Basic Electrical Engineering (TEE 101)

Lecture 10: Star – Delta Transformation

Content

This lecture covers:

- Transformation of Star to Delta and Delta to Star type formation of the Electric Circuits
- Numerical on Star – Delta Transformation

Introduction

In electrical Circuits, the circuit elements are connected in various ways. Such as: series, parallel or a complex formation.

It is very easy to solve the series or parallel electrical circuits

However, the complex formation may involve the connections of circuit elements in STAR (or Y or T) formation or DELTA (or Pi) formation.

It is not easy to solve electrical circuits with such types of formations

Such types of formations are generally found in Three – Phase Circuits.

Hence, to simplify such electrical circuits, we have a mathematical concept in Electrical Systems which we call as STAR – DELTA Transformation (or DELTA – STAR Transformation)

The Transformation of one type of formation into other type is based on the concept of Equivalent Resistance across any two same terminals in the two types of formation.

Delta to Star Conversion

A Delta Formation (also known as Pi formation) of circuit elements in an electrical circuits can be transformed or converted into its equivalent Star formation (also known as Y or T formation)

For convenience, we will understand this concept through a resistive network

The conversion of a Delta to Star has been explained below:

Let us consider a Delta formation of resistances, as shown below, which is to be converted into a Star formation.

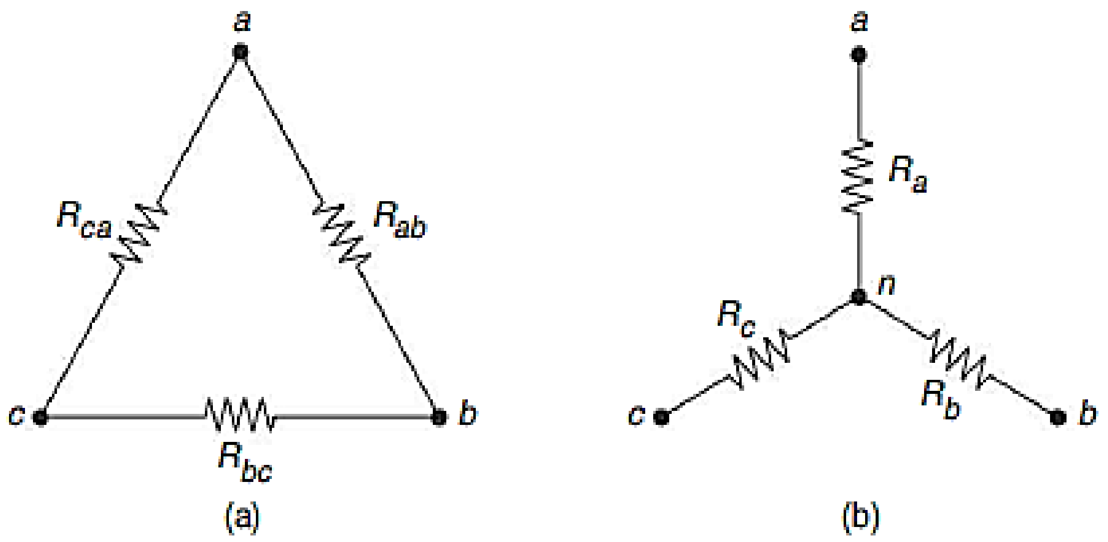


Figure (a) Delta-connected and (b) Star-connected networks

Figure (a) shows three Delta connected resistances connected between three nodes a, b and c.

On the other hand in Figure (b), there are three Y-connected resistances.

Equating the Resistances

Equating resistance between node pairs:

Node pair ab

- $R_a + R_b = R_{ab} \parallel (R_{bc} + R_{ca})$ (1)

Node pair bc

- $R_b + R_c = R_{bc} \parallel (R_{ca} + R_{ab})$ (2)

Node pair ca

- $R_c + R_a = R_{ca} \parallel (R_{ab} + R_{bc})$ (3)

Where

- Ra, Rb and Rc are the resistances of STAR Formation and
- Rab, Rbc and Rca are the resistances of DELTA Formation

Equating the Resistances

These Equations can be written as:

$$R_a + R_b = \frac{R_{ab} \times (R_{bc} + R_{ca})}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab} \times R_{bc} + R_{ab} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \qquad (4)$$

$$R_b + R_c = \frac{R_{bc} \times (R_{ca} + R_{ab})}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{bc} \times R_{ca} + R_{bc} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}} \qquad (5)$$

$$R_c + R_a = \frac{R_{ca} \times (R_{ab} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ca} \times R_{ab} + R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}} \qquad (6)$$

