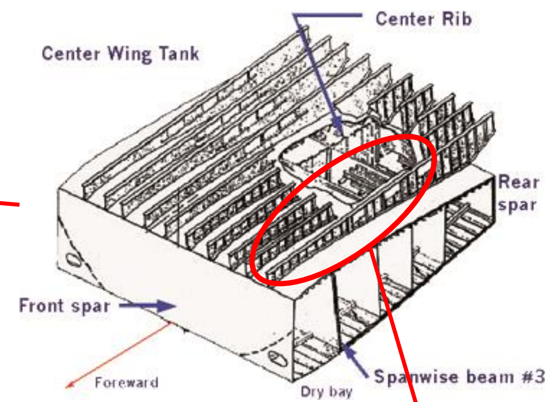
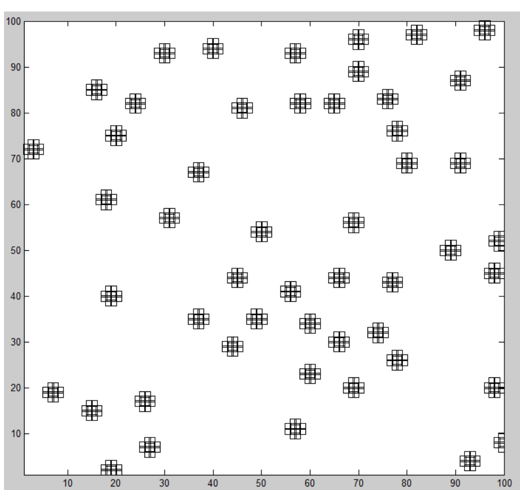
Modeling

Context

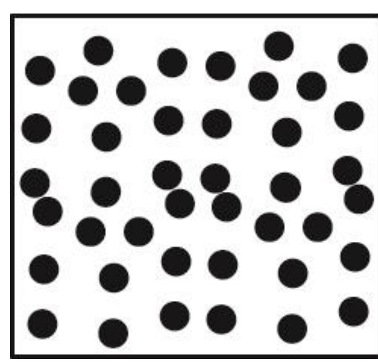
A380



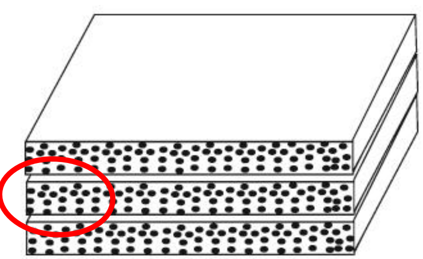
Composit wingbox



Meshing of the element (finite difference model)



Selection of the studied element

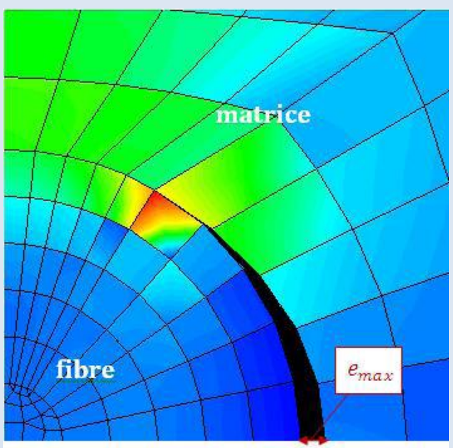


composit carbon/epoxy ply

Modeling

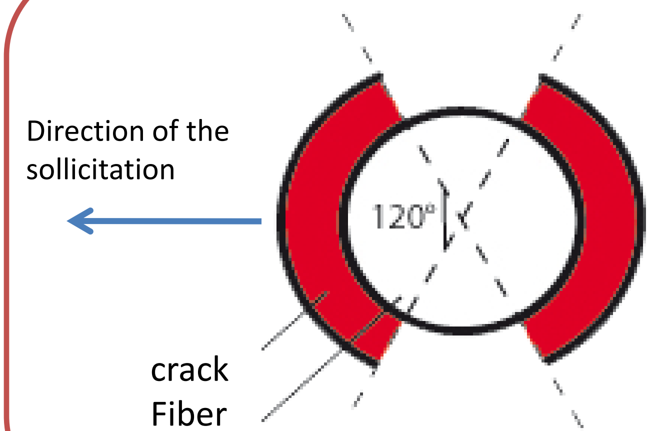
diffuse damage modeling

A test on the meshing and analysis software CASTEM gives us the geometry and the localization of the diffuse damage, around à fiber under axial stress.



