# Basic Electrical Engineering (TEE 101)

# Lecture 46: Efficiency of Transformer

By: Er. Parvesh Saini

# Content

**This lecture covers:**

- **Transformer Efficiency**
- **Condition for maximum efficiency in Transformer**
- **All Day Efficiency**

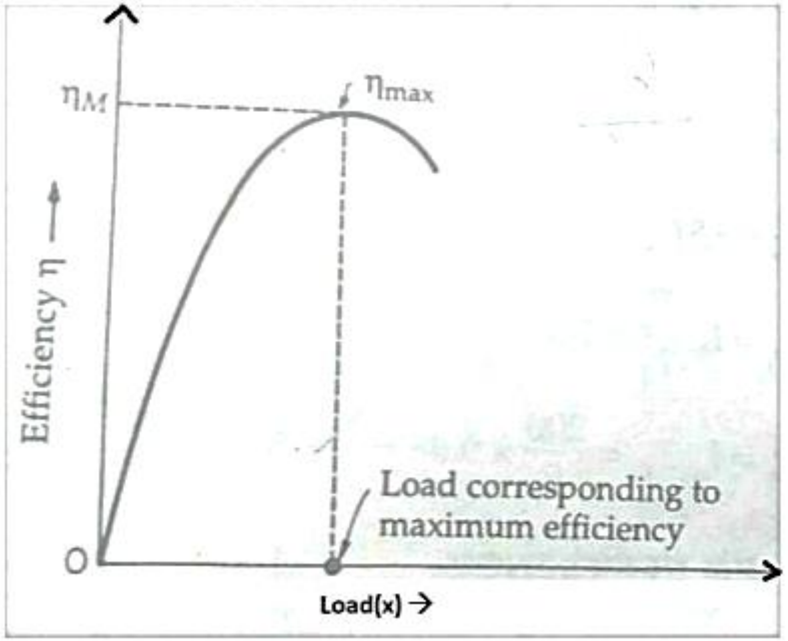
# Transformer Efficiency

The efficiency of a transformer at a particular load and power factor is defined as *the ratio of power output to power input*.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{output}}{\text{output} + \text{cu loss} + \text{iron loss}}$$

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}}$$

- It may be noted that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to volt-amperes.
- Hence at any volt-ampere load, the efficiency depends on power factor, being *maximum at unity power factor*.
- Efficiency can be calculated by determining core losses from open-circuit test and copper losses from short-circuit test.



# Condition for maximum efficiency

**Statement:**

- **In an electrical transformer, the maximum efficiency occurs when the Iron Losses are equal to Copper Losses of the transformer**
- **i.e. $P_i = P_{cu}$**

- Iron losses, $P_i$ = hysteresis loss + eddy current loss = $P_h + P_e$
- Copper losses, $P_c = I_1^2R_1$ or $I_2^2R_2$

**PROOF**

Input Power to the trasnformer = $V_1 I_1 cos\phi_1$

*Output Power*= $V_2 I_2 cos\phi_2$

$$\text{Efficiency, } \eta = \frac{\textit{Input Power} - Losses}{\textit{Input Power}}$$

$$\text{Efficiency, } \eta = \frac{V_1 I_1 \cos\phi_1 - P_{cu} - P_i}{V_1 I_1 \cos\phi_1}$$

$$\text{Efficiency, } \eta = 1 - \frac{P_{cu} + P_i}{V_1 I_1 \cos\phi_1}$$

$$\text{Efficiency, } \eta = 1 - \frac{P_{cu}}{V_1 I_1 \cos\phi_1} - \frac{P_i}{V_1 I_1 \cos\phi_1}$$

Where, $P_{cu}$ and $P_i$ are the copper and iron losses respectively

$$P_{cu} = I_1^2R_1 \text{ or } I_2^2R_2$$

$$\text{Efficiency, } \eta = 1 - \frac{I_1^2R_1}{V_1 I_1 \cos\phi_1} - \frac{P_i}{V_1 I_1 \cos\phi_1}$$

$$\text{Efficiency, } \eta = 1 - \frac{I_1R_1}{V_1 \cos\phi_1} - \frac{P_i}{V_1 I_1 \cos\phi_1} \qquad \textbf{(1)}$$