Now, by using equations (4) to (6), we can determine the values of Ra, Rb and Rc in terms of Rab, Rbc and Rca (this is DELTA to STAR Conversion)

Or, Values of Rab, Rbc and Rca in terms of Ra, Rb and Rc (This is STAR to DELTA Conversion)

Delta to Star Conversion Conti…

To get, a STAR Equivalent from a DELTA, apply following operations on equation (4) and (5):

Subtract eq. (5) from eq. (4)

$$R_a + R_b - R_b - R_c = \frac{R_{ab} \times R_{bc} + R_{ab} \times R_{ca} - R_{bc} \times R_{ca} - R_{bc} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

Or, $$R_a - R_c = \frac{R_{ab} \times R_{ca} - R_{bc} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \qquad \textbf{(7)}$$

Now, add equation (6) to equation (7), we get

$$R_a - R_c + R_c + R_a = \frac{R_{ab} \times R_{ca} - R_{bc} \times R_{ca} + R_{ca} \times R_{ab} + R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

Or, $$R_a + R_a = \frac{R_{ab} \times R_{ca} + R_{ca} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

Or, $$2R_a = \frac{2R_{ab} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

Or, $$R_a = \frac{R_{ab} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \qquad \textbf{(8)}$$

Equation (8) gives the value of R_a (which is the resistance of STAR Formation, in terms of R_{ab}, R_{bc} and R_{ca} (which are the resistances of DELTA formation of resistances.

Similarly, we can obtain the values of R_b and R_c in terms of R_{ab}, R_{bc} and R_{ca} to get the complete DELTA to STAR conversion.

Delta to Star Conversion Conti…

Now, to Subtract eq. (6) from eq. (5) and add the result to equation (4). This will give us the value of R_b

$$R_b = \frac{R_{bc} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}} \qquad \textbf{(9)}$$

Now, to Subtract eq. (4) from eq. (6) and add the result to equation (5). This will give us the value of R_c

$$R_c = \frac{R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}} \qquad \textbf{(10)}$$

From equations (8), (9) and (10), we can conclude that the while converting a DELTA formation of circuit elements (resistances in this case) to the STAR formation, the values of STAR resistances depend upon the the product of two out of three DELTA resistances divided by the sum of all three DELTA resistances.

Now, using the values of Ra, Rb and Rc (as given by equations (8), (9) and (10), we can obtain the values of Rab, Rbc and Rca in terms of Ra, Rb and Rc. This is STAR to DELTA conversion.

Star to Delta Conversion

To obtain a DELTA equivalent of a STAR network, we can use equations (8), (9) and (10).

Multiply Equation (8) and (9), multiply (9) and (10) and multiply (10) and (8)

$$R_aR_b = \frac{R_{ab} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \times \frac{R_{bc} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R^2_{ab} \times R_{bc} \times R_{ca}}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})} \qquad \textbf{(11)}$$

$$R_bR_c = \frac{R_{bc} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}} \times \frac{R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab} \times R^2_{bc} \times R_{ca}}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})} \qquad \textbf{(12)}$$

$$R_cR_a = \frac{R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}} \times \frac{R_{ab} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab} \times R_{bc} \times R^2_{ca}}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})} \qquad \textbf{(13)}$$

Now, add equations (11), (12) and (13)

Star to Delta Conversion conti…

$$R_aR_b + R_bR_c + R_cR_a = \frac{R^2_{ab} \times R_{bc} \times R_{ca}}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})} + \frac{R_{ab} \times R^2_{bc} \times R_{ca}}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})} + \frac{R_{ab} \times R_{bc} \times R^2_{ca}}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})}$$

Or $$R_aR_b + R_bR_c + R_cR_a = \frac{(R^2_{ab} \times R_{bc} \times R_{ca}) + (R_{ab} \times R^2_{bc} \times R_{ca}) + (R_{ab} \times R_{bc} \times R^2_{ca})}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})}$$

Or $$R_aR_b + R_bR_c + R_cR_a = \frac{R_{ab} \times R_{bc} \times R_{ca}(R_{ab} + R_{bc} + R_{ca})}{(R_{ab} + R_{bc} + R_{ca}) \times (R_{ab} + R_{bc} + R_{ca})}$$

Or $$R_aR_b + R_bR_c + R_cR_a = \frac{R_{ab} \times R_{bc} \times R_{ca}}{(R_{ab} + R_{bc} + R_{ca})} \qquad \textbf{(14)}$$

We can use equation (14) to calculate the values of R_{ab}, R_{bc} and R_{ca} in terms of R_a, R_b and R_c.

