PUC
RIO

Trabalho Final INF1608

Pêndulo via Runge-Kutta-Fehlberg

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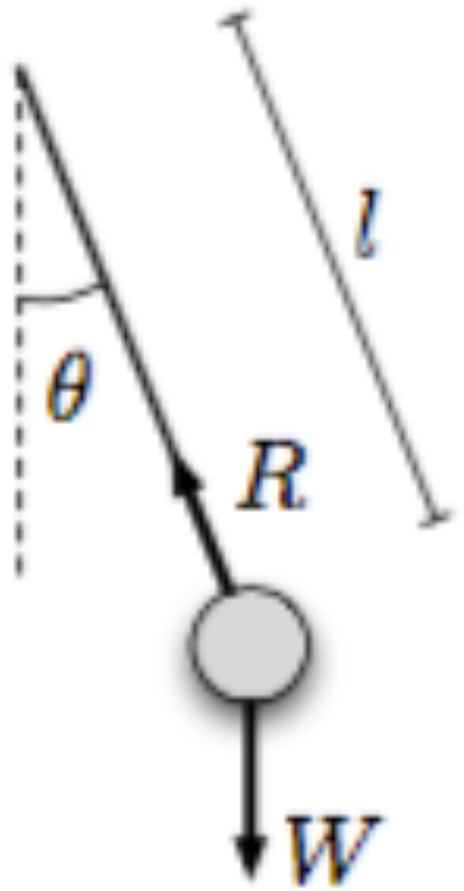
Problema

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$



Primeiros passos

$$w_{i+1} = w_i + \frac{h}{6}(s_1 + 2s_2 + 2s_3 + s_4)$$

Passo = 0.0001

$$s_1 = f(t_i, w_i)$$
$$s_2 = f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}s_1\right)$$
$$s_3 = f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}s_2\right)$$
$$s_4 = f(t_i + h, w_i + hs_3).$$

$$\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t\right)$$

Segunda ordem

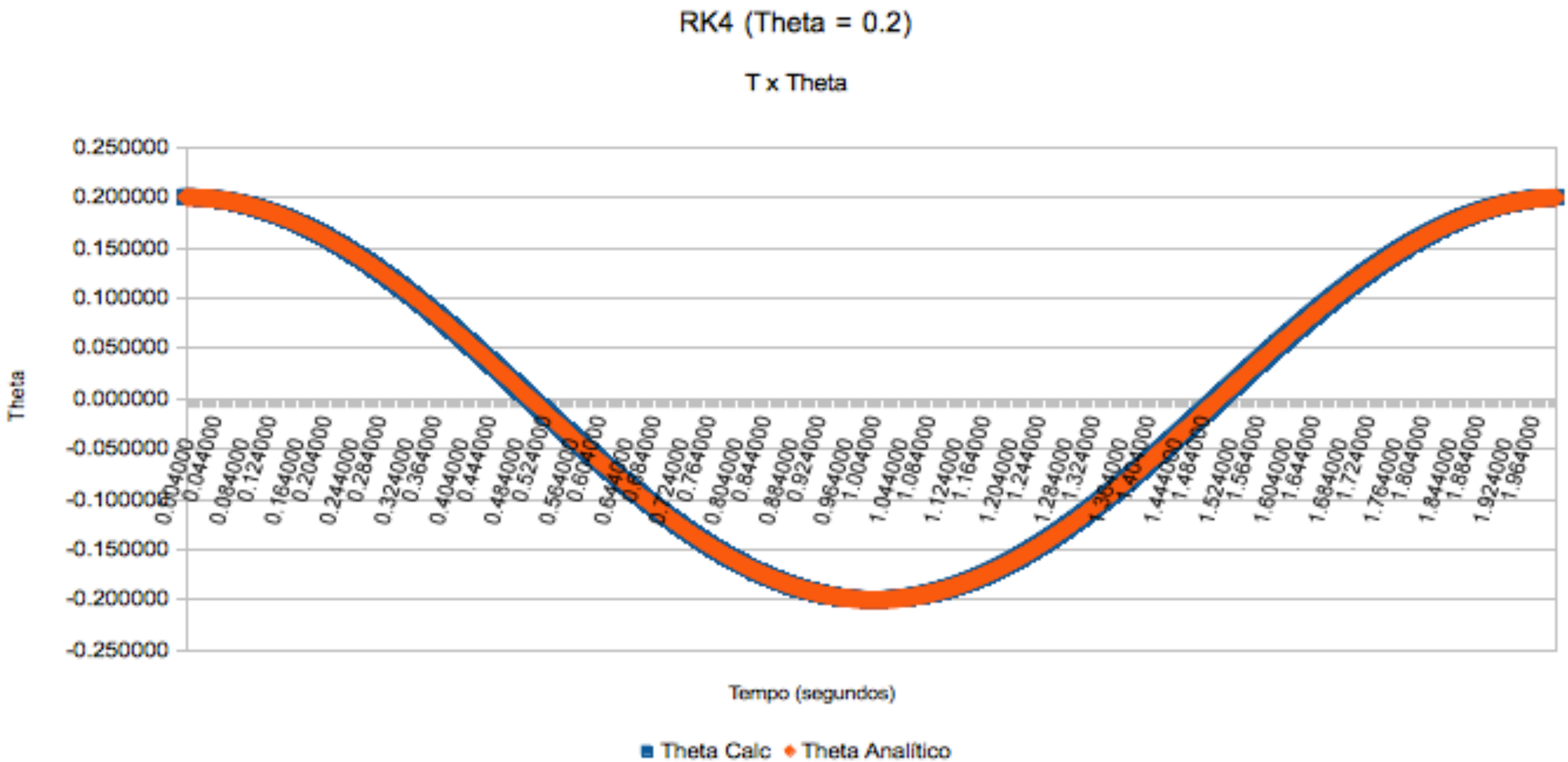
$$y^{(n)} = f(t, y, y', y'', \ldots, y^{(n-1)})$$

