

UNIT IV

[Topics mentioned in Syllabus:

Superconductivity: Essential properties of Superconductors, Zero resistivity, Type I, Type II superconductors and their properties.

Electromagnetism: Displacement current, Three electric vectors (**E, P, D**), Maxwell's equations in integral and differential forms. Electromagnetic wave propagation in free space.]

Students

- We start this UNIT IV with **Electromagnetism** for your perusal.
- The basics essential to understand and required for the derivations have been elaborated so that an understanding of the topic can be developed. In this unit you learn how the speed of light c has been derived.

ELECTROMAGNETISM

EM waves are the waves on EM spectrum whether visible or invisible to human.

Application of Maxwell's equations

- Help to analyze the permittivity (ϵ) and permeability (μ) of any medium for propagation of electromagnetic waves.
- Maxwell Equations proves that in vacuum electromagnetic waves (EM waves) travels with the speed of light.
- To calculate the energy dissipated due to propagation of EM waves in any medium
- To calculate the depth of penetration of EM waves

E, P and D vectors and Displacement Current:

If we consider a parallel plate capacitor, it is well known that it works for AC source (as it used in ceiling fan to give initial gust of charge to motor to start the rotations). When a DC source is applied across it, it gets charged but further conduction stops because this device offers infinite impedance for DC source (refer Fig. (a) and (b) below).

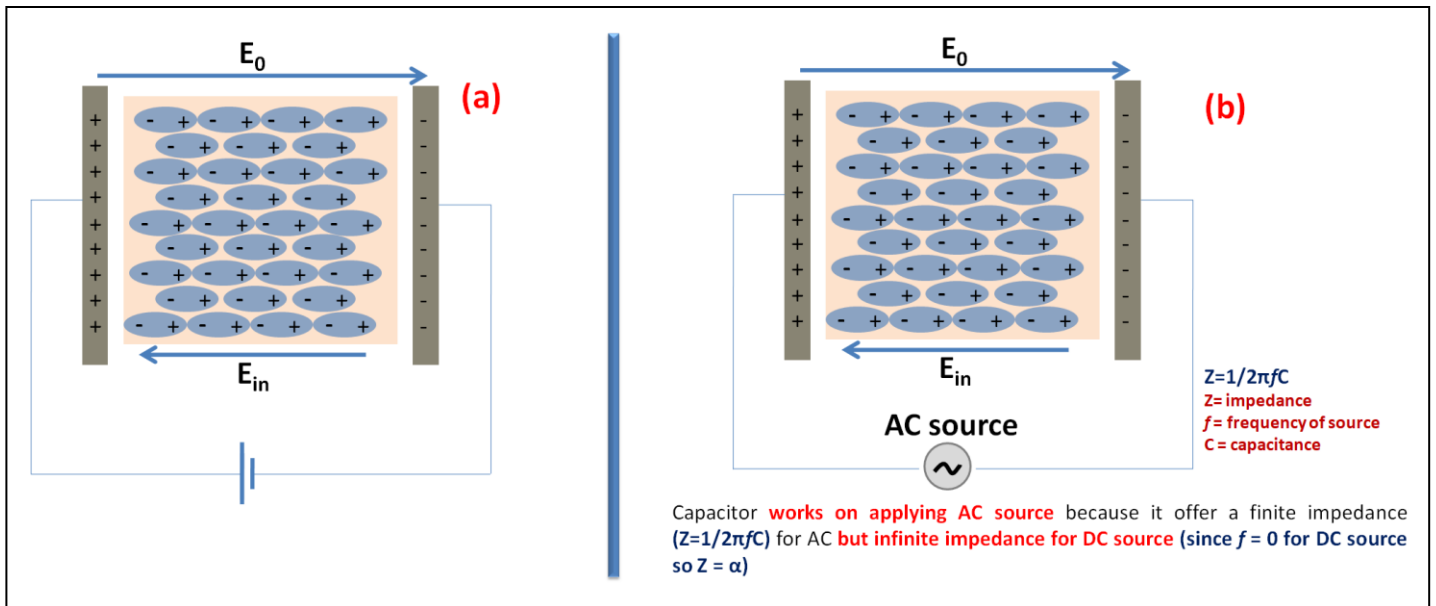


Fig. 1 (a) and (b)

