

might be useful

$$\frac{d}{dx} \left( \int_0^x f(x, y) dy \right) = f(x, x) + \int_0^x \frac{\partial}{\partial x} f(x, y) dy$$

care e obtinuta din formula Leibniz:

$$\frac{d}{dx} \left( \int_{a(x)}^{b(x)} f(x, t) dt \right) = f(x, b(x)) \cdot \frac{d}{dx} b(x) - f(x, a(x)) \cdot \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt$$

- laplace operator (laplacian):  $\Delta = \nabla^2 = \nabla \cdot \nabla = \sum_i \frac{\partial^2}{\partial x_i^2}$

- field theory shit: (curl=rotor)

$$\operatorname{div} \operatorname{grad} f \equiv \nabla \cdot \nabla f \equiv \nabla^2 f$$

$$\operatorname{curl} \operatorname{grad} f \equiv \nabla \times \nabla f = \mathbf{0}$$

$$\operatorname{div} \operatorname{curl} \mathbf{A} \equiv \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\operatorname{curl} \operatorname{curl} \mathbf{A} \equiv \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\nabla^2(fg) = f\nabla^2 g + 2\nabla f \cdot \nabla g + g\nabla^2 f$$

- Gauß-Ostrogradskiy:

$$\int_{\Omega} \frac{\partial u}{\partial x_i} = \int_{\partial\Omega} u \cdot \nu_i d\sigma$$

also:

$$\int_{\Omega} \operatorname{div} F(x) dx = \int_{\partial\Omega} F \cdot \nu d\sigma$$

- Green formula:

$$\int_{\Omega} (\nabla u \cdot \nabla v + v \Delta u) dx = \int_{\partial\Omega} v \sum_i \frac{\partial u}{\partial x_i} \nu_i d\sigma$$

- Green formula 1:

$$-\int_{\Omega} \Delta u v dx = \int_{\Omega} \nabla u \cdot \nabla v dx - \int_{\partial\Omega} \frac{\partial u}{\partial \nu} v d\sigma$$

- Green formula 2:

$$\int_{\Omega} (\Delta u v u \Delta v) dx = \int_{\partial\Omega} \frac{\partial u}{\partial \nu} v - u \frac{\partial v}{\partial \nu} d\sigma$$

- convoluție:

$$f * g(x) = \int_{\mathbb{R}^d} f(x - y) g(y) dy$$

- REMEMBER THE NORM FOR THE FOURIER THING:

$$u(x) = \sum_k \frac{1}{\|\varphi_k\|^2} \int_a^b u(x) \varphi_k(x) dx$$

## misc

- proiectie pe subspatii inchise:

$$\exists! Pu \in V \text{ a\hat{t}} \|Pu - u\| = \inf_{v \in V} \|v - u\|$$

- în plus  $(u - Pu) \perp V$  (ie  $\langle u - Pu, v \rangle = 0, \forall v \in V$ )
- bessel inequality:  $V = \text{span}\{v_1, \dots, v_n\}$

$$\|u\|^2 \geq \|P_V u\|^2 = \sum_{j=1}^n \frac{|\langle u, v_j \rangle|^2}{\|v_j\|^2}$$

- daca are loc ineg parseval și  $\{f_j\}$  ortongolală, at e bază Hilbertiană
- scalar product with functions  $f : [a, b] \rightarrow \mathbb{C} \in L^2([a, b])$ :

$$\langle f, g \rangle = \int_a^b f(x) \overline{g(x)} dx$$

- weak convergence:

$$u^n \rightharpoonup u \text{ dac\u0103 } \langle u^n - u, v \rangle \rightarrow 0, \forall v \in H$$

- subarmonica?

## Sturm-Liouville - S1

### valori proprii

- c3 - pg 5
- Melnig thing pg 7
- do the  $|\cdot| \varphi, \int$  to get  $\lambda \geq 0$
- if  $\varphi = 0$  we ignore that one. we dont want null solutions
- we get the characteristic equation<sup>1</sup> (polinom characteristic:

$$\varphi''(x) + \lambda \varphi(x) = 0 \text{ becomes } r^2 + \lambda \cdot 1 = 0$$

