# UNIT II Polarization (Lecture-1)

[Topics mentioned in Syllabus:

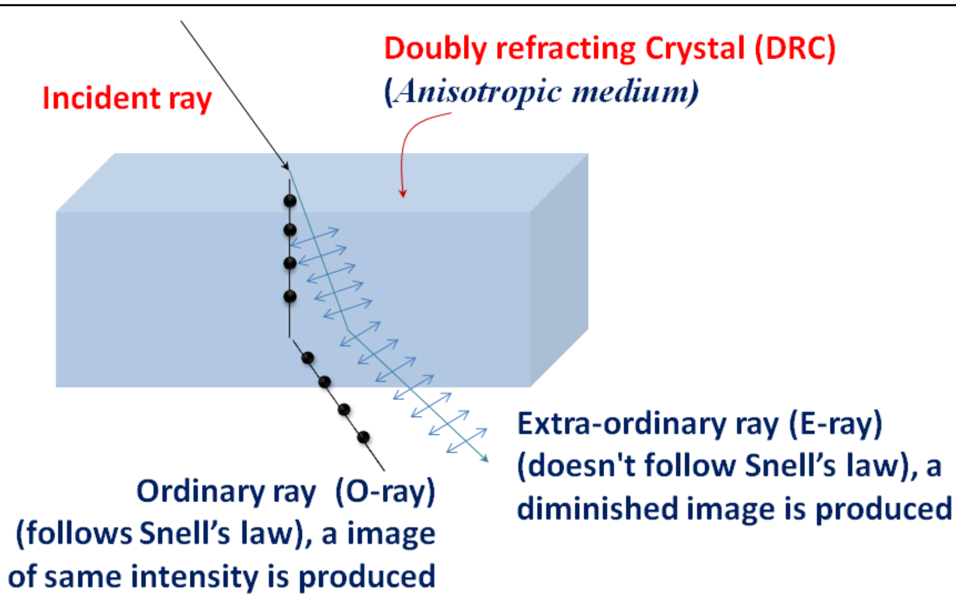
Polarization: Basic theory of double refraction, Malus Law, Ordinary and Extra-ordinary ray, Production and Detection of plane, circularly and elliptically polarized light, specific rotation and polarimeters.

Laser: Spontaneous and stimulated emission of radiation, Einstein Coefficient's, Principle of laser action, Construction and working of ruby and He-Ne laser, Photovoltaic effect.

Fibre Optics: Introductiom to Fibre Optics, Types of Fibre, Acceptance angle and cone, Numerical Aperture]

Q.1 What is double refraction phenomenon?

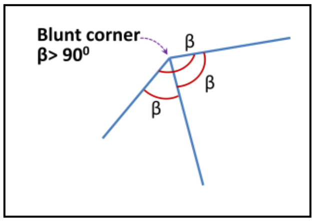
**Doubly-refracting crystal (DRC):** There are such crystals in nature which *split the incident light ray in ordinary (O-ray) and extra-ordinary (E-ray) rays* due to anisotropic property of such crystals. Such crystals are called **doubly-refracting crystals**.



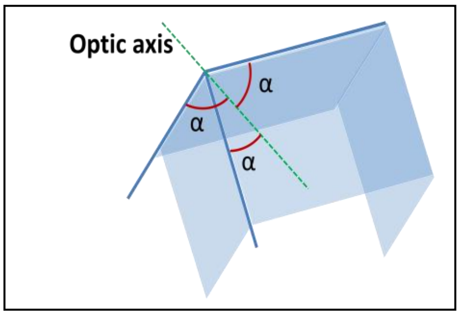

**Bifurcation of light into two rays due to *Anisotropic property* ( at least two crystallographic arrangement of atoms are present in DRCs)**

## Terminology used:

1. **Blunt corner:** It is such corner in which all three edges make same angle with each other (>$90^0$).



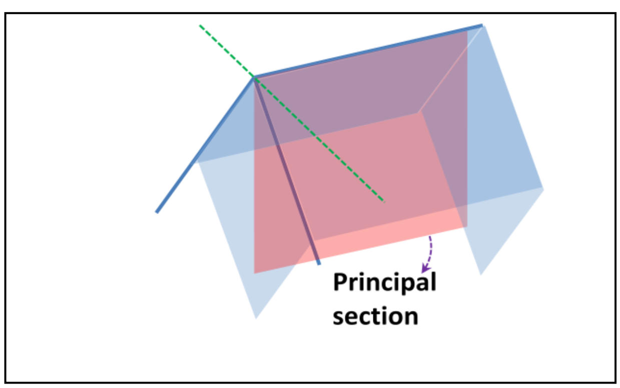

2. **Optic axis:** It is the direction from blunt corner which makes same angle from three edges.




**Property of optics axis:** If the light is incident along the optic axis or directions parallel to it then O-ray and E-ray travel in same directions. In other words, the phenomenon of travelling of O-ray and E-ray in different directions does not occur.

3. **Principal Section:** It is such a plane which contains the optic axis but it is perpendicular to the two opposite refracting faces. There are always three principal sections in a doubly refracting crystal.

