
Statistical Analysis of Reliability and Survival Data: Rotterdam Dataset

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Leuven. May, 2022

1 Exploratory Data Analysis

These data sets are used in the paper by Royston and Altman that is referenced below. The Rotterdam data is used to create a fitted model, and the GBSG data for validation of the model. The paper gives references for the data source.

There are 43 subjects who have died without recurrence, but whose death time is greater than the censoring time for recurrence. A common way that this happens is that a death date is updated in the health record sometime after the research study ended, and said value is then picked up when a study data set is created. But it raises serious questions about censoring. For instance subject 40 is censored for recurrence at 4.2 years and died at 6.6 years; when creating the endpoint of recurrence free survival (earlier of recurrence or death), treating them as a death at 6.6 years implicitly assumes that they were recurrence free just before death. For this to be true we would have to assume that if they had progressed in the 2.4 year interval before death (while off study), that this information would also have been noted in their general medical record, and would also be captured in the study data set. However, that may be unlikely. Death information is often in a centralized location in electronic health records, easily accessed by a programmer and merged with the study data, while recurrence may require manual review. How best to address this is an open issue.

Table 1: Data description

pid	Patient identifier
year	Year of surgery
age	Age at surgery
meno	Menopausal status (0 = premenopausal, 1 = postmenopausal)
size	Tumor size, a factor with levels ≤ 20 , 20-25, > 50
grade	Differentiation grade
nodes	Number of positive lymph nodes
pgr	Progesterone receptors (fmol/l)
er	Estrogen receptors (fmol/l)
hormon	Hormonal treatment (0=no, 1=yes)
chemo	Chemotherapy
rtime	Days to relapse or last follow-up
recur	0 = no relapse, 1 = relapse
dtime	Days to death or last follow-up
death	0 = alive, 1 = dead

Table 1 explains the covariates in the Rotterdam dataset.

2 Further Analysis

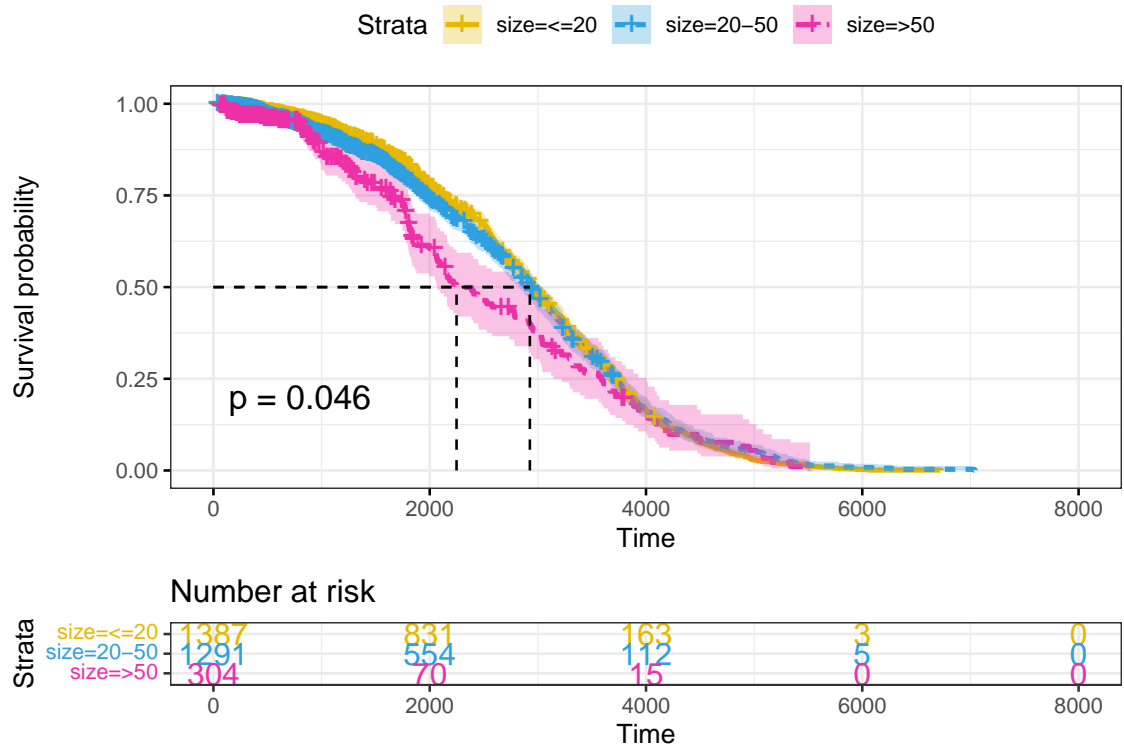
Now the focus will be on the response variable, the censoring indicator, and the categorical variable.