E, P and D vectors: following above discussion. Refer Fig (a), when DC is applied across the capacitor the dielectric medium (insulating medium) between the plates gets polarized due to electric field (E_0). As shown in Fig (a), there exists an internal electric field (E_{in}) inside the dielectric medium. Hence a net electric field (E) is established across the plate.

$$\vec{E} = \vec{E}_0 - \vec{E}_{in}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{in}}{\epsilon_0} \quad (\text{since the electric (E) of charged sheet is } \frac{\sigma}{\epsilon_0})$$

Here **Displacement vector (D)** is defined as

$$\vec{D} = \sigma$$

It is equal to the **free charge per unit area** or equal to the **surface density of free charges**.

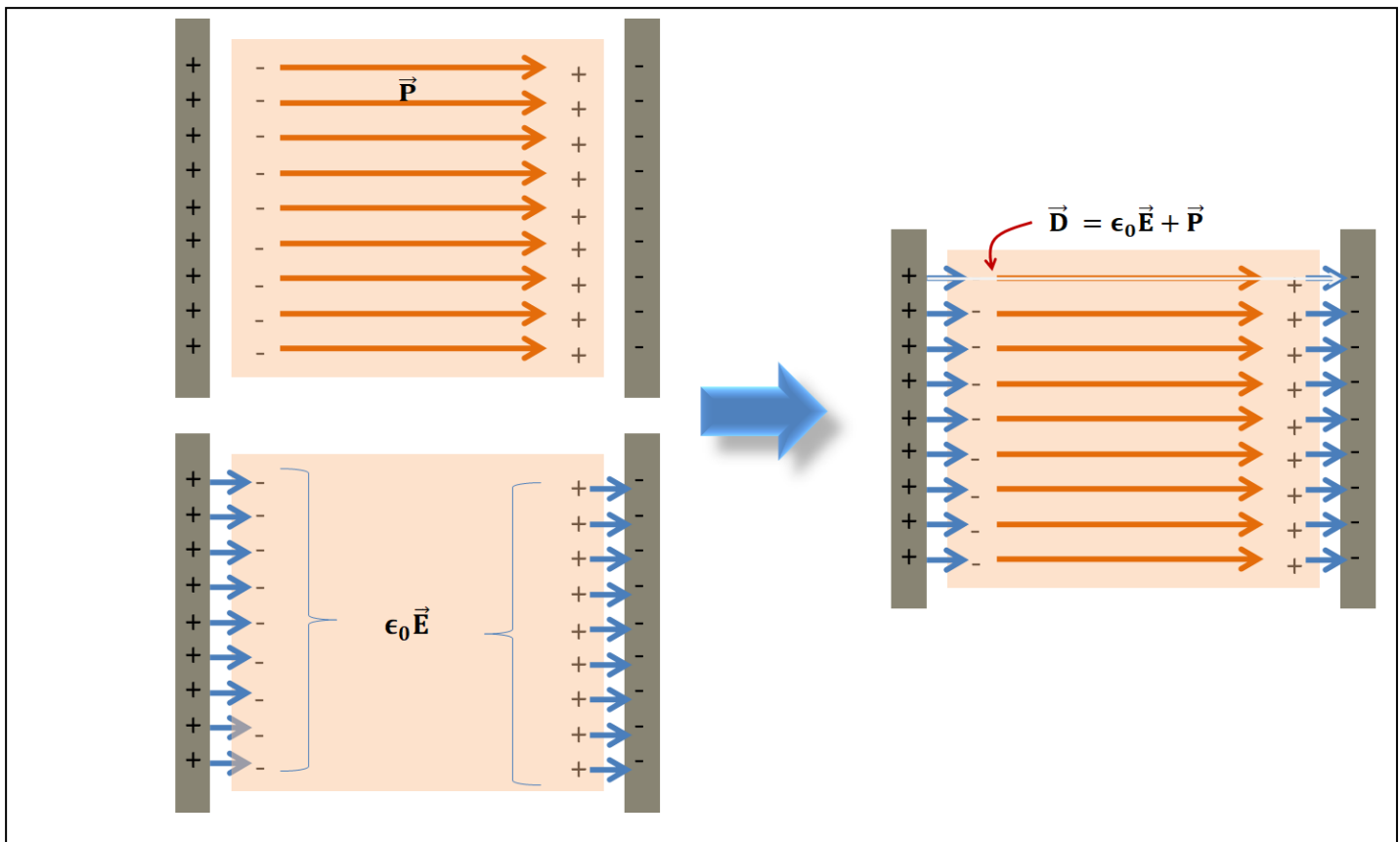
$$\epsilon_0 \vec{E} = \sigma - \sigma_{in}$$