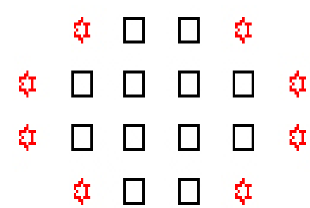
Modèle d'étude CASTEM

Modeling



Diffuse damage model

Meshing



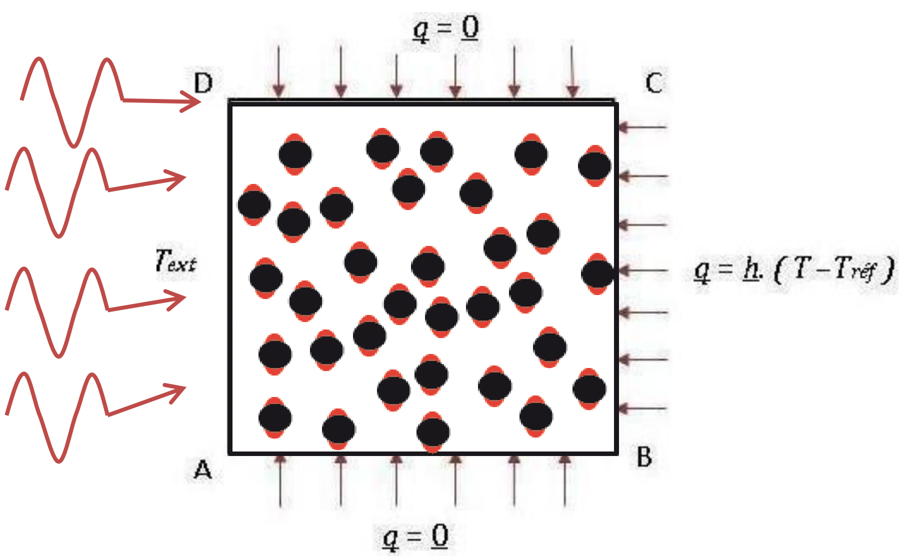
Meshed diffuse damage model

Modeling

Local equations, limit and Initial conditions

Limit conditions

On AD: sinusoid thermal solicitation
On DC & AB: adiabatic behavior
ON BC: thermal exchange with ambient air



Studied element with limit conditions

Initial conditions

$$\begin{cases} \vec{T}^{\,0}\ (\grave{a}\ t=0) = \vec{T}_i \\ \dfrac{\delta\, \vec{T}^{\,0}}{\delta\, t}(\grave{a}\ t=0) = 0 \end{cases}$$

Local equations

Equilibre thermique

Thermal equilibrium $\rho\,\dfrac{\delta\, e}{\delta\, t} = div\, q$

Loi de comportement (loi de Fourier)

Thermal behavior $q = -\lambda\ grad\, T$

On obtient l'équation de diffusion thermique :

$$\frac{\delta\, T}{\delta\, t} = \alpha\, \Delta\, T$$

Resolution procedure

Integration Schemes

Equation de diffusion

$$\boxed{\frac{\delta T^{\,n+1}}{\delta t} = \alpha_{\,matériau} \Delta\, T^{n+1}}$$

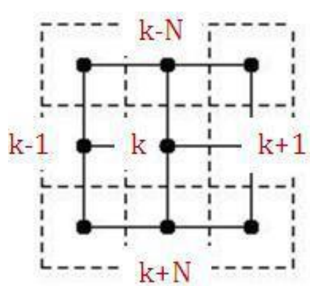
Spatial discretization

$$\Delta\, T(M,t) = \frac{\delta^2 T}{\delta x^2} + \frac{\delta^2 T}{\delta y^2}$$