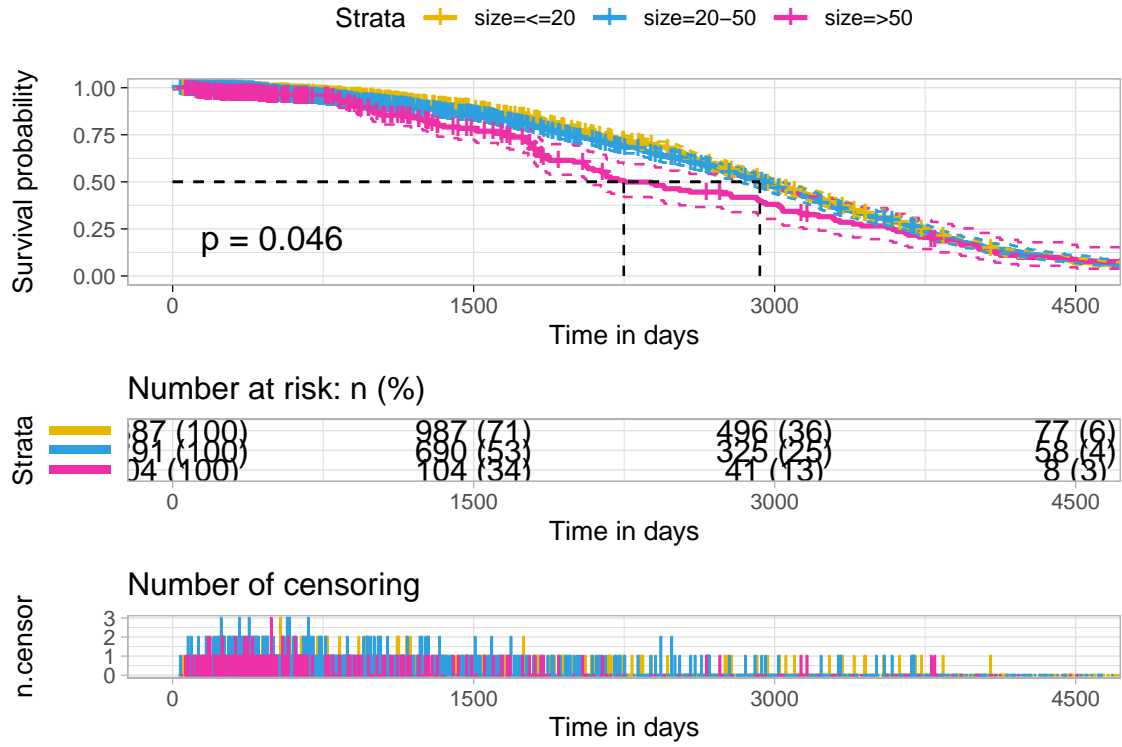
2.1 Survival Distrubution by Levels of ...

- For each of the levels of the categorical variable, compute the survival distribution. Plot them on the same graph. What do the graphs suggest?

2.1.1 Size

##	time	n.risk	n.event	n.censor	surv	upper	lower
## 1	36	1387	1	0	0.9992790	1.0000000	0.9978674
## 2	49	1386	1	0	0.9985580	1.0000000	0.9965631
## 3	50	1385	0	1	0.9985580	1.0000000	0.9965631
## 4	64	1384	1	0	0.9978365	1.0000000	0.9953940
## 5	76	1383	0	1	0.9978365	1.0000000	0.9953940
## 6	87	1382	1	0	0.9971145	0.9999422	0.9942949





The horizontal axis represents time in days, and the vertical axis shows the probability of surviving, or the proportion of people surviving. The lines represent survival curves of the three groups. A vertical drop in the curves indicates an event. The vertical tick mark on the curves means that a patient was censored at this time.

Table 2: Summary of the model.

	records	n.max	n.start	events	rmean	se(rmean)	median	0.95LCL	0.95UCL
Size (≤ 20)	1387	1387	1387	1048	2922.224	34.31951	2951	2891	3045
Size (20–50)	1291	1291	1291	734	2869.170	43.69240	2926	2826	3031
Size (> 50)	304	304	304	123	2558.339	111.69108	2248	2070	2918

At time zero, the survival probability is 1.0 (or 100% of the participants are alive). At time 2250, the probability of survival is approximately 0.625 for size ≥ 50 , and 0.85 for size < 50 .

From Table 2 can be seen that the median survival for Size 50 is 2951, for Size 20-50 is 2926, and for Size > 50 is 2248. This suggests slightly worse survival for patients with tumor of larger size. However, to evaluate whether this difference is statistically significant requires a formal statistical test, a subject that is discussed in the next sections.