**Significance or property of a Principal Section:** light passing through principal section gives maximum intensity.



# Doubly refracting crystal (DRC)

**Uni –axial** (One optic-axis)

- Calcite (Negative crystal)
- Quartz (Positive crystal)
- Tourmaline (Negative crystal)

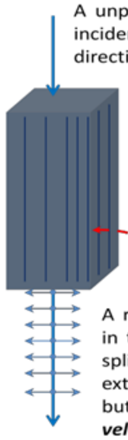
**Bi -axial** (Two optic-axis)

- Topaz
- Mica
- Copper sulphate

## Properties of DRC slabs whose optic axis is parallel to its refracting surface (or these are the DRC slabs in which Optic Axis is lying Parallel to Refracting Surface)

**(a)**

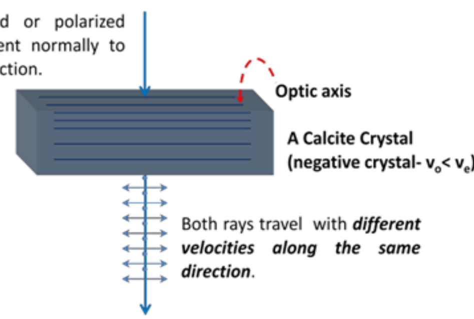
A unpolarised or polarized light is incident along the direction in direction of optic axis .




A ray of unpolarised light incident in the direction of optic axis is not split up into ordinary (O-ray) and extraordinary (E-ray) components, but both rays travel with ***same velocities along the same direction***.

**(b)**

A unpolarised or polarized light is incident normally to optic axis direction.



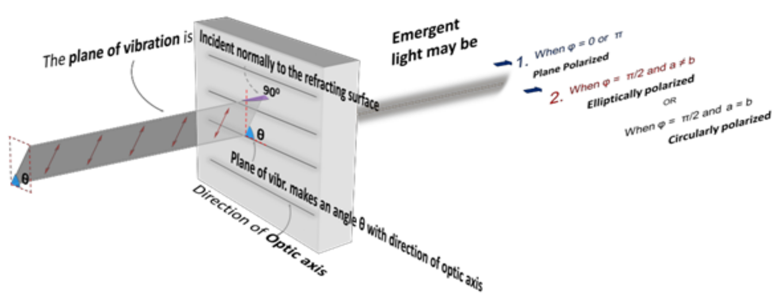

Both rays travel with ***different velocities along the same direction***.

**Application** of this property of such a slab:

This property is used to create a desired path difference between E and O ray.
Thus, ***Half wave plate*** and ***Quarter wave plate*** are designed which creates path difference of λ/2 and λ/4 respectively. These plates are also known as **retardation plates**.

✓**Half wave plate :** A plane polarized beam incident on a half-wave plate emerges as a plane polarized beam but rotated such that its angle to the optical axis is twice that of the incident beam. Therefore, half-wave plates can be used as continuously adjustable polarization rotators. Half-wave plates are ***used in rotating the plane of polarization***.

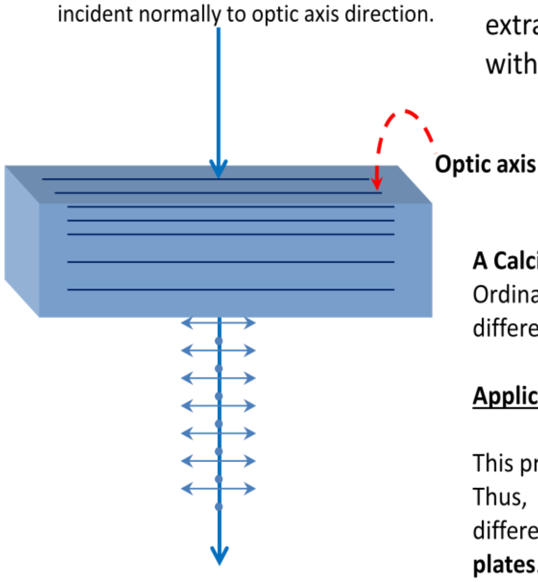
✓**Quarter wave plate :** It is **used to produce the *Elliptically* or *circularly* polarized** light .




Properties of a doubly refracting crystal whose **optic axis** is cut parallel to the refracting surface.

*When light is incident normally on the refracting surface then nature of emergent light depends upon the thickness of the plate (t).*

A unpolarised or polarized light is incident normally to optic axis direction.




A ray of unpolarised light incident normally to the optic axis is not split up into ordinary (O-ray) and extraordinary (E-ray) components, but both rays travel with ***different velocities along the same direction***.

**A Calcite Crystal (negative crystal- $v_o < v_e$)**
Ordinary (O-ray) and extraordinary (E-ray) components, but both rays travel with different velocities along the same direction.

**Application** of this property of such a slab:

This property is used to create a desired path difference between E and O ray.
Thus, ***Half wave plate*** and ***Quarter wave plate*** are designed which creates path difference of $\lambda/2$ and $\lambda/4$ respectively. These plates are also known as **retardation plates**.

✓**Half wave plate :** A plane polarized beam incident on a half-wave plate emerges as a plane polarized beam but rotated such that its angle to the optical axis is twice that of the incident beam. Therefore, half-wave plates can be used as continuously adjustable polarization rotators. Half-wave plates are ***used in rotating the plane of polarization***.

✓**Quarter wave plate :** It is **used to produce the *Elliptically* or *circularly* polarized** light .

# UNIT II Polarization (Lecture-2)

[Topics mentioned in Syllabus:

Polarization: Basic theory of double refraction, Malus Law, Ordinary and Extra-ordinary ray, Production and Detection of plane, circularly and elliptically polarized light, specific rotation and polarimeters.

Laser: Spontaneous and stimulated emission of radiation, Einstein Coefficient's, Principle of laser action, Construction and working of ruby and He-Ne laser, Photovoltaic effect.

Fibre Optics: Introductiom to Fibre Optics, Types of Fibre, Acceptance angle and cone, Numerical Aperture]

Dear students:

In this lecture notes you will study

- ✓ How the linear, circularly and elliptical lights are produced using a DRC and how to detect the nature of the light produced by specially designed DRCs.
- ✓ Especially designed DRC i.e. half wave plate and quarter wave plate.
- ✓ Laurent's Half Shade polarimeter (Only Laurent's Half Shade polarimeter is in your syllabus).
- ✓ Need of half shade device.