$$\epsilon_0 \vec{E} = \vec{D} - \vec{P}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

This is the relation between E, P and D vectors.

The representation of the displacement vector is shown in figure below



Concept of Displacement Current:

As discussed above that a capacitor works on applying AC source; although the circuit is open due to gap between the plates still the circuit is completed means a current between the plates exist. The polarization of dielectric medium established the field across the plates. Maxwell assumed the **existence of a current called displacement current between the plates**. $\mathbf{J_D} = d\mathbf{D}/dt$ (i.e., current density of displacement current ($\mathbf{J_D}$) is equal to the rate of change of displacement vector (\mathbf{D})). This concept is used in deriving Maxwells fourth equation.

Things to Remember:

Following basics and identities are essential to start with derivation of Maxwell's equations.

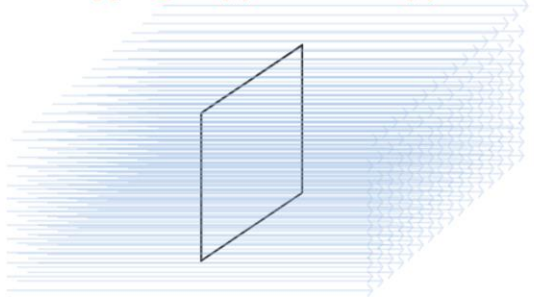
The **Maxwell equations are given differential forms** which are again in notation of divergence and curl. So we **need to understand the physical meaning of these**. Following is **how the divergence and curl are assessed and what their physical meaning is**.

1. **Curl:** Curl is the **maximum line integration** (of vector A) of a **unit surface area**. It is a **vectorial quantity**.

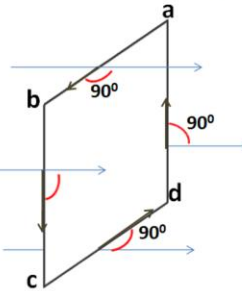
It is assessed by
$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

There is a **certain orientation of the area**, for which the **line integral of the vector field is maximum**. It is shown in a Fig. below.

Here the **field in which a rectangular coil is placed may be a electric (E), magnetic (B) or a fluid vector (A)**



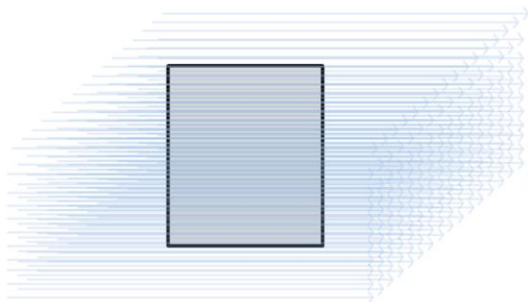
In this figure the **area of plane rectangular plane** (say a coil) is **perpendicular to the field**



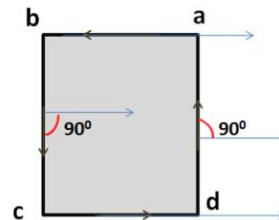
Here the **vector field is normal to each side of the rectangle**. i.e. the angle between **dl vector** and **field vector** is 90° so the net line integration is

$$\begin{aligned}\oint \mathbf{A} \cdot d\mathbf{l} &= \int_a^b \mathbf{A} \cdot d\mathbf{l} + \int_b^c \mathbf{A} \cdot d\mathbf{l} + \int_c^d \mathbf{A} \cdot d\mathbf{l} + \int_d^a \mathbf{A} \cdot d\mathbf{l} \\ &= A dl \cos 90^\circ + A dl \cos 90^\circ + A dl \cos 90^\circ + A dl \cos 90^\circ \\ &= 0\end{aligned}$$

