A hybrid local/non-local model for the study of damage and fracture: a Ph.D. defense

Yan Azdoud

COHMAS laboratory-PSE division
King Abdullah University of Science and Technology
23955-6900 Thuwal, Saudi Arabia

http://cohmas.kaust.edu.sa
yan.azdoud@kaust.edu.sa

Outline

- Treating failure
- The peridynamic theory
- The hybrid paradigm
- The morphing method
- A numerical solution
- Application to fracture
- Application to damage
- Concluding remarks

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Treating failure

One purpose of solid mechanics is the study of failure.

failed bike crank

broken leg

On challenge in the simulation of failure is the prediction of the propagation of discontinuities within structure.

The purpose of this thesis is to use a hybrid local/non-local model for the simulation of fracture of damage.

Treating failure

Multiple strategies exist for simulation of failure:

Methods that explicitly introduce discontinuities:

- **Linear elastic fracture mechanics, *Griffith* (1921)**
- **XFEM/GFEM, review by *Belytschko et* al (2009)**
- **Cohesive Zone model, review by *Elices et* al (2002)**

Methods that smear the discontinuities

- **Damage mechanics, Katchanov (1971)**
- **Phase field method, *Dugdall* (1960)**

A new framework:

the peridynamic theory, *Silling* (2000)

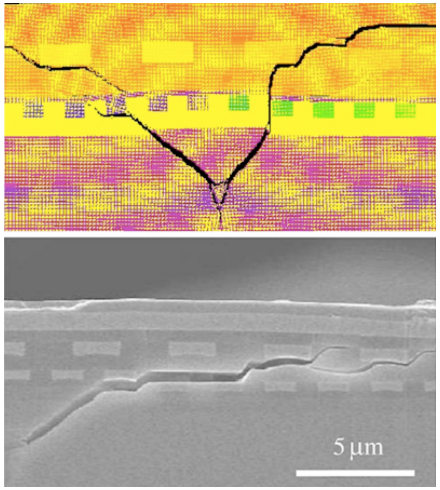
Treating failure

Multiple strategies exist for simulation failure induced discontinuities:

- Methods that explicitly introduce discontinuities
- Methods that smear the discontinuities
- **A new framework: the peridynamic theory, *Silling* (2000)**

The motivation to study the peridynamic theory:

- good qualitative results
- discontinuities exist natively in the framework



failure in Si chips, from *Madenci* (2009)

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The peridynamic theory

- Peridynamics is a long range interaction non-local integral continuum model:

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathscr{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{u}(\mathbf{x}',t)-\mathbf{u}(\mathbf{x},t),\mathbf{x}'-\mathbf{x})\,\mathrm{d}V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t),$$

- Each point x interacts through long distance forces with a neighborhood Hx limited by δ , the cut-off radius of interaction. the link between x and x' is called a *bond*

- Integral formulation allows discontinuities, contrarily to gradient models

- Bonds can irremediably break to simulate damage

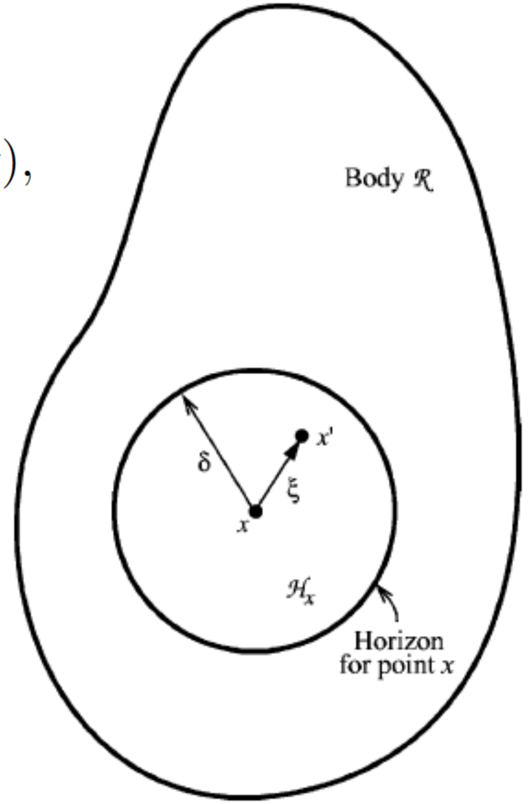


Fig. 1. Each point x in the body interacts directly with points in the sphere $\mathscr{H}_{\mathbf{x}}$ through *bonds*. from Silling (2005)

The Peridynamic theory

Peridynamic

- Bond-based theory *Silling* (2000)

Central force model, "continuum" version of a molecular mechanics approach, limited to specific isotropic and anisotropic models, allowing damage.

- State-based theory *Silling & al.* (2007)

Generalization of the Bond-Based theory using a second level of neighbors that works for all anisotropic models

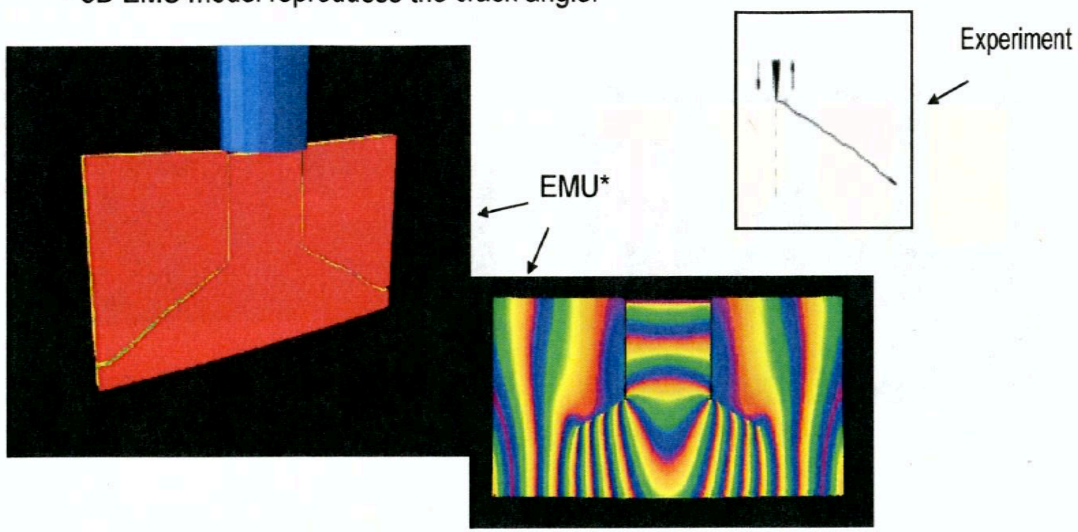
Similar models

- Kroner (1967)
- Di Paola (2008)

+ The Peridynamic theory

- Dynamic fracture in a hard steel plate

- Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)
    - Mode-II loading at notch tips results in mode-I cracks at 70deg angle.
    - 3D EMU model reproduces the crack angle.



Courtesy of S.A. Silling

S. A. Silling, Dynamic fracture modeling with a meshfree peridynamic code, in *Computational Fluid and Solid Mechanics 2003*, K.J. Bathe, ed., Elsevier, pp. 641-644.

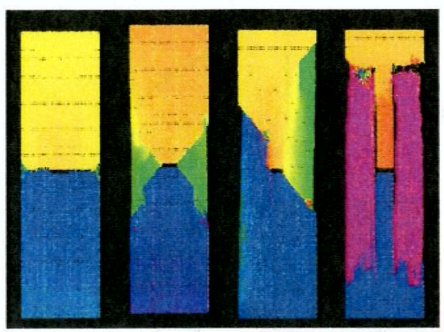
frame 9

Sandia National Laboratories

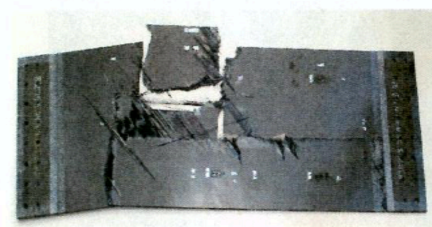
The Peridynamic theory

- Splitting and fracture mode change in composites

• Distribution of fiber directions between plies strongly influences the way cracks grow.



EMU simulations for different layups



Typical crack growth in a notched laminate
(photo courtesy Boeing)

Courtesy of S.A. Silling

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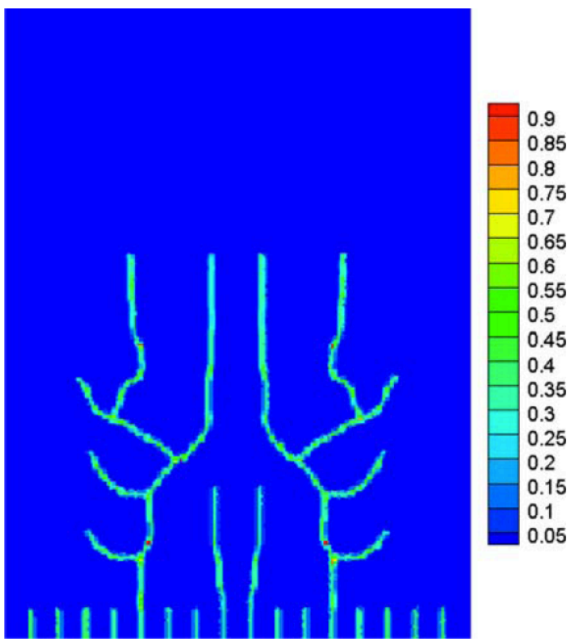
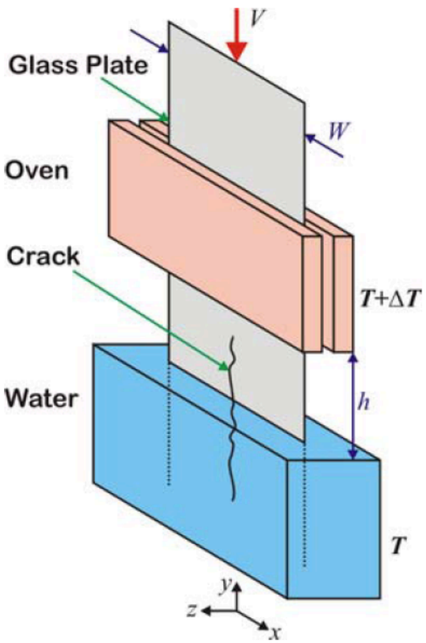
frame 11

The Peridynamic theory

- Branching in brittle fracture of glass



Branching of cracks



setup

From Kilic & Madenci (2009)

Peridynamic theory

Additional information can be found in:

Theory:

Silling **et al (2007),** ***Silling and Lehoucq*** **(2010)**

Boundary conditions:

Gunzburger and Lehoucq **(2010)**

Discretization:

Chen and Gunzburger **(2011)**

Applications:

Ha and Bobaru **(2011),*****Littlewood*** **(2010),** ***Askari*** **et al (2008),** ***Silling*** **(2003)**

Peridynamic theory

Bond-Based theory

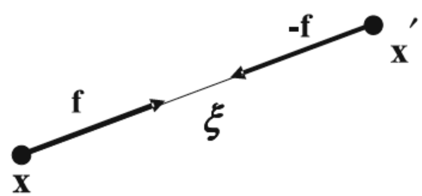
The bond interaction depends only on the kinematic of this bond

☐ **Kinematic admissibility and compatibility**

$$\eta_{\xi}(\underline{p}-\underline{x}) = u_{\xi}(\underline{p}) - u_{\xi}(\underline{x}) \quad \forall(\underline{x},\underline{p}) \in \Omega$$

☐ **Static admissibility**

$$\int_{H_\delta(\underline{x})} \left\{ \underline{\hat{f}}[\underline{x}] < \underline{p}-\underline{x} > - \underline{\hat{f}}[\underline{p}] < \underline{x}-\underline{p} > \right\} dV_{\underline{p}} = \underline{0} \quad \forall \underline{x} \in \Omega$$



Bond based

☐ **Constitutive equations**

$$\underline{\hat{f}}[\underline{x}] < \underline{p}-\underline{x} > = \frac{c[\underline{x}]}{2} \eta_{\xi}(\underline{p}-\underline{x}) \underline{e}_{\xi} \quad \forall \underline{x} \in \Omega$$

Peridynamic theory

Bond-Based theory

The bond interaction depends only on the kinematic of this bond

☐ **Kinematic admissibility and compatibility**

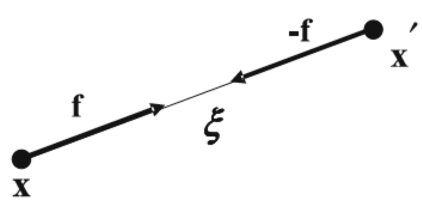
$$\boxed{\eta_{\xi}(\underline{p}-\underline{x})} = u_{\xi}(\underline{p}) - u_{\xi}(\underline{x}) \quad \forall(\underline{x},\underline{p}) \in \Omega$$

stretch of a bond

☐ **Static admissibility**

$$\int_{H_{\delta}(\underline{x})} \left\{ \boxed{\hat{\underline{f}}[\underline{x}] < \underline{p}-\underline{x} > - \hat{\underline{f}}[\underline{p}] < \underline{x}-\underline{p} >} \right\} dV_{\underline{p}} = \underline{0} \quad \forall \underline{x} \in \Omega$$

Anti-symmetric form



Bond based

☐ **Constitutive equations**

$$\hat{\underline{f}}[\underline{x}] < \underline{p}-\underline{x} > = \frac{\boxed{c[\underline{x}]}}{2} \eta_{\xi}(\underline{p}-\underline{x}) \underline{e}_{\xi} \quad \forall \underline{x} \in \Omega$$

Stiffness operator: micro-modulus

The peridynamic theory

Important points:

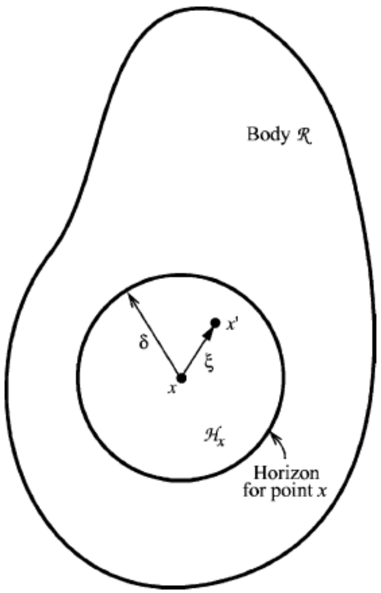
- it allows discontinuities

but

- it is non-local and *a priori* computationally costly

- it is challenging to define internal parameters and boundary conditions

- There are surface effects



A peridynamic point x and its Horizon Hx, from *Silling* (2005)

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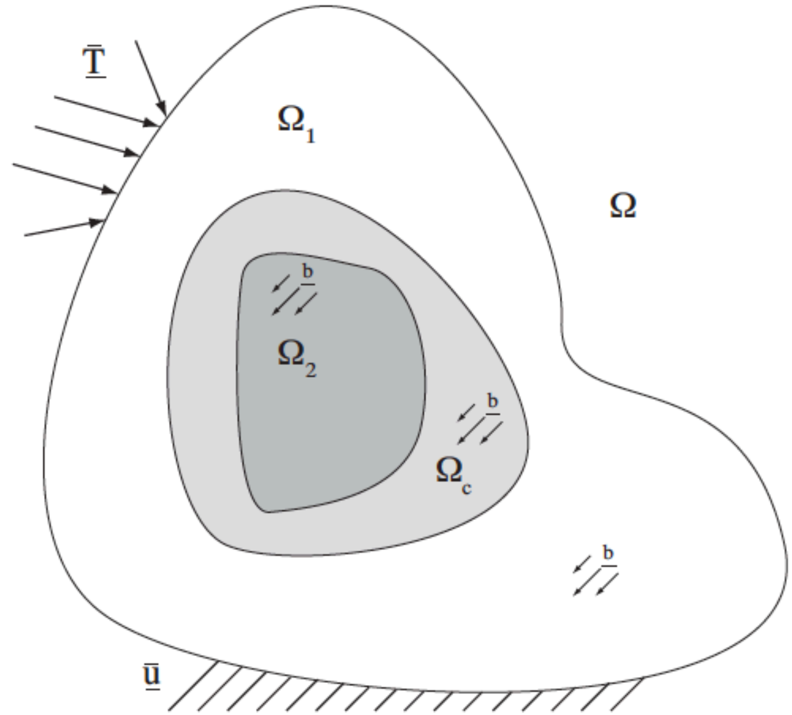
- The hybrid paradigm
- The morphing method
- A numerical solution
- Application to fracture
- Application to damage localization
- Concluding remarks

The hybrid paradigm

The computational cost of peridynamics justifies the introduction of a hybrid framework, in which the non-local model is used only in an area of interest. In the rest of the structure, we use classic continuum mechanics.



