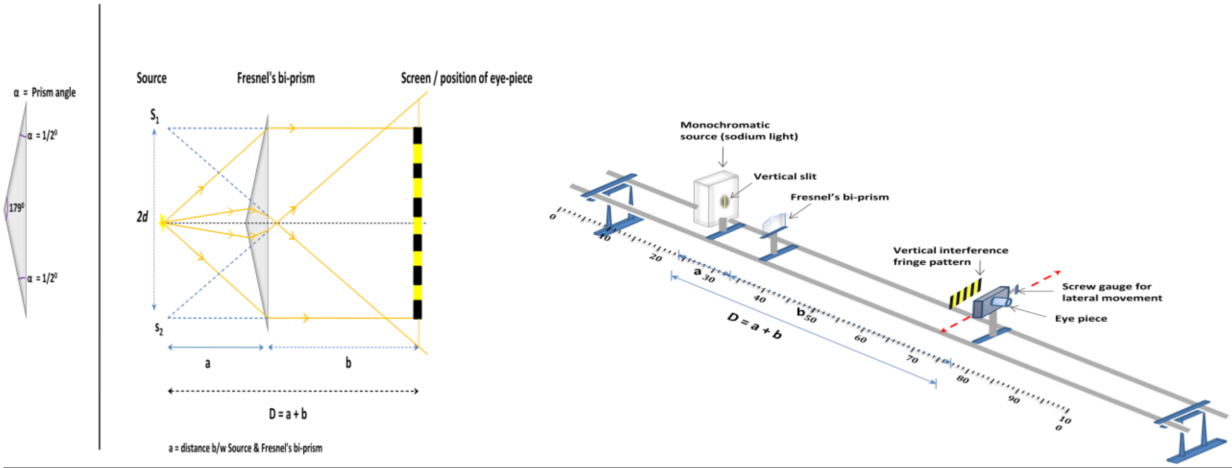
UNIT I INTERFERENCE:

Q.1 Determine the wavelength (λ) of a monochromatic light using Fresnel's biprism experiment.



Fresnel's bi-prism: It is a device to produce interference pattern by division of wave front. The upper portion and lower portion of device act as a prism therefore it is called bi-prism. Since its interference is due to division of wave front therefore, the relation $\omega = D\lambda/2d$ is applicable to find the wavelength.

$\lambda = (\omega.2d) / D$

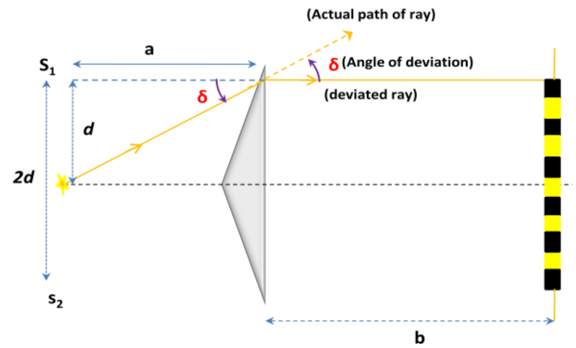
Here the parameters ω (fringe width), D (distance between source and screen) and 2d (distance between two virtual light sources) are to be determined experimentally. In figure, the mounted source, bi-prism and eyepiece are shown.

- ✓ **Determination of fringe width (ω):** The cross-wire of eye piece is set accurately at any of the nth dark fringe and then the corresponding screw gauge reading is noted (initial reading).Now moving on laterally at arbitrary no. of dark fringes (say 10) on either side of the nth dark fringe, again the corresponding screw gauge reading is noted (final reading).

  Thus the fringe width (ω) = **(final reading ~ initial reading)/10**

- ✓ **Determination of D** distance between source and screen (position of eye-piece): It is determined directly on optical bench.
- ✓ **Determination of 2d** distance between two virtual light sources (i.e. S1 and S2)]**:** There are two methods to find the 2d.

  1. **Deviation method:** It uses the property of a prism [$\delta = (\mu-1)\alpha$].



