Non Parametric Full Bayesian Significance Testing for bayesian histograms

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2. Bayesian Histograms
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Parametric FBST

Let Θ be a finite dimensional space such as a subset of $\mathbb{R}^p$, $p \in \mathbb{N}$ The FBST [BSW08] consists on deciding whether to reject some statistical hypothesis $H_0 : \theta = \theta_0$ based on the posterior probability of the tangent set $T(\theta_0)$.

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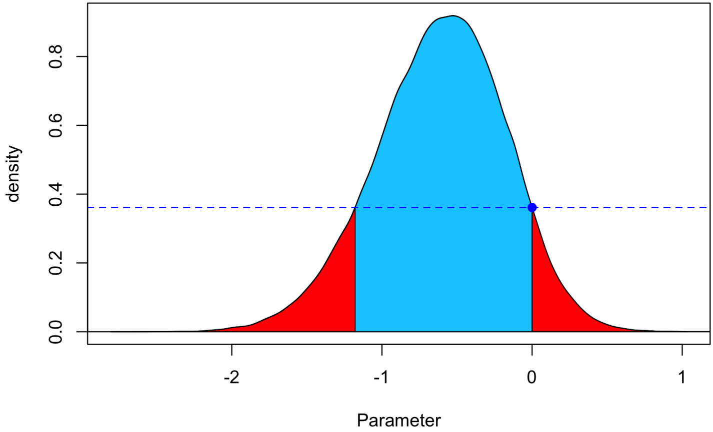
- Then we find the set in which $f_{\theta|\mathbf{x}_n}(\theta)$ is smaller than $f_{\theta|\mathbf{x}_n}(\theta_0)$. This is $T(\theta_0)$
- Finally calculate the posterior probability of $T(\theta_0)$. For point hypothesis we shall write:

$$ev(\theta_0) = \int_{T(\theta_0)} f_{\theta|\mathbf{x}_n}(t)dt = \Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}(T(\theta_0))$$

FBST in a nutshell: Are there many θ that are less likely than θ_0 (regarding the posterior)? If there are many, $ev >> 0$ then we don't reject H_0. If there are few, $ev \approx 0$, then we reject H_0 (almost all Θ are more likely than θ_0)



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$f_{\theta|\mathbf{x}_n}$ is the density of $\Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}$ with reference to the Lebesgue Measure

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Remark

When Θ is infinite dimensional there is no Lebesgue Measure (translation invariant Borel measures exists, but are pathologic) $\implies$ we don't have a canonical version of the density of $\Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}$ even when we have some posterior measure over infinite dimensional parameters spaces (e.g. Gaussian Process, Bayesian Kernel Density Estimation, Mixture of Dirichlet etc)

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FBST depends on some notion of posterior density. In infinite dimensions we must look for alternatives

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Bottom up approach — Bayesian Histograms

Let's start with an example of an histogram

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$(n_1, n_2, ..., n_{10}) = (50, 114, 161, 173, 196, 165, 99, 42)$

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Bayesian histograms perform inference on the probability of each bin:

$$(Prior)(\theta_1, ..., \theta_{10}) \sim Dirichlet(\alpha_{1,10}, ..., \alpha_{10,10})$$

$$(Likelihood)(n_1, ..., n_{10})|(\theta_1, ..., \theta_{10}) \sim Multinom(n, (\theta_1, ..., \theta_{10}))$$

$$(\theta_1, ..., \theta_{10})|(n_1, ..., n_{10}) \sim Dirichlet(n_1 + \alpha_{1,10}, ..., n_{10} + \alpha_{10,10})$$

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Also, consider a partition of $[0, 1]$ defined by $I_j = [(j-1)/k(n), j/k(n))$ for $j = 1, 2, \ldots, k(n)$. Let $\mathcal{S}_{k(n)}$ be the unit simplex in $\mathbb{R}^{k(n)}$ and denote by:

$$\mathcal{H}_{k(n)} = \left\{ f : f(x) = k(n) \sum_{j=1}^{k(n)} w_j \mathbb{I}_{I_j}(x), w \in \mathcal{S}_{k(n)} \right\} \tag{1}$$

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We'll define a prior on $\mathcal{H}_{k(n)}$ by defining a prior on $(w_1, \ldots, w_{k(n)})$.
Conditionally on $k(n)$, we consider the following Dirichlet priors:

$$w \sim \mathcal{D}(\alpha_{1,k(n)}, \ldots, \alpha_{k(n),k(n)}), \quad c_1 \leq \alpha_{j,k} \leq c_2 \tag{2}$$