## Doubly refracting crystal

**Uni –axial**
(One optic-axis)

- Calcite (Negative crystal)
- Quartz (Positive crystal)
- Tourmaline (Negative crystal)

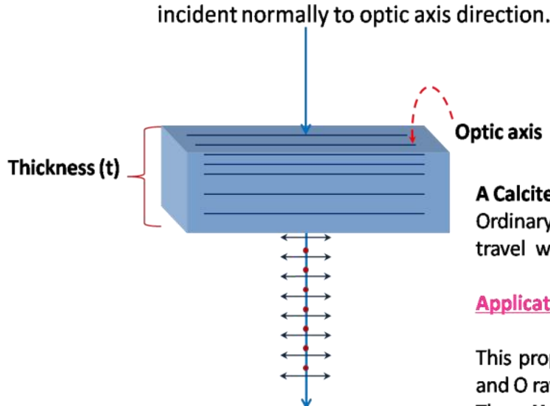
**Bi -axial**
(Two optic-axis)

- Topaz
- Mica
- Copper sulphate

## Specially designed DRC slab (Calcite – $V_0 < V_e$) whose optic axis is lying parallel to refracting slab

**(specially designed DRC slab to introduce path difference between E and O ray . The path difference depends upon thickness of slab (t))**

A unpolarised or polarized light is incident normally to optic axis direction.




A ray of unpolarised light incident normally to the optic axis is not split up into ordinary (O-ray) and extraordinary (E-ray) components, but both rays travel with ***different velocities along the same direction***.

**A Calcite Crystal (negative crystal- $v_o < v_e$)**
Ordinary (O-ray) and extraordinary (E-ray) components, but both rays travel with different velocities along the same direction.

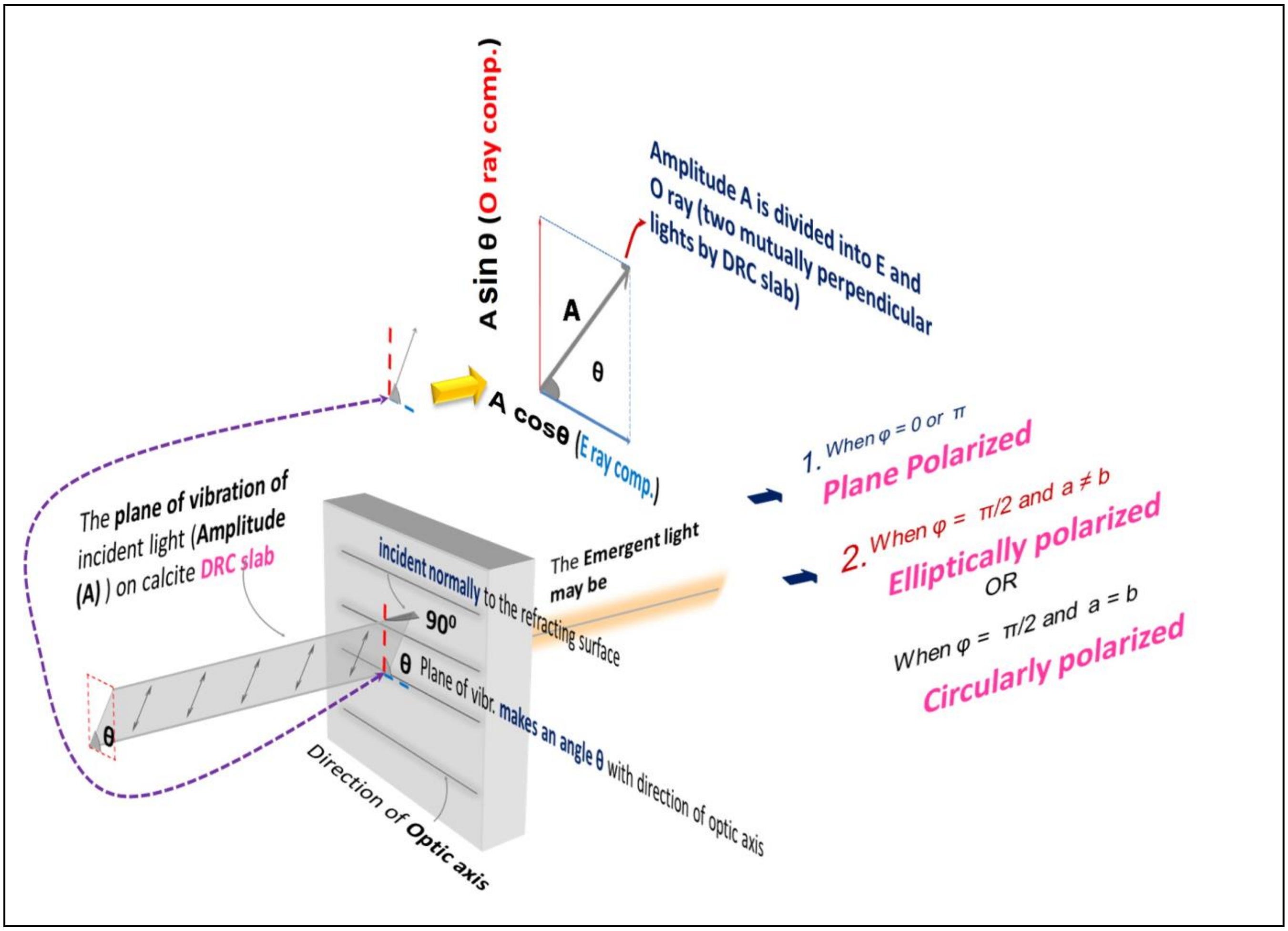
**Application of this property of such a slab:**

This property is used to create a desired path difference between E and O ray.
Thus, ***Half wave plate*** and ***Quarter wave plate*** are designed which creates path difference of λ/2 and λ/4 respectively. These plates are also known as **retardation plates**.