But line integration is maximum when plane is place parallel to the field vector



In this figure the **area of plane rectangular plane** (say a coil) is **parallel to the field**



Here the **vector field is normal to two side of the rectangle**. And other two sides are having zero angles with field vector so net line integration is not zero and is **maximum** in this case

$$\begin{aligned}\oint \mathbf{A} \cdot d\mathbf{l} &= \int_a^b \mathbf{A} \cdot d\mathbf{l} + \int_b^c \mathbf{A} \cdot d\mathbf{l} + \int_c^d \mathbf{A} \cdot d\mathbf{l} + \int_d^a \mathbf{A} \cdot d\mathbf{l} \\ &= A dl \cos 0^\circ + A dl \cos 90^\circ + A dl \cos 0^\circ + A dl \cos 90^\circ \\ &= A dl \cos 0^\circ + A dl \cos 0^\circ \\ &= A dl + A dl\end{aligned}$$

2. **Divergence:** Divergence of a vector function A (i.e., $\nabla \cdot A$) is the rate of flow of a vector function through unit Gaussian volume (It may be a magnetic, electric or any fluid vector) of a Gaussian surface S containing volume V . It is a scalar quantity.

It is denoted by notation **Del dot**($\nabla \cdot$), since ∇ (Del Operator) = $\left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right)$

When a Del operator is operated over a vector function A then

$$\text{therefore } \nabla \cdot A = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot (iA_x + jA_y + kA_z)$$

$$= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right). \text{ Since this quantity is independent of } i, j \text{ and } k \text{ unit vectors therefore it is a scalar quantity.}$$

3. In deriving the Maxwell's equations, we need to convert **line integration** to **surface integration** and **surface integration** to **volume integration**. Below is given how to do it

(It must be mentioned here that in these literature symbol \int denotes line integration, \iint denotes surface integration and \iiint denotes volume integration. Otherwise, it is understood when we write (suppose) $\oint E \cdot dl$, $\oint E \cdot ds$ and $\oint E \cdot dv$ is for line, surface and volume closed integration.)

Conversion of Line integral to surface integral

Stock's theorem enables to convert line integral into a surface integral as

$$\int E \cdot dl = \iint (\nabla \times E) \cdot ds$$

For this we require to take curl of that vector (say E) and then write it in surface integral form.

Conversion of surface integral to volume integral

Gauss divergence theorem enables to convert surface integral into a volume integral as

$$\iint E \cdot ds = \iiint \nabla \cdot E \cdot dv$$

For this we require to take divergence of that vector (say E) and then write it in volume integral form.

4. **Emf (Electromotive force):** Emf is the work done to carry a unit charge over a closed circuit (or closed ckt. loop). In integration form it is denoted as

$$\text{emf} = \int E \cdot dl$$

5. $\nabla \cdot (\nabla \times A) = 0$ [div curl $A = 0$, Divergence of a curl is always zero]

6. $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$ [It is called as **Curl of Curl $A = \text{grad div } A - \nabla^2 A$**]

Maxwell's equations in differential form:

These equations are four laws in differential form in Electrostatics and Magnetostatics.

Gauss Law in Electrostatics:

The electric flux (ϕ_E) diverging from a Gaussian surface S is equal to the $1/\epsilon_0$ times of charge enclosed by that surface. Integral form of law is $\phi_E = \iint E \cdot ds = \frac{1}{\epsilon_0} q$. Differential form of law is given below. This is known as Maxwell's Ist equation.

$$\nabla \cdot \mathbf{D} = \rho$$

Gauss Law in Magnetostatics: Magnetic flux (ϕ_B) diverging from a Gaussian surface of S always zero.

It is because magnetic flux has always two poles (North-south) thus any flux diverging from Gaussian surface re-enters the surface therefore net flux diverging is zero. Integral form of law is $\phi_B = \iint B \cdot ds = 0$. Differential form of law is given below. This is known as Maxwell's IInd equation.