A Bayesian histogram posterior on $\mathcal{H}_{k(n)}$ is the conjugated Dirichlet-Multinomial posterior based how many sample points ended up in I_j, the counts n_j

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We define the posterior random histogram as:

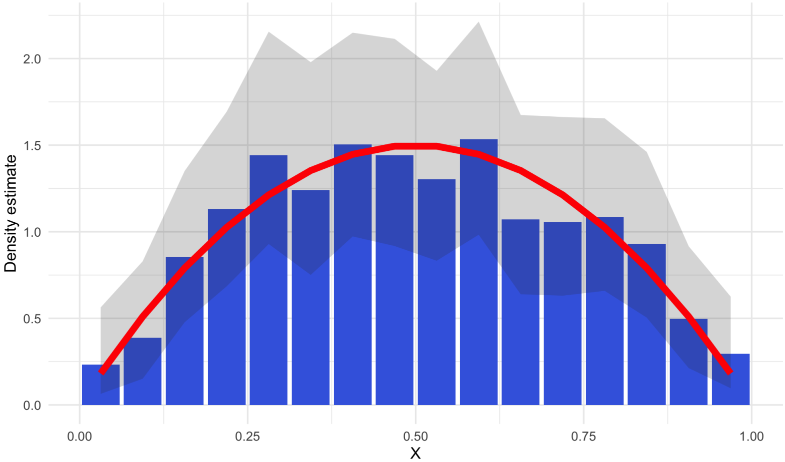
$$h(t) = k(n) \sum \theta_j \mathbb{I}_{I_j}(t), \tag{3}$$

where $(\theta_1, \ldots, \theta_{k(n)})$ follows the posterior Dirichlet distribution previously highlighted. The posterior estimate of the histogram is $\hat{h}$ pluging the MAP on 3:

$$\hat{h}(t) = \sum \frac{n_i + \alpha_{i,k(n)}}{n + \sum \alpha_{j,k(n)}} \mathbb{I}_{I_j}(t) \tag{4}$$

Bayesian Histograms (example)

The shaded area represents the uncertainty encoded in the Dirichlet posterior, the blue bars represents $\hat{h}$ and the red line represents f^*, a $Beta(2,2)$. Sample size $N = 1000$, $k(n) = 16$.



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Why bayesian histograms? — Relation to entropy

Log likelihood in this model is cross-entropy $\implies$ e-value test statistic for $H_0 : \theta = p$ is the Kullback-Leibler Divergence between $\hat{\theta}$ and p, one element of the simplex

$$ev(p) = \Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}(f_{\theta|\mathbf{x}_n}(\theta) \leq f_{\theta|\mathbf{x}_n}(p)) =$$

$$\Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}\left(\prod_{i=1}^{k(n)} \theta_i^{\alpha_{i,k(n)}+n_i} \leq \prod_{i=1}^{k(n)} p_i^{\alpha_{i,k(n)}+n_i}\right) =$$

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$$\Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}\left(\sum_{i=1}^{k(n)} (\alpha_{i,k(n)} + n_i) \log \theta_i \leq \sum_{i=1}^{k(n)} (\alpha_{i,k(n)} + n_i) \log p_i\right) =$$

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$$\Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}\left(H(\hat{\theta},\theta) > H(\hat{\theta},p)\right) = \Pi_{\theta|\mathbf{X}_n=\mathbf{x}_n}\left(D_{KL}(\hat{\theta},\theta) > D_{KL}(\hat{\theta},p)\right) \tag{5}$$

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The continuous version of the last line might be used on any density estimation problem!

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Lemma adapted from [Gho00]

If f^* is a Lipchitz continuous density on $[0,1]$ and $k(n) = O\left(\frac{n^{1/6}}{(\log n)^{1/6+\epsilon}}\right)$ for any $\epsilon > 0$ then:

1. θ posterior distribution is approximately a multivariate gaussian $N(\hat{\theta}, \hat{F}/\sqrt{n})$, where $\hat{F}$ is the Fisher Information Matrix evaluated at $\hat{\theta}$. The approximation holds in the total variation distance between both measures.
2. $||\hat{\theta} - \theta^{f^*}|| \to 0$ in f^* probability
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The proof is very sophisticated, but it mostly guarantees that the lowest eigenvalue of the Fisher Information Matrix is large in comparison to $1/\sqrt{n}$ so the model is *approximately* parametric

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Sketch of proof — Theorem 1

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2. By 2. we are able to detect differences between density g and any Lipchitz continuous f^*

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3. As Gaussian ellipsoids are χ^2 distributed the results follows.

