UNIT III Special theory of relativity

Special theory of relativity: Inertial and non-inertial frames, Galilean transformation, Michelson-Morley experiment, Einstein postulates of special theory of relativity, Lorentz transformation equation, length contraction, time dilation, variation mass of velocity, Mass energy relation.

Please refer prescribed book Engg Physics by Hitendra K Malik (The pdf has already been shared) for topics "Inertial and non-inertial frames".

UNIT 3 Special Theory of Relativity:

1. Explain the Michelson Morley experiment.
2. Discuss the phenomenon of length contraction and time dilation.
3. Derive Lorentz transformation equation and discuss the significance.
4. Derive mass-energy relation.

Unit at Glance (Basics to start this unit)

Frame of reference:

A coordinate system with respect to which we measure the position of a point object of an event is called a **frame of reference**.

In order to specify the location of a point object in space, we require a coordinate system. For location of a point 'P' we need three co -ordinate x, y and z. For complete identification of an event we must know 't' also, i.e., the time of the occurrence. Hence an event in characterized by four co-ordinates (x, y, z, t).

The frame of reference is of two types

- **Non-inertial frames of reference:**
  Accelerating frames are considered non-inertial frame because the law of inertia and other laws of Newtonian mechanics does not follow in such frames.
  Merry-go-round is non-inertial frame because rotating bodies are accelerating. similarly, revolving earth is not considered as non-inertial frame because the speed of motion of bodies on earth is not comparable to revolving speed of earth i.e. of order 10^4 m/sec

- **Inertial frames of reference:**
  The non-accelerating frames or frames moving with constant velocity are considered as inertial frame of reference because Newton's Law of Inertia and other laws of Newtonian mechanics hold good in such frames.

Relativistic particle is the particle which moves with comparable speed of light (i.e. v ~c). Only the elementary particle as electron proton, neutron (particles residing in an atom) can travel with comparable speed of light.

In numerical, the speed of a particle mentioned as 0.2c…0.9c should be considered as **relativistic speed and** the particle **as relativistic particle**.

Non-relativistic particle is the particle whose speed is not comparable to speed of light.

General idea about topics in this Unit

- **Michelson–Morley experiment:**

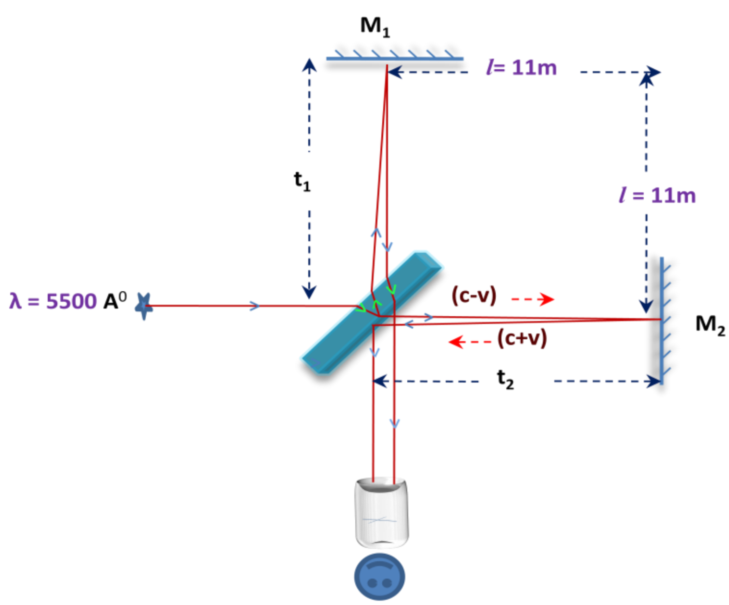
Michelson-Morley experiment was performed in 1887 to detect the presence of an ***ether hypothetical medi***um (through which the light travels from here to there even in vacuum) or to ***find the velocity of earth relative to ether medium***. It was a large interferometer which could detect fringe shift in interference pattern less than 0.4. This set-up was sensitive enough to detect the fringe shift of 0.4 but no shift was observed.
Thus Michelson –Morley concluded that ether does not exist.
(This is known as negative results of Michelson–Morley experiment).
The theoretical analysis of this experiment produced theoretical fringe shift of 0.4. But experimentally no shift was observed.

Therefore, to remove the discrepancy in calculated theorical value ($\Delta n = 0.4$) and experimental shift ($\Delta n = 0$), Einstein laid down two postulates.

Einstein explained it in 1905 using two postulates.

1. **The law of physics is same in all inertial frames of reference moving with a constant velocity (without acceleration) with respect to one another.**
2. **The speed of light c in free space (vacuum) is always same in all inertial frames.**

Q.1 Discuss the Michelson-morley experiment and its negative results.



Michelson Interferometer

Michelson-Morley experiment was performed in 1887 to detect the presence of an ***ether hypothetical medi***um (through which the light travels from here to there even in vacuum) or to ***find the velocity of earth relative to ether medium***.