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's induction law: The rate of change of magnetic flux is equals to induced electromotive force (emf) in the ckt. Integral form of law is $\text{emf} = \oint E \cdot dl = -\frac{d\phi_B}{dt}$. Differential form of law is given below. This is known as Maxwell's IIIrd equation.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Modified Ampere's law for time varying fields:

As the Ampere's law in integral form is given as $\oint B \cdot dl = \mu_0 I$. The modified **Ampere's law for time varying fields'** in differential form is given below. This is known as Maxwell's IVth equation.

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$$

Derivation of Maxwell's Ist equation: As the **Gauss Law of Electrostatics in integral form** is given as

$$\varphi_E = \iint E \cdot ds = \frac{1}{\epsilon_0} q$$

$$\iint (\epsilon_0 E) \cdot ds = q$$

$$\text{since } \iiint \rho \cdot dv = q \text{ therefore } \iint (\epsilon_0 E) \cdot ds = \iiint \rho \cdot dv$$

$$\text{and } \epsilon_0 E = D$$

$$\iint D \cdot ds = \iiint \rho \cdot dv$$

Using **Gauss divergence** theorem to convert surface integral into volume integral $\iint D \cdot ds = \iiint \nabla \cdot D \cdot dv$

$$\iiint \nabla \cdot D \cdot dv = \iiint \rho \cdot dv$$

$$\text{or } \iiint (\nabla \cdot D - \rho) \cdot dv = 0$$

$$\text{or } \nabla \cdot D - \rho = 0$$

$$\text{or } \nabla \cdot D = \rho$$

Derivation of Maxwell's IInd equation: As the **Gauss Law of Magnetostatics in integral form** is given as

$$\varphi_B = \iint B \cdot ds = 0.$$

Using **Gauss divergence** theorem to convert surface integral into volume integral $\iint B \cdot ds = \iiint \nabla \cdot B \cdot dv$

$$\iint B \cdot ds = \iiint \nabla \cdot B \cdot dv = 0$$

$$\nabla \cdot B = 0$$

Derivation of Maxwell's IIIrd equation:

Faraday's induction law in integral form is given as $\oint E \cdot dl = - \frac{d\varphi_B}{dt}$

$$\oint E \cdot dl = - \frac{d\varphi_B}{dt} \quad \text{since } \varphi_B = \iint B \cdot ds$$

$$\text{therefore } \oint E \cdot dl = - \frac{d(\iint B \cdot ds)}{dt}$$

Here operator $-d/dt$ can be operated over B in $\iint B \cdot ds$. Therefore

$$\int E \cdot dl = - \iint \frac{dB}{dt} \cdot ds$$

Using **Stock's theorem** to convert surface integral into volume integral $\int E \cdot dl = \iint (\nabla \times E) \cdot ds$, then above can be written as

$$\iint (\nabla \times E) \cdot ds = - \iint \frac{dB}{dt} \cdot ds$$

$$\iint \left[(\nabla \times E) - \frac{dB}{dt} \right] \cdot ds = 0$$

$$\left[(\nabla \times E) - \frac{dB}{dt} \right] = 0$$

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

Derivation of Maxwell's IVth equation: To derive this refer Fig. 1(a) of capacitor

As the **Ampere's law** is given as $\int B \cdot dl = \mu_0 I$

Or $\int B \cdot dl = \mu_0 \iint J \cdot ds$ since $I = \iint J \cdot ds$ and $B = \mu_0 H$ or $\frac{B}{\mu_0} = H$ ($J =$ surface current density)

$$\int \frac{B}{\mu_0} \cdot dl = \iint J \cdot ds$$

$$\text{or } \int H \cdot dl = \iint J \cdot ds$$

Using **Stock's theorem** to convert surface integral into volume integral $\int H \cdot dl = \iint (\nabla \times H) \cdot ds$

$$\iint (\nabla \times H) \cdot ds = \mu_0 \iint J \cdot ds$$

$$\text{or } \nabla \times H = J \quad \dots \dots \dots [1]$$

Taking divergence of above expression as

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J$$

As the **divergence of a curl is always zero** [$\nabla \cdot (\nabla \times A) = 0$] therefore, $\nabla \cdot (\nabla \times H) = 0$