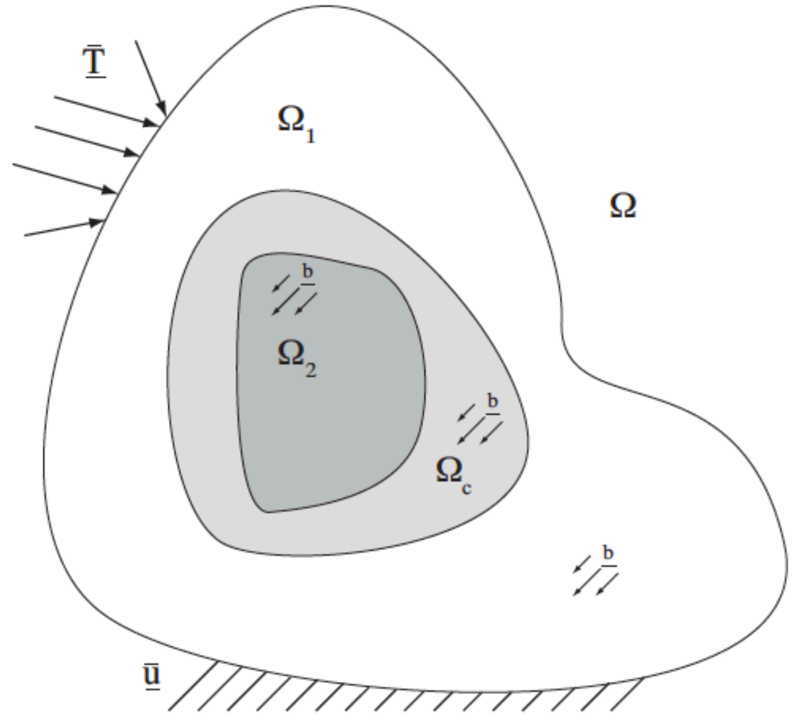
hybrid model

The hybrid paradigm

Other advantages are:

- defining Boundary condition on the local part of the model

- Use the local model as a predictor of the local/non-local partition



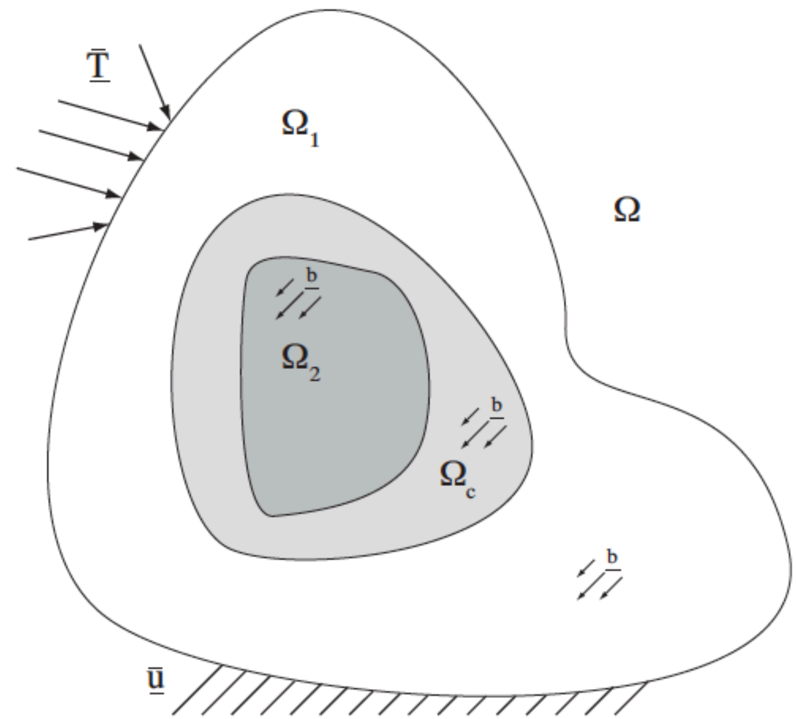
hybrid model

The hybrid paradigm

This explains the purpose of this thesis:

To develop a hybrid local/ non-local model for the simulation of fracture and damage.

first, we should develop a coupling method between the local and the non-local models



hybrid model

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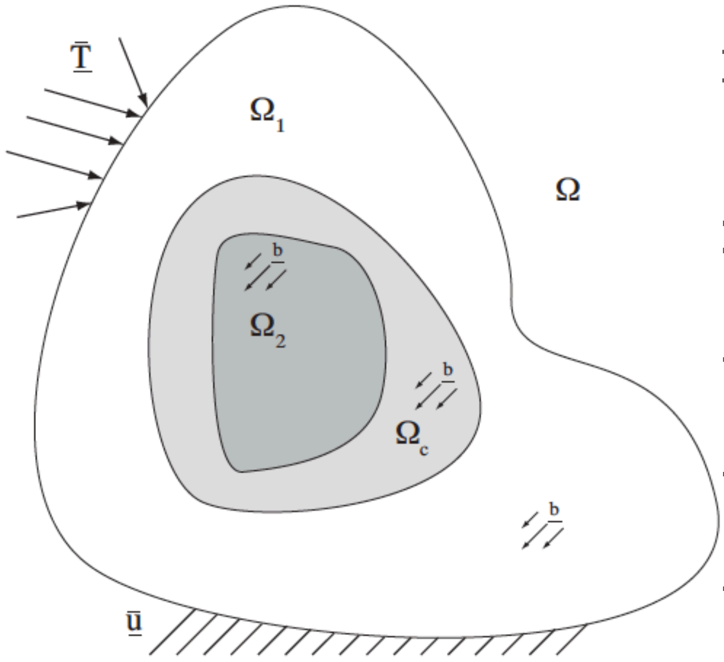
The morphing method

The morphing method couples this model with the classical local continuum model only where needed.



Coupled model

Available coupling techniques

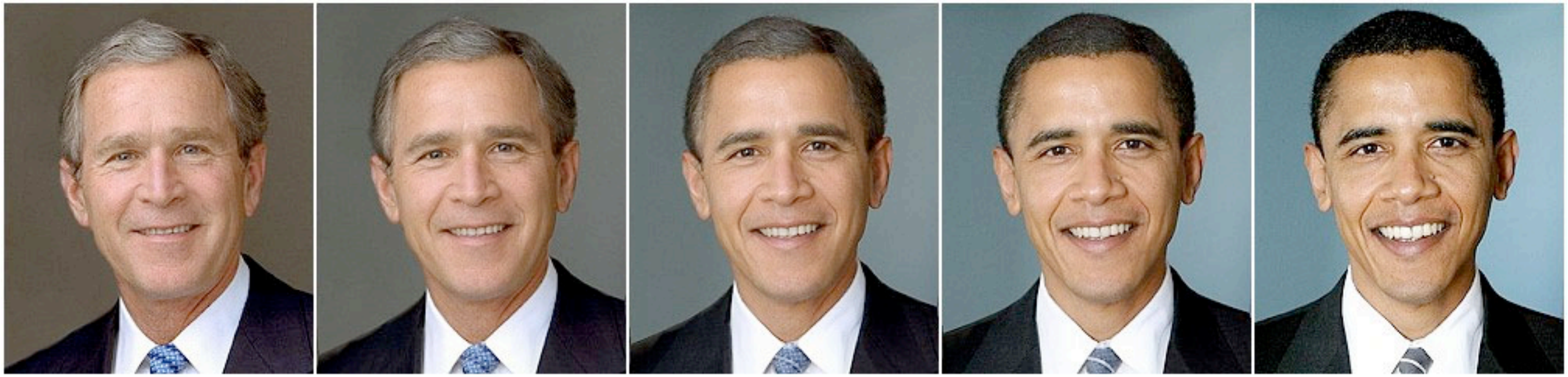
Discrete/continuum: [*Belytschko & Xiao* (2003), *Curtin & Miller* (2003), *BenDhia* (1998)].

Non-local/local continua:

- Seleson, Beneddine and Prudhomme (2013)
- Liu and Hong (2013)
- Han and Lubineau (2011)

The morphing method

The idea behind morphing is to create a coupling method that is not a traditional coupling, but rather a hybrid model containing both descriptions of the coupled models, that varies through space:



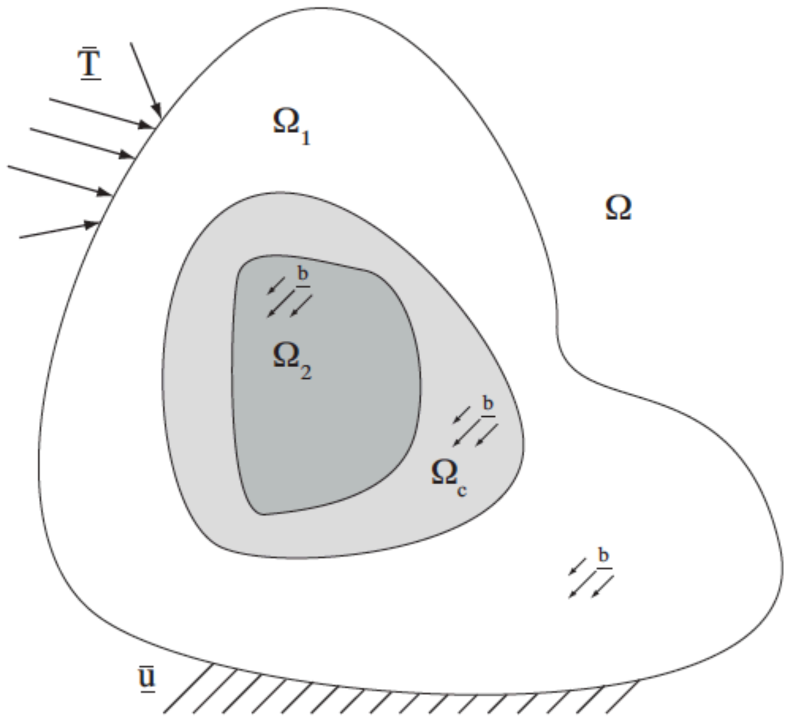
Example of morphing in pictures… and policies

The morphing method

Key point 1 : a single equilibrium equation that involves both contact and long range forces

Key point 2 : a single kinematic space

Key point 3 : a progressive gradient of constitutive parameters inducing a natural transition or *Morphing* between contact and long range forces



Coupled model

The morphing method

- Non-local part
- Local part

It leads to:

☐ **Kinematic admissibility and compatibility**

$$\underline{\underline{\varepsilon}}(\underline{x}) = \frac{1}{2}\left(\underline{\nabla}\cdot\underline{u}(x) + {}^{t}\underline{\nabla}\cdot\underline{u}(x)\right) \quad \forall \underline{x} \in \Omega$$

$$\eta_{\underline{\xi}}(\underline{p}-\underline{x}) = u_{\underline{\xi}}(\underline{p}) - u_{\underline{\xi}}(\underline{x}) \quad \forall(\underline{x},\underline{p}) \in \Omega$$

$$\underline{u} = \underline{\bar{u}} \quad \forall \underline{x} \in S_{\underline{\bar{u}}}$$

☐ **Static admissibility**

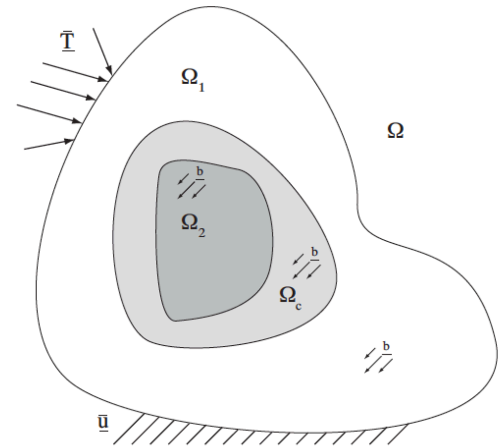
$$\underline{div\underline{\sigma}} + \int_{H_\delta(x)} \left\{ \underline{\hat{f}}[\underline{x}] < \underline{p}-\underline{x} > - \underline{\hat{f}}[\underline{p}] < \underline{x}-\underline{p} > \right\} dV_{\underline{p}} = \underline{0} \quad \forall \underline{x} \in \Omega$$

$$\underline{\underline{\sigma}}\cdot\underline{n} = \underline{\bar{T}} \quad \forall \underline{x} \in S_{\underline{\bar{T}}}$$

☐ **Constitutive equations**

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{K}}}}(\underline{x}) : \underline{\underline{\varepsilon}} \quad \forall \underline{x} \in \Omega$$

$$\underline{\hat{f}}[\underline{x}] < \underline{p}-\underline{x} > = \frac{c[\underline{x}](\|\underline{\xi}\|)}{2}\eta_{\underline{\xi}}(\underline{p}-\underline{x})\underline{e}_{\underline{\xi}} \quad \forall \underline{x} \in \Omega$$