$$\begin{aligned} y_1 &= y \\ y_2 &= y' \\ y_3 &= y'' \\ &\vdots \\ y_n &= y^{(n-1)}, \end{aligned}$$

v=dTheta/dt

a=dv/dt

RK4 - Resultados



Avançando

$$
\begin{aligned}
s_1 &= f(t_i, w_i) \\
s_2 &= f\left(t_i + \frac{1}{4}h, w_i + \frac{1}{4}hs_1\right) \\
s_3 &= f\left(t_i + \frac{3}{8}h, w_i + \frac{3}{32}hs_1 + \frac{9}{32}hs_2\right) \\
s_4 &= f\left(t_i + \frac{12}{13}h, w_i + \frac{1932}{2197}hs_1 - \frac{7200}{2197}hs_2 + \frac{7296}{2197}hs_3\right) \\
s_5 &= f\left(t_i + h, w_i + \frac{439}{216}hs_1 - 8hs_2 + \frac{3680}{513}hs_3 - \frac{845}{4104}hs_4\right) \\
s_6 &= f\left(t_i + \frac{1}{2}h, w_i - \frac{8}{27}hs_1 + 2hs_2 - \frac{3544}{2565}hs_3 + \frac{1859}{4104}hs_4 - \frac{11}{40}hs_5\right) \\
w_{i+1} &= w_i + h\left(\frac{25}{216}s_1 + \frac{1408}{2565}s_3 + \frac{2197}{4104}s_4 - \frac{1}{5}s_5\right) \\
z_{i+1} &= w_i + h\left(\frac{16}{135}s_1 + \frac{6656}{12825}s_3 + \frac{28561}{56430}s_4 - \frac{9}{50}s_5 + \frac{2}{55}s_6\right).
\end{aligned}
$$

Tolerância

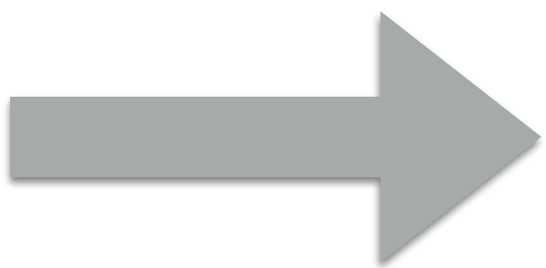
$$w_{i+1} = w_i + h\left(\frac{25}{216}s_1 + \frac{1408}{2565}s_3 + \frac{2197}{4104}s_4 - \frac{1}{5}s_5\right)$$

$$z_{i+1} = w_i + h\left(\frac{16}{135}s_1 + \frac{6656}{12825}s_3 + \frac{28561}{56430}s_4 - \frac{9}{50}s_5 + \frac{2}{55}s_6\right).$$

