INTERFERENCE OF LIGHT

The re-distribution of light intensity due to the superposition of two light waves is called the interference of light.

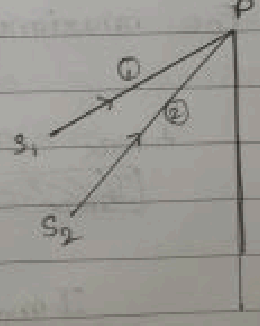
At some points the intensity of these parts are maximum and these points ~~are known as~~ of interference is known as instructive interfence and at some parts the intensity is minimum and its interference is known as destructive interference

Two types of interface:
- Interference produced by division of wavelength
- Interference produced by division of amplitude

Coherent sources - Sources which have same frequency, wavelength and amplitude which remains constant with time

Methods to produce coherent sources
- Llod's single mirror method
- Fresnel's double mirror method
- Fresnel's biprism experiment
- ~~Michael~~ Michelson interferometer experiment
- Young's double slit experiment

Phase difference and path difference



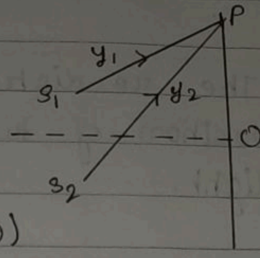
Path difference $(\Delta P) = S_2P - S_1P$

Phase difference $= \frac{2\pi}{\lambda}(\Delta P)$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}(S_2P - S_1P)}$$

Resultant intensity due to superposition of two interfering waves

$$\Delta\phi = \frac{2\pi}{\lambda}(S_2P - S_1P)$$



$$y_1 = a_1 \sin\omega t$$
$$y_2 = a_2 \sin(\omega t + \Delta\phi)$$
$$y = y_1 + y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \Delta\phi)$$
$$= a_1 \sin\omega t + a_2 [\sin\omega t \cos\Delta\phi + \cos\omega t \sin\Delta\phi]$$
$$y = a_1 \sin\omega t + a_2 \sin\omega t \cos\Delta\phi + a_2 \cos\omega t \sin\Delta\phi$$
$$= \sin\omega t[a_1 + a_2\cos\Delta\phi] + a_2 \cos\omega t \sin\Delta\phi$$

let $a_1 + a_2\cos\Delta\phi = A\cos\delta$

$$a_2 \sin\Delta\phi = A\sin\delta$$
$$y = \sin\omega t[A\cos\delta] + \cos\omega t[A\sin\delta]$$
$$= A[\sin\omega t\cos\delta + \cos\omega t\sin\delta]$$
$$\boxed{y = A\sin(\omega t + \delta)}$$

A = Resultant intensity
δ = Phase difference

Determine the value of resultant amplitude (A)

$$a_1 + a_2\cos\Delta\phi = A\cos\delta$$
$$a_2\sin\Delta\phi = A\sin\delta$$
$$(a_1 + a_2\cos\Delta\phi)^2 + (a_2\sin\Delta\phi)^2 = A^2 \cos^2\delta + A^2\sin^2\delta$$
$$a_1^2 + a_2^2\cos^2\Delta\phi + 2a_1a_2\cos\Delta\phi + a_2^2\sin^2\Delta\phi = A^2(1)$$
$$A^2 = a_1^2 + a_2^2(\cos^2\Delta\phi + \sin^2\Delta\phi) + 2a_1a_2\cos\Delta\phi$$
$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi$$

$$\boxed{I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi}$$

For maximum intensity, $\cos\Delta\phi = +1$

$\Delta\phi = 2n\pi$ where $n = 0, 1, 2, 3, \dots$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$

$$\boxed{I_{max} = (a_1 + a_2)^2}$$

$a_1 = a_2 = a$

$$I_{max} = (2a)^2 = 4a^2$$

For minimum intensity, $\cos\Delta\phi = -1$

$$\Delta\phi = (2m+1)\pi \qquad \text{where } m = 0, 1, 2, 3, \ldots$$

$$\boxed{I_{mini} = (a_1 - a^2)}$$

$$a_1 = a_2 = a$$

$$I_{min} = 0$$

Average intensity

$$I_{av} = \frac{\int_0^{2\pi} I \, d(\Delta\phi)}{\int_0^{2\pi} d(\Delta\phi)}$$

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi$$

$$I = \frac{\int_0^{2\pi}(a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi)\,d\Delta\phi}{\int_0^{2\pi} d(\Delta\phi)}$$

$$= \frac{\int_0^{2\pi} a_1^2\,d(\Delta\phi) + \int_0^{2\pi} a_2^2\,d(\Delta\phi) + \int_0^{2\pi} 2a_1a_2\cos(\Delta\phi)\,d(\Delta\phi)}{[\Delta\phi]_0^{2\pi}}$$

$$= \frac{a_1^2(\Delta\phi)_0^{2\pi} + a_2^2(\Delta\phi)_0^{2\pi} + 2a_1a_2(\sin\Delta\phi)_0^{2\pi}}{2\pi - 0}$$

$$= \frac{a_1^2(2\pi - 0) + a_2^2(2\pi - 0) + 2a_1a_2(\sin 2\pi - \sin 0)}{2\pi}$$

$$= \frac{a_1^2 \cdot 2\pi + a_2^2 \cdot 2\pi + 2a_1a_2 \cdot 0}{2\pi}$$

$$I_{av} = \frac{(a_1^2 + a_2^2)\cancel{2\pi}}{\cancel{2\pi}} = a_1^2 + a_2^2$$

$$\boxed{I_{av} = a_1^2 + a_2^2}$$

if $a_1 = a_2 = a$

$$I_{av} = (a)^2 + (a)^2$$

$$= 2a^2$$

Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes. Deduce the ratio of maximum to minimum intensity of the fringe system.

$$\frac{I_1}{I_2} = \frac{81}{1}$$

$$\frac{a_1^2}{a_2^2} = \frac{81}{1} = \frac{9^2}{1^2} \Rightarrow \frac{a_1}{a_2} = \frac{9}{1}$$

$$a_1 = 9a_2$$

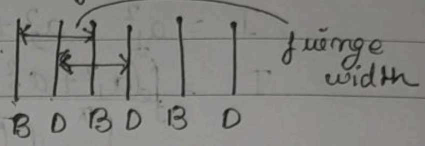
$$\frac{I_{max}}{I_{mini}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$$

$$= \frac{(9a_2+a_2)^2}{(9a_2-a_2)^2} = \frac{(10a_2)^2}{(8a_2)^2} = \frac{100a_2^2}{64a_2^2}$$

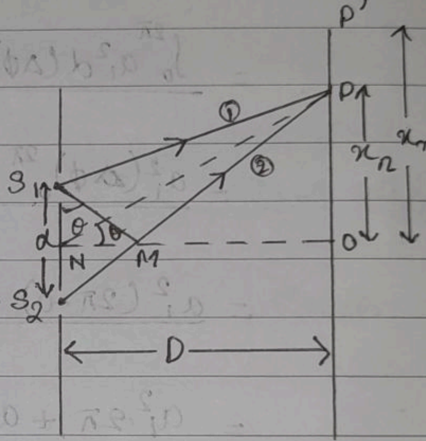
$$= \frac{100}{64} = \frac{25}{16}$$

$$I_{max} : I_{mini} = 25:16$$

There are two types of fringes formed:
i) Bright fringe
ii) Dark fringe



Fringe width



P (Bright or dark) n^{th} no. of fringe

x_n – n^{th} fringe

Δx = Path difference between ① and ②

$$S_2M = \Delta x$$

$$\sin\theta = \frac{S_2M}{S_1S_2} = \frac{\Delta x}{d} \quad \text{--- (i)}$$

$$\Delta NPO \;\tan\theta = \frac{PO}{NO} = \frac{x_n}{D} \quad \text{--- (ii)}$$

$\tan\theta \approx \sin\theta$ $\quad$ ($\because$ θ is really small)

$$\frac{\Delta x}{d} = \frac{x_n}{D}$$

$$x_n = \frac{\Delta x \cdot D}{d}$$

for constructive interference, $\Delta x = n\lambda$

$$x_n = \frac{n\lambda \cdot D}{d}$$

$$x_{n+1} = \frac{(n+1)\lambda D}{d}$$

$w = x_{n+1} - x_n$ $\quad$ (w = fringe width)

$$w = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d}$$

$$= \frac{(n+1)D\lambda - nD\lambda}{d} = \frac{nD\lambda + D\lambda - nD\lambda}{d} = \frac{D\lambda}{d}$$

$$\boxed{\text{Fringe width for bright fringe, } w = \frac{D\lambda}{d}}$$

For destructive fringe,

$$\Delta x = (2n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2, 3, \dots$$

$$x_n = \frac{\Delta x \cdot D}{d} = \frac{(2n+1)\lambda D}{2d}$$

$$x_{n+1} = \frac{(2n+2+1)\lambda D}{2d} = \frac{(2n+3)\lambda D}{2d}$$

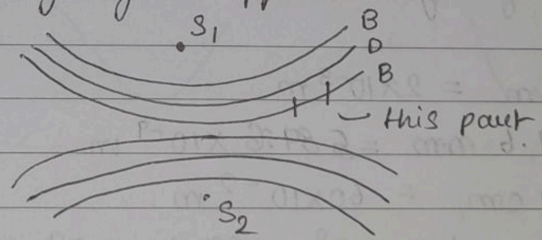
$$w = x_{n+1} - x_n$$

$$= \frac{(2n+3)\lambda D}{2d} - \frac{(2n+1)\lambda D}{2d}$$

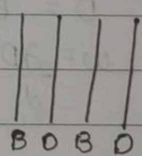
$$= \frac{\cancel{2}\lambda D}{\cancel{2}d} = \frac{\lambda D}{d}$$

$$\boxed{\text{Fringe width for dark fringe, } w = \frac{\lambda D}{d}}$$

In practical, fringes appear as



But what we see is



Due to large eccentricity of the hyperbola we see only the linear part in our experiment.

Conditions for interference of light and for sustained interference

i) the two interfering waves should be coherent

ii) The two waves should have the same frequency.

iii) If the waves are polarised, they have to be in the same state of polarisation

iv) The distance between the two light sources should be as small as possible as $\omega = \frac{D\lambda}{d}$
if d is small the ω becomes large.

v) The distace of the screen and two sources 'D' should be large, as $\omega = \frac{D\lambda}{d}$ if D is large, ω is large

vi) The amplitudes of Interfering waves should be equal or at least nearly equal, as we know that maximum intensity is $(a_1+a_2)^2$ if the amplitude of both the waves are same then it becomes $4a^2$ and for minimum it becomes 0.

No interference is possible by independent source as there is no steady ~~fa~~ phase difference between the light waves ~~and~~ emitted from them.

1. Two coherent sources are 2mm apart and are illuminated with a monochromatic light of wavelength 589.6 nm. Fringes are observed at 60 cm from the sources. Find the fringe width. Comment on the shape of fringes.

$d = 2mm = 2\times10^{-3}$ m

$\lambda = 589.6$ nm $= 589.6\times10^{-9}$ m

$D = 60$ cm $= 60\times10^{-2}$ m.

$$\omega = \frac{\lambda D}{d} = \frac{60\times10^{-2}\times589.6\times10^{-9}}{2\times10^{-3}}$$

$$= \frac{60\times589.6\times10^{-11}}{2\times10^{-3}}$$

$$= 1.76\times10^{-4} \text{ m}$$

2. In a two slit interference pattern with $\lambda = 600$ nm, the zero order and tenth order maxima fall at 12.34 mm and 14.73 mm respectively. If λ is to 500 nm, deduce

the positions of the zero order and twentieth fringes, other arrangements remaining the same.

$\lambda = 600\ nm = 600 \times 10^{-9}\ m$

width of the fringes $= 14.73 - 12.32$
$= 2.39\ mm$

14.73 — 10th order
10 fringes
12.32 — zero order

width of one fringe $= \frac{2.39}{10} = 0.239\ mm$

$$w = \frac{\lambda D}{d}$$

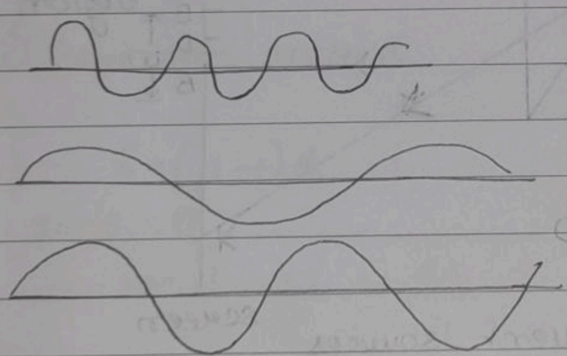
$$w \propto \lambda$$

$$\frac{w_1}{w_2} = \frac{\lambda_1}{\lambda_2} \qquad \frac{0.239}{w_2} = \frac{\cancel{600}}{500} = \frac{6}{5}$$

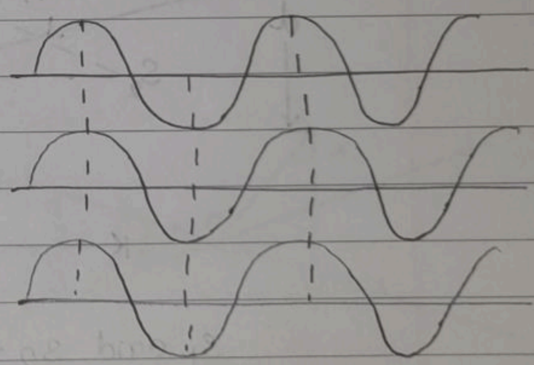
$$w_2 = \frac{5}{6} \times 0.239 = 0.199\ mm$$

Position of zero order $= 12.32 + (0.199 \times 20)$
$= 16.32\ mm$

Ordinary rays



Coherent rays



same phase difference.

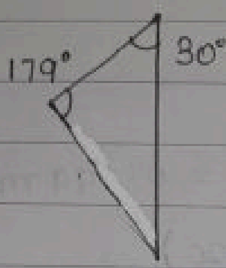
Two types of coherence:
- Temporal coherence
- Spatial Coherance

Coherence – It is a measure of the correlation between the phases measured at different points on a wave.

Temporal coherence: If the phase difference of waves crossing the two ~~[illegible]~~ points lying along the direction of propagation of the bem is time dependent then a beam of light is said to posses temporal or time. It is also known longidudinal coherance.

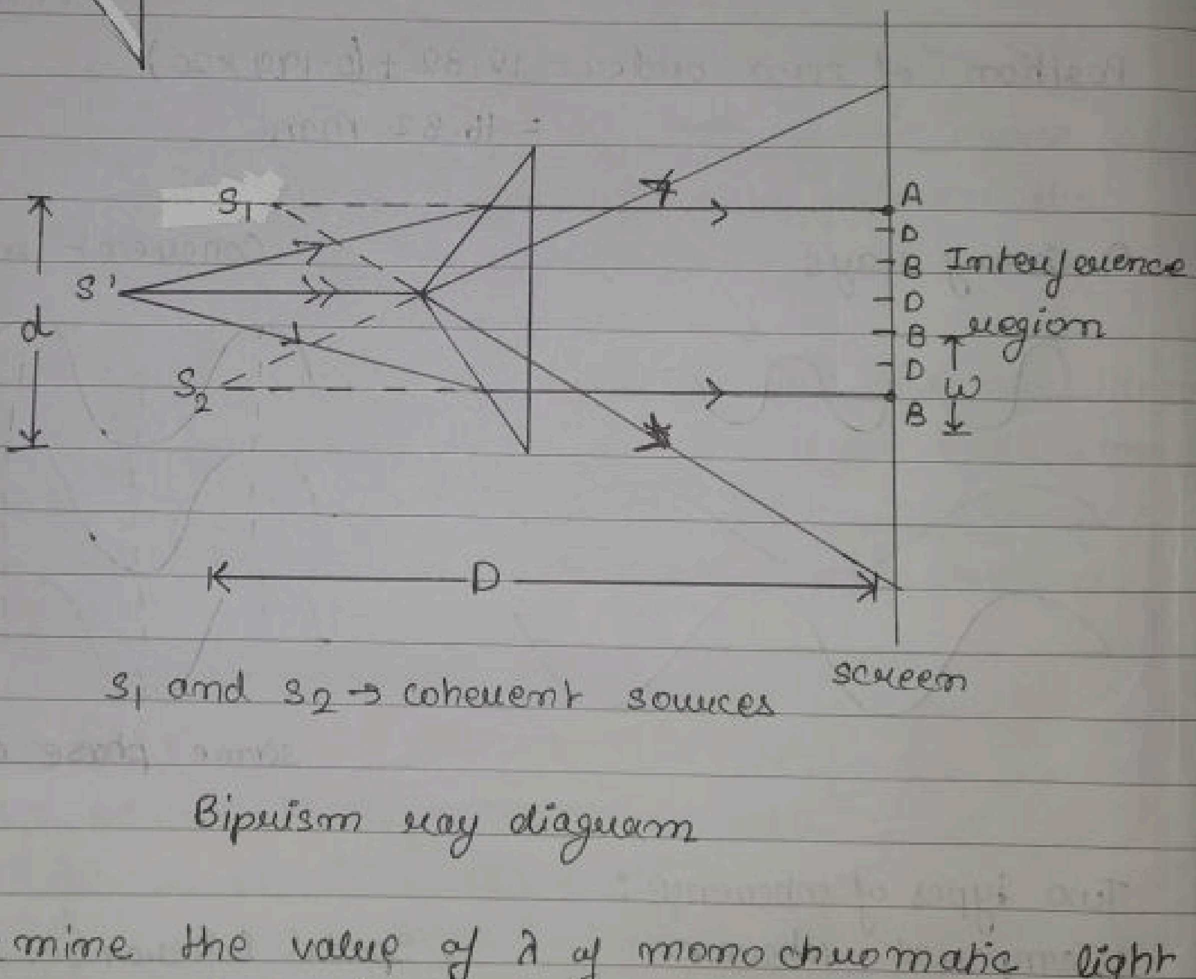
Spatial coherence – If the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of the propagation of the ~~beam~~ beam ~~beam~~ is time dependant. Also known as lateral and transverse coherence.

Freshnel's Biprism



biprism

we see the fringes by eyepeice



S_1 and S_2 → coherent sources

Biprism ray diagram

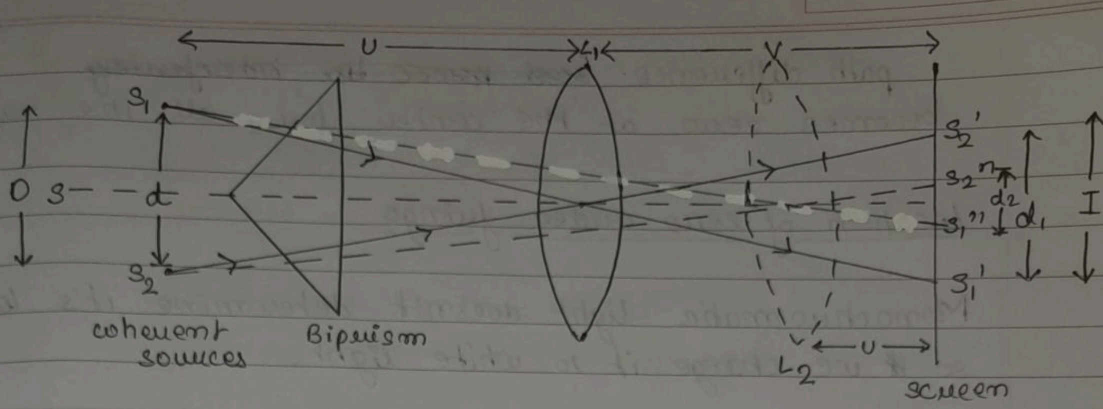
Determine the value of λ of monochromatic light

$$\lambda = \frac{wd}{D}$$

Determination of d

i) Displacement method (if $D > 4f$)

S is the monochromatic light (yellow light)



$$m = \frac{I}{O} = \frac{v}{u}$$

$$\frac{d_1}{d} = \frac{v}{u} \quad \text{—— (i)}$$

$$\frac{d_2}{d} = \frac{u}{v} \quad \text{—— (ii)}$$

$$\frac{d_1}{d} \times \frac{d_2}{d} = \frac{\cancel{v}}{\cancel{u}} \times \frac{\cancel{u}}{\cancel{v}}$$

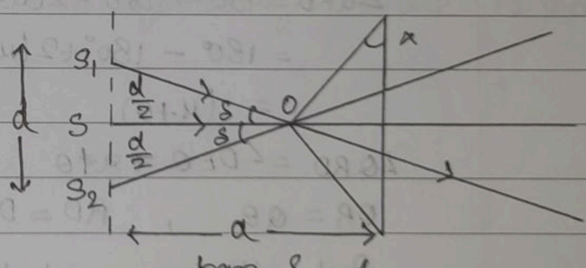
$$\frac{d_1 d_2}{d^2} = 1 \qquad d^2 = d_1 d_2 \qquad \boxed{d = \sqrt{d_1 d_2}}$$

ii) Deviation method

$\delta = (\mu - 1)\alpha.$

In $S_1 S O$

$\tan \delta = \frac{S_1 S}{SO} = \frac{d/2}{a}$



$\tan \delta = \frac{d}{2a}$

If δ is ~~been~~ very small,

$\tan \delta \simeq \delta$

$\delta = \frac{d}{2a}$ $\qquad d = 2\delta a$

$$\boxed{d = 2(\mu - 1)\alpha a}$$

Freshnel's biprism experiment

We use displacement method.

If we replace the monochromatic light by white light then the interference will consiste of colour fringes.

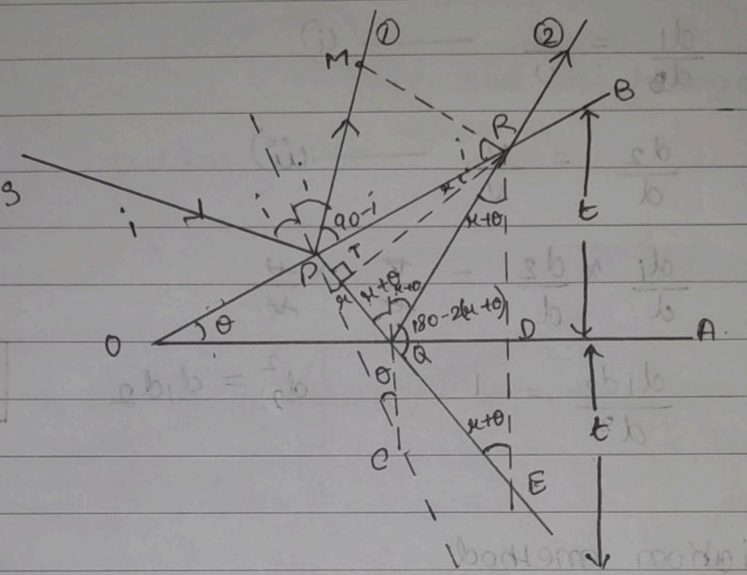
∴ path difference ~~and hence the interfering~~ becomes zero at the center for all the rays

Location of zero order fringe

Monochromatic light doesn't determine it's location so if we change it to white light.

Interference due to wedge shaped thin film

RM ⊥ to R has ray 1



RD = DE = t (thickness of film)

$$\angle QRD = 180° - (180° - 2(r+\theta)) - (r+\theta)$$
$$= 180° - 180° + 2(r+\theta) - (r+\theta)$$
$$= (r+\theta)$$

∴ $\angle QRD = \angle DEQ = r+\theta$

$QR = QE$, $RD = DE$

Path difference between reflected rays ① and ②

$$\Delta P = PQ + QR - PM$$
$$= \mu(PQ + QR) - PM \cdot 1$$
$$= \mu(PQ + QR) - PM$$
$$= \mu(PT + TQ + QR) - PM$$

In $\triangle PTR$,

$$\sin r = \frac{PT}{PR}$$

$$\sin i = \frac{PM}{PR}$$

rays

location

Page:
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By Snell's law,

$$\mu = \frac{\sin i}{\sin r} = \frac{PM/PR}{PT/PR}$$

$$\mu = \frac{PM}{PT} , \quad PM = \mu PT$$

$$\Delta P = \mu(PT + TQ + QR) - \mu PT$$

$$= \mu PT + \mu TQ + \mu TR - \mu PT$$

$$= \mu TQ + \mu TR$$

$$= \mu(TQ + TR)$$

$$\Delta P = \mu (TE)$$

In ΔETR

$$\cos(r+\theta) = \frac{TE}{RE}$$

$$TE = RE\cos(r+\theta)$$

$$TE = RE\cos(r+\theta) = (RD+DE)\cos(r+\theta)$$

$$= 2t\cos(r+\theta)$$

$$\Delta P = \mu(2t\cos(r+\theta))$$

$$\boxed{\Delta P = 2\mu t\cos(r+\theta)}$$

In incident and refracted ray there is a phase difference of $\frac{\lambda}{2}$ and in incident ray and reflected ray there is no phase difference.

Path difference:

$$\Delta P = 2\mu t\cos(r+\theta) + \frac{\lambda}{2}$$

Constructive interference or maximum intensity.

$$\Delta P = n\lambda$$

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

$$\boxed{2\mu t\cos(r+\theta) = n\lambda - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}} \quad n = 1, 2, 3, \ldots$$

Destructive interference or minimum intensity

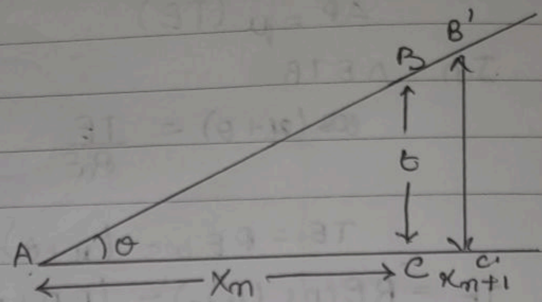
$$\Delta P = (2n+1)\frac{\lambda}{2}$$

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = n\lambda + \frac{\lambda}{2}$$

$$2\mu t\cos(r+\theta) = n\lambda \quad \text{where } n=4$$

In fringe width,



In ΔABC,

$$\tan\theta = \frac{BC}{AC} = \frac{t}{x_n}$$

$$t = x_n \tan\theta$$

$$2\mu t\cos(r+\theta) = n\lambda$$

$$2\mu(x_n\tan\theta)\cos(r+\theta) = n\lambda$$

$$x_n = \frac{n\lambda}{2\mu\tan\theta\cos(r+\theta)}$$

$$x_{n+1} = \frac{(n+1)\lambda}{2\mu\tan\theta\cos(r+\theta)}$$

$$w = x_{n+1} - x_n = \text{Fringe width}$$

$$= \frac{(n+1)\lambda}{2\mu\tan\theta\cos(r+\theta)} - \frac{n\lambda}{2\mu\tan\theta\cos(r+\theta)}$$

$$= \frac{(n+1)\lambda - n\lambda}{2\mu\tan\theta\cos(r+\theta)}$$

$$\boxed{w = \frac{\lambda}{2\mu\tan\theta\cos(r+\theta)}}$$

(t is very small)
(θ becomes small)

for normal incidence,

$r = 0$ $\qquad \cos(r+\theta) \approx 1$

$\tan\theta \approx \theta$

$$w = \frac{\lambda}{2\mu(\theta)(1)} = \frac{\lambda}{2\mu\theta}$$

$$\boxed{w = \frac{\lambda}{2\mu\theta}}$$

1. A parallel beam of light of wavelength 5890 Å is incident on a thin glass plate of refractive index $\mu = 1.5$ such that the angle of refraction is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.

$\mu = 1.5$

$\lambda = 5890 \times 10^{-10}$ m

$r = 60°$

$n = 1$

$$2\mu t \cos r = n\lambda$$

$$2(1.5)t \cos 60° = 1 \times 5890 \times 10^{-10}$$

$$3t \times \frac{1}{2} = 5890 \times 10^{-10}$$

$$t = \frac{5890 \times 10^{-10}}{3} \times 2$$

$$= 3.926 \times 10^{-7} \text{ m}$$

2. Soap film of refractive index 1.33 is illuminated with light of different wavelength at an angle of 45°. There is complete destructive inference for $\lambda = 5890$ Å. Find the thickness of the film.

$\mu = 1.33$

$i = 45°$

$\lambda = 5890 \times 10^{-10}$ m

$$\mu = \frac{\sin i}{\sin r}$$

$$1.33 = \frac{\sin 45°}{\sin r}$$

$$\sin r = \frac{\sin 45°}{1.33} = \frac{1}{\sqrt{2} \times 1.33}$$

$$= \frac{1}{1.414 \times 1.33}$$

$$\sin r = 0.5317$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.5317)^2} = 0.8469$$

$$2\mu t \cos r = n\lambda$$

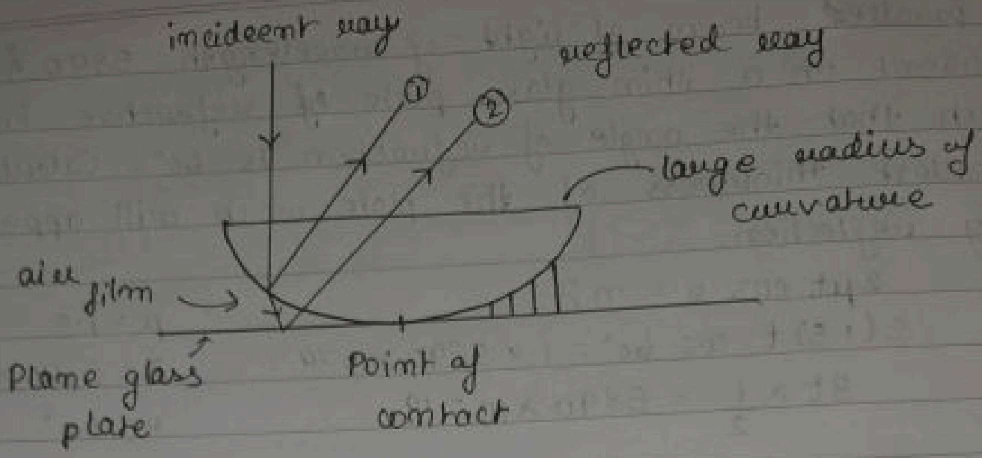
$$2(1.33)t \times 0.8469 = 1 \times 5890 \times 10^{-10}$$

$$t = \frac{5890 \times 10^{-10}}{2 \times 1.33 \times 0.8469}$$

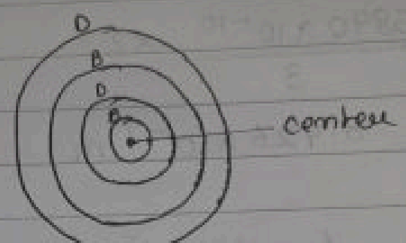
$$\boxed{= 2.614 \times 10^{-7} \text{ m}}$$

<u>Newtons Rings formation</u>

Newton rings are formed as a result of light rays



we can see this by the help of low power microscope



Newton's rings

reflected from the upper and the lower surfaces. They are concentric and circular because of the points equal to the thickness of film lie on the circles.

Path difference between two reflected ray

$$\Delta P = \cancel{2\mu t}\ 2\mu t \cos(r+\theta) + \frac{\lambda}{2}$$

where, θ is negligible

$$\Delta P = 2\mu t \cos r + \frac{\lambda}{2}$$

for film $\mu = 1$

for normal incidence $r = 0$

$$\Delta P = 2(1)\, t \cos 0 + \frac{\lambda}{2}$$

$$\boxed{\Delta P = 2t + \frac{\lambda}{2}} \quad \text{—— (i)}$$

For constructive interference or bright rings

$$\Delta P = n\lambda$$

$$\therefore n\lambda = 2t + \frac{\lambda}{2} \Rightarrow 2t = n\lambda - \frac{\lambda}{2} \Rightarrow (2n-1)\frac{\lambda}{2}$$

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad \text{(ii)}$$

For destructive interference or dark rings

$$\Delta P = (2n+1)\frac{\lambda}{2}$$

$$(2n+1)\frac{\lambda}{2} = 2t + \frac{\lambda}{2}$$

$$n\lambda + \frac{\lambda}{2} = 2t + \frac{\lambda}{2}$$

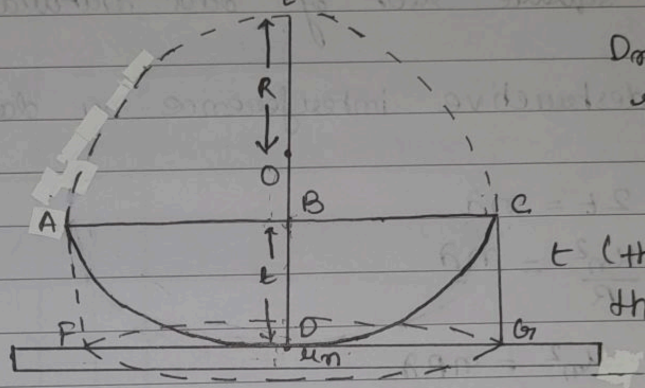
$$\boxed{2t = n\lambda} \quad \text{—— (iii)}$$

To determine the value of t

R - radius of curvature of lens

$BD = t$

$BE = 2R - t$



D_n = diameter of n^{th} ring

t (the film thickness)

r_n - radius of n^{th} ring

$DG = r_n = FD$

$DG = BC$, $FD = AB$

$AB \times BC = BE \times BD$

$r_n \times r_n = (2R - t)t$

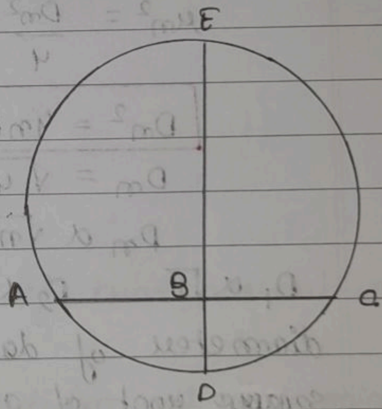
$r_n^2 = 2Rt - t^2$

t is thickness of film

as t is very small, t^2 is ngligible

$r_n^2 = 2Rt$

$$\boxed{2t = \frac{r_n^2}{R}}$$



$AB \times BC = BE \times BD$

$D_n = 2 r_n$ $\qquad r_n = \frac{D_n}{2}$

$$\boxed{r_n^2 = \frac{D_n^2}{4}}$$

for constructive interference or bright rings

$$2t = (2n-1)\frac{\lambda}{2}$$

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{D_n^2 = 2(2n-1)\lambda R}$$

$$\frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$$

$$D_n^2 = (2n-1)\frac{\lambda}{2} \times 4R = 2(2n-1)\lambda R$$

$$D_n^2 \propto (2n-1) \quad \text{or} \quad D_n \propto \sqrt{2n-1}$$

$n = 1,$

$D_1 \propto \sqrt{1}$ $D_2 \propto \sqrt{3}$ $D_3 \propto \sqrt{5}$ $D_4 \propto \sqrt{7}$

diameter of bright rings are directly proportional to square root of odd natural numbers.

for destructive interference or dark rings

$$2t = n\lambda$$

$$\frac{r_n^2}{R} = n\lambda$$

$$r_n^2 = nR\lambda$$

$$r_n^2 = \frac{D_n^2}{4} \qquad \frac{D_n^2}{4} = nR\lambda$$

$$\boxed{D_n^2 = 4nR\lambda}$$

$$D_n = \sqrt{4nR\lambda}$$

$$D_n \propto \sqrt{n}$$

$D_1 \propto \sqrt{1}$ $D_2 \propto \sqrt{2}$ $D_3 \propto \sqrt{3}$ $D_4 \propto \sqrt{4}$

diameter of dark rings are directly proportional to square root of all natural numbers.

<u>Applications of Newton's rings experiment</u>

i) To determine λ of monochromatic light

$$D_n^2 = 4nR\lambda$$

$$D_{n+p}^2 = 4(n+p)R\lambda$$

$$D_{n+p}^2 - D_n^2 = 4(n+p)R\lambda - 4nR\lambda$$

$$D_{n+p}^2 - D_n^2 = 4nR\lambda + 4pR\lambda - 4nR\lambda$$

p = difference in no. of two rings

2nd 6th

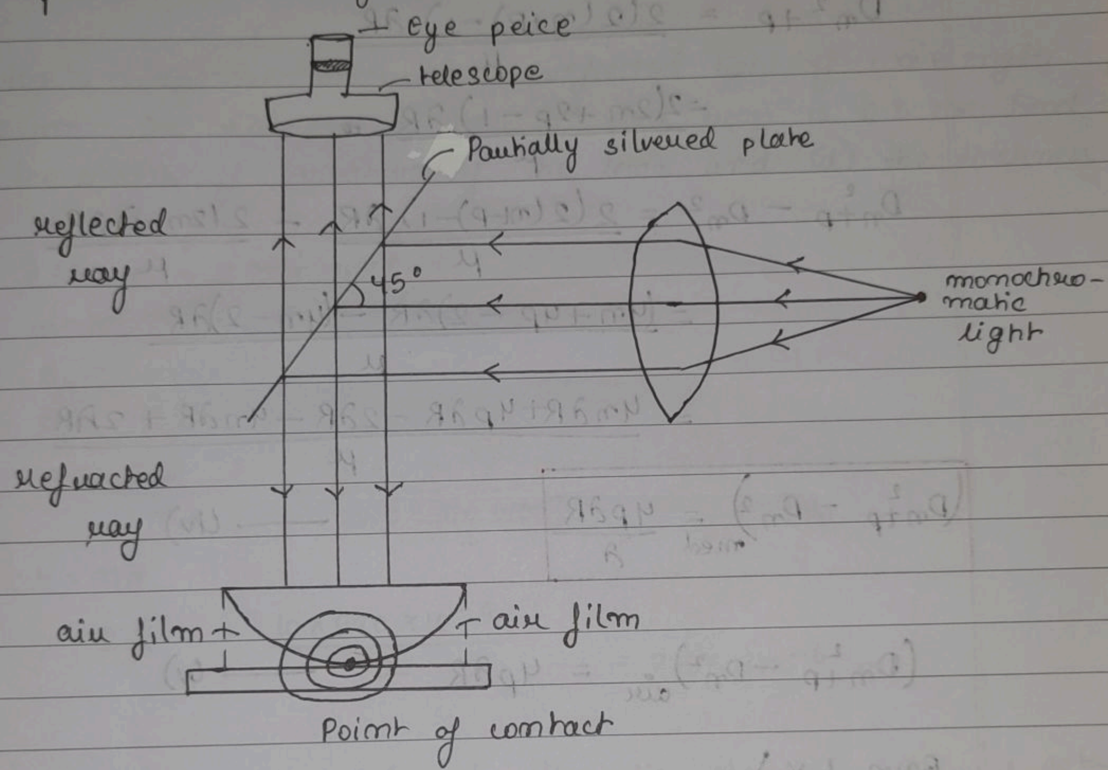
p = 6 − 2 = 4

$$\boxed{D_{m+p}^2 - D_m^2 = 4pR\lambda}$$

$$\boxed{\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}}$$

$$\boxed{R_1 = \frac{D_{m+p}^2 - D_m^2}{4p\lambda}}$$

Experimental arrangement



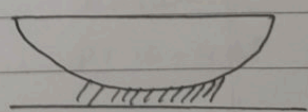
ii) To determine the refractive index of transparent medium

if we introduce a transparent medium at the point of contact then it replaces the air film

$2t = (2m-1)\frac{\lambda}{2}$ (for bright rings)

$\frac{r_m^2}{R} = (2m-1)\frac{\lambda}{2}$

$r_m^2 = (2m-1)\frac{\lambda R}{2}$



$2\mu t = (2m-1)\frac{\lambda}{2}$

$\mu\left(\frac{r_m^2}{R}\right) = (2m-1)\frac{\lambda}{2}$

$r_m^2 = (2m-1)\frac{\lambda R}{2\mu}$

$D_m^2 = 4r_m^2$

$r_m^2 = \frac{D_m^2}{4}$

$\frac{D_m^2}{4} = (2n-1)\frac{\lambda R}{2\mu}$

$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$

$D_{m+p}^2 = \frac{2(2(n+p)-1)\lambda R}{\mu}$

$= \frac{2(2n+2p-1)\lambda R}{\mu}$

$D_{n+p}^2 - D_m^2 = \frac{2(2(n+p)-1)\lambda R}{\mu} - \frac{2(2n-1)\lambda R}{\mu}$

$= \frac{(4n+4p-2)\lambda R - (4n-2)\lambda R}{\mu}$

$\Rightarrow \frac{4n\lambda R + 4p\lambda R - 2\lambda R - 4n\lambda R + 2\lambda R}{\mu}$

$$\boxed{(D_{n+p}^2 - D_m^2)_{med} = \frac{4p\lambda R}{\lambda}} \quad \text{—— (iv)}$$

$(D_{n+p}^2 - D_m^2)_{air} = 4p\lambda R$ —— (v)

Eqn $\left(\frac{v}{iv}\right)$

$\frac{(D_{m+p}^2 - D_m^2)_{air}}{(D_{m+p}^2 - D_m^2)_{med}} = \frac{4p\lambda R}{4p\lambda R} \times \mu = \mu.$

~~$\mu = \frac{(D_{n+p}^2) - (D_n^2)}{}$~~

$$\boxed{\mu = \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{med}}}$$

1. In Newton's ring experiment the diameter of the 15th ring was found to be 0.59 cm and that of the 5th ring 0.336 cm. If the radius of the plano convex lens is 100 cm. Calculate the wavelength of the light used.

$D_{n+p} = D_{15} = 0.59\ cm = 0.59 \times 10^{-2} m$

$D_n = D_5 = 0.336\ cm = 0.336 \times 10^{-2}\ m$

$R = 100\ cm = 1m$

$\lambda = \dfrac{D_{n+p}^2 + D_n^2}{4pR}$ $\qquad p = 15 - 5 = 10$

$\lambda = \dfrac{(0.59 \times 10^{-2})^2 + (0.336 \times 10^{-2})^2}{4 \times 10 \times 1}$

$= 5880\ \mathring{A}$

2. Newton's rings are observed in reflected light of wavelength 590 nm. The diameter of the 10th dark ring is 0.5 cm. Find (i) the radius of curvature of the lens and (ii) the thickness of the air film.

$\lambda = 590\ nm = 590 \times 10^{-9}\ m$

$n = 10$

$D_{10} = 0.5 \times 10^{-2}\ m$

i) $D_n^2 = 4nR\lambda$

$R = \dfrac{D_n^2}{4n\lambda} = \dfrac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 590 \times 10^{-9}} = 1.059\ m$

ii) $2t = n\lambda$

$t = \dfrac{n\lambda}{2} = \dfrac{10 \times 590 \times 10^{-9}}{2} = 2.95 \times 10^{-6}\ m$

3. In newton's rings experiment, the diameter of 4th and 12th dark rings are 0.4 and 0.7 cm respectively. Find the diameter of the 20th dark ring.

$D_4 = 0.4\ cm$

$D_{12} = 0.7\ cm$ $\qquad p = (n+p) - n$

$\dfrac{D_{n+p}^2 - D_n^2}{4pR} = \lambda$ $\qquad n+p = 12$

$\qquad n = 12$

$D_{n+p}^2 - D_n^2 = 4pR\lambda$ $\qquad p = 12 - 4 = 8$

$D_{12}^2 - D_4^2 = 4 \times 8 \times R\lambda$ ——— (i)

$D_{20}^2 - D_4^2 = 4 \times 16 \times R\lambda$ ——— (ii) $\qquad n+p = 20$

$\qquad n = 4$

~~Divide~~ so, $\dfrac{(i)}{(ii)}$ $\qquad p = 20 - 4 = 16$

$\dfrac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \dfrac{\cancel{4} \times \cancel{8} \times \cancel{R\lambda}}{\cancel{4} \times \cancel{16} \times \cancel{R\lambda}}$

2

$$2(D_{12}^2 - D_4^2) = D_{20}^2 - D_4^2$$
$$2D_{12}^2 - 2D_4^2 = D_{20}^2 - D_4^2$$
$$D_{20}^2 = 2D_{12}^2 - 2D_4^2 + D_4^2$$
$$= 2D_{12}^2 - D_4^2$$
$$= 2(0.7)^2 - (0.4)^2$$
$$D_{20}^2 = 2\times(0.49) - 0.16$$
$$= 0.9 \text{ cm}$$

4. The lower surface of a lens resting on a plane glass plate has a radius of curvature of 400 cm. When illuminated by monochromatic light, the arrangement produces newton's rings and 15th bright ring has a diameter of 1.16 cm. Calculate the wavelength of the monochromatic light.

$n = 15$

$R = 400$ cm

$D_{15} = 1.16$ cm

$$D_n^2 = 2(2n-1)\lambda R$$
$$= 2(2\times15-1)\lambda\times400$$
$$(1.16)^2 = 2\times29\times\lambda\times100$$
$$\lambda = \frac{1.16\times1.16}{2\times29\times100}$$
$$\lambda = 5800\times10^{-8} \text{ cm}$$
$$= 5800\times10^{-10} \text{ m}$$
$$= 5800 \text{ Å}$$

5. Newton's rings are formed in reflected light of wavelength 600 nm with a liquid between the plane and curved surfaces. If the diameter of the 6th bright ring is 3.1 mm and the radius of curvature of the curved surface is 100 cm. Calculate the refractive index of the liquid.

$\lambda = 600\times10^{-9}$ m

$D_6 = 3.1$ mm

$R = 100$ cm

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

$$\mu = \frac{2(2n-1)\lambda R}{D_n^2}$$

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$$= \frac{2\left(\left((2\times 6)-1\right)600\times 10^{-9}\times 100\times 10^{-2}\right)}{(3.1\times 10^{-3})^2}$$

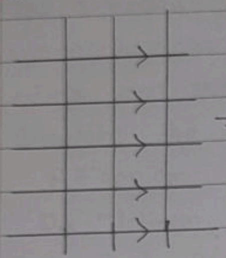
$$= \frac{2\times 11\times 600\times 10^{-9}}{3.1\times 3.1\times 10^{-6}} = 1.373$$

DIFFRACTION OF LIGHT

The word diffraction is derived from a latin word deffractus which is break to pieces.

Diffraction is when waves encounter obsticles they bend round the edges of the obstucles. In the dimensions of the obstucles are comparable to the wavelength of the waves. The bending of waves around the edges of obstucles are diffraction.

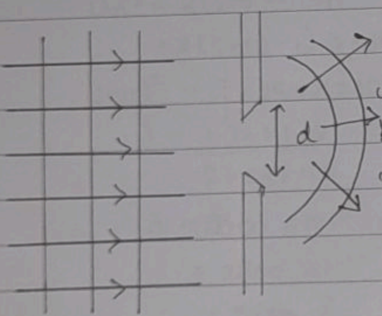
Case (i)



$d >> \lambda$ λ = wavelength of incident light

when the oppening is large compared to the wavelength, the waves do not bend round the edge

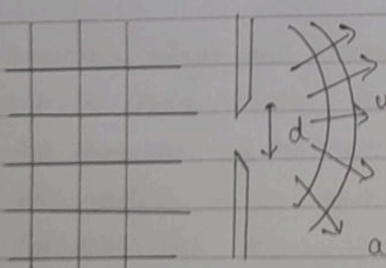
Case (ii)



$d \approx \lambda$

when the opening is small, the bending effect round the edges is not differenciable

Case (iii)

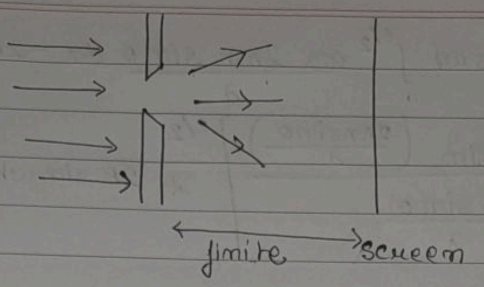


$d << \lambda$

when the opening is very small the waves spread over all the surface of the opening, the opening acts as a independent source of rays which propagate in all direction of the opening.

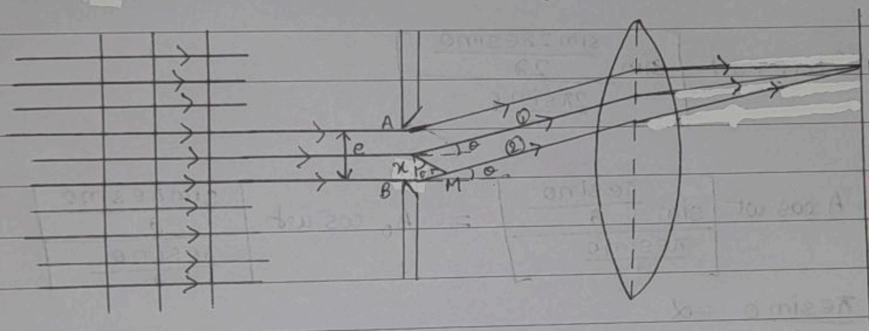
Types of diffraction:

- Fresnel's diffraction - either the point source or screen or both are at finite distances from the obstrucle. It deals with non-plane wavefronts.



- Fraunhofer diffraction – the surface and the screen are effectively at infinite distances from the obstacles. It deals with parallel rays and plane wavefronts. It's conditions can be easily achived by the help of lenses.

Fraunhofer diffraction at single slit



Phase difference

$\phi = \frac{2\pi}{\lambda} (\Delta P)$

$\boxed{\phi = \frac{2\pi}{\lambda} x \sin\theta}$

$AB = x$

BM = Path difference between diffracted rays ① and ②

$BM = AB \sin\theta = x\sin\theta$

$\boxed{y = A\cos\omega t}$, before diffraction.

After defraction, $\boxed{dy = A\,dx \cos(\omega t + \phi)}$

$$y = \int_{-e/2}^{+e/2} dy = 2\int_0^{e/2} dy$$

$$y = 2\int_0^{e/2} A\cos(\omega t + \phi)\,dx$$

$$= 2A\int_0^{e/2} (\cos\omega t + \cos\phi - \sin\omega t \sin\phi)\,dx$$

$$= 2A\int_0^{e/2} \cos\omega t \cos\phi\,dx - 2A\int_0^{e/2} \sin\omega t \sin\phi\,dx$$

$$= 2A\int_0^{e/2} \cos\omega t \cos\left(\frac{2\pi x\sin\theta}{\lambda}\right)dx - 2A\int_0^{e/2} \sin\omega t \sin\left(\frac{2\pi x\sin\theta}{\lambda}\right)dx$$

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$$= 2A\cos\omega t \int_0^{e/2} \cos \frac{2\pi x \sin\theta}{\lambda}\, dx - 2A \sin\omega t \int_0^{e/2} \sin \frac{2\pi x \sin\theta}{\lambda}\, dx$$

$$y = \left[\frac{2A\cos\omega t \sin\left(\frac{2\pi x \sin\theta}{\lambda}\right)}{\frac{2\pi \sin\theta}{\lambda}}\right]_0^{e/2} + 2A\sin\omega t \left[\frac{\cos\left(\frac{2\pi x \sin\theta}{\lambda}\right)}{\frac{2\pi\sin\theta}{\lambda}}\right]_0^{e/2}$$

$$y = 2A\cos\omega t \left[\frac{\sin \frac{2\pi e \sin\theta}{\lambda}}{\frac{2\pi\sin\theta}{\lambda}} - \frac{\sin 0}{\frac{2\pi\sin\theta}{\lambda}}\right] + 2A\sin\omega t \left[\frac{\frac{\cos 2\pi e \sin\theta}{2\lambda}}{\frac{2\pi\sin\theta}{\lambda}} - \frac{\cos 0}{\frac{2\pi\sin\theta}{\lambda}}\right]$$

$$y = 2A\cos\omega t \left[\frac{\frac{\sin 2\pi e \sin\theta}{2\lambda}}{\frac{2\pi \sin\theta}{\lambda}}\right]$$

$$= A\cos\omega t \left[\frac{\sin \frac{\pi e \sin\theta}{\lambda}}{\frac{\pi e\sin\theta}{\lambda}}\right] = A_0 \cos\omega t \left[\frac{\sin \frac{\pi e \sin\theta}{\lambda}}{\frac{\pi e \sin\theta}{\lambda}}\right]$$

let $\frac{\pi e \sin\theta}{\lambda} = \alpha$

$$y = A_0 \cos\omega t \frac{\sin\alpha}{\alpha}$$

Amplitude of resultant wave $= \frac{A_0 \sin\alpha}{\alpha}$

$$\boxed{y = A_0 \frac{\sin\alpha}{\alpha} \cos\omega t}$$

Resultant intensity,

$$\boxed{I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}}$$