Phasor diagram of transformer on Load

In this section we will discuss about the phasor diagram and Equivalent circuit of practical transformer on LOAD.

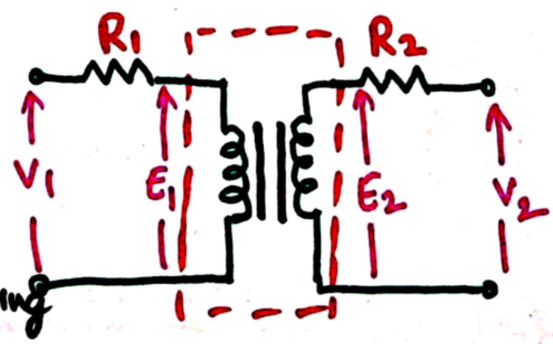
A practical transformer has following charactrustics :-
→ windings have resistance
→ core of transformer has finite permeability.
→ It has leakage ~~flux~~ of flux.
→ It possess finite losses.
→ The efficiency of practical transformer is not 100%..

So; while analysing a practical transformer on load we will consider all the above mentioned characteristics.

(a) Winding Resistance :→

An Ideal transformer does not have winding resistance But a practical transformer has.

The winding resistances are always shown in series with each winding as shown.



R_1 = Resistance of Primary winding
R_2 = " " Secondary "

ⓑ **Leakage Reactance** :- It is assumed that ALL the flux that is established by the primary winding must be confined within the core. And the full flux should reach the secondary winding.

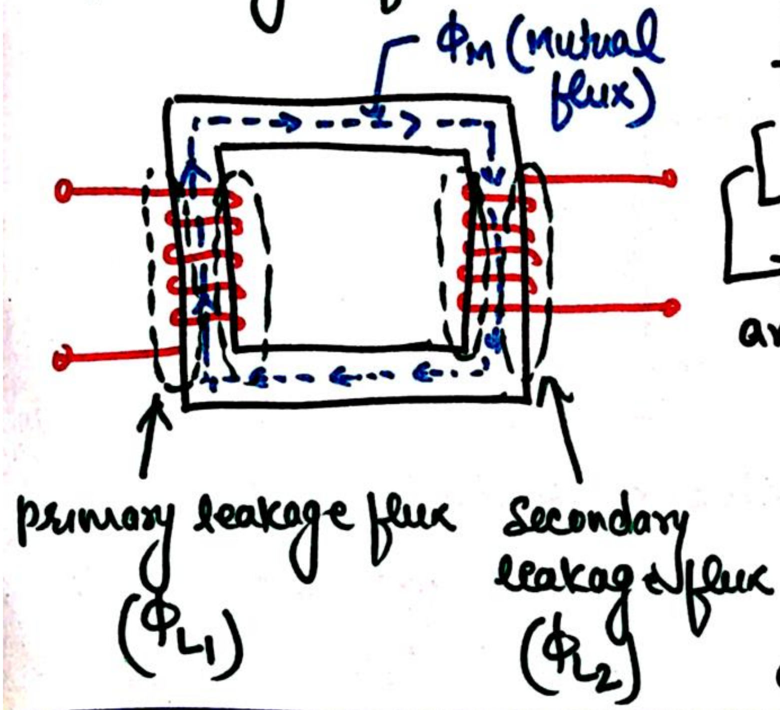
But, In a practical transformer this condition is not achieved and some amount of flux is leaked from the core. This happens because the surrounding of transformer core has finite permeability and hence it can conduct flux. So; the flux can also flow through such mediums.

Now; the flux leakage in primary winding is known as primary leakage flux. Let it is denoted by ϕ_{L_1}. This flux Induces an emf (E_{L_1}) in the primary winding.

Also; the current that flows in secondary is I_2. This current establishes flux ϕ_2 in secondary winding. ϕ_2 always opposes the main flux ϕ_M.