✓**Half wave plate :** A plane polarized beam incident on a half-wave plate emerges as a plane polarized beam but rotated such that its angle to the optical axis is twice that of the incident beam. Therefore, half-wave plates can be used as continuously adjustable polarization rotators. Half-wave plates are ***used in rotating the plane of polarization***.

✓**Quarter wave plate :** It is **used to produce the *Elliptically* or *circularly* polarized** light .

# Superposition of linearly polarized vibrations:




**When a plane polarized is incident on a calcite crystal which is specially designed DRC as discussed in Figure above. The incident light divides into two components called E-ray and O-ray by the DRC (because DRC bifurcate the light in E and O ray). And the thickness (t) produces a desired path difference between E and O ray so emergent ray on other side may be Plane, Elliptical or Circularly polarized light. Mathematical analysis is discussed below.**

Since the calcite crystal is a negative crystal means velocity of O-ray is less than the velocity of E-ray. In other words E-ray leads the O- ray thus here phase (ωt + δ) of E-ray is shown to lead by δ than O - ray. E- ray is in the direction of optic axis.

**E** – Ray → $x = \mathrm{A}\ \cos\theta\ \sin(\omega t + \delta)$ …… [1]

**O**– Ray → $y = \mathrm{A}\sin\theta\ \sin\omega t$ …… [2]

Let $\mathrm{A}\ \cos\theta\ = a$ and $\mathrm{A}\sin\theta\ = b$ then

$$x = a\sin(\omega t + \delta) \ \ or \ \ \frac{x}{a} = \sin\omega t \ \cos\delta \ + \ \sin\delta \ \cos\omega t \ \ \ldots\ldots[3]$$

$$y = b\sin\omega t \ \ or \ \ \frac{y}{b} = \sin\omega t \ \ \ldots\ldots[4] \ \ and \ \ \cos\omega t \ = \sqrt{1-\left(\frac{y}{b}\right)^2}$$

Substituting $\sin\omega t = \frac{y}{b}$ and $\cos\omega t \ = \sqrt{1-\left(\frac{y}{b}\right)^2}$ into equation [3]

$$\frac{x}{a} = \frac{y}{b}\cos\delta \ + \ \sin\delta\sqrt{1-\left(\frac{y}{b}\right)^2}$$

$$\left(\frac{x}{a} - \frac{y}{b}\cos\delta\right)^2 \ = \ \sin\delta\sqrt{1-\left(\frac{y}{b}\right)^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\,cos^2\,\delta \ - 2\frac{x}{a}\frac{y}{b}\cos\delta \ = \ \sin^2\delta\left[1-\left(\frac{y}{b}\right)^2\right]$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\,cos^2\,\delta \ - 2\frac{x}{a}\frac{y}{b}\cos\delta \ = \ \sin^2\delta - \sin^2\delta\,\frac{y^2}{b^2}$$

$$\frac{x^2}{a^2} + \left(\frac{y^2}{b^2}\,cos^2\,\delta \ + \ \sin^2\delta\,\frac{y^2}{b^2}\right) - 2\frac{x}{a}\frac{y}{b}\cos\delta \ = \ \sin^2\delta$$

$$\boldsymbol{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\cos\delta \ = \ \sin^2\delta}$$

This is the equation of oblique ellipse or general equation which governs the nature of emergent ray from this DRC.

**Case I**: If the thickness of the plate is such that it introduces a phase difference **(δ) = 0 or 2nπ between E and O ray (or Δx = 0)** then cosδ = 1 and sinδ = 0

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b} = \ 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 \ = \ 0$$

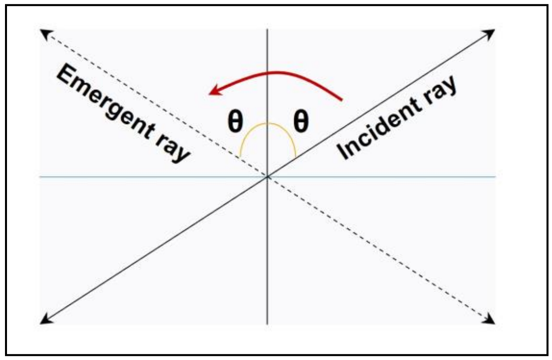
$$\frac{x}{a} - \frac{y}{b} = \ 0 \ or \ \ \frac{x}{a} = \frac{y}{b}$$

$$\pm\boldsymbol{y} = \pm\left(\frac{\boldsymbol{b}}{\boldsymbol{a}}\right)\boldsymbol{x}$$

This is the equation of a **straight line**. It shows that emergent light will be **straight line.**

**Case II:** If the thickness of the plate is such that δ = π or (2n+1) π **(or Δx = λ/2 or called Half wave plate)** then cosδ = -1 and sinδ = 0

$$\pm y = \pm\left(\frac{b}{a}\right)x$$




$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+2\frac{x}{a}\frac{y}{b}= 0$$

$$\left(\frac{x}{a}+\frac{y}{b}\right)^2 = 0$$

$$\frac{x}{a}+\frac{y}{b}= 0\ or\ \rightarrow\frac{x}{a}=-\frac{y}{b}$$

$$\boldsymbol{\pm y = \mp\left(\frac{b}{a}\right)x}$$

**This shows that emergent ray is again a plane polarized light, and it is rotated by angle 2θ from direction of incident ray.**

