

NP-Hard: A problem L is NP-Hard iff satisfiability reduces to L i.e., any nondeterministic polynomial time problem is satisfiable and reducible then the problem is said to be NP-Hard.

NP-Complete: A problem L is NP-Complete iff L is NP-Hard and L belongs to NP (nondeterministic polynomial).

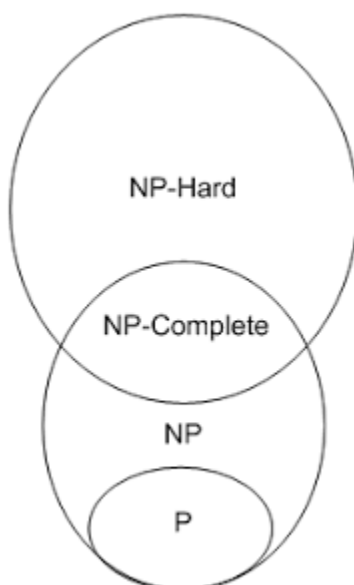
A problem that is NP-Complete has the property that it can be solved in polynomial time iff all other NP-Complete problems can also be solved in polynomial time. ($NP=P$)

Reducability:

A problem $Q1$ can be reduced to $Q2$ if any instance of $Q1$ can be easily rephrased as an instance of $Q2$. If the solution to the problem $Q2$ provides a solution to the problem $Q1$, then these are said to be reducible problems.

Let $L1$ and $L2$ are the two problems. $L1$ is reduced to $L2$ iff there is a way to solve $L1$ by a deterministic polynomial time algorithm using a deterministic algorithm that solves $L2$ in polynomial time and is denoted by $L1 \leq L2$.

If we have a polynomial time algorithm for $L2$ then we can solve $L1$ in polynomial time. Two problems $L1$ and $L2$ are said to be polynomially equivalent iff $L1 \leq L2$ and $L2 \leq L1$.



Cook's Theorem shows that the Boolean Satisfiability Problem (SAT) is NP-complete. To formally connect this to " $P = NP$," here is the logical reasoning:

1. Key Insight from Cook's Theorem:

- Cook's Theorem states that SAT is NP-complete. That means SAT is one of the hardest problems in NP.
- If SAT can be solved in polynomial time ($SAT \in P$), then every NP problem can also be solved in polynomial time, making $P = NP$.

2. Proof Breakdown:

- Suppose $SAT \in P$:
 - By the definition of NP-completeness, every problem in NP can be reduced to SAT in polynomial time.
 - If SAT can be solved in polynomial time, the solution to every NP problem can be derived in polynomial time (via reduction to SAT + solving SAT).
 - Therefore, $P=NP$.
- Conversely, suppose $P=NP$:
 - By definition, every NP-complete problem, including SAT, would belong to P, as all problems in NP would have polynomial-time solutions.
 - Thus, $SAT \in P$.

3. Conclusion:

- SAT is in P if and only if $P = NP$.

This is why Cook's Theorem is often tied to the P vs. NP question—it identifies SAT as the "representative" problem of NP-completeness. Solving SAT efficiently would imply efficient solutions for all problems in NP.