The Math Wars

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During the 1990s, the teaching of mathematics became the subject of heated controversies known as the math wars. The immediate origins of the conflicts can be traced to the "reform" stimulated by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics. *Traditionalists fear that reform-oriented, "standards-based" curricula are superficial and undermine classical mathematical values; reformers claim that such curricula reflect a deeper, richer view of mathematics than the traditional curriculum. An historical perspective reveals that the underlying issues being contested—Is mathematics for the elite or for the masses? Are there tensions between "excellence" and "equity"? Should mathematics be seen as a democratizing force or as a vehicle for maintaining the status quo?—are more than a century old. This article describes the context and history, provides details on the current state, and offers suggestions regarding ways to find a productive middle ground.*

Keywords: *math wars; mathematics curriculum; standards; mathematics reform*

> Let the war rage.
>
> —Wayne Bishop

> Can't we all just get along?
>
> —Rodney King

The phrase *Math Wars* seems oxymoronic, a category error. According to popular belief at least, the facts of mathematics are universally true, its procedures universally correct, and both completely independent of culture.

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Suppose, for example, that you draw a triangle on a flat surface. No matter what triangle you draw, the sum of its interior angles will be a straight angle. That is the case whether you draw it on a table in Tunisia, a floor in France, a whiteboard in Waterloo, or a blackboard in Boston. In short, math is math. What could there possibly be to fight about? How could things get to the point where U.S. Secretary of Education Richard Riley (1998) felt compelled to address the annual Joint Mathematics Meetings[1] in January of 1998 and to plead, à la Rodney King, for civility and respectful behavior in what had become a knock-down-drag-out battle between advocates of "traditional" and "reform" mathematics?

This article relates the story of the math wars. The story is told from the perspective of a participant-observer who sits squarely in the middle of the territories claimed by both sides. I am a mathematician by training and inclination, hence, comfortable with the core mathematical values cherished by traditionalists. I have also, for more years than I would care to admit, conducted research on mathematical thinking, learning, and teaching; I am thus equally at home with the "process orientation" cherished by reformers. One critical issue for the future (and for this article) will be, just how large is the middle ground?

I begin this essay with some historical perspective, a distillation of 100 years of curricular trends and controversies in mathematics instruction. Wars have histories, and the current controversies are best understood when viewed in the context of prior controversies. The historical context is followed by a description of the changing research and epistemological base regarding mathematical thinking and learning that began in the 1970s, which provided the context for the creation of the *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) (National Council of Teachers of Mathematics [NCTM], 1989) in particular and reform in general. I then move to a description of the origins and substance of the current controversies and the possibilities for their resolution. Resolution is essential because the math wars, like all wars, involve casualties to innocent parties. When extremists battle, the "collateral damage" to those in the middle (in this case, America's children, who should be well served by mathematics education) can be significant. I believe that resolution is possible, although it will not come easily.

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HISTORICAL BACKGROUND AND CONTEXT

The counterpoint to the mathematics-is-independent-of-culture perspective expressed above is that knowledge of any type, but specifically mathematical knowledge, is a powerful vehicle for social access and social mobility. Hence, lack of access to mathematics is a barrier—a barrier that leaves people socially and economically disenfranchised. For these reasons, noted civil rights worker Robert Moses (2001) declared that

> the most urgent social issue affecting poor people and people of color is economic access. . . . I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of registered Black voters in Mississippi was in 1961. (p. 5)

Who gets to learn mathematics, and the nature of the mathematics that is learned, are matters of consequence. This fact is one of the underpinnings of the math wars. It has been true for more than a century.

Anthropologists and historians may differ with regard to details, depending on their focus. Rosen (2000), an anthropologist, argued that during the past century or more there have been three "master narratives" (or myths) regarding education in America,

> each of which celebrates a particular set of cultural ideals: *education for democratic equality* (the story that schools should serve the needs of democracy by promoting equality and providing training for citizenship); *education for social efficiency* (the story that schools should serve the needs of the social and economic order by training students to occupy different positions in society and the economy); and *education for social mobility* (the story that schools should serve the needs of individuals by providing the means of gaining advantage in competitions for social mobility). (p. 4)

Stanic (1987), a historian of mathematics education, described four perspectives on mathematics that battled for dominance in the early 1900s and then throughout the century. Humanists believed in "mental discipline," the ability to reason, and the cultural value of mathematics. That is, learning mathematics is (by virtue of transfer) learning to think logically in general; mathematics is also one of civilization's greatest cultural achievements and merits study on that basis. Developmentalists focused on the alignment of school curricula with the growing mental capacities of children. (During the heyday of Piagetian stage theory, some developmentalists argued that topics such as algebra should not be taught until students became "formal thinkers.") Social efficiency educators, identified above by Rosen (2000), thought of schools as the place to prepare students for their predetermined social roles. In opposition, social meliorists (similar to those who believed in education for social mobility) focused on schools as potential sources of social justice, calling for "equality of opportunity through the fair distribution of extant knowledge" (Stanic, 1987, p. 152).

To these social forces shaping mathematics curricula, I would add one more. Mathematics has been seen as a foundation for the nation's military and economic preeminence, and in times of perceived national crisis mathematics curricula have received significant attention. This was the case before and during both World Wars, the cold war (especially the post-Sputnik era, which gave rise to the *new math*), and the U.S. economic crises of the 1980s (see *A Nation at Risk*, National Commission on Excellence in Education, 1983).

With this as background, let us trace both numbers and curricular trends.

The 20th century can be viewed as the century of democratization of schooling in the United States. In 1890, fewer than 7% of the 14-year-olds in the United States were enrolled in high school, with roughly half of those going on to graduate (Stanic, 1987, p. 150). High school and beyond were reserved for the elite, with fewer students graduating from high school back then than earn master's and Ph.D. degrees today. In short, "education for the masses" meant elementary school. In line with the ideas of the social efficiency educators, an elementary school education often meant instruction in the very very basics. For example, one set of instructions from a school district in the 1890s instructed teachers that their students were to learn no more mathematics than would enable them to serve as clerks in local shops (L. B. Resnick, personal communication, January 10, 1987). In contrast, the high school curriculum was quite rigorous. High school students studied algebra, geometry, and physics and were held to high standards. In the 1909-1910 school year, roughly 57% of the nation's high school students studied algebra and more than 31% studied geometry (a negligible 1.9% studied trigonometry, which was often studied at the college level; calculus was an upper-division college course) (Jones & Coxford, 1970, p. 54).

By the beginning of World War II, almost three fourths of the children aged 14 to 17 attended high school, and 49% of the 17-year-olds graduated (Stanic, 1987, p. 150). This expanding population put pressure on the system. Broadly speaking, the curriculum remained unchanged, whereas the student body facing it was much more diverse and ill prepared than heretofore. (The percentage of students enrolled in high school mathematics dropped steadily from 1909 to 1949, from 57% to 27% in the case of algebra and from 31% to 13% in the case of geometry. It must be remembered, however, that there was a ten-fold increase in the proportion of students enrolled in high school, as well as general population growth. In purely numerical terms, then, far more students were enrolled in algebra and geometry than previously [Klein, 2003, Historical Outline: 1920 to 1980 section, para. 20].) As always, in times of national crisis, the spotlight tends to focus on mathematical and scientific preparation for the military and for the economy:

> In the 1940s it became something of a public scandal that army recruits knew so little math that the army itself had to provide training in the arithmetic needed for basic bookkeeping and gunnery. Admiral Nimitz complained of mathematical deficiencies of would-be officer candidates and navy volunteers. The basic skills of these military personnel should have been learned in the public schools but were not. (Klein, 2003, Historical Outline: 1920 to 1980 section, para. 14)

The truth be told, however, there was not a huge amount of change in the actual curriculum, before or after these complaints.