Star to Delta Conversion conti…

Now, arrange equation 14 in such a way that we can get the value of R_{ab}

$$R_a R_b + R_b R_c + R_c R_a = R_{ab} \times \left(\frac{R_{bc} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \right) \quad (15)$$

From equation (10), we have

$$R_c = \frac{R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

Substitute this in equation (15), we get..

$$R_a R_b + R_b R_c + R_c R_a = R_{ab} \times R_c$$

Or, $$R_{ab} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c} \quad (16)$$

Equation (16) gives the value of R_{ab} in terms of R_a, R_b and R_c . Similarly, we can obtain the value of R_{bc} and R_{ca} in terms of R_a, R_b and R_c .

Star to Delta Conversion conti…

On doing so, the values of R_{bc} and R_{ca} obtained are given below:

$$R_{bc} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a} \qquad \textbf{(17)}$$

and

$$R_{ca} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b} \qquad \textbf{(18)}$$

Hence, equations (16 to 18) give the value of R_{ab} R_{bc} and R_{ca} in terms of R_a, R_b and R_c .

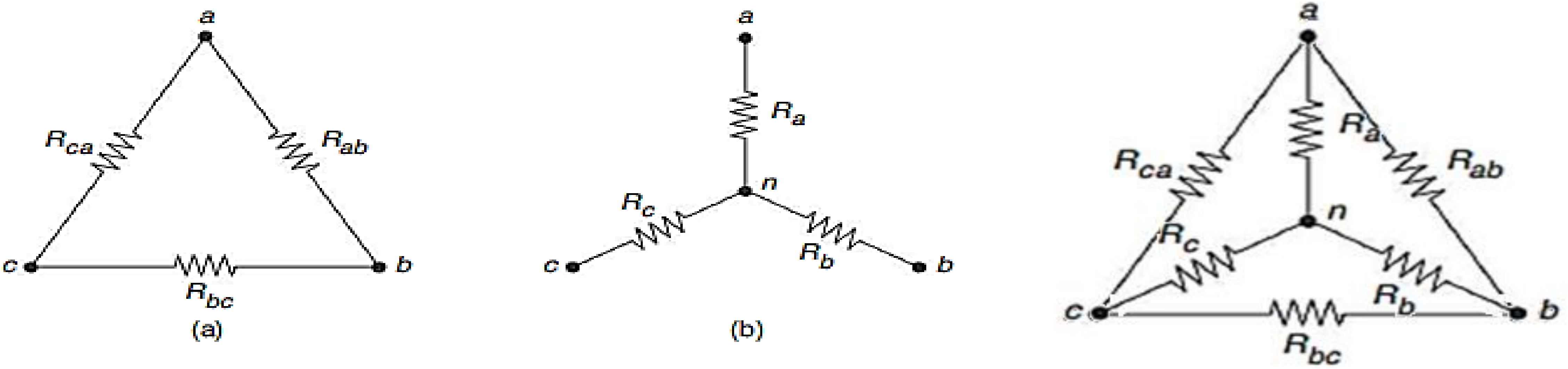
Summary

DELTA to STAR Conversion

$$R_a = \frac{R_{ab} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \qquad R_b = \frac{R_{bc} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}} \qquad R_c = \frac{R_{ca} \times R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

