Basic Electrical Engineering (TEE 101)

Lecture 6: Kirchhoff's Current Law, and Nodal Analysis

Content

This lecture covers

- Kirchhoff's Current Law
- Nodal Analysis
- Numerical

Kirchhoff’s Current Law (KCL)

Kirchhoff’s Laws were developed by a German Physicist “Gustav Kirchhoff” in 1845.

These laws deal with the law of conservation of energy and charge in electrical circuits. The pair of laws developed by Kirchhoff are:

Kirchhoff’s Current Law and Kirchhoff’s Voltage Law

The first law given by Kirchhoff is: **Kirchhoff’s Current Law (KCL)**

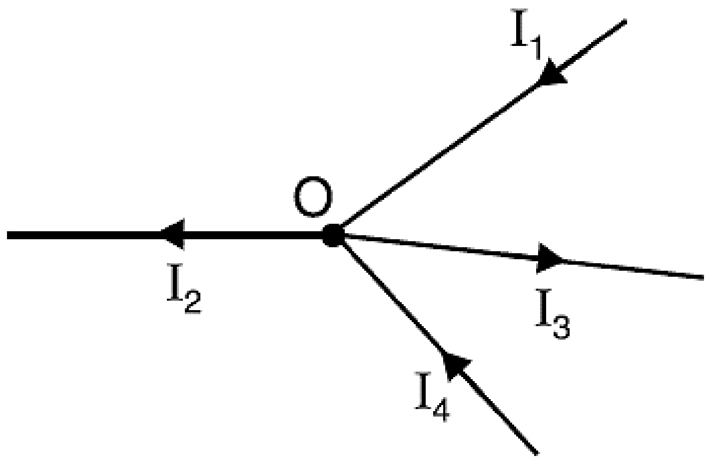
KCL is also known as “**Kirchhoff’s Junction Rule**”

According to KCL, ***“The algebraic sum of the currents meeting at a junction in an electrical circuit is zero”***

Or it can also be stated as **“Net current on a junction in an electrical network is ZERO”**

An algebraic sum is one in which the sign of the quantity is taken into account.

For example, consider four conductors carrying currents I_1, I_2, I_3 and I_4 and meeting at point O as shown in the Figure.



If we take the signs of currents flowing towards point *O* as positive, then currents flowing away from point *O* will be assigned negative sign.

Thus, applying **Kirchhoff's Current Law** to the junction *O* in Figure above, we have,

$$(I_1) + (I_4) + (-I_2) + (-I_3) = 0$$

$$I_1 + I_4 = I_2 + I_3$$

***i.e.*, Sum of incoming currents = Sum of outgoing currents**

- Hence, Kirchhoff's current law may also be stated as under :
- ***The sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction.***

NODAL ANALYSIS

Nodal Analysis is a method used to determine branch currents in an electrical network. Hence, it is used to solve the complex network

In this method, one of the nodes is taken as the ***reference node***.

The potentials of all the points in the circuit are measured w.r.t. this reference node.

Hence **nodal analysis** *essentially aims at choosing a reference node in the network and then finding the unknown voltages at the independent nodes w.r.t. reference node*.

Nodal Analysis primarily make use of Kirchhoff's Current Law (KCL).

Nodal Analysis is also used to determine the voltage at the independent junctions/nodes of the given electrical network. Hence, Nodal analysis is also called as. **Node-voltage method**

Procedure to solve a network using Nodal Analysis

Any electrical circuit can be solved using following four major steps

Step 1 − Identify the **principal nodes**** and choose one of them as **reference node**. Consider that **reference node** as the **Ground**.

Step 2 − Label the **node voltages** with respect to Ground from all the principal nodes except the **reference node**.

Step 3 − Write **nodal equations** at all the principal nodes except the reference node. **Nodal equation** is obtained by applying KCL first and then Ohm's law.

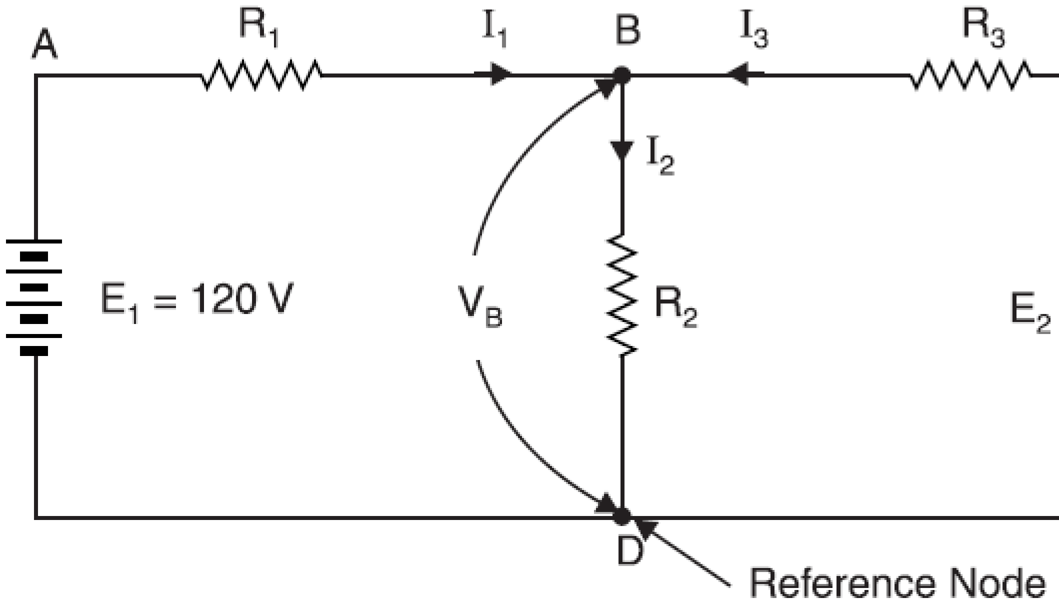
Step 4 − Solve the nodal equations obtained in Step 3 in order to get the node voltages.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using **node voltages**.