From above figure: $\tan\delta = \frac{d}{a}$ or $\delta = \frac{d}{a}$

Now substituting the value of δ in prism relation δ = (μ-1) α we get

$$\frac{d}{a} = (\mu - 1)\alpha$$

$$d = a(\mu - 1)\alpha$$

$$2d = 2a(\mu - 1)\alpha$$

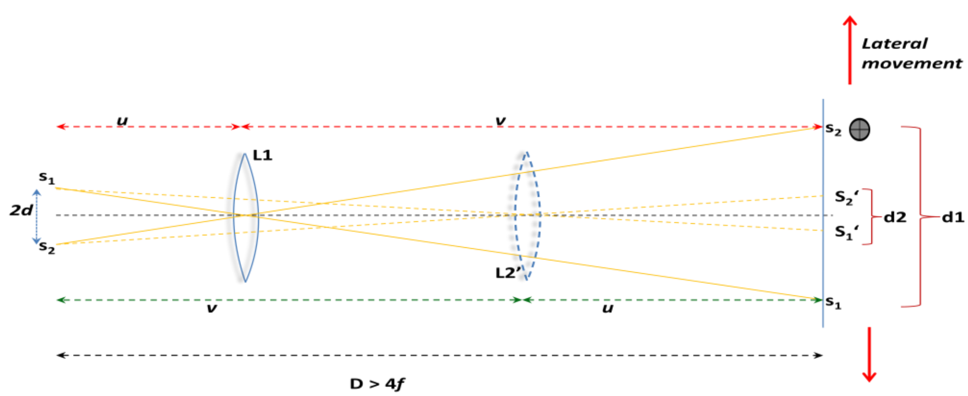
or

$$\boxed{2d = 2a(\mu - 1)\alpha \cdot \frac{\pi}{180}\ cm}$$

When μ =1.5 , α =1/2^0 then $2d = (0.0087)\alpha\ cm$ (For numerical point of view)

This is expression to find the 2d by deviation method.

2. **<u>Displacement method:</u>** This method utilizes the property of a convex lens.



[This method is based upon the fact that real and same size images are obtained when the distance between object (2d) and its image is 4f, thus if two small and large images are to be obtained then the distance must be taken greater than 4f. Here we take

D>4f thus just displacing the lens (as shown above position L1 and L2') the large (d1 image) and small image (d2) can be obtained.]

Since the L1 and L2' positions are ***conjugate*** therefore,

[Conjugate means once in position L1 the object is small and its corresponding image is large but converse is obtained in position L2' , the image is small and object is large, therefore in L2' position the image serves the purpose of a object and object (2d) as its corresponding image. Thus position of L1and L2' is ***conjugate****]*

Applying magnification relation for L1 position,

$$\frac{2d}{u} = \frac{d_1}{v} \; \boldsymbol{or} \; \frac{\boldsymbol{u}}{\boldsymbol{v}} = \frac{\boldsymbol{2d}}{\boldsymbol{d_1}} \; \ldots \ldots \ldots [\mathbf{1}]$$

Applying magnification relation for L2' position,

$$\frac{d_2}{u} = \frac{2d}{v} \; \boldsymbol{or} \; \frac{\boldsymbol{u}}{\boldsymbol{v}} = \frac{\boldsymbol{d_2}}{\boldsymbol{2d}} \; \ldots \ldots \ldots [\mathbf{2}]$$

Therefore, from [1] / [2]