Differentiating equation (1) both sides w.r.t. $I_1$, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_1}{V_1 \cos\phi_1} + \frac{P_i}{V_1 I_1^2 \cos\phi_1} \qquad (2)$$

Now the condition for maxima is that

$$\frac{d\eta}{dI_1} = 0 \quad and \quad \frac{d^2\eta}{dI_1^2} < 0$$

Hence, doing so we get $$-\frac{R_1}{V_1 cos\phi_1} + \frac{P_i}{V_1 I_1^2 \; cos\phi_1} = 0$$

$$\frac{R_1}{V_1 cos\phi_1} = \frac{P_i}{V_1 I_1^2 \; cos\phi_1}$$

$$R_1 = \frac{P_i}{I_1^2}$$

$$I_1^2 R_1 \left(or, I_2^2 R_2\right) = P_i$$

Hence proved, **Copper Loss=Iron Loss** (3)

The output current corresponding to maximum efficiency is $$I_2 = \sqrt{\frac{P_i}{R_2}} \qquad (4)$$

By: Dr. Parvesh Saini

## Variation of efficiency with power factor.

We know that transformers efficiency, $$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}$$

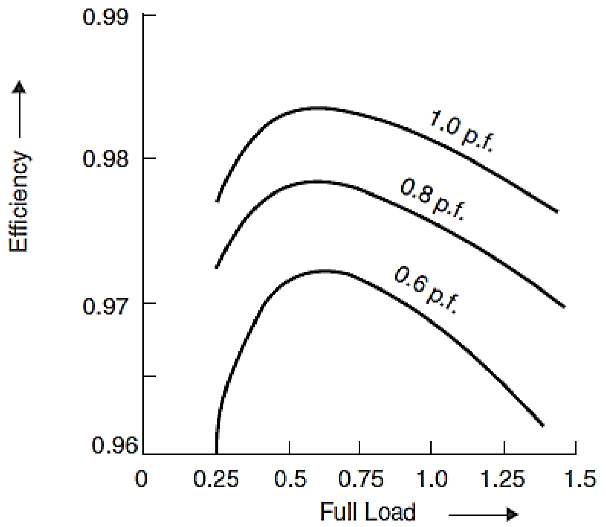
$$= 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{(V_2 I_2 \cos\phi + \text{losses})}$$

$$\eta = 1 - \left(\frac{\text{losses}/V_2 I_2}{\cos\phi + \text{losses}/V_2 I_2}\right) \qquad (5)$$

Let, $$\frac{\text{losses}}{V_2 I_2} = \beta$$

$$\eta = 1 - \frac{\beta}{\cos\phi_2 + \beta}$$

The variations of efficiency with power factor at different loadings on a typical transformer are shown in Figure below



# All Day Efficiency

*All-day efficiency* is the *ratio of energy (kWh) delivered in a 24 hour period divided by the energy (kWh) input in the same length of time*.

$$\text{ordinary efficiency} = \frac{\text{output (in watts)}}{\text{input (in watts)}}$$

$$\eta_{\text{all-day}} = \frac{\text{output in kWh}}{\text{input in kWh}} \text{ (for 24 hours)}$$

- Transformers used on residence-lighting circuits (and distribution circuits generally) are either idle or only lightly loaded during much of 24-hour period.
- Because a transformers are connected to the line all the time, so that the core losses are being supplied continually.
- It is therefore very important that such transformers be designed for minimum core loss.
- The copper losses are relatively less important, as they are considerable only when transformers are loaded.
- To calculate all-day efficiency, it is necessary to know how the load on the transformer varies from hour to hour.
- The quotient obtained by dividing the energy output by the energy output plus energy losses over a 24-hour period yields the efficiency expressed as a decimal fraction.

# Thank You