We get

$$0 = \nabla \cdot J$$

Interpretation of $\nabla \cdot J = 0$ is that **charge does not flow**, or **current is zero** but as the current is established between the plates of a capacitor (ckt. completes for an AC voltage) therefore here the equation of continuity $\nabla \cdot J = - \frac{d\rho}{dt} \dots \dots \dots [2]$ must be followed

Maxwell suggested adding a term J_D in RHS of equation [1] and then again takes the divergence.

$$\nabla \times H = J + J_D \dots \dots \dots [3]$$

Take the divergence of above as $\nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot J_D$. As above $\nabla \cdot (\nabla \times H) = 0$ therefore

$$\nabla \cdot J_D = -\nabla \cdot J$$

Now, substituting the equation of continuity $\nabla \cdot J = -\frac{d\rho}{dt}$ in above expression we get

$$\nabla \cdot J_D = \frac{d\rho}{dt}$$

Now substituting the value of ρ from equation [1] (i.e. $\nabla \cdot D = \rho$)

$$\nabla \cdot J_D = \frac{d(\nabla \cdot D)}{dt}$$

Here operator d/dt can be operated over D in above expression.

$$\nabla \cdot J_D = \frac{\nabla \cdot d(D)}{dt}$$

From above we get

$$J_D = \frac{d(D)}{dt}$$

The value of density of **displacement current** (J_D) is equal to rate of change of displacement vector (D).

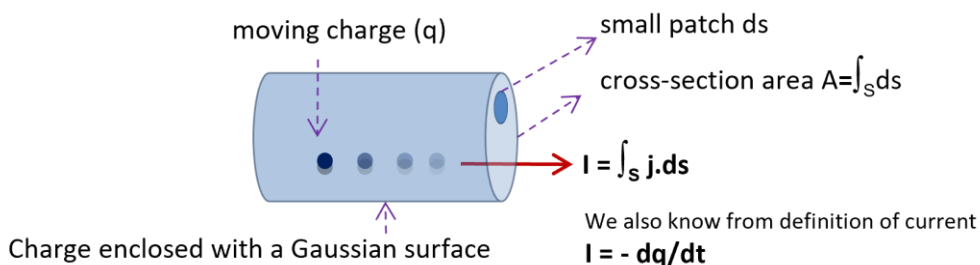
At last, substituting the value of $J_D = \frac{d(D)}{dt}$ into equation [3] we get the Maxwell's IVth equation as

$$\nabla \times H = J + \frac{d(D)}{dt}$$

Equation of continuity ($\nabla \cdot J = -\frac{d\rho}{dt}$)

(Divergence of current density is equal to rate of change of volume charge density)

As shown, a current is constituted only when a charge moves or leaves the volume V enclosed by the Gaussian surface S .



Therefore, $I = \iint \mathbf{J} \cdot d\mathbf{s} \dots \dots \dots [1]$

And as we know that current $I = -\frac{dq}{dt} \dots \dots \dots [2]$

Negative sign shows that charge (q) leaves the volume V enclosed by the Gaussian surface S.

$$I = \iint \mathbf{J} \cdot d\mathbf{s} = -\frac{dq}{dt} \text{ since } q = \iiint \rho \cdot dv$$

Therefore $\iint \mathbf{J} \cdot d\mathbf{s} = -\frac{d(\iiint \rho \cdot dv)}{dt}$

Here operator d/dt can be operated over ρ in above expression

$$\iint \mathbf{J} \cdot d\mathbf{s} = \iiint -\frac{d\rho}{dt} \cdot dv$$

Using **Gauss divergence** theorem to convert surface integral into volume integral $\iint \mathbf{J} \cdot d\mathbf{s} = \iiint \nabla \cdot \mathbf{J} dv$

$$\iiint \nabla \cdot \mathbf{J} dv = \iiint -\frac{d\rho}{dt} \cdot dv$$

From above we find that

$$\nabla \cdot \mathbf{J} = -\frac{d\rho}{dt}$$

Interpretation of the expression is that divergence of current density (J) is equal to rate of change of volume charge density (ρ). Negative sign shows that charge (q) leaves the unit volume ρ .