The next major crisis did affect curricula, at least temporarily. In October 1957, the Soviet Union caught the United States off guard with its successful launch of the satellite Sputnik. That event came amidst the cold war and Soviet threats of world domination. (It was more than 40 years ago, but I still remember Nikita Khrushchev banging his shoe on a table at the United Nations, and his famous words "We will bury you.") Sputnik spurred the American scientific community into action. With support from the National Science Foundation (NSF), a range of curricula with "modern" content were developed in mathematics and the sciences. Collectively, the mathematics curricula became known as the new math. For the first time, some of the content really was new: aspects of set theory, modular arithmetic, and symbolic logic were embedded in the curriculum.

The full story of the new math should be told (though not here); it shows clearly how curricular issues can become social issues. Specifically, it provides a cautionary tale for reform. One of the morals of the experience with the new math is that for a curriculum to succeed, it needs to be made accessible to various constituencies and stakeholders. If teachers feel uncomfortable with a curriculum they have not been prepared to implement, they will either shy away from it or bastardize it. If parents feel disenfranchised because they do not feel competent to help their children and they do not recognize what is in the curriculum as being of significant value (And what value is someone trained in standard arithmetic likely to see in studying "clock arithmetic" or set theory?) they will ultimately demand change.

The following one-liner is an oversimplification but represents accepted wisdom: "By the early 1970s New Math was dead" (Klein, 2003). In a reaction to what were seen as the excesses of the new math, the nation's mathematics classrooms went "back to basics"—the theme of the 1970s. In broadbrush terms, the curriculum returned to what it had been before: arithmetic in the 1st through 8th grades, algebra in the 9th grade, geometry in the 10th grade, a 2nd year of algebra and sometimes trigonometry in the 11th grade, and precalculus in the 12th grade.[2] There are various opinions about the level of standards and rigor demanded of students—some will argue that less was being asked of students than before, and some will disagree—but in broad outline, the curricula of the 1970s resembled those of the pre-Sputnik years. In compensation for the "excesses" of the 1960s, however, the back-to-basics curricula focused largely on skills and procedures.

By 1980, the results of a decade of such instruction were in. Not surprisingly, students showed little ability at problem solving—after all, curricula had not emphasized aspects of mathematics beyond mastery of core mathematical procedures. But performance on the "basics" had not improved either. Whether this was due to back-to-basics curricula being watered-down versions of their pre-Sputnik counterparts, to a different social climate after the 1960s where schooling (and discipline) were de-emphasized, or because it is difficult for students to remember and implement abstract symbolic manipulations in the absence of conceptual understanding, was (and is) hotly debated. What was not debated, however, is that the mathematical performance of U.S. students was not what it should have been.

In response, the NCTM (1980) published *An Agenda for Action*. NCTM proposed that an exclusive focus on basics was wrongheaded, and that a primary goal of mathematics curricula should be to have students develop problem-solving skills. Back to basics was to be replaced by "problem solving."

From the jaundiced perspective of a researcher in mathematical thinking and problem solving, what passed for problem solving in the 1980s was a travesty. Although research on problem solving had begun to flower, the deeper findings about the nature of thinking and problem solving were not generally known or understood. As a result, the problem solving "movement" was superficial. In the early 1980s, problem solving was typically taken to mean having students solve simple word problems instead of (or in addition to) performing computations. Thus a sheet of exercises that looked like

$7 - 4 = ?$

might be replaced by a sheet of exercises that looked like

John had 7 apples. He gave 4 apples to Mary. How many apples does John have left?

But otherwise, things remained much the same. Part of the difficulty lay in the mechanisms for producing textbooks, a topic discussed in greater detail below. Soon after the publication of *An Agenda for Action*, major publishers produced problem-solving editions of their textbooks. But the changes were trivial, typically consisting of insertions of problem-solving sections (a page or two long) at the end of chapters that were otherwise essentially unchanged.

CONDITIONS IN THE 1980s—
THE BACKDROP FOR THE *STANDARDS*

It is important to understand the context that made the *Standards* (and their impact) possible. Here are some of the factors involved.

Yet Another Crisis

In the 1980s, the crisis was economic rather than military, but major nonetheless. Japanese and other Asian economies waxed as the American economy waned. The national deficit soared, and the nation felt besieged and vulnerable. In 1981, U.S. Secretary of Education T. H. Bell appointed the National Commission on Excellence in Education, which produced the very influential report, *A Nation at Risk*. The National Commission on Excellence in Education (1983) report began as follows:

> Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world. . . . The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people. . . .
>
> If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in student achievement made in the wake of the Sputnik challenge. Moreover, we have dismantled essential support systems which helped make those gains possible. We have, in effect, been committing an act of unthinking, unilateral educational disarmament. (p. 1)

In mathematics specifically, the very poor showing of U.S. students on the Second International Mathematics Study (McKnight et al., 1987; McKnight, Travers, Crosswhite, & Swafford, 1985; McKnight, Travers, & Dossey, 1985) gave license for change—although what direction that change might take was anybody's guess.

The (Non)role of the NSF

The NSF had played a significant role in supporting post-Sputnik curricula in mathematics and other fields, but it could not play such a role in direct response to *A Nation at Risk* and the sentiments it reflected. The tide regarding federal funding of innovative education efforts had turned, thanks to a political controversy over an NSF-supported elementary school science and social science curriculum called Man: A Course of Study (MACOS). MACOS met with initial success, and then a strong political backlash:

> The first sign of impending trouble appeared in Lake City, a small market town in northern Florida (population 10,000), in the fall of 1970. Shortly after school opened in September, Reverend Don Glenn, a Baptist minister who had recently moved to Lake City visited his daughter's sixth-grade class. . . . The school was under a court ordered integration plan. The teachers had chosen the materials because they felt they might help ease racial tensions. However, when Reverend Glenn saw the materials he formed a study group to examine MACOS in detail. Glenn claimed that the materials advocated sex education, evolution, a "hippie-yippee philosophy," pornography, gun control, and Communism. With support of a local radio station he broadcast four hour-long programs criticizing MACOS. He read excerpts from the student and teacher materials and warned that MACOS was a threat to democracy. This set off a growing series of attacks on MACOS over several years that led to a full scale Congressional debate of MACOS in both houses in 1975. NSF launched an internal review of its Education Directorate activities including an audit of the fiscal management of the project at EDC. While the audit revealed little to complain about, the damage in a sense was done. Dow quotes the former acting assistant director for science education, Harvey Averch, "It was the worst political crisis in NSF history." (Lappan, 1997, The MACOS Materials: How Success Can Go Awry section, para. 3)

In the 1980s, the NSF did not dare to engage in the support of what might be seen as a potential national curriculum. To do so would risk the wrath of Congress.

This story is important for at least two reasons. First, it establishes the context for private action by the NCTM. Had there been a federal presence in the reform arena, NCTM might or might not have gotten involved. But there was a clear perception of crisis in mathematics education and an absence of leadership in dealing with it. Thus the stage was set. Second, the MACOS affair shows how strongly politics can intervene in matters of curriculum. A well-organized group with no curricular expertise can get members of Congress to take up its cause, resulting in "the worst political crisis in NSF history."

The National Curriculum Context and the Role of Publishers

One of the American myths (in the anthropological sense of master narrative) related to "states rights" is that every one of the 50 states is educationally autonomous and can do as it sees fit with regard to educational goals and standards. This autonomy was especially prized after the MACOS affair, when even a hint of a federally sponsored curriculum would cause serious problems. In fact, the myth of independence goes even further: There are roughly 15,000 school districts in the United States, and (at least in 1980[3]) each district had significant freedom in setting goals and in selecting instructional materials to achieve those goals.