2.1.2 Menopause

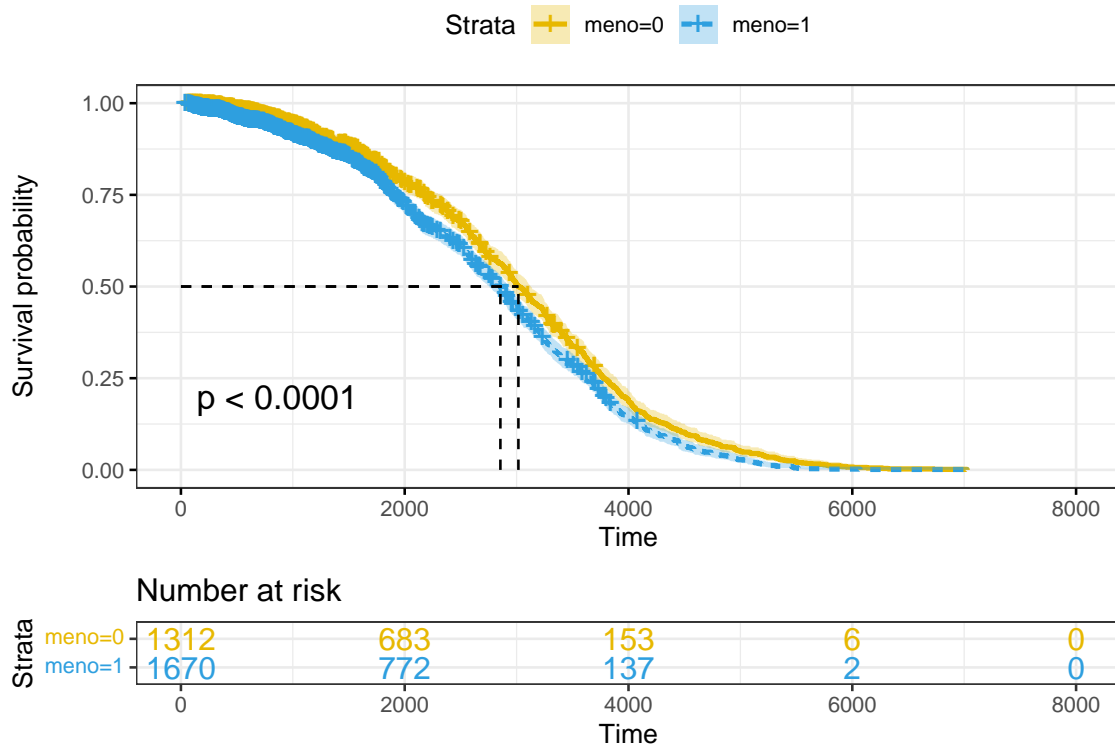


Table 3: Median survival times for each group.

	records	n.max	n.start	events	rmean	se(rmean)	median	0.95LCL	0.95UCL
Meno = 0	1312	1312	1312	855	2997.972	39.83041	3016	2945	3128
Meno = 1	1670	1670	1670	1050	2770.741	34.82820	2855	2755	2918

The horizontal axis represents time in days, and the vertical axis shows the probability of surviving, or the proportion of people surviving. The lines represent survival curves of the three groups. A vertical drop in the curves indicates an event. The vertical tick mark on the curves means that a patient was censored at this time.

At time zero, the survival probability is 1.0 (or 100% of the participants are alive). At time 2200, the probability of survival is approximately 0.75 for premenopausal patients, and 0.825 for postmenopausal patients.

From Table 3 can be seen that the median survival is 3016 for premenopausal patients, and 2855 for postmenopausal, suggesting slightly worse survival for patients that have gone through menopause. However, to evaluate whether this difference is statistically significant requires a formal statistical test, a subject that is discussed in the next sections.