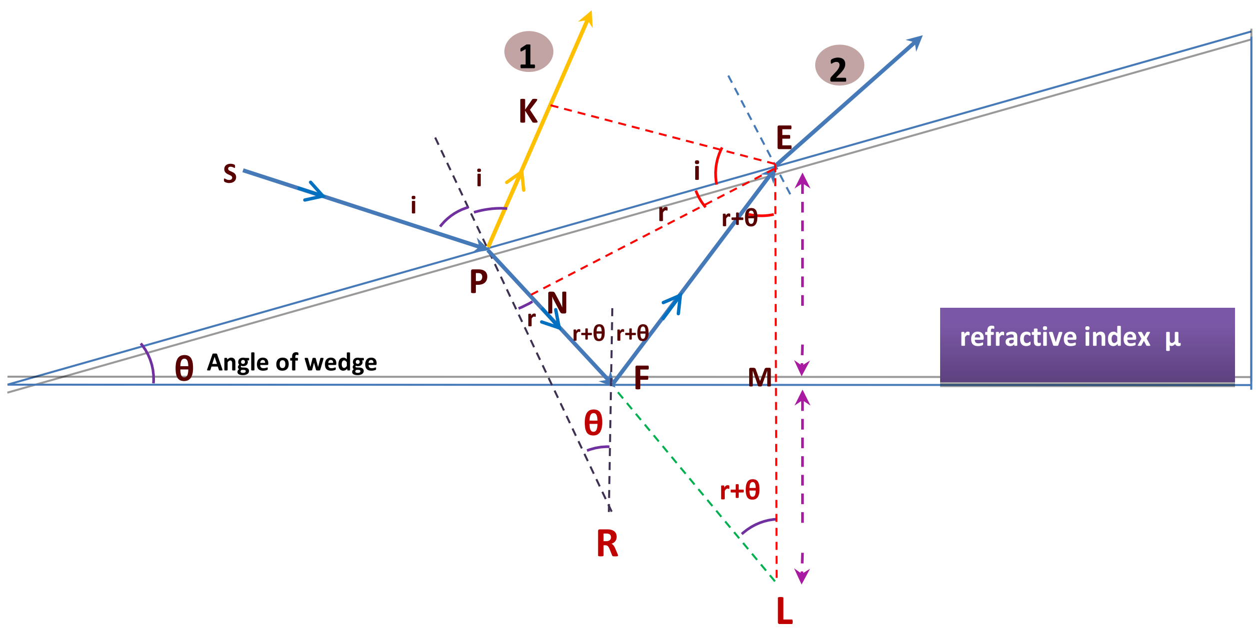
$$\frac{\cancel{\boldsymbol{u}}}{\cancel{\boldsymbol{v}}} \times \frac{\cancel{\boldsymbol{v}}}{\cancel{\boldsymbol{u}}} = \frac{\boldsymbol{2d}}{\boldsymbol{d_1}} \times \frac{\boldsymbol{2d}}{\boldsymbol{d_2}}$$

$$(\boldsymbol{2d})^{\boldsymbol{2}} = \boldsymbol{d_1}\, \boldsymbol{d_2}$$

$$\boxed{\boldsymbol{2d} = \sqrt{\boldsymbol{d_1}\, \boldsymbol{d_2}}}$$

To find d1 experimentally, the screw gauge is positioned at S1 and S2 respectively then the corresponding screw gauge reading is noted. The difference of these gives us the d1. Similarly, the d2 is obtained. Substituting d1 and d2 in above formula enable us to find the 2d.

Q2. Derive the condition of bright and dark fringe in wedge shaped film.



*[**Thing to remember:** According to **Stoke's treatment**, when a ray reflects from a denser medium then the reflected ray suffers a phase difference (Δφ) of π with incident ray or a path difference (Δx) of λ/2 with incident ray [since $\Delta\varphi = \frac{2\pi}{\lambda}\Delta x$ therefore, $\pi = \frac{2\pi}{\lambda}\Delta x$ or $\Delta x = \frac{\lambda}{2}$]. Here, incident ray S reflects from a denser medium (point P) (reflected ray is ray 1). Thus ray1 has phase difference of π with incident ray S. similarly, since the ray 2 reflects from a rarer medium (air) (point F) therefore it doesn't suffers any phase difference with incident ray PF. Therefore, ray 2 emerged from point E, is in same phase as the phase of incident ray from S.*

*Consequently, we can say that **ray 1 and ray 2 has phase difference of π or path difference of λ/2(this path difference is associated with ray1)**]*

A light ray incident from source S, reflects from upper surface (point P) and lower internal surface (point F).The light from F emerges from E, thus 1 and 2 light rays interfere(other rays reflected rays from inner surface don't have much intensity to get interfere).

$$\mu = \frac{\sin i}{\sin r} = \frac{\frac{PK}{\cancel{PE}}}{\frac{PN}{\cancel{PE}}} = \frac{PK}{PN}$$

Or

$$\boxed{\boldsymbol{PK = \mu PN} \qquad \dots\dots\dots\dots [1]}$$

<u>The path difference between 1 and 2 ray:</u> *[since the ray 1 travels an optical path $\mu_{(air)}$ PK in air than the ray 2 (with respect to upper surface). But the ray 2 travels an optical path μ (PF+FE) in medium of refractive index μ with respect to upper surface. Here, it is to mention that according to **Stoke's treatment,** a path difference of λ/2 is already associated with ray 1. Therefore, the net optical path of ray 1 become **$\mu_{(air)}$ PK + λ/2** . Thus, the net expression of path difference **(Δx)** between 1 and 2 interfering waves becomes:*