The reality was otherwise. There was indeed a free market, but market forces and the fact that teaching tended to be textbook driven constrained district choices to the point where the United States had a de facto national curriculum in mathematics, and one that changed slowly at that. Three states—California, Texas, and New York—were "textbook adoption states" in which mathematics curricula were specified and books meeting the curricular goals were identified. In California, for example, it is true that any school district could buy any books it wanted to. However, the district was reimbursed by the state for its textbook purchases only if the books purchased were listed on the state-approved textbook adoption list. Thus, although independence was theoretically available, the price was steep. Relatively few districts were willing to bear the cost.

At the national level, it was considered financial suicide for any major text series to fail to meet California, Texas, and New York's adoption criteria. Each state held a significant part of the national market share, and it was fiscally irresponsible for a publisher to make itself ineligible for that part of the market. (In 1992, publishers' representatives told members of the California Mathematics Framework Committee that it cost approximately U.S.$25 million to develop a K-8 textbook series and bring it to market. Such huge development costs tend to lead to conservative business practices.) As a result, the major publishers' textbook series were all designed to meet California, Texas, and New York's textbook adoption criteria. School district text selection committees in Georgia, or in Kansas, or in Montana typically chose their local texts from among the major publishers' textbook series—but all of those bore a strong resemblance to each other because of the constraints just discussed. Thus, in many ways independence—although theoretically available—was an illusion.

Some other facts about K-8 mathematics textbook publishing through the 1980s must be understood. Textbooks typically come packaged in series (for all of kindergarten through Grade 8, or separate packages for elementary school and middle school) rather than being targeted for individual grades. This makes sense in terms of coherence: Lining up the content of books written by individual authors and targeted for individual grades would be a nightmare. Textbook adoptions tend to take place about once every 7 years, after which a district is (unless it wants to bear undue expense) locked into its texts until the next adoption cycle. So, publishers tie their development to the major states' frameworks and textbook adoption criteria cycles. Their modus operandi for producing a series bears the stamp of the industrial revolution. For the sake of consistency, a "production-line approach" to generating a series results in a much more consistent product than, say, autonomy and artisanship in the books at each of the different grade levels. The major publishers typically seek a high profile series editor or editorial team to shape the series—to make decisions about style, scope, and sequence and to produce design specs for each section of each book. The name of this person or these persons graces the cover of the series. But much of the writing is done in piecework fashion by individual authors hired to write to the specs. As one can imagine, this kind of division of labor works well for the production of a standard product (which textbooks were, for many years). It is, however, a very conservative mechanism. It is extremely difficult to train a team of writers distributed across the country to produce innovative materials aimed at a new set of intellectual goals.

Cognitive and Epistemological Revolutions

In the 1970s and 1980s, scholars from disparate disciplines including anthropology, artificial intelligence, cognitive psychology, computer science, education, linguistics, and philosophy joined to form the new interdisciplinary field of cognitive science. It is always dangerous to say "this time things were different," but this time, things were different. What has been called the cognitive revolution (see Gardner, 1985) produced significantly new ways of understanding knowledge, thinking, and learning. Here is the core idea.

The naïve view is that mathematical competence is directly related to what one "knows" (facts, procedures, conceptual understandings) and that knowledge accumulates with study and practice. This is hard to argue with as far as it goes. It is, however, dramatically incomplete. Studies of expert mathematicians show that there are other, equally critical aspects of mathematical competence. The mark of powerful learning is the ability to solve problems in new contexts or to solve problems that differ from the ones one has been trained to solve. Competent mathematicians have access to a wide range of problem-solving strategies. They use these strategies to make sense of new problem contexts or to make progress toward the solution of problems when they do not have ready access to solution methods for them. Research shows that students often fail to solve problems because they use the resources at their disposal (including time and what they do know) inefficiently. For example, they would doggedly pursue inappropriate solution directions, thus depriving themselves of the opportunity to try other, potentially more effective methods. In contrast, more competent problem solvers would periodically take stock of their solution attempts and make decisions about what to pursue on the basis of those evaluations. Students would often stop working on problems, believing (on the basis of their prior experience with school mathematics) that problems should be solvable within a few minutes—and thus that a problem they failed to solve in short order was basically unsolvable. Competent problem solvers persevered. They were also able to communicate the results of their mathematical work effectively, both orally and in writing.

In short, mathematical competence was shown to depend on a number of factors:

— having a strong knowledge base;
— having access to productive problem-solving strategies;
— making effective use of the knowledge one has (this is known as *metacognition*); and
— having a set of productive beliefs about oneself and the mathematical enterprise (which position the individual to act in mathematically appropriate ways).

Research indicates that classroom instruction, which tends to focus almost exclusively on the knowledge base, deprives students of problem-solving knowledge. It gives them little experience grappling with tough challenges and fosters the development of numerous unproductive beliefs. (See de Corte, Verschaffel, and Greer (1996) and Schoenfeld (1992) for detailed summaries of the literature.) The research does not, in general, say how these problems should be fixed—but it does make it clear that such problems need to be fixed. On the basis of what was known by the middle of the 1980s, it was clear that goals for mathematics instruction had to be much broader than mere content mastery. Students needed to learn to think mathematically as well as to master the relevant mathematical content.

Demographics and National Context

The demographic trends identified above (specifically, increasing numbers of students staying in high school and graduating) continued from the post–World War II period through 1970. Enrollments in elementary and high school increased from 25 million to about 45 million from 1950 through 1970, at which point they leveled off (from 1970 to 2000 there was a slight dip and then a rise in enrollments, starting at 45 million in 1970, dropping to 40 million in 1985 and rising to 48 million in 2000) (Rosen, 2000, pp. 105-107). These demographic pressures exacerbated the tensions discussed earlier. There was, in essence, one viable track through high school mathematics: the traditional course sequence described in the previous section that was designed for the "college intending." De facto, high school mathematics was (still) for the elite. Students who took the standard sequence were prepared for postsecondary education. Most states required only 1 or 2 years of high school mathematics. Those who found the college track inhospitable were generally placed in courses such as "business math" or "shop math." Although there may have been some useful mathematics in such courses, they tended not to lead anywhere—but to satisfy requirements that enabled the students in them to graduate. Mathematics for the college intending and mathematics for the rest were separate and unequal.

In 1985, the National Research Council (NRC) established the Mathematical Sciences Education Board as a mechanism for devoting sustained attention to issues of mathematics instruction. The NRC produced a series of reports, including *Everybody Counts* (NRC, 1989) and *A Challenge of Numbers* (Madison & Hart, 1990), which documented some of the more troubling demographics surrounding the traditional curriculum and set the stage for something new (although not specifying what it might be).

A Challenge of Numbers (Madison & Hart, 1990) shows that the attrition rate from mathematics, from 9th grade on, was roughly 50% per year; worse still, the attrition rate for Latinos and African Americans was significantly larger. For example, African Americans composed 12% of the population of 8th graders, 11% of 12th graders, 5% of those earning a bachelor's degree in mathematics, and 2% of those earning master's and Ph.D. degrees in mathematics. Madison and Hart (1990) also noted that "in the new-doctoral-degree population . . . the fraction of U.S. citizens has tumbled from four-fifths to less than one-half" (p. 36). A major point of the volume was that the nation's preeminence in mathematics and science was in jeopardy because of declining numbers and interest.

The NRC's (1989) *Everybody Counts* took a broader perspective, also pressing the urgency of the situation. Some relevant quotes include

— "Mathematical literacy is essential as a foundation for democracy in a technological age." (p. 8)

— "Public acceptance of deficient standards contributes significantly to poor performance in mathematics education." (p. 9)
— "We are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond reach. Unless corrected, innumeracy and illiteracy will drive America apart." (p. 14)
— "Equity for all requires excellence for all; both thrive when expectations are high." (p. 29)
— "Mathematics curricula at all levels must introduce more of the breadth and power of the mathematical sciences." (p. 43)
— "America needs to reach consensus on national standards for school mathematics." (p. 46)
— "All high school students should study a common core of broadly useful mathematics." (NRC, 1989, p. 49)

Released in the spring of 1989, *Everybody Counts* set the stage for the release of the NCTM *Standards* that fall.