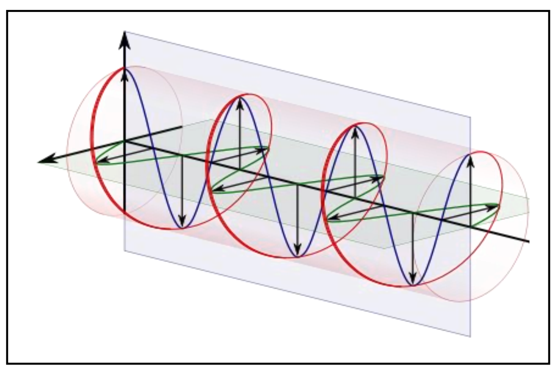
**Case III:** If the thickness of the plate is such that it introduces a phase difference (**δ)** = π/2 or (2n+1) π/2 **(or Δx = λ/4 or called Quarter wave plate)** then cosδ = 0 and sinδ = 1

$$\boldsymbol{\frac{x^2}{a^2}+\frac{y^2}{b^2}= 1}$$

This is the equation of an ellipse. It shows that emergent light will be **an elliptically polarized light**.

**Special case:** In above equation there can be a=b = A cosθ = A sinθ, only when θ = $45^0$; let b = a then above equation reduces to

$$\boldsymbol{\frac{x^2}{a^2}+\frac{y^2}{a^2}= 1\ or\ x^2+y^2 = a^2}$$

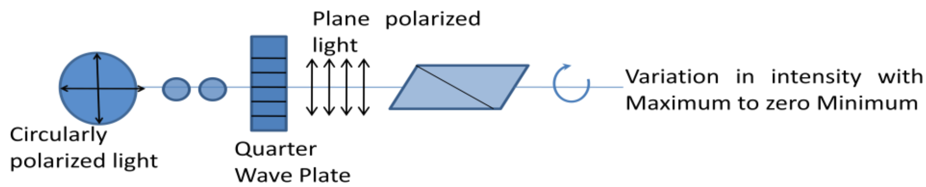


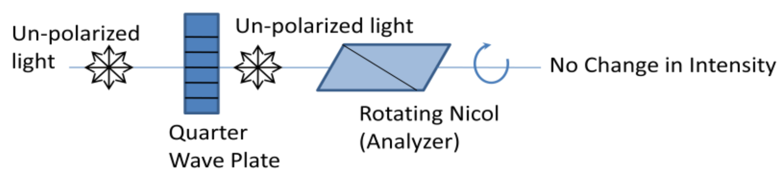
This is the equation of a circle. It shows that if the incident plane polarized light makes an angle θ=$45^0$ with optic axis then emergent light is always **a circularly polarized light**.

## Detection:

***Plane Polarized light:*** *When passed through a rotating Nicol prism the intensity of emergent light varies from* ***Maximum to Zero Minimum****.*

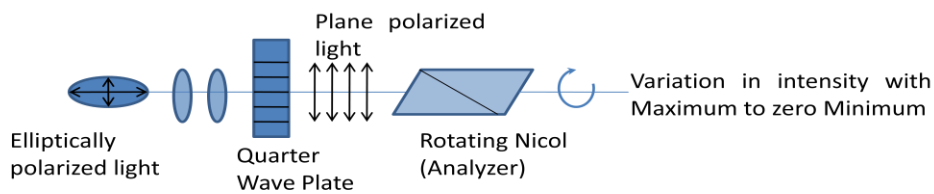

***Circularly Polarized light:*** *Generally, on rotating a Nicol (analyser) in front of a* ***unpolarized*** *and* ***a circular polarized light*** *gives same impression of no variation in intensity because of their same amplitudes in all directions. Therefore, to distinguish between these two these are passed through a quarter wave plate (as shown in figure).*

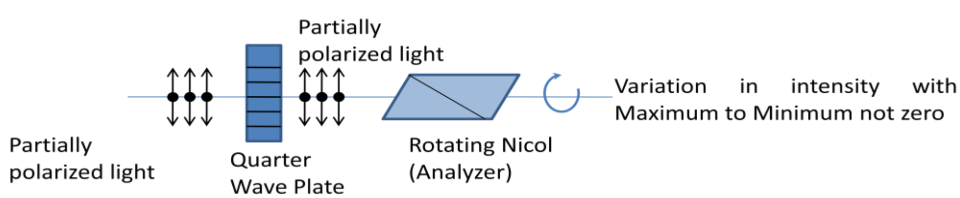



On Passing through quarter wave, a circularly light is converted into plane polarized light. It shows variation in intensity with maximum to zero minimum. And an un-polarized light passes as it is; thus, it shows no change in intensity.

***Elliptically Polarized light:*** *On rotating a Nichol (analyzer) in front of an* ***Elliptically polarized light*** *and* ***a partially plane polarized light*** *shows variation in intensity from maximum to minimum not zero. Therefore, to distinguish between these two these are passed through a* ***quarter wave plate*** *(as shown in figure).*



On Passing through quarter wave, an elliptically polarized light is converted into plane polarized light. It shows variation in intensity with maximum to zero minimum. And a partially polarized light passes as it is; thus, it shows variation in intensity from maximum to minimum not zero.

# Phase Retardation Plates:

Retardation plates are used **for the introduction of phase difference** between **ordinary (o-ray) and extraordinary (E-ray) rays** during their normal transmission through doubly refracting crystal. (**provided** that in doubly refracting crystal, the optic axis must be cut parallel to the refracting surface. In this case when the light is incident normally to the optics axis (or refracting surface), the **O-ray and E-ray travel both in the same direction** but with a **certain path deference** between them which does **depend upon** the **thickness of the plate**.