Δx = (optical path travelled by ray 2 – optical path traveled by ray 1)]

$$\boldsymbol{\Delta x = \mu\,(PF\ + FE) - \left[\mu_{air} PK + \frac{\lambda}{2}\right]}$$

$$\Delta x = \mu\,(PF\ + FE) - \left[PK + \frac{\lambda}{2}\right]$$

$$\Delta x = \mu\,[(PN\ + NF)\ + FE] - \mu PN - \frac{\lambda}{2} \qquad from\ fig.\ PF\ = PN\ + PF\ and\ PK = \mu PN\ (eq.1)$$

$$\Delta x = \cancel{\mu PN} + \mu\,(NF + FE) - \cancel{\mu PN} - \frac{\lambda}{2} \qquad from\ geometry\ \ FE\ = FL$$

$$\Delta x = \mu\,(NF + FL) - \frac{\boldsymbol{\lambda}}{\boldsymbol{2}}$$

$$\Delta x = \mu\,(NL) - \frac{\boldsymbol{\lambda}}{\boldsymbol{2}} \qquad \dots\dots\dots\dots\dots..[2]$$

From ***right triangle ENL*** $\cos(\mathrm{r} + \theta) = \frac{NL}{EL}$

Or $NL = \cos(\mathrm{r} + \theta)\,EL$

Or $NL = 2t\cos(\mathrm{r} + \theta) \qquad from\ fig.\ EL\ = EM + ML = 2t$

Substituting value of NL into eq. [2]

$$\Delta x = 2\mu t\cos(\mathrm{r} + \theta) - \frac{\boldsymbol{\lambda}}{\boldsymbol{2}}$$

This is the expression of path difference between ray1 and ray2. It depends upon the refractive index (μ) of wedge medium, thickness of the film (t) as well as refractive angle (r) and angle of wedge (θ).

✓ If at thickness **t**, there is **any bright fringe** then applying the condition of a bright fringe

$$\Delta x = 2n\frac{\boldsymbol{\lambda}}{\boldsymbol{2}}$$

$$\text{or } \Delta x = 2n\frac{\lambda}{2} = 2\mu t\cos(\mathrm{r} + \theta) - \frac{\lambda}{2}$$

$$\boxed{\text{or } \mathbf{2\mu t\cos(r + \theta) = (2n + 1)\frac{\lambda}{2}}}$$

This expression shows that the condition of bright fringe is that the ***term* 2μt cos (r+θ)** must be equal to ***odd multiple of λ/2.***

✓ If at thickness **t**, there is **any dark fringe** then applying the condition of a dark fringe

$$\Delta x = (2n + 1)\frac{\boldsymbol{\lambda}}{\boldsymbol{2}}$$

or $\Delta x = \ (2n+1)\frac{\lambda}{2} = \ 2\mu t\cos(\mathrm{r}+\theta) - \frac{\lambda}{2}$

$$\boxed{\text{or } \mathbf{2\mu t\cos(r+\theta)} = \mathbf{(2n)}\frac{\boldsymbol{\lambda}}{\mathbf{2}} \quad or\ n\boldsymbol{\lambda}}$$

This expression shows that the condition of dark fringe is that the ***term*** **2μt cos (r+θ)** must be equal to ***even multiple of λ/2.***

Learning outcome:

The Condition of Bright Fringe in Wedge Shaped Film:

$$\boxed{\text{or } \mathbf{2\mu t\cos(r+\theta)} = \mathbf{(2n+1)}\frac{\boldsymbol{\lambda}}{\mathbf{2}}}$$

This expression shows that the condition of bright fringe is that the ***term*** **2μt cos (r+θ)** must be equal to ***odd multiple of λ/2.***

The Condition of Dark Fringe in Wedge Shaped Film

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This expression shows that the condition of dark fringe is that the ***term*** **2μt cos (r+θ)** must be equal to ***even multiple of λ/2.***

Fringe Width (Ω) in Wedge Shaped Film:

from figure , we have $\tan\theta = t/x_n$ or $\mathbf{t = \tan\theta . x_n}$

Since the condition of bright fringe in wedge shaped film is given by 2μ **t** co (r+ θ) = (2n+1)λ/2 . Therefore, we ca n establish two relations such that