THE *STANDARDS*

In 1986, the NCTM's board of directors established the Commission on Standards for School Mathematics, chaired by Thomas Romberg. The following year, NCTM President John Dossey appointed a team of 24 writers to produce the *Standards*. The group produced a draft in the summer of 1987, obtained feedback on the draft during the 1987-1988 working year, and revised the draft in the summer of 1988. NCTM published the *Standards* in the fall of 1989 and began a major effort to bring the work to the attention of its membership. Copies of the *Standards* were mailed to all members, and various aspects of the *Standards* became the themes for regional and national NCTM meetings.

Before we turn to the substance of the document itself, a few things should be noted. The context for its production was described immediately above. In a sense, NCTM was jumping into the breach: There was a nationwide problem and no federal mechanism for dealing with it. There were, of course, antecedent reports: Through the years, various professional groups had issued proclamations about what the nature of the curriculum should be. Generally speaking, however, such documents turned out to have little lasting (or even short-term) impact on the curriculum and were mostly of historical interest. Although it is the case that efforts were made to position the *Standards* well in terms of national dialogue—NCTM and the NRC coordinated the releases of *Everybody Counts* and the *Standards*, for example—such reports tend to come and go. There was little reason to expect things to be different this time around. Various mathematicians invited by Dossey to participate in the writing effort declined; from their perspective, this was an in-house affair for NCTM and not necessarily worth the effort from outsiders. None of the authors or others involved in the production of the *Standards* had any idea of what the ultimate magnitude of the response to their document would be. Indeed, what turned out to be some of the most controversial aspects of the *Standards*, such as the lists of topics that should receive increased and decreased attention in the curriculum, were last-minute additions during the 1988 editing process.

As the introduction to the *Standards* notes, the goal of the writers (including the commission) was to "create a coherent vision of what it means to be mathematically literate" in a rapidly changing world and to "create a set of standards to guide the revision of the school mathematics curriculum" (NCTM, 1989, p. 1). The definition of *standard* given by the authors was as follows: "*Standard*. A standard is a statement that can be used to judge the quality of a mathematics curriculum or methods of evaluation. Thus, standards are statements about what is valued" (NCTM, 1989, p. 2). That is, the *Standards* were intended to be a set of desiderata for curriculum development and assessment. They were not intended to be a "scope-and-sequence" document (a blueprint for curriculum development); nor were they intended to be a set of specifications for examinations that would say whether students "met the standard."

The NCTM's (1989) *Standards* focused on new goals for society at large and for students in particular: "New social goals for education include (1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate" (p. 3). The *Standards* were oriented toward

> five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically. (NCTM, 1989, p. 5)

They were grounded in assumptions about learning being an active process rather than one of memorization and practice:

> This constructive, active view of the learning process must be reflected in the way much of mathematics is taught. Thus, instruction should vary and include opportunities for:

— appropriate project work;
— group and individual assignments;
— discussion between teacher and students and among students;
— practice on mathematical methods;
— exposition by the teacher. (NCTM, 1989, p. 10)

The document was divided into four sections, three devoted to defining content and process standards for three grade bands (kindergarten through Grade 4; Grades 5 through 8; Grades 9 through 12) and a fourth for defining standards for student and program evaluation. As much of the controversy has focused on content and process standards, I shall focus on those as well. I also emphasize the parts of the document that led to controversy.

In many ways, the *Standards* was both a radical and a conservative document. On the conservative side, the document was written for the broad NCTM constituency: mathematics teachers across the nation. It represented a consensus among the writers as to what was possible—and thus fell far short (as it should have) of what some visionaries might have thought possible. The cost of consensus was precision: Michael Apple (1992) called the *Standards* a "slogan system" encompassing "a penumbra of vagueness so that powerful groups or individuals who would otherwise disagree can fit under the umbrella" (p. 413). Indeed, some years later, one state assessment system would assess students' mathematical competency via portfolios containing the students' work on extended projects, whereas another state would employ multiple-choice tests that focused on basic skills. Both did so in the name of the *Standards*.

On the radical side, the *Standards* challenged (or was seen as challenging) many of the assumptions underlying the traditional curriculum. As noted above, the traditional curriculum bore the recognizable traces of its elitist ancestry: The high school curriculum was designed for those who intended to pursue higher education. Yes, it is true that half the students dropped out of the mathematics pipeline each year after Grade 9—but as some see it, this is because honest-to-goodness mathematics is hard. The 50% annual attrition rate was taken by some as confirmation of the difficulty of mathematics. For them, there was the suspicion that the curriculum would have to be dumbed down in order for more students to succeed. That is, only a bastardized curriculum (a lowering of real standards) could result in greater success rates. These fears were exacerbated by, among other things, the "increased attention/decreased attention" charts in the *Standards*. Topics to receive decreased attention included the following: complex paper-and-pencil computations, long division, rote practice, rote memorization of rules, teaching by telling, relying on outside authority (teacher or an answer key), memorizing rules and algorithms, manipulating symbols, memorizing facts and relationships, the use of factoring to solve equations, geometry from a synthetic viewpoint, two column proofs, the verification of complex trigonometric identities, and the graphing of functions by hand using tables of values. These can be seen as the meat and potatoes of the traditional curriculum.

The *Standards*, buttressed by NCTM's call for "mathematics for all" and the equity agenda in *Everybody Counts*, clearly sat in the education-for-democratic-equality and education-for-social-mobility camps. In contrast, whatever the intention may have been, the reality was that the traditional curriculum was a vehicle for social efficiency and the perpetuation of privilege. Statistically speaking, the rich stayed rich and the poor got disenfranchised. There is a long history of data indicating that race and socioeconomic status correlate with mathematics performance, with drop-out rates, and with economic opportunity (Kozol, 1992; National Action Committee for Minorities in Engineering, 1997; NSF, 2000). Thus the *Standards* could be seen as a threat to the current social order.

Epistemologically, with its focus on process, the *Standards* could be seen as a challenge to the "content-oriented" view of mathematics that predominated for more than a century. Each of the three grade bands began with the following four standards: mathematics as problem solving, mathematics as communication, mathematics as reasoning, and mathematical connections. Only after these four process standards were described did the *Standards* turn to what has traditionally been called mathematical content.

In short, the seeds for battle were sown—not that anyone at the time could predict that the *Standards* would have much impact or that the battle would rage. The *Standards* were vague. This was part of their genius and part of what caused so much trouble. Because of their vagueness, they served as a Rorschach test of sorts—people tended to read much more into them than was there. (For many years, people would claim this or that was in the *Standards*, when a close examination showed it was not.) The genius is that the *Standards* set in motion a highly creative design process during the following decade, far transcending what the authors of the *Standards* could have produced in 1989. Because it was in essence a vision statement rather than a set of design specs, it proved remarkably enfranchising: During the coming years, different groups produced very different sets of materials "in the spirit of the *Standards*." And there is the rub. Some of the materials produced would be considered pretty flaky. Some of the classroom practices employed in the name of the *Standards* would appear pretty dubious. And the *Standards* would be blamed for all of them.

THE REACTION, PART 1: THE FIRST FEW POST-*STANDARDS* YEARS

As noted above, there was no reason to expect, a priori, that the *Standards* would have any more impact than myriad curriculum documents that had preceded it, in mathematics or in other fields. But for some reason, the time was right. The epistemological underpinnings of the *Standards* were consistent with contemporary research; soon (and not, as far as I could tell, by design) it became de rigueur to invoke the *Standards* in research proposals. The NSF, which had had its hands tied ever since the MACOS fiasco, found itself in a new position. There was a compelling national need: The sense of crisis following *A Nation at Risk* was still strong. A solution had been proposed—but not a solution that (shades of MACOS) would result in a national curriculum. Because it offered a vision statement that could support radically different instantiations, the *Standards* could not be seen as offering a de facto national curriculum. Indeed, different groups could be supported in developing very different curricula. Hence, local autonomy could be supported. Moreover, given the character of curriculum development described above and the great cost in developing new curricula (for a market that might or might not ever develop), commercial publishers were most unlikely to step up to the plate. Thus the NSF decided to play a catalytic role. In 1990 and 1991, the NSF issued requests for proposals for the development of curricula consistent with the *Standards*. It went on to fund four centers devoted to the support of standards-based curricula.[4]

Beyond that, a "standards movement" took the nation by storm. Motivated by NCTM's example in mathematics, the NRC undertook the creation of the *National Science Education Standards* (NRC, 1995); fields as diverse as English and social studies took on similar challenges.