If $\nabla \cdot \mathbf{J} = 0$

It shows that rate of flow charge from unit volume is zero means there is no flow of charge therefore we can say charge is stagnated or not moving.

It can also be written as

$$\nabla \cdot \mathbf{J} - \frac{d\rho}{dt} = 0$$

Propagation of Plane Electromagnetic Waves in Free Space:

Characteristics of free space:

As there is no free charge in free space (vacuum) thus charge density (ρ)=0 and therefore conductivity (σ) =0 or current density (J) = σE =0. Consequently, Maxwell's equations reduces to

$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{D} = 0$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$

$$\nabla \times \nabla \times \mathbf{H} = \nabla \times \left(\frac{\partial \mathbf{D}}{\partial t} \right)$$

$$\text{since } \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \text{ therefore } \nabla \times (\nabla \times \mathbf{H}) = \nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H}$$

$$\text{and } \mathbf{D} = \epsilon_0 \mathbf{E}$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = \frac{\partial}{\partial t} \nabla \times (\epsilon_0 \mathbf{E})$$

$$\nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = \epsilon_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) \quad \text{since } \nabla(\nabla \cdot \mathbf{H}) = 0 \text{ gradient of a divergence is always zero.}$$

$$\text{Since } \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$-\nabla^2 \mathbf{H} = -\epsilon_0 \frac{\partial}{\partial t} \left(\frac{\partial \mathbf{B}}{\partial t} \right)$$

$$\text{Since } \mathbf{B} = \mu_0 \mathbf{H}$$

$$\nabla^2 \mathbf{H} = -\epsilon_0 \frac{\partial}{\partial t} \left(\frac{\partial \mu_0 \mathbf{H}}{\partial t} \right) \rightarrow \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}}{\partial^2 t}$$

$$\nabla^2 \mathbf{H} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}}{\partial^2 t}$$

Comparing it with standard equation of motion

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial^2 t}$$

$$\text{We find that } \frac{1}{v^2} = \epsilon_0 \mu_0$$

$$\text{or } v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} \text{ farad/m}) \times (4\pi \times 10^{-7} \text{ Henry/m})}} = 2.99 \times 10^8 \text{ m/sec} = c \text{ (speed of light in vacuum)}$$

Similarly, from taking the curl of Maxwell's IIIrd equation will give equation of motion of electric field (E) in vacuum

$$\nabla \times \nabla \times \mathbf{E} = \frac{\nabla \times \partial \mathbf{B}}{\partial t}$$

$$\text{since } \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \text{ therefore } \nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\text{and } \mathbf{B} = \mu_0 \mathbf{H}$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \frac{\partial}{\partial t} \nabla \times (\mu_0 \mathbf{H})$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \mu_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{H})$$

$$\text{Since } \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \text{ and } \nabla(\nabla \cdot \mathbf{H}) = 0 \text{ (gradient of a divergence is always zero).}$$

$$-\nabla^2 H = -\mu_0 \frac{\partial}{\partial t} \left(\frac{\partial D}{\partial t} \right)$$

Since $D = \epsilon_0 E$

$$\nabla^2 H = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(\frac{\partial \epsilon_0 E}{\partial t} \right) \Rightarrow \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial^2 t}$$

$$\nabla^2 E = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial^2 t}$$

Comparing it with standard equation of motion

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial^2 t}$$

We find that $\frac{1}{v^2} = \epsilon_0 \mu_0$ or $\frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \frac{\text{Henry}}{\text{m}} \times 8.85 \times 10^{-12} \frac{\text{farad}}{\text{m}}}} v = c$ (speed of light)

Above, shows that electric field and magnetic field travels with the same speed in vacuum.

Facts:

Maxwell also proved that

1. Electric field (E) and magnetic field (H) are bound with each other at 90° (or mutually perpendicular) and the direction flow of electromagnetic (EM) energy is again perpendicular to plane of E and B field.
2. The ratio of electric field (E) and magnetic field (H) is $E_0/H_0 = Z_0$ is defined as the *characteristic impedance or intrinsic impedance or wave impedance of the free space*.