2.1.3 Hormonal Treatment

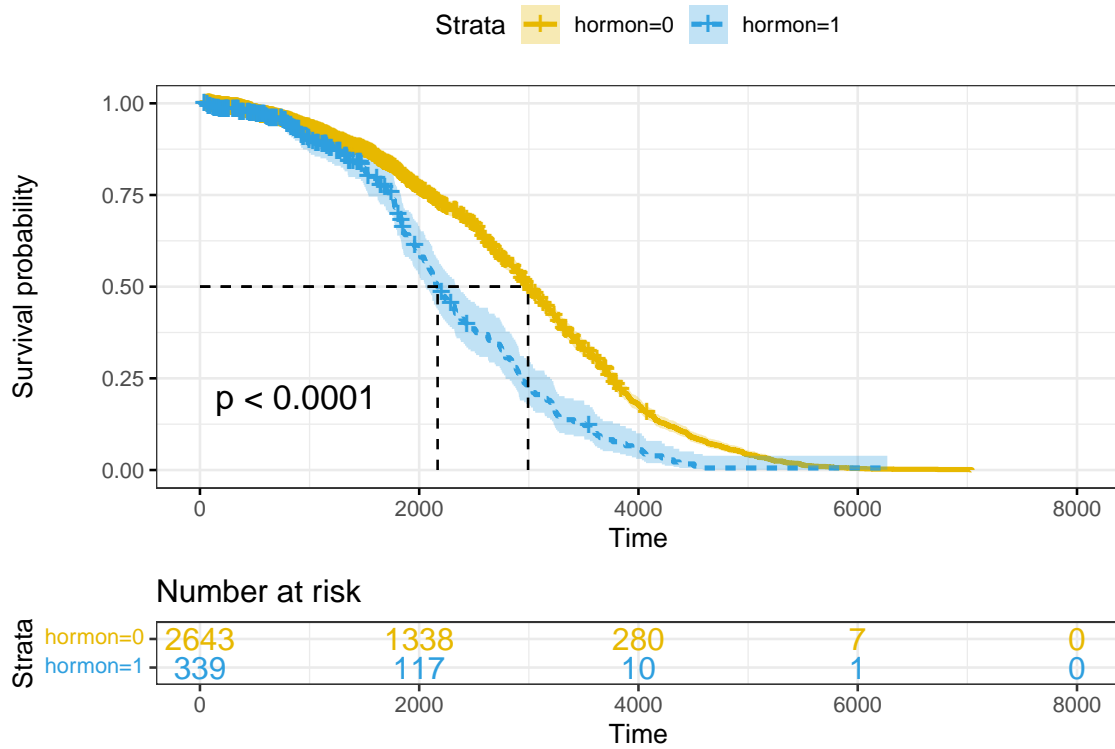


Table 4: Median survival times for each group.

	records	n.max	n.start	events	rmean	se(rmean)	median	0.95LCL	0.95UCL
hormon=0	2643	2643	2643	1701	2938.101	27.98564	2993	2945	3074
hormon=1	339	339	339	204	2307.286	66.70770	2168	2056	2340

The horizontal axis represents time in days, and the vertical axis shows the probability of surviving, or the proportion of people surviving. The lines represent survival curves of the three groups. A vertical drop in the curves indicates an event. The vertical tick mark on the curves means that a patient was censored at this time.

At time zero, the survival probability is 1.0 (or 100% of the participants are alive). At time 2200, the probability of survival is approximately 0.75 for premenopausal patients, and 0.825 for postmenopausal patients.

From Table 4 can be seen that the median survival is 2993 for patients that went through hormonal therapy, and 2168 for those who did not, suggesting slightly worse survival for patients that have gone through the hormonal therapy. However, to evaluate whether this difference is statistically significant requires a formal statistical test, a subject that is discussed in the next sections.