Now; some amount of ϕ_2 also gets leaked out from the secondary winding. Thus, this leaked flux is known as secondary flux leakage (ϕ_{L_2})

ϕ_{L_2} is only linked with the secondary winding. Hence, this flux leakage induces an emf E_{L_2} in the secondary winding. The following diagrams shows the phenomenon of leakage flux.



This is to be noted that:

- E_{L_1} is due to ϕ_{L_1}
- E_{L_2} is due to ϕ_{L_2}

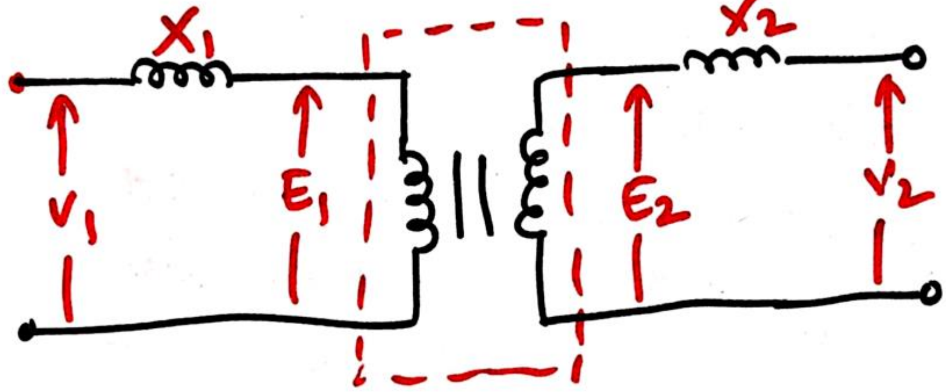
→ These two emfs are not same as E_1 and E_2

E_1 & E_2 are Induced due to ϕ_M and not due to leakage flux.

Now the question is that how to show these flux leakages in the equivalent circuit of the transformer.

From the above analysis it is clear that the flux leakages ϕ_{L_1} and ϕ_{L_2} Induces E_{L_1} and E_{L_2} in primary and secondary windings respectively. That means these flux leakages produce voltage drops in the windings of the transformer. Hence; the effect

Of these flux leakages is equivalent to the Inductances in series with each winding such that the voltage drop in each series Inductance is same as that produced by the flux leakages in each winding. These leakage reactances are shown in the diagram below.



X_1 = Primary leakage reactance

X_2 = Secondary " "

X_1 and X_2 are Imaginary quantities.

(C) Referred values :→ In a transformer there are two windings. In other words we can say that a transformer has two circuits. One on the primary side and other on the secondary side of the transformer.

Now; theoretically through Mathematical Analysis; It is possible convert these two circuit in one circuit. In this section we will study about how to reduce a transformer's in one circuit.

When a transformer is operational, the parameters of primary side of the transformer (Such as: R_1, X_1, I_1, E_1, V_1 etc) have effect on the secondary side of the transformer.

Similarly the parameters of secondary side of the transformer (such as:- R_2, X_2, I_2, E_2 etc) have their effect on the primary side of the transformer.

Technically, we call them referred values.

To understand this concept; let us proceed with the ~~steps~~ analysis :→

The winding resistance of secondary is R_2. Let us assume that this resistance has its reflected or referred value in primary winding of the transformer. Let this referred value of R_2 is R_2'.

This means that R_2' produces same effect in the primary winding as produced by the R_2 in secondary winding.

This means that the power consumed by R_2 in secondary and power consumed by R_2' in primary is same.

i.e $\boxed{I_2'^2 R_2' = I_2^2 R_2}$ where I_2' is the referred value of I_2 reflecting in primary.

Now; rearranging the above equation we get.

$$\boxed{R_2' = \left(\frac{I_2}{I_2'}\right)^2 R_2}$$

Now; we know that In a transformer the Ampere-Turns on both sides of the transformer is equal

i.e $I_2' T_1 = I_2 T_2$

or $\left(\frac{I_2}{I_2'}\right) = \left(\frac{T_1}{T_2}\right) = a$

where a = Transformation Ratio.