Finite difference method

$$\frac{\delta^2 T(x,y)}{\delta x^2} = \frac{T(x-h,y) + T(x+h,y) - 2T(x,y)}{h^2}$$

$$\Delta\, T_k = (T_{k-1} + T_{k+1} + T_{k-N} + T_{k+N}) - 4\, T_k$$



temporal discretization

Time step $\Delta\, t$ $t_n \in [0, T]$

$$(1-\theta)\,\frac{\delta\, T^{\,n+1}}{\delta\, t} + \theta\,\frac{\delta\, T^{\,n}}{\delta\, t} = \frac{T^{\,n+1} - T^{\,n}}{\Delta\, t}$$

Crank-Nicholson scheme: $\boxed{\theta = 1/2}$

$$\frac{\delta\, T^{\,n+1}}{\delta\, t} = 2\,\left(\frac{T^{\,n+1} - T^{\,n}}{\Delta\, t}\right) - \frac{\delta\, T^{\,n}}{\delta\, t}$$

Implicit scheme

The temperature at the point k and at the time t_n respects this relation

$$\frac{2}{\Delta t}\, T_k^{\,n} + \frac{\delta T_k^{\,n}}{\delta t} = T_k^{\,n+1}\left(\frac{2}{\Delta\, t} + \frac{4\,\alpha_{\,i}}{h^2}\right) - \frac{\alpha_{\,i}}{h^2}\,(T_{k-1}^{\,n+1} + T_{k+1}^{\,n+1} + T_{k-N}^{\,n+1} + T_{k+N}^{\,n+1})$$

Resolution procedure

Determination of the matrix equations to resolve

Construction of the matrix of rigidity

$$\underbrace{\begin{pmatrix} -\dfrac{\alpha}{h^2} \\ -\dfrac{\alpha}{h^2} \\ \dfrac{2}{\Delta t} + \dfrac{4\,\alpha}{h^2} \\ -\dfrac{\alpha}{h^2} \\ -\dfrac{\alpha}{h^2} \end{pmatrix}^T}_{} \underbrace{\begin{pmatrix} T_{k-1} \\ T_{k+1} \\ T_k \\ T_{k+N} \\ T_{k-N} \end{pmatrix}^{n+1}}_{} = \frac{2}{\Delta t} T_k^n + \frac{\delta}{\delta t} T_k^n$$

K th « elementary » line of $[M]$

« elementary » Vector of temperature

Table of connectivity

Matrix of rigidity $[M]$
length N^2

CL

Iterative resolution

$$\begin{cases} [M]\,\vec{T}^{\,n+1} = \left(\dfrac{2}{\Delta t}\,\vec{T}^{\,n} + \dfrac{\delta \vec{T}^{\,n}}{\delta t} \right) \\[2ex] \dfrac{\delta \vec{T}^{\,n}}{\delta t} = - \dfrac{\delta \vec{T}^{\,n-1}}{\delta t} + 2 \left(\dfrac{\vec{T}^{\,n} - \vec{T}^{\,n-1}}{\Delta T} \right) \end{cases}$$

Method of penalization

$$\vec{T}_{global} = \begin{pmatrix} \vec{T}_u \\ \vec{T}_i \end{pmatrix} \qquad \vec{T}_i = \begin{pmatrix} T_{excitationt} \\ T_{ambiant} \end{pmatrix}$$

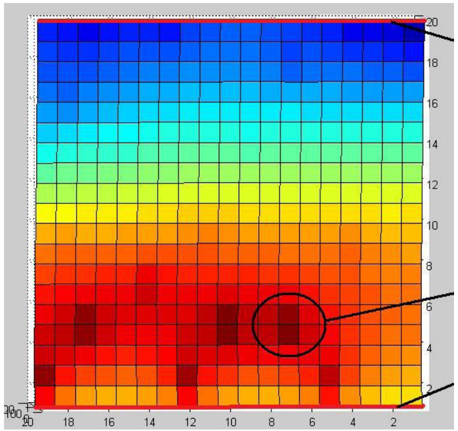
$$\vec{T}_i^{\,n+1} \approx \vec{T}_i^{\,n}$$

$$\boxed{[M_{globale}] = \begin{pmatrix} [M] & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{pmatrix}}$$