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7} \text{ Henry/m}}{8.85 \times 10^{-12} \text{ farad /m}}} = 376.72 \Omega$$

$$\frac{E_0}{H_0} = 376.72 \Omega$$

It shows that vacuum offers 376.72 Ω resistance to the propagation of Electric field (E) and magnetic field (H).

$$\frac{E_0}{H_0} = \mu_0 c \quad \text{or} \quad \frac{E}{B} = c \quad \text{since } B = \mu_0 H$$

This relation shows that ratio of electric field and magnetic density (B) in vacuum is equal to c (speed of light)

3. Skin depth δ is a measure of the penetration of a plane **electromagnetic wave** into a material. It is the distance over which the amplitude of wave drops by 1/e

$$\delta = \frac{1}{\alpha} \quad \text{For good conductors } \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

Assignment

1. Derive all four Maxwell's equations.
2. Derive the equation of continuity $\nabla \cdot \mathbf{J} = -\frac{d\rho}{dt}$ and interpret the expression.
3. Show that electric field and magnetic fields travel with same speed (as that of c) or propagation of plane electromagnetic waves in free space:
4. If the magnitude of \mathbf{H} (vector) in a plane wave is 1 amp/meter, find the magnitude of \mathbf{E} (vector) for plane wave in free space. Then find the \mathbf{B} (vector).
[Hint: since $\mathbf{E}/\mathbf{H} = 376 \text{ ohm}$, find E field. Units: $\mathbf{E} = \text{volt /meter}$ and $\mathbf{H} = \text{turns-amp/meter}$]
5. The maximum electric field (\mathbf{E}) in a plane electromagnetic wave is 10^2 Newton/coulomb. The wave is proceeding in X direction and the electric field is in the Y direction. Find the maximum magnetic field in the wave and its direction.
[Hint: since $\mathbf{E}/\mathbf{B} = c = 3 \times 10^8$, find B field. Units: $\mathbf{B} = \text{Weber/m}^2$. Units: $\mathbf{E} = \text{volt /meter}$ and $\mathbf{H} = \text{turns-amp/meter}$]
6. If the earth receives $3 \text{ cal min}^{-1} \text{ cm}^2$ solar energy, what are the amplitudes of electric (\mathbf{E}) and magnetic (\mathbf{H}) fields of radiation.
[Hint: 1 calorie = 4.2 joule, 1m = 100cm and $\mathbf{E}/\mathbf{H} = 376 \text{ ohm}$ (known)]
7. Given rate of transmission/Flow of electromagnetic radiation (\mathbf{S}) = $\mathbf{E} \times \mathbf{H} = \mathbf{EH} = 3 \text{ cal min}^{-1} \text{ cm}^2 = (3 \times 4.2 \times 10^4)/60 \text{ joule m}^{-2} \text{ sec}^{-1}$. Find the field E using $\mathbf{E}/\mathbf{H} = 376$ and $\mathbf{S} = \mathbf{EH} = (3 \times 4.2 \times 10^4)/60 \text{ joule m}^{-2} \text{ sec}^{-1}$.
Units: $\mathbf{E} = \text{volt /meter}$ and $\mathbf{H} = \text{turns-amp/meter}$
8. A lamp radiates energy of 1500 watt uniformly. Calculate the average values of electric (\mathbf{E}) and magnetic (\mathbf{H}) fields of radiation at a distance of 3 m from the lamp.
[Hint: rate of transmission/Flow of electromagnetic radiation (\mathbf{S}) = $\mathbf{E} \times \mathbf{H} = \mathbf{EH} = \text{power/total surface area of sphere of radius 3 meter} = \mathbf{P}/4\pi r^2 = 1500/(4 \cdot 3.14 \cdot 3^2)$ and $\mathbf{E}/\mathbf{H} = 376 \text{ ohm}$ (known).
Using $\mathbf{E}/\mathbf{H} = 376 \text{ ohm}$ and $\mathbf{S} = 1500/(4 \cdot 3.14 \cdot 3^2)$, find E and H fields]
Here $\mathbf{B}/\mathbf{H} = \mu$ (permeability of material) and $\mathbf{M}/\mathbf{H} = \chi_m$ (susceptibility of magnetic material).