Principle node is the one where 3 or more than 3 branches are connected. (Also called as Junction)

Numerical Example on Nodal Analysis

Consider an electrical circuit as shown below. Let R_1 = 10Ω, R_2 = 20Ω and R_3 = 10Ω. Using Nodal Analysis, determine the current through the 20 Ω resistance



Step 1 – Identify the number of principle nodes in the given circuit. There are two principle nodes in this case.

Step 2 – Mark all the principle nodes (such as x, y or a, b or 1, 2 etc.) and consider one as reference node

Step 3 – The reference node is considered to be at zero potential w.r.t all principle nodes

Step 4 – Assume some voltage on each Junction (such as V_x, V_y etc.)

Step 5 – Now take the currents at each Junction (principle node) as identified

Step 6 – Mark the direction of each current around each Junction. ***(You have free hand to assume the current direction around the Junctions)***

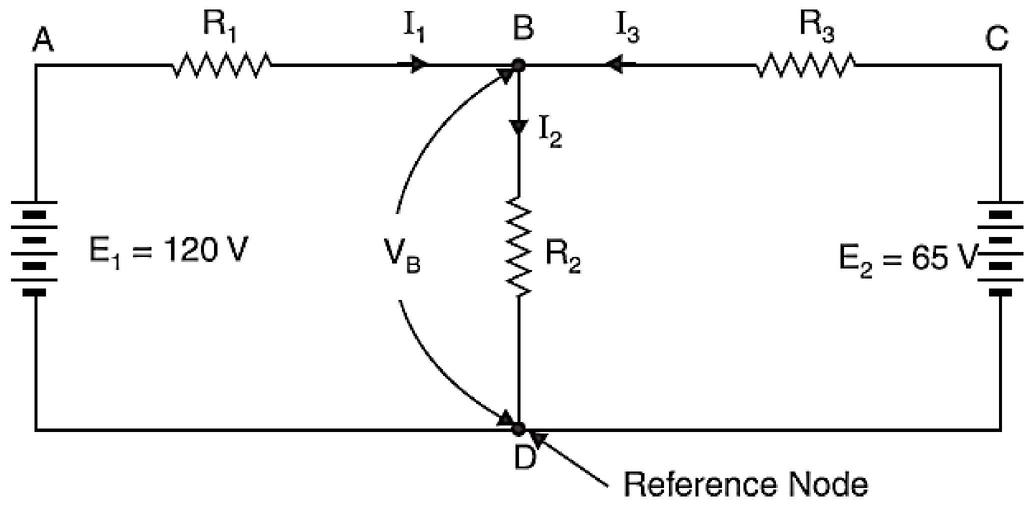
Step 7 – Give some notation to each current (such as I_1, I_2, I_3 etc.)

Step 8 – Now apply KCL on each Junction and write the current equation of each Junction

Step 9 – Now, using Ohms's Law, convert the current equations into equivalent V/R equation.

Step 10 – Now, solve the derived equations (as obtained in step 9) for node voltages algebraically

Step 11 – Using the values of node voltages, we can obtain the current in any circuit element / Branch



In this figure, **Node B** the independent nodes. And **node D** as reference node (Ground).

Let I_1, I_2 and I_3 be the currents at Junction B

Let us assume the voltage at Junction B is V_B

Now, apply KCL at Junction B to obtain the current equation

The KCL equation is $I_1 + I_3 = I_2$ **(1)**

Where, I_1 , I_2 and I_3 are the branch currents

Now apply Ohm's Law to get the equivalent V/R of each current i.e. $I_1 + I_3 = I_2$

$$I_1 = \frac{E_1 - V_B}{R_1} \qquad I_2 = \frac{V_B}{R_2} \qquad I_3 = \frac{E_2 - V_B}{R_3} \qquad \textbf{(2)}$$

Now, substitute the values of all currents in equation (1), we get:

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2} \qquad (3)$$

Now, put the values of E_1, E_2, R_1, R_2 and R_3 in equation (3), we get:

$$\frac{120 - V_B}{10} + \frac{65 - V_B}{10} = \frac{V_B}{20} \qquad (4)$$

We can solve equation (4) to obtain the node voltage V_B

$$2\left(120 - V_B + 65 - V_B\right) = V_B \qquad (5)$$

Or, $$\left(240 - 2V_B + 130 - 2V_B\right) = V_B$$

Or, $$\left(370 - 4V_B\right) = V_B$$

Or, $$5V_B = 370$$

Or, $$V_B = 74V \qquad (6)$$

Now, we can calculate the current in each branch using the node voltage as:

$$I_1 = \frac{120 - V_B}{10} \qquad I_2 = \frac{V_B}{20} \qquad I_3 = \frac{65 - V_B}{10}$$

Substitute the value of V_B from equation (6) in to obtain I_1, I_2 and I_3. Hence, the values of branch current are:

$$I_1 = \frac{120 - 74}{10} = \frac{46}{10} \qquad I_2 = \frac{74}{20} \qquad I_3 = \frac{65 - 74}{10} = \frac{-9}{10}$$

$$I_1 = 4.6A \qquad I_2 = 3.7A \qquad I_3 = -0.9A$$

Thank You