A Key Datum: A Paucity of Data (That Is, War in Absence of Real Ammunition)

We are now about to turn to the origins of the math wars themselves, in California. One thing that must be understood as we do is that a decade of battle was conducted in the absence of any real data. Here is the chronology. The *Standards* was published in 1989 by the NCTM, and the California Department of Education (1992) published the *Mathematics Framework for California Public Schools, Kindergarten Through Grade 12* (the *Frameworks*) (which took the *Standards* somewhat further along the lines of reform). In the early 1990s, NSF released its curriculum requests for proposals. Spurred on by the *Frameworks*, some mainstream textbook publishers made reform texts available for textbook adoptions in 1993 and 1994. No large-scale data were gathered on the effectiveness of these curricula. Many of the NSF-supported reform curricula were in either alpha or beta testing at that point; it was not until the mid-1990s that they became widely available, and testing data on those curricula tended to use "home-grown" measures. It was not until the late 1990s that full cohorts of students had worked their way through the entire reform curricula. Only at the turn of the 21st century did large-scale data evaluating the impact of those curricula begin to become available. As it happens, the evidence at this point is unambiguously in favor of reform (see, e.g., ARC Center, 2003; Senk & Thompson, 2003). But such data turn out to be largely irrelevant to the story of the math wars. When things turn political, data really do not matter.

For the record (and in anticipation of what follows below) I wrote the following in 1994:

> In the United States we have no real existence proofs—that is, there do not exist substantial numbers of students who have gone through the reform curricula and emerged demonstrably competent to do further work either in collegiate mathematics or in the workplace. . . . Why abandon the old content specifications, some would say, until you can replace them with something that is demonstrably better? The more conservative among us want to save the rest of us from going off the deep end.
>
> The fact is that, in the absence of either large-scale empirical proof of success or the existence of compelling and documentable standards, there *is* reason to be cautious. The traditionalists are nervous for good reason. It should be noted, however, that the resistance to change is not based on the purported success of current curricula (one is hard-pressed to find people who say that we are doing things well!), but on the fear that the replacement will be even worse. Here it is worth returning to the notion of a zero-based curriculum planning process. Suppose we declared that any proposed curriculum must, in order to be implemented, make a plausible case that it would do well. The reform curricula would fail because they cannot yet produce real proof, or real standards. But current mainstream curricula would fail even more strongly because there exists a massive body of evidence indicating that they do not work. Conclusion: we cannot and must not inhibit the extensive field testing of well-designed reform curricula, but we must at the same time be vigilant. (Schoenfeld, 1994, pp. 73-74)

WAR(S)!

Before the math wars in California, there were reading wars (see Pearson, 2004 [this issue], for substance and details). That makes some sense at least: What the state mandates as reading matter can be controversial. Indeed, the battles quickly polarized around right versus left politics. The Religious Right got up in arms when books it found objectionable were put on the state's reading lists and when the California Learning Assessment System asked students to write about their feelings in response to what they read. From the (very) traditionalists' point of view, the role of schooling should be to provide authoritative knowledge. Certain things are right or wrong; it is the responsibility of the teacher to say what is right and to make sure the students learn it. What the students feel is irrelevant and inappropriate for discussion in school; what counts is what the students are taught and what they should know. Protests began and began to be effective.

But how does one get from objecting to what children are given to read to doing battle nationally over whether to teach phonics or whole language? A major factor is the view of authority described above. The concomitants of a phonics approach are discipline, structure, and authority. A concomitant of the whole-language approach is *invented spelling*, which from the perspective of the traditionalists is as undisciplined and individualistic as you can get ("You don't know how to spell a word? Make it up!"). Thus the conflict of phonics versus whole language can be seen as a microcosm of the larger historical traditional-versus-progressive debate about American schooling. As such, it fit the agenda of the New Right, a powerfully connected and politically influential group that included the far right and neoconservatives such as Diane Ravitch, Chester Finn, E. D. Hirsch, and Lynn Cheney (Berliner & Biddle, 1995).

But that is reading. What about mathematics? There are interesting parallels and interesting differences.

For a more detailed history of the math wars in California, see Rosen (2000); a nicely balanced account can also be found in Jackson (1997a, 1997b). A brief capsule history is as follows. In the 1980s, California was at the forefront of the reform movement catalyzed by calls for change such as those in *A Nation at Risk*. The California Department of Education's (1985) *Mathematics Framework for California Public Schools, Kindergarten Through Grade 12* was considered a mathematically solid and progressive document, in many ways the antecedent of the 1989 NCTM *Standards*. State Superintendent of Education Bill Honig pushed educational reform strongly, and the California Mathematics Council (the state affiliate of NCTM) actively supported *Standards*-based practices. The *Frameworks* represented a next incremental step in the change agenda, grounded in the positive national reaction to the NCTM *Standards* and the growing research base on mathematical thinking and problem solving.[5] Publishers, seeing the positive national response to the *Standards* and seeing California as being on the leading edge of a national trend, created texts in line with their view of *Standards*- and *Frameworks*-based mathematics. In 1994, the California State Board of Education approved instructional materials consistent with the *Frameworks*.

As noted above, the *Standards* and the *Frameworks* were "vision statements" (or "slogan systems" if you prefer the pejorative) regarding the substance and character of instruction rather than blueprints for it. The upside of such documents is that they permit significant creativity, allowing designers to create innovative materials beyond the imagination of the *Standards'* and the *Frameworks'* authors themselves. But there is a significant downside as well. As Rosen (2000) noted,

> the new textbooks were radically different from the traditional texts' orderly, sequential presentation of formulas and pages of practice problems familiar to parents. New texts featured colorful illustrations, assignments with lively, fun names and sidebars discussing topics from the environment to Yoruba mathematics (prompting critics to dub new programs with names such as "Rainforest Algebra" and "MTV Math"). (p. 61)

In their alien appearance and inaccessibility to parents, the texts repeated some of the mistakes of the new math. Once the rhetorical battles heated up, they were easily caricatured as the "new-new math."

Moreover, reform called for new teaching practices. As quoted above, the *Standards* in particular and reform in general called for a balance of "appropriate project work; group and individual assignments; discussion between teacher and students and among students; practice on mathematical methods; [and] exposition by the teacher" (NCTM, 1989, p. 10). This too seems alien to people who have experienced mathematics instruction only in traditional ways. Teaching in the ways envisioned by the authors of the reform documents is hard. It calls for both knowledge and flexibility on the part of the teacher, who must provide support for students as they engage in mathematical sense making. This means knowing the mathematics well, having a sense of when to let students explore and when to tell them what they need to know, and knowing how to nudge them in productive directions. To pick one example, think about having students work together in groups. If you just tell a group of students to "work together," just about anything can happen. Structuring problems for groups so that interactions over them can be productive is a difficult design task. Structuring and supervising student interactions so that students can make progress on the problems, learn from each other, and know when they need more expert advice, is very hard. When these things are done well, students can learn a great deal. When superficial aspects of reform are implemented without the underlying substance, students may not learn much at all. The logistical problems of supporting reform in substantive ways should not be underestimated. Teachers who had themselves been taught in traditional ways were now being asked to teach in new ways and not given much support in doing it.

