Basic Electrical Engineering (TEE 101)

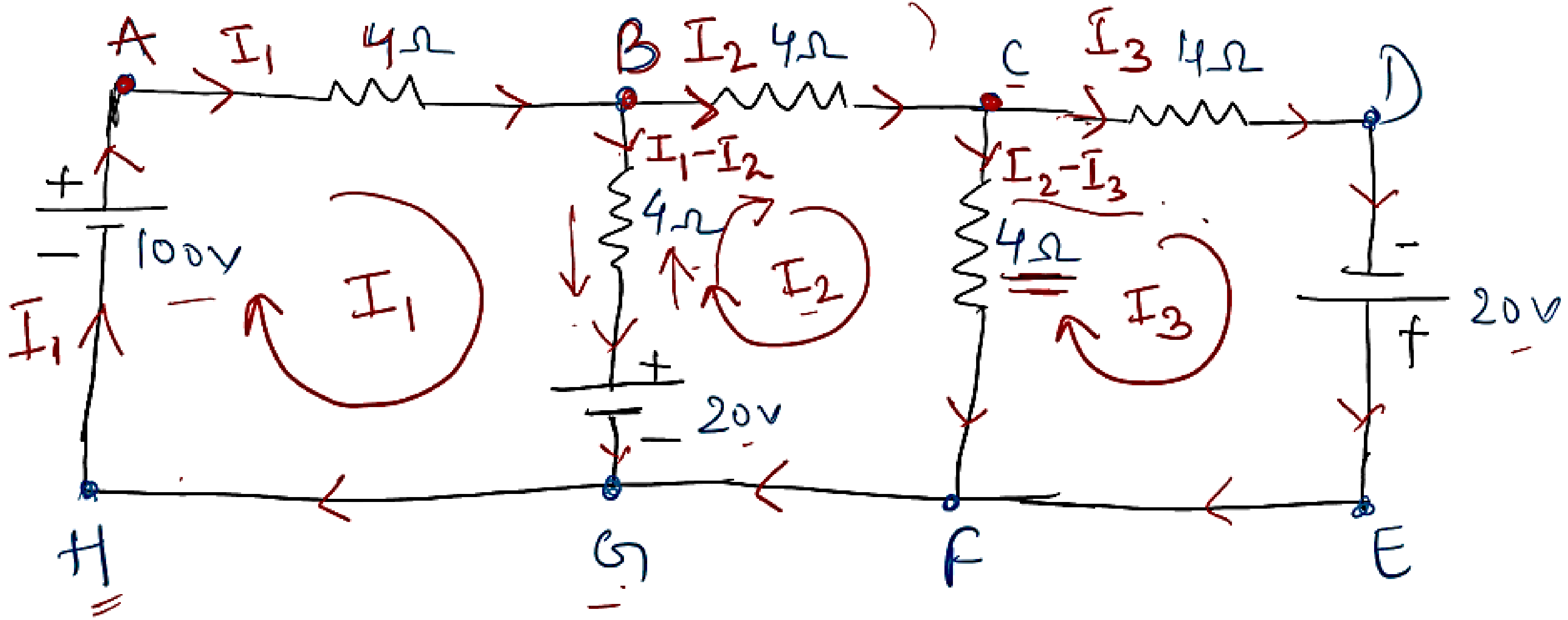
Lecture 11 (a): Numerical Practice

Content

This lecture covers Numerical on:

- Mesh Analysis
- Star – Delta Transformation

Determine the current in each mesh of the given electric circuit



KVL eqn of mesh ABGHA

$$+100 - 4I_1 - 4(I_1 - I_2) - 20 = 0$$

$$100 - 4I_1 - 4I_1 + 4I_2 - 20 = 0$$

$$(-8I_1 + 4I_2 + 80 = 0)/4$$

$$-2I_1 + I_2 = -20$$

or, $2I_1 - I_2 = 20$ —— (1)

KVL eq^n of mesh BCFGB

$$-4I_2 - 4(I_2 - I_3) + 20 + 4(I_1 - I_2) = 0$$

$$-4I_2 - 4I_2 + 4I_3 + 20 + 4I_1 - 4I_2 = 0$$

$$(4I_1 - 12I_2 + 4I_3 = -20)/4$$

$$\boxed{I_1 - 3I_2 + I_3 = -5} \text{ --- } (2)$$

Similarly, we can apply the KVL in last mesh

So, the KVL eq^n of mesh CDEFC

$$-4I_3 + 20 + 4(I_2 - I_3) = 0$$

$$-4I_3 + 20 + 4I_2 - 4I_3 = 0$$

or $(4I_2 - 8I_3 = -20)/4$

or, $I_2 - 2I_3 = -5$

or, $\boxed{0I_1 + I_2 - 2I_3 = -5}$ — (3)

$$\left[\begin{array}{l} 2I_1 - I_2 + 0I_3 = 20 \quad \text{— (1)} \\ I_1 - 3I_2 + I_3 = -5 \quad \text{— (2)} \\ 0I_1 + I_2 - 2I_3 = -5 \quad \text{— (3)} \end{array}\right]$$

