind)	25.4.40	<a href="#">Chebyshev-Gauss quadrature</a>
$[0, \infty)$	$e^{-x}$	<a href="#">Laguerre polynomials</a>	25.4.45	<a href="#">Gauss-Laguerre quadrature</a>
$[0, \infty)$	$x^\alpha e^{-x}, \quad \alpha > -1$	<a href="#">Generalized Laguerre polynomials</a>		<a href="#">Gauss-Laguerre quadrature</a>
$(-\infty, \infty)$	$e^{-x^2}$	<a href="#">Hermite polynomials</a>	25.4.46	<a href="#">Gauss-Hermite quadrature</a>

cum cere pe 3 noduri,  $n = 3$   
 deci avem nevoie de al 3-lea polinom lagrange - notat de profa cu  $T_3$   
 pe care-l calculam asa

The [Rodrigues representation](#) provides the formula

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l,$$

which yields upon expansion

si ar trebui sa dea atat

$$P_3(x) = \frac{1}{2} (5x^3 - 3x)$$

(sunt mai multe valori aici:  
<https://mathworld.wolfram.com/LegendrePolynomial.html>)

si trebuie sa afli radacinile polinomului

aici are o expresie mai simpla si se vede aproape automat ca radacinile sunt:  
 $0, \pm\sqrt{5/3}$

deci in formula asta stim  $n = 3$ , stim  $x_i$  = radacinile, stim functia definita pe  $[-1, 1]$

$[-1, 1]$ , the rule takes the form:

$$\int_{-1}^1 f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

where

- $n$  is the number of sample points used,
- $w_i$  are quadrature weights, and
- $x_i$  are the roots of the  $n$ th [Legendre polynomial](#).

deci mai avem nevoie doar de  $w_i$

care au minunata forma:

weights are given by the formula

$$w_i = \frac{2}{(1 - x_i^2) [P'_n(x_i)]^2}.$$

cu  $P_n = T_3$  notatia profei  
adica asta:

$$P_3(x) = \frac{1}{2} (5x^3 - 3x)$$

si acum ai tot ce-ti trebuie, doar inlocuiesti (si speri la ce e mai bun :))) )  
si asta e restul:

The error of a Gaussian quadrature rule can be stated as follows (Stoer & Bulirsch 2002, Thm 3.6.24). For an integrand which has  $2n$  continuous derivatives,

$$\int_a^b \omega(x) f(x) dx - \sum_{i=1}^n w_i f(x_i) = \frac{f^{(2n)}(\xi)}{(2n)!} (p_n, p_n)$$

for some  $\xi$  in  $(a, b)$ , where  $p_n$  is the monic (i.e. the leading coefficient is 1) orthogonal polynomial of degree  $n$  and where

$$(f, g) = \int_a^b \omega(x) f(x) g(x) dx.$$

aici calculam cu functia pe care am facut-o pe  $[-1, 1]$  si acel  $p_n$  (cred că) e polinomul lagrange la care impartim prin coef lui  $x^3$  (adica  $\frac{5}{2}$ ):

$$x^3 - \frac{6}{5}x$$

(de aici [https://en.wikipedia.org/wiki/Gaussian\\_quadrature#Error\\_estimates](https://en.wikipedia.org/wiki/Gaussian_quadrature#Error_estimates))

si acum ultima parte din problema:  
se poate itera daca  $w_i > 0, \forall i = 1..n$

atentie,  $x_i$  urile sunt radacinile polinomului, astea nu se mai iau de la ex1  
ca iti si zice (sau mai degraba nu zice) "pe 3 noduri", nu "pe nodurile din A"

sanity check:  
calculeaza integrala (cu wolfram alpha eventual) si verifica daca „seamană” cu ce ti-a dat

btw; in cazul de aici  $\int_{-1}^2 a/x + b dx$  nu converge, deci e vina profei :p