2.1.4 Chemotherapy

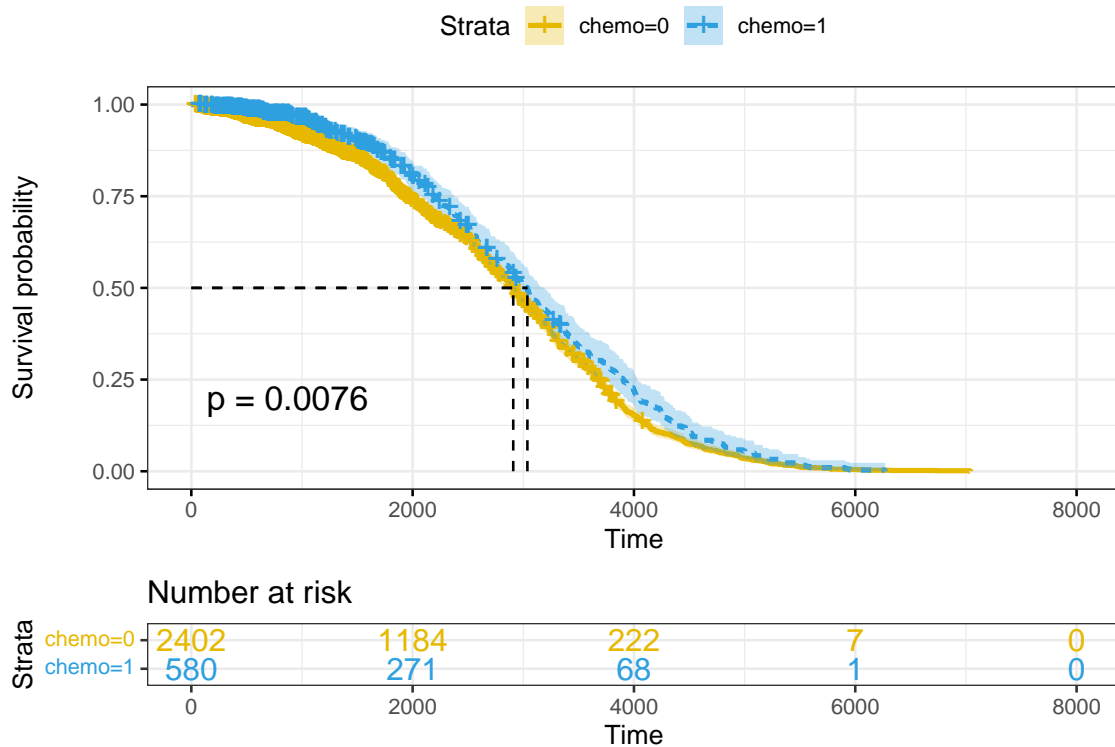


Table 5: Median survival times for each group.

	records	n.max	n.start	events	rmean	se(rmean)	median	0.95LCL	0.95UCL
chemo=0	2402	2402	2402	1576	2835.109	29.13472	2909	2843	2966
chemo=1	580	580	580	329	3040.167	62.10873	3037	2879	3186

The horizontal axis represents time in days, and the vertical axis shows the probability of surviving, or the proportion of people surviving. The lines represent survival curves of the three groups. A vertical drop in the curves indicates an event. The vertical tick mark on the curves means that a patient was censored at this time.

At time zero, the survival probability is 1.0 (or 100% of the participants are alive). At time 2200, the probability of survival is approximately 0.75 for premenopausal patients, and 0.825 for postmenopausal patients.

From Table 5 can be seen that the median survival is 2993 for patients that went through hormonal therapy, and 2168 for those who did not, suggesting slightly better survival for patients that have gone through chemotherapy. However, to evaluate whether this difference is statistically significant requires a formal statistical test, a subject that is discussed in the next sections.

2.2 Confidence Intervals and Estimators by Levels of ...

- (b) For each level obtain an appropriate estimator and confidence interval for the 3 quartiles of the survival curves. Interpret the results.

2.2.1 Size

\$quantile

```

##           25    50    75
## size=<=20 2120 2951 3716
## size=20-50 1969 2926 3713
## size=>50   1641 2248 3588
##
## $lower
##           25    50    75
## size=<=20 2011 2891 3642
## size=20-50 1869 2826 3632
## size=>50   1310 2070 3143
##
## $upper
##           25    50    75
## size=<=20 2236 3045 3782
## size=20-50 2108 3031 3816
## size=>50   1808 2918 4006

```

2.2.2 Menopause

```

## $quantile
##           25    50    75
## meno=0 2182 3016 3780
## meno=1 1905 2855 3654
##
## $lower
##           25    50    75
## meno=0 2064 2945 3703
## meno=1 1838 2755 3557
##
## $upper
##           25    50    75
## meno=0 2329 3128 3896
## meno=1 2002 2918 3725

```

2.2.3 Hormonal Treatment

```

## $quantile
##           25    50    75
## hormon=0 2097 2993 3754
## hormon=1 1752 2168 2914
##
## $lower
##           25    50    75
## hormon=0 2002 2945 3710
## hormon=1 1538 2056 2788
##
## $upper
##           25    50    75

```



```
## hormon=0 2182 3074 3813
## hormon=1 1834 2340 3194
```

2.2.4 Chemotherapy

```
## $quantile
##          25    50    75
## chemo=0 1956 2909 3676
## chemo=1 2197 3037 3907
##
## $lower
##          25    50    75
## chemo=0 1869 2843 3629
## chemo=1 2083 2879 3723
##
## $upper
##          25    50    75
## chemo=0 2050 2966 3741
## chemo=1 2378 3186 4031
```

2.3 Test of Differences Between the Survival Curves

(c) Conduct a single test of differences between the survival curves. Justify your choice of test.

2.3.1 I need to rephrase all of this text, because it's been copy-pasted!!!

Now, the questions that arises is if these two curves are statistically equivalent. For answering it, we can use the log-rank test (Mantel 1966; Peto and Peto 1972). This is the most well-known and widely used method to test the null hypothesis of no difference in survival between two or more independent groups. It is a large-sample chi-square test that is obtained by constructing a two by two contingency table at each distinct event time, and comparing the failure rates between the two groups, conditional on the number at risk in each group. The test compares the entire survival experience between groups and can be thought of as a test of whether the survival curves are identical or not.