To cut to the chase, new materials and new practices raised concerns among some parents, some of whom enlisted outside help (from mathematicians, legislators, etc.) in combating the new practices and materials:

> Partly on the advice of sympathetic state legislators, parents started forming organizations to alert others to their cause. They designated representative spokespersons, created internet web pages, and adopted memorable names such as Mathematically Correct (see http://ourworld.compuserve.com/homepages/mathman/), H.O.L.D. (Honest Open Logical Debate) (see http://www.dehnbase.org/hold/), B.O.L.D. (Brea Open Logical Debate), and QED. (Rosen, 2000, p. 62)

Amid all this, advocates of reform committed some mistakes that were the public relations equivalent of handing the traditionalists a gun and saying "shoot me." For example, the California Learning Assessment System released a sample mathematics test item in 1994 in which students were asked to arrive at an answer and then write a memo justifying it. The sample response from a student who got the right answer but failed to write a coherent memo was given a low score, whereas a sample student response that contained a computation error but a coherent explanation was given a high score. Editorial comments raked the California Learning Assessment System (and reform in general) over the coals, saying that in the new, "fuzzy" math, being able to write baloney counted more than getting the right answer.

The Web sites listed above, and the political connections of those who ran them, facilitated the evolution of the antireform collectives into a potent political force. With some speed, what started out as a collection of local oppositional movements became a statewide movement—with the support of conservatives such as Governor Pete Wilson and California Assemblyman Steve Baldwin who, as chairman of the Assembly Education Committee, held public hearings on the *Frameworks* in 1995 and 1996. Ultimately the conservatives prevailed: State Board of Education President Yvonne Larson and State Superintendent of Public Instruction Delaine Eastin agreed to convene a new mathematics *Frameworks* writing team ahead of schedule. The state legislature enacted Assembly Bill 170, which according to the official legislative summary,

> requires the State Board of Education to ensure that the basic instructional materials it adopts for reading and mathematics in grades 1 to 8, inclusive, are based on the fundamental skills required by these subjects, including, but not limited to, systematic,

explicit phonics, spelling, and basic computational skills. (See http://www.cde.ca.gov/board/readingfirst/exhibit-i.pdf)

As a step toward a new framework, the state appointed a committee to produce mathematics content standards, which were ultimately adopted in December 1997 and which served as the content foundation for the framework. This process by which the document was adopted was unprecedented. Here is how it was described by David Klein (1998), an antireform activist:

> Question: What would happen if California adopted the best, grade-by-grade mathematics achievement standards in the nation for its public schools?
>
> Answer: The education establishment would do everything in its power to make them disappear.
>
> In December 1997, the State Board of Education surprised the world by not accepting extremely bad, "fuzzy" math standards written by one of its advisory committees, the Academic Standards Commission. Instead, in a few short weeks and with the help of four Stanford University math professors, the state board developed and adopted a set of world-class mathematics standards of unprecedented quality for California's public schools. (p. 15)

Note the rhetoric, both in tone and language: The draft standards submitted to the board were "extremely bad" and "fuzzy," and truly high standards would be challenged by "the education establishment." This kind of rhetoric had become common. *San Francisco Chronicle* columnist Debra Saunders (1995a, 1995b), for example, titled some of her columns "New-New Math: Boot Licking 101" and "Creatures from the New-New Math Lagoon." Maureen DiMarco, California state secretary of child development and education, and one-time candidate for superintendent of public instruction, referred to the new curricula as "fuzzy crap." Moreover, when the math wars entered the public arena, standards for truth and honesty declined substantially. Saunders' "boot licking" column contains the following excerpt:

> Witness "Mathematical Power," a handbook for parents put together by teachers at Jane Lathrop Stanford Middle School in Palo Alto. Under the heading "Mathematical Communication"—as in, it is better to write well about math than do it—teachers featured excerpts from student assignments, all glowing tributes to their new-new math classes.

Her column implies that there is no mathematical content in the handbook and precious little mathematical content in the classrooms at Jane Lathrop Stanford Middle School. A rejoinder by Cathy Humphreys (1995), the instructional supervisor at the school, indicates that the reality is otherwise:

> It is disturbing to see what Ms. Saunders chose to selectively extract and omit from the students' writing. For example, she selected a sentence from a piece of work called "A Day in Math Class," in which a student (a real 7th-grader named Kathy) describes being "startled and intrigued" by connections she had noticed between probability, Chaos Theory and a famous fractal called Sierpinski's Gasket. Why did Ms. Saunders ignore the mathematical context which inspired Kathy's comments? The cynicism Ms. Saunders used to describe this excerpt of work ("fishing for a compliment") is belittling and demeaning. Why does she presume that Kathy is only writing this to impress or influence her teacher?

Similarly, Jackson (1997b) discussed the tactics used:

> The scramble to find numbers to support certain viewpoints has led to some questionable uses of data. For example, the anti-reform group HOLD . . . recently circulated an email message containing data on the Elementary Level Mathematics Examination, which is administered in the California State University system to place students in appropriate mathematics courses. The data were stunning: In 1989, 28.2% of the students failed; in 1992, 45.0% failed; and in 1994, 54.0% failed. The email message [also posted on the HOLD web site] blamed California mathematics education reform for this poor showing. What the message neglected to say was that the examination was changed in 1992 to include not just two but three years of high school mathematics. (p. 817)

To return to Klein's (2003) discussion of the rewriting of the California Mathematics Standards: The truth is, shall we say, more complex than Klein made it out to be. The draft standards had been a year and a half in development and although far from perfect, had undergone significant public review and comment. They reflected current research—but not the perspective of the conservative majority of the state board. The board summarily rejected the draft, rewrote much of the elementary grades section itself, and commissioned mathematics faculty from Stanford (who had negligible experience with K-12 classrooms or curricula) to rewrite the standards for the secondary grades. It went on to disregard written comments from notables such as Hyman Bass, research mathematician and director of the NRC's Mathematical Sciences Education Board, and William Schmidt, who had conducted curriculum content analyses for the Third International Mathematics and Science Study, to the effect that the draft standards were of high caliber and that the alternative was not. In his letter to the board, Bass also commented on the tone of the debate:

> The tragedy of the current debate in California is that political forces and agendas, and the belligerent and scoffing rhetoric they employ, have usurped the stage for the kind

> of honest, probing, and multidisciplinary discourse which is now so desperately needed. . . . I hope, for the sake of California and of the nation, that your Board will find the means to rescue educational debate from the hands of abusive extremists. The alternative ill serves us all. (H. Bass, personal communication, November 13, 1997)

For examples of extremism in public discourse, readers may wish to look at the thesaurus of derogatory terms for reform curricula and texts on the Mathematically Correct Web site http://www.mathematicallycorrect.com/glossary.htm. For examples of extreme and intemperate dialogue from both sides of the math wars, follow the threads of math-wars or standards discussions in the amte, math-learn, and math-teach discussion groups at the Mathematics Forum, http://mathforum.org/discussions/.

The gloves were off and those who held power did not hesitate to use it. The revision of the *Standards* was one example, the late-1996 composition of the curriculum framework and criteria committee another. Appointments are made by the state board. The established process for creating such committees is for the board to solicit recommendations for membership from its curriculum commission—in this case the commission that had earlier recommended the approval of reform-oriented texts.

> However, newly appointed board member Janet Nicholas led a successful campaign to overturn the Commission's recommendations for the Framework Committee and replace them with a new group containing leaders of the tradition campaign. . . . The California Mathematics Council and several allied groups loudly protested this move, decrying the Board's disregard for an established process and the expertise of its own commissioners. (Rosen, 2000, pp. 64-65)

Not only did the complaints fail, but Governor Wilson also went on to appoint another archtraditionalist, Marion Joseph, to the board (Joseph had been a leader of the anti–whole language campaign).

