University of the Pacific ECPE 127 Notes

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Contents

1 Unit 1 - Probability Theory

1.1 Introduction to Set Theory

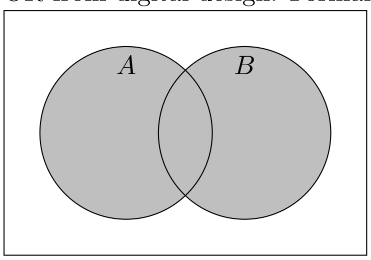
A **set** is a collection of things. For example:

- Collection of all natural numbers $\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$
- Even natural numbers less than or equal to 6: $E = \{2, 4, 6\}$.

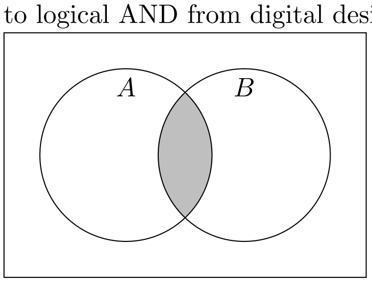
Elements of a set are denoted using lowercase letters. For example if $x = 4$ belngs to E we would denote that with $x \in E$. To denote an element is **not** in a set you dash the epsilon: $5 \notin E$.

1.1.1 Set Operations

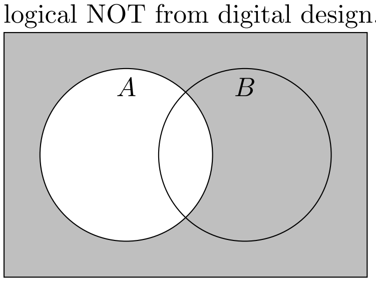
Set Union:
The union of two sets A and B is the set of all elements which is either in A or B. Behaves similar to logical OR from digital design. Formal Definiton: $A \cup B = \{x|(x \in A) \vee (x \in B)\}$.



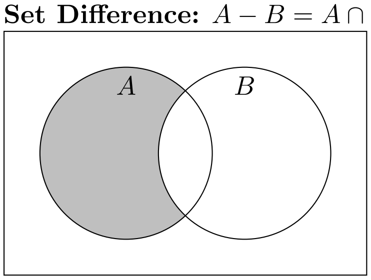
Set Intersection:
$A \cap B$ is the intersection of two sets, and contains every element that is both in A and B. Behaves similarly to logical AND from digital design. Formal Defintion: $A \cap B = \{x|(x \in A) \wedge (x \in B)\}$



Set Compliment: A^c is the compliment of A and contains every element not in A. Behaves similar to logical NOT from digital design. Formal Definition: $A^c = \{x|x \notin A\}$.



Set Difference: $A - B = A \cap B^c$. Contains every element of A that is not in B.



1.1.2 Other Definitions

A collection of sets $A_1, ..., A_n$ is **mutually exlusive** if and only if:

$$A_i \cap A_j = \emptyset \qquad i \neq j$$

A collection of sets $A_1, \ldots, A_n$ is **collectively exhaustive** if and only if

$$A_1 \cup A_2 \cup \cdots \cup A_n = S$$

Two sets are equal to each other if and only if

$$(A \subseteq B) \wedge (B \subseteq A)$$

De Morgan's Law: De Morgan's law relates all three basic set operations

$$(A \cup B)^c = A^c \cap B^c$$

Proof: Let $x \in (A \cup B)^c$. Then as $x \notin A \cup B$ therefore $x \notin A$ and $x \notin B$ Therefore $x \in A^c \cap B^c$ Therefore $(A \cup B)^c \subseteq A^c \cap B^c$. Now assume $x \in A^c \cap B^c$. Then $x \notin A$ and $x \notin B$, and therefore $x \notin (A \cup B)$. Thus $x \in (A \cup B)^c$. Therefore $(A \cup B)^c = A^c \cap B^c$. $\square$

$$(A \cap B)^C = A^c \cup B^c$$

Proof: Let $x \in (A \cap B)^c$. Then x is either in A not in B, in B not in A, or not in either A or B. Therefore $x \in A^c \cup B^c$ and thus $(A \cap B)^c \subseteq A^c \cup B^c$. Now let $x \in A^c \cup B^c$. Then by definition x is either in A^c or B^c. Thus x is not in both A and B. Therefore $x \in (A \cap B)^c$ and thus $A^c \cup B^c \subseteq (A \cap B)^c$. As $A^c \cup B^c \subseteq (A \cap B)^c$ and $(A \cap B)^c \subseteq A^c \cup B^c$, $(A \cap B)^c = A^c \cup B^c$. $\square$

1.2 Applying Set Theory to Probability

An **experiment** consists of a procedure and observations.

Experiment	Procedure	Observation
Coin Flip	Flip the coin	heads or tails
Dice Rolls	Roll the die	the number face up on the die
Networking	Send packets	Record the packets that successfully get transmitted

The **sample space** of an experiment is the finest-grain, mutually exclusive, collectively exhaustive set of all possible outcomes.

Roll a die	$S = D = \{1, 2, 3, 4, 5, 6\}$
Flip a coin	$S = C = \{H, T\}$
Flip a coin twice	$S = F = \{HH, HT, TH, TT\}$

An **event** is a set of desired outcomes of an experiment. Example: Roll a die, you win if you roll an even number. $E = \{2, 4, 6\}$.

1.3 Axioms

Probability P maps the events from a sample space to real numbers such that

1. $P(A) \geq 0$ where A is an event in the sample space S
2. $P(S) = 1$ where S is the universal set
3. For a countable collections of mutually exclusive sets $A_1, A_2, A_3, ...A_n \in S$, $P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

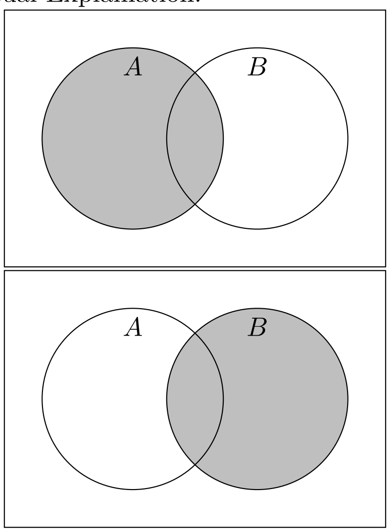
1.4 Theorems

Theorem 1.4
$P(A^c) = 1 - P(A)$.

For any two sets A and B not necessarily mutually exclusive:

$$P(A \cup B) = P(A) + B(B) - P(A \cap B)$$

Visual Explaination:



Here we see the intersection of A and B could be counted twice if we add $P(A)$ and $P(B)$ so we have to subtract the intersection so it is only counted once.

Theorem 1.5
The probability of event $B = \{s_1, s_2, \ldots s_m\}$ is the sum of probabilities contained in the event:

$$P(B) = \sum_{i=1}^{m} P(\{s_i\})$$

Follows from axiom 3 as each s_i is mutually exclusive.

Theorem 1.6
For an experiment with sample space $S = \{s_1, \ldots, s_n\}$ in which each outcome s_i is equally likely,

$$P(s_i) = 1/n \quad 1 \leq i \leq n$$

Where n is the number of outcomes in the sample space (same as n is equal to the cardinality of S)

Theorem 1.7
If outcomes in an experiment are equally likely, then probability of event A is given by:

$$P(A) = \frac{|A|}{|S|}$$

Where || denotes the cardinality of the set.

1.5 Conditional Probability