$$e_{i+1} = |z_{i+1} - w_{i+1}| = h\left|\frac{1}{360}s_1 - \frac{128}{4275}s_3 - \frac{2197}{75240}s_4 + \frac{1}{50}s_5 + \frac{2}{55}s_6\right|.$$

$$\frac{e_i}{|w_i|} < T$$

hMin
hMax
eMax
eMin

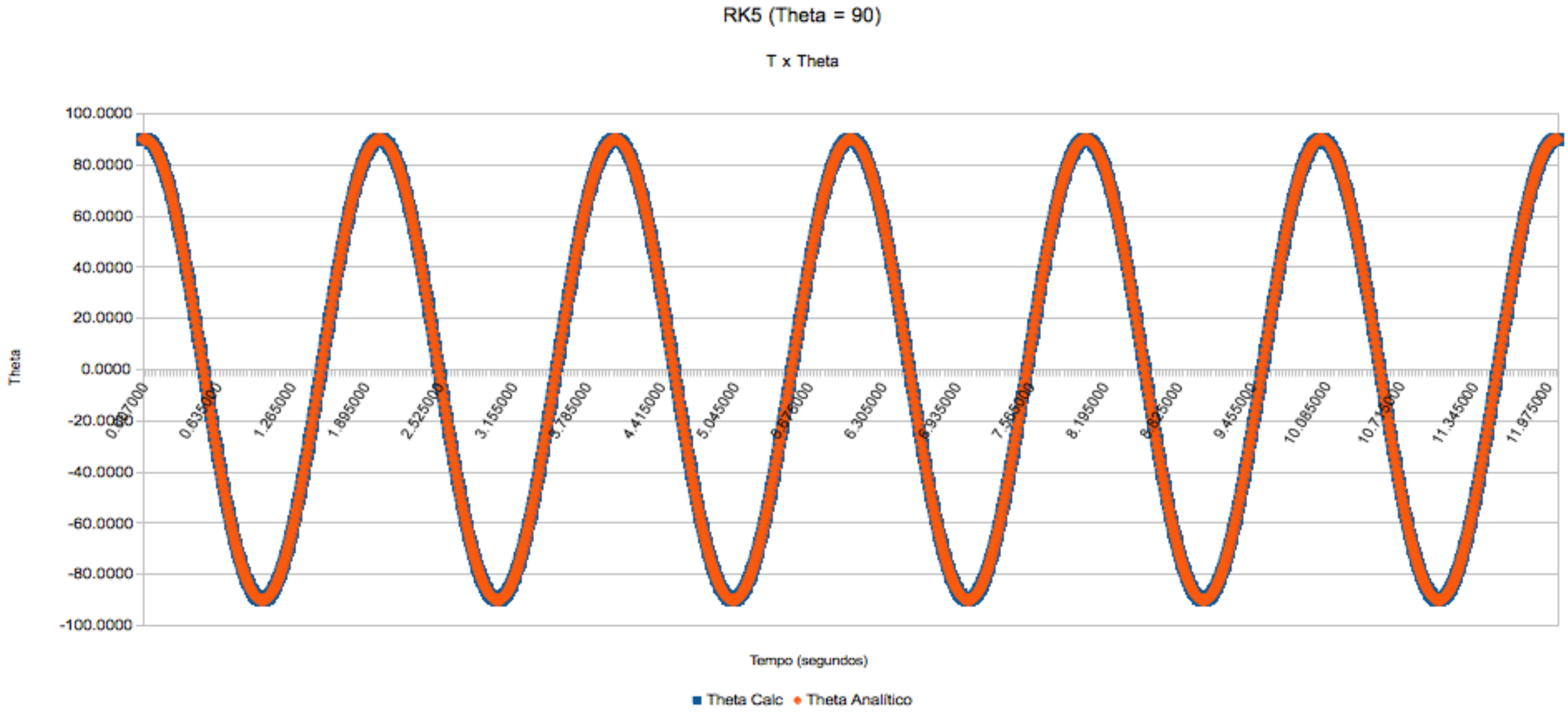


hAnterior

Variação dos passos

Passo Var t	Theta Calc	Theta Analítico	Passo
0.00100	0.20000	0.20000	0.00100
0.00300	0.19999	0.19999	0.00200
0.00700	0.19995	0.19995	0.00400
0.01500	0.19978	0.19978	0.00800
0.02500	0.19939	0.19939	0.01000
0.03500	0.19880	0.19880	0.01000
0.04500	0.19802	0.19802	0.01000
0.05500	0.19704	0.19704	0.01000
0.06500	0.19587	0.19587	0.01000
0.07500	0.19451	0.19451	0.01000
0.08500	0.19295	0.19295	0.01000
0.09500	0.19121	0.19121	0.01000
0.10500	0.18928	0.18928	0.01000
0.11500	0.18717	0.18717	0.01000
0.12500	0.18487	0.18487	0.01000
0.13500	0.18239	0.18239	0.01000
0.14500	0.17973	0.17973	0.01000
0.15500	0.17689	0.17689	0.01000
0.16500	0.17388	0.17388	0.01000
0.17500	0.17070	0.17070	0.01000
0.18500	0.16735	0.16735	0.01000
0.19500	0.16384	0.16384	0.01000
0.20500	0.16017	0.16017	0.01000
0.21500	0.15634	0.15634	0.01000
0.22500	0.15236	0.15236	0.01000
0.23500	0.14823	0.14823	0.01000
0.24500	0.14395	0.14395	0.01000
0.25500	0.13953	0.13953	0.01000
0.26500	0.13498	0.13498	0.01000