The matrix representation of (1), (2) & (3) is

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ -5 \\ -5 \end{bmatrix} \rightarrow (4)$$

Cramer's rule can be used to determine the value of I_1, I_2 and I_3.

where;

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

Using Cramer's Rule, $\rightarrow I_1$, I_2 and I_3 are given as:

$$I_1 = \frac{\det\begin{bmatrix} 20 & -1 & 0 \\ -5 & -3 & 1 \\ -5 & 1 & -2 \end{bmatrix}}{\det(A)} \qquad \text{—— } 5$$

$$I_2 = \frac{\det\begin{bmatrix} 2 & 20 & 0 \\ 1 & -5 & 1 \\ 0 & -5 & -2 \end{bmatrix}}{\det(A)} \qquad \text{—— } 6$$

$$I_3 = \frac{\det\begin{bmatrix} 2 & -1 & 20 \\ 1 & -3 & -5 \\ 0 & 1 & -5 \end{bmatrix}}{\det(A)} \qquad \text{—— } 7$$

Now, let us find out $|A|$ (i.e $\det(A)$) because it is required in eq^n 5, 6 and 7 to calculate the value of I_1, I_2 and I_3 resp.

$$|A| = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$|A| = 2\begin{bmatrix} -3 & 1 \\ 1 & -2 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} + 0\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

$$|A| = 2[6-1] + [-2-0] + 0$$

$$|A| = 2 \times 5 - 2 = 10 - 2 = 8$$

$$\boxed{|A| = 8} \qquad \text{—— } 8$$

$$I_1 = \frac{20\begin{bmatrix}-3 & 1\\ 1 & -2\end{bmatrix} - (-1)\begin{bmatrix}-5 & 1\\ -5 & -2\end{bmatrix} + 0\begin{bmatrix}-5 & 3\\ -5 & 1\end{bmatrix}}{8} = \frac{20(6-1)+(10+5)+0}{8}$$

$$I_1 = \frac{20\times 5+15}{8} = \boxed{\frac{115}{8}\,A}$$

$$I_2 = \frac{2\begin{bmatrix}-5 & 1\\ -5 & -2\end{bmatrix} - 20\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix} + 0\begin{bmatrix}1 & -5\\ 0 & -5\end{bmatrix}}{8} = \frac{2\,[10+5] - 20(-2-0)+0}{8}$$

$$I_2 = \frac{2(15)-20(-2)}{8} = \frac{30+40}{8} = \boxed{\frac{70}{8}\,A}$$

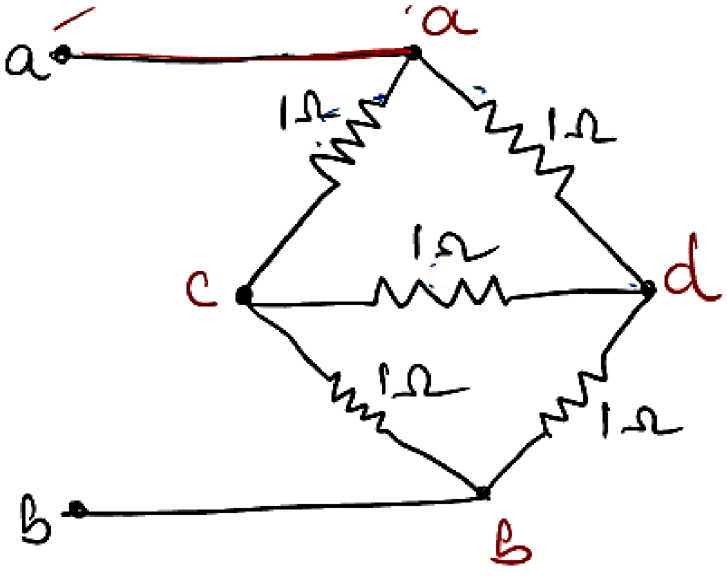
$$I_3 = \frac{2\begin{bmatrix}-3 & -5\\ 1 & -5\end{bmatrix} - (-1)\begin{bmatrix}1 & -5\\ 0 & -5\end{bmatrix} + 20\begin{bmatrix}1 & -3\\ 0 & 1\end{bmatrix}}{8} = \frac{2(15+5)+(-5+0)+20(1+0)}{8}$$

$$I_3 = \frac{2(20)+(-5)+20}{8} = \boxed{\frac{55}{8}}\,A$$

Hence, mesh currents are:

$I_1 = \frac{115}{8}\,A$, $I_2 = \frac{70}{8}\,A$ and $I_3 = \frac{55}{8}\,A$

Que :→ Determine the equivalent resistance across a–b in the circuit shown below :



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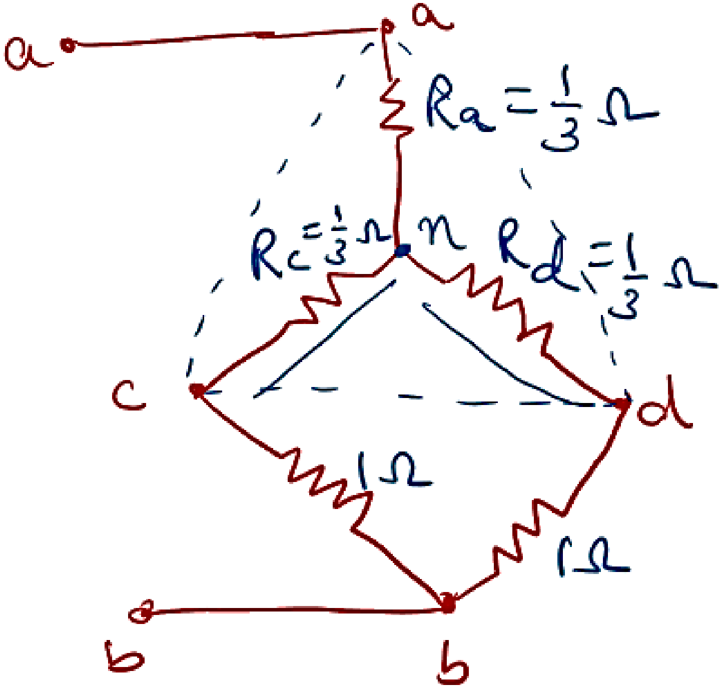


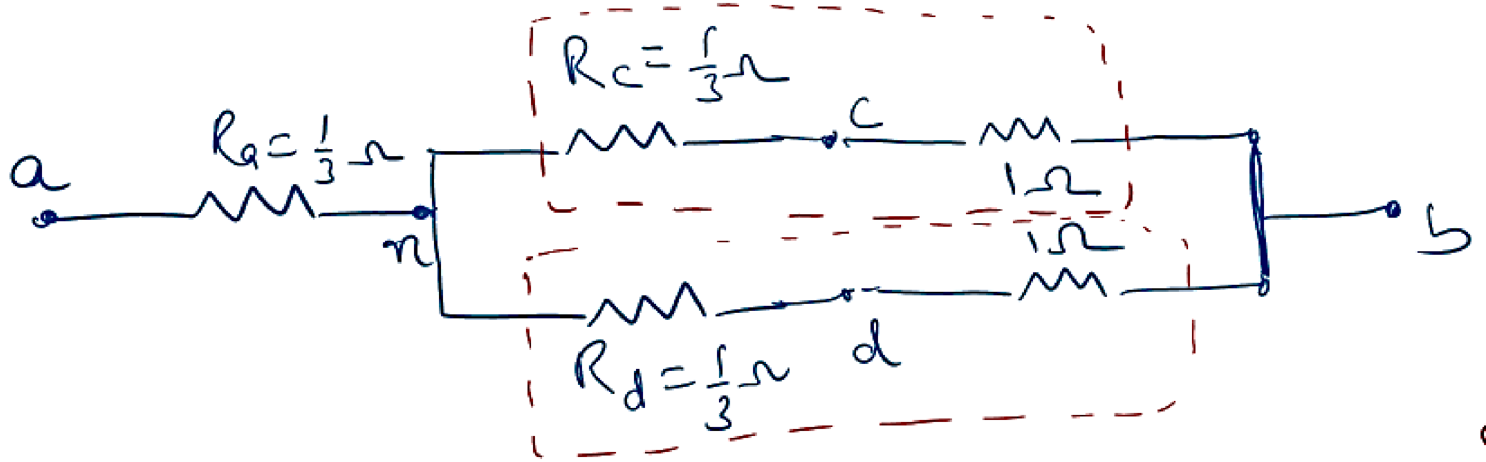
Solution

$\Delta \rightarrow Y$

$$R_a = \frac{1 \times 1}{1+1+1} = \frac{1}{3}\Omega$$

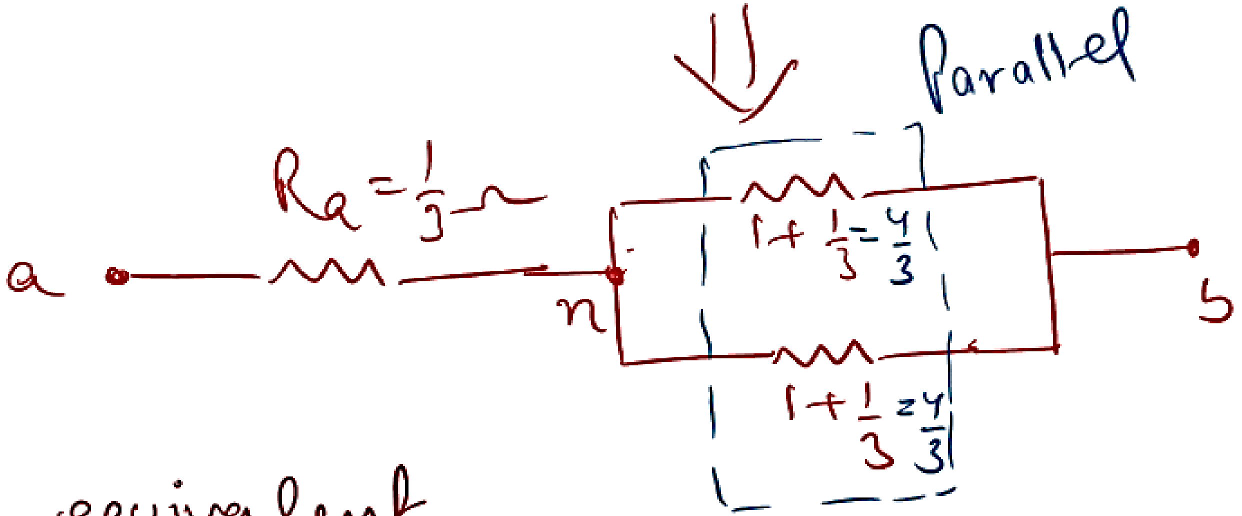
$$R_c = \frac{1}{3}\Omega$$

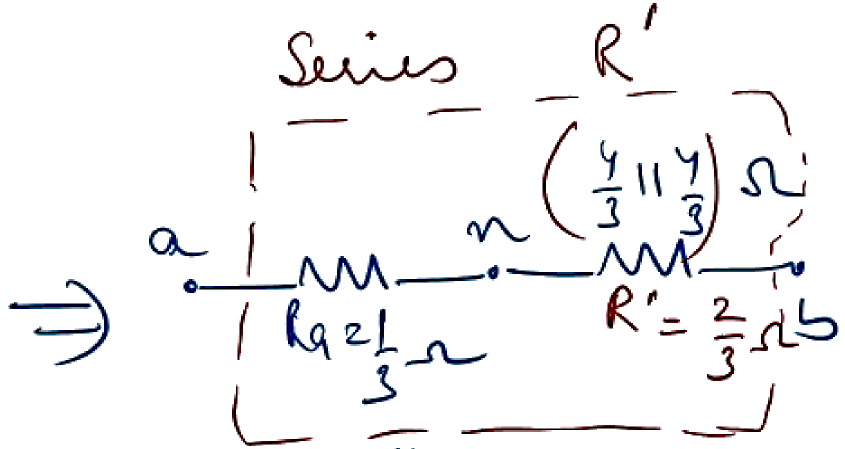
$$R_d = \frac{1 \times 1}{1+1+1} = \frac{1}{3}\Omega$$

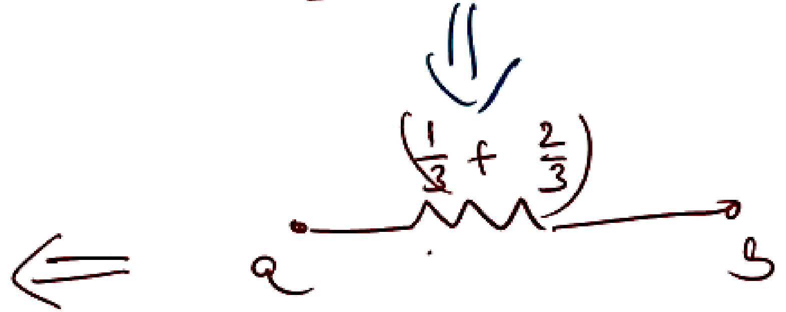


$R' = \frac{4}{3} \,||\, \frac{4}{3}$

$R' = \frac{2}{3}\Omega$







Hence equivalent resistance b/w a-b is 1Ω.

Thank You