By all reports, the discussions within the Curriculum Framework and Criteria Committee, and its interactions with the state board, were highly contentious. A series of "minority reports" by Bill Jacob (Becker & Jacob, 2000; Jacob, 1999, 2001; Jacob & Akers, 2003), a reform-oriented member of the committee, provide the gory details. I shall skip over most of those details, which make interesting reading, but focus on one highly public and controversial set of actions by the state board:

> California law requires that state-adopted instructional materials "incorporate principles of instruction reflective of current and confirmed research" (CA Education Code 60200c-3). But even in such an apparently noncontroversial area, California has opened new categories of dispute. For example, the state board invited E. D. Hirsch,

Jr., to speak on this issue in April 1997. In the written version of his comments, Hirsch ridiculed "mainstream educational research," as found in "journals such as the *Educational Researcher*," explicitly stating, "This is a situation that is reminiscent of what happened to biology in the Soviet Union under the domination of Lysenkoism, which is a theory that bears similarities to constructivism. . . ."

Citing math education experts John Anderson, David Geary, and Robert Siegler on the matter of what research shows that math students need, he goes on, "They would tell you that only through intelligently directed and repeated practice, leading to fast, automatic recall of math facts, and facility in computation and algebraic manipulation can one do well at real-world problem solving." Hirsch received a standing ovation from the state board, and then the board moved forward in line with his recommendations. (Becker & Jacob, 2000, p. 535)

With all due respect, Hirsch's claim about Lysenkoism is unmitigated nonsense, indicating his total misunderstanding of and disregard for serious educational research. His claim about real-world problem solving is a dubious extrapolation from work that is not at all central to the problem-solving literature (my own area of expertise).

Becker and Jacob (2000) continued:

In spite of the state board's instructions to base the framework on research, the framework committee never discussed any research articles. Instead, in July 1997 state board member Janet Nicholas announced a contract award to Douglas Carnine, a professor at the University of Oregon, to provide a review of high-quality mathematics research on which the framework's instructional strategies would be based. (p. 535)

Some background is appropriate here. Carnine, like Hirsch, is anti-research; see Carnine's (2000) "Why Education Experts Resist Effective Practices (And What It Would Take to Make Education More Like Medicine)," written for the Fordham Foundation. Carnine advocates direct instruction, and he is an author of two direct instruction programs currently being marketed in California: *DISTAR* and *Connecting Math Concepts*, produced by Science Research Associates (SRA)/McGraw-Hill. Thus, Carnine stood to profit financially from a state board endorsement of direct instruction. That would appear to be a conflict of interest, but the state board proceeded in any case—with Carnine being the sole purveyor of research on effective instruction to the board. Not to put too fine a point on it, the report was shoddy at best. The methodology was questionable, so much so that the American Educational Research Association's Special Interest Group for Research in Mathematics Education, a collection of specialists in the area, wrote a public letter to the state board disputing Carnine's methods. Summaries of many of the papers reviewed were inaccurate, and some of the report's conclusions were not clearly related to the research summary. Nonetheless, Carnine's report went on to serve as the basis for the section on instructional strategies in the board-approved 1999 *Frameworks*.

There is more, but that gives the flavor of events in California—events that continued with the development of a high-stakes accountability system tied to the *Frameworks*. But we should turn our eyes to the national scene, if only briefly. The traditionalist base in California became a base for a national antireform movement.

As noted in the introduction, the math wars had grown to national scale by 1998. Diane Ravitch, Chester Finn, and Lynne Cheney among others had weighed in against fuzzy, new-new math in the national media. The dispute was so vitriolic that Secretary of Education Richard Riley pleaded for civility in his 1998 address to the joint mathematics societies. Klein (2003) wrote about the events that followed:

> In October 1999, the U.S. Department of Education recommended to the nation's 15,000 school districts a list of math books, including several that had been sharply criticized by mathematicians and parents of school children across the country for much of the preceding decade. Within a month of that release, 200 university mathematicians added their names to an open letter to Secretary Riley calling upon his department to withdraw those recommendations. The list of signatories included seven Nobel laureates and winners of the Fields Medal, the highest international award in mathematics, as well as math department chairs of many of the top universities in the country, and several state and national education leaders. By the end of the year 1999, the U.S. Secretary of Education had himself become embroiled in the nation's math wars. (Introduction, para. 1)

What Klein neglects to mention is that his was the first signature on the letter, and that the powerful antireform network in which he plays a central role had orchestrated the signature gathering. The signature gathering was a highly political act. Ralston observed that

> the letter was signed by 6 mathematicians and "endorsed" by 219 others, almost all of whom were research mathematicians, among them many eminent ones. The 6 signers were, I assume, familiar with all 10 programs but it is essentially certain that at most a handful of the endorsers were familiar with all 10 and almost certain that most were familiar with none. (A. Ralston, personal communication, July 10, 2003, regarding draft article for publication)

By the late 1990s, the antireform movement had reached a level of organization and efficiency that enabled it to quickly mount high profile, large-scale efforts such as the one described above. Antireform Web sites promise quick help for those who find their own school districts "threatened" by reform. And, they deliver. Shortly after an article describing the successes of standards-based instruction in Pittsburgh, Pennsylvania was published (Schoenfeld, 2002), antireformers mounted a campaign that resulted in a "showdown" before Pittsburgh's Board of Education. This past year, New York City's mathematics curriculum choices were embroiled in so much controversy that it became front-page news across the country.

As noted above, tactics employed in the math wars can be rather nasty. Robert Megginson (personal communication, July 5, 2003) has noted a strong similarity between the tactics used by some antireformers and the antievolution tactics used by "creation scientists." Referring to Michael Shermer's (2002) discussion of what might be called the "creation wars," Megginson asked:

> Has anyone noticed that the more extreme members of Mathematically Correct have taken their strategy and tactics, almost line for line, straight out of the creation scientists' playbook? In particular—
>
> 1. Number one tactic—Go after the boards in the big states, particularly California and Texas, that evaluate and approve textbooks. If you can get the books with your point of view at the top of the playlists, you are in great shape.
> 2. Number two tactic—Plant fear in the minds of well-meaning parents, who genuinely and understandably want the best for their children, that the schools are subjecting their children to unproven theories that may result in their not getting into heaven or Berkeley.
> 3. Number three tactic—Constantly demand proof of their position (which you never intend to accept) from those who disagree with you, attempting to create the impression that they do not really have much. When they offer any evidence, poke and prod hard at every facet and in every crevice of it until something is found that seems not fully justified or a bit controversial or counterintuitive, and use that to discard the entire piece of evidence.
> 4. Above all, treat any disagreement among your opponents, or modification in position due to new evidence ("even so-and-so doesn't believe in her former position on this anymore") as an indication that your opponents have it wrong, and therefore (!) that you have it right. (R. Megginson, personal communication, July 5, 2003)

Generally speaking, those on the reform side of the wars have been slow to develop effective techniques to counter the most extreme attacks of the antireformers. Some (e.g., Jacob, 1999, 2001) have tried to provide public documentation of abuses of process. Some (e.g., Susan Ohanian, see http://www.susanohanian.org/) raise loud voices of protest. Some (e.g., Mathematically Sane, see http://mathematicallysane.com/home.asp) seek to provide both evidence and counter-terrorist tools. Major reform organizations have gotten smarter about dotting their *i*s and crossing their *t*s in public documents. The most significant reform document since the 1989 *Standards* is its successor, NCTM's (2000) *Principles and Standards for School Mathematics*. It was clear from the beginning that this document would be controversial, and that the only hope for it to get a fair reception was to involve all the relevant constituencies from the very beginning—asking for their input and then taking it seriously. NCTM created a series of association review groups that were asked for ongoing input and commentary. NCTM distributed thousands of copies of a draft version of the document (roughly 30,000 hard-copy versions of the draft were distributed, and roughly 50,000 copies were downloaded from NCTM's Web site). It commissioned the NRC to review the process by which it responded to the thousands upon thousands of comments it received. A result was a commendation by the NRC for the integrity of its process and an unprecedented "Letter of Appreciation" from the chief officers of 15 major mathematical societies. The letter ends as follows:

> With this letter, representatives of the following member organizations of CBMS [the Conference Board of the Mathematical Sciences] wish to register their appreciation to the NCTM for the design, and implementation with integrity, of this process. With this, the NCTM has established a model, all too rare, of how to stage civil, disciplined, and probing discourse among diverse professionals on matters of mathematics education. (NCTM, 2000, p. xv)

Will it work? Will civil, disciplined, and probing discourse prevail, and will there be a return to balance? That remains to be seen.