Coupled model

The morphing method

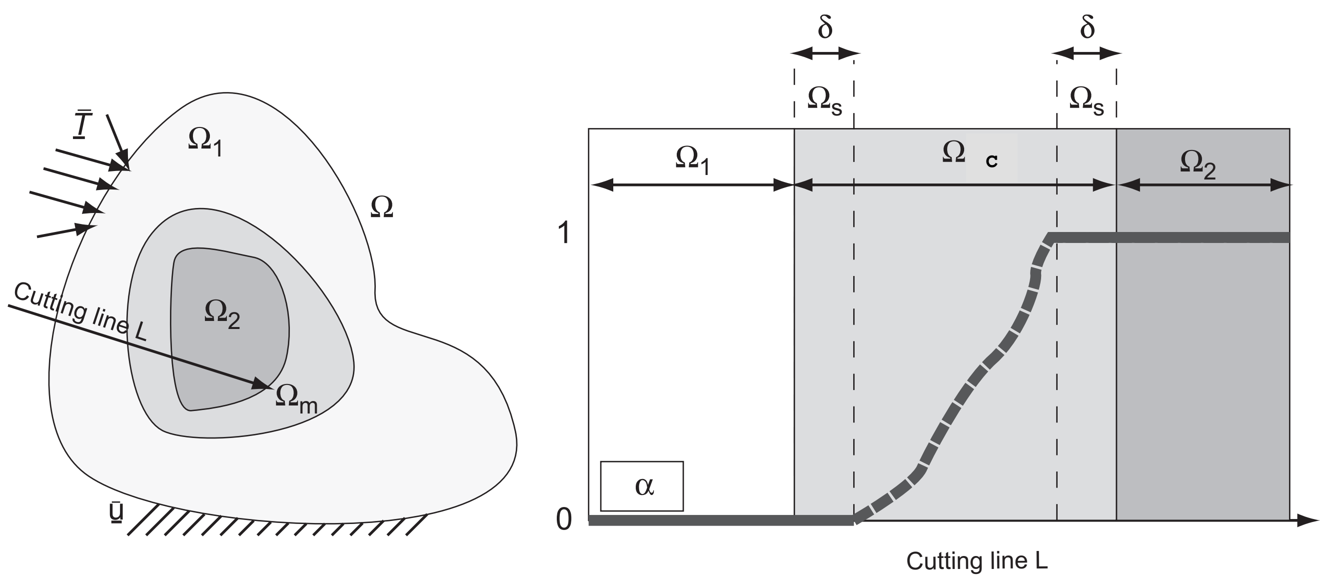
- Non-local part
- Local part

The coupling parameter

K(x) and c[x] define the local material properties of the local and non-local model respectively. We introduce coupling parameters α:

$$\boxed{c[\underline{x}](\|\underline{\xi}\|) = \alpha(\underline{x})c^0(\|\underline{\xi}\|)}$$

$$\boxed{\underline{\underline{K}}(\underline{x}) = \underline{\underline{K}}^0} \quad \text{in } \Omega_1$$



Cut of a coupling zone and qualitative variation of alpha

Morphing Method

- Non-local part
- Local part

Constraining the coupling parameter

- The local strain energy density is conserved

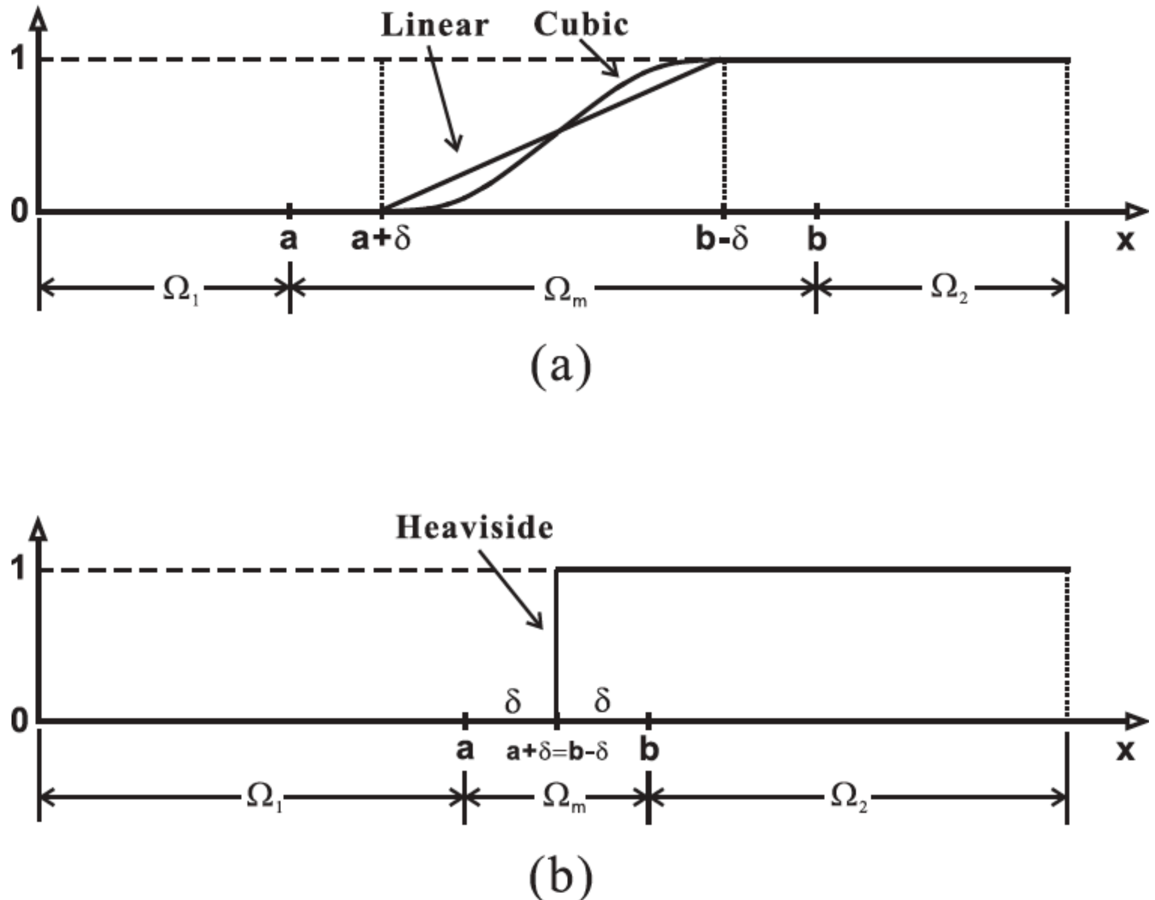
$$W(\underline{x}) = \boxed{\frac{1}{2}\underline{\underline{\varepsilon}}(\underline{x}) : \underline{\underline{\underline{\underline{K}}}}(\underline{x}) : \underline{\underline{\varepsilon}}(\underline{x})} + \boxed{\frac{1}{4}\int_{H_\delta(\underline{x})} c^o(\| \underline{\xi} \|)\frac{\alpha(\underline{x}) + \alpha(\underline{p})}{2}\left(u_{\underline{\xi}}(\underline{p}) - u_{\underline{\xi}}(\underline{x})\right)^2 dV_{\underline{p}}}$$

- Under the hypothesis of smooth strain field, it yields :

$$\underline{\underline{\underline{\underline{K}}}}(\underline{x}) = (1-\alpha(\underline{x}))\underline{\underline{\underline{\underline{K}}}}^0 + \int_{H_\delta(\underline{x})} c^0(\|\underline{\xi}\|)\frac{(\alpha(\underline{x})-\alpha(\underline{p}))}{2}\frac{\underline{\xi}\otimes\underline{\xi}\otimes\underline{\xi}\otimes\underline{\xi}}{2\|\underline{\xi}\|^2}dV_{\underline{p}}$$

The morphing method

Choice of alpha



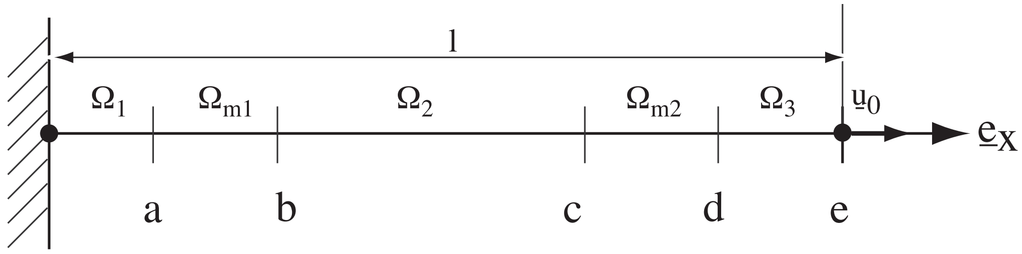
Heaviside (b), Linear and Cubic (a) choices of alpha

+ The morphing method: ghost forces

Analytic evaluation of ghost forces

-Ghost forces are a spurious effect that unbalance the equilibrium equation.

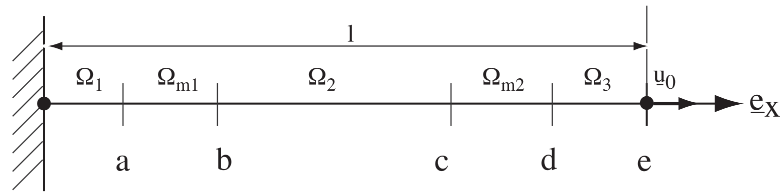
-The morphing method ensures that energy is conserved locally, but not that no ghost forces appear.



1D model for ghost force analytical evaluation

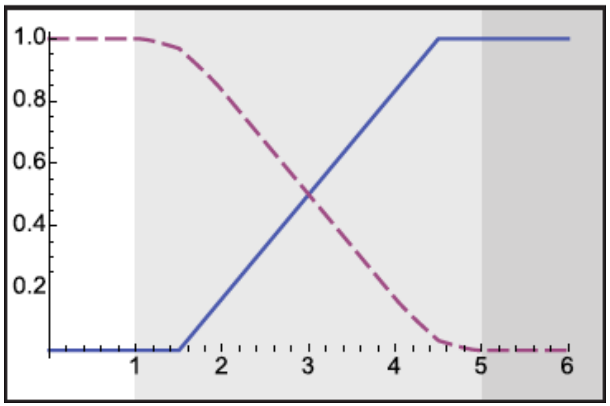
+ The morphing method: ghost forces

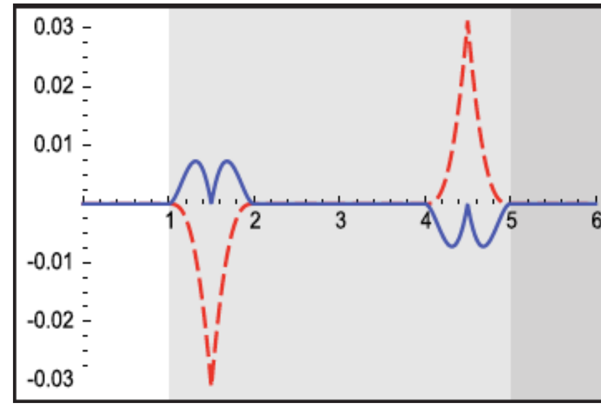
1D analytical benchmark



For a linear coupling parameter:

$$\frac{f_g(\tilde{x})}{\sigma} = o\left(\frac{\delta}{L_c}\right)$$



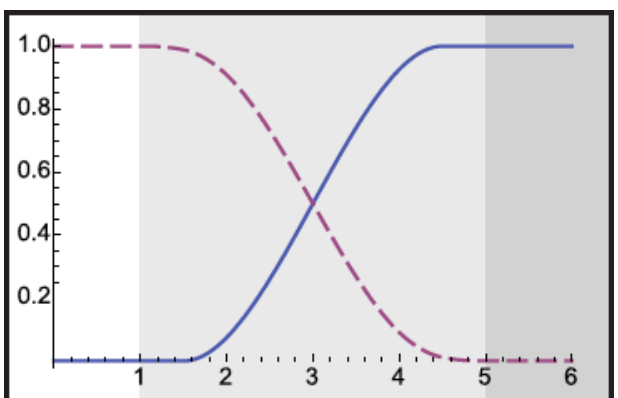


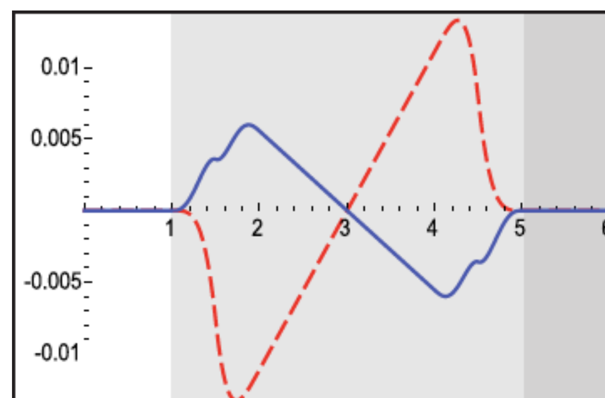
For a cubic coupling parameter:

α

f_g / σ

$$\frac{f_g(\tilde{x})}{\sigma} = o\left(\frac{\delta^2}{L_c^2}\right)$$

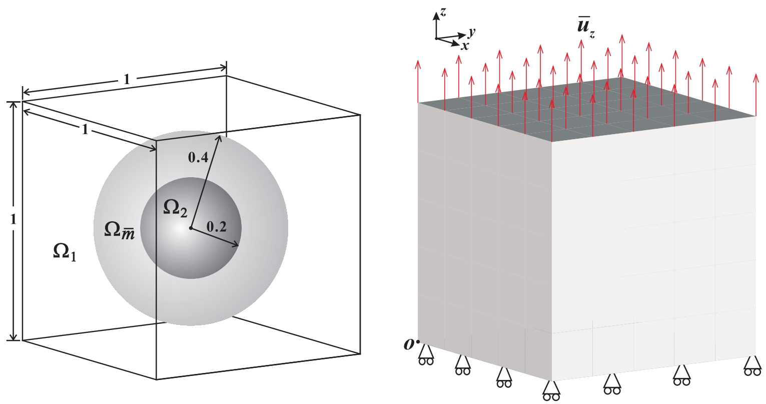




+ The morphing method: ghost forces

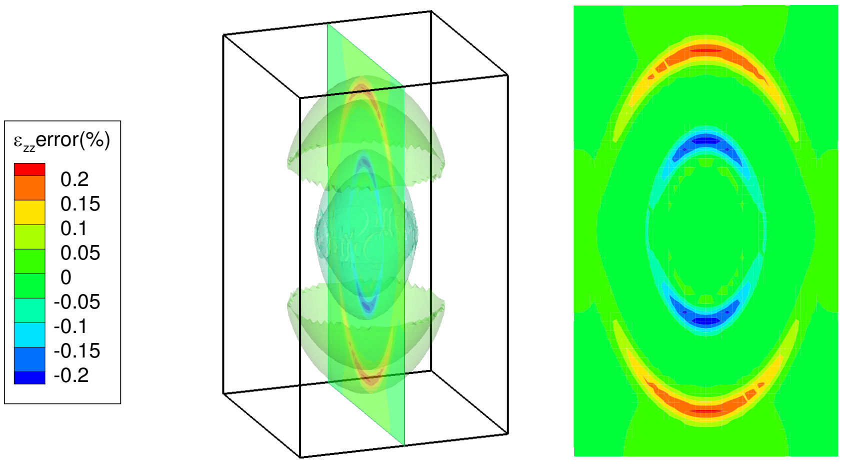
3D numerical benchmark

3D numerical results are compatible with 1D analytical results:



(a) (b)

Boundary conditions for an homogeneous traction test with $\delta = 0.06$ and a linear α



(a) (b)

Error in ε_{zz} for the traction test. (a) iso-contours (b) slice for linear choice of alpha