When we state that two KM curves are statistically equivalent, we mean that, based on a testing procedure that compares the two curves in some overall sense, we do not have evidence to indicate that the true (population) survival curves are different. The null hypothesis of the testing procedure is that there is no overall difference between the two (or k) survival curves. Under this, the log-rank statistic is approximately a chi-square with $k-1$ degree of freedom. Thus, tables of the chi-square distribution are used to determine the p-value. This test is the one with most power to test differences that fit the proportional hazards model - so works well as a set-up for subsequent Cox regression. It gives equal weight to early and late failures.

An alternative test that is often used is the Peto & Peto (Peto and Peto 1972) modification of the Gehan-Wilcoxon test (Gehan 1965). This last one is a variation of the log-rank test statistic and is derived by applying different weights at the f -th failure time. This approach is most sensitive to early differences (or earlier time points) between survival. This type of weighting may be used to assess whether the effect of a treatment/marketing campaign on survival is strongest in the earlier phases of administration/contacto and tends to be less effective over time. (Marta Sestelo 2017)

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ size, data = data)
##
##           N Observed Expected (O-E)^2/E (O-E)^2/V
## size=<=20 1387      1048      1062    0.175    0.397
## size=20-50 1291       734       744    0.145    0.240
## size=>50   304       123        99    5.825    6.169
##
##  Chisq= 6.2  on 2 degrees of freedom, p= 0.05
```

We fail to reject the null hypothesis, hence we do not have evidence to indicate that the three survival curves are different.

2.3.2 Log-rank Test

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ meno, data = data,
##          rho = 0)
##
##           N Observed Expected (O-E)^2/E (O-E)^2/V
## meno=0 1312      855      946    8.67    17.4
## meno=1 1670     1050     959    8.54    17.4
##
##  Chisq= 17.4  on 1 degrees of freedom, p= 3e-05
```

Using the log-rank test, we reject the null hypothesis. Hence, it is concluded that there is statistically significant difference in survival curves between patients who have gone through menopause, and those who have not.

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ hormon, data = data,
##          rho = 0)
##
##           N Observed Expected (O-E)^2/E (O-E)^2/V
## hormon=0 2643     1701     1788    4.23    70
## hormon=1  339      204      117   64.72    70
##
##  Chisq= 70  on 1 degrees of freedom, p= <2e-16
```

Using the log-rank test, we reject the null hypothesis. Hence, it is concluded that there is statistically significant difference in survival curves between patients who have gone through hormonal therapy, and those who have not.

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ chemo, data = data,
##          rho = 0)
##
##           N Observed Expected (O-E)^2/E (O-E)^2/V
```

```
## chemo=0 2402      1576      1530      1.4      7.12
## chemo=1  580       329       375       5.7       7.12
##
## Chisq= 7.1  on 1 degrees of freedom, p= 0.008
```

Using the log-rank test, we reject the null hypothesis. Hence, it is concluded that there is statistically significant difference in survival curves between patients who have gone through chemotherapy, and those who have not.

2.3.3 Peto & Peto Test

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ meno, data = data,
##          rho = 1)
##
##              N Observed Expected (O-E)^2/E (O-E)^2/V
## meno=0 1312      421      474      5.9      16.3
## meno=1 1670      580      528      5.3      16.3
##
## Chisq= 16.3  on 1 degrees of freedom, p= 5e-05
```

Using the Peto & Peto test, we reject the null hypothesis. Hence, it is concluded that there is statistically significant difference in survival curves between patients who have gone through menopause, and those who have not.

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ hormon, data = data,
##          rho = 1)
##
##              N Observed Expected (O-E)^2/E (O-E)^2/V
## hormon=0 2643      865     924.7      3.89     67.2
## hormon=1  339      137      76.7     46.88     67.2
##
## Chisq= 67.2  on 1 degrees of freedom, p= 2e-16
```

Using the Peto & Peto test, we reject the null hypothesis. Hence, it is concluded that there is statistically significant difference in survival curves between patients who have gone through hormonal therapy, and those who have not.

```
## Call:
## survdiff(formula = Surv(rtime, censored) ~ chemo, data = data,
##          rho = 1)
##
##              N Observed Expected (O-E)^2/E (O-E)^2/V
## chemo=0 2402      841      811      1.10      8.4
## chemo=1  580      160      190      4.67      8.4
##
## Chisq= 8.4  on 1 degrees of freedom, p= 0.004
```

Using the Peto & Peto test, we reject the null hypothesis. Hence, it is concluded that there is statistically significant difference in survival curves between patients who have gone through chemotherapy, and those who have not.