REFLECTIONS

To a disinterested outsider, aspects of the reading wars and the math wars just make no sense. Consider the controversy of phonics versus whole language, for example. Of course children need to learn to sound out words—a healthy dose of phonics at the right time is salutary. Of course children need to make sense of what they are reading—learning to use context is an essential skill and motivational as well. Any sensible person would realize that children need both phonics and reading for understanding. Either of the two perspectives, taken to extremes, is nonsensical. The polarization that resulted in a "winner take all" battle between the two extremes is equally nonsensical. The same is the case in mathematics. An exclusive focus on basics leaves students without the understandings that enable them to use mathematics effectively. A focus on "process" without attention to skills deprives students of the tools they need for fluid, competent performance. The extremes are untenable. So, why have so many people taken extreme positions, and why are things as polarized as they are? More important, what might be done about it?

Neither the extreme reform camp nor the extreme traditionalist camp is monolithic; each can be considered a confederation of strange bedfellows. Some of the reformers are committed to a pure vision of discovery learning, consistent with the traditionalists' caricatures. Some are committed to the democratic, equity vision enunciated in the *Standards*. Some were provoked by the traditionalists and took a strong defensive stance. Similarly, some of the traditionalists feel that the mathematical values they cherish are being challenged. Some feel that equity and excellence are in tension, and that the reform curricula, whatever their social goals, must perforce weaken mathematics (and U.S. national security and the economy). Some like a good fight. Some like the opportunity to take the limelight. And some see opportunities to make their political fortunes.

Nonetheless, it is interesting to see the wars shape up along the 100-year-old fault lines described by Rosen (2000) and Stanic (1987). The democratic language of the *Standards* and its successor, *Principles and Standards for School Mathematics*, clearly situates core reform efforts under the umbrella of education for democratic equality (schools should serve the needs of democracy by promoting equality and providing training for citizenship) and education for social mobility (schools should serve the needs of individuals by providing the means of gaining advantage in competitions for social mobility). In contrast, the traditional curriculum, with its filtering mechanisms and high drop-out and failure rates (especially for certain minority groups) has had the effect of putting and keeping certain groups "in their place." Thus the traditionalist agenda can (at least by its likely impact) be seen as situated under the umbrella of education for social efficiency (schools should serve the needs of the social and economic order by training students to occupy different positions in society and the economy). In a zero-sum game, those who hold privilege are best served by the perpetuation of the status quo.

Even though the wars rage, partly because there are some true believers on both sides and partly because some stand to profit from the conflict, I remain convinced that there is a large middle ground. I believe that the vocal extremes, partly by screaming for attention and partly by claiming the middle ground ("it's the other camp that is extreme"), have exerted far more influence than their numbers should dictate.

One way to reclaim the middle ground, suggested by Phil Daro (2003), is to define it clearly—to specify a set of propositions that will call for some degree of compromise from reformers and traditionalists alike. That middle ground would be broadly encompassing, containing propositions that most people would find reasonable (or at least livable). Daro offered a draft “Math Wars Peace Treaty” (or perhaps “Math Wars Disarmament Treaty”) that includes the following stipulations:

We have among ourselves various agreements and disagreements. But about these things we agree:

- The status quo is unacceptable. Its defenders are wrong, mathematics instruction must improve.
- Teachers, especially K-8 teachers, should learn more mathematics throughout their careers.
- No students should be denied a fair chance to learn mathematics because they have been assigned unqualified mathematics teachers.
- All students should have a copy of the basic instructional materials (textbooks, handouts, etc.) to take home.
- Research and evidence should be used whenever it is available to inform decisions.

We also agree that students should learn to:

- add, subtract, multiply, and divide single-digit numbers automatically and accurately;
- add, subtract, multiply, and divide integers, decimals, and fractions accurately, efficiently, and flexibly without calculators;
- understand the mathematics they study and use;
- use the mathematics they know to solve problems with calculators and computers;
- be fluent with the symbolic language of algebra and understand how to use the basic laws of algebra when solving mathematics problems;
- explain and justify their reasoning and understand the reasoning of others;
- reason with increasing rigor and mathematical maturity as they advance through the curriculum; [and much more]. (pp. 1-2)

The hope is that if such a list is put together well, most people will feel comfortable with most of it and be willing to part with a few things they would rather keep in the interests of making peace and working together in the interests of our children. If so, those who refuse to sign on will reveal themselves for the extremists they are.

It is not clear how optimistic one should be. There already exist documents that appear to have some consensus behind them (e.g., Conference Board of the Mathematical Sciences, 2001; NCTM, 2000; NRC, 2001). The tactic of the extremists has been to ignore such volumes and to attack what they can attack. To date, they have been fairly successful. Not only have some of the major state boards (Texas and California, for example) made the traditional choice but current federal legislation (e.g., the No Child Left Behind Act of 2001) puts substantial force behind rather narrow and traditional assessments as well. Moreover, much of the public, ill served by media that seek to profit from conflict, sees curriculum choice as dichotomous—it is either traditional or reform. At the same time, there are grounds for some optimism. At the college level, "calculus reform" stimulated a great deal of controversy but then settled in as part of the mainstream. The same may well happen with regard to standards-based mathematics. One cannot simply turn the clock back; too much is known about mathematical thinking and learning. Despite extremist proposals (and mandates), there is a rational middle ground, and many teachers seek it. (Some years ago, amid the reading wars, a friend who teaches elementary school expressed the sentiment I voiced above: Any sane person knows that both sounding out words and making sense of what you read are essential. No matter what mandates came down, she would continue to help her students do both.) The short-term goal, however, must be to capture the middle ground for the majority. Efforts must be made publicly to identify the extremists for what they are and to marginalize them. The math wars have casualties—our children, who do not receive the kind of robust mathematics education they should.

NOTES

1. The meetings are jointly sponsored by the American Mathematical Society, Mathematical Association of America, Association for Symbolic Logic, Association for Women in Mathematics, National Association of Mathematicians, and Society for Industrial and Applied Mathematics.

2. There was one lasting change due to the post-Sputnik reforms. Calculus entered the high school curriculum for those students on a more accelerated track.

3. In the past 6 years or so, districts' independence has diminished substantially as a result of high-stakes accountability measures at the state level; states' independence has diminished substantially as a result of the No Child Left Behind Act at the federal level. Massive consolidation in the publishing industry has also limited the options available to districts.

4. The NSF curriculum centers can be accessed as follows on the Web: The K-12 mathematics curriculum center is at http://www.edc.org/mcc, the elementary grades center is at http://www.arccenter.comap.com, the middle grades center is at http://showmecenter.missouri.edu, and the high school center is at http://www.ithaca.edu/compass.

5. I was one of the authors of the 1992 California Mathematics *Frameworks* (California Department of Education, 1992). As noted above, there were no large-scale data supporting the

practices suggested in the *Frameworks*—it would be a decade before such data became available. The recommendations in the document were made on the basis of ample evidence that traditional practices had not succeeded and that research suggested strongly that the goals and practices described in the document were appropriate. In many ways the *Frameworks*, like the 1989 NCTM *Standards*, was as much a vision statement as it was a prescription for curriculum design.

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