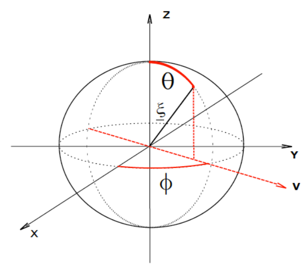
Azdoud et al 2013, IJSS

+ The morphing method: anisotropy

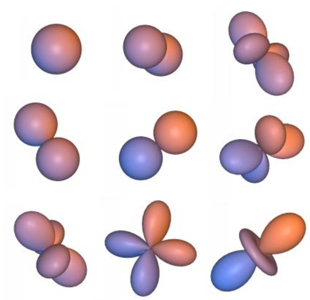
Introduction of angular and radial variables

$$f(\underline{p} \to \underline{x}) = \frac{c[\underline{x}](\|\underline{\xi}\|) + c[\underline{p}](\|\underline{\xi}\|)}{2} \left\{ u_\xi(\underline{p}) - u_\xi(\underline{x}) \right\} \underline{e_\xi} \quad \forall \underline{x}, \underline{p} \in \Omega$$

$$c[\underline{x}] = c[\underline{x}](\theta, \phi, \|\underline{\xi}\|)$$



Spherical coordinates of the bond



First spherical harmonics

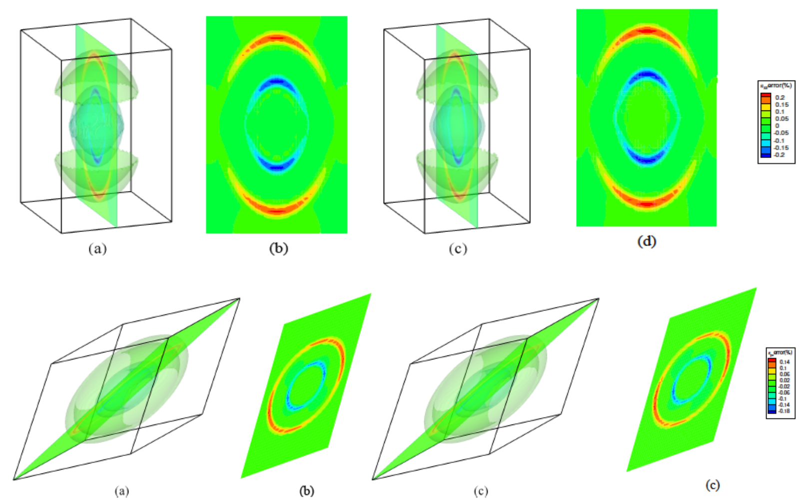
Decomposition of the micromodulus on spherical harmonics

$$c(\theta, \phi, \|\underline{\xi}\|) = c_\xi(\|\underline{\xi}\|) \left[a_{00} + \sum_{k=1}^{+\infty} \left[\sum_{m=0}^{k} P_k^m(cos(\theta))(a_{km}cos(m\phi) + b_{km}sin(m\phi)) \right] \right]$$

Centrosymmetry

$$c(\theta, \phi, \|\underline{\xi}\|) = c(\pi - \theta, \pi + \phi, \|\underline{\xi}\|)$$

Benchmark results: pure traction, pure shear



Azdoud et al 2013, IJSS

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A numerical solution: set of equations

Hybrid coupling model

Non-local part
Local part

☐ Kinematic admissibility and compatibility

$$\underline{\underline{\varepsilon}}(\underline{x}) = \frac{1}{2}\left(\underline{\underline{\nabla}} \cdot \underline{u}(x) + {}^{t}\underline{\underline{\nabla}} \cdot \underline{u}(x)\right) \quad \forall \underline{x} \in \Omega$$

$$\eta_{\underline{\xi}}(\underline{p}-\underline{x}) = u_{\underline{\xi}}(\underline{p}) - u_{\underline{\xi}}(\underline{x}) \quad \forall(\underline{x},\underline{p}) \in \Omega$$

$$\underline{u} = \underline{\bar{u}} \quad \forall \underline{x} \in S_{\underline{\bar{u}}}$$

☐ Static admissibility

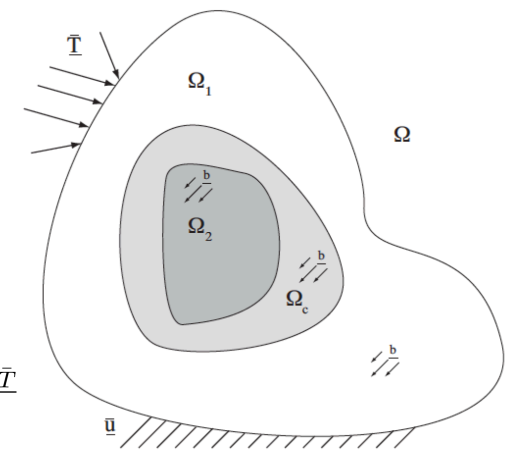
$$\underline{div\underline{\sigma}} + \int_{H_\delta(x)} \left\{\underline{\hat{f}}[\underline{x}] < \underline{p}-\underline{x} > - \underline{\hat{f}}[\underline{p}] < \underline{x}-\underline{p} >\right\} dV_{\underline{p}} = \underline{b} \quad \forall \underline{x} \in \Omega$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = \underline{\bar{T}} \quad \forall \underline{x} \in S_{\underline{\bar{T}}}$$

☐ Constitutive equations

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{K}}}}(\underline{x}) : \underline{\underline{\varepsilon}} \quad \forall \underline{x} \in \Omega$$

$$\underline{\hat{f}}[\underline{x}] < \underline{p}-\underline{x} > = \frac{c[\underline{x}](\|\underline{\xi}\|)}{2}\eta_{\underline{\xi}}(\underline{p}-\underline{x})\underline{e}_{\underline{\xi}} \quad \forall \underline{x} \in \Omega$$



Coupled model

+ A numerical solution: weak formulation

Galerkin formulation

$$Find \quad \underline{u}_s \in W \quad | \quad \forall \underline{v} \in W^o \quad a(\underline{u}_s, \underline{v}) = l(\underline{v})$$

Bilinear form:

Introduction of a virtual admissible field

$$a(\underline{u}, \underline{v}) = \boxed{\int_\Omega \underline{\sigma}(\underline{u}) : \underline{\varepsilon}(\underline{v}) d\Omega +}$$

$$\boxed{\frac{1}{2} \int \int_\Omega \frac{c[x](\theta, \phi, \|\underline{\xi}\|) + c[p](\theta, \phi, \|\underline{\xi}\|)}{2} \eta_{\underline{\xi}}[\underline{u}](\underline{x}, \underline{\xi}) \eta_{\underline{\xi}}[\underline{v}](\underline{x}, \underline{\xi}) d\underline{x} d\underline{\xi}}$$

$$\forall (\underline{u}, \underline{v}) \in W \times W^o$$

linear form:

$$l(\underline{v}) = \int_\Omega \underline{b} \cdot \underline{v} d\Omega + \boxed{\int_{S_{\underline{T}}} \underline{\bar{T}} \cdot \underline{v} dS} \; \forall \underline{v} \in W^o$$

A numerical solution: set of equations

Introduction of shape functions

$$\{e_i\} \qquad \{\phi_k^i\} \qquad \underline{\tilde{v}}(x) = \sum_k \underline{v}(\underline{x}_k)\phi_k^i(\underline{x})$$

Billinear form:

Partition of integrals on elements

$$a(\underline{\tilde{u}}, \underline{\tilde{v}}) = \boxed{\sum_i \int_{e_i} \underline{\underline{\varepsilon}}(\underline{\tilde{v}})\underline{\underline{\underline{\underline{C}}}}\,\underline{\underline{\varepsilon}}(\underline{\tilde{u}})de_i}$$

$$\boxed{+\frac{1}{2}\sum_i \int_{e_i} \sum_j \int_{e_j} \frac{\alpha(\underline{x}) + \alpha(\underline{p})}{2} c^o(\theta, \phi, \|\underline{\xi}\|)(\underline{\tilde{v}}(\underline{p}) - \underline{\tilde{v}}(\underline{x}))(\underline{e}_{\xi} \otimes \underline{e}_{\xi})(\underline{\tilde{u}}(\underline{p}) - \underline{\tilde{u}}(\underline{x}))d\underline{p}d\underline{x}}$$

linear form:

$$l(\underline{v}) = \sum_i \int_{e_i} \underline{b}\underline{\tilde{v}}d\underline{x} + \boxed{\sum_i \int_{S_{\underline{T}}^i} \underline{\bar{T}}\underline{\tilde{v}}dS^i}$$

+ A numerical solution: discretization

Discretized form

$$[K]\{u^d\} = \{F\} \qquad \underline{\tilde{u}}(\underline{x}) = [\Phi(\underline{x})]\{u^d\}$$

Stiffness matrix

Differential operator

$$[K] = \sum_i \int_{e_i} ([H][\Phi(\underline{x})])^T [C][H][\Phi(\underline{x})] d\underline{x}$$

$$+ \sum_i \sum_j \int_{e_i} \int_{e_j} \frac{\alpha(\underline{x}) + \alpha(\underline{p})}{2} c^o(\theta, \phi, \|\underline{\xi}\|) [\Phi(\underline{x})]^T [G][\Phi(\underline{x})] d\underline{p} d\underline{x}$$

$$- \sum_i \sum_j \int_{e_i} \int_{e_j} \frac{\alpha(\underline{x}) + \alpha(\underline{p})}{2} c^o(\theta, \phi, \|\underline{\xi}\|) [\Phi(\underline{x})]^T [G][\Phi(\underline{p})] d\underline{p} d\underline{x}$$

Integrals solved by Gauss quadrature

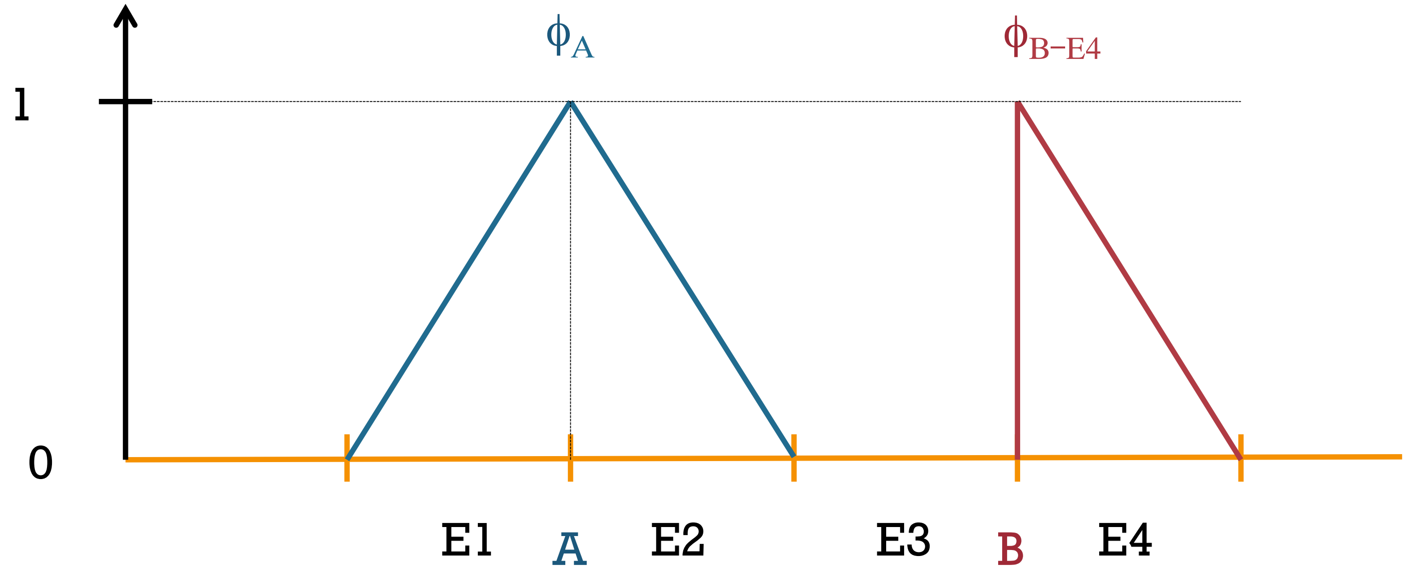
$$[G] = \underline{e_\xi} \otimes \underline{e_\xi}$$

Force vector

$$\{F\} = \sum_i \int_{e_i} [\Phi(\underline{x})]^T \{b\} d\underline{x} + \sum_i \int_{S_T^i} [\Phi(\underline{x})]^T \{T\} dS^i$$

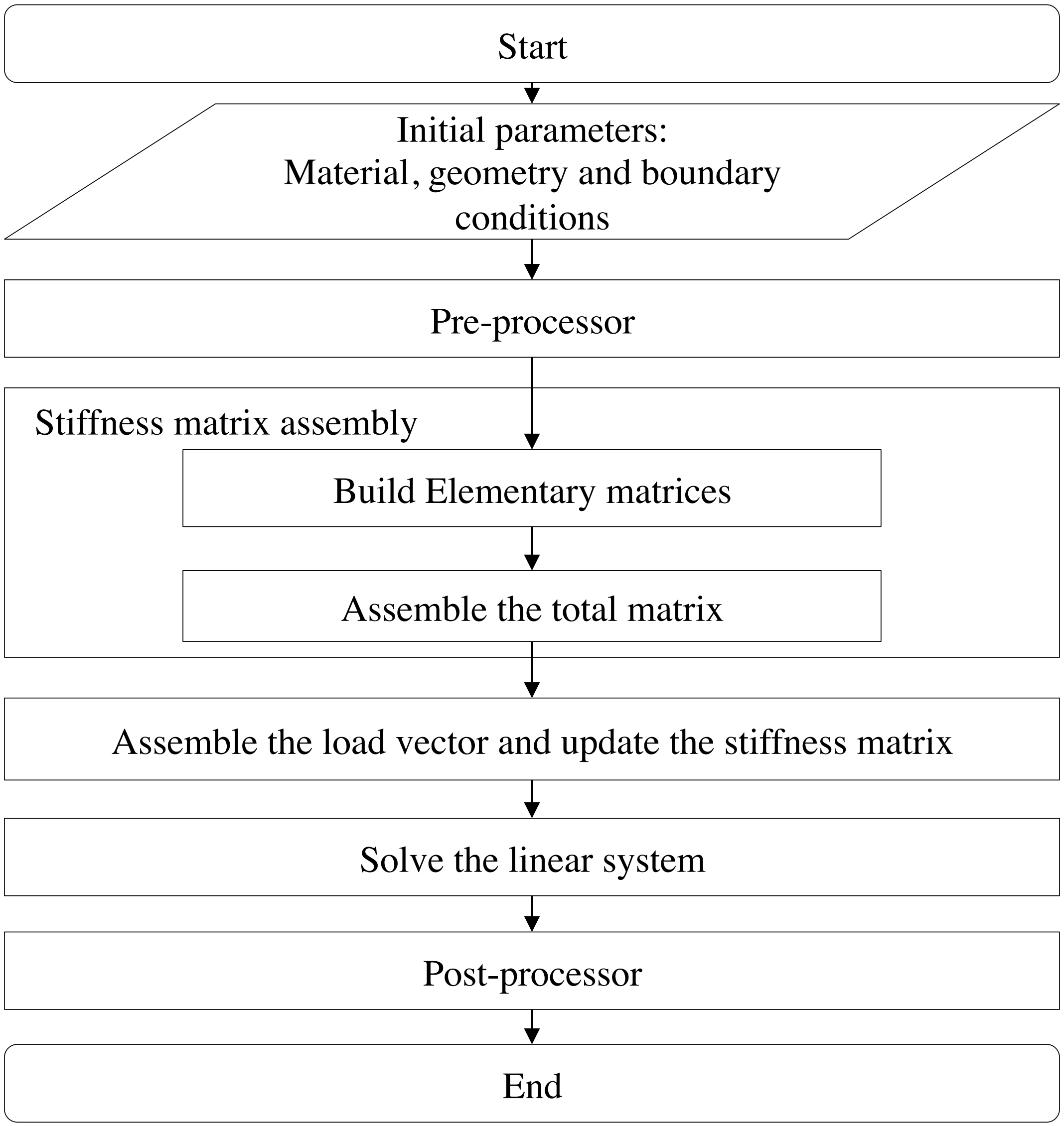
+ A numerical solution: shape functions

DGFEM vs FEM: 1D example



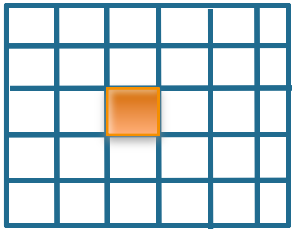
In DGFEM, discontinuities are handled at the interface between elements