$2\mu\, \mathbf{\tan\theta}.\, x_n \cos(r+\theta) = (2n+1)\lambda/2$ ...(1)

$2\mu\, \mathbf{\tan\theta}.\, x_{n+1} \cos(r+\theta) = [2(n+1)+1]\lambda/2$ ...(2)

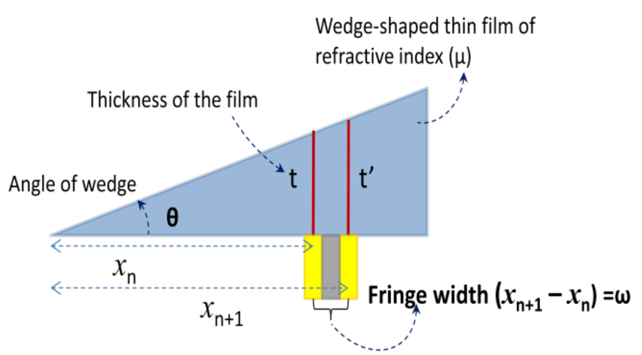
Therefore , eq. (2) –(1) gives fringe width $\mathbf{(x_{n+1} - x_n) = \omega}$

$2\mu \tan\theta\cos(r+\theta)\, [x_{n+1} - x_n] = [(2n+3) - (2n+1)]\lambda/2$

$[x_{n+1} - x_n] = \lambda/2\mu\tan\theta\cos(r+\theta)$

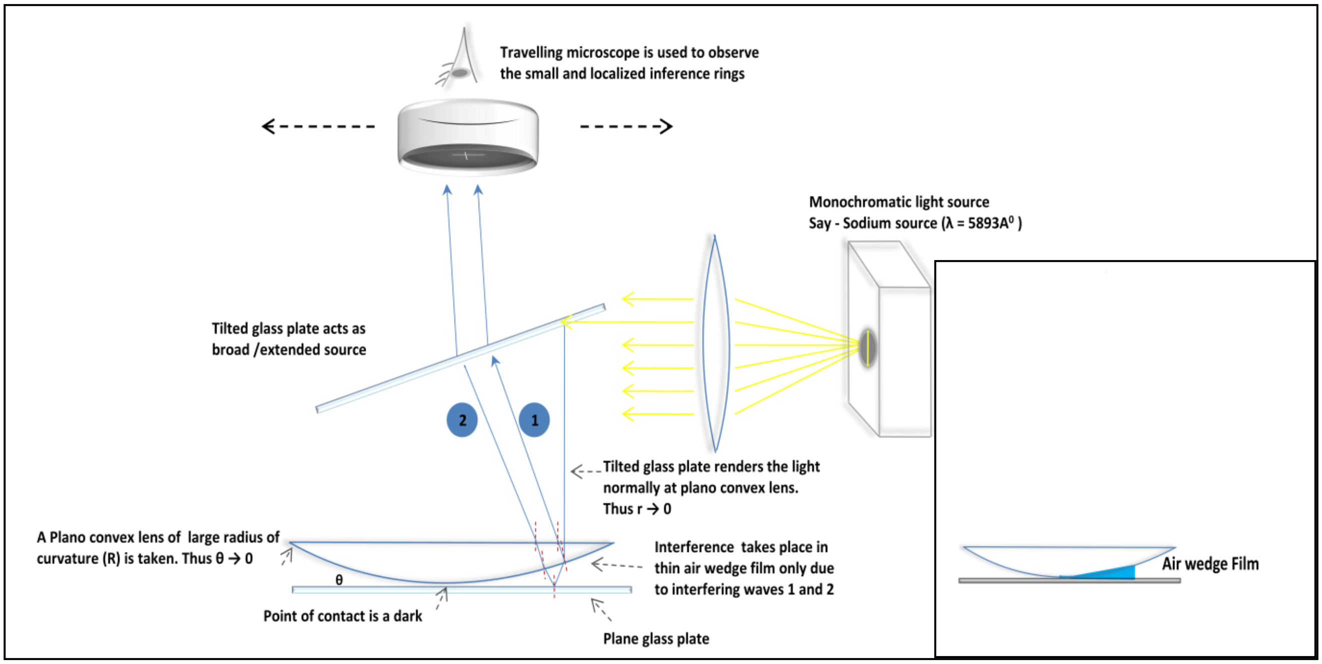
$\omega = \lambda/2\mu\sin\theta$

$\omega = \lambda/2\mu\theta$ here θ is in degree, it must be in radian. Thus for numerical point of view $\mathbf{\omega = (\lambda.\ 180)/(2\mu\theta)}$ **cm**