Thus, the path difference **($\Delta$) = ($\mu_0 \sim \mu_e$) t**

[It is the optical path **($\Delta$)** travelled in doubly refracting crystal's medium which offers two refractive indices one for O-ray ($\mu_0$) and another for E-ray ($\mu_e$), t = thickness of retardation plate]

## *There are **two types** of **retardation plates:***

1. ***Quarter wave plate:*** *The retardation plate which introduces a path deference of $\lambda/4$ (or phase difference = $\pi/2$ between O-ray and E-ray (therefore, **$\Delta$=** $\lambda/4$)*

$$\Delta = \lambda/4 = (\mu_0 \sim \mu_e)\, t$$

$$t = \lambda/4\,(\mu_0 \sim \mu_e)$$

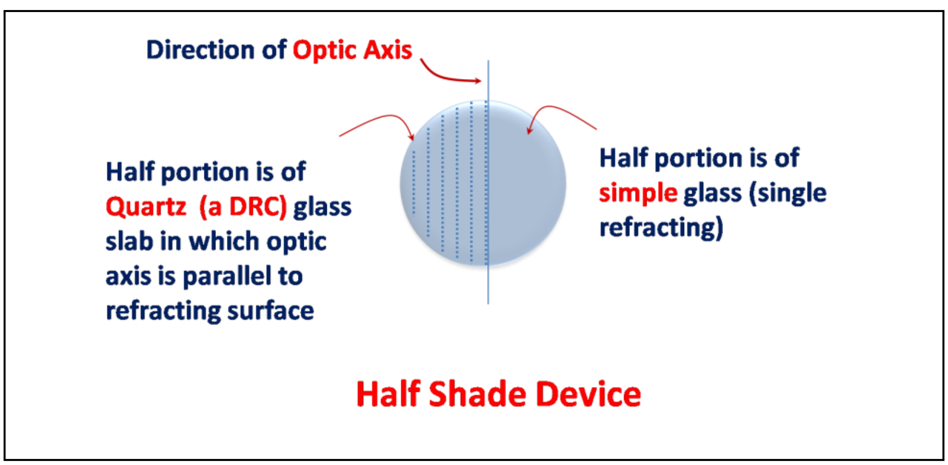
**Applications:**

- ✓ *It is used to produce **elliptically polarized light.***
  *If the vibrations of incident plane polarized light **do not make an angle** of the $45^0$, $0^0$ or $90^0$ with the optic axis, the **emergent light is elliptically polarized**.*
- ✓ *It is used to produce **circularly polarized light.***
  *When plane polarized light is incident on quarter wave plate **making an angle $45^0$ with the optic axis**, the emergent light is **circularly polarized**.*

2. ***Half wave plate:*** *The retardation plate which introduces a path deference of $\lambda/2$ between O-ray and E-ray (therefore, **$\Delta$=** $\lambda/2$*

$$\Delta = \lambda/2 = (\mu_0 \sim \mu_e)\, t$$

$$t = \lambda/2(\mu_0 \sim \mu_e)$$



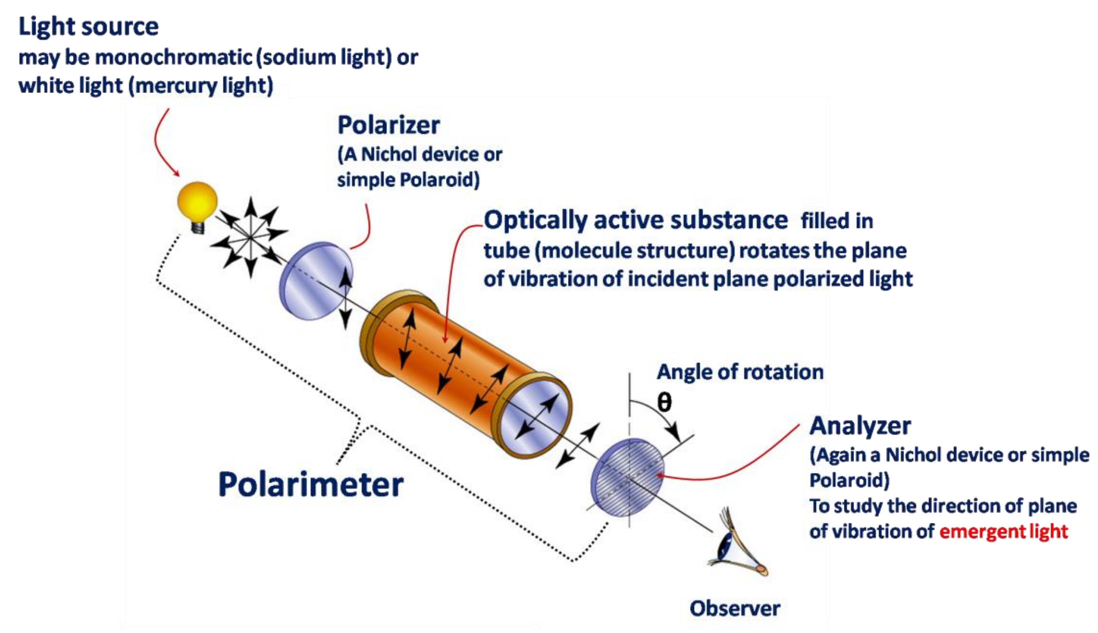
**Applications:**

- ✓ *It produces **Plane polarized light.***
  *Whatever the plane of vibration of incident plane polarized light makes an angle ϑ with the optic axis the emergent light is also plane polarized light with vibrations inclined at an angle 2ϑ with the optic axis.*
- ✓ *It is used in **Half-Shade polarimeter.***