A numerical solution: linear elastic algorithm

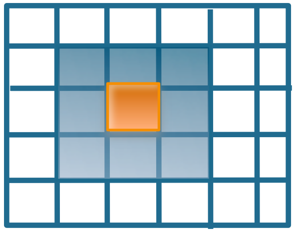


A numerical solution: matrix assembly

- Computational need: matrix assembly



Calculation of the interactions within the element



Calculation of the interactions with the surrounding elements

For every Elements

For all Gauss Point

Build Keg

Build Ke

Build K (NxN)

For every Elements : parallel loop

For all Gauss Point

For neighbors G.P

Build Kegg

Build Keg

Build Ke

Build K (NxN)

+ A numerical solution: software

We developed a solution based on C++

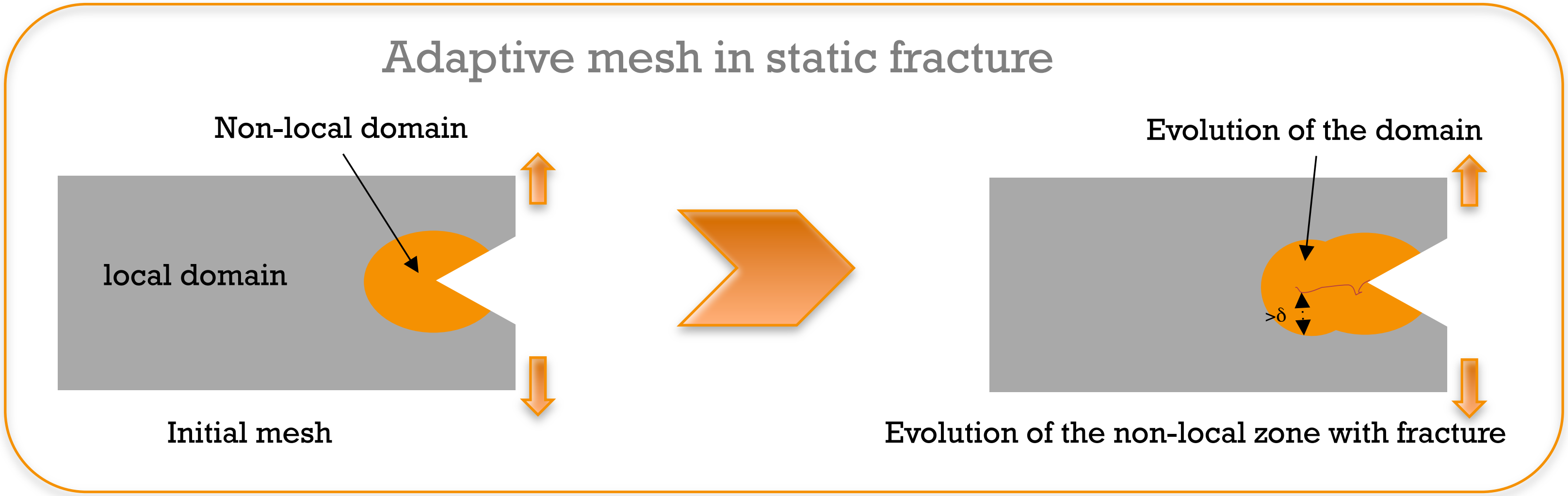
- 2D and 3D capability
- Supports local and non-local models
- Supports FEM and DGFEM
- Parallelized (openMP) and Portable (C++)
- Adaptable for various constitutive equations

Outline

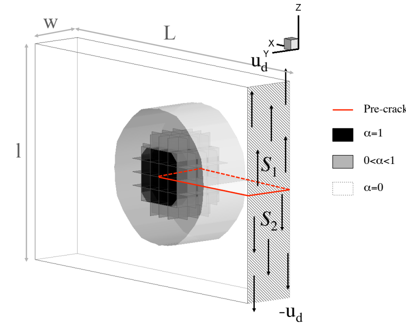
- Treating failure
- The peridynamic theory
- The hybrid paradigm
- The morphing method
- A numerical solution
- Application to fracture
- Application to damage
- Concluding remarks

Application to fracture

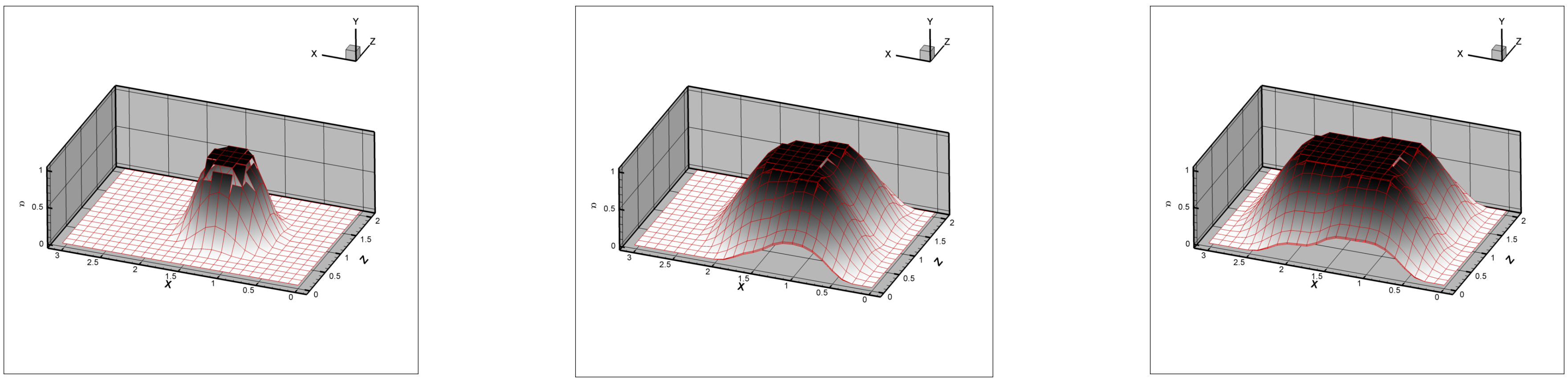
- Principle

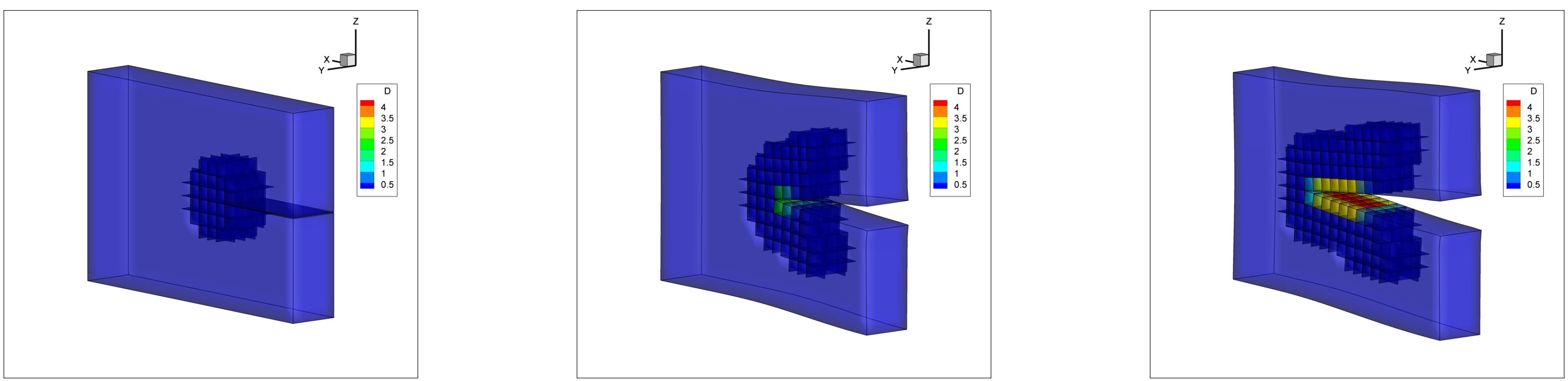


Application to fracture



mapping of the coupling parameter with increasing loading

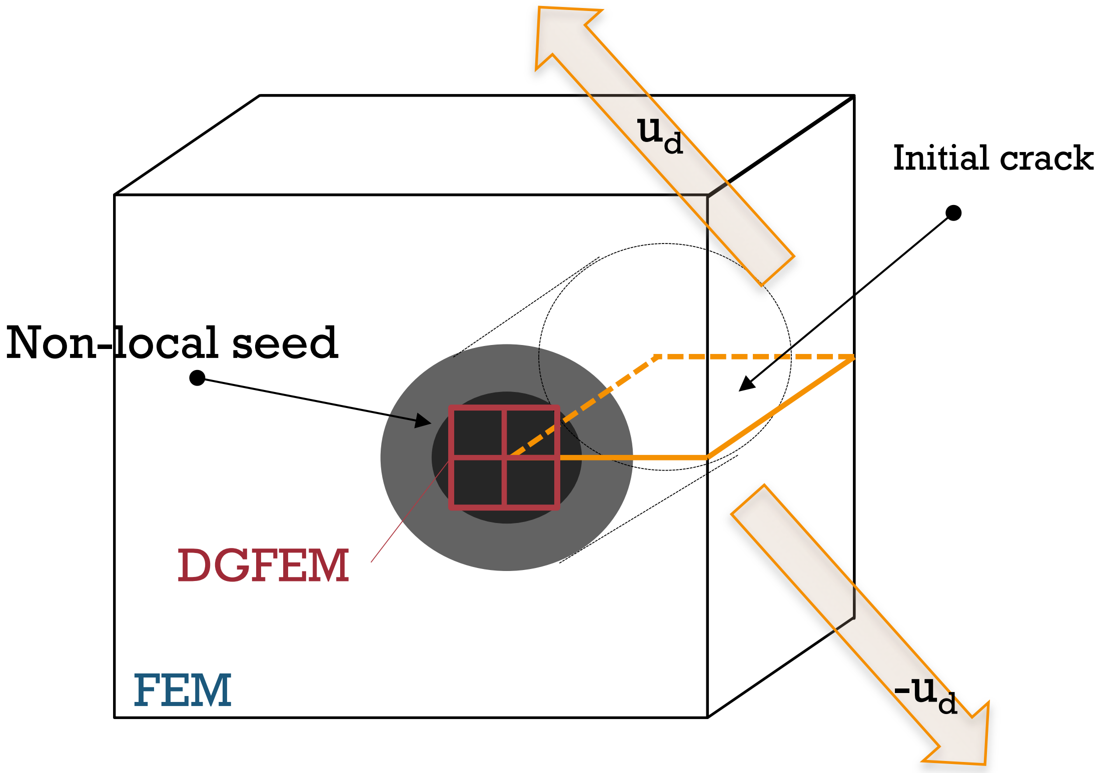




map of dissipated energy with increasing loading

Application to fracture

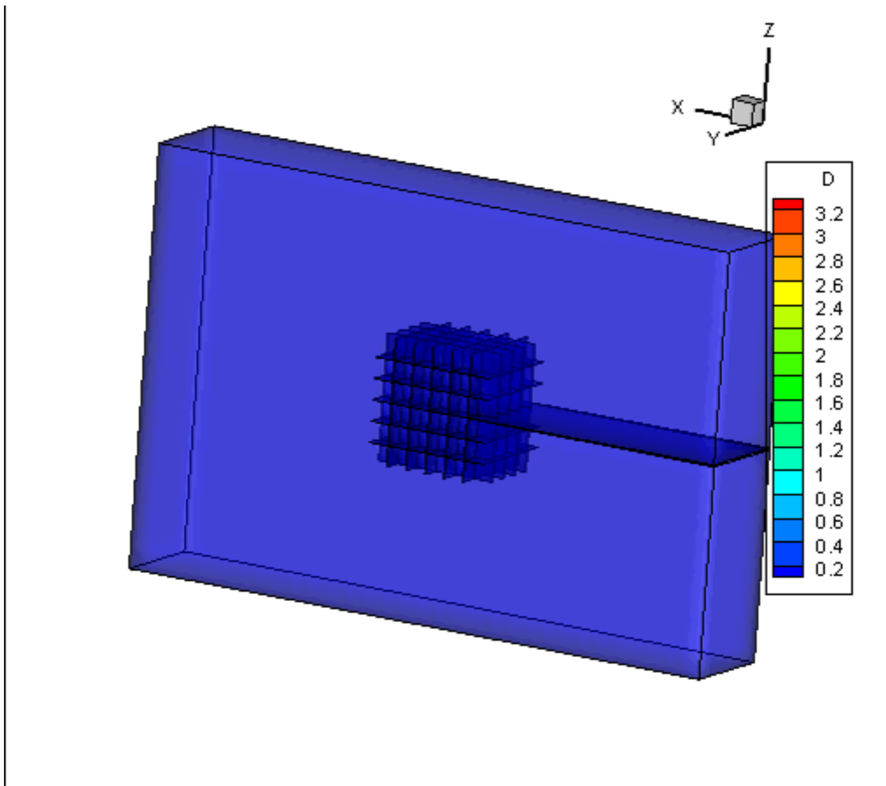
Mode mixing boundary condition



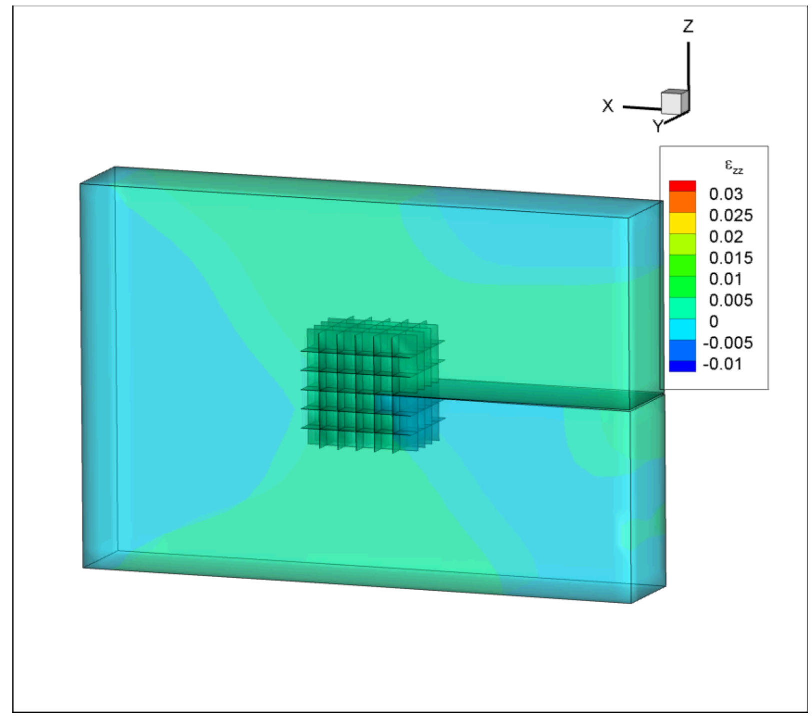
We perform a CT test with mixed mode fracture