So; $\boxed{R_2' = a^2 R_2}$ — (1)

Similarly; Let X_2' is the referred value of X_2 reflecting in primary winding of the transformer.

This means that X_2' will produce same effect in the primary side of the transformer as X_2 will produce on the secondary side.

i.e The Reactive power (VAr) must be same on both sides of the transformer. Hence;

$$\underbrace{(I_2')^2 X_2'}_{\text{VAr of Primary winding}} = \underbrace{I_2^2 X_2}_{\text{VAr of Secondary winding.}}$$

or;

$$x_2' = \left(\frac{I_2}{I_2'}\right)^2 x_2$$

But $\frac{I_2}{I_2'} = a$; hence

$$\boxed{x_2' = a^2 x_2} \quad \text{--- (2)}$$

Similarly; we can find z_2'. (which is the referred value of the secondary impedance z_2 reflecting in primary). Hence;

$$\boxed{z_2' = a^2 z_2} \quad \text{--- (3)}$$

Now let us find the effective values of resistance/reactance/ Impedance of the transformer through the values of secondary winding referred to Primary winding.

Let Re_1 = effective Resistance

Xe_1 = effective Reactance

Ze_1 = " Impedance

then;

$$\left.\begin{aligned} Re_1 &= R_1 + R_2' \\ Xe_1 &= X_1 + X_2' \\ \text{and} \quad Ze_1 &= Z_1 + Z_2' \end{aligned}\right\} \text{--- (4)}$$

where; R_1, X_1 and Z_1 are the actual values of resistance, Reactance and impedance of the primary winding.

So; substituting the values of R_2', X_2' and Z_2' from eqn (1), (2) & (3) respectively to eqn (4) we get.

$$\left.\begin{aligned} Re_1 &= R_1 + a^2 R_2 \\ Xe_1 &= X_1 + a^2 X_2 \\ Ze_1 &= Z_1 + a^2 Z_2 \end{aligned}\right\} \text{--- (5)}$$

Also; $\boxed{Z_{e_1} = R_{e_1} + jX_{e_1}}$ —— (6)

Now; Let Z_L' is the value of Z_L referred to primary. Then $\boxed{Z_L' = a^2 Z_L}$ —— (7)

From the above analysis; you must have noticed that all the resistance, reactance and Impedance of secondary winding reflecting in primary winding are multiplied by the square of the transformation ratio(a).

Similarly It can be proved that the values of primary resistance, reactance and Impedance reflecting in secondary winding are divided by the square of the transformation ratio (a).

Let R_1' is the value of R_1 referred to the secondary winding (i.e R_1' is the value of R_1 reflecting in the secondary winding). So; R_1' and R_1 will produce same effect in their respective windings.

i.e $\boxed{I_1^2 R_1 = I_1'^2 R_1'}$ where I_1' is the value of I_1 reflecting in secondary.

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Now; re-arranging the equation we get

$$\boxed{R_1' = \left(\frac{I_1}{I_1'}\right)^2 R_1} \quad \text{—— (8)}$$

But $I_1 T_1 = I_1' T_2$

or $\frac{T_1}{T_2} = \frac{I_1'}{I_1} = a$

or $\boxed{\frac{I_1}{I_1'} = \frac{1}{a}}$

$R_1' = \frac{1}{a^2} R_1$

or $\boxed{R_1' = \frac{R_1}{a^2}}$ —— (9)

Similarly we can prove that;

$\boxed{X_1' = \frac{X_1}{a^2}}$ and —— (10)

$\boxed{Z_1' = \frac{Z_1}{a^2}}$ —— (11)

where; X_1' = value of X_1 referred to secondary

Z_1' = value of Z_1 referred to secondary.