Total time taken by the light ray from P $\rightarrow M_2$ with velocity (c-v) and then back from $M_2 \rightarrow$ P with velocity (c+v) -

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$= \frac{l\,(c+v)+l\,(c-v)}{c^2-v^2}$$

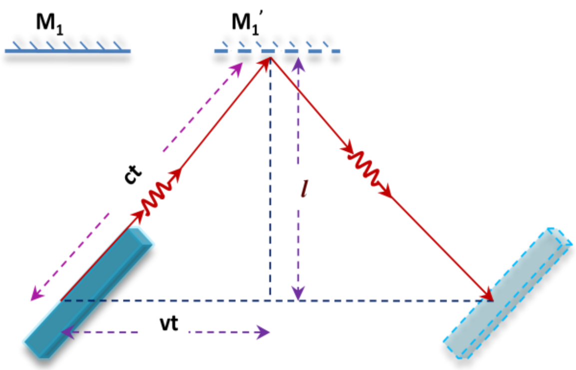
$$= \frac{2lc}{c^2\left(1-\frac{v^2}{c^2}\right)}$$

$$= \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

$Expenading\ with\ Binomail\ expansion\ i.e. (1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} \ldots \ldots \ldots$

$$\boxed{t_2 = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)}$$

$here\ x = \frac{v^2}{c^2} and\ n = -1$



Above is the situation of the light ray as seen by an observer outside the frame.

(Explanation: Since in above figure there are two mutually perpendicular relative motions i.e. at x-direction, the motion of revolving earth (v) and in y direction the moving photon (c). Since both the motion are comparable therefore, the relative direction of photon comes in the diagonal direction. This situation is realized by the observer standing outside the frame. Basically, this is the cause of two mutually perpendicular relative motions)

Assessment of time taken by the light ray from P $\rightarrow M_1$

$$c^2t^2 = v^2t^2 + l^2$$

$$c^2t^2 - v^2t^2 = l^2$$

$$(c^2 - v^2)t^2 = l^2$$

$$t^2 = \frac{l^2}{(c^2 - v^2)}$$

$$t^2 = \frac{l^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

Taking square root:

$$t = \frac{l}{c\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Or $t = \frac{l}{c\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$

$or\ t = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ $\quad Expenading\ with\ Binomail\ expansion\ i.e.(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} \ldots \ldots \ldots.$

$$t = \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

Or the total time taken by the light ray from P→M_1'and back to M_1'→ P

$$\therefore\ t_1 = 2t = \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\Delta t = t_2 - t_1 = \frac{2l}{c}\left\{\cancel{1} + \frac{v^2}{c^2} - \cancel{1} - \frac{1}{2}\frac{v^2}{c^2}\right\}$$

$$\Delta t = \frac{\cancel{2}l}{c}\left\{\frac{1}{\cancel{2}}\frac{v^2}{c^2}\right\}$$

$$\Delta t = \frac{lv^2}{c^3}$$

Therefore, the corresponding path difference between 1and 2 interfering waves

$$\Delta = c.\Delta t = c.\frac{lv^2}{c^3}$$

$$\boxed{\Delta = \frac{lv^2}{c^2}}$$

The actual experimental set was rotated through 90^0 so that PM_1 become longer than the path PM_2 by an amount $\frac{lv^2}{c^2}$. Thus on rotating the apparatus through 90^0, introduces a path difference of the same amount in opposite direction so that the total difference between the 1 and 2 interfering waves become $\Delta = \frac{2lv^2}{c^2}$. Hence a shift of $\Delta n = \frac{2lv^2}{c^2\lambda}$.

Therefore, the **fringe shift** $(\Delta \boldsymbol{n}) = \frac{2lv^2}{c^2\lambda} = \frac{2x\ 11x(3x\ 10^4)^2}{(3x\ 10^8)^2} \cdot \frac{1}{5.5\ x10^{-72}}$ = **0.4**

This set-up was sensitive enough to detect the fringe shift of 0.4 but no shift was observed.

Thus Michelson –Morley concluded that ether does not exist.

(This is known as negative results of Michelson–Morley experiment).

Hence, in Michaelson-Morley (MM) experiment, to remove the theoretical (Δn = 0.4) and experimental shift (Δn = 0), Einstein laid down two postulates.

Einstein explained it in 1905 using two postulates.

3. **The law of physics is same in all inertial frames of reference moving with a constant velocity (without acceleration) with respect to one another.**
4. **The speed of light c in free space (vacuum) is always same in all inertial frames.**

*[(It is to mention here that the **transformation equations** used in explaining the **MM experiment** is called **Galilean transformation**.*

***Trans formation equation** are those set of equations which establish relation between physical quantities measured in stationary (represented by X frame) and moving frame (represented by X' frame).]*