- and we get the functions of form (may differ depending on the characteristic equation):

$$\{e^{r_i}, \dots, x^m e^{r_i}\}$$

or, for our example:

$$\sin(\sqrt{\lambda}x), \cos(\sqrt{\lambda}x)$$

so

$$\varphi(x) = \alpha \sin(\sqrt{\lambda}x) + \beta \cos(\sqrt{\lambda}x)$$

- with the initial conditions:  $\varphi(0) = \varphi(l) = 0$  we get some restrictions for  $\alpha$  and  $\beta$

and, tada, ya get some  $\lambda_k, \varphi_k$

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<sup>1</sup>[https://en.wikipedia.org/wiki/Characteristic\\_equation\\_\(calculus\)](https://en.wikipedia.org/wiki/Characteristic_equation_(calculus))

## Green's function:

- for n-th order differential equations: see green-kurzgesagt

## separation of variables

- see s6 - pg 2
- we have:

$$\begin{cases} -\Delta u = f, & \text{in } \Omega = (a, b) \times (c, d) \\ \text{some condition like } u = 0, & \text{pe } \partial\Omega \end{cases}$$

- we write stuff with respect to  $x$ :

$$\begin{cases} -\varphi'' = \lambda\varphi, & \text{in } \Omega = (a, b) \\ \text{some condition like } u(a) = u(b) = 0 \end{cases}$$

- and we get some eigen functions and values:  $\{\varphi_k\}, \{\lambda_k\}$
- we write things with the new functions:

$$u(x, y) = \sum_k^\infty u_k(y) \varphi_k(x)$$

$$u_{xx} = \dots, u_{yy} = \dots$$

$$f(x, y) = \sum f_k(y) \varphi_k(x) = \sum \frac{1}{\|\varphi_k\|^2} \left( \int_a^b f(t, y) \varphi_k(t) dt \right) \varphi_k(x)$$

- then we solve it for some  $k$

$$\begin{cases} -\Delta u_k(y) = f_k(y), & \text{in } (c, d) \\ \text{some condition like } u(c) = u(d) = 0 \end{cases}$$

and we get

$$u_k(y) = \int_c^d G_k(y, s) f_k(s) ds$$

- sum things together and we get a  $G((x, y), (t, s))$ :

$$u(x, y) = \int_c^d \sum_k G_k(y, s) \frac{1}{\|\varphi_k\|^2} \left( \int_a^b f(t, y) \varphi_k(t) dt \right) \varphi_k(x) ds$$

aka

$$u(x, y) = \int_c^d \int_a^b \left( \sum_k G_k(y, s) \frac{1}{\|\varphi_k\|^2} \varphi_k(t) \varphi_k(x) \right) f(t, s) dt ds$$

and, tada

$$G((x, y), (t, s)) = \sum_{k=1}^\infty G_k(y, s) \frac{1}{\|\varphi_k\|^2} \varphi_k(t) \varphi_k(x)$$

## toc

### course

- C1: basic shit
- C2:
  - basic shit (prod scalar and norm)
  - projections
  - besel inequality
- C3:
  - more besel
  - hilbert basis
  - problem with Green's function
  - hilbert spaces examples
- C4:
  - proprietati Green's thing - pg 2
  - Riesz representation theorem - pg 5 (dual stuff)
  - autoadjunct daca  $T = T^*$
- C5:
  - weak convergence
  - hilbert basis proprieties & stuff
- C6:
  - more weird abstract shit
  - sturm liouville in general form - pg 11
- C7:
  - differential subvariety stuff
  - green's formulas
  - convolutions
  - that weird *Elementary* thing

### seminaries

#### S1

- tldr normal differential equations

$$\begin{cases} u'_k(t) + \lambda_k u_k(t) = f_k(t), t > 0 \\ u_k(0) = u_k^0 \end{cases}$$

$$u_k(t) = e^{-\lambda_k t} u_k^0 + \int_0^t \exp(-\lambda_k(t-s)) f_k(s) ds$$

- sturm-liouville stuff

## **S2, s3**

- sturm-liouville and fourier exercises

## **s4:?**

- met sep variabilelor pg 4
- fundamental solution pg 10

## **s5**

- green shit

## **s6**

- separation of variables for sturm-liouville problems + green - pg 3

## **that old book**

- green - pg 39

## **melnig thing**

- 7 - val proprii
- 15 - parseval stuff

## **things to know**

- sp Hilbert, serii Fourier, pb Sturm-Liouville
- separarea variabilelor (pb val proprii, hip, parab, eliptice - serii fourier)
- fct Green (op laplace+ sturm liouville)
- pp maxim (op eliptici + aplicatii - unicitatea sol si estimari)
- formularea variationala a pb eliptice (si parab si hip) => sep variabilelor
- transformata fourier - calcul + cateva proprietati
- oral: he asks bout some theory bit

## **todo:**

- do some fourier shit
- "sep variabilelor"?
- green shit for liouville and