Also; let R_0' and X_0' are the values of R_0 and X_0 respectively which are referred to the secondary winding. Then

$$\left.\begin{aligned} R_0' &= \frac{R_0}{a^2} \text{ and} \\ X_0' &= \frac{X_0}{a^2} \end{aligned}\right\} \text{—— (12)}$$

R_0 and X_0 are the Resistance and Reactance of the core of the transformer.

Now we can obtain the effective Resistance / Reactance / Impedance of the transformer based on the values of R_1, X_1 and Z_1 referred to the secondary winding. So;

$$\left.\begin{aligned} Re_2 &= R_2 + R_1' \\ Xe_2 &= X_2 + X_1' \text{ and} \\ Ze_2 &= Z_2 + Z_1' \end{aligned}\right\} \text{—— (13)}$$

Hence;

$$\left.\begin{aligned} Re_2 &= R_2 + \frac{R_1}{a^2} \\ Xe_2 &= X_2 + \frac{X_1}{a^2} \\ \text{and} \\ Ze_2 &= Z_2 + \frac{Z_1}{a^2} \end{aligned}\right\} \quad \text{--- (14)}$$

Also

$$\boxed{Ze_2 = Re_2 + j\,Xe_2}$$

Also; It can be proved that

$$Re_2 = \frac{Re_1}{a^2}\,;\; Xe_2 = \frac{Xe_1}{a^2}$$

and; $Ze_2 = \frac{Ze_1}{a^2}$

Now let us see what happens to the voltages of secondary winding reflecting in the primary winding.

let E_2' is the value of E_2 referred to the Primary winding

i.e. E_2' is the value of E_2 reflecting in the primary winding. so; we know that;

$$E_2' I_2' = E_2 I_2$$

or $E_2' = \left(\frac{I_2}{I_2'}\right) E_2$

where: $\frac{I_2}{I_2'} = a$;

so;

$$\boxed{E_2' = a\,E_2} \quad \text{--- (15)}$$

Similarly It can be proved that

$$\boxed{V_2' = a\,V_2} \quad \text{--- (16)}$$

where; V_2' is the value of V_2 referred to the primary winding.

E_2 and V_2 are the RMS value of emf Induced in secondary winding and voltage on load terminals respectively.

Similarly we can prove that the values of E_1 and V_1 referred to secondary winding are divided by the transformation ratio (a).

Let E_1' is the value of E_1 referred to the secondary winding. Hence; from the concept of voltage per turn ratio we have:

$$\frac{E_1}{T_1} = \frac{E_1'}{T_2} \quad \text{or} \quad E_1' = \left(\frac{T_2}{T_1}\right) E_1$$

But $\frac{T_2}{T_1} = \frac{1}{a}$; so; $\boxed{E_1' = \frac{E_1}{a}}$ —— (17)

also; $\boxed{V_1' = \frac{V_1}{a}}$ —— (18) where E_1 and V_1 are the RMS value of the emf Induced in the Primary winding and the Input voltage respectively.

Now; from the concept of Ampere-Turns we have;

$$I_1 T_1 = I_1' T_2 \quad \text{or} \quad I_1' = \left(\frac{T_1}{T_2}\right) I_1$$

~~or~~ But $\frac{T_1}{T_2} = a$; so $\rightarrow$ $I_1' = a I_1$.