Application to fracture

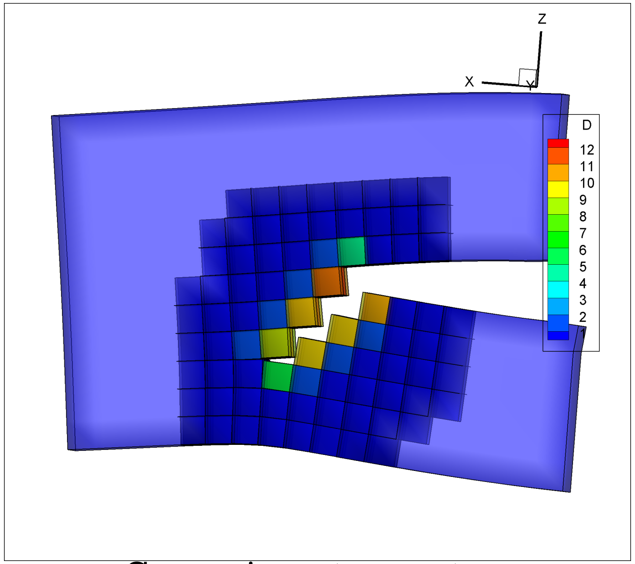
Dissipated energy map



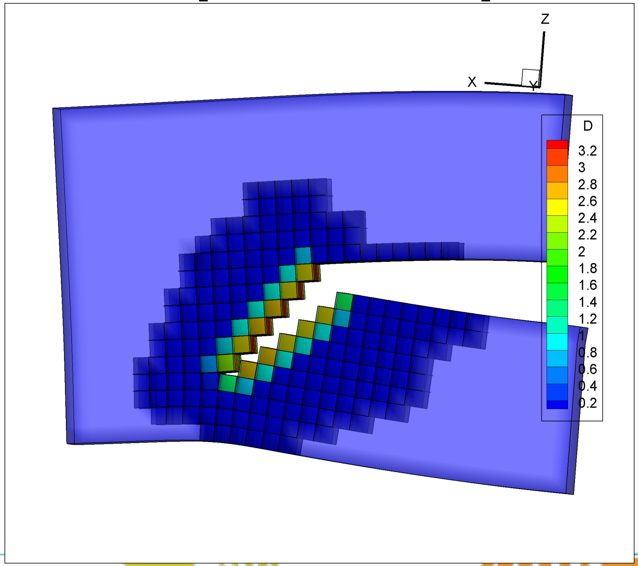
Strain component ezz

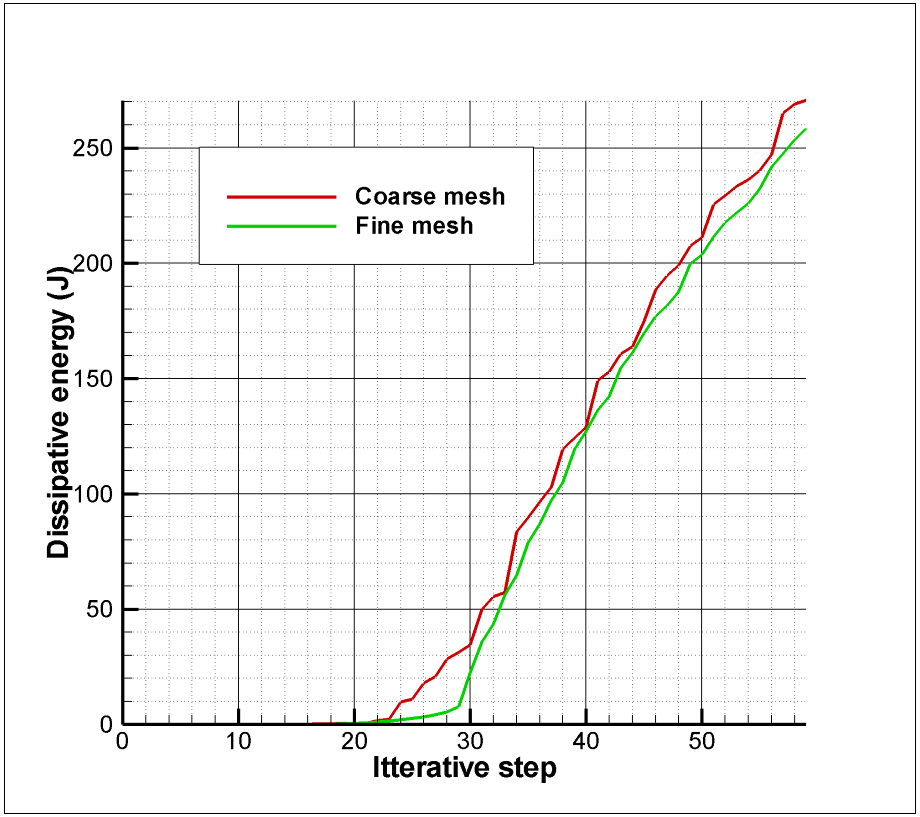


Application to fracture



Comparison at same step





Application to fracture

Key points

- The adaptive method allows to follow evolution of the non-local effects with better usage of computational resources

- The coupling does not affect the quality of the observed phenomena (spurious effects are negligible)

- The overall algorithm presents good convergence properties

Outline

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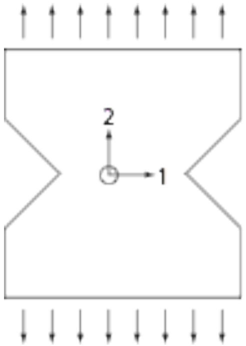
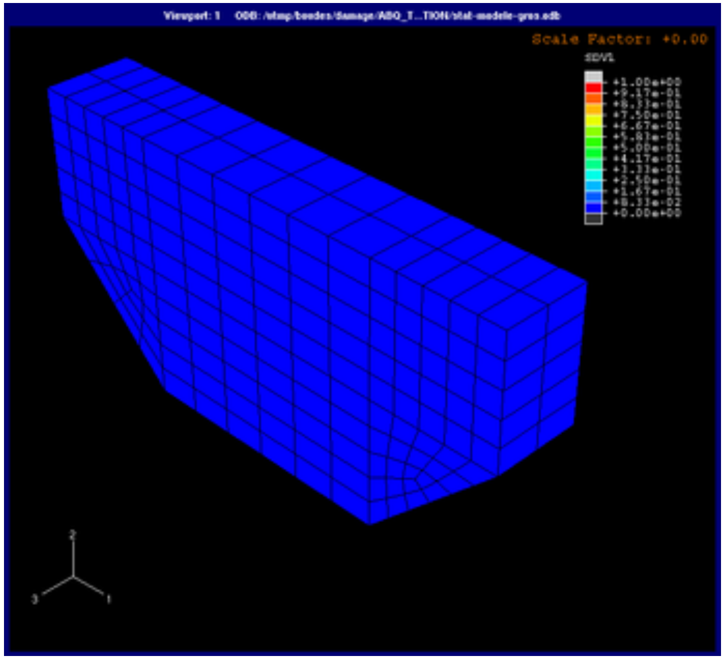
- Application to damage
- Concluding remarks

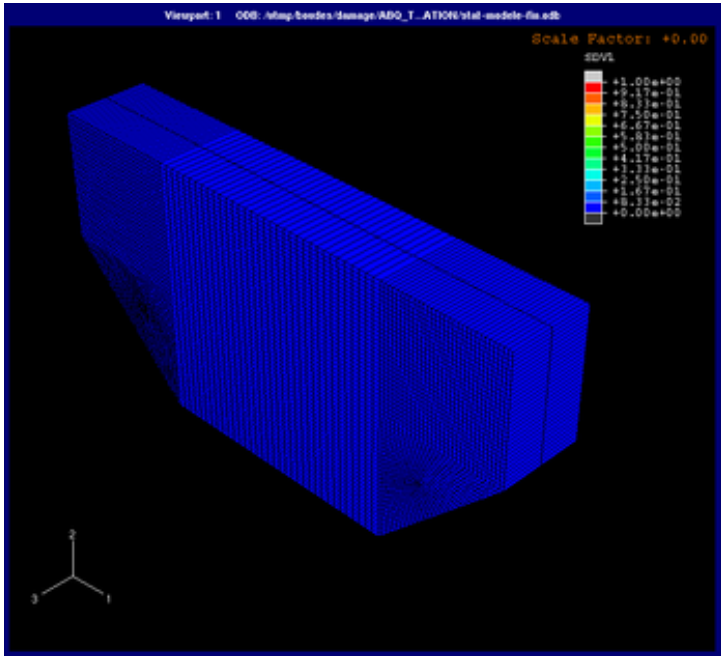
+ Application to damage: Overview



Damage mechanics becomes ill-posed in the softening region due to localization:



coarse mesh



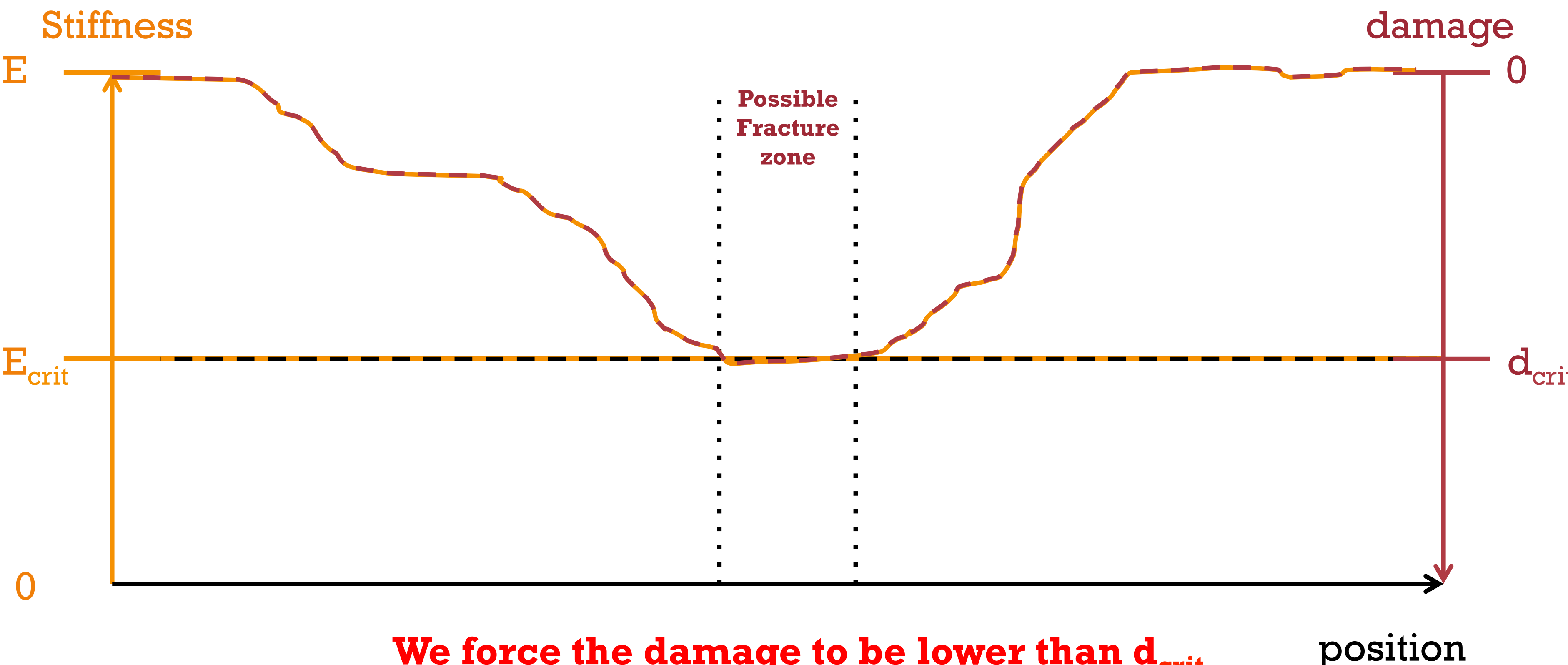
fine mesh

courtesy of G. Lubineau

We use damage only when it is valid: in the initial phase for d<d_{crit}

Application to damage: Overview

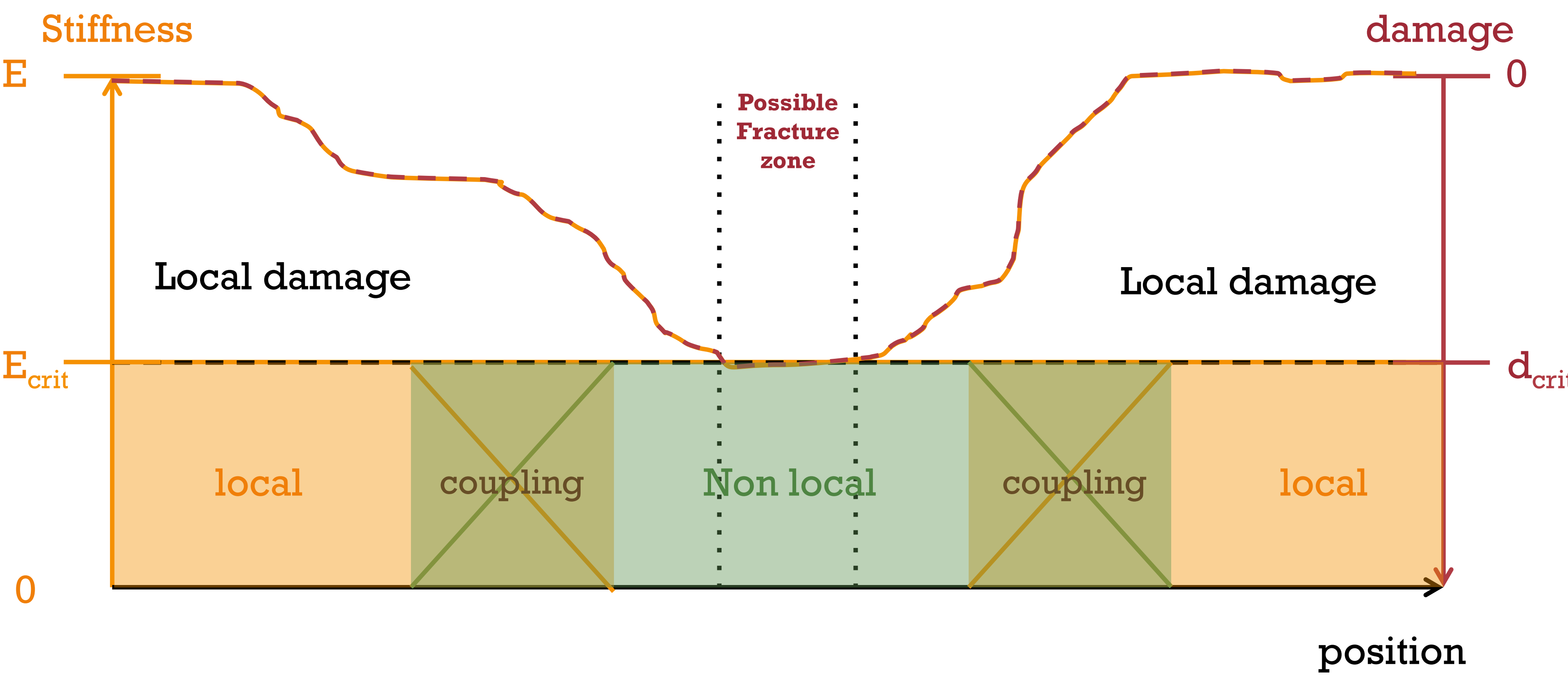
When $d = d_{crit}$, we replace the residual local model by a non-local model: here a one dimensional problem:



We force the damage to be lower than d_{crit}

Application to damage: Overview

When $d=d_{crit}$, we replace the residual local model by a non-local model: here a one dimensional problem:



Application to damage: model

Free strain energy as a function of d

$$\psi(d) = \frac{1}{2}\Lambda(1-d)tr(\underline{\underline{\varepsilon}})^2 + \Lambda(1-d)tr(\underline{\underline{\varepsilon}}^2)$$

Free strain energy as a function of d and d_{crit}

$$\begin{aligned}
\psi(d) &= \hat{\psi}(d, d_{crit}) \\
&= \psi(d_{crit}) + \psi(1 - d_{crit} + d) \\
&= \frac{1}{2}\Lambda(1-d_{crit})tr(\underline{\underline{\varepsilon}})^2 + \Lambda(1-d_{crit})tr(\underline{\underline{\varepsilon}}^2)' \\
&+ \frac{1}{2}\Lambda(d_{crit}-d)tr(\underline{\underline{\varepsilon}})^2 + \Lambda(d_{crit}-d)tr(\underline{\underline{\varepsilon}}^2)
\end{aligned}$$

Application to damage: model

Recasting the constitutive equation we get:

$$\underline{\underline{\sigma}} = \Lambda(1 - d_{crit}) tr(\underline{\underline{\varepsilon}})\mathbb{I} + 2\Lambda(1 - d_{crit})\underline{\underline{\varepsilon}}$$

$$+ \Lambda(d_{crit} - d) tr(\underline{\underline{\varepsilon}})\mathbb{I} + 2\Lambda(d_{crit} - d)\underline{\underline{\varepsilon}}$$

$$= \underline{\underline{\underline{\underline{K_o}}}}^{d_{crit}} : \underline{\underline{\varepsilon}} + \underline{\underline{\underline{\underline{K}}}}^{d}(d) : \underline{\underline{\varepsilon}}$$

Residual stiffness

"damageable" stiffness

Application to damage: model

Evolution law

$$f = Y - (k_1 * d + k_0),$$

$$Y = -\frac{\partial \psi}{\partial d}\Big|_{\varepsilon=const.}$$
$$= \frac{1}{2}\Lambda tr(\underline{\underline{\varepsilon}})^2 + \Lambda tr(\underline{\underline{\varepsilon}}^2)$$
$$= \psi^o \geq 0.$$

$$\begin{cases} \dot{d} = \frac{1}{k_1}\dot{Y}, & \text{if } d < d_{crit}, \quad and \quad f \geq 0 \\ \dot{d} = 0, & \text{if } d \geq d_{crit} \quad or \quad f < 0 \end{cases}$$

Application to damage: model

Final model

☐ **Kinematic admissibility and compatibility:**

$$\underline{\underline{\varepsilon}}(\underline{x}) = \frac{1}{2}\left(\underline{\nabla}\cdot\underline{u}(x) + {}^t\underline{\nabla}\cdot\underline{u}(x)\right) \quad \forall \underline{x}\in\Omega$$

$$\eta_{\underline{\xi}}(\underline{p}-\underline{x}) = u_{\underline{\xi}}(\underline{p}) - u_{\underline{\xi}}(\underline{x}) \quad \forall \underline{x},\underline{p}\in\Omega$$

$$\underline{u} = \underline{\bar{u}} \quad \forall \underline{x}\in S_{\underline{u}}$$

☐ **Static admissibility:**

$$\underline{div\underline{\sigma}} + \int_{H_\delta(\underline{x})} \underline{f}(\underline{p}\rightarrow\underline{x})dV_{\underline{p}} = -\underline{b} \quad \forall \underline{x}\in\Omega$$

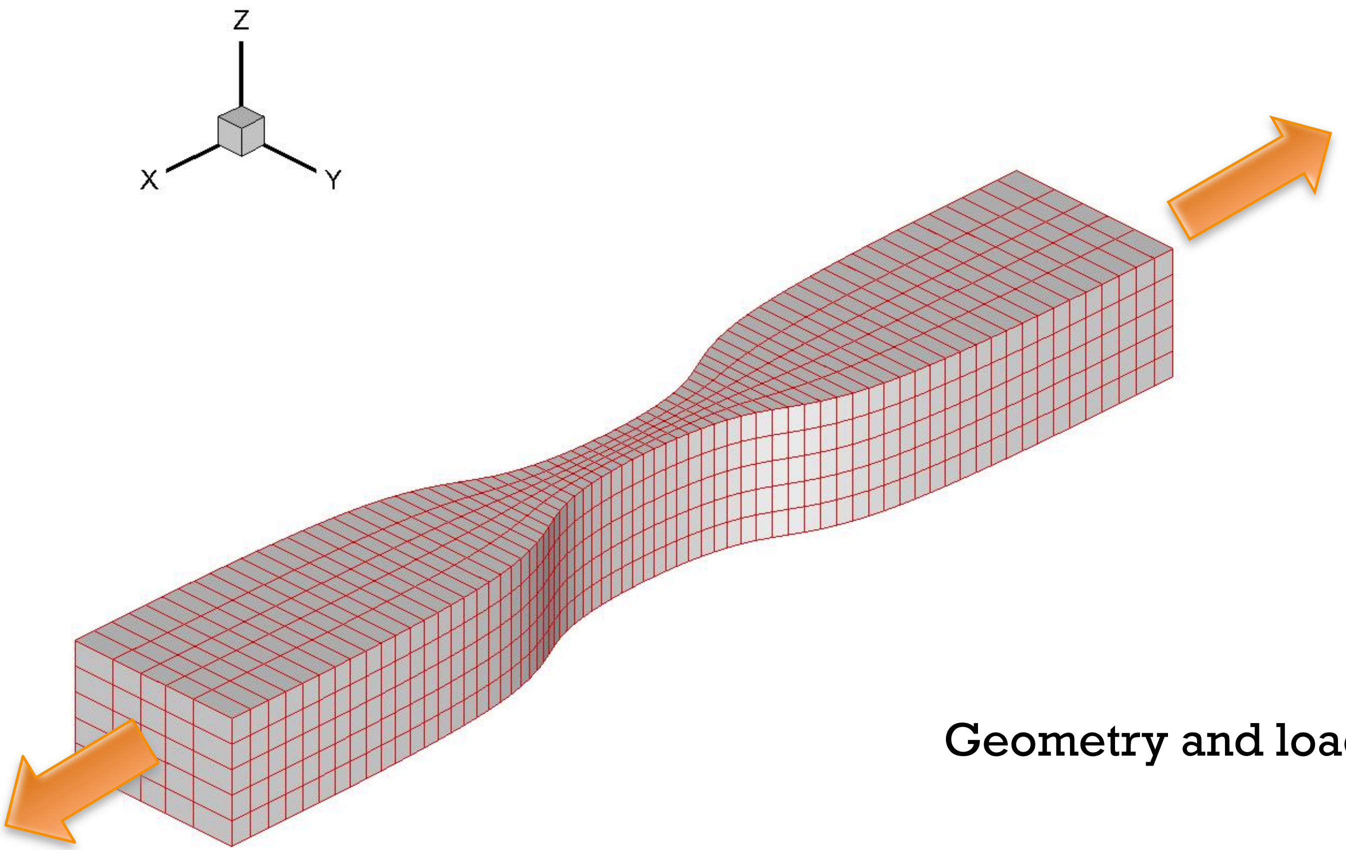
$$\underline{\underline{\sigma}}\cdot\underline{n} = \underline{\bar{T}} \quad \forall \underline{x}\in S_{\underline{T}}$$

☐ **Constitutive equations:**

$$\underline{\underline{\sigma}} = \left[\underline{\underline{\underline{\underline{K}}}}^{d_{crit}}(\underline{x}) + \underline{\underline{\underline{\underline{K}}}}^{d}(d(\underline{x}))\right] : \underline{\underline{\varepsilon}} \quad \forall \underline{x}\in\Omega$$

$$\underline{f}(\underline{p}\rightarrow\underline{x}) = \frac{(\alpha(\underline{x})+\alpha(\underline{p}))c(\|\underline{\xi}\|)}{2}\eta_{\underline{\xi}}(\underline{p}-\underline{x})\underline{e}_{\underline{\xi}} \forall \underline{x},\underline{p}\in\Omega$$