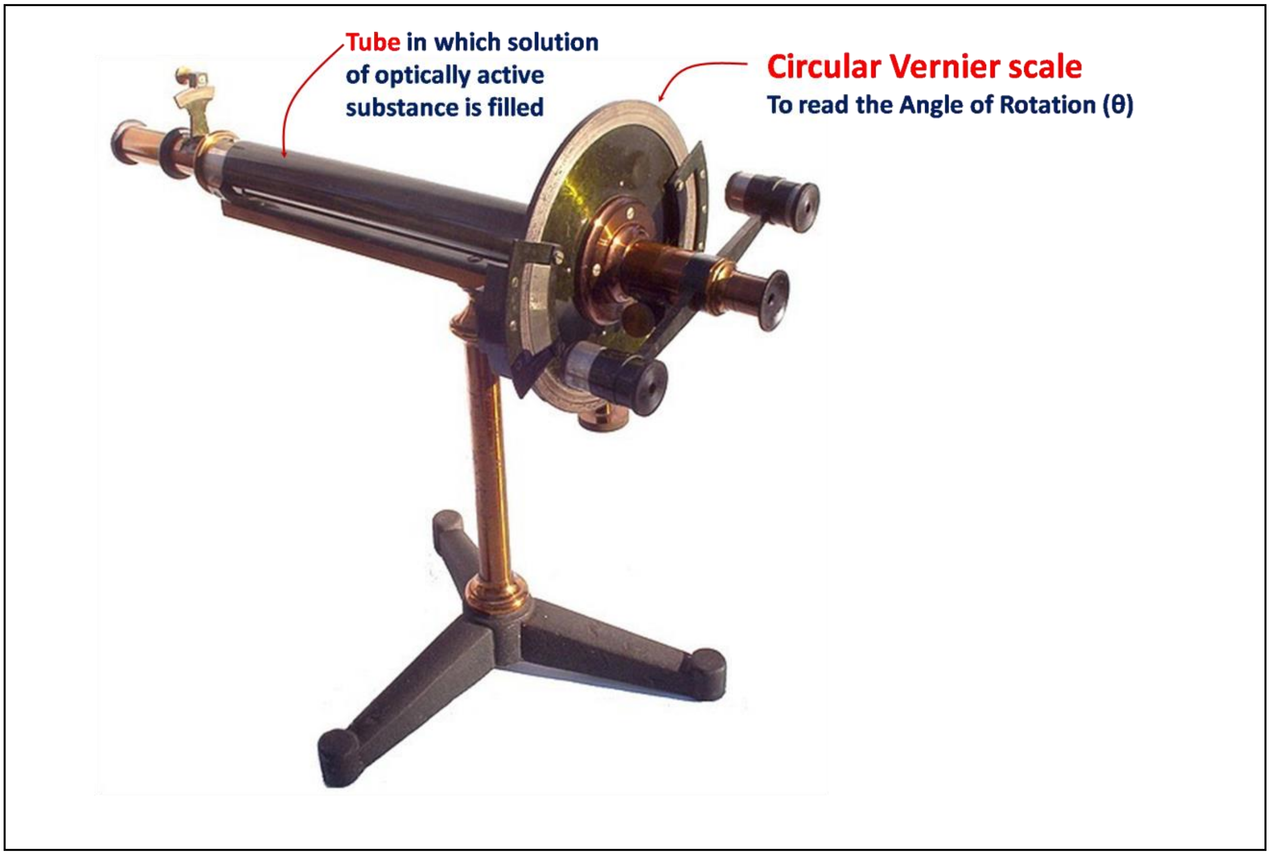
# Phenomenon of Optical Activity:

**Optically Active substance:** Those substances which respond to the plane polarized light are called ***optically active*** means they have the ability rotate the plane of vibration of incident light.

**Phenomenon**: A*ccording to Fresnel's theory,* when a plane polarized light is incident on an *optically active substance, on passing through the optically active substance, the plane of vibration of incident plane polarized plane is rotated either on **right hand side** (Dextrorotatory-**sugar solution**) or on **left hand side** (laevorotatory- **D-Fructose**). This* **phenomenon is called optical rotation**.




**Polarimeter :** *It is an arrangements of a light source, polarizer , tube and an analyzer which is used to determine **angle of rotation**. The determination of **angle of rotation** enables to determine specific rotation (S) of the optically active substance filled in tube.*




*The **various derivative of sugar are optically active**. The optical rotation of few are given below*

- D-Fructose −92.4°
- D-Glucose +52.5°
- D-Sucrose +66.47°
- D-Lactose +52.3°
- Camphor +44.1°

*+ Sign denotes dextro rotation and - sign denotes laevo rotation. All values are given in units of deg $dm^{-1}cm^{3}\ g^{-1}$.*

**Specific rotation (S)** *is the angle of rotation* (**θ**) *when Unit concentration* (1 gm/c.c.) *is filled in unit length of polarimeter tube* (in decimeter).