3 Modeling

3.1 Semiparametric PH modeling

```
## Call:
## coxph(formula = Surv(rtime, censored) ~ chemo + meno + hormon +
##       age + as.factor(grade) + nodes + pgr, data = data)
##
##      n= 2982, number of events= 1905
##
##              coef exp(coef)    se(coef)      z Pr(>|z|)
## chemo          -1.234e-01  8.839e-01  6.825e-02 -1.809  0.07050 .
## meno           -1.877e-01  8.288e-01  7.931e-02 -2.367  0.01793 *
## hormon          5.446e-01  1.724e+00  7.993e-02  6.814  9.51e-12 ***
## age             1.528e-02  1.015e+00  3.269e-03  4.674  2.95e-06 ***
## as.factor(grade)3 1.630e-01  1.177e+00  5.123e-02  3.182  0.00146 **
## nodes           1.310e-02  1.013e+00  7.705e-03  1.700  0.08908 .
## pgr             1.478e-04  1.000e+00  7.341e-05  2.013  0.04410 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##              exp(coef) exp(-coef) lower .95 upper .95
## chemo             0.8839    1.1314    0.7732    1.0104
## meno              0.8288    1.2065    0.7095    0.9682
## hormon            1.7240    0.5801    1.4740    2.0163
## age               1.0154    0.9848    1.0089    1.0219
## as.factor(grade)3 1.1771    0.8496    1.0646    1.3014
## nodes             1.0132    0.9870    0.9980    1.0286
## pgr               1.0001    0.9999    1.0000    1.0003
##
## Concordance= 0.571 (se = 0.008 )
## Likelihood ratio test= 111.4 on 7 df,  p=<2e-16
## Wald test              = 121.7 on 7 df,  p=<2e-16
## Score (logrank) test = 124.2 on 7 df,  p=<2e-16

##              chemo          meno          hormon          age
## chemo          4.658302e-03  7.529186e-04 -7.890467e-05  3.174654e-05
## meno           7.529186e-04  6.289348e-03 -4.562636e-04 -1.973409e-04
## hormon         -7.890467e-05 -4.562636e-04  6.388817e-03 -2.098779e-06
## age            3.174654e-05 -1.973409e-04 -2.098779e-06  1.068697e-05
## as.factor(grade)3 -7.523294e-05 -2.767068e-04 -1.191548e-04  2.175279e-06
## nodes          -1.275770e-04 -2.779044e-05 -1.744326e-04 -1.297557e-06
## pgr            -4.609931e-07  4.269145e-07  2.636117e-07 -1.901573e-08
## as.factor(grade)3          nodes          pgr
```

```

## chemo          -7.523294e-05 -1.275770e-04 -4.609931e-07
## meno          -2.767068e-04 -2.779044e-05  4.269145e-07
## hormon        -1.191548e-04 -1.744326e-04  2.636117e-07
## age           2.175279e-06 -1.297557e-06 -1.901573e-08
## as.factor(grade)3  2.624501e-03 -2.333973e-05  6.726537e-07
## nodes        -2.333973e-05  5.936775e-05  2.657625e-08
## pgr           6.726537e-07  2.657625e-08  5.388883e-09

## Call:
## coxph(formula = Surv(rtime, censored) ~ chemo + meno + hormon +
##       age + as.factor(grade) + nodes + pgr, data = data)
##
##      n= 2982, number of events= 1905
##
##              coef exp(coef) se(coef)      z Pr(>|z|)
## chemo        -1.234e-01  8.839e-01  6.825e-02 -1.809  0.07050 .
## meno        -1.877e-01  8.288e-01  7.931e-02 -2.367  0.01793 *
## hormon        5.446e-01  1.724e+00  7.993e-02  6.814  9.51e-12 ***
## age          1.528e-02  1.015e+00  3.269e-03  4.674  2.95e-06 ***
## as.factor(grade)3  1.630e-01  1.177e+00  5.123e-02  3.182  0.00146 **
## nodes        1.310e-02  1.013e+00  7.705e-03  1.700  0.08908 .
## pgr          1.478e-04  1.000e+00  7.341e-05  2.013  0.04410 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##              exp(coef) exp(-coef) lower .95 upper .95
## chemo          0.8839      1.1314    0.7732    1.0104
## meno          0.8288      1.2065    0.7095    0.9682
## hormon        1.7240      0.5801    1.4740    2.0163
## age           1.0154      0.9848    1.0089    1.0219
## as.factor(grade)3  1.1771      0.8496    1.0646    1.3014
## nodes         1.0132      0.9870    0.9980    1.0286
## pgr           1.0001      0.9999    1.0000    1.0003
##
## Concordance= 0.571 (se = 0.008 )
## Likelihood ratio test= 111.4 on 7 df,  p=<2e-16
## Wald test            = 121.7 on 7 df,  p=<2e-16
## Score (logrank) test = 124.2 on 7 df,  p=<2e-16