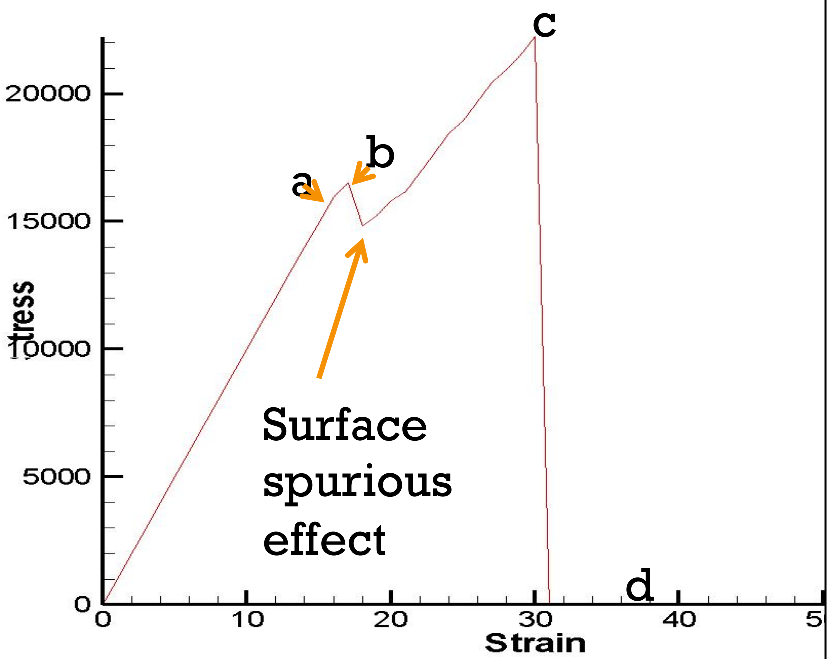
Application to damage: simulation



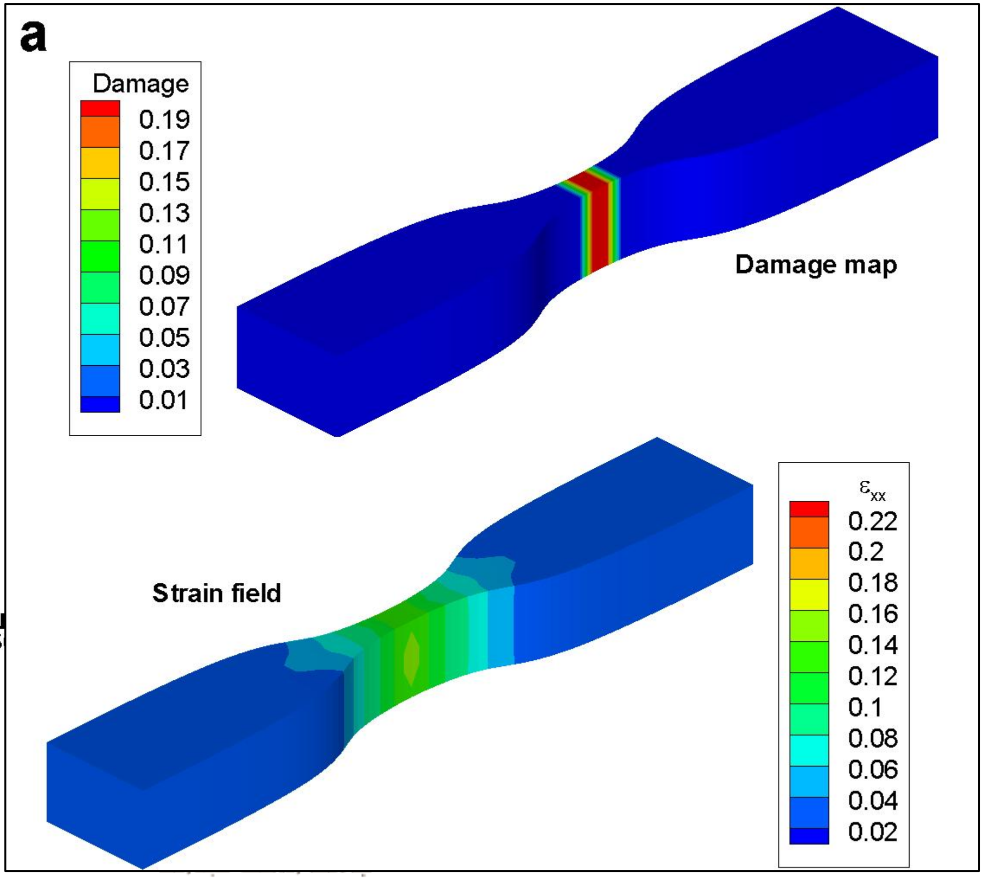
Geometry and loading

Application to damage: simulation

a: start of damage



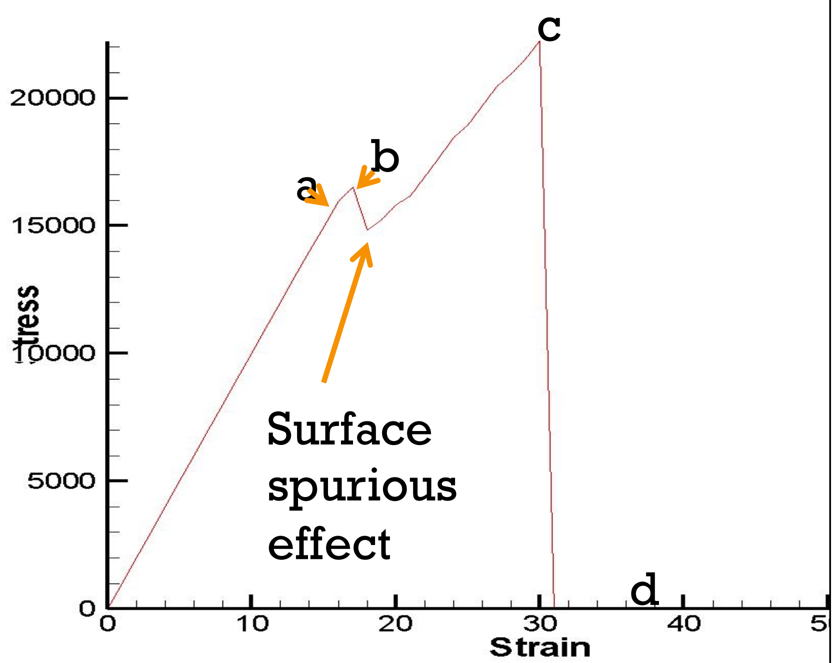
Stress/strain curve



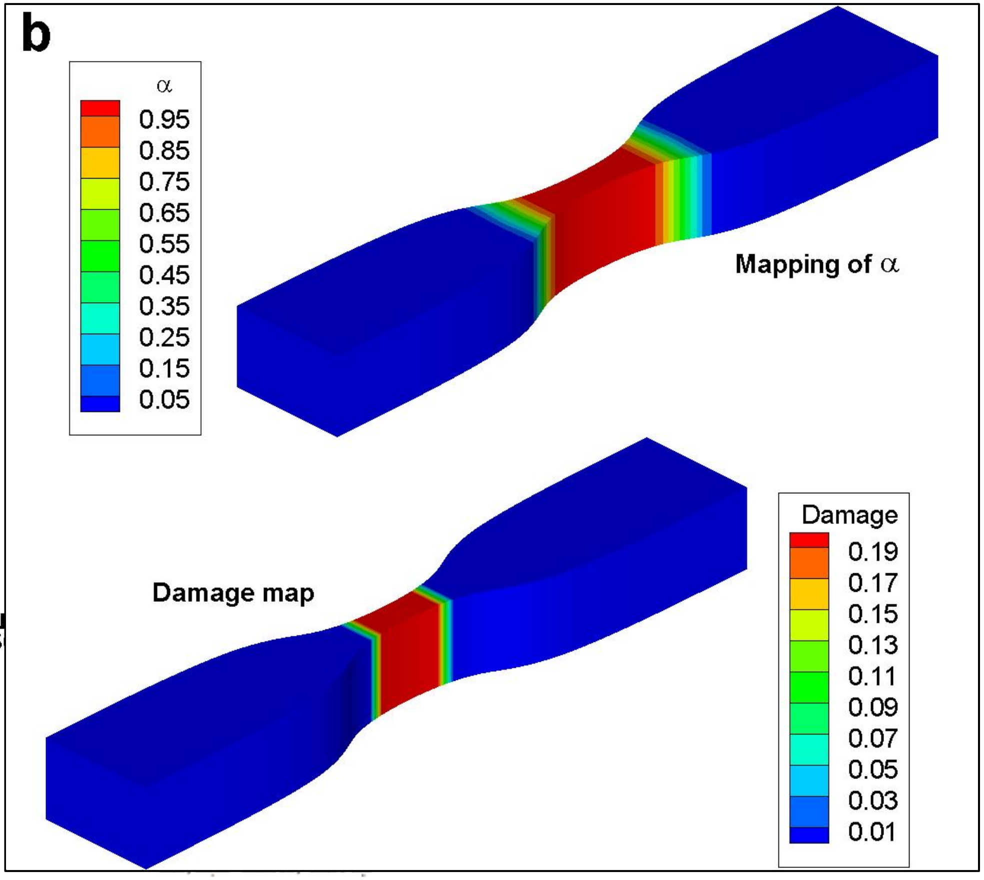
Application to damage: simulation

a: start of damage

b: introduction of a non-local zone

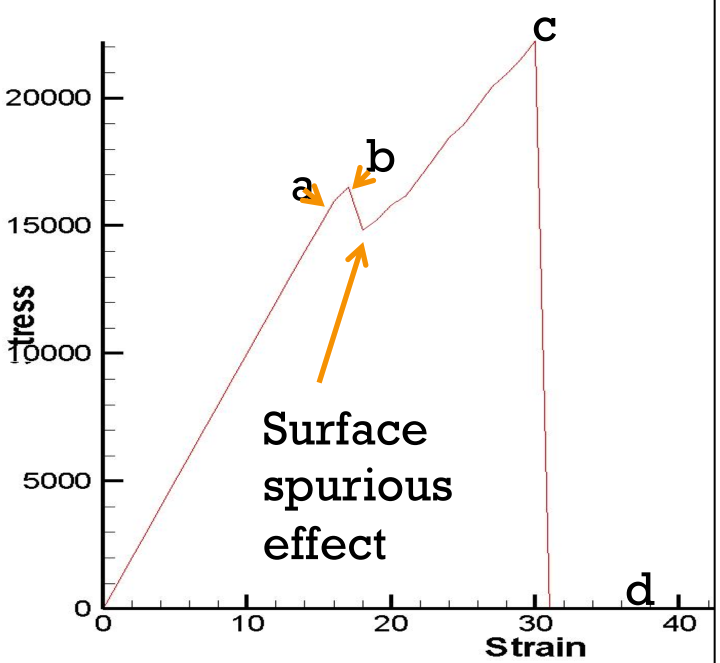


Stress/strain curve

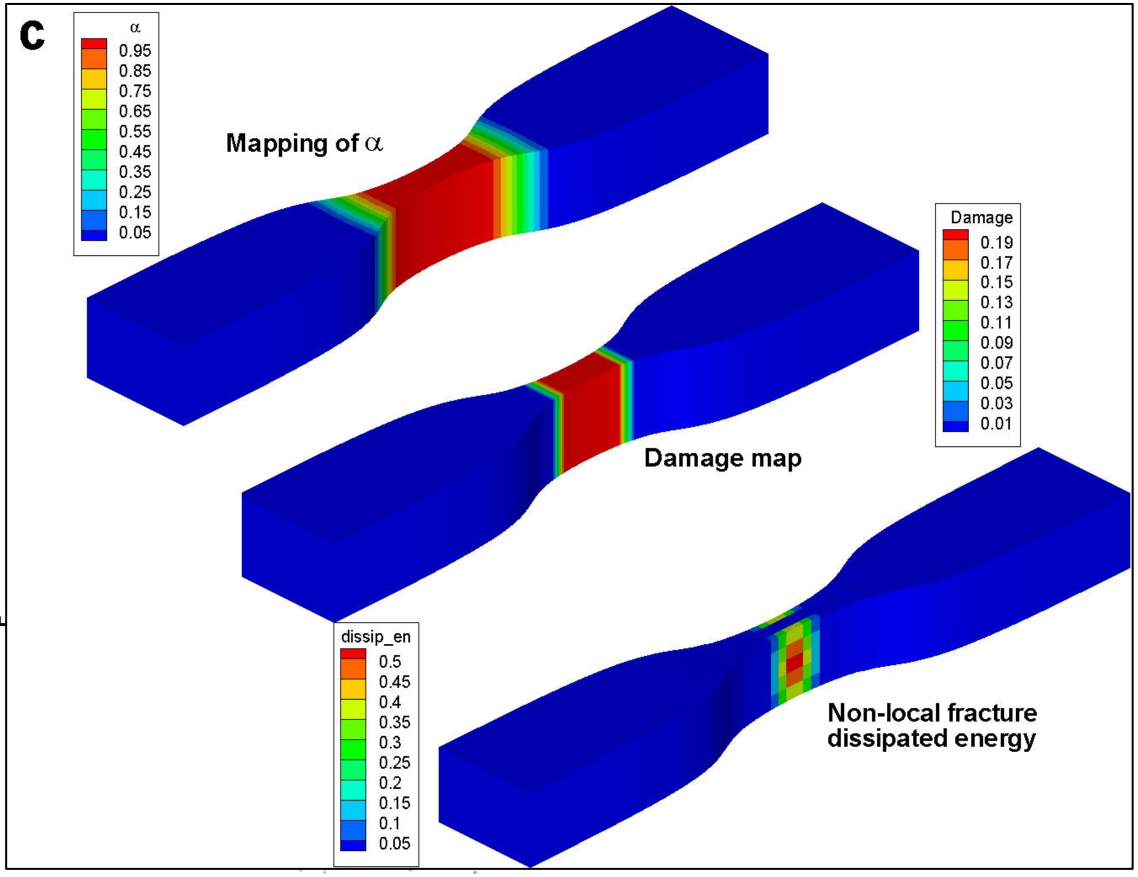


Application to damage: simulation

a: start of damage
b: introduction of a non-local zone
c: start of fracture

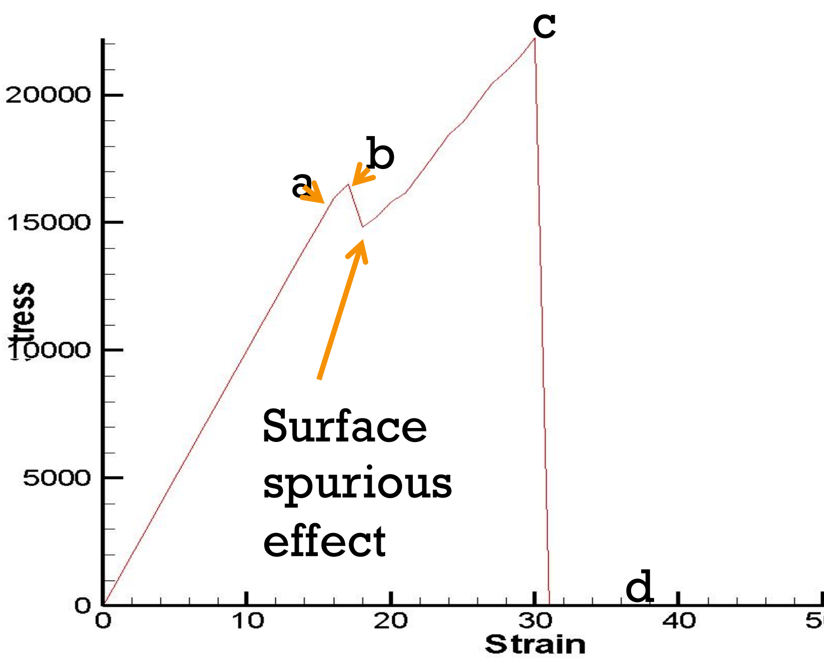


Stress/strain curve

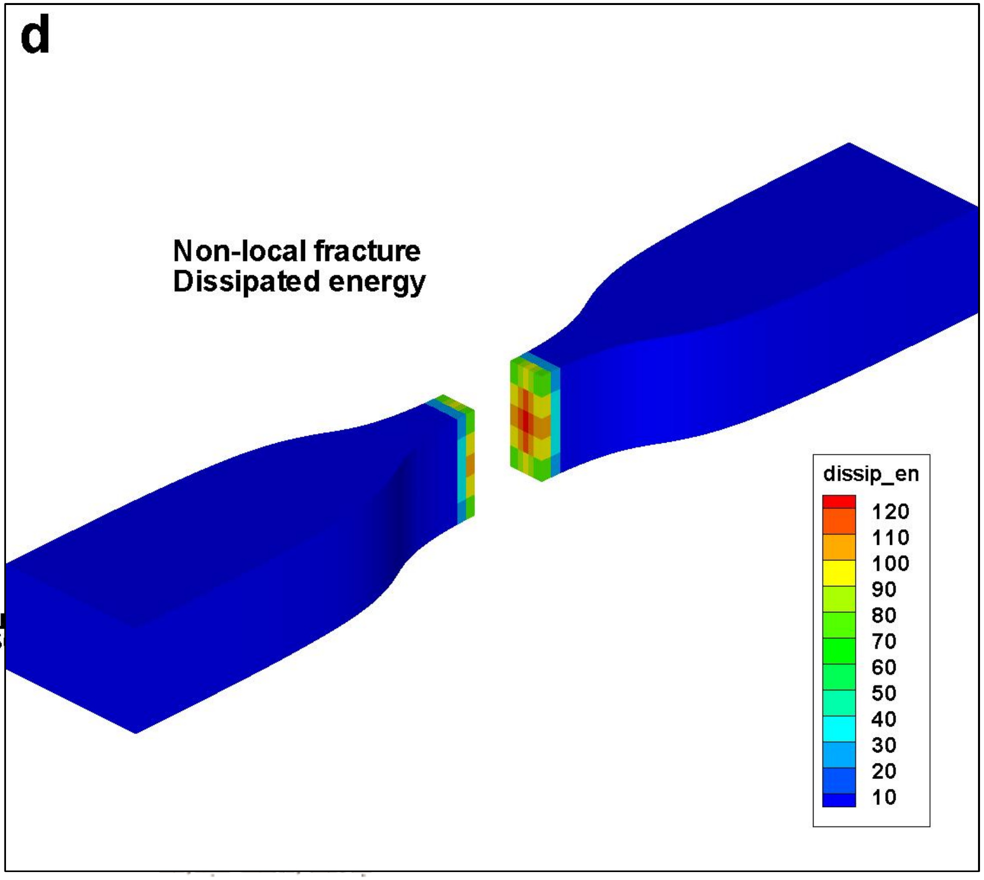


Application to damage: simulation

a: start of damage
b: introduction of a non-local zone
c: start of fracture
d: complete failure



Stress/strain curve



Application to damage: numerical results

Strain field

Application to damage: future work

The simulation tool is currently limited for computational reasons. We should:

- Pass to 2D
- Increase the size of the structure compared to delta
- Try more complex geometries

This will give use more detailed data to analyze

Application to damage: future work

Key points:

- coupling local damage/non-local fracture allows to use a non-local fracture model in the localization part of the damage model
- this alleviate the need for regularization and gives the opportunity of introducing complex behavior after the localization point.

Outline

- Treating failure
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Concluding remarks: summary

- a hybrid paradigm to ease use of peridynamics

- a coupling method based on a transformation of the constitutive equation, that is robust and adaptive

- an adaptive algorithm that follows non-local fracture

- a local damage/non-local fracture, to treat localization

Concluding remarks: future works

- Extending the numerical software to a better parallel solution to be useful for real life application

- Looking deeper into the local damage/non-local fracture model

- Extending to more complex constitutive behaviors

Concluding remarks: publications

Adaptive non-local fracture:

Y.Azdoud, F. Han, G. Lubineau, The morphing method as a flexible tool for adaptive local/non-local simulation of static fracture, (2014), minor revision, Computational Mechanics

Alternative usage of a local/non-local hybrid: nano-composite simulation:

F. Han, Y. Azdoud, G. Lubineau, Computational modeling of elastic properties of carbon nanotube/polymer composites with interphase regions. Part I: Microstructural characterization and geometric modeling, Computational Materials Science, Vol 81, 641-651, (2014)

F. Han, Y. Azdoud, G. Lubineau, Computational modeling of elastic properties of carbon nanotube/polymer composites with interphase regions. Part II: Mechanical modeling, Computational Materials Science, Vol 81, 652-661, (2014)

Extension to anisotropy:

Y.Azdoud, F. Han, G. Lubineau, A morphing framework to couple non-local and local anisotropic continua, International Journal of Solids and Structures, Vol 50, 9, 1332-1341, (2013)

The morphing method:

G. Lubineau, Y. Azdoud, F. Han, C. Rey and A. Askari, A morphing strategy to couple local to non-local continuum mechanics, Journal of the Mechanics and Physics of Solids, Vol 60, 6, 1088-1102, (2012)

Thank You.

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http://cohmas.kaust.edu.sa

yan.azdoud@kaust.edu.sa