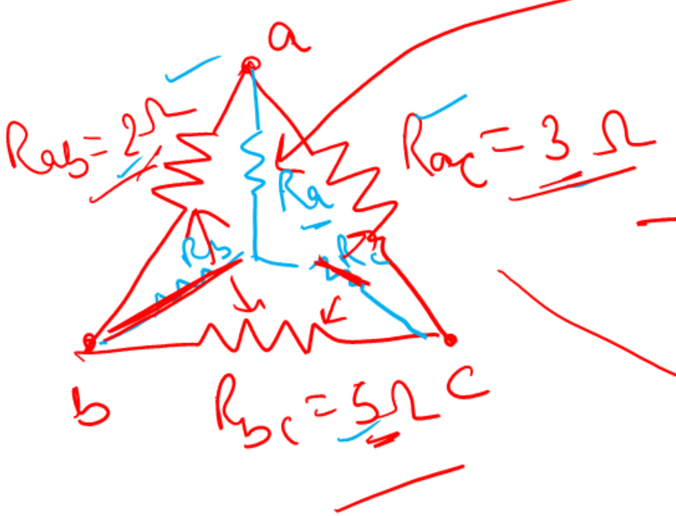
STAR to DELTA Conversion

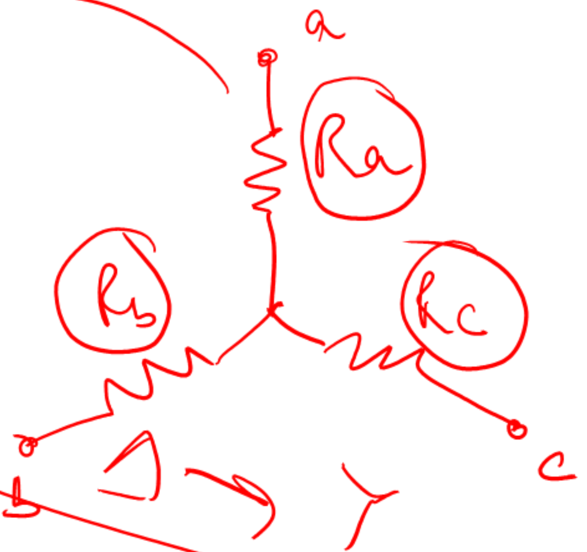
$$R_{ab} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c} \qquad R_{bc} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a} \qquad R_{ca} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$



Numerical Problem

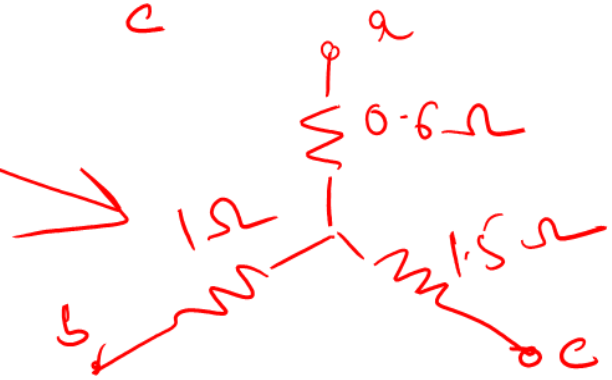
Question: 03 resistances of 2Ω, 3Ω and 5Ω are connected in DELTA formation. Obtain the STAR equivalent of these resistances.





$$R_b = \frac{2\times 5}{2+3+5} = \frac{10}{10} = 1\Omega$$

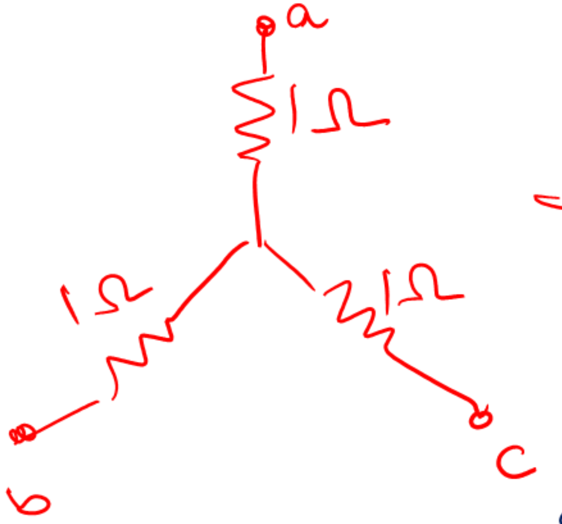
$$R_c = \frac{3\times 5}{2+3+5} = \frac{15}{10} = 1.5\Omega$$



$$R_a = \frac{2\times 3}{2+3+5} = \frac{2\times 3}{10} = 0.6\ \Omega$$

Numerical Problem

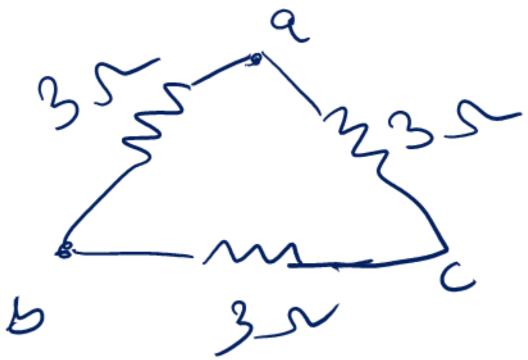
Question: 03 resistances of 1Ω each are connected in STAR formation. Obtain the DELTA equivalent of these resistances.



$$R_{ab} = \frac{1\times1+1\times1+1\times1}{1} = \frac{1+1+1}{1} = \frac{3}{1} = 3\Omega$$

$$R_{bc} = \frac{1\times1+1\times1+1\times1}{1} = 3\Omega$$

$$R_{ca} = \frac{1\times1+1\times1+1\times1}{1} = 3\Omega$$



Thank You