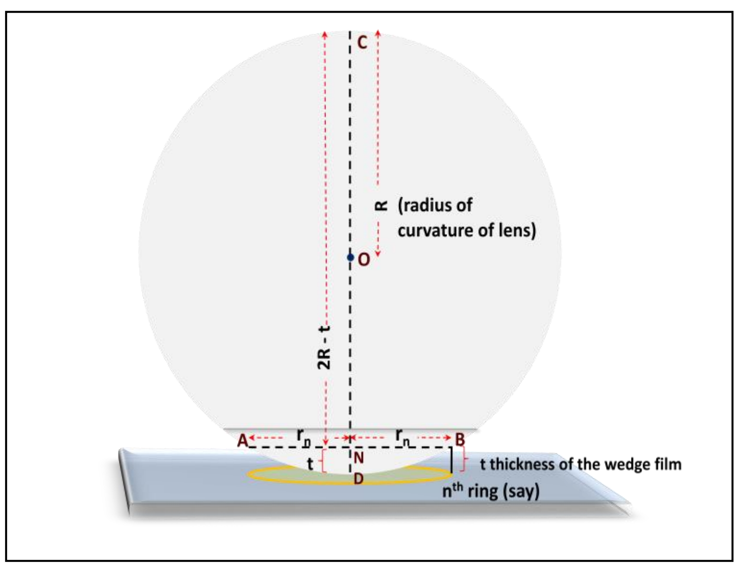


The n^{th} and $n+1^{th}$ bright fringes are shown above . The distances from O are x_n and x_{n+1} and the thickness of the film are t and t' respectively.

Newton's Ring Experiment:



<u>Newton's Ring:</u> In Newton's ring experiment the interference takes place in air wedge film (μ=1), using a tilted glass plate the light is incident normally (thus r→0) on Plano convex lens and the lens is taken of large radius of curvature (thus θ→0). Therefore, the condition of path difference (2μtcos(r+θ) –λ/2) between interfering waves **1** and **2** become Δ = **2t - λ/2.** Therefore



Condition of Dark ring:

$\Delta = 2t - \lambda/2 = (2n-1)\,\lambda/2$

Or $\mathbf{2t = n\lambda}$ .....[1]

Condition of Bright ring:

$\Delta = 2t - \lambda/2 = 2n\,\lambda/2$

Or $\mathbf{2t = (2n-1)\,\lambda/2}$ .....[2]

Diameter of Bright and Dark rings:

From fig 2, using property of chords (product of segment of two intersecting chords is equal to each other. Here, AB and CD chords are intersecting at point N) Therefore:

$$\boldsymbol{AN \times NB = CN \times ND}$$

$$r_n \times r_n = (2R - t) \times t$$

$$r_n^2 = 2Rt - t^2$$

t (thickness of wedge film) is very small as comaperdd to R (Rad. of curvature of lens). Therefore $t^2 \to 0$

$$r_n^2 = 2Rt$$

$$t = \frac{r_n^2}{2R} \qquad \text{since} \left[r_n(\text{radius}) = \frac{D_n\,(\text{diameter})}{2}\right] \to \left[r_n^2 = \frac{D_n^2}{4}\right]$$

..... [3]

$$\boxed{\boldsymbol{t = \frac{D_n^2}{8R}}}$$

Substituting it into eq. [1] and [2] will establish a relation between radius (**r_n**) of **n** **th** ring and thickness **t** of the wedge film at that point.