Also; $I_0' = a I_0$; $I_w' = a I_w$ and

$I_\mu' = a I_\mu$

Similarly; if I_2' is the value of I_2 referred to the primary winding. So;

$$I_2' T_1 = I_2 T_2 \quad \text{or} \quad I_2' = \left(\frac{T_2}{T_1}\right) I_2$$

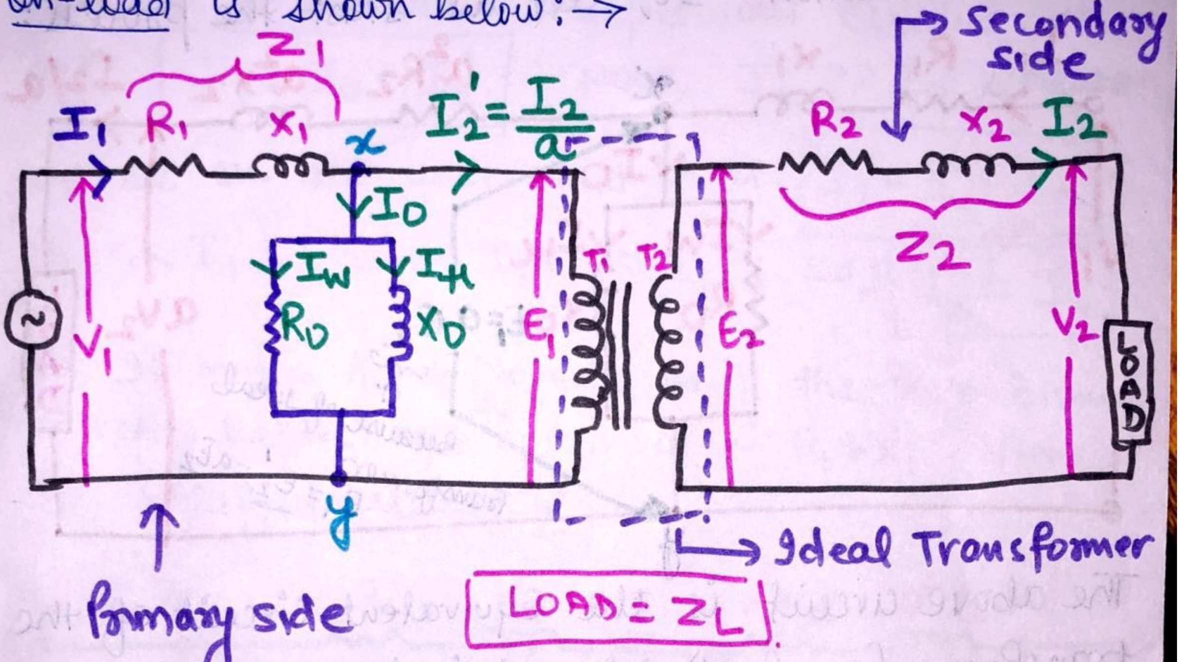
But $\frac{T_2}{T_1} = \frac{1}{a}$. So $\rightarrow$ $\boxed{I_2' = \frac{I_2}{a}}$

Summary

Values of primary referred to secondary	Values of secondary referred to primary.
a) $R_1' = \frac{R_1}{a^2}$ b) $X_1' = \frac{X_1}{a^2}$ $Z_1' = \frac{Z_1}{a^2}$	$R_2' = a^2 R_2$; $X_2' = a^2 X_2$ and $Z_2' = a^2 Z_2$ $Z_L' = a^2 Z_L$
Voltages. $E_1' = \frac{E_1}{a}$; and $V_1' = \frac{V_1}{a}$	$E_2' = a E_2$ $V_2' = a V_2$
Currents $I_1' = a I_1$ $I_0' = a I_0$ $I_w' = a I_w$ $I_\mu' = a I_\mu$	$I_2' = \frac{I_2}{a}$

Equivalent circuit of a practical transformer ON LOAD

The complete equivalent circuit of a practical transformer on-load is shown below: →



