UWI

Design and Analysis of Algorithms

Tutorial 5

Divide and Conquer Approach

It is important that we learn the theory behind the divide and conquer approach and its algorithms.

1. Explain the theory behind the divide and conquer approach
   (a) What is meant by the divide and conquer approach?
   (b) Is divide and conquer more efficient than brute force?
2. Master's theorem can be used to calculate the time complexities of recurrence relations.
   (a) State Master's Theorem and its uses.
   (b) Calculate the time complexity of the Quick Hull algorithm (which has the following recurrence relation) using Master's Theorem.
   $$T(n) = 2T(\frac{n}{2}) + n \tag{1}$$
   (c) Calculate the time complexity of the Merge Sort algorithm (which has the following recurrence relation) using Master's Theorem.
   $$C_w(n) = 2C_w(\frac{n}{2}) + C_{merge}(n), \text{ for } n > 1 \tag{2}$$
   $$C_{merge}(n) = n - 1 \tag{3}$$
   $$C_w(1) = 0 \tag{4}$$
3. Solve the following recurrence relation for the Quick Sort algorithm using the backward substitution method.
   $$C_b(n) = 2C_b(\frac{n}{2}) + n, \text{ for } n > 1 \tag{5}$$
   $$C_b(1) = 0 \tag{6}$$
4. Solve the following recurrence relation for the n-digit multiplication algorithm using the backward substitution method.
   $$M(n) = 3M(\frac{n}{2}), \text{ for } n > 1 \tag{7}$$
   $$M(1) = 1 \tag{8}$$
5. Solve the following recurrence relation using the backward substitution method.
   $$T(n) = T(\sqrt{n}) + 1, \text{ for } n > 2, \text{ where } T(n) \text{ is constant for } n \leq 2 \tag{9}$$