```

0.8854536

Estimation and CI of relative risks for every pair of levels of the categorical variables are as below: HR of chemo vs non-chemo: 1.0423 with 95%CI [0.91353 1.18914] HR of meno vs non-meno: 0.8160 with 95%CI [0.69415 0.95916] HR of hormon treatment vs no treatment: 1.7815 with 95%CI [0.69415 0.95916] HR of size20-50 vs size<=20: 0.8650 with 95%CI [0.78503 0.95308] HR of size>50 vs size<=20: 0.9678 with 95%CI [0.79736 1.17457] HR of size>50 vs size20-50: $\exp(\text{Beta}(\text{size}>50) - \text{Beta}(\text{size}20-50)) = \{\exp(\text{fit_ph4coefficients}['\text{as.factor}(\text{size}) > 50']) / \exp(\text{fit_ph4coefficients}['\text{as.factor}(\text{size})20-50'])\} = 1.118818$ with 95%CI [??] HR of age i vs age i-1: 1.0075 with 95%CI [1.00110 1.01387] HR of grade 3 vs grade 2: 1.0575 with 95%CI [0.95655

Table 6: Estimators and confidence intervals for relative risks under xxxxx model.

	$\exp(\hat{\beta})$	5%	95%
Chemo	0.885	0.774	1.012
Meno	0.817	0.698	0.956
Hormon	1.741	1.488	2.037
Size (20-50)	0.936	0.849	1.032
Size (>50)	1.096	0.902	1.331
Age	1.015	1.009	1.022
Grade = 3	1.178	1.064	1.303
Nodes	1.013	0.997	1.029
Pgr	1.000	1.000	1.000
Er	1.000	1.000	1.000

1.16910] HR of recur vs non-recur: 0.0131 with 95%CI [0.01043 0.01646] HR of rtime i vs rtime i-1: 0.9985 with 95%CI [0.99841 0.99854]

```
##               chisq df      p
## chemo         0.168  1 0.682
## meno         0.155  1 0.694
## hormon       0.151  1 0.697
## age          2.206  1 0.137
## as.factor(grade) 1.989  1 0.158
## nodes        1.715  1 0.190
## pgr          2.014  1 0.156
## GLOBAL       14.397  7 0.045
```

We run a statistical test based on Schoenfeld residuals for proportional hazard assumption for each covariate included in the cox fit. From the output, one can see that test is statistically significant for some covariates like size, grade, rtime and recur and so is the global test. So we can state that proportional hazard assumption is violated since there is significant dependency between Schoenfeld residuals and time.

##next step Q: should we also check outliers and nonlinearity between log hazard and covariates??

3.2 Parametric Regression Models

```
##      log(normal)      weibull      exponential      log(logistic)
##      33751.81      32795.40      34660.59      33279.98

##      (Intercept)      as.factor(chemo)1      as.factor(size)20-50
##      8.138095e+00      6.052914e-02      1.071216e-02
##      as.factor(size)>50      as.factor(grade)3      as.factor(hormon)1
##      -7.925056e-02      -5.976519e-02      -2.246120e-01
##      pgr      er
##      -4.370997e-05      -4.317483e-05

##      2.5 %      97.5 %
```

```
## (Intercept)          8.0980324003  8.178157e+00
## as.factor(chemo)1    0.0122476499  1.088106e-01
## as.factor(size)20-50 -0.0277547692  4.917910e-02
## as.factor(size)>50   -0.1549881990 -3.512918e-03
## as.factor(grade)3    -0.0998557070 -1.967467e-02
## as.factor(hormon)1    -0.2835217996 -1.657023e-01
## pgr                  -0.0001042701  1.685014e-05
## er                   -0.0001056779  1.932824e-05

##          (Intercept)    as.factor(chemo)1 as.factor(size)20-50
##      -2.031924e+01      -1.511295e-01      -2.674619e-02
## as.factor(size)>50    as.factor(grade)3    as.factor(hormon)1
##      1.978732e-01      1.492220e-01      5.608125e-01
##              pgr              er
##      1.091353e-04      1.077991e-04

## as.factor(chemo)1 as.factor(size)20-50 as.factor(size)>50
##      -0.1736958736      -0.0400272548      0.1711272964
## as.factor(grade)3 as.factor(hormon)1      pgr
##      0.1671662471      0.6146832178      0.0001263394
##              er
##      0.0001396488
```

4 Next Steps for Aleks

- Keep researching to decide which test is appropriate for survival curves diff testing
- Format text
- Make all ugly R outputs into nice, coherent tables

5 References

Marta Sestelo. 2017. *A Short Course on Survival Analysis*. https://bookdown.org/sestelo/sa_financial/comparing-survival-curves.html.