RKF5 - Resultados

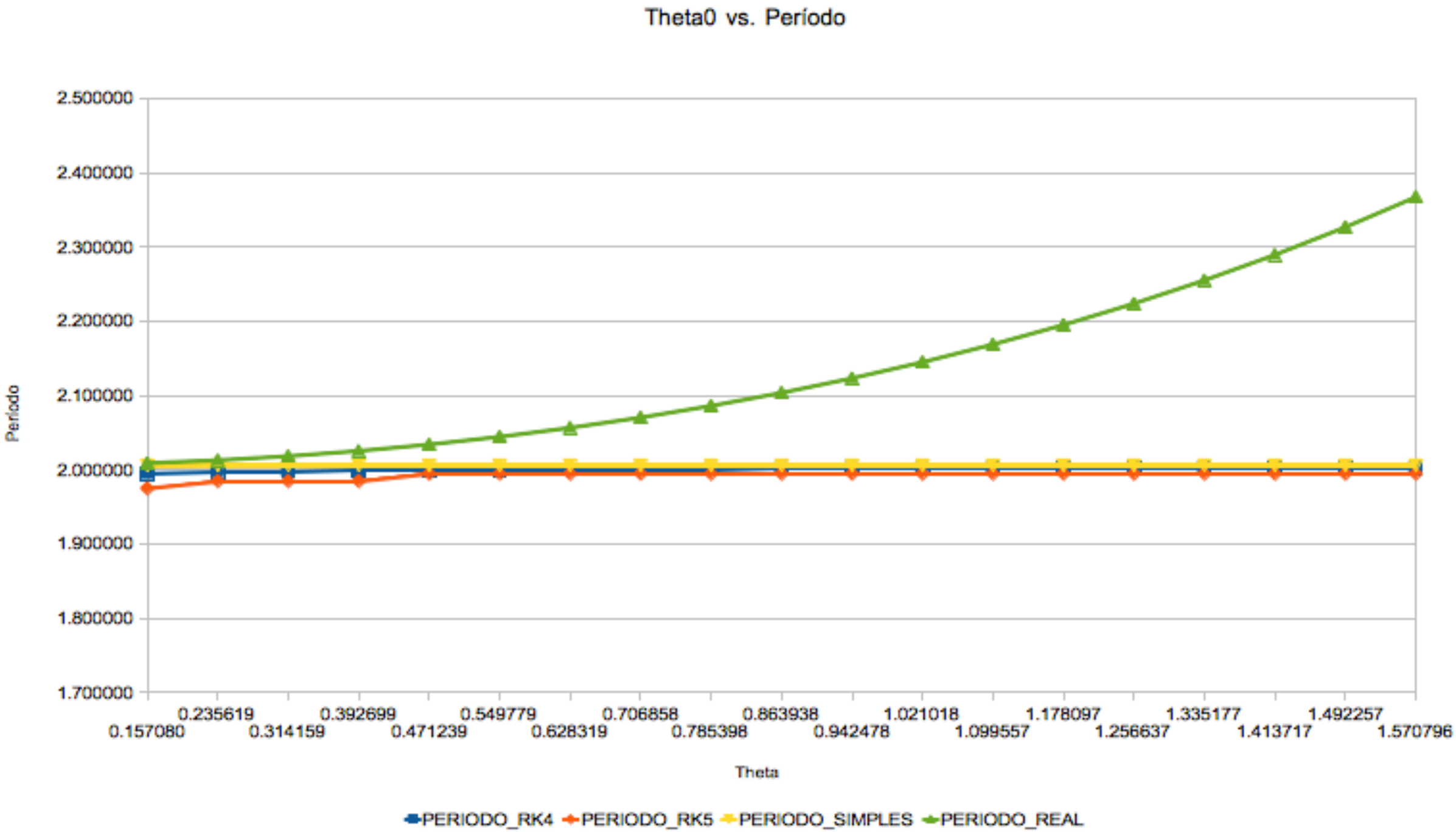


Número de passos

Theta0	NUM_PASSOS_RK4	NUM_PASSOS_RK5
0.157080	19947	200
0.235619	19968	201
0.314159	19981	201
0.392699	19989	201
0.471239	19995	202
0.549779	20000	202
0.628319	20004	202
0.706858	20007	202
0.785398	20010	202
0.863938	20013	202
0.942478	20015	202
1.021018	20016	202
1.099557	20018	202
1.178097	20020	202
1.256637	20021	202
1.335177	20022	202
1.413717	20023	202
1.492257	20024	202
1.570796	20025	202

Theta: 1.000000
  RK4 - Esperado:2.0061 - Encontrado:2.0016 - Erro:0.004467 - NumPassos:20015
  RK5 - Esperado:2.0061 - Encontrado:1.9950 - Erro:0.011067 - NumPassos:202
Theta: 4.000000
  RK4 - Esperado:2.0061 - Encontrado:2.0039 - Erro:0.002167 - NumPassos:20038
  RK5 - Esperado:2.0061 - Encontrado:2.0050 - Erro:0.001067 - NumPassos:203
Theta: 7.000000
  RK4 - Esperado:2.0061 - Encontrado:2.0044 - Erro:0.001667 - NumPassos:20043
  RK5 - Esperado:2.0061 - Encontrado:2.0050 - Erro:0.001067 - NumPassos:203

E ângulos grandes?



Considerações finais

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \frac{173}{737280}\theta_0^6 + \frac{22931}{1321205760}\theta_0^8 + \frac{1319183}{951268147200}\theta_0^{10} + \frac{233526463}{2009078326886400}\theta_0^{12} + \ldots\right)$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Theta0	PERIODO_RK4	PERIODO_RK5	PERIODO_SIMPLES	PERIODO_REAL	NUM_PASSOS_RK4	NUM_PASSOS_RK5