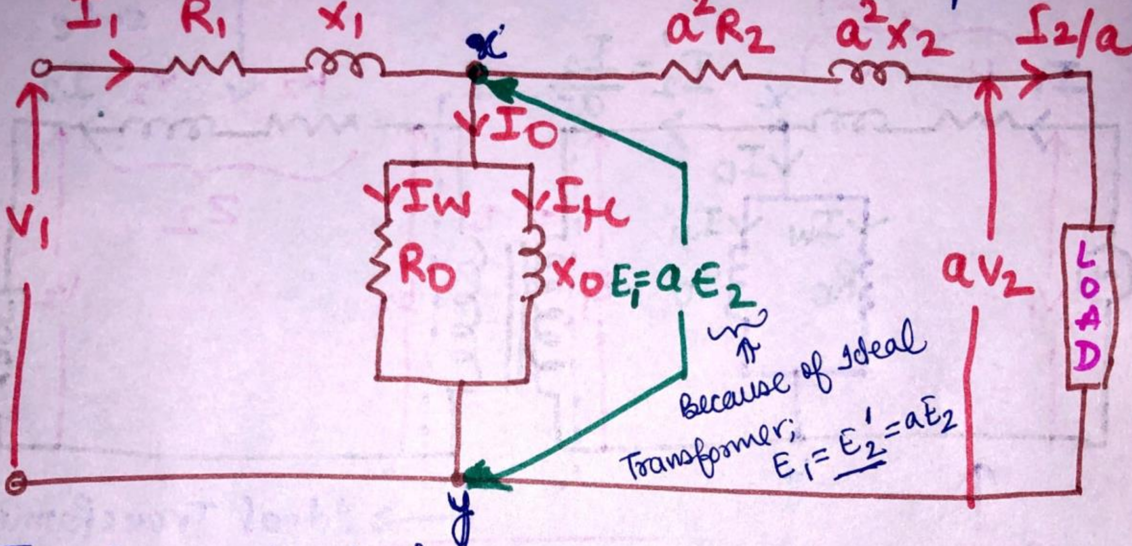
Now let us simplify the above circuit into a single circuit.

Firstly, let us find out the equivalent circuit of the transformer on-load referred to primary. This means that we will consider the values of secondary parameters (i.e R_2, x_2, I_2, E_2, V_2 and Z_L) which are referred to primary.

We know that the values of secondary parameters referred to primary are: $R_2' = a^2 R_2$; $x_2' = a^2 x_2$; $Z_L' = a^2 Z_L$

$I_2' = \frac{I_2}{a}$; $E_2' = a E_2$ and $V_2' = a V_2$.

Now we will merge the secondary side of the transformer into the primary side of the transformer by taking all the referred values into consideration. So; let us start the process:



The above circuit is the Equivalent Circuit of the transformer (on-load) referred to primary. You can Notice in the circuit that the values of secondary parameters referred to primary have been taken into consideration.

Now the above circuit can be simplified further by shifting the x-y branch (i.e the branch representing the core parameters) ahead of the primary impedance (i.e $Z_1 = R_1 + jx_1$).

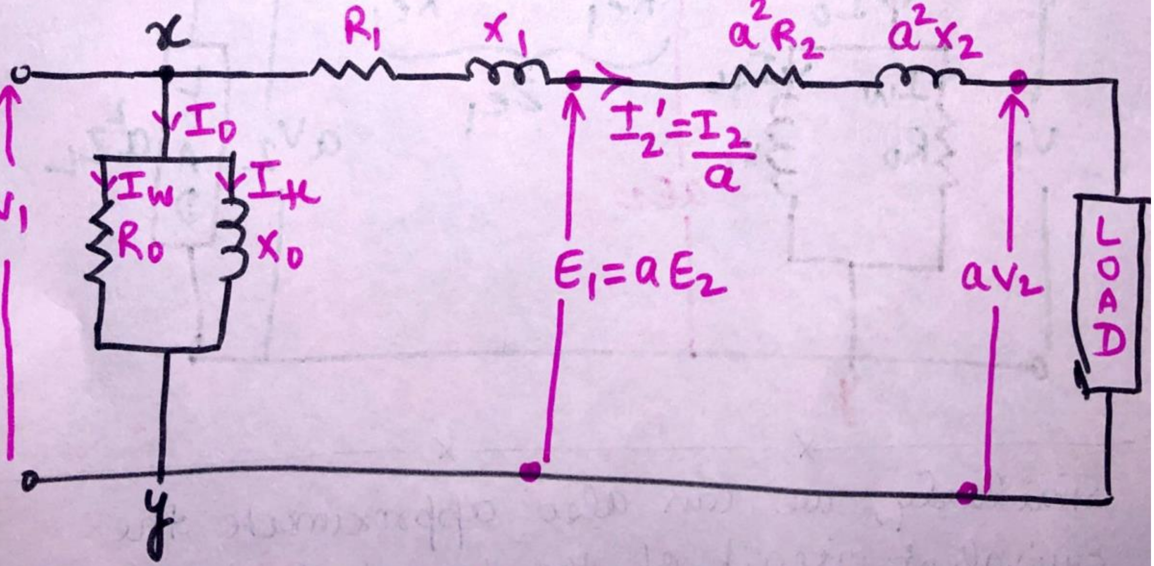
The reason behind this is the no-load current (I_0) which is (2-5)% of the rated primary current. Hence; when the transformer is ON-load