Exploitation of the results

Observation tools

Observation the temperature in the model



Output surface

Visualisation of a crack

Input surface

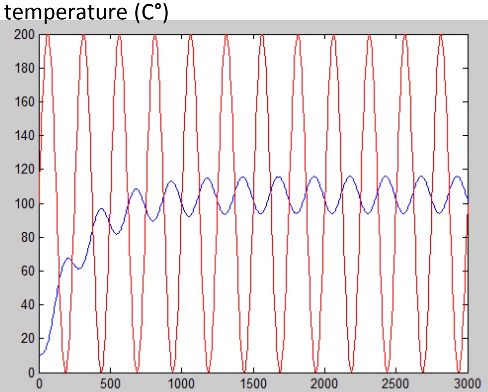
Visualization of the mesh at a certain time

The cracks make the thermal diffusion slower

We can see a phase shift between the two surfaces

Observation of the temporal output

temperature (C°)



time(s)

Visualization of the input surface temperature (red) and of the output surface temperature (blue)

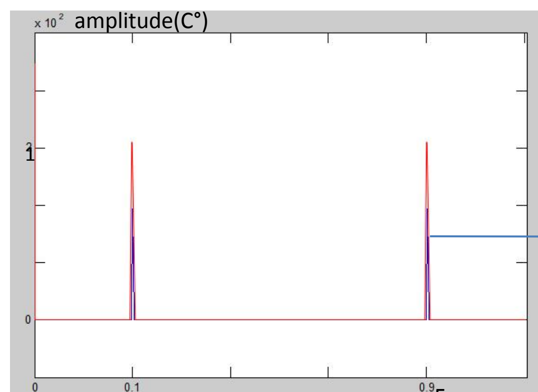
It gives:

The reponse time

The phase shift

The amplitude shift

Observation the spectrum



Frequency (Hz)

Visualization of the amplitudes of the thermal solicitation (red) and of the thermal response (blue)

It gives:

The amplitude for each solicitation frequency

Aliasing of the Spectrum

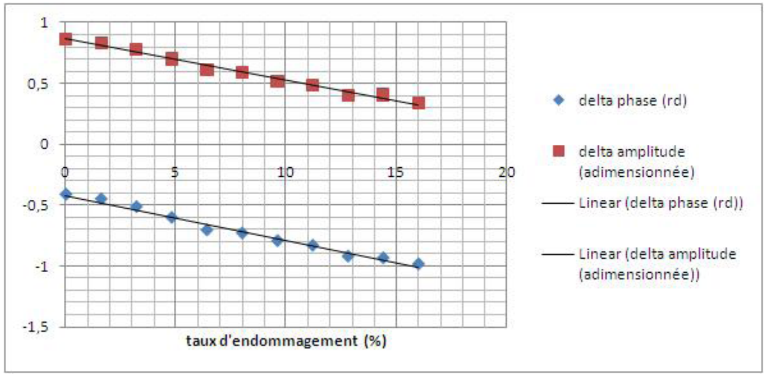
The theorem of Shannon is validated

$$2 f_{max} \leq f_e$$

Exploitation of the results

Influence of the frequency and of the cracks ratio

Influence of the cracks ratio



Phase et amplitude relative en fonction du taux d'endommagement

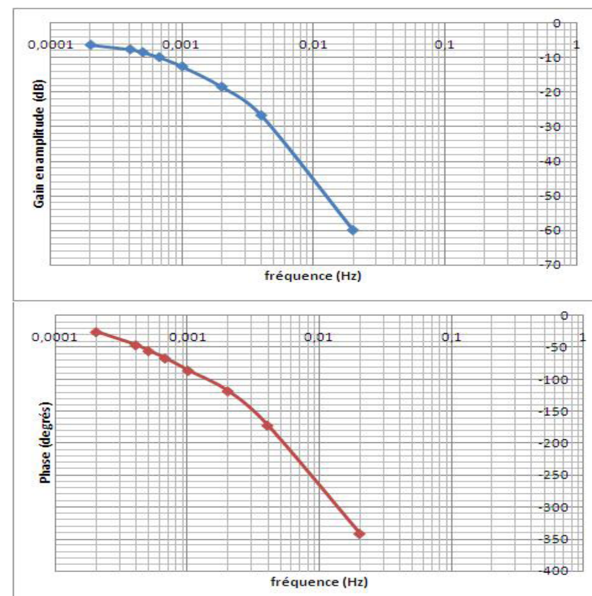
We change the cracks ratio between 0 and 16% while the frequency of the solicitation stay the same