Condition of Bright ring:

$$2t = (2n+1)\frac{\lambda}{2}$$

Substituting the value of t from eq. [3]

$$\cancel{2}\frac{D^2_{n(\text{bright})}}{\cancel{8}R} = (2n+1)\frac{\lambda}{2}$$

$$\frac{D^2_{n(\text{bright})}}{4R} = (2n+1)\frac{\lambda}{2}$$

$$D^2_{n(\text{bright})} = \cancel{4}(2n+1)\frac{\lambda R}{\cancel{2}}$$

$$\boxed{\boldsymbol{D^2_{n(bright)} = 2\,(2n+1)\,\lambda R}}$$

$$\boxed{\boldsymbol{or\ \ D_{n(bright)}\ \alpha\ \sqrt{(2n+1)}}}$$

It show that diameter of bright ring is proportional to square root of odd integer number.

$$\boxed{\begin{array}{l} \boldsymbol{D_{1(bright)}\ \alpha\ \sqrt{3} = 1.73} \\ \boldsymbol{D_{2(bright)}\ \alpha\ \sqrt{5} = 2.25} \\ \boldsymbol{D_{3(bright)}\ \alpha\ \sqrt{7} = 2.64} \end{array}}\ \begin{array}{l} \} \ \boldsymbol{0.52} \\ \} \ \boldsymbol{0.39} \end{array} \quad \downarrow \text{Decrement}$$

It shows that spacing between two successive bright rings decreases as we move away from the point of contact. Or bright rings contracts.

Condition of Dark ring:

$$\boldsymbol{2t = n\lambda}$$

Substituting the value of t from eq. [3]

$$\cancel{2}\frac{D^2_{n(dark)}}{\cancel{8}R} = n\ \lambda$$

$$\frac{D_n^2}{4R} = n\ \lambda$$

$$\boxed{\boldsymbol{D^2_{n(dark)} = 4\ n\ \lambda R}}$$

$$\boxed{\boldsymbol{D_{n(dark)}\ \alpha\ \sqrt{n}}}$$

It show that diameter of bright ring is proportional to square root of an integer number.

$$\boxed{\begin{array}{l} \boldsymbol{D_{1(dark)}\ \alpha\ \sqrt{1} = 1} \\ \boldsymbol{D_{2(dark)}\ \alpha\ \sqrt{2} = 1.41} \\ \boldsymbol{D_{3(dark)}\ \alpha\ \sqrt{3} = 1.73} \end{array}} \begin{array}{l} \} \ \mathbf{0.41} \\ \} \ \mathbf{0.32} \end{array} \Big\downarrow \text{Decrement}$$

It shows that spacing between two successive dark rings decreases as we move away from the point of contact. Or dark rings contracts.

Q.3 Discuss the Applications of Newton's ring experiment set-up (*i.e.* wavelength of a monochromatic light (λ) and refractive index of a transparent liquid (μ).

(1) Determination of a monochromatic light (λ) -

When Newton's ring experiment is performed in the air

Then for any ***n^th^*** dark ring $D_n^2 = 4n\lambda R$ ....(1)

And for any ***n+p^th^*** dark ring $D_{n+p}^2 = 4(n + p)\lambda R$ ....(2)

Then (2) – (1) gives us the relation

$$\boxed{(D_{n+p}^2 - D_n^2)_{(air)} = 4p\lambda R}$$

(2) **Determination of refractive index (μ) of a transparent liquid –**

Once the experiment is performed in air wedge film then we have

$$\boxed{(D_{n+p}^2 - D_n^2)_{(air)} = 4p\lambda R}$$

And again the experiment is performed by introducing the transparent liquid between the wedge shaped film (between the lens and plane glass plate). Again the diameter of ***nth*** dark (say D'^2_n) and ***n+pth*** dark ring (say D'^2_{n+p}) is determined. Then

Then for any $\boldsymbol{n^{th}}$ dark ring $D'^2_n = 4n\lambda R/\mu$ ....(3)

And for any $\boldsymbol{n+p^{th}}$ dark ring $D'^2_{n+p} = 4(n + p)\lambda R/\mu$ .....(4)

Then (4) – (3) gives us the relation

$$\boxed{(D'^2_{n+p} - D'^2_n)_{(liquid)} = 4p\lambda R/\mu}$$

From above two expressions in box we have

$$\boxed{\mu = (D_{n+p}^2 - D_n^2)_{(air)} / (D'^2_{n+p} - D'^2_n)_{(liquid)}}$$

Thus by determining the value $(D_{n+p}^2 - D_n^2)$ in air and in liquid wedge film, we can determine the refractive index.