The the rated current flows in the primary winding. That is the value of I_1 (rated primary current) is very large as compared to $\underline{\underline{I_0}}$ (i.e the No-load current). Now If we apply KCL on node "x" as shown in the previous circuit, we get $\boxed{I_1 = I_0 + I_2'}$ Now; $I_2' >> I_0$;

or we can also understand it as: $\boxed{I_2' = I_1 - I_0}$

$I_0 << I_1$; hence $I_1 - I_0 \approx I_1$; so; $\boxed{I_2' = I_1}$

So; It makes no difference if the x-y branch is kept before R_1 & X_1 or after R_1 & X_1. Hence, the simplified circuit is:



Now, we can see that in the simplified circuit as shown above all the resistances and reactances have come in series. So; we can replace these with the respective effective values.

The effect resistance is, $\boxed{R_{e_1} = R_1 + a^2 R_2}$.

Similarly; the effective reactance is;

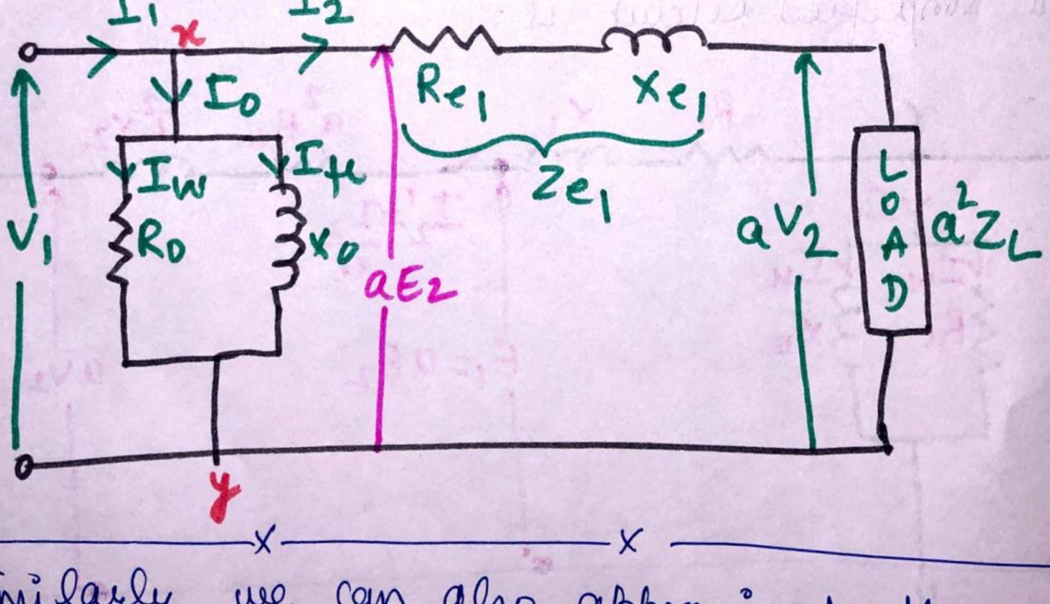
$$\boxed{X_{e_1} = X_1 + a^2 X_2}$$

and the effective impedance is $\boxed{Z_{e_1} = R_{e_1} + jX_{e_1}}$

The primary current now be calculated as:

$$I_1 = \frac{V_1}{Z_{e_1} + a^2 Z_L}$$

The simplified circuit of transformer on load referred to primary is shown below:



Similarly, we can also approximate the Equivalent circuit of the practical transformer on-load by taking the parameters of primary side of the transformer referred to secondary. i.e we can merge primary side to the secondary side.

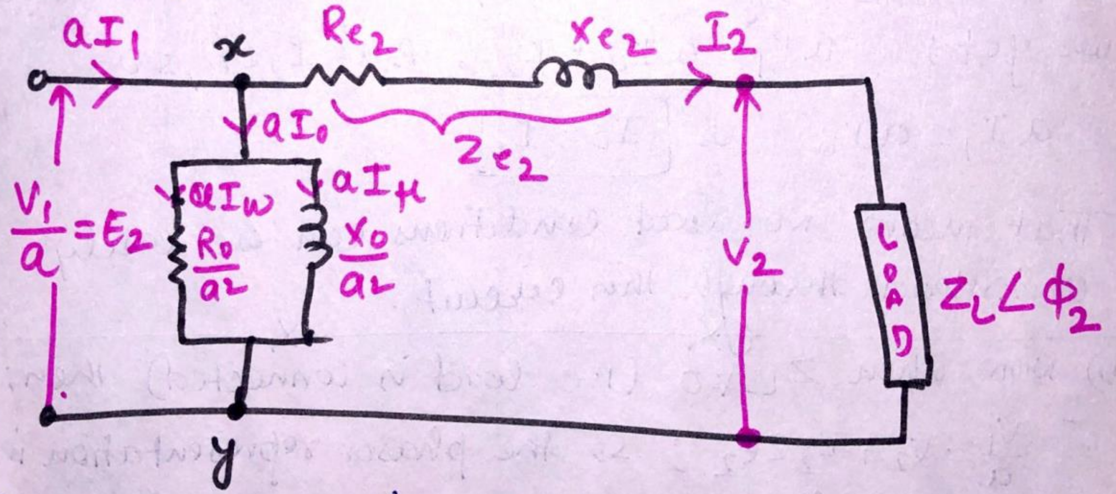
We know that the parameters of primary side of the transformer referred to secondary are :

$R_1' = \frac{R_1}{a^2}$; $X_1' = \frac{X_1}{a^2}$; $X_0' = \frac{X_0}{a^2}$; $R_0' = \frac{R_0}{a^2}$

$V_1' = \frac{V_1}{a}$; $E_1' = \frac{E_1}{a}$;

$I_1' = aI_1$; $I_0' = aI_0$; $I_w' = aI_w$ and $I_\mu' = aI_\mu$

So; In the same manner ; on transferring ~~the~~ all the primary parameters to the secondary ; the approximate equivalent circuit referred to the secondary is shown below :



From the above circuit :

$$\boxed{Re_2 = R_2 + R_1' = R_2 + \frac{R_1}{a^2}}$$

$$\boxed{Xe_2 = X_2 + X_1' = X_2 + \frac{X_1}{a^2}}$$

$$\boxed{Ze_2 = Re_2 + jXe_2}$$

$$\boxed{I_2 = \frac{V_2}{Z_L} = \frac{E_2}{Ze_2 + Z_L}}$$

$$\boxed{E_2 = \frac{V_1}{a} = V_2 + I_2 Ze_2}$$