0.157080	1.994700	1.975000	2.006100	2.009165	19947	200
0.235619	1.996800	1.985000	2.006100	2.013050	19968	201
0.314159	1.998100	1.985000	2.006100	2.018512	19981	201
0.392699	1.998900	1.985000	2.006100	2.025574	19989	201
0.471239	1.999500	1.995000	2.006100	2.034269	19995	202
0.549779	2.000000	1.995000	2.006100	2.044633	20000	202
0.628319	2.000400	1.995000	2.006100	2.056714	20004	202
0.706858	2.000700	1.995000	2.006100	2.070566	20007	202
0.785398	2.001000	1.995000	2.006100	2.086256	20010	202
0.863938	2.001300	1.995000	2.006100	2.103857	20013	202
0.942478	2.001500	1.995000	2.006100	2.123456	20015	202
1.021018	2.001600	1.995000	2.006100	2.145152	20016	202
1.099557	2.001800	1.995000	2.006100	2.169060	20018	202
1.178097	2.002000	1.995000	2.006100	2.195307	20020	202
1.256637	2.002100	1.995000	2.006100	2.224040	20021	202
1.335177	2.002200	1.995000	2.006100	2.255426	20022	202
1.413717	2.002300	1.995000	2.006100	2.289655	20023	202
1.492257	2.002400	1.995000	2.006100	2.326938	20024	202
1.570796	2.002500	1.995000	2.006100	2.367521	20025	202