There is an affine relation between:
-the phase and the crack ratio
-the amplitude and the crack ratio

Influence of the solicitation frequency

We change the frequency between 0,02 and 2.10-4 Hz with a 10% cracks ratio

The element has a low pass behavior



Lieu de Bode de la réponse

Exploitation of the results

Validation of the ratio between the fiber diameter and the model length

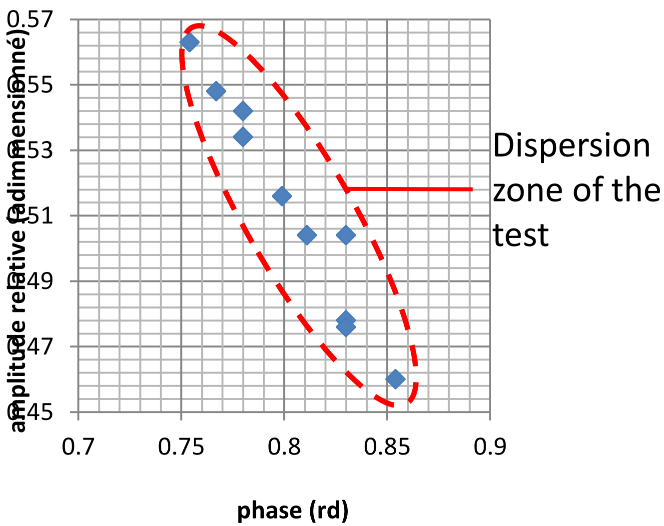
Dispersion of the results

We try 10 random draw of the fibers positions with a 10% crack ratio and a 1/2500 Hz frequency

The ration between the model length and the fiber diameter is 1 to 10, as a result:

