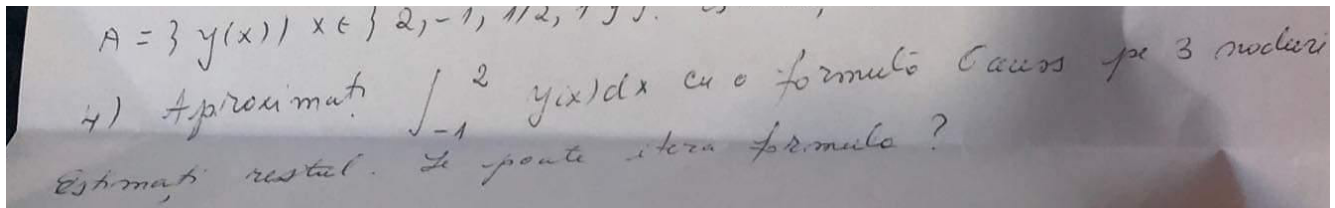


problema:



BTW, y de aici e y -ul aflat la ex 1

avem ponderea $p(x) = 1$

pentru ca iti cere $\int y(x)dx$ aka $\int y(x) \cdot 1dx$

daca cerea de ex $\int e^{-x}y(x)dx$, atunci ponderea era e^{-x}

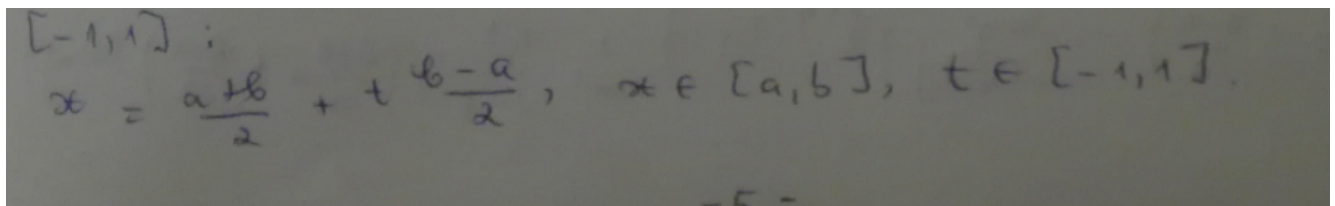
si cum ponderea e 1, "o formula gauss" inseamna Gauss-Legendre
de ex daca era e^{-x} era gauss-laguerre

si acum ar trebui sa convertim functia de la una pe $[-1, 2]$ la una pe $[-1, 1]$ (practic e schimbare de variabila)

(Gauss-Legendre cere ca functia sa fie pe intervalul $[-1, 1]$)

https://en.wikipedia.org/wiki/Gauss%E2%80%93Legendre_quadrature

deci folosim:



deci avem dupa schimbarea de variabila:

$$\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^1 f(t)dt$$

formula e (noi lucram cu $\int_{-1}^1 f(t)dt$, nu $\int_{-1}^1 f(x) dx$)

$[-1, 1]$, the rule takes the form:

$$\int_{-1}^1 f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

where

- n is the number of sample points used,
- w_i are quadrature weights, and
- x_i are the roots of the n th Legendre polynomial.

aici e un tabel pt alte ponderi (am vazut doar ponderea 1 in exercitii)
(sau newton cotes- dar asta e alta treaba)

https://en.wikipedia.org/wiki/Gaussian_quadrature

| Interval | $w(x)$ | Orthogonal polynomials | A & S | For more information, see ... |
|---------------------|---|---|-------------------------|---|
| $[-1, 1]$ | 1 | Legendre polynomials | 25.4.29 | § Gauss–Legendre quadrature |
| $(-1, 1)$ | $(1-x)^\alpha(1+x)^\beta, \quad \alpha, \beta > -1$ | Jacobi polynomials | 25.4.33 ($\beta = 0$) | Gauss–Jacobi quadrature |
| $(-1, 1)$ | $\frac{1}{\sqrt{1-x^2}}$ | Chebyshev polynomials (first kind) | 25.4.38 | Chebyshev–Gauss quadrature |
| $[-1, 1]$ | $\sqrt{1-x^2}$ | Chebyshev polynomials (second kind) | 25.4.40 | Chebyshev–Gauss quadrature |
| $[0, \infty)$ | e^{-x} | Laguerre polynomials | 25.4.45 | Gauss–Laguerre quadrature |
| $[0, \infty)$ | $x^\alpha e^{-x}, \quad \alpha > -1$ | Generalized Laguerre polynomials | | Gauss–Laguerre quadrature |
| $(-\infty, \infty)$ | e^{-x^2} | Hermite polynomials | 25.4.46 | Gauss–Hermite quadrature |

cum cere pe 3 noduri, $n = 3$
deci avem nevoie de al 3-lea polinom lagrange - notat de profa cu T_3
pe care-l calculam asa

The [Rodrigues representation](#) provides the formula

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l,$$

which yields upon expansion

si ar trebui sa dea atat

$$P_3(x) = \frac{1}{2} (5x^3 - 3x)$$

(sunt mai multe valori aici:
<https://mathworld.wolfram.com/LegendrePolynomial.html>)

si trebuie sa afli radacinile polinomului

aici are o expresie mai simpla si se vede aproape automat ca radacinile sunt:
 $0, \pm\sqrt{5/3}$

deci in formula asta stim $n = 3$, stim x_i = radacinile, stim functia definita pe $[-1, 1]$

$[-1, 1]$, the rule takes the form:

$$\int_{-1}^1 f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

where

- n is the number of sample points used,
- w_i are quadrature weights, and
- x_i are the roots of the n th [Legendre polynomial](#).

deci mai avem nevoie doar de w_i

care au minunata forma:

weights are given by the formula

$$w_i = \frac{2}{(1 - x_i^2) [P'_n(x_i)]^2}.$$

cu $P_n = T_3$ notatia profei
adica asta:

$$P_3(x) = \frac{1}{2} (5x^3 - 3x)$$

si acum ai tot ce-ti trebuie, doar inlocuiesti (si speri la ce e mai bun :))))

si asta e restul:

The error of a Gaussian quadrature rule can be stated as follows (Stoer & Bulirsch 2002, Thm 3.6.24). For an integrand which has $2n$ continuous derivatives,

$$\int_a^b \omega(x) f(x) dx - \sum_{i=1}^n w_i f(x_i) = \frac{f^{(2n)}(\xi)}{(2n)!} (p_n, p_n)$$

for some ξ in (a, b) , where p_n is the monic (i.e. the leading coefficient is 1) orthogonal polynomial of degree n and where

$$(f, g) = \int_a^b \omega(x) f(x) g(x) dx.$$

aici calculam cu functia pe care am facut-o pe $[-1, 1]$ si acel p_n (cred că) e polinomul lagrange la care impartim prin coef lui x^3 (adica $\frac{5}{2}$):

$$x^3 - \frac{6}{5}x$$

(de aici https://en.wikipedia.org/wiki/Gaussian_quadrature#Error_estimates)

si acum ultima parte din problema:

se poate itera daca $w_i > 0, \forall i = 1..n$

atentie, x_i urile sunt radacinile polinomului, astea nu se mai iau de la ex1
ca iti si zice (sau mai degraba nu zice) "pe 3 noduri", nu "pe nodurile din A"

sanity check:

calculeaza integrala (cu wolfram alpha eventual) si verifica daca „seamană” cu ce ti-a dat