NP-FBST for composite hypothesis

Theorem 2

Let $\mathcal{F}_0$ be a set of densities f such that $\inf ||f^* - f|| > \delta$ for some $\delta > 0$, Θ_0 the projection of $\mathcal{F}_0$ on Θ_0 and $ev(\Theta_0)$ be the FBST for testing $H_0 : \theta \in \Theta_0$. If f^* is a Lipchitz continuous density on $[0, 1]$ and $k(n) = O\left(\frac{n^{1/6}}{(\log n)^{1/6+\epsilon}}\right)$ then $ev(\Theta_0) \to^P 0$ in f^* probability

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1. This theorem qualifies FBST as a test for goodness-of-fit to parametric families, median tests, mean tests etc. Is suffices to note that on all those examples the densites being tested are not arbitrary close fo f^*
2. This results does not show how to control Type I error, so pratical applications should apply e.g. bootstrap techniques

Uniformity tests — Power comparisons

Theorem 1 might be fully applied to test if some sample where sampled from a uniform density. We shall compare the power of our procedure to the power of other existing procedures such as Kolmogorov Smirnov, Anderson-Darling and its more powerful versions by [Zha02]

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3. Our procedure is never worse than Kolmogorov-Smirnov
4. The Lipchitz continuity hypothesis seems very important and hurts the statistical power of the test

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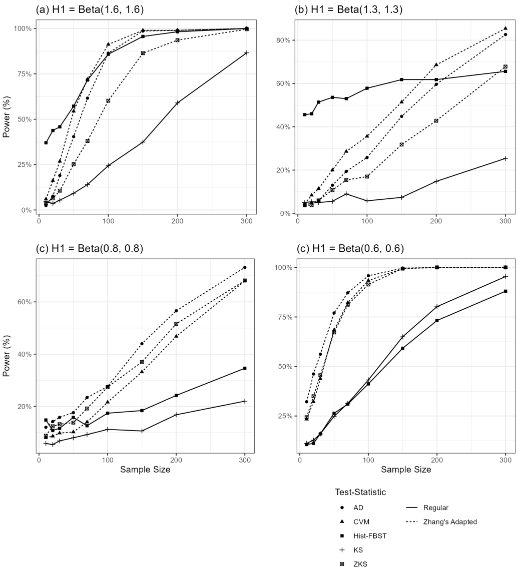

Caveats

The results are useful in their current state but there are some important remarks to be made:

1. The bin size grows slowly, so the choice of ϵ is important
2. Boostrap methods for obtaining the null distribution might be more reliable than Theorem 1, as our results shows that the test is well suited for small sample sizes
3. The Lipchitz is restrictive. There are other Bernstein Von Mises Theorems [CN14] for bayesian histograms that allow for less smooth f^*, but then $k(n)$ would grow even slower
4. The null distribution for composite hypothesis remains to be studied

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2. The same approach applied to the Dirichlet-Multinomial model could be applied to other exponential family parametric distributions, as the Bernstein Von Mises results already exists.
3. For practical applications more refined algorithms for building $k(n)$ should be developed.

Bibliography I

Carlos A. de B. Pereira, Julio Michael Stern, and Sergio Wechsler. "Can a significance test be genuinely Bayesian?" In: *Bayesian Analysis* 3.1 (2008), pp. 79–100. DOI: 10.1214/08-BA303. URL: https://doi.org/10.1214/08-BA303.

Ismaël Castillo and Richard Nickl. "On the Bernstein–von Mises phenomenon for nonparametric Bayes procedures". In: *The Annals of Statistics* 42.5 (2014), pp. 1941–1969. DOI: 10.1214/14-AOS1246. URL: https://doi.org/10.1214/14-AOS1246.

Bibliography II

Subhashis Ghosal. “Asymptotic Normality of Posterior Distributions for Exponential Families when the Number of Parameters Tends to Infinity”. In: *Journal of Multivariate Analysis* 74.1 (2000), pp. 49–68. URL: https://EconPapers.repec.org/RePEc:eee:jmvana:v:74:y:2000:i:1:p:49-68.

Jin Zhang. “Powerful Goodness-of-Fit Tests Based on the Likelihood Ratio”. In: *Journal of the Royal Statistical Society. Series B (Statistical Methodology)* 64.2 (2002), pp. 281–294. ISSN: 13697412, 14679868. URL: http://www.jstor.org/stable/3088800 (visited on 05/16/2024).