- A 10% error on the phase

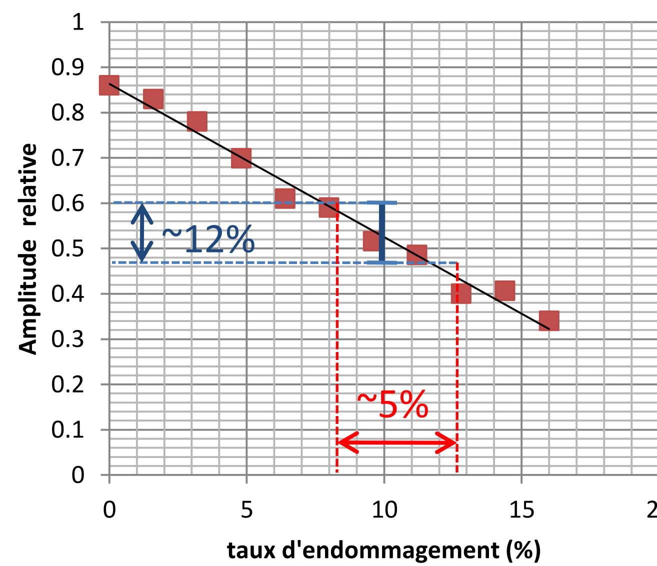
- A 10% error on the amplitude



Relative amplitude in function of the phase

Influence on the accuracy

The cracks ratio is known at more or less 3%

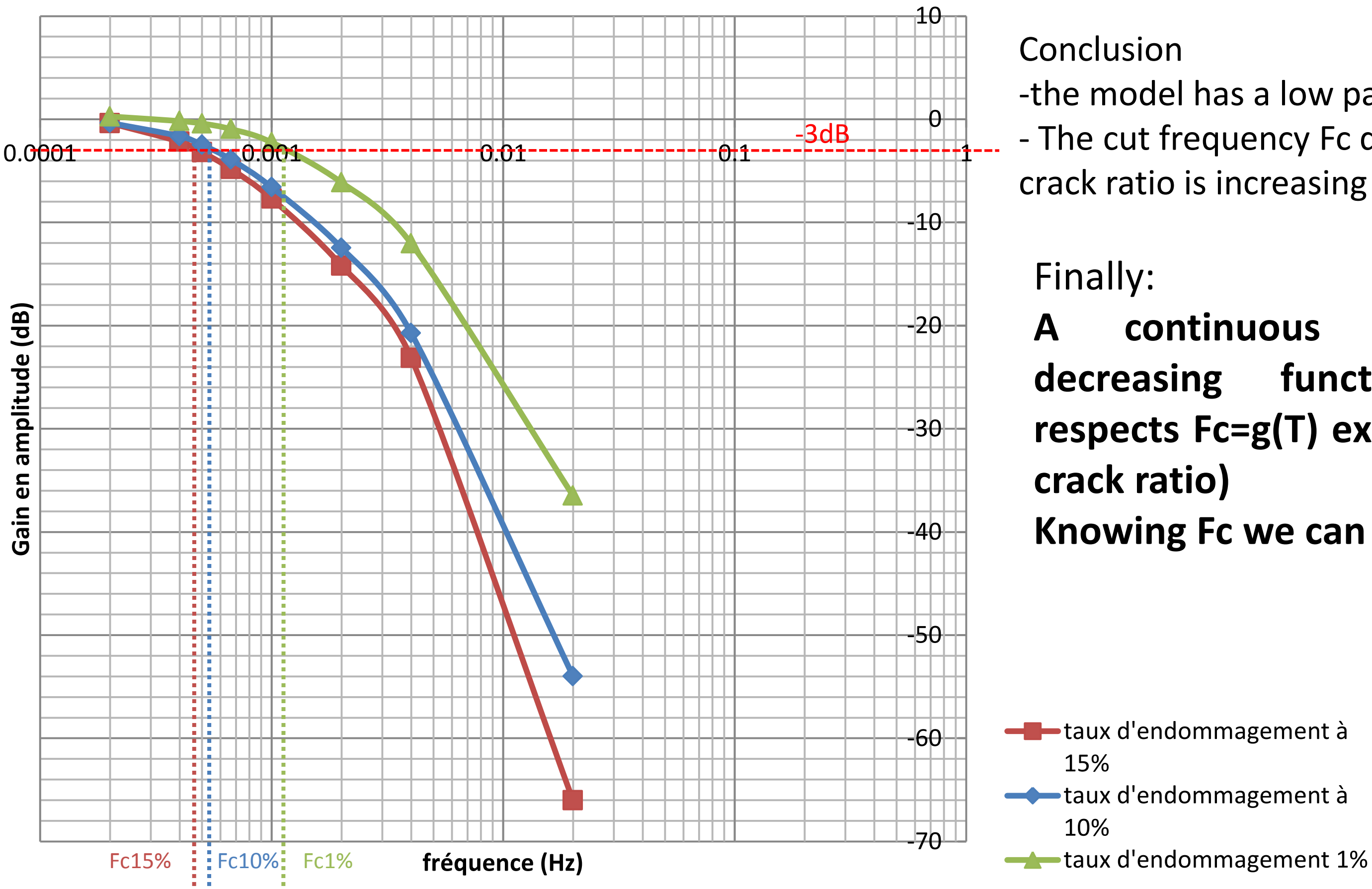


Relative amplitude in function of the cracks ratio

Exploitation of the results

Conclusion on the results

Amplitude in function of the frequency for 3 different crack ratio



Lieu de Bode en gain de trois réponses

Conclusion

-the model has a low pass behavior

- The cut frequency Fc decrease when the crack ratio is increasing

Finally:

A continuous and strictly decreasing function g that respects Fc=g(T) exist. (with T the crack ratio)

Knowing Fc we can find T using:

$$\boxed{T=g^{-1}(F_c)}$$