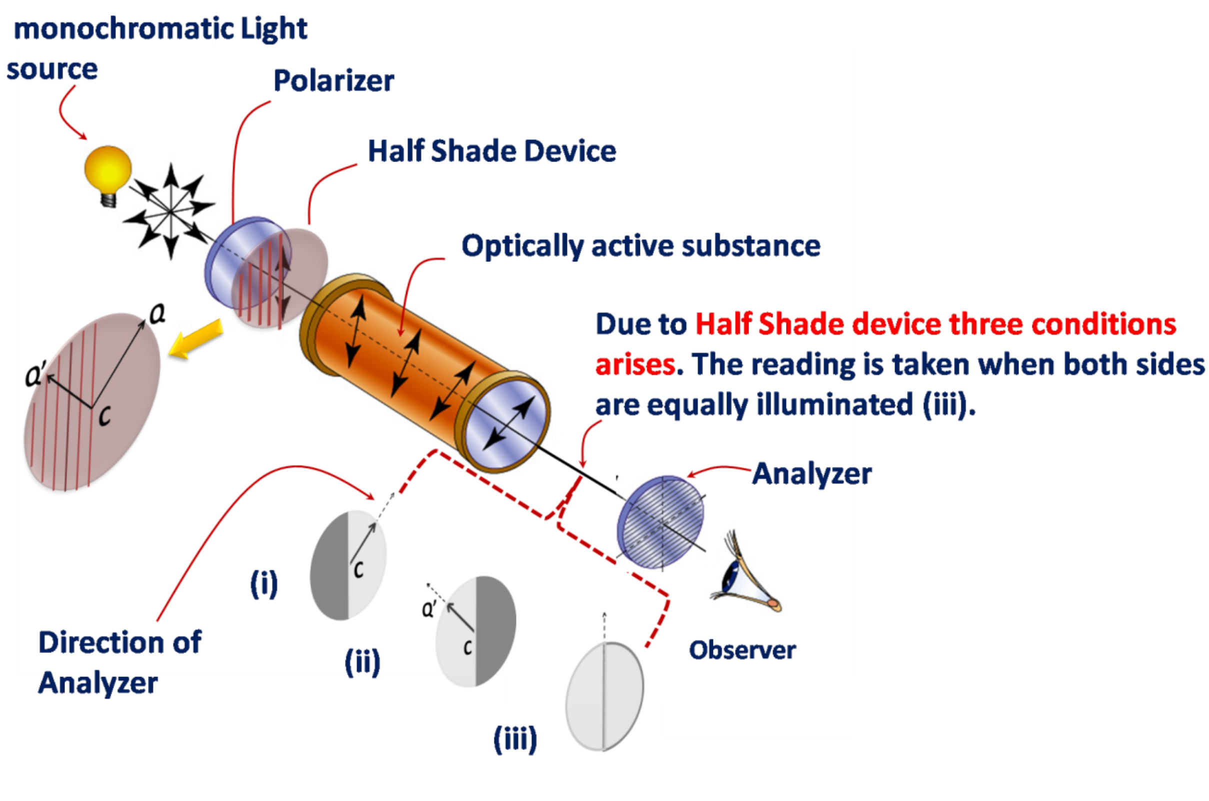
Specific rotation (**S) = θ/ (l. c)** **... [1]**

**Determination of concentration of sugar in a solution:**

Here **θ (angle of rotation) is determined experimentally** using a Laurent's polarimeter in Laboratory. Since the specific rotation (s) of a sugar +66.47° is known so substituting the values of S, θ and l (length of polarimeter tube in decimeter) in eq. [1], the **concentration (c)** can be determined. This is one of the practical applications of plane polarized light.

**Q.2 What is Laurent's Half Shade Polarimeter? What is the use of half-shade device? Explain the working of half-shade polarimeter.**

Ans. It is the polarimeter in which a monochromatic light and a half shade device (i.e. *λ/2 retardation plate)* is used. This device rotates the light incident (CQ') in CQ direction. Hence, the light is observed in CQ' and CQ both directions (refer figure below). Following three conditions arises (see Fig read further) among which condition (iii) when both sides of Half shade device are equally illuminated.



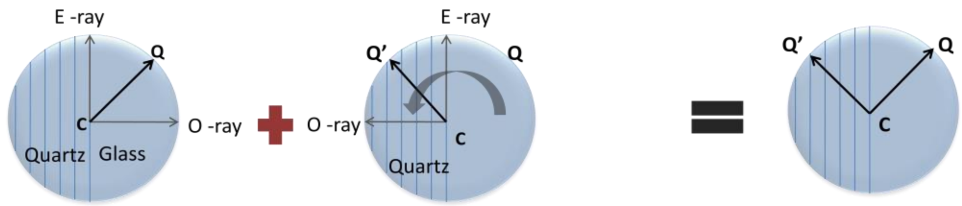

Laurent's Polarimeter

## Need of introducing the Half Shade Device in a half shade polarimeter:

It is not possible for us to distinguish between the intensity variations of a light therefore, in polarimeter a **half- shade device** is installed which **provides a reference condition** (condition of equally illuminated portion of both sides of a half shade device) **to measure an angle on circular scale.**

Condition at ***entering*** of light in **CQ direction** in glass half

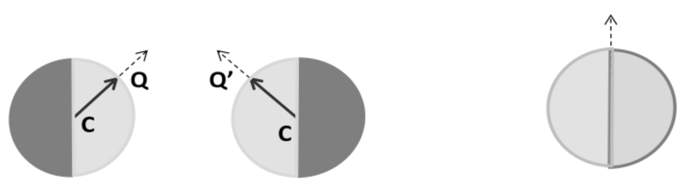
Condition of **O-component** travelling through Quartz




when plane polarized light **enters in CQ** direction . On entering in Quartz, the E component is in the direction of Optic Axis and O-component is perpendicular to it. The light passes in CQ direction as it is through glass half.

Quartz is positive doubly refracting crystal
During travelling trough Quartz Half wave Plate (produces path deference of λ/2 or phase difference of π between E and O components) the faster travelling O-component is rotated by π as shown above. Thus on emergence, a new resultant is obtained in CQ' direction.

Thus, on emergence, two resultant component in **CQ and CQ'** direction are obtained.




When principal section of rotating Nicol (analyzer) is in CQ direction then Glass half is illuminated

When principal section of rotating Nicol (analyzer) is in CQ' direction then Quartz half is illuminated

When principal section of rotating Nicol (analyzer) is in the